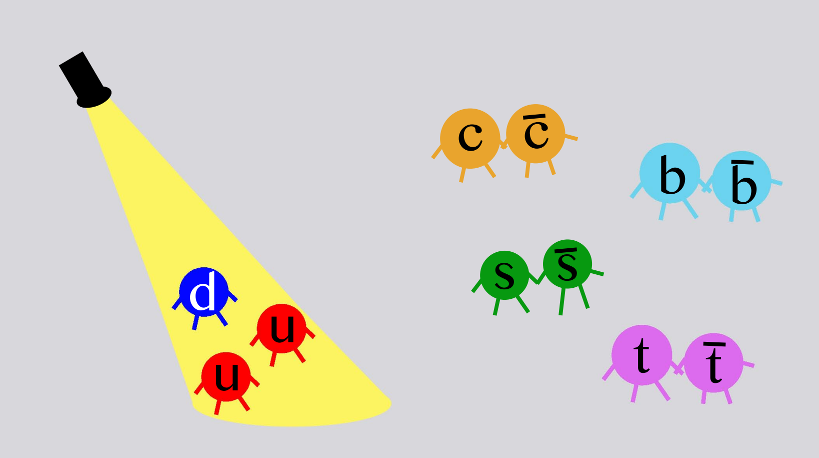
Indirect determination of the strange nucleon form factors from lattice QCD

Phiala Shanahan

Collaborators: Anthony Thomas, Ross Young
James Zanotti and QCDSF/UKQCD lattice group

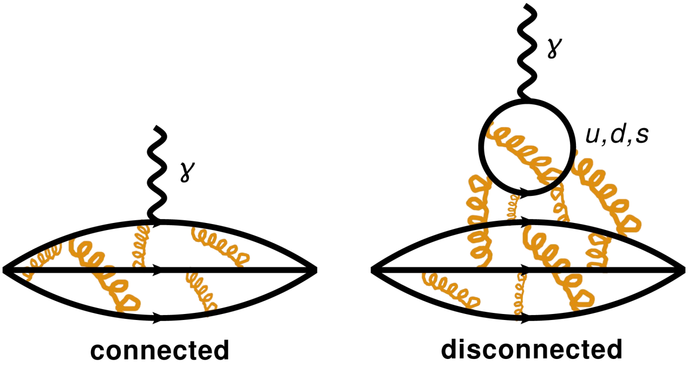
April 2014

How much do 'hidden flavours' contribute to nucleon observables?



Why are hidden flavour contributions interesting?

Generated **entirely** by interactions with the vacuum



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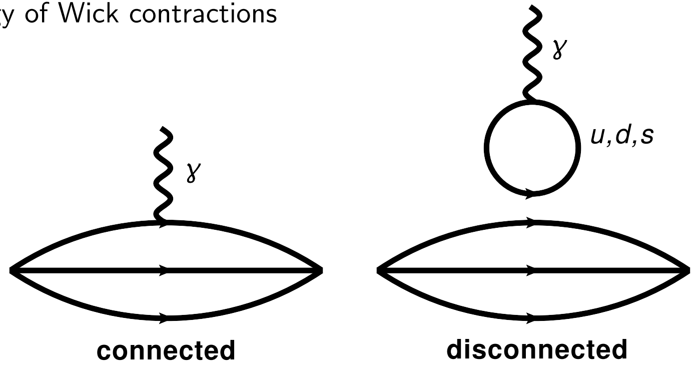
'Skeleton' diagram:
topology of Wick contractions



Why are hidden flavour contributions interesting?

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$\rightarrow$ play the largest role

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Test for nonperturbative QCD

strange quarks

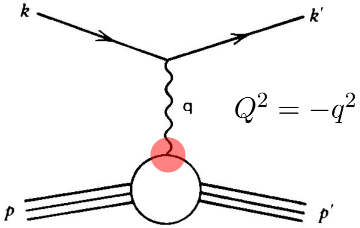
lightest of the 'sea only' quarks

$\rightarrow$ play the largest role

- strange sigma terms - needed for calculation/prediction of dark matter cross sections
- strange contribution to proton spin
- strange PDFs
- **strange electromagnetic form factors**

Electromagnetic form factors

Form factors characterize the extended nature of composite particles

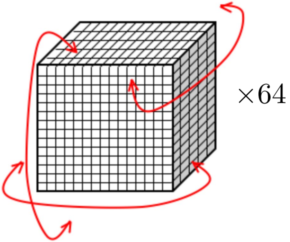


Lattice QCD (Ken Wilson 1974)

Numerical first-principles approach

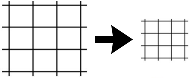
Discretise space-time (4D box)

Lattice spacing a, volume $L^3 \times T$
order $32^3 \times 64 \approx 2 \times 10^6$ lattice sites



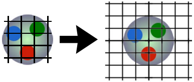
Lattice QCD - systematics and limitations

- **Finite lattice spacing** a
  discretisation artifacts
  Continuum extrapolation



$$a \to 0$$

- **Finite box size** L
  $\Rightarrow$ momentum quantized, finite-volume effects
  Finite-volume corrections



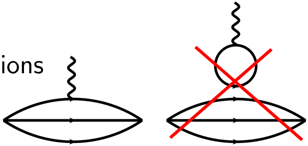
$$L \to \infty$$

- **Large pion mass** m_π
  Chiral extrapolation

$$m_\pi \to 140\mathrm{MeV}$$

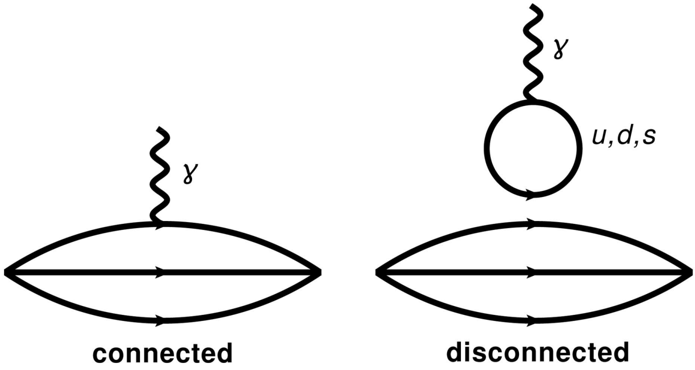
  BUT: Can map out m_ϕ-dependence of observables

- **Omitted disconnected loops**
  BUT: can separate 'valence' and 'sea' contributions
  This is the key here.



Method

Use lattice results to **deduce** the strange form factors of the nucleon.



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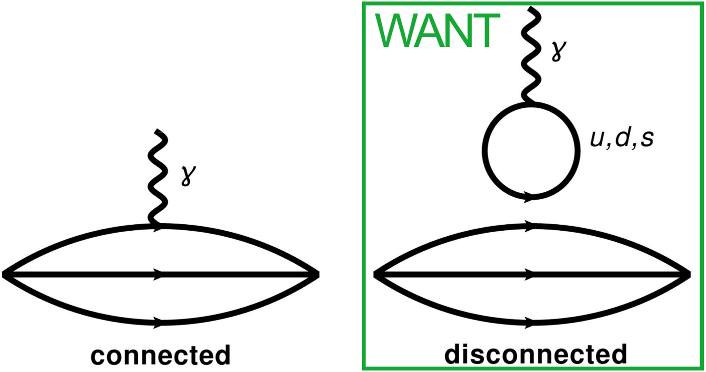

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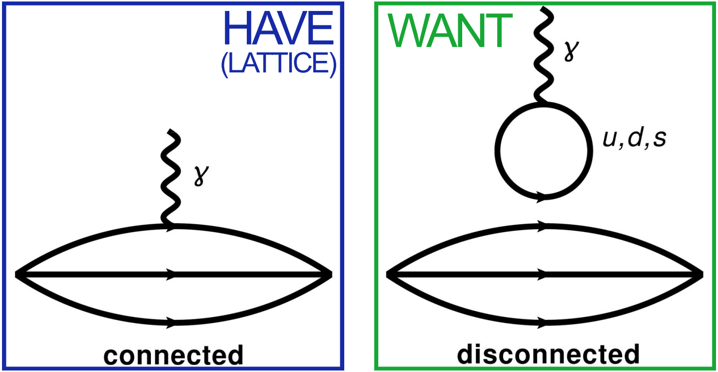

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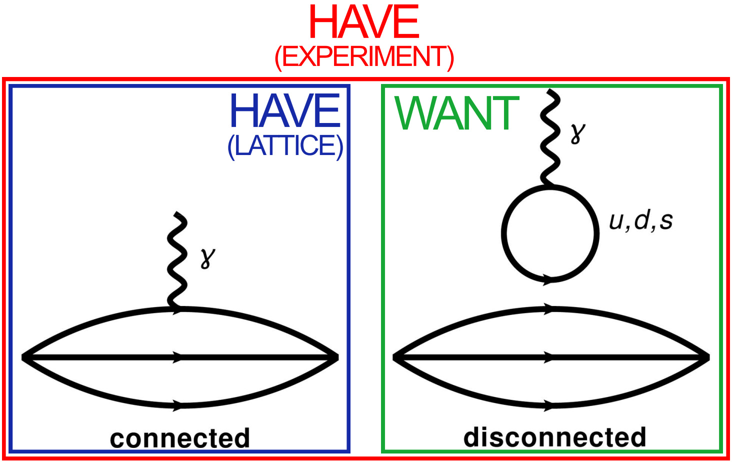

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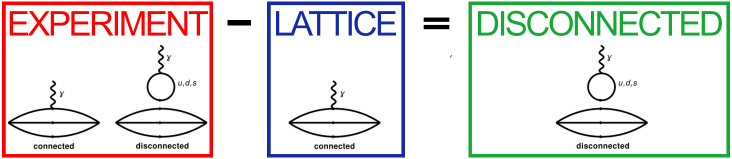

Difference between lattice and experiment gives disconnected terms

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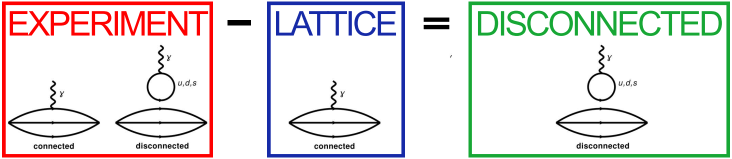

Difference between lattice and experiment gives disconnected terms

IF all systematics are under control . . .

Method - Details

Use **charge symmetry**:

Experimental form factors

$$p = e^u u^p + e^d d^p + O_N$$

$$n = e^d u^p + e^u d^p + O_N$$

Disconnected quark loop term

Connected u and d quark contributions

$$O_N = \frac{2}{3} {}^{\ell}G^u - \frac{1}{3} {}^{\ell}G^d - \frac{1}{3} {}^{\ell}G^s$$

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Method - Details

Rearrange:

$$ {}^{\ell}G^{s} = \left(\frac{{}^{\ell}R_d^s}{1 - {}^{\ell}R_d^s} \right) \left[\frac{3}{2}(p+n)_{\mathrm{Exp.}} - \frac{1}{2}(u^p + d^p)_{\mathrm{Latt.}} \right]. $$

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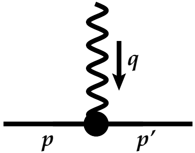
Have: Connected lattice simulations for G_E and G_M

- Quark contributions to outer-ring octet baryons $G^{p,u}$, $G^{p,d}$, $G^{\Sigma,u}$, $G^{\Sigma,s}$, $G^{\Xi,s}$, $G^{\Xi,u}$
- Six sets of pseudoscalar masses (m_π, m_K)
- Six values of the momentum transfer $q = p' - p$



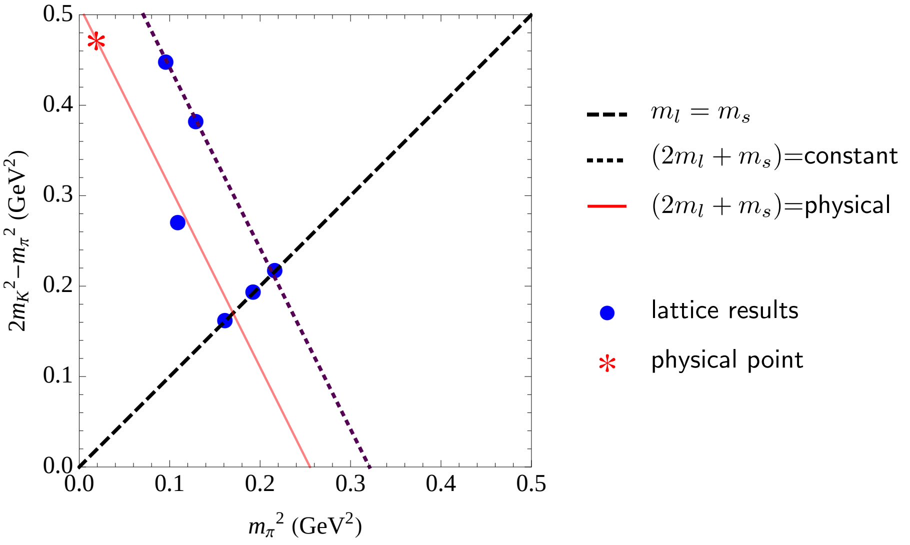
Want: G_E and G_M at the physical point

- Finite-volume corrections
- Chiral extrapolation - simultaneous fit to all baryons

CSSM/QCDSF/UKQCD Collaborations

P.E. Shanahan *et al.* arXiv:1401.5862, 1403.1965, 1403.6537

The lattice simulations: $2m_l - 2m_s$ plane



Chiral extrapolation at *fixed* Q^2

q quark contribution to magnetic form factor of the baryon B

$$G_M^{B,q}(Q^2) = \text{terms analytic in } m_\phi^2$$

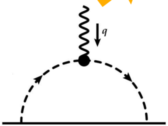
$$+ \text{chiral loop corrections}$$

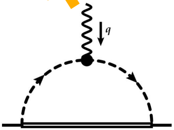
Chiral extrapolation at *fixed* Q^2

q quark contribution to magnetic form factor of the baryon B

Linear in m_ϕ^2

$$G_M^{B,q}(Q^2) = \alpha^{Bq} + \sum_{q'} \overline{\alpha}^{Bq(q')} \underline{\mathcal{B} m_{q'}} + \frac{m_N}{16\pi^3 f^2} \sum_\phi \left(\beta_O^{Bq(\phi)} \underline{\mathcal{I}_O(m_\phi, Q^2)} + \beta_D^{Bq(\phi)} \underline{\mathcal{I}_D(m_\phi, Q^2)} \right)$$





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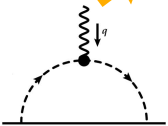
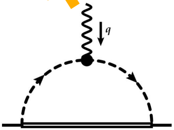
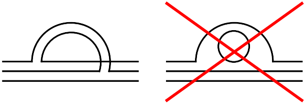




Chiral coefficients: choose **connected** contributions only

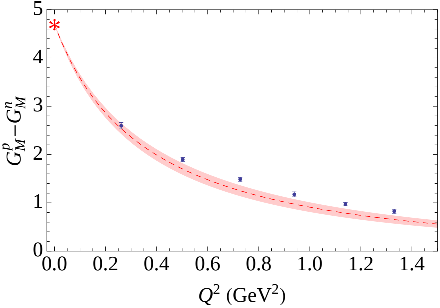


Isovector nucleon form factors

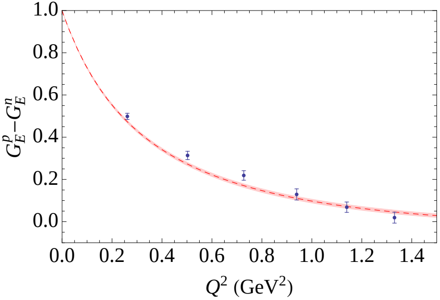
Systematics under control ⇒ Isovector nucleon FFs agree with experiment

$$
\begin{aligned}
(p-n)_{\text{total}} &= (p_{\text{connected}} + O_N) - (n_{\text{connected}} + O_N) \\
&= (p-n)_{\text{connected}}
\end{aligned}
$$

Magnetic



Electric

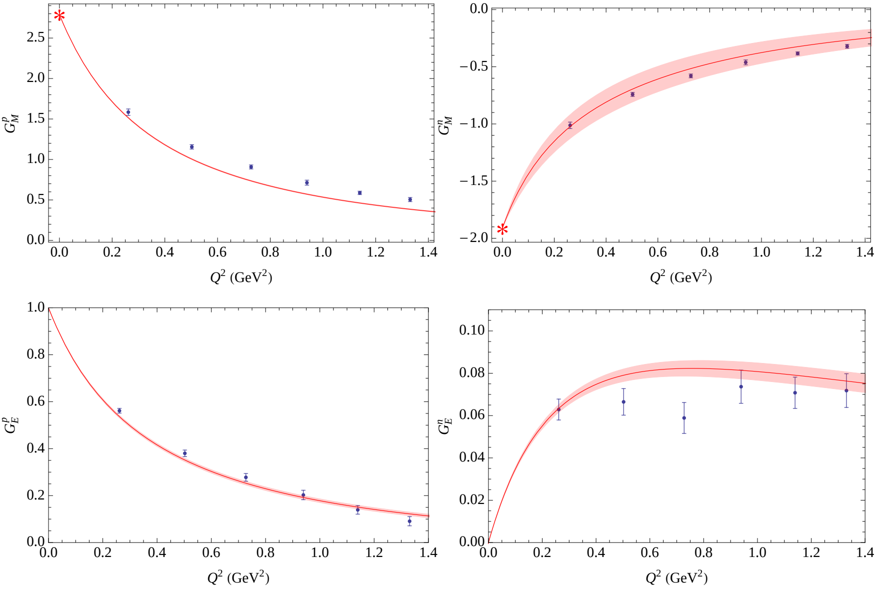


Kelly experimental parameterization: J.J. Kelly, PR C70, 068202 (2004)

Proton **Neutron**

M

E



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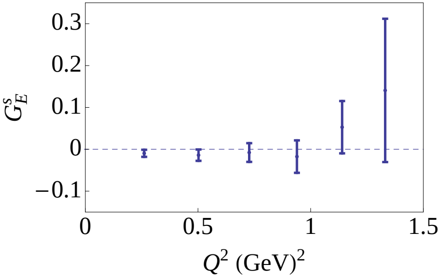
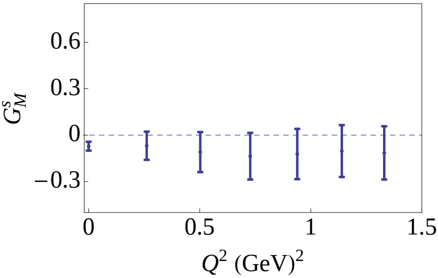
Results - no charge factor ($\times -\frac{1}{3}$ for physical units)

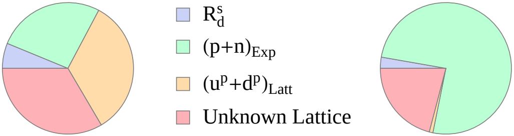
Electric



Magnetic



Uncertainties:



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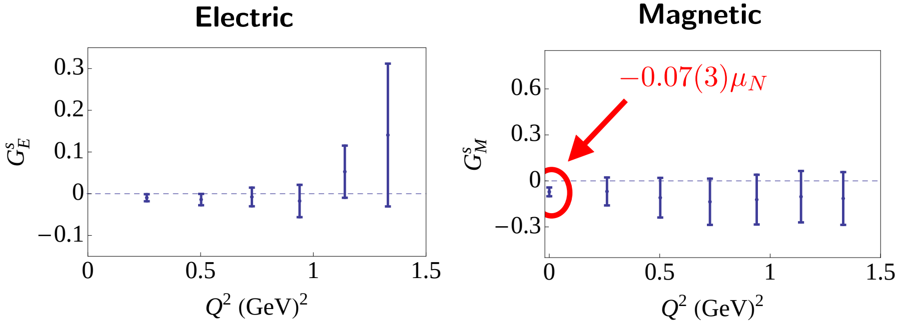

Strange quarks contribute $0.8(3)\%$ to the proton magnetic moment.

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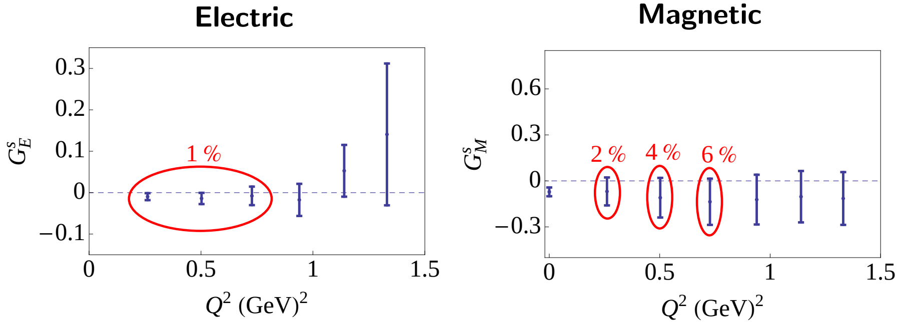

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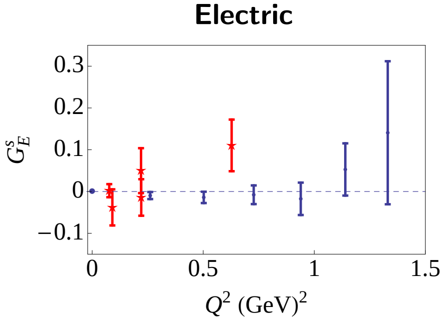
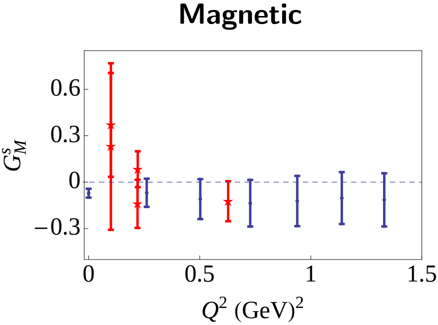
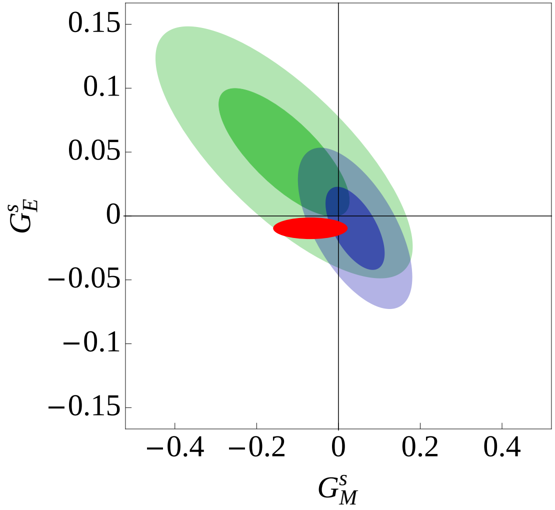




Red stars: G0, SAMPLE, HAPPEX, A4.

At $Q^2 \approx 0.26$ GeV^2



This work: 0.26 GeV^2
A4: 0.23 GeV^2
G0: 0.23 GeV^2

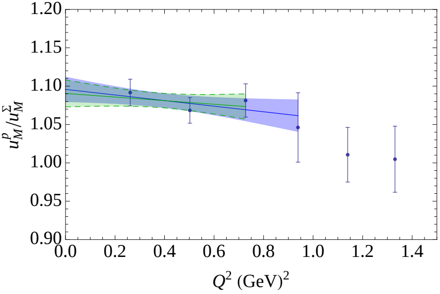
Dark: 1σ
Light: 2σ

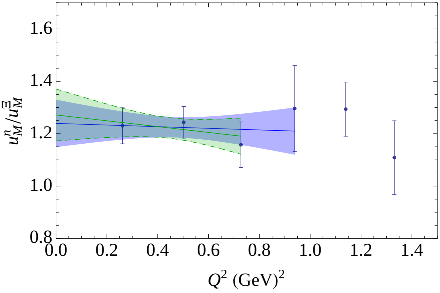
Strange magnetic moment $G_M(Q^2 = 0)$

Additional information: hyperon magnetic moments have been measured.
Use the assumption of charge symmetry:

$$ {}^{\ell}G^s = \left(\frac{{}^{\ell}R_d^s}{1 - {}^{\ell}R_d^s} \right) \left[2p + n - \frac{u^p}{u^{\Sigma}} \left(\Sigma^+ - \Sigma^- \right) \right], $$
$$ {}^{\ell}G^s = \left(\frac{{}^{\ell}R_d^s}{1 - {}^{\ell}R_d^s} \right) \left[p + 2n - \frac{u^n}{u^{\Xi}} \left(\Xi^0 - \Xi^- \right) \right]. $$

Take ratios of form factors u^p/u^{Σ}, u^n/u^{Ξ} from lattice QCD.





Experimental determinations of $G^s_{E/M}$

EM and **weak** vector currents give access to different combinations of $G^{p,(u/d/s)}$:

$$G^{p,\gamma} = \frac{2}{3}G^{p,u} - \frac{1}{3}\left(G^{p,d} + G^{p,s}\right)$$
$$G^{p,Z} = \left(1 - \frac{8}{3}\sin^2\theta_W\right)G^{p,u} - \left(1 - \frac{4}{3}\sin^2\theta_W\right)\left(G^{p,d} + G^{p,s}\right)$$

Assume **charge symmetry** ($G^{p,u} = G^{n,d}$, $G^{p,d} = G^{n,u}$, $G^{p,s} = G^{n,s}$)

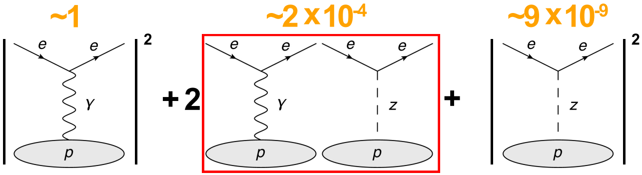
$$G^{p,s}_{E/M} = \left(1 - \underbrace{4\mathrm{Sin}^2\theta_W}_{\text{well determined}}\right)\underbrace{G^{p,\gamma}_{E/M}}_{\text{well determined}} - \underbrace{G^{n,\gamma}_{E/M}}_{\text{well determined}} - \underbrace{G^{p,Z}_{E/M}}_{\text{PVES}}$$

Parity-violating electron scattering

JLab (*G0, HAPPEX*), MIT-Bates (*SAMPLE*), Mainz (*A4*)

Accessing the neutral weak current G^Z

Elastic $e-p$ scattering cross sections $\propto |\mathcal{M}_\gamma + \mathcal{M}_Z|^2$, **BUT** γ **dominates**



Parity-violating cross-term $\rightarrow$ form observable sensitive to G^Z:

$$A_{PV} = \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L} \sim \frac{2M_\gamma^* M_Z^{PV}}{|M_\gamma|^2} \sim 10^{-5}$$

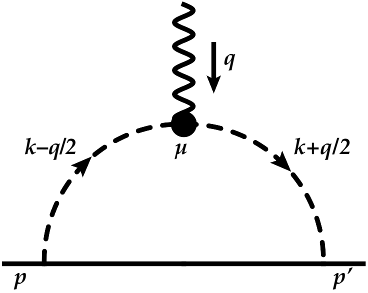
spin-dependent DIS

$$= -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \frac{\epsilon G_E^\gamma G_E^Z + \tau G_M^\gamma G_M^Z - (1 - 4\mathrm{Sin}^2\theta_W)\epsilon' G_M^\gamma G_A^e}{\epsilon (G_E^\gamma)^2 + \tau (G_M^\gamma)^2}$$

Different targets (proton, deuteron, helium-4), different kinematic configurations $\rightarrow$ different ϵ, ϵ', i.e., different linear combinations of G_E^s and G_M^s

3. The ratio of strange to light disconnected loops ${}^{\ell}R^s_d = \frac{{}^{\ell}G^s}{{}^{\ell}G^d}$ **from a model based on chiral perturbation theory.**

Approximate ${}^{\ell}R^s_d = \frac{{}^{\ell}G^s}{{}^{\ell}G^d}$ by the ratio of the strange to light **disconnected** contributions to the loops:



Loop contribution $\mathcal{I}(m_\phi, Q^2)$
for meson ϕ in loop

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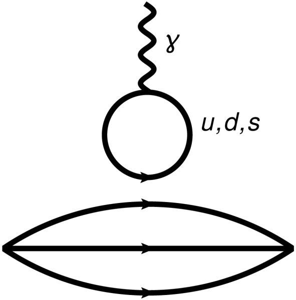

disconnected

Disconnected contribution depends only on m_q for quark q in the loop

3. The ratio of strange to light disconnected loops ${}^{\ell}R_d^s = \frac{{}^{\ell}G^s}{{}^{\ell}G^d}$ **from a model based on chiral perturbation theory.**

Approximate ${}^{\ell}R_d^s = \frac{{}^{\ell}G^s}{{}^{\ell}G^d}$ by the ratio of the strange to light **disconnected** contributions to the loops:

$$ {}^{\ell}R_d^s(Q^2) = \frac{{}^{\ell}G^s}{{}^{\ell}G^d} = \frac{\mathcal{I}(m_K, Q^2)}{\mathcal{I}(m_\pi, Q^2)} $$

Uncertainties:

- Model-dependence (range of regulator masses Λ in FRR scheme)
- Higher-order terms (use decuplet intermediate state loops to estimate)

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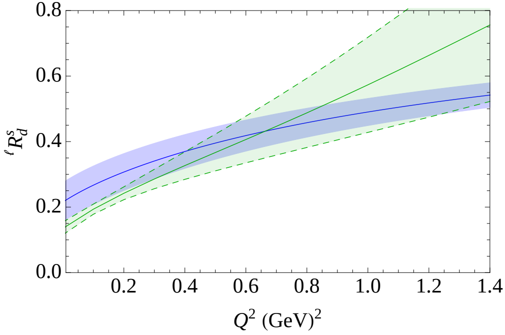


Magnetic (solid)
Electric (dashed)

Chiral perturbation theory (χPT)

The **effective description** of a strongly coupled theory

$\uparrow$ χPT $\qquad$ $\uparrow$ Low-energy QCD

Goldstone bosons (pions) become the fundamental degrees of freedom

- Built on the symmetries of QCD
- Preserves non-analyticity of loops (correct chiral behavior of QCD)
- Same IR behaviour as underlying theory, different UV behaviour

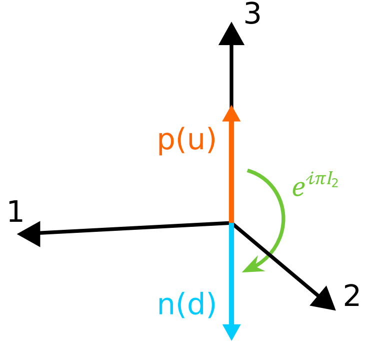
Expansion in small momenta and light quark masses:

$$\mathcal{L} = \mathcal{L}_2 + \frac{1}{\Lambda_\chi^2}\mathcal{L}_4 + \ldots$$

ASSUMPTION: Charge symmetry

Equivalence of u quarks in the proton
and d quarks in the neutron.

Precisely: invariance of strong interaction under a rotation of 180° about the '2' axis in isospin space.



ASSUMPTION: Charge symmetry

Equivalence of u quarks in the proton
and d quarks in the neutron.

In terms of form factors:

$$G^{p,u}_{E/M} = G^{n,d}_{E/M}$$
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ASSUMPTION: Charge symmetry

Equivalence of u quarks in the proton
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$$u^p = d^n$$

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