Energy-momentum Tensor Form Factors and Transverse Charge Densities of the Pion and the Kaon from the Instanton Vacuum

Hyeon-Dong Son
Inha University
In collaboration with
Prof. H.-Ch. Kim

APFB 2014
April 7-11, 2014 in Hahndorf, SA, Australia

Outline

- Motivation
- Energy momentum tensor form factors & Transverse Charge Densities
- Nonlocal chiral quark model from the Instanton Vacuum
- Numerical Result
- Summary

Motivation

- The internal structure of hadrons
- Generalized Parton Distributions(GPDs)
- Generalized Form Factors(GFFs)
  A10, A20, B10, B20...
- Energy-momentum tensor form factors
- Transverse Charge Densities(TChDs)
  : Spatial distribution at the transverse momentum plane
- SU(3) symmetry breaking in the mesonic sector: pi, K
- Reliability of a model: Low-energy theorem

Generalized Form Factors

$$\psi^\dagger(x - \frac{\epsilon}{2})\gamma_\mu\psi(x + \frac{\epsilon}{2})$$

[R.L. Jaffe, arXiv:hep-ph/9602236]

$$= \psi^\dagger(x)\gamma_\mu\psi(x) + \epsilon_\nu\psi^\dagger(x)\frac{1}{2}(\gamma_\mu\overrightarrow{\partial}_\nu - \gamma_\mu\overleftarrow{\partial}_\nu)\psi(x) + \ldots$$

$$\equiv J_\mu(x) + \epsilon_\nu\Theta_{\mu\nu}(x) + \ldots$$

The Hadronic Matrix elements of each operator define the generalized form factors.

Generalized Form Factors

$$\langle \phi^a(p_f)|\psi^\dagger(0)\gamma_{\{\mu} i \overleftrightarrow{D}_{\mu_1} \ldots \overleftrightarrow{D}_{\mu_{n-1}\}}\psi(0)|\phi^b(p_i)\rangle$$

$$= 2P_{\{\mu}P_{\mu_1}\ldots P_{\mu_{n-1}\}}A_{n0}(t)$$

$$+ \sum_{k=2\ even}^{n} q_{\{\mu}q_{\mu_1}\ldots q_{\mu_{k-1}}P_{\mu_k}P_{\mu_{n-1}\}}2^{-k}A_{nk}(t)$$

$$\langle \phi(p_f)|\psi^\dagger(0)\sigma_{[\mu\nu} i \overleftrightarrow{D}_{\mu_1} \ldots i \overleftrightarrow{D}_{\mu_{n-1}]}\psi(0)|\phi(p_i)\rangle$$

$$= \frac{p_{[\mu}q_\nu - q_\mu p_\nu}{m_\phi} \sum_{i=even}^{n-1} q_{\mu_1}\ldots q_{\mu_i}P_{\mu_{i+1}}P_{\mu_{n-1}]}B_{ni}(t)$$

$$(\overleftrightarrow{D}_\mu = \frac{1}{2}(\overrightarrow{D}_\mu - \overleftarrow{D}_\mu), P_\mu = (p_i + p_f)_\mu, q_\mu = (p_f - p_i)_\mu, t = -q^2)$$

n=1	$2P_\mu A_{10}(t)$
n=2	$2P_{\{\mu}P_{\nu\}}A_{20}(t) + \frac{1}{2}q_{\{\mu}q_{\nu\}}A_{22}(t)$

n=2: Energy-momentum Tensor Form Factors

- H.Pagels: Energy-Momentum Structure Form Factors of Particles (Phys.Rev.144,1250.,1966)
- Momentum distribution of the partons inside of hadrons
- Accessible from the vector GPDs by using the polynomiality

$$\int_{-1}^{1} dx x H(x,t,\xi) = 2A_{20}(t) + 2A_{22}(t)\xi^2$$

- Lattice calculation:

POS(LAT2005)360 D. Brommel et al(2005)

PhD Thesis, D. Brommel(2007)

PRL.101,122011 D. Brommel et al(2008)
(QCDSF-UKQCD Collaborations)

n=2: Energy-momentum Tensor Form Factors

$$\langle \phi^a(p_f)|\Theta_{\mu\nu}(0)|\phi^b(p_i)\rangle$$

[H.Pagels ,Phys.Rev.144,1250.,1966]

$$= \frac{\delta^{ab}}{2}\left[(q^2\delta_{\mu\nu} - q_\mu q_\nu)\Theta_1(t) + 4P_\mu P_\nu \Theta_2(t)\right]$$

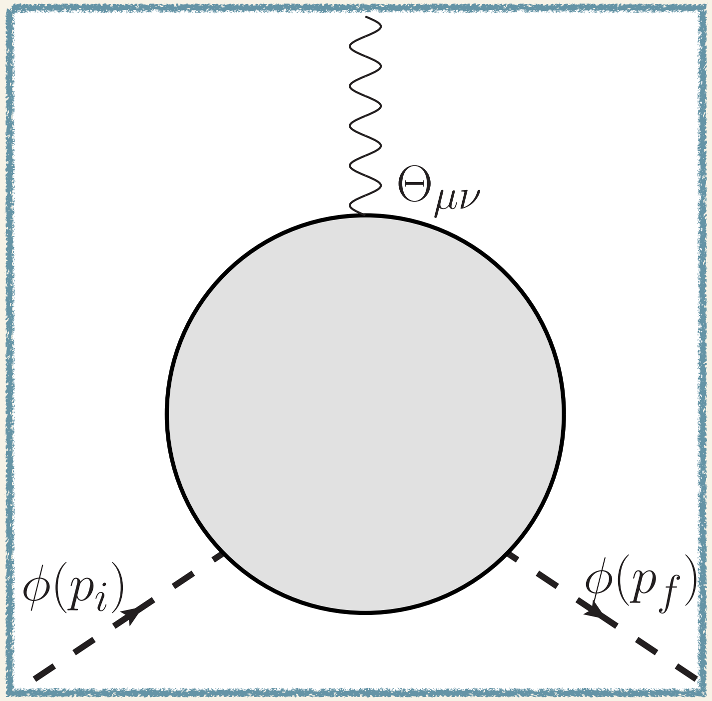
$$\Theta_2(t) = 2A_{20}(t)$$

$$\Theta_1(t) = -2A_{22}(t)$$

$$\Theta_2(0) = 1$$

$$\Theta_1(0) - \Theta_2(0) = \mathcal{O}(m_\pi^2)$$

[J.F. Donoghue and H. Leutwyler, Z.Phys.C(1991) 52, 343]



Transverse Charge Densities

P-pole parametrization

$$A_{20}(t) = \frac{A_{20}(0)}{(1 - \frac{t}{pM^2})^p}$$

Transverse charge density

$$A_{n0}(b_\perp) = \frac{1}{(2\pi)^2} \int d^2 q_\perp \, e^{-ib_\perp \cdot q_\perp} A_{n0}(q_\perp^2)$$

$$= \frac{1}{2\pi} \int_0^\infty Q dQ J_0(b_\perp Q) A_{n0}(Q^2)$$

Polarized quark spin structure

$$\rho_n^\phi(b_\perp, s_\perp) = \frac{1}{2}\left[A_{n0}^\phi(b_\perp^2) - \frac{s_\perp^i \epsilon^{ij} b_\perp^j}{m_\phi} \frac{\partial B_{n0}^\phi(b_\perp^2)}{\partial b_\perp^2}\right]$$

Transverse Charge Densities

P-pole parametrization

$$A_{20}(t) = \frac{A_{20}(0)}{(1 - \frac{t}{pM^2})^p}$$

Transverse charge density

$$A_{n0}(b_\perp) = \frac{1}{(2\pi)^2} \int d^2 q_\perp \, e^{-ib_\perp \cdot q_\perp} A_{n0}(q_\perp^2)$$

$$= \frac{1}{2\pi} \int_0^\infty Q dQ J_0(b_\perp Q) A_{n0}(Q^2)$$

$$\frac{2^{-p} b_\perp^{-1+p} (\sqrt{M^2 p})^{p+1} K_{1-p}(b_\perp M p)}{\pi \Gamma(p)}$$

Logarithmically divergent
at b=0 for p=1.
Singular when p<1.5

Polarized quark spin structure

$$\rho_n^\phi(b_\perp, s_\perp) = \frac{1}{2}\left[A_{n0}^\phi(b_\perp^2) - \frac{s_\perp^i \epsilon^{ij} b_\perp^j}{m_\phi} \frac{\partial B_{n0}^\phi(b_\perp^2)}{\partial b_\perp^2}\right]$$

Transverse Charge Densities

P-pole parametrization

$$A_{20}(t) = \frac{A_{20}(0)}{(1-\frac{t}{pM^2})^p}$$

Transverse charge density

$$A_{n0}(b_\perp) = \frac{1}{(2\pi)^2}\int d^2q_\perp\, e^{-ib_\perp\cdot q_\perp} A_{n0}(q_\perp^2)$$

$$= \frac{1}{2\pi}\int_0^\infty Q dQ J_0(b_\perp Q) A_{n0}(Q^2)$$

$$\frac{2^{-p} b_\perp^{-1+p} (\sqrt{M^2 p})^{p+1} K_{1-p}(b_\perp M p)}{\pi\Gamma(p)}$$

Logarithmically divergent at b=0 for p=1.
Singular when p<1.5

p=1 for A_{no}, p=1.6 for B_{no}

Polarized quark spin structure

$$\rho_n^\phi(b_\perp, s_\perp) = \frac{1}{2}\left[A_{n0}^\phi(b_\perp^2) - \frac{s_\perp^i \epsilon^{ij} b_\perp^j}{m_\phi}\frac{\partial B_{n0}^\phi(b_\perp^2)}{\partial b_\perp^2}\right]$$

Transverse Charge Densities

P-pole parametrization

$$A_{20}(t) = \frac{A_{20}(0)}{(1 - \frac{t}{pM^2})^p}$$

Transverse charge density

$$A_{n0}(b_\perp) = \frac{1}{(2\pi)^2} \int d^2 q_\perp \, e^{-ib_\perp \cdot q_\perp} A_{n0}(q_\perp^2)$$

$$= \frac{1}{2\pi} \int_0^\infty Q dQ J_0(b_\perp Q) A_{n0}(Q^2)$$

Polarized quark spin structure

$$\rho_n^\phi(b_\perp, s_\perp) = \frac{1}{2}\left[A_{n0}^\phi(b_\perp^2) - \frac{s_\perp^i \epsilon^{ij} b_\perp^j}{m_\phi} \frac{\partial B_{n0}^\phi(b_\perp^2)}{\partial b_\perp^2} \right]$$

Spatial distribution of the quark polarized in the transverse plane inside the pion and the kaon.

Nonlocal Chiral Quark Model From the Instanton Vacuum

$$S_{\mathrm{eff}} = -N_c \mathrm{Tr} \log \left[i\not{\partial} + i\hat{m} + i\sqrt{M(i\partial)} U^{\gamma_5} \sqrt{M(i\partial)} \right]$$

$$U^{\gamma_5} = \exp\left[\frac{i\gamma_5}{f_\phi} (\lambda \cdot \phi) \right]$$

- The chiral effective action derived from the instanton vacuum
- No free parameter
  -Average Instanton size & separation $\bar{\rho} \approx \frac{1}{3}\ \mathrm{fm} \quad \bar{\mathrm{R}} \approx 1\ \mathrm{fm}$
- Nonlocality
  -Momentum-dependent dynamical quark mass
- Nicely reproduces pion properties: Fpi, EMFF
- SU(3) symmetry breaking

$$\hat{m} = \mathrm{diag}(\mathrm{m_u}, \mathrm{m_d}, \mathrm{m_s}) \quad m_u = m_d = 5\ [\mathrm{MeV}] \quad m_s = 180\ [\mathrm{MeV}]$$

[D. Diakonov, Instantons at work, arXiv:hep-ph/0212026v4]

Nonlocal Chiral Quark Model & Gauge Invariance

- The gauge invariance is broken by the nonlocal interaction of the model
- The ordinary Noether currents are not conserved due to the broken gauge invariance
- The gauge invariance is restored: The conserved vector current

$$J_\mu = \psi^\dagger(\gamma_\mu + i\frac{\partial\sqrt{M}}{\partial\hat{p}_\mu}U^{\gamma_5}\sqrt{M} - i\sqrt{M}U^{\gamma_5}\frac{\partial\sqrt{M}}{\partial\hat{p}_\mu})\psi$$

[M.M. Musakhanov and H.-Ch. Kim, Phys. Lett. B572(2003)]

Energy-momentum Tensor Operator for the Nonlocal Chiral Quark Model

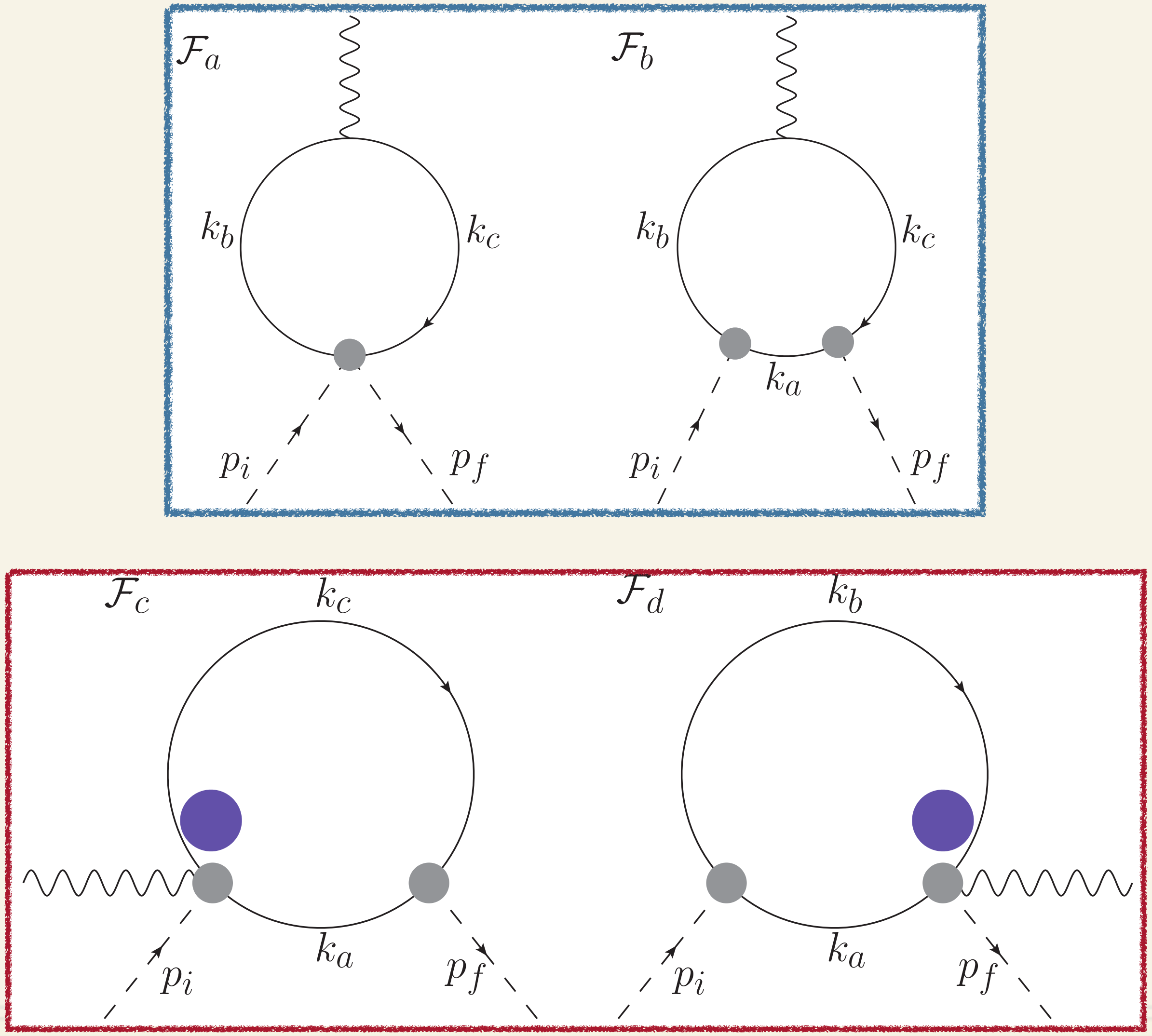
$$\Gamma_\mu = \gamma_\mu + i\frac{\partial\sqrt{M}}{\partial\hat{p}_\mu}U^{\gamma_5}\sqrt{M} - i\sqrt{M}U^{\gamma_5}\frac{\partial\sqrt{M}}{\partial\hat{p}_\mu}$$

$$\psi^\dagger(x-\frac{\epsilon}{2})\Gamma_\mu\psi(x+\frac{\epsilon}{2}) = \hat{\Gamma}(x)_\mu + \epsilon_\nu\hat{\Theta}(x)_{\mu\nu} + \ldots$$

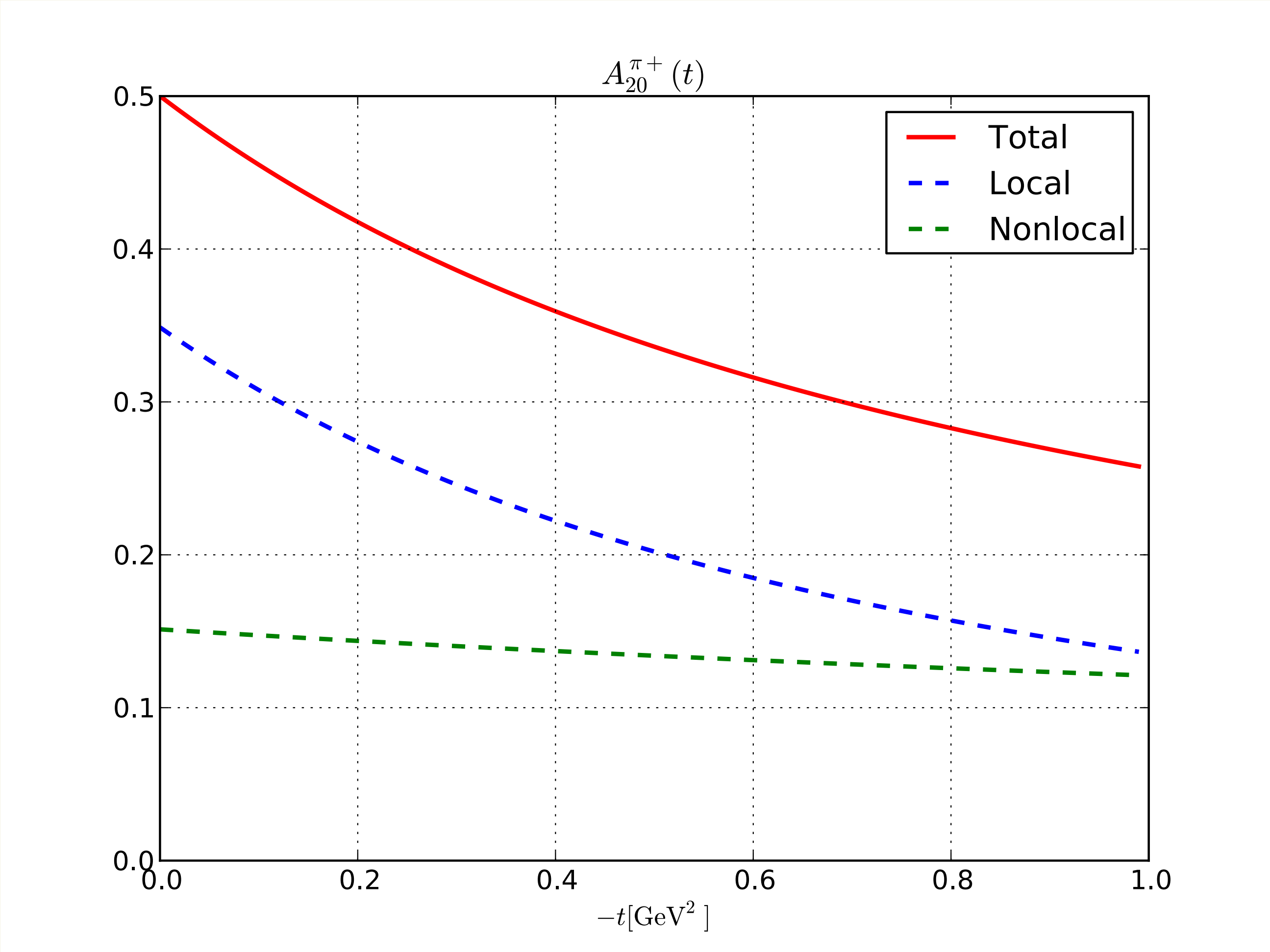
$$\begin{aligned}\hat{\Theta}(x)_{\mu\nu} = \frac{i}{4}\psi^\dagger(x)\Big[\gamma_\mu\overrightarrow{\partial}_\nu - \gamma_\mu\overleftarrow{\partial}_\nu \\ &+i\frac{\partial\sqrt{M}}{\partial\hat{p}_\mu}U^{\gamma_5}\sqrt{M}\overrightarrow{\partial}_\nu - i\sqrt{M}U^{\gamma_5}\frac{\partial\sqrt{M}}{\partial\hat{p}_\mu}\overrightarrow{\partial}_\nu \\ &-i\overleftarrow{\partial}_\nu\frac{\partial\sqrt{M}}{\partial\hat{p}_\mu}U^{\gamma_5}\sqrt{M} + i\overrightarrow{\partial}_\nu\sqrt{M}U^{\gamma_5}\frac{\partial\sqrt{M}}{\partial\hat{p}_\mu}\Big]\psi(x)\end{aligned}$$

Calculation of the matrix element

$$\langle \phi^a(p_f) | \hat{\Theta}_{\mu\nu}(0) | \phi^b(p_i) \rangle$$



Pion Form Factors



m_π [MeV]	f_π [MeV]	$\sqrt{\langle r^2 \rangle}$ [fm]	p	M_π [GeV]
140	92.4	0.337	1	1.01

Pion Form Factors

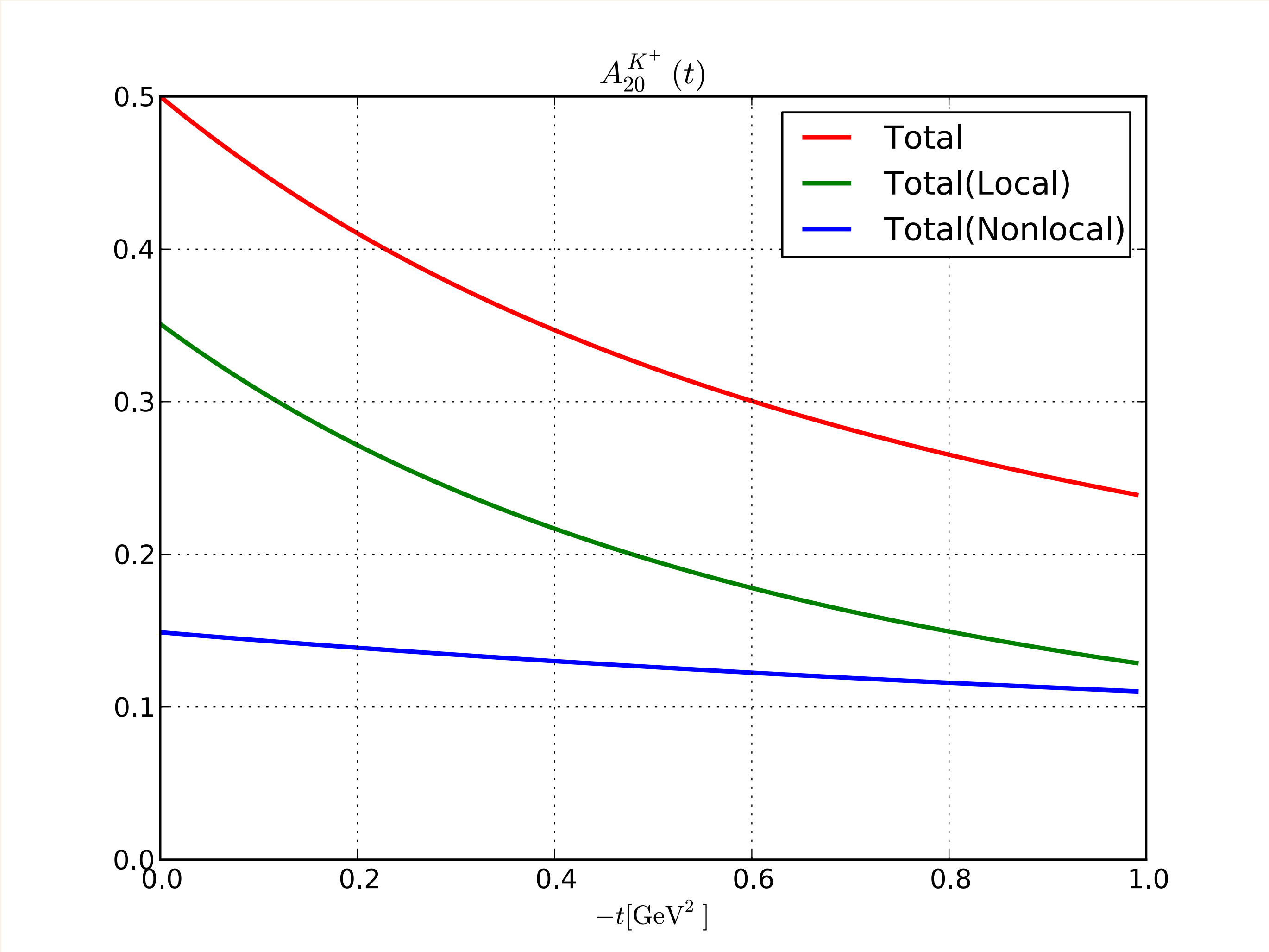
A10 (EM) [S.-i.Nam & H.-Ch.Kim Phys.Rev.D77(2008),094014]

	Local	Nonlocal	Total	Exp. [61]
$\langle r^2\rangle_{\pi^+}^{1/2}$ [fm]	0.594	0.319	0.675	0.672 ± 0.008
$\langle r^2\rangle_{K^+}^{1/2}$ [fm]	0.658	0.318	0.731	0.560 ± 0.031

A20 (present work)

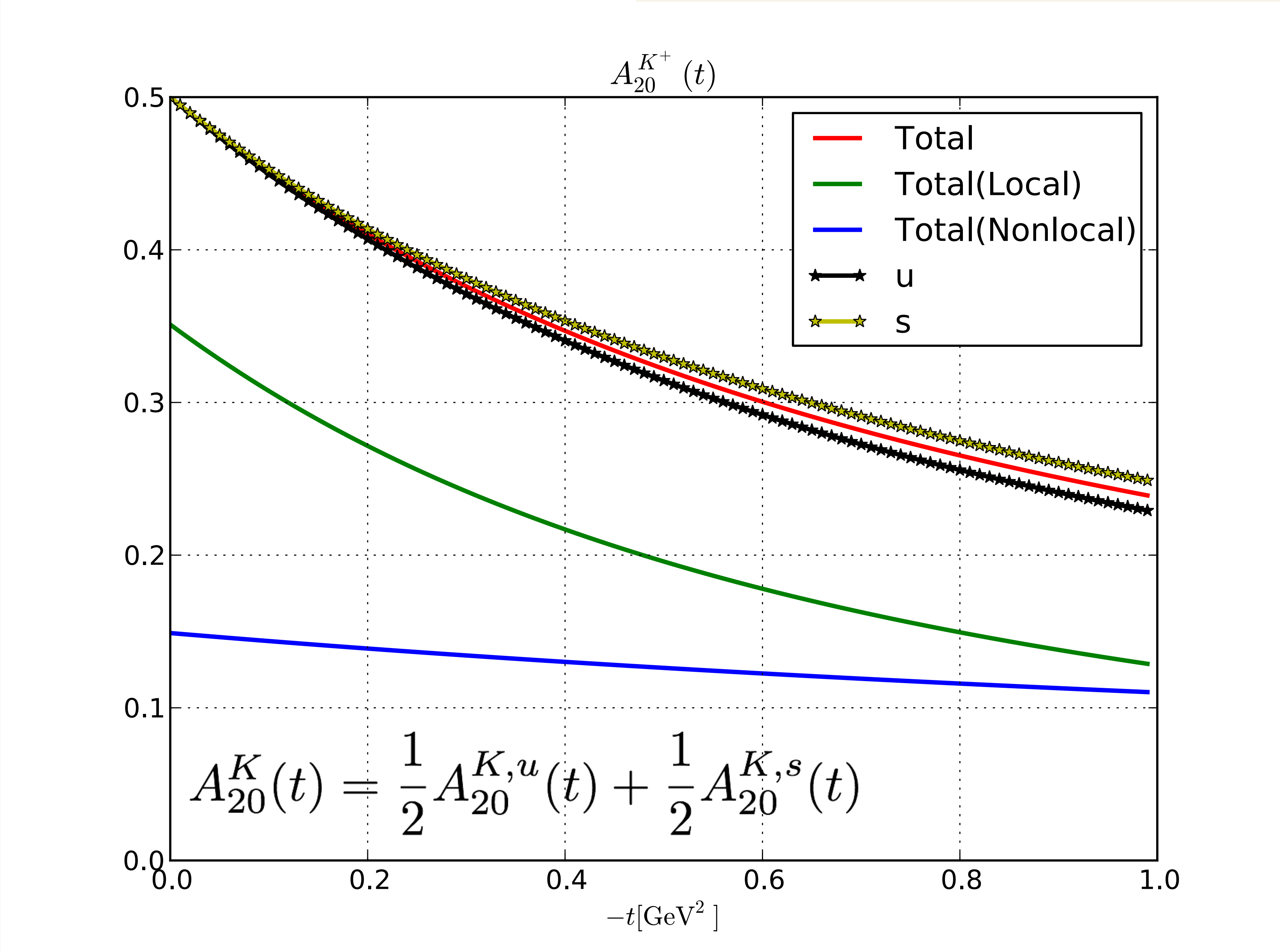
m_π [MeV]	f_π [MeV]	$\sqrt{\langle r^2\rangle}$ [fm]	p	M_π [GeV]
140	92.4	$\boxed{0.337}$	1	1.01

Kaon Form Factors

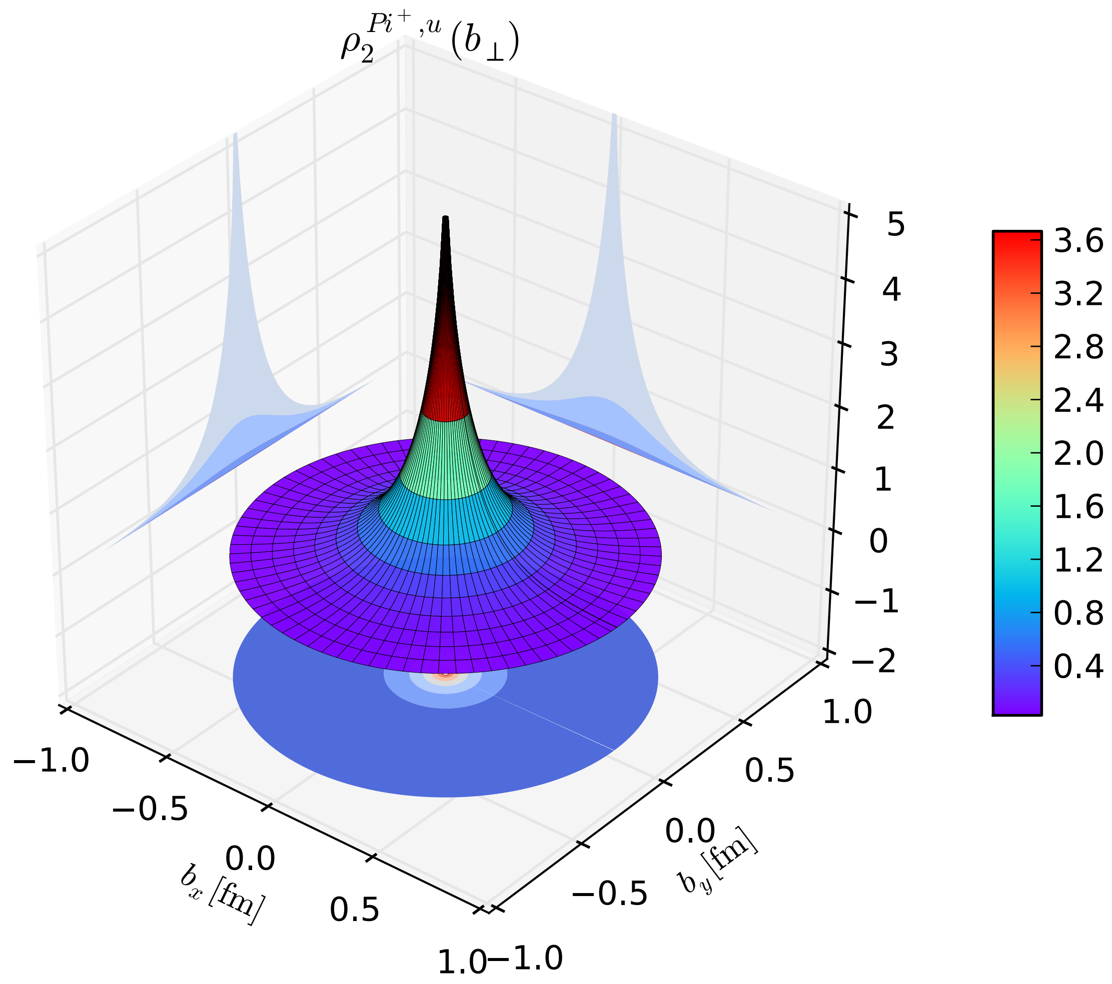


$m_K\,[\mathrm{MeV}]$	$f_K\,[\mathrm{MeV}]$	$\sqrt{\langle r^2\rangle}\,[\mathrm{fm}]$	p	$M_K\,[\mathrm{GeV}]$
493.6	114	0.352	1	0.97

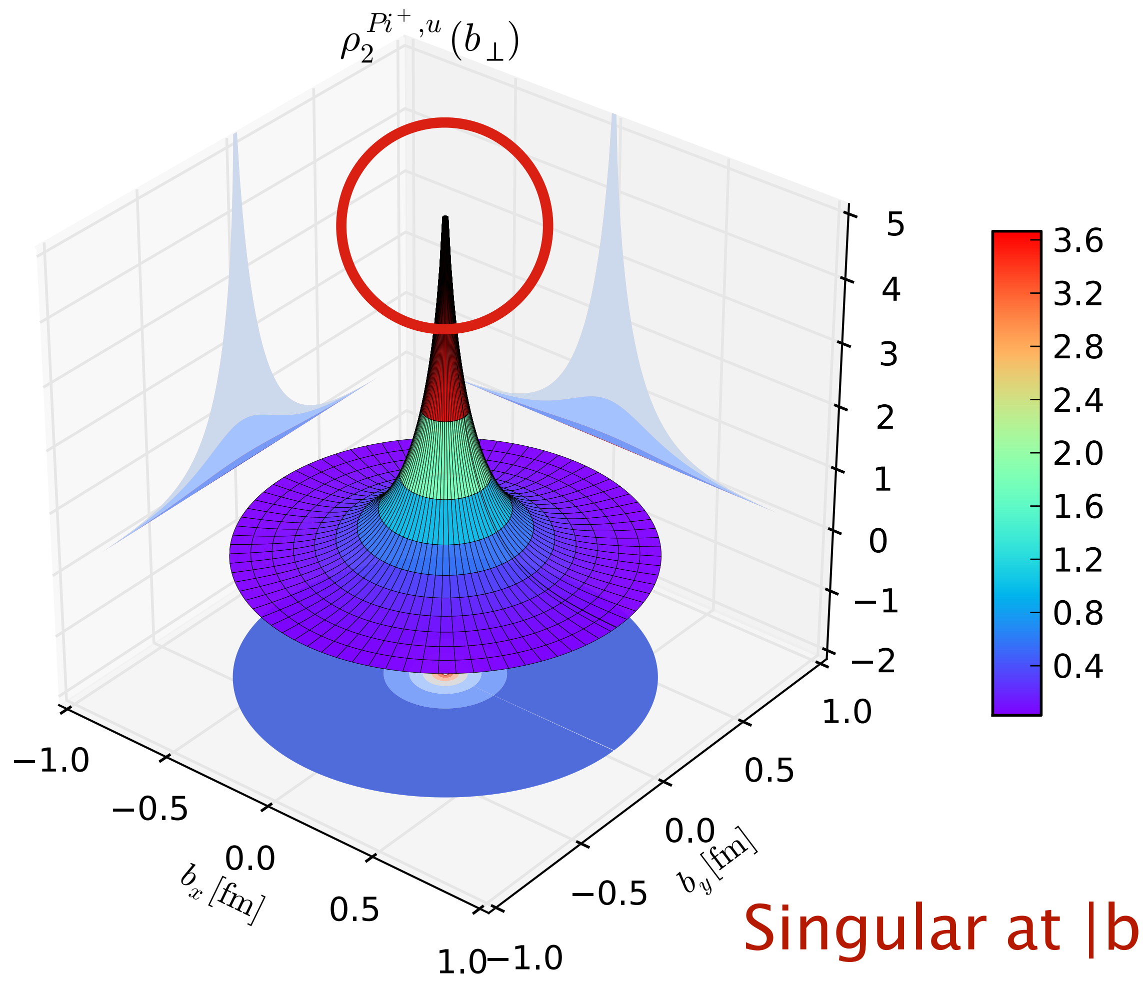
Kaon Form Factors



Transverse Charge Densities of the Pion



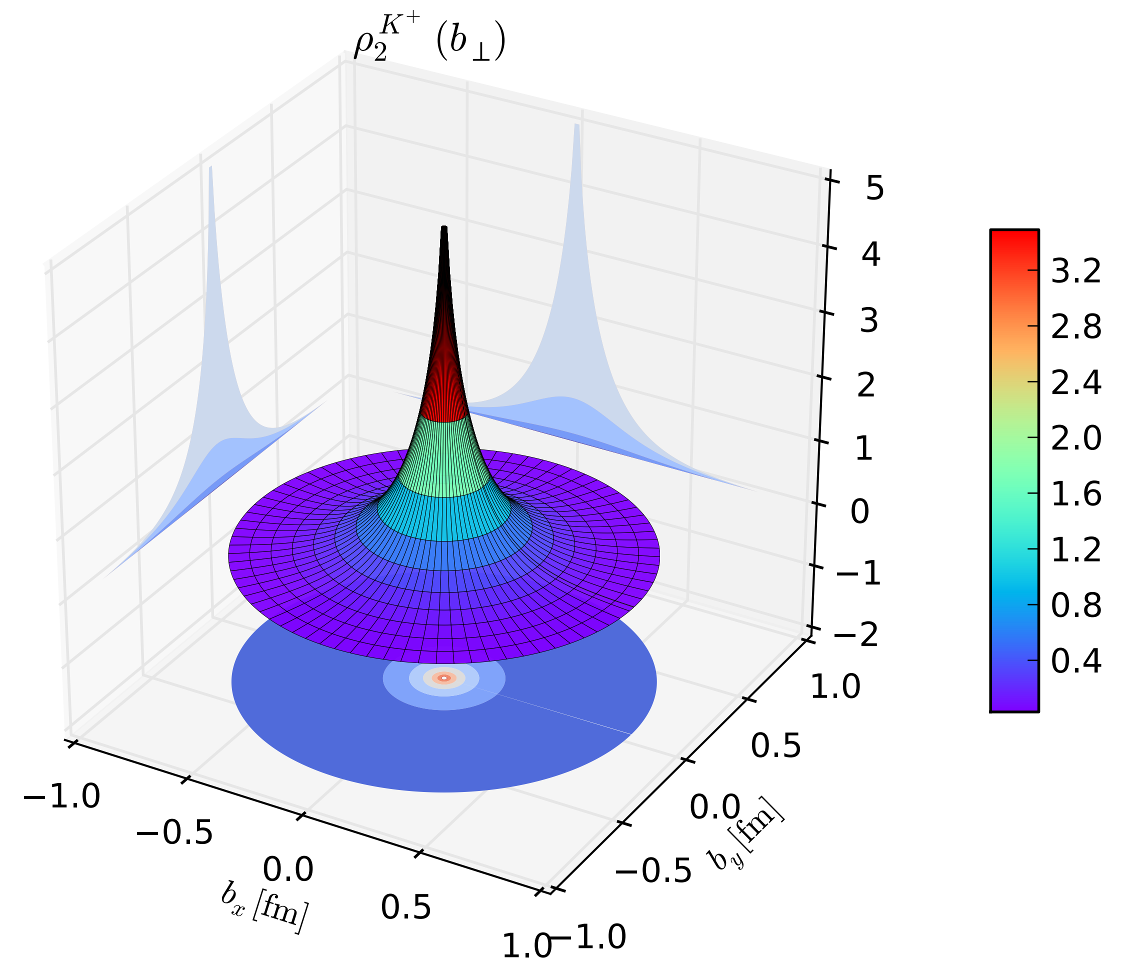
Transverse Charge Densities of the Pion



Singular at |b| = 0

[G.A. Miller, Phys.Rev.C 79, 055204(2009)]

Transverse Charge Densities of the Kaon



Quark Spin Structure

$$\rho_n^{\phi}(b_\perp, s_\perp) = \frac{1}{2}\left[A_{n0}^{\phi}(b_\perp^2) - \frac{s_\perp^i \epsilon^{ij} b_\perp^j}{m_\phi}\frac{\partial B_{n0}^{\phi}(b_\perp^2)}{\partial b_\perp^2}\right] \qquad \langle b_y^{\phi,f}\rangle = \frac{\int d^2 b_\perp b_y \rho_n^{\phi,f}(b_\perp, s_\perp)}{\int d^2 b_\perp \rho_n^{\phi,f}(b_\perp, s_\perp)} = \frac{1}{2m_\phi}\frac{B_{n0}^{\phi,f}(0)}{A_{n0}^{\phi,f}(0)}$$

$\boxed{s_\perp = (1,0)}$ [S.-i.Nam & H.-Ch.Kim Phys.Lett.B707(2012),546]

$\rho_n^{\pi,u}$

	$A_{10}^{\pi,u}(0)$	$M_{A_{10}^{\pi,u}}$ [GeV]	$B_{10}^{\pi,u}(0)$	$M_{B_{10}^{\pi,u}}$ [GeV]	$\langle b_y^{\pi,u}\rangle$ [fm]
n=1	1	0.748	0.244	0.761	0.172

	$A_{20}^{\pi,u}(0)$	$M_{A_{20}^{\pi,u}}$ [GeV]	$B_{20}^{\pi,u}(0)$	$M_{B_{20}^{\pi,u}}$ [GeV]	$\langle b_y^{\pi,u}\rangle$ [fm]
n=2	0.5	1.01	0.049	0.864	0.069

$\rho_n^{K,u}$

	$A_{10}^{K,u}(0)$	$M_{A_{10}^{K,u}}$ [GeV]	$B_{10}^{K,u}(0)$	$M_{B_{10}^{K,u}}$ [GeV]	$\langle b_y^{K,u}\rangle$ [fm]
n=1	1.045	0.647	0.880	0.726	0.168

	$A_{20}^{K,u}(0)$	$M_{A_{20}^{K,u}}$ [GeV]	$B_{20}^{K,u}(0)$	$M_{B_{20}^{K,u}}$ [GeV]	$\langle b_y^{K,u}\rangle$ [fm]
n=2	0.5	0.919	0.199	0.748	0.080

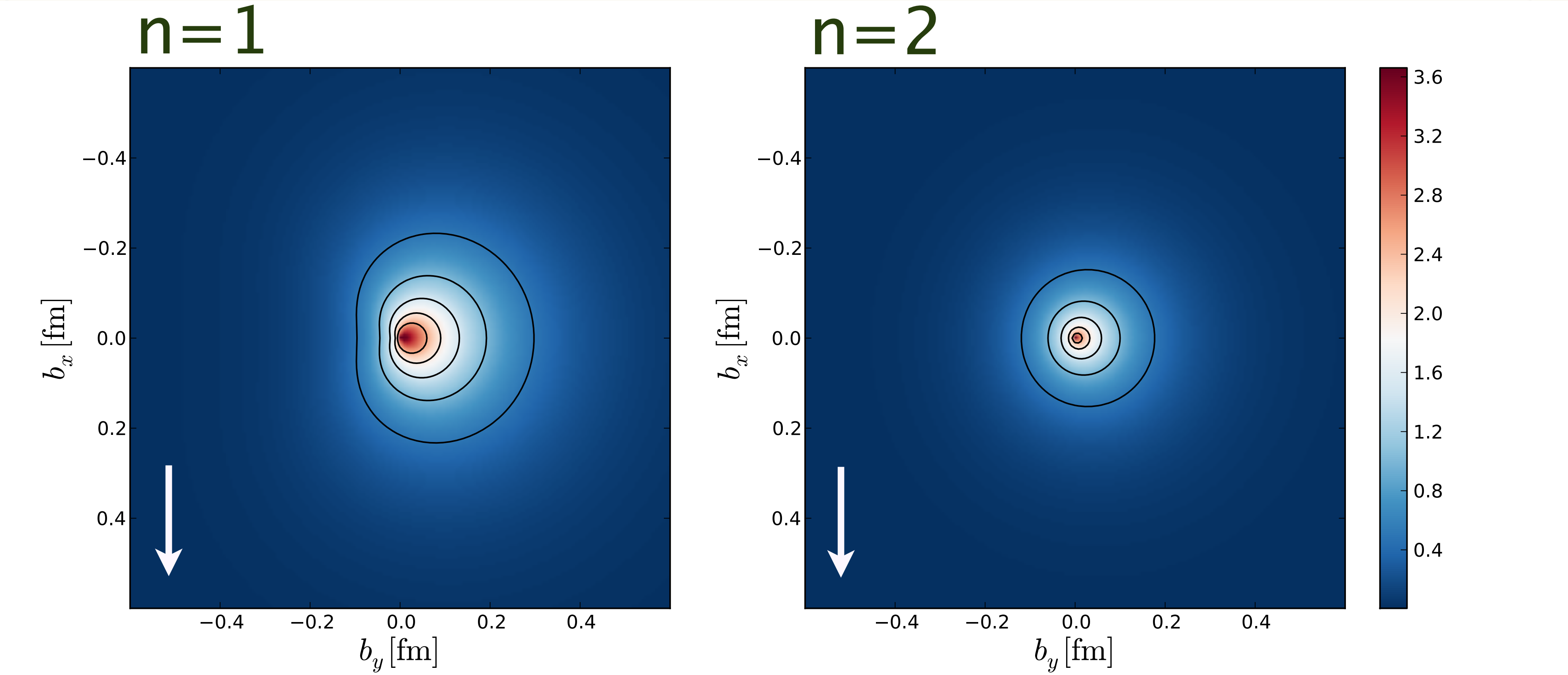
$\rho_n^{K,s}$

	$A_{10}^{K,s}(0)$	$M_{A_{10}^{K,s}}$ [GeV]	$B_{10}^{K,s}(0)$	$M_{B_{10}^{K,s}}$ [GeV]	$\langle b_y^{K,s}\rangle$ [fm]
n=1	-0.909	0.772	-0.760	0.709	0.166

	$A_{20}^{K,s}(0)$	$M_{A_{20}^{K,s}}$ [GeV]	$B_{20}^{K,s}(0)$	$M_{B_{20}^{K,s}}$ [GeV]	$\langle b_y^{K,s}\rangle$ [fm]
n=2	0.5	0.985	-0.143	0.806	-0.057

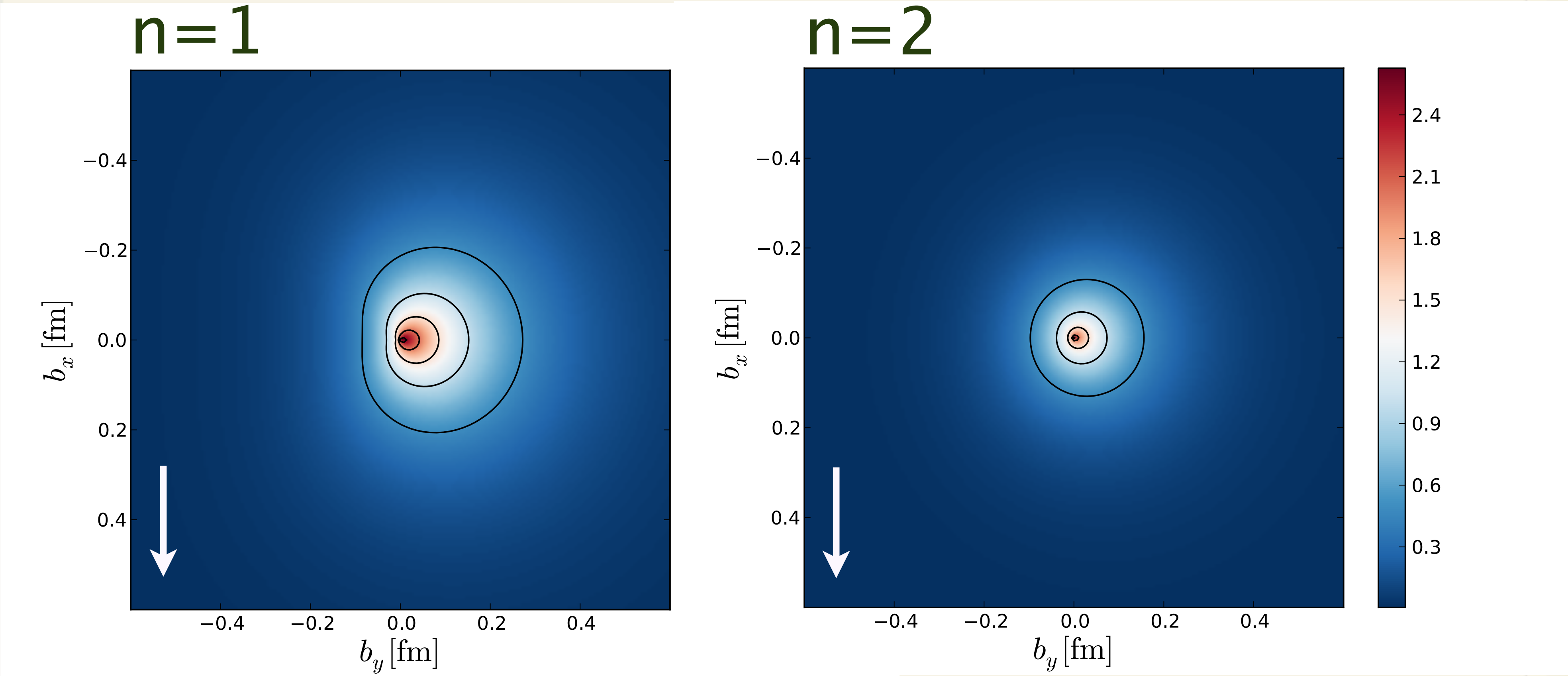
Quark Spin Structure of the Pion

$\rho_n^{\pi,u}$



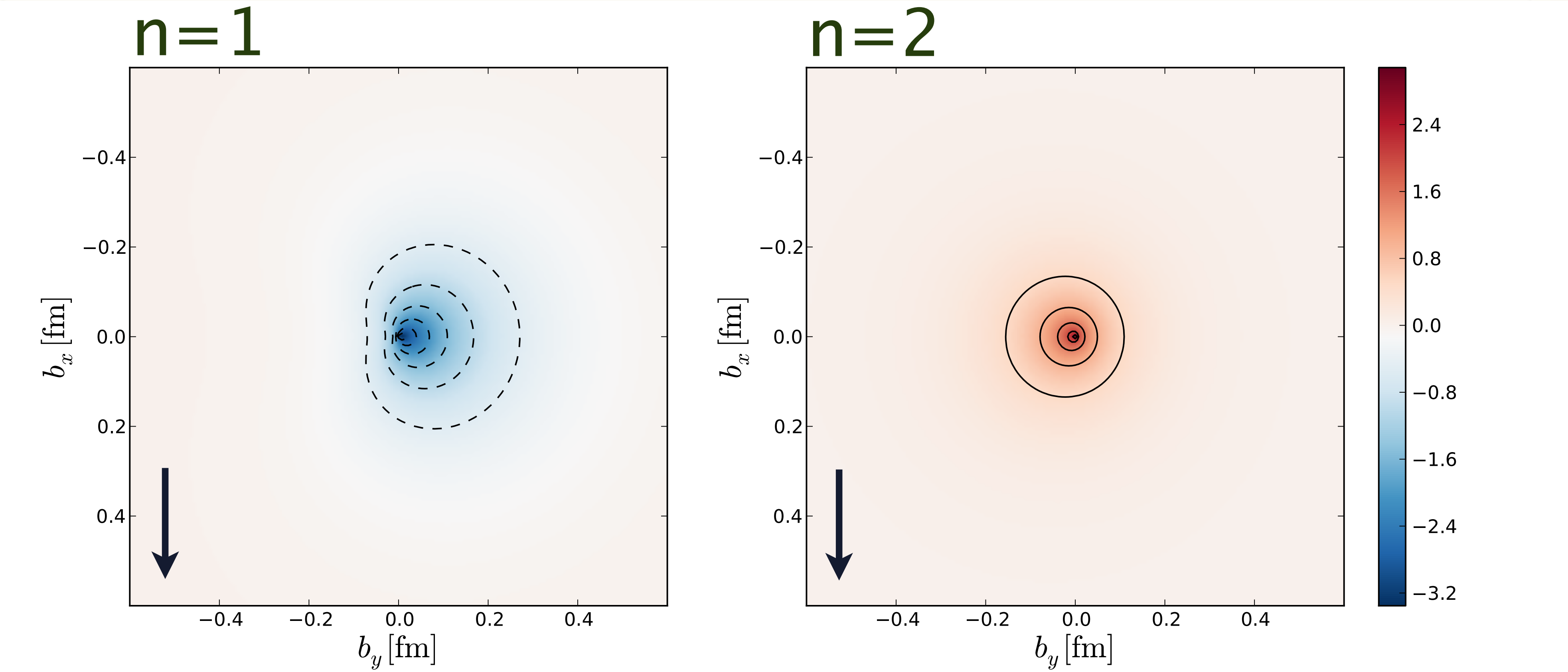
Quark Spin Structure of the Kaon

$\rho_n^{K,u}$



Quark Spin Structure of the Kaon

$\rho_n^{K,s}$



Summary and outlook

- Generalized FFs A20 of the pion and the kaon have been studied.
- Transverse charge density
  : Singular behavior at the center
- n=2 distribution is narrower than n=1.
- A22 is under way.
- Generalized form factors with higher n (n=3, n=4) are under investigation.

Thank you very much!

Calculation of the matrix element

$$\langle \phi^a(p_f) | \hat{\Theta}_{\mu\nu}(0) | \phi^b(p_i) \rangle = \delta^{ab} \frac{2N_c}{f_\phi^2} \int \frac{d^4k}{(2\pi)^4} \sum_i \mathcal{F}_i(k, p_i, p_f)_{\mu\nu} + (\mu \leftrightarrow \nu)$$

$$\mathcal{F}_{a,\mu\nu} = \frac{\sqrt{M_b M_c}}{(k_b^2 + \bar{M}_b^2)(k_c^2 + \bar{M}_c^2)} (\bar{M}_b k_{c\mu} + \bar{M}_c k_{b\mu})(k_{b\nu} + k_{c\nu})$$

$$\mathcal{F}_{b,\mu\nu} = \frac{2M_a\sqrt{M_b M_c}}{(k_a^2 + \bar{M}_a^2)(k_b^2 + \bar{M}_b^2)(k_c^2 + \bar{M}_c^2)} (-k_{a\mu}(k_b \cdot k_c + \bar{M}_b \bar{M}_c)$$

$$+k_{b\mu}(k_c \cdot k_a + \bar{M}_c \bar{M}_a) + k_{c\mu}(k_a \cdot k_b + \bar{M}_a \bar{M}_b))(k_{b\nu} + k_{c\nu})$$

$$\mathcal{F}_{c,\mu\nu} = \frac{\sqrt{M_a M_c}(k_a \cdot k_c + \bar{M}_a \bar{M}_c)}{(k_a^2 + \bar{M}_a^2)(k_c^2 + \bar{M}_c^2)} (\sqrt{M_{a_\mu}}\sqrt{M_b} - \sqrt{M_a}\sqrt{M_{b_\mu}})$$

$$\mathcal{F}_{d,\mu\nu} = \frac{\sqrt{M_a M_b}(k_a \cdot k_b + \bar{M}_a \bar{M}_b)}{(k_a^2 + \bar{M}_a^2)(k_b^2 + \bar{M}_b^2)} (\sqrt{M_{a_\mu}}\sqrt{M_c} - \sqrt{M_a}\sqrt{M_{c_\mu}})$$

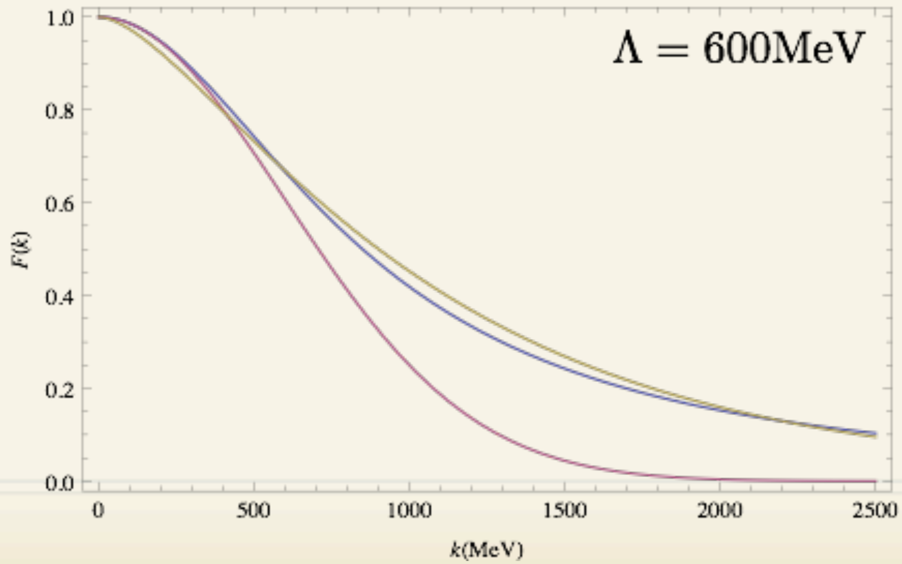
Nonlocal Chiral Quark Model From the Instanton Vacuum

The Dynamical Quark Mass and the Form-factors

$$\sqrt{M(i\partial)} = \sqrt{M_0 f(m) F^2(i\partial)}$$

Instanton Model $\Lambda = 1/\bar{\rho}$

- $F(k) = \frac{k}{\Lambda}\left[I_0(\frac{k}{2\Lambda})K_1(\frac{k}{2\Lambda}) - I_1(\frac{k}{2\Lambda})K_0(\frac{k}{2\Lambda}) - \frac{2\Lambda}{k}I_1(\frac{k}{2\Lambda})K_1(\frac{k}{2\Lambda})\right]$



Approximation

- $F_N(k) = \left(\frac{2N\Lambda^2}{2N\Lambda^2 + k^2}\right)^N$
- $F(k) = \exp\left(-\frac{k^2}{2\Lambda^2}\right)$

Nonlocal Chiral Quark Model From the Instanton Vacuum

The Dynamical Quark Mass and the Form-factors

$$\sqrt{M(i\partial)} = \sqrt{M_0 f(m) F^2(i\partial)}$$

Current quark mass correction f(m)

Zero-momentum dynamical quark mass M0

$$f(m) = \sqrt{1 + \frac{m^2}{d^2}} - \frac{m}{d}, d \approx 198 MeV \qquad M_0 \approx 350 MeV$$

[M. Musakhanov Eur.Phys.J.C9,235(1999)]

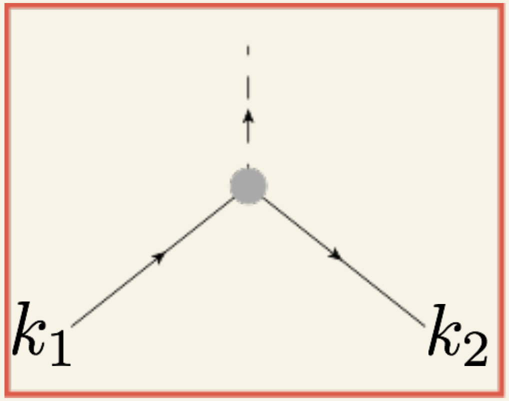
Pion decay constant $\quad f_\pi = 0.0924\mathrm{GeV}$

Calculation of the matrix element

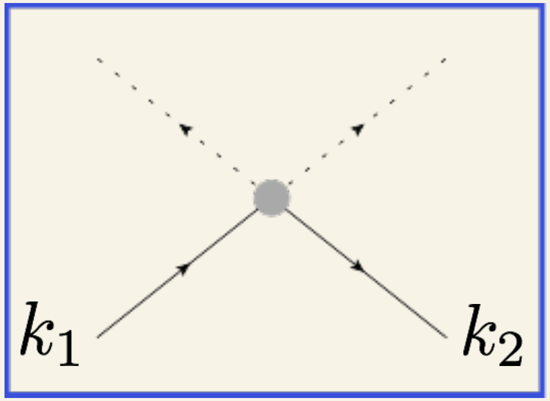
The Quark-Pseudoscalar meson interactions

$$\mathcal{L}_{int} = -\boxed{i\psi^{\dagger}}\, i\sqrt{M(\hat{p})}\,U^{\gamma_5}\sqrt{M(\hat{p})}\,\boxed{\psi}$$

$$U^{\gamma_5} = \exp[i\frac{\gamma_5}{f_\phi}(\lambda\cdot\phi)] = 1 + \boxed{i\frac{\gamma_5}{f_\phi}(\lambda\cdot\phi)} - \boxed{\frac{1}{2f_\phi^2}(\lambda\cdot\phi)^2} + \ldots$$

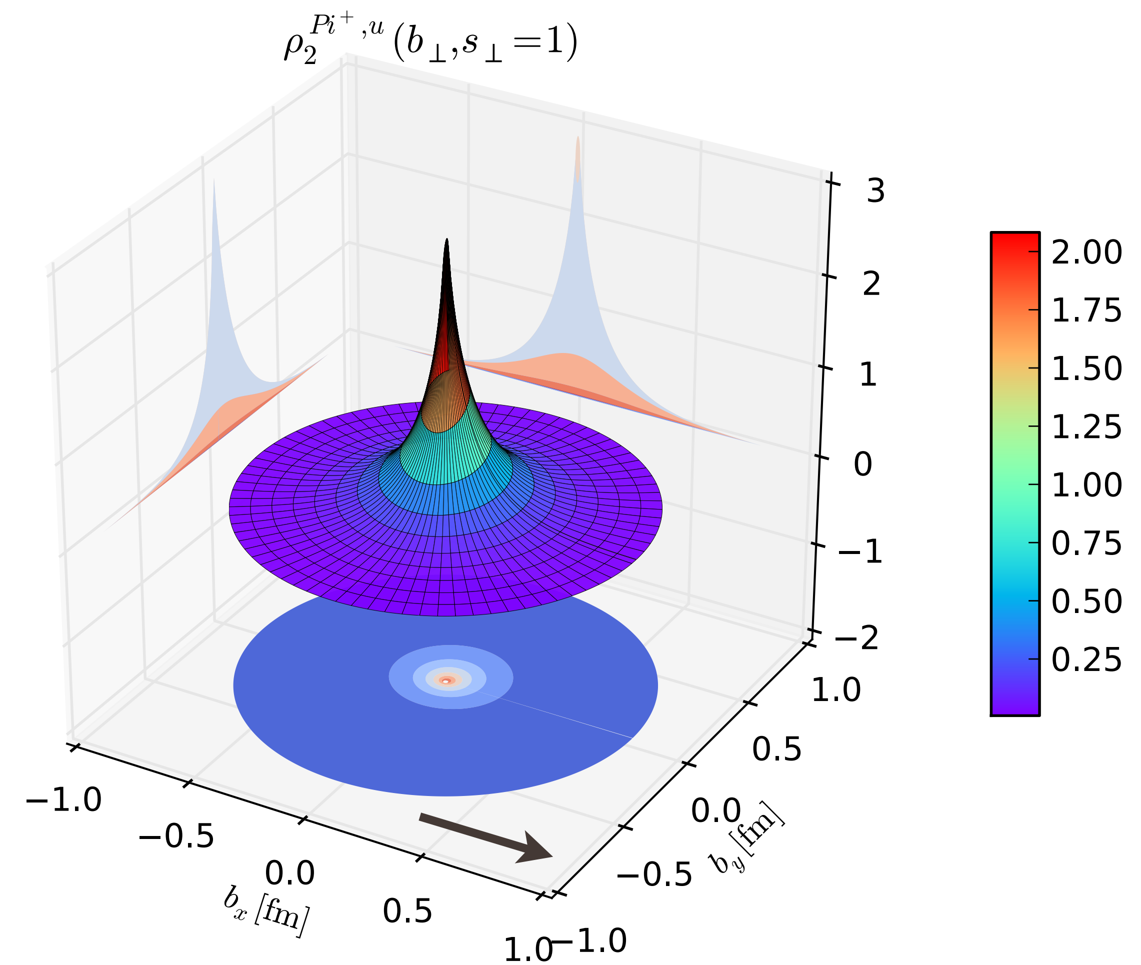


$$i\frac{\gamma_5}{f_\phi}\sqrt{M(k_1)}\sqrt{M(k_2)}$$

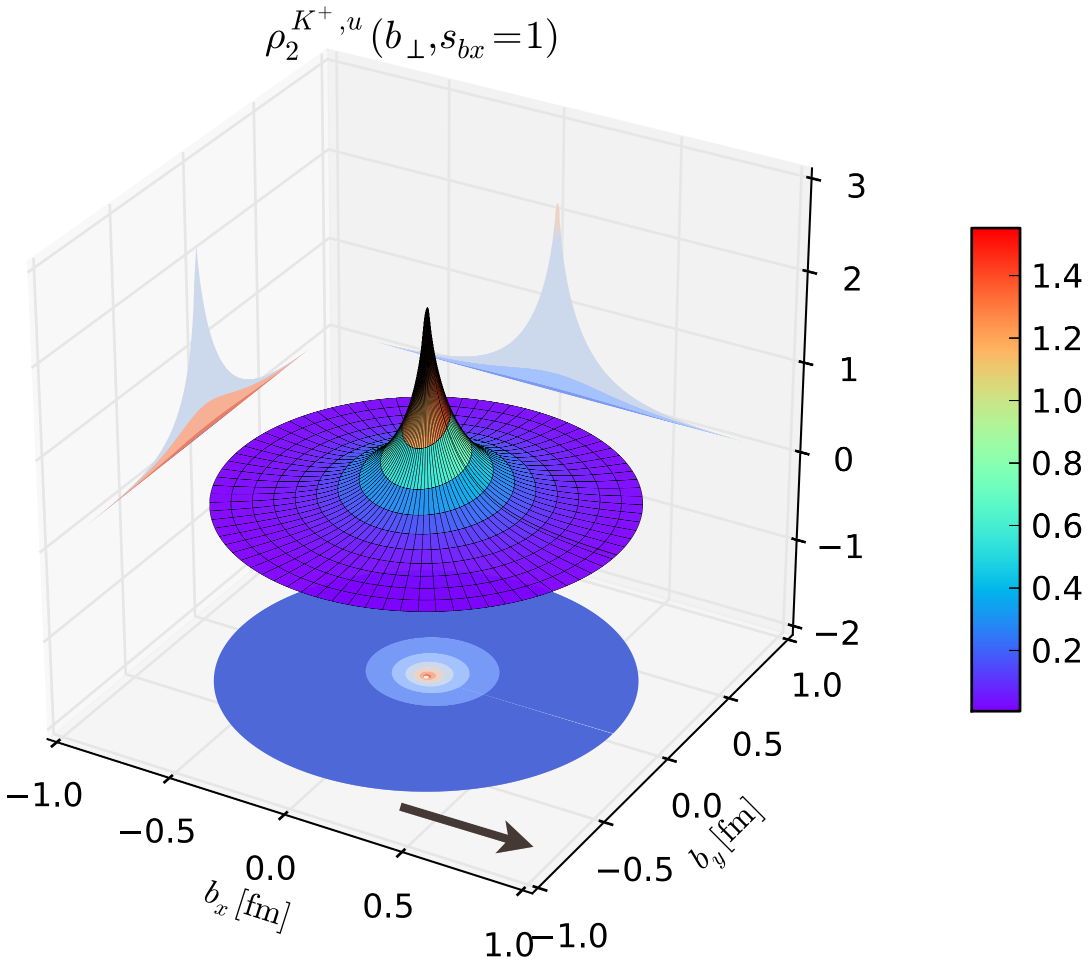


$$-\frac{1}{2f_\phi^2}\sqrt{M(k_1)}\sqrt{M(k_2)}$$

Quark Spin Structure of the Pion



Quark Spin Structure of the Kaon



Quark Spin Structure of the Kaon