The influence of centre vortices on the quark propagator in lattice QCD

Daniel Trewartha
Derek Leinweber and Waseem Kamleh

CSSM, School of Chemistry and Physics
University of Adelaide

Monday, 7th April 2014

Centre Vortices

- Topological defect associated with centre group Z_3 of $SU(3)$
- Appear as surfaces of centre flux, loops $U(C)$ linked to the surface acquire a centre flux

$$U(C) \to zU(c) \quad (1)$$

with z a centre element

Centre Vortices

- Topological defect associated with centre group Z_3 of $SU(3)$
- Appear as surfaces of centre flux, loops $U(C)$ linked to the surface acquire a centre flux

$$U(C) \to zU(c) \tag{1}$$

with z a centre element

Centre Vortices on the lattice

- Transform to Maximal Centre Gauge, where links are brought close to centre elements

$$Z_\mu = \exp\left[\frac{2\pi i}{3} m_\mu(x)\right] \mathrm{I}, \quad m_\mu(x) \in \{-1, 0, 1\} \tag{2}$$

- Require gauge transformation $\Omega(x)$ maximising overlap between gauge links and centre elements

$$\sum_{x,\mu} \mathrm{Re}\,\mathrm{Tr}\left[U_\mu^\Omega(x) Z_\mu^\dagger(x)\right] \to \mathrm{Max} \tag{3}$$

Centre Vortices on the lattice

- Transform to Maximal Centre Gauge, where links are brought close to centre elements

$$Z_\mu = \exp\left[\frac{2\pi i}{3} m_\mu(x)\right] \mathrm{I}, \quad m_\mu(x) \in \{-1, 0, 1\} \tag{2}$$

- Require gauge transformation $\Omega(x)$ maximising overlap between gauge links and centre elements

$$\sum_{x,\mu} \mathrm{Re}\,\mathrm{Tr}\left[U_\mu^\Omega(x) Z_\mu^\dagger(x)\right] \to \mathrm{Max} \tag{3}$$

Centre Vortices on the Lattice

- Implemented through 'mesonic' centre gauge fixing condition

$$R_{mes} = \sum_{x,\mu} |\mathrm{Tr}\, U_\mu(x)^\Omega|^2 \to \mathrm{Max} \quad (4)$$

- Then we project onto Z_3

$$\frac{1}{3}\mathrm{Tr}U_\mu^\Omega(x) = r_\mu(x)\exp(i\phi_\mu(x)) \quad (5)$$

So choose $m_\mu(x) \in \{-1, 0, 1\}$ closest to $\frac{3\phi_\mu(x)}{2\pi}$

- Then a plaquette is pierced by a centre vortex if

$$P_{\mu\nu} = Z_\mu Z_\nu(x+\mu) Z_\mu^\dagger(x+\nu) Z_\nu^\dagger(x) \quad (6)$$

$$= z\mathrm{I}, \quad z \neq 1 \quad (7)$$

$$(8)$$

Centre Vortices on the Lattice

- Implemented through 'mesonic' centre gauge fixing condition

$$R_{mes} = \sum_{x,\mu} |\mathrm{Tr}\, U_\mu(x)^\Omega|^2 \to \text{ Max} \tag{4}$$

- Then we project onto Z_3

$$\frac{1}{3}\mathrm{Tr} U_\mu^\Omega(x) = r_\mu(x) \exp(i\phi_\mu(x)) \tag{5}$$

So choose $m_\mu(x) \in \{-1, 0, 1\}$ closest to $\frac{3\phi_\mu(x)}{2\pi}$

- Then a plaquette is pierced by a centre vortex if

$$P_{\mu\nu} = Z_\mu Z_\nu(x+\mu) Z_\mu^\dagger(x+\nu) Z_\nu^\dagger(x) \tag{6}$$

$$= z\mathrm{I}, \quad z \neq 1 \tag{7}$$

(8)

Centre Vortices on the Lattice

- Implemented through 'mesonic' centre gauge fixing condition

$$R_{mes} = \sum_{x,\mu} |\mathrm{Tr}\, U_\mu(x)^\Omega|^2 \to \mathrm{Max} \tag{4}$$

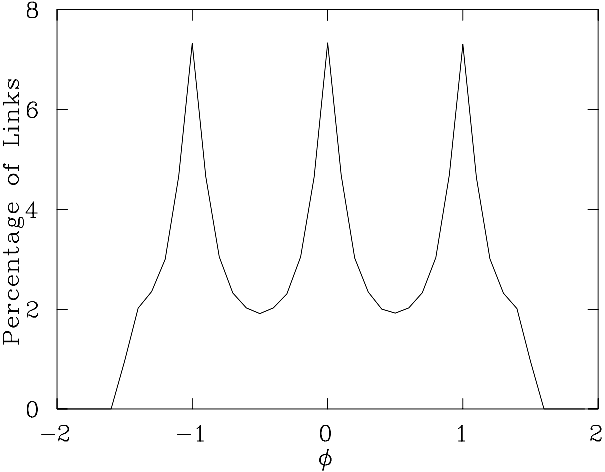
- Then we project onto Z_3

$$\frac{1}{3}\mathrm{Tr}U_\mu^\Omega(x) = r_\mu(x)\exp(i\phi_\mu(x)) \tag{5}$$

So choose $m_\mu(x) \in \{-1, 0, 1\}$ closest to $\frac{3\phi_\mu(x)}{2\pi}$

- Then a plaquette is pierced by a centre vortex if

$$\begin{aligned} P_{\mu\nu} &= Z_\mu Z_\nu(x+\mu) Z_\mu^\dagger(x+\nu) Z_\nu^\dagger(x) && (6) \\ &= z\mathrm{I}, \quad z \neq 1 && (7) \\ & && (8) \end{aligned}$$



Percentage of Links
8
6
4
2
0
−2
−1
0
1
2
ϕ

Centre Vortices on the Lattice

- 3 sets of configurations:
  - Untouched configurations $U_\mu(x)$
  - Vortex only configurations $Z_\mu(x) = \exp\left[\frac{2\pi i}{3} m_\mu(x)\right]$
  - Vortex Removed configurations $Z^\dagger_\mu(x) U^\Omega_\mu(x)$

Centre Vortices on the Lattice

- 3 sets of configurations:
  - Untouched configurations $U_\mu(x)$
  - Vortex only configurations $Z_\mu(x) = \exp\left[\frac{2\pi i}{3} m_\mu(x)\right]$
  - Vortex Removed configurations $Z^\dagger_\mu(x) U^\Omega_\mu(x)$

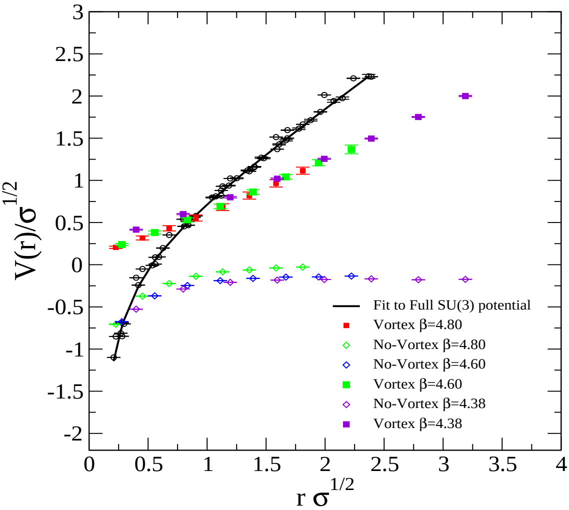
Centre Vortices on the Lattice

- 3 sets of configurations:
  - Untouched configurations $U_\mu(x)$
  - Vortex only configurations $Z_\mu(x) = \exp\left[\frac{2\pi i}{3} m_\mu(x)\right]$
  - Vortex Removed configurations $Z^\dagger_\mu(x) U^\Omega_\mu(x)$

Centre Vortices on the Lattice

- 3 sets of configurations:
  - Untouched configurations $U_\mu(x)$
  - Vortex only configurations $Z_\mu(x) = \exp\left[\frac{2\pi i}{3} m_\mu(x)\right]$
  - Vortex Removed configurations $Z^\dagger_\mu(x) U^\Omega_\mu(x)$

Centre Vortices



From P. O. Bowman et. al., Phys. Rev. D ***84****, 034501 (2011) [arXiv:1010.4624 [hep-lat]].*

Dynamical Mass Generation

- Non-trivial topology gives rise to a non-zero topological charge, Q
- Intersections and writhing points of centre vortices associated with topological charge, centre of instanton-like objects
- Atiyah-Singer index theorem;

$$Q = n_+ - n_- \tag{9}$$

- Linked to dynamical chiral symmetry breaking and dynamical mass generation by Casher-Banks relation

$$< \bar{q}q >= -\pi\rho(0) \;\; \text{as} \;\; m_q \to 0 \tag{10}$$

Dynamical Mass Generation

- Non-trivial topology gives rise to a non-zero topological charge, Q
- Intersections and writhing points of centre vortices associated with topological charge, centre of instanton-like objects
- Atiyah-Singer index theorem;

$$Q = n_+ - n_- \tag{9}$$

- Linked to dynamical chiral symmetry breaking and dynamical mass generation by Casher-Banks relation

$$< \bar{q}q >= -\pi\rho(0) \ \text{ as } \ m_q \to 0 \tag{10}$$

Dynamical Mass Generation

- Non-trivial topology gives rise to a non-zero topological charge, Q
- Intersections and writhing points of centre vortices associated with topological charge, centre of instanton-like objects
- Atiyah-Singer index theorem;

$$Q = n_+ - n_- \tag{9}$$

- Linked to dynamical chiral symmetry breaking and dynamical mass generation by Casher-Banks relation

$$< \bar{q}q >= -\pi\rho(0) \ \text{as} \ m_q \to 0 \tag{10}$$

Dynamical Mass Generation

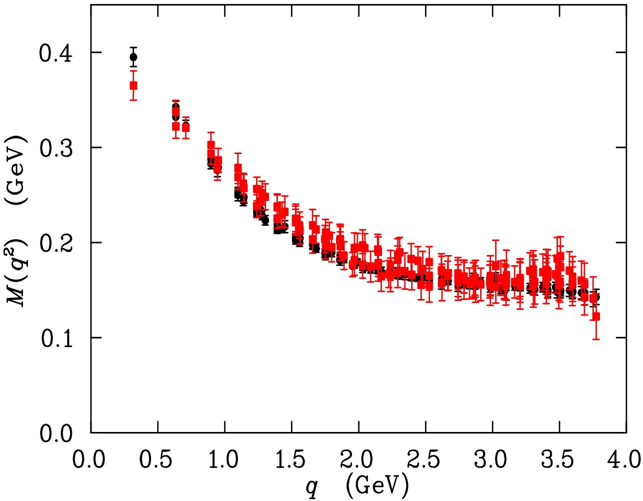
- Non-trivial topology gives rise to a non-zero topological charge, Q
- Intersections and writhing points of centre vortices associated with topological charge, centre of instanton-like objects
- Atiyah-Singer index theorem;

$$Q = n_+ - n_- \tag{9}$$

- Linked to dynamical chiral symmetry breaking and dynamical mass generation by Casher-Banks relation

$$< \bar{q}q >= -\pi\rho(0) \;\; \text{as} \;\; m_q \to 0 \tag{10}$$

Previous Studies



From P. O. Bowman et. al., Phys. Rev. D **84**, *034501 (2011) [arXiv:1010.4624 [hep-lat]].*

Overlap Quark Propagator

- Operator with lattice-deformed version of chiral symmetry
- Allows us to write momentum-space propagator as

$$S(p) = \frac{Z(p)}{i\not{q} + M(p)} \tag{11}$$

Overlap Quark Propagator

- Operator with lattice-deformed version of chiral symmetry
- Allows us to write momentum-space propagator as

$$S(p) = \frac{Z(p)}{i\not{q} + M(p)} \tag{11}$$

Simulation Details

- 20 configurations using Lušcher-Weisz mean-field improved action
- $20 \times 20 \times 20 \times 40$ lattice with a spacing of 0.125 fm
- Spacial extent of 2.5 fm, temporal $5fm$

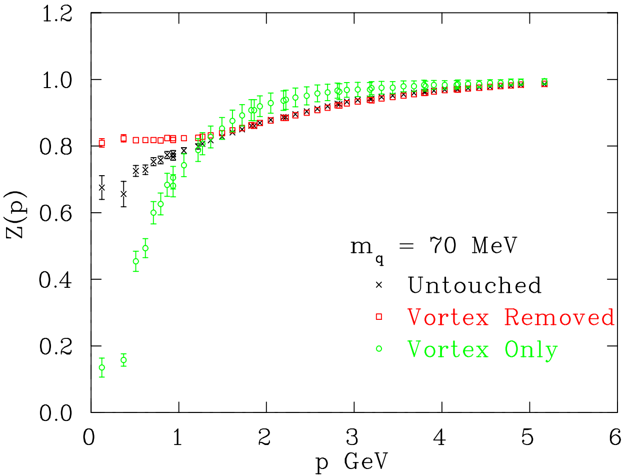
Simulation Details

- 20 configurations using Lušcher-Weisz mean-field improved action
- $20 \times 20 \times 20 \times 40$ lattice with a spacing of 0.125 fm
- Spacial extent of 2.5 fm, temporal $5fm$

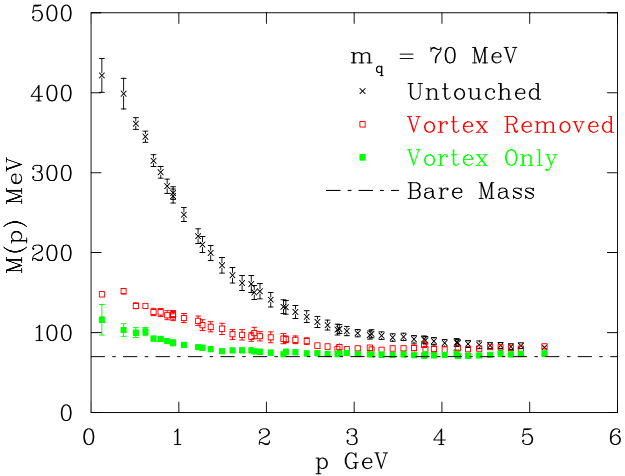
Simulation Details

- 20 configurations using Lušcher-Weisz mean-field improved action
- $20 \times 20 \times 20 \times 40$ lattice with a spacing of 0.125 fm
- Spacial extent of 2.5 fm, temporal $5fm$

Renormalization function



Mass function



Conclusion

- Removal of centre vortices removes instanton-like objects and dynamical mass generation
- A centre vortex background alone cannot reproduce dynamical mass generation

Conclusion

- Removal of centre vortices removes instanton-like objects and dynamical mass generation
- A centre vortex background alone cannot reproduce dynamical mass generation