NΞ and ΞΞ interactions from Lattice QCD

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for HAL QCD collaboration

HAL (Hadrons to Atomic nuclei from Lattice) QCD Collaboration

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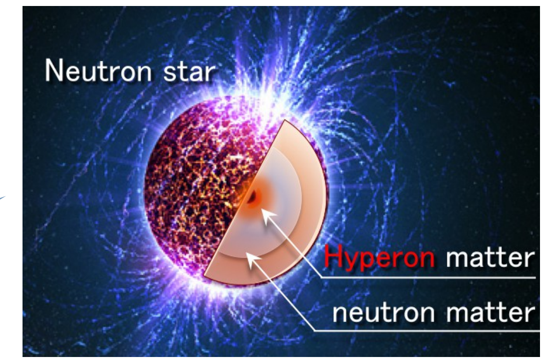
- Introduction
- HAL QCD method
- NΞ and ΞΞ potentials
- Summary and Outlook

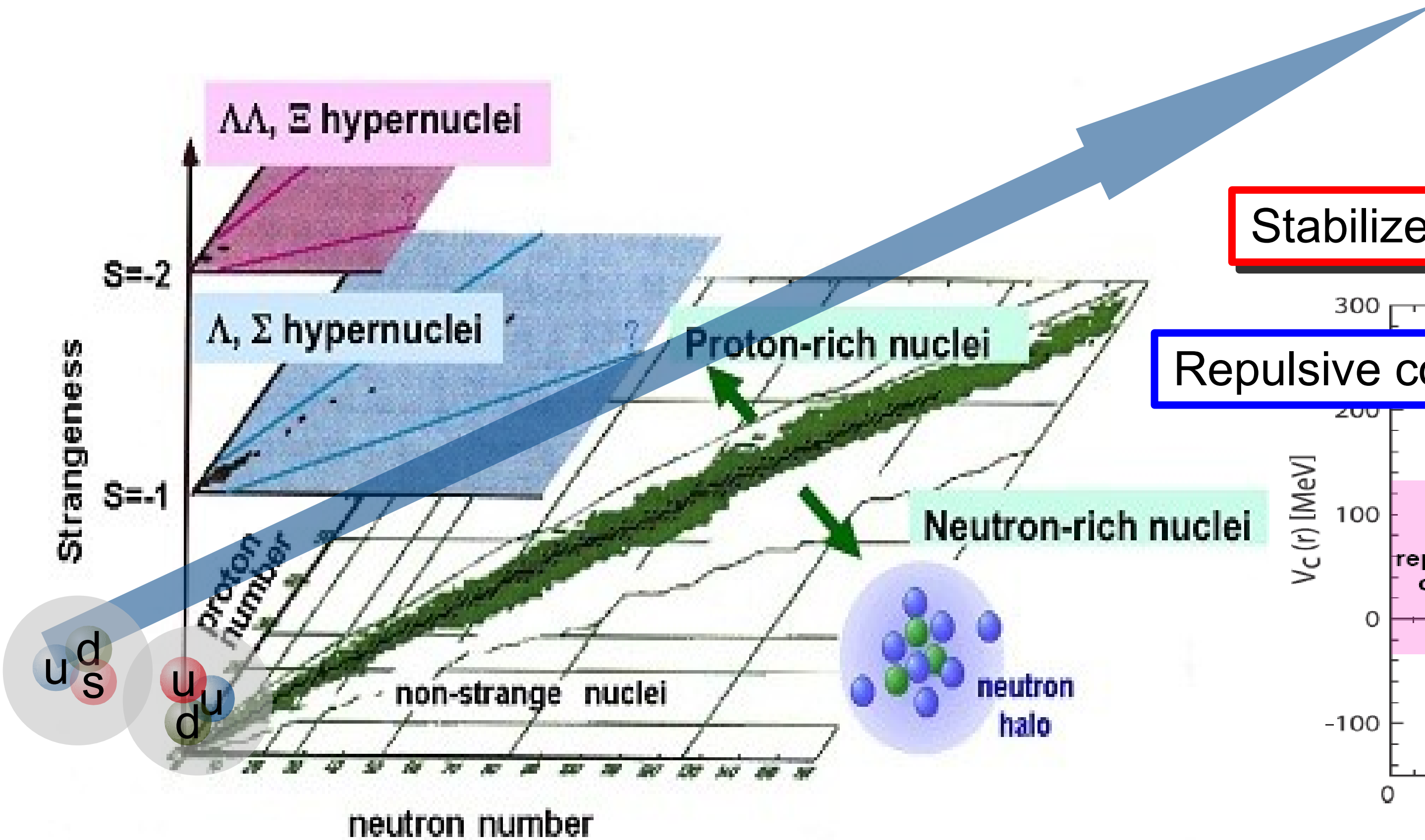
Introduction

Introduction

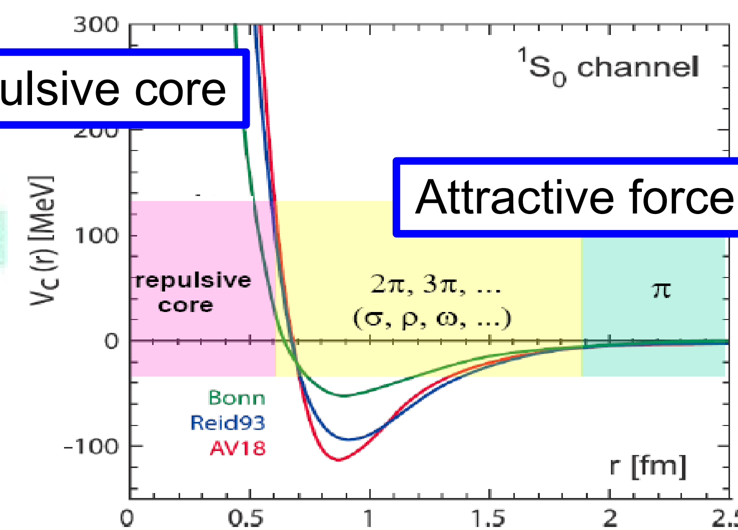
<u>BB interactions are inputs for nuclear structure, astrophysical phenomena</u>

Once we obtain a “reliable” nuclear potential,
we apply them to the structure of (hyper-) nucleus
and neutron star calcration.





Stabilize the nucleus and the matter



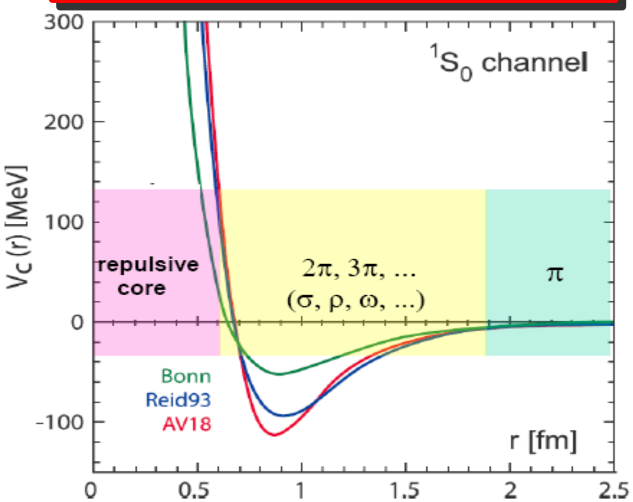
How do we obtain the nuclear force?

Introduction

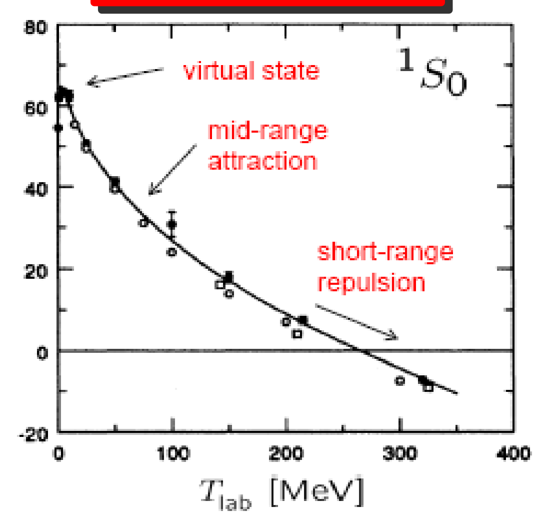
Baryon-baryon interactions are key to understand nuclear structures and astrophysical phenomena

Traditional way to research the BB interaction / potential

BB interaction (potential)



BB phase shift



NN interaction

Large amount of scattering data to determine theoretical parameters

YN / YY interaction

More strangeness, more difficult to access experimentally. Experimental data are scarce.

Lattice QCD results for YN and YY interactions are highly awaited

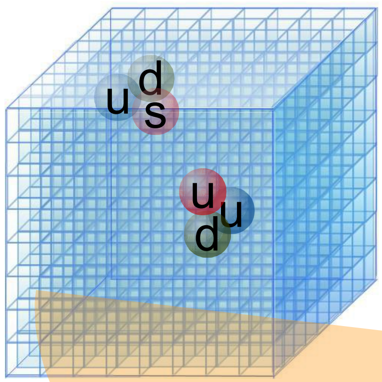
HAL QCD method

QCD to hadronic interactions

Start with the fundamental theory,QCD, to obtain a “reliable” interaction

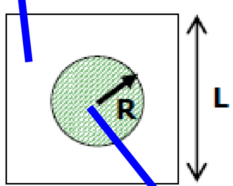
$$L_{QCD} = \bar{q}\left(i\gamma_\mu D^\mu - m\right)q + \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}$$

Lattice QCD simulation

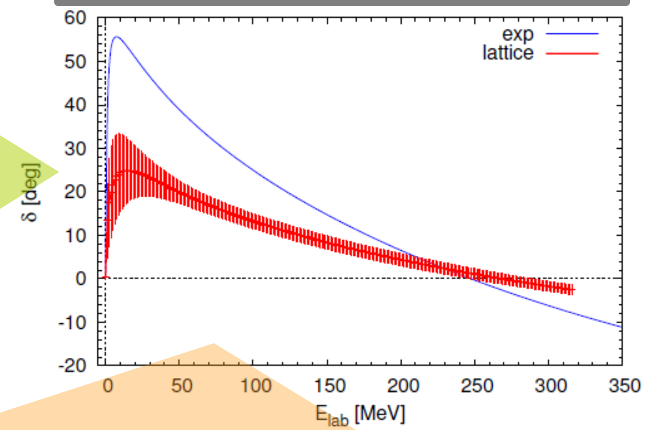


Lüscher's finite volume method

M. Lüscher, NPB354(1991)531



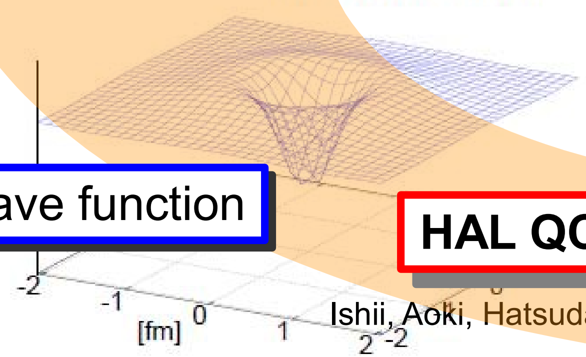
BB scattering phase shift



Guaranteed to be the same

Kurth et al JHEP 1312 (2013) 015

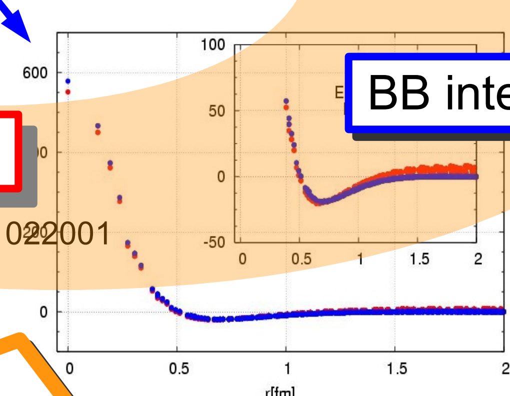
NBS wave function



HAL QCD method

Ishii, Aoki, Hatsuda, PRL**99** (2007) 022001

BB interaction (potential)



The potential is proper for the phase shift by QCD

Aoki, Hatsuda, Ishii, PTP123, 89 (2010).

Nambu-Bethe-Salpeter wave function

Definition : equal time NBS w.f.

$$\Psi(E,\vec r)e^{-E(t-t_0)}=\sum_{\vec x}\langle 0|B_i(t,\vec x+\vec r)B_j(t,\vec x)|E,\nu,t_0\rangle$$

E : Total energy of system

Local composite interpolating operators

$$B=\epsilon^{abc}(q_a^T C\gamma_5 q_b)q_c \quad \begin{cases} p=udu \quad n=udd \quad \Xi^0=sus \quad \Xi^-=sds \\ \Lambda=\sqrt{\frac{1}{6}}[dsu+sud-2uds] \\ \Sigma^+=-usu \quad \Sigma^0=-\sqrt{\frac{1}{2}}[dsu+usd] \quad \Sigma^-=-dsd \end{cases}$$

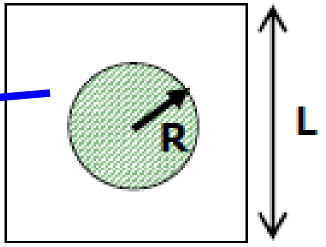
Four point correlator

$$F_{B_1B_2}(\vec r,t)=\langle 0|T[B_1(\vec r,t)B_2(0,t)(\bar B_2\bar B_1)_{t_0}]|0\rangle=\sum_n A_n\Psi(E_n,\vec r)e^{-E_n t}$$

- It satisfies the Helmholtz eq. in asymptotic region : $(p^2+\nabla^2)\Psi(E,\vec r)=0$
- In interaction region : $(p^2+\nabla^2)\Psi(E,\vec r)=K(E,\vec r)$

NBS wave function has a same asymptotic form with quantum mechanics.
(NBS wave function is characterized from phase shift)

$$\Psi(E,\vec r)\simeq A\frac{\sin(pr+\delta(E))}{pr}$$



N. Ishii et al Phys. Lett. B**712**(2012)437

Time-dependent method

Let's start with the normalized four-point correlator.

$$R_I^{B_1B_2}(t,\vec{r}) = F_{B_1B_2}(\vec{r},t)\, e^{(m_1+m_2)t}$$

$$= A_0 \Psi(\vec{r}, E_0)\, e^{-(E_0-m_1-m_2)t} + A_1 \Psi(\vec{r}, E_1)\, e^{-(E_1-m_1-m_2)t} + \cdots$$

$$\left(\frac{p_0^2}{2\mu} + \frac{\nabla^2}{2\mu}\right)\Psi(\vec{r},E_0) = \int U(\vec{r},\vec{r}\,')\Psi(\vec{r}\,',E_0)\, d^3r'$$

Each wave functions satisfy Schroedinger eq. with proper energy

$$\left(\frac{p_1^2}{2\mu} + \frac{\nabla^2}{2\mu}\right)\Psi(\vec{r},E_1) = \int U(\vec{r},\vec{r}\,')\Psi(\vec{r}\,',E_1)\, d^3r'$$

$$E_n - m_1 - m_2 \approx \frac{p_n^2}{2\mu}$$

$$\left(-\frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu}\right) R_I^{B_1B_2}(t,\vec{r}) = \int U(\vec{r},\vec{r}\,')\, R_I^{B_1B_2}(t,\vec{r})\, d^3r'$$

A single state saturation is not required!!

BB interaction from NBS wave function

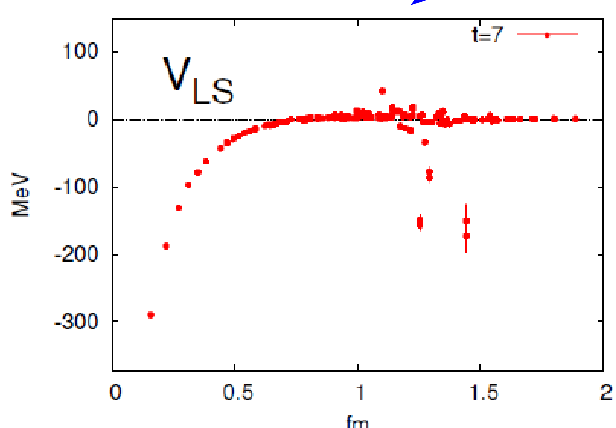
$$\left(-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu}\right)R_I^{B_1B_2}(t,\vec{r})=\int U(\vec{r},\vec{r}\,')R_I^{B_1B_2}(t,\vec{r})d^3r\,'$$

Nonlocality of U(r,r')

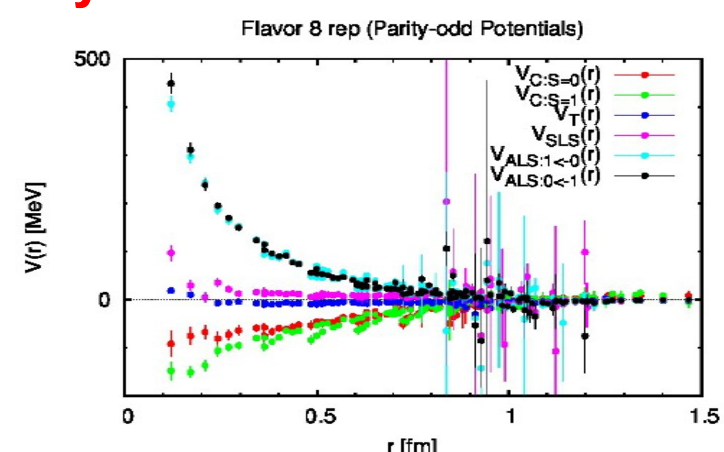
$$U(\vec{r},\vec{r}\,') = V_C(r)+S_{12}V_T(r)+\vec{L}\cdot\vec{S}_s V_{LS}(r)+\vec{L}\cdot\vec{S}_a V_{ALS}(r)+O(\nabla^2)$$
$$= V_C^{eff}(r) + \vec{L}\cdot\vec{S}_s V_{LS}(r)+\vec{L}\cdot\vec{S}_a V_{ALS}(r)+O(\nabla^2)$$

Effective central potential

For negative parity sector



K. murano et al arXiv:1305.2293 [hep-lat]

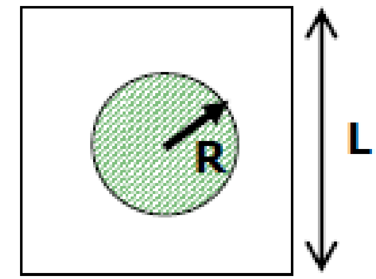


N.Ishii @ Lat2013 conference

Coupled channel Schrödinger equation

Preparation for the NBS wave function

Two-channel coupling case



$$\Psi^{\alpha}(E,\vec{r})e^{-Et} = \sum_{\vec{x}} \langle 0 | (B_1 B_2)^{\alpha}(t,\vec{r}) | E \rangle$$

$$\Psi^{\beta}(E,\vec{r})e^{-Et} = \sum_{\vec{x}} \langle 0 | (B_1 B_2)^{\beta}(t,\vec{r}) | E \rangle$$

The same "in" state

Inside the interaction range

In the *leading order of derivative expansion* of non-local potential, $U(\vec{r},\vec{r}\,')=V(\vec{r})\delta(\vec{r}\,'-\vec{r})$

Coupled channel Schrödinger equation.

Factorization of kernel function

$$\left(\frac{p_{\alpha}^2}{2\mu_{\alpha}}+\frac{\nabla^2}{2\mu_{\alpha}}\right)\Psi^{\alpha}(\vec{r},E)=V^{\alpha}{}_{\alpha}(\vec{r})\Psi^{\alpha}(\vec{r},E)+V^{\alpha}{}_{\beta}(\vec{r})\Psi^{\beta}(\vec{r},E)$$

μ_{α} : *reduced mass*

p_{α} : asymptotic momentum.

Asymptotic momentum are replaced by the time-derivative of *R*.

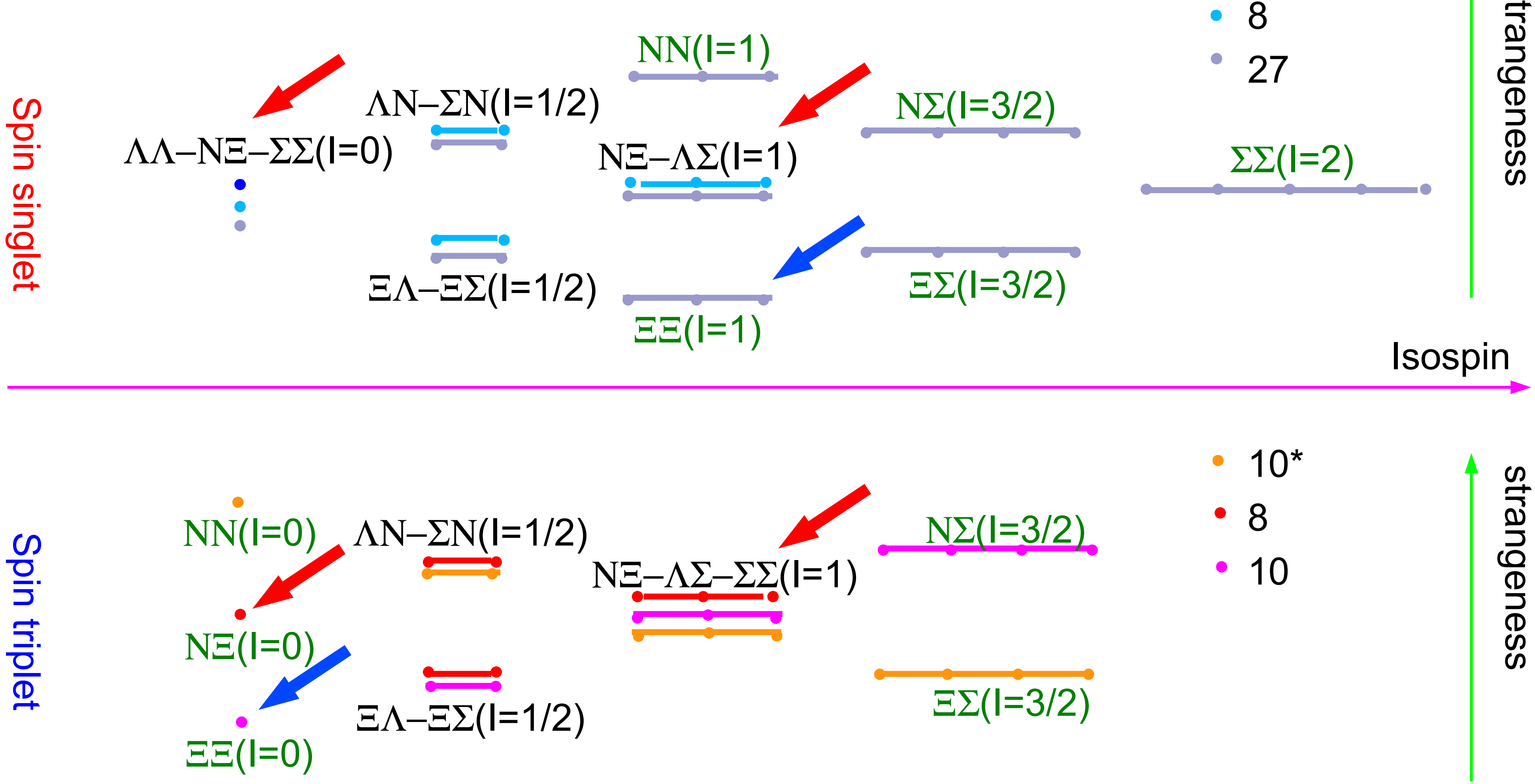
$$R_I^{B_1 B_2}(t,\vec{r})=\sum_{\vec{x}} \langle 0 | B_1(t,\vec{x}+\vec{r}) B_2(t,\vec{x}) \bar{I}(0) | 0 \rangle e^{(m_1+m_2)t}$$

$$\begin{pmatrix} V^{\alpha}{}_{\alpha}(\vec{r}) & V^{\alpha}{}_{\beta}(\vec{r})x \\ V^{\beta}{}_{\alpha}(\vec{r})x^{-1} & V^{\beta}{}_{\beta}(\vec{r}) \end{pmatrix} = \begin{pmatrix} \left(\frac{\nabla^2}{2\mu_{\alpha}}-\frac{\partial}{\partial t}\right)R_{I1}^{\alpha}(\vec{r},t) & \left(\frac{\nabla^2}{2\mu_{\beta}}-\frac{\partial}{\partial t}\right)R_{I2}^{\beta}(\vec{r},t) \\ \left(\frac{\nabla^2}{2\mu_{\alpha}}-\frac{\partial}{\partial t}\right)R_{I1}^{\alpha}(\vec{r},t) & \left(\frac{\nabla^2}{2\mu_{\beta}}-\frac{\partial}{\partial t}\right)R_{I2}^{\beta}(\vec{r},t) \end{pmatrix} \begin{pmatrix} R_{I1}^{\alpha}(\vec{r},t) & R_{I1}^{\beta}(\vec{r},t) \\ R_{I2}^{\alpha}(\vec{r},t) & R_{I2}^{\beta}(\vec{r},t) \end{pmatrix}^{-1}$$

$$x=\frac{\exp(-(m_{\alpha_1}+m_{\alpha_2})t)}{\exp(-(m_{\beta_1}+m_{\beta_2})t)}$$

Target of this work

Three flavor (u,d,s) world : broken SU(3) symmetry

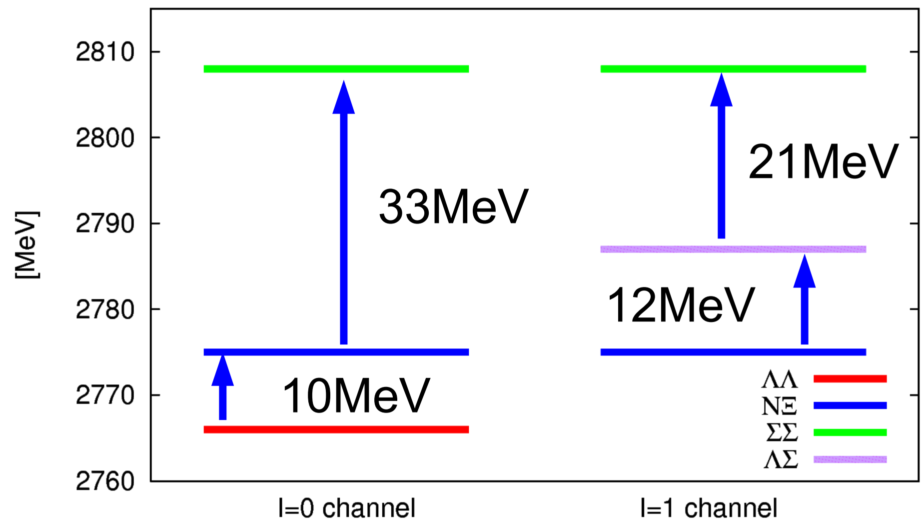


- In order to extract NΞ interaction, we have to solve a coupled channel.
- Only NΞ (I=0) is single channel.

Numerical setup

- 2+1 flavor gauge configurations by PACS-CS collaboration.
  - Iwasaki gauge action & O(a) improved Wilson quark action
  - *a* = 0.08995(40) [*fm*], a^{-1} = 2.194(10) GeV, determined with m_Ω
  - *32^3x48 lattice, L* = 2.8784 [*fm*], T = 4.3176 [*fm*].
  - Kud = 0.1373316, Ks = 0.1367526
  - 402 confs x 24 sources.
- Flat wall source is considered to produce S-wave B-B state.
- BG/Q computer resources had been used.

	Mass [MeV]
π	509±1
K	612±1
m_π/m_K	0.83
N	1319±3
Λ	1383±2
Σ	1404±3
Ξ	1456±2



NΞ potential

Lists of channels

I=0 states

Spin	BB channels			SU(3) representation		
$^1S_0$	ΛΛ	**NΞ**	ΣΣ	1	8s	27
$^3S_1$	--	**NΞ**	--	8a	--	--

I=1 states

Spin	BB channels			SU(3) representation		
$^1S_0$	**NΞ**	--	ΛΣ	--	8s	27
$^3S_1$	**NΞ**	ΣΣ	ΛΣ	8a	10	10*

I=2 states

Spin	BB channels			SU(3) representation		
$^1S_0$	ΣΣ			--	--	27
~~$^3S_1$~~						

Strong attraction (H-dibaryon)

Aattraction

Strong repulsion

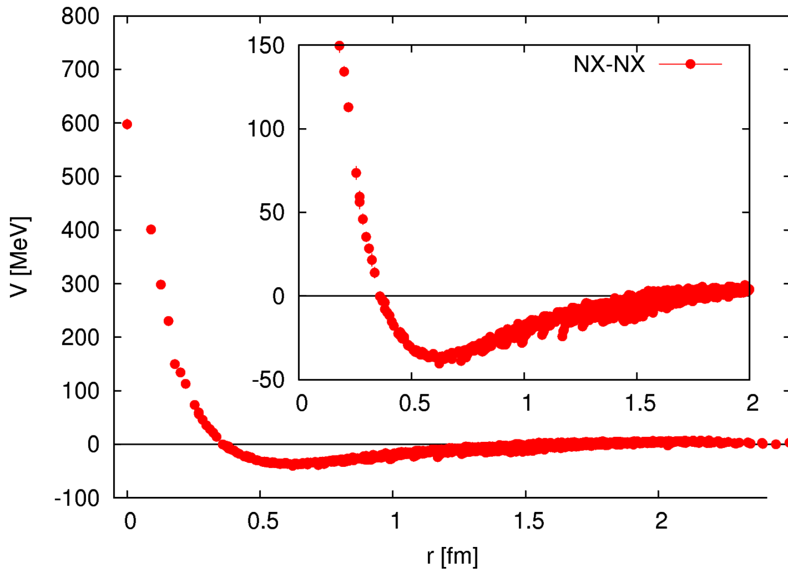
Similar to The NN potential

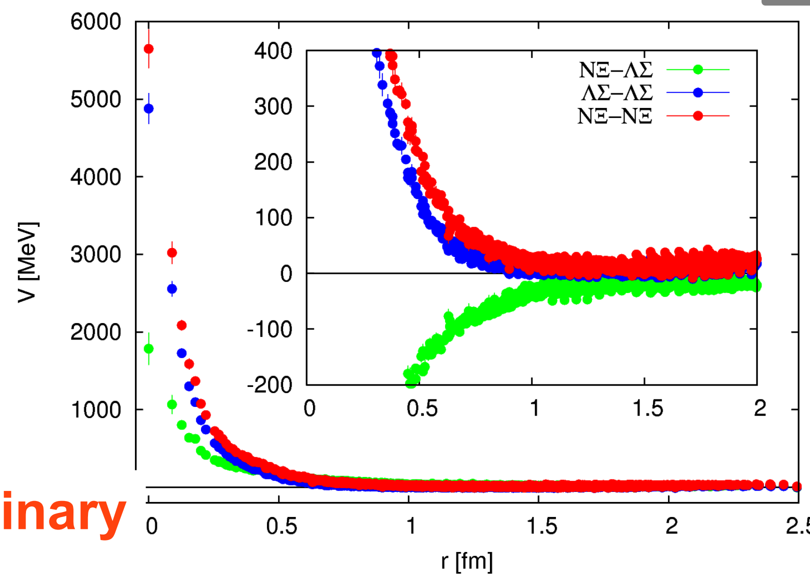
Repulsion

$N\Xi$ (I=0) $^3S_1$ channel

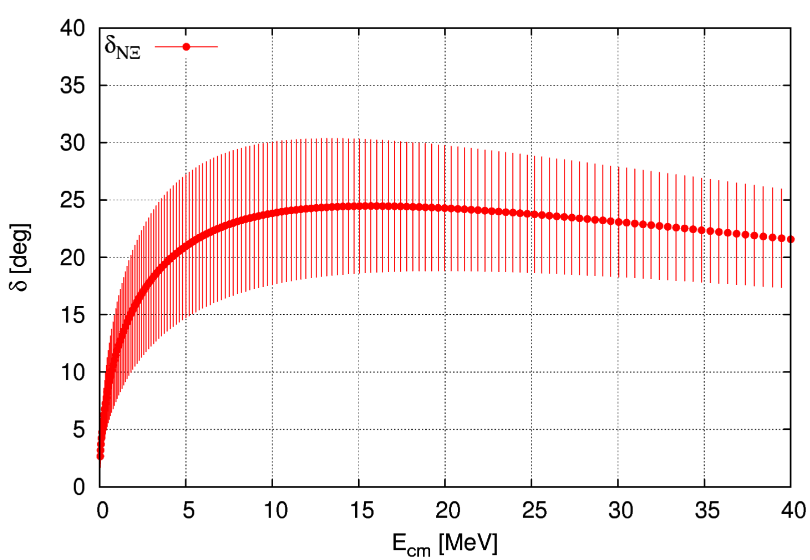
$N\Xi$, $\Lambda\Sigma$ (I=1) $^1S_0$ channel

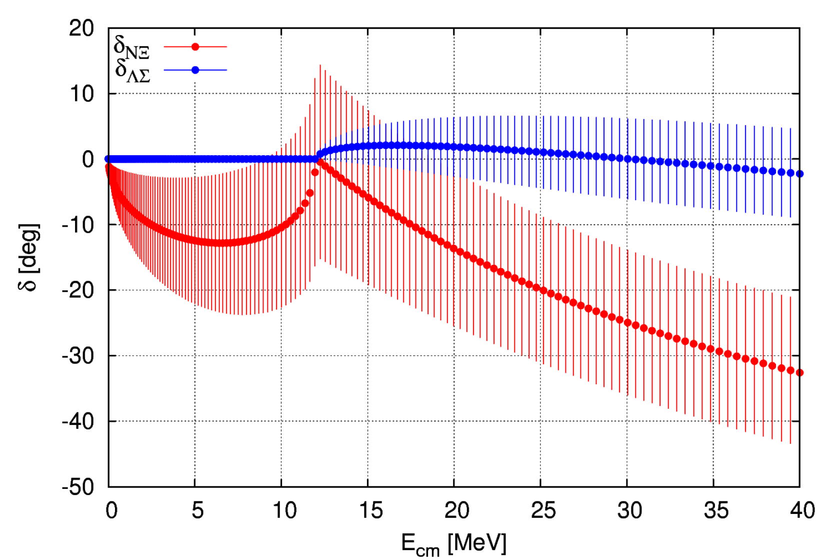
$m\pi$= 510 MeV





preliminary

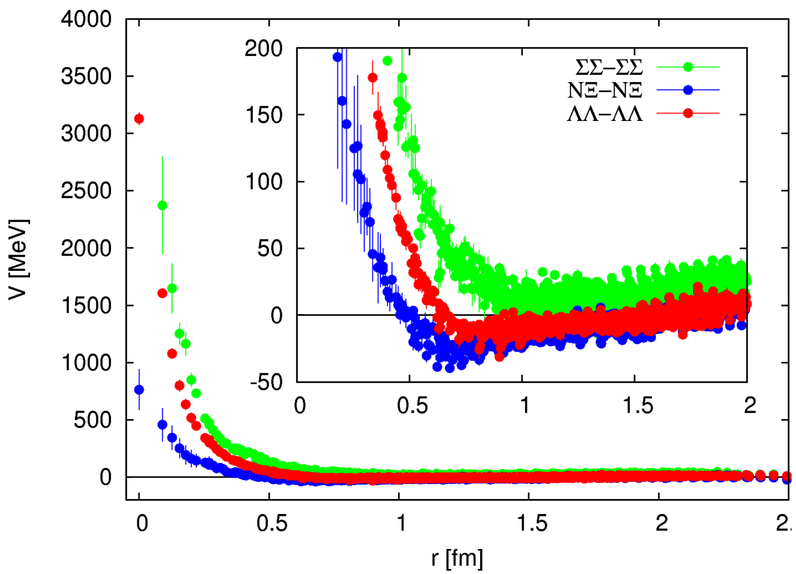




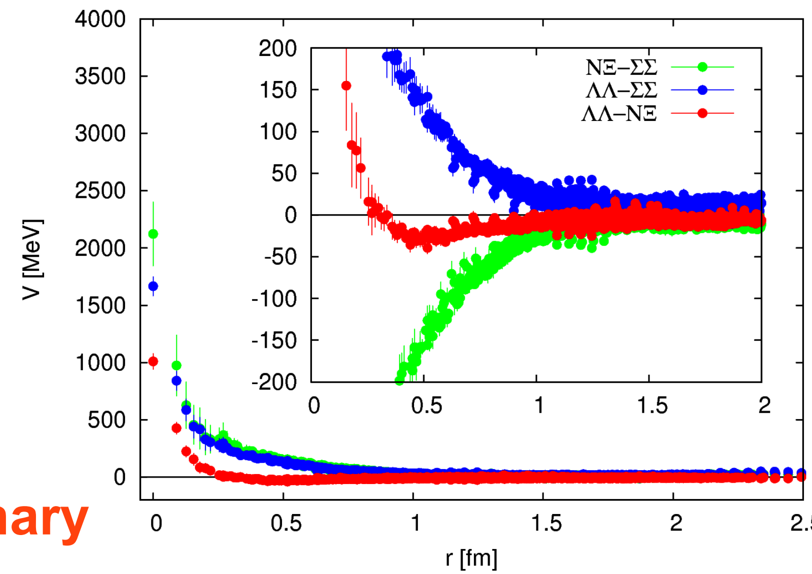
ΛΛ, NΞ, ΣΣ (I=0) $^1S_0$ channel

mπ= 510 MeV

Diagonal elements

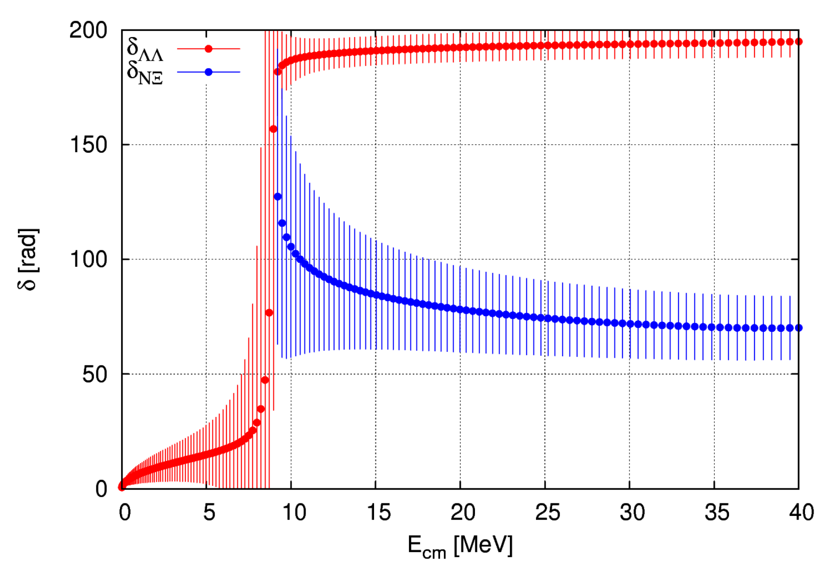


Off-diagonal elements



preliminary

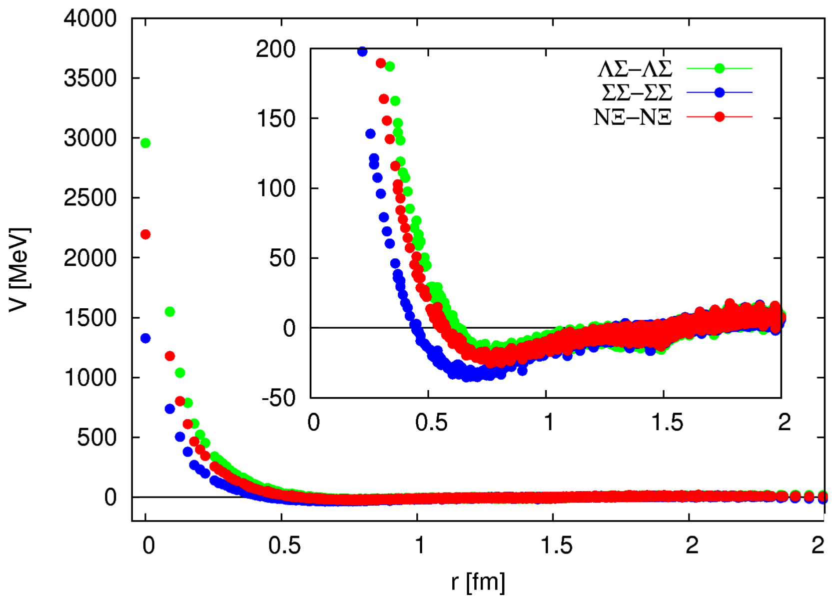
- All diagonal element have a repulsive core.
  ΣΣ–ΣΣ potential is strongly repulsive.
- Off-diagonal potentials are relatively strong.
  except for ΛΛ–NΞ transition
- Existence of H-dibaryon can be seen
  in ΛΛ phase-shift as a sharp resonance.



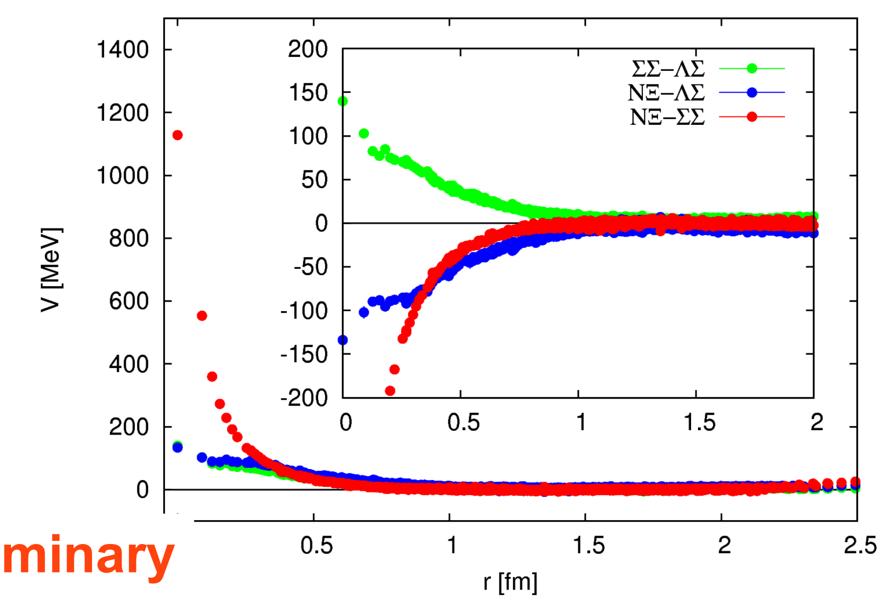
NΞ, ΛΣ, ΣΣ (I=1) $^3S_1$ channel

mπ= 510 MeV

Diagonal elements

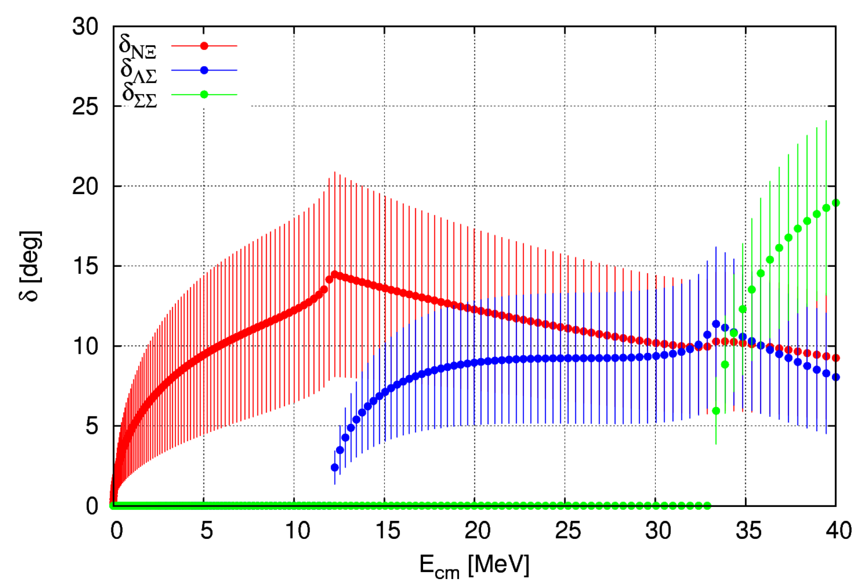


Off-diagonal elements



preliminary

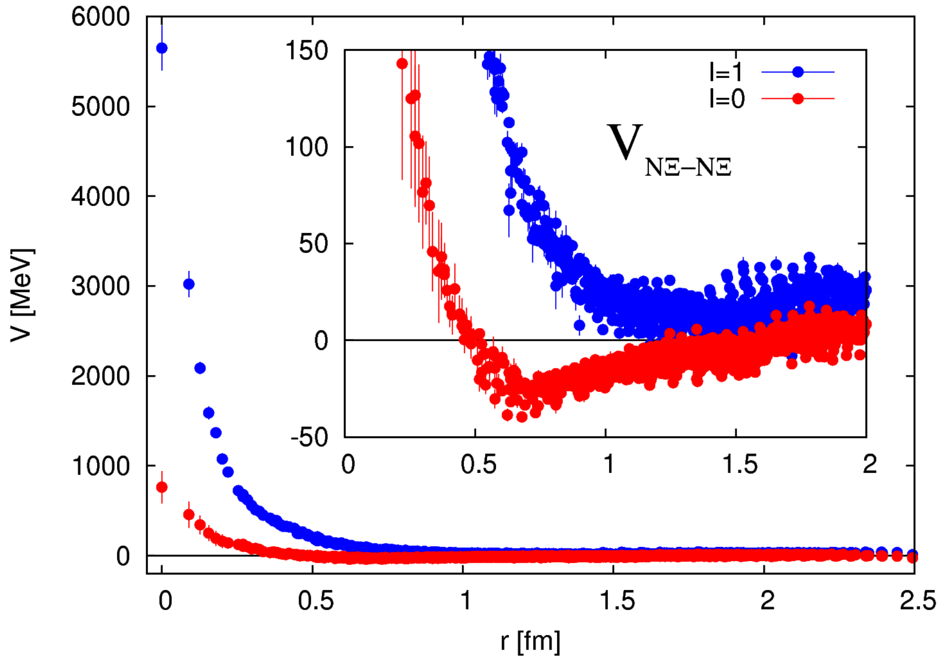
- All diagonal element have a repulsive core and attractive pocket.
- Off-diagonal potentials are are weak except for NΞ–ΣΣ transition.



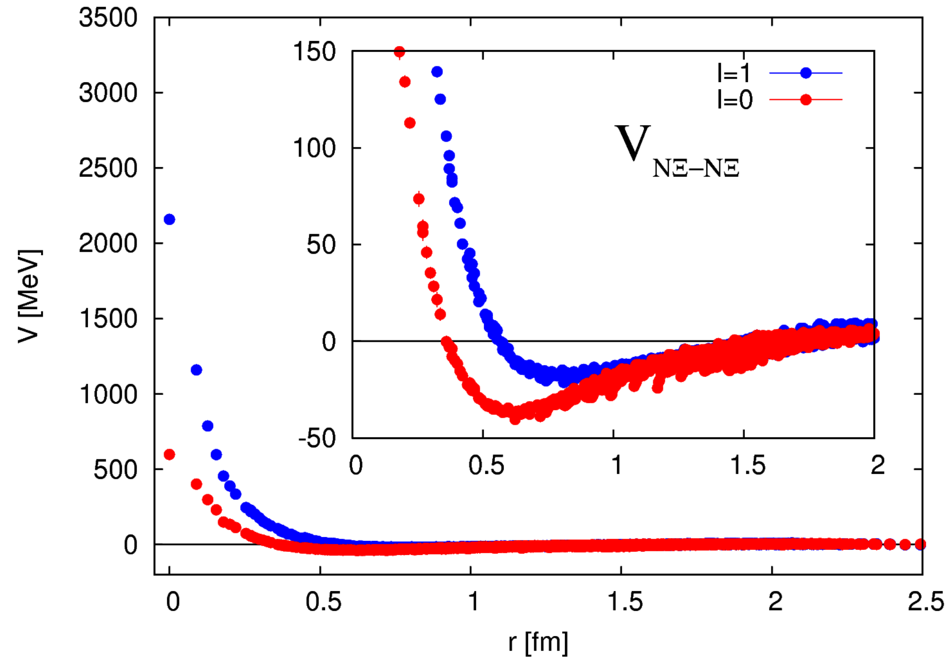
NΞ channel

$m\pi$= 510 MeV

$^1S_0$



$^3S_1$

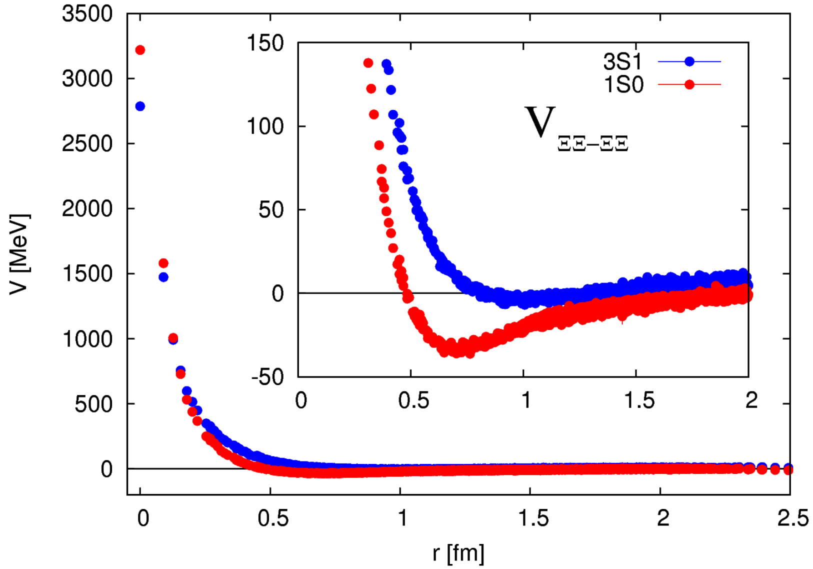


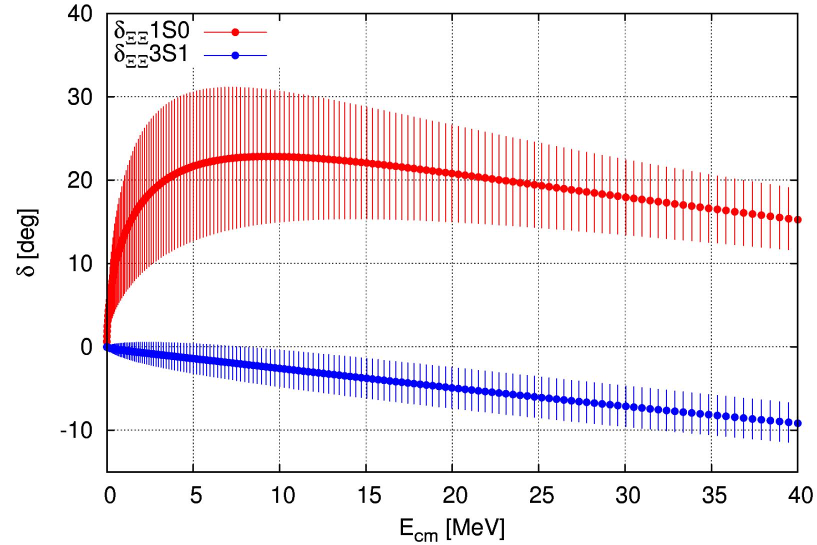
- Potentials in spin-triplet channel have an attractive pocket.
- Repulsive cores in iso-singlet channel are not so high.
- Strongly repulsive potential is found in 1S0(I=1).
- Relatively small ΛΛ–NΞ transition potential would be help to stabilize a Ξ-hypernucleus

$\Xi\Xi$ *potential*

ΞΞ channel

m_π= 510 MeV





Iso-singlet channel

Potential of 10 irrep is expected to be repulsive due to the quark Pauli effect.

Iso-triplet channel

- EFT calculation found that the bound ΞΞ state in 1S0 channel [1].
- Meson exchange model calculations: bound.[2] or unbound [3]
- Bound ΞΞ state was found by Lattice QCD simulation at m=389MeV [4]

1). J. Haidenbauer Nucl.Phys.A881(2012)44
2). M. Yamaguchi PTP105(2001)627
3). Y. Fujiwara PPNP 58(2007)439
4). S.R. Beane PRD85(2012)054511

Search for the bound ΞΞ state at the physical point is interesting!

Summary and outlook

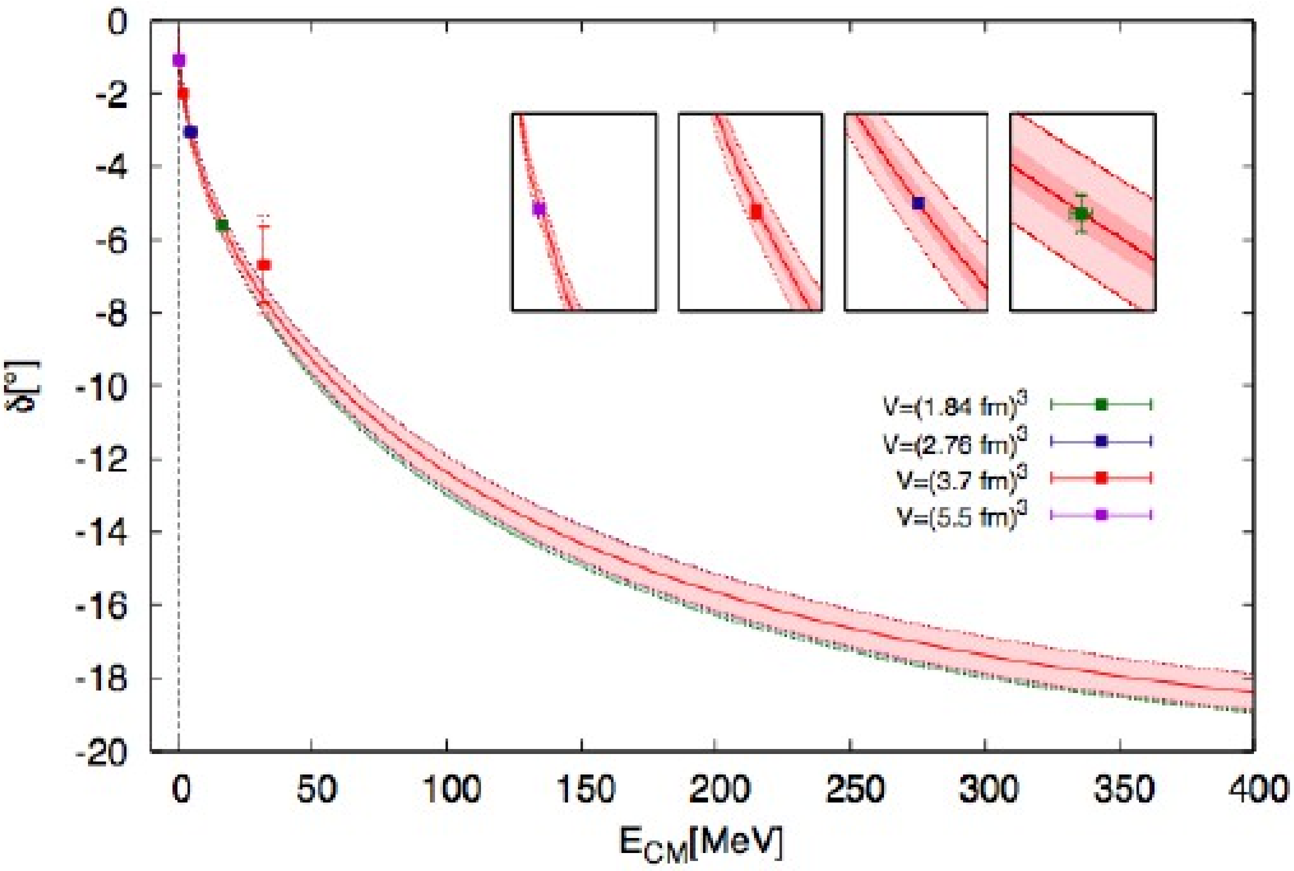
- We have investigated $N\Xi$ and $\Xi\Xi$ interactions from lattice QCD.
- In order to deal with a variety of interactions, we extend our method to the coupled channel formalism.
- $N\Xi$ and $\Xi\Xi$ potentials are derived from NBS wave functions calculated with PACS-CS configurations
- These interactions (potentials) will be used to explore the Ξ-hypernucleus and $\Xi\Xi$ hypernucleus.
- We need a well-established technique to tackle for study of hypernucler system.
- We want to perform this simulation at physical situation m_π/m_K=0.28.

Backup slides

Nuclear Force from Lattice QCD

Comparison between the potential method and Lueshcer’s method.

ππ scattering in I = 2 channel



Ns=16,24,32,48, Nt=128, a=0.115

m_{pi} = 940 MeV by Quenched QCD

Kurth et al., arXiv:1305.4462[hep-lat]

Effective Schrodinger equation with E-independent potential

$K(\vec{x};E) \equiv \left(\vec{\nabla}^2 + k^2\right)\psi(\vec{x};E)$ **[START] local but E-dep pot. ($L^3 x L^3$ dof)**

(1) We assume ψ(x; E) for different E is linearly independent with each other.

(2) ψ(x; E) has a "left inverse" as an integration operator as

$E \equiv 2\sqrt{m_N^2 + k^2}$

$$\int d^3x \tilde{\psi}(\vec{x};E')\psi(\vec{x};E) = 2\pi\delta(E-E')$$

(3) K(x; E) can be factorized as

$$\begin{aligned} K(\vec{x};E) &= \int \frac{dE'}{2\pi} K(\vec{x};E') \times \int d^3y \tilde{\psi}(\vec{y};E')\psi(\vec{y};E) \\ &= \int d^3y \left\{ \sum_\alpha \int \frac{dE'}{2\pi} K(\vec{x};E')\tilde{\psi}(\vec{y};E') \right\} \psi(\vec{y};E) \end{aligned}$$

$\equiv m_N U(\vec{x},\vec{y})$

(4) We are left with an effective Schrodinger equation with an E−independent potential U.

$$\left(\vec{\nabla}^2 + k^2\right)\psi(\vec{x};E) = m_N \int d^3y\, U(\vec{x},\vec{y})\psi(\vec{y};E)$$

Intuitive understanding

[GOAL] non-local but E-indep pot. ($L^3 x L^3$ dof)

Asymptotic form of BS wave function

[C.-J.D.Lin et al., NPB619,467(2001)]

For simplicity, we consider BS wave function of two pions

$$\psi_{\bar{q}}(\vec{x}) \equiv \langle 0 | N(\vec{x})\, N(\vec{0}) | N(\vec{q}) N(-\vec{q}), in \rangle$$

complete set

$$1 = \int \frac{d^3 p}{(2\pi)^3 2E_N(\vec{p})} |N(\vec{p})\rangle\langle N(\vec{p})| + \cdots$$

$$= \int \frac{d^3 p}{(2\pi)^3 2E_N(\vec{p})} \langle 0 | N(\vec{x}) | N(\vec{p}) \rangle \langle N(\vec{p}) | N(\vec{0}) | N(\vec{q}) N(-\vec{q}), in \rangle + I(\vec{x})$$

$$Z^{1/2} e^{i\vec{p}\cdot\vec{x}}$$

$$disc. + Z^{1/2} \frac{T(\vec{p};\vec{q})}{m_N^2 - \left(2E_N(\vec{q}) - E_N(\vec{p})\right)^2 + \vec{p}^2 - i\varepsilon}$$

$$= Z\left(e^{i\vec{q}\cdot\vec{x}} + \frac{1}{(2\pi)^3} \int \frac{d^3 p}{2E_N(\vec{p})} \frac{T(\vec{p};\vec{q})}{4E_N(\vec{q}) \cdot (E_N(\vec{p}) - E_N(\vec{q}) - i\varepsilon)} e^{i\vec{p}\cdot\vec{x}} \right)$$

Integral is dominated by the on-shell contribution $E_N(\vec{p}) \approx E_N(\vec{q})$

➔ T-matrix becomes the on-shell T-matrix

$$T^{(s-wave)}(s) = \frac{E(\vec{q})}{2|\vec{q}|}(-i)\left(e^{2i\delta_0(s)} - 1\right)$$

$$= Z\left(e^{i\vec{q}\cdot\vec{x}} + \frac{1}{2i}\left(e^{2i\delta_0(s)} - 1\right)\frac{e^{iqr}}{qr} \right) + \cdots$$

The asymptotic form

$$\psi_{\vec{q}}(\vec{x}) = Z e^{i\delta_0(s)} \frac{\sin(qr + \delta_0(s))}{qr} + \cdots \quad \text{(s-wave)}$$

This is analogous to a non-rela. wave function

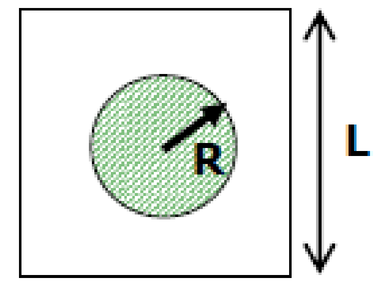
Energy indep. potential in coupled channel S.E.

Inside the interaction range, we can define the interaction kernel

$$\begin{pmatrix} p^2+\nabla & q^2+\nabla \\ p^2+\nabla & q^2+\nabla \end{pmatrix}\begin{pmatrix} \psi_a^a(\vec{x},E) & \psi_a^b(\vec{x},E) \\ \psi_a^b(\vec{x},E) & \psi_a^b(\vec{x},E) \end{pmatrix} = \begin{pmatrix} K_a^a(\vec{x},E) & K_b^a(\vec{x},E) \\ K_a^b(\vec{x},E) & K_b^b(\vec{x},E) \end{pmatrix} \xrightarrow{|x|>R} 0$$

Factorization of the interaction kernel

$$\begin{pmatrix} K_a^a(\vec{x},E) & K_b^a(\vec{x},E) \\ K_a^b(\vec{x},E) & K_b^b(\vec{x},E) \end{pmatrix} = \int dy \begin{pmatrix} U_a^a(\vec{x},\vec{y}) & U_b^a(\vec{x},\vec{y}) \\ U_a^b(\vec{x},\vec{y}) & U_b^b(\vec{x},\vec{y}) \end{pmatrix}\begin{pmatrix} \psi_a^a(\vec{y},E) & \psi_b^a(\vec{y},E) \\ \psi_a^b(\vec{y},E) & \psi_b^b(\vec{y},E) \end{pmatrix}$$



$$\int dx \begin{pmatrix} \tilde{\psi}_a^a(\vec{x},E') & \tilde{\psi}_b^a(\vec{x},E') \\ \tilde{\psi}_a^b(\vec{x},E') & \tilde{\psi}_b^b(\vec{x},E') \end{pmatrix}\begin{pmatrix} \psi_a^a(\vec{x},E) & \psi_b^a(\vec{x},E) \\ \psi_a^b(\vec{x},E) & \psi_b^b(\vec{x},E) \end{pmatrix} = 2\pi\delta(E-E')$$

$$\begin{pmatrix} U_a^a(\vec{x},\vec{y}) & U_b^a(\vec{x},\vec{y}) \\ U_a^b(\vec{x},\vec{y}) & U_b^b(\vec{x},\vec{y}) \end{pmatrix} = \int \frac{dE}{2\pi}\begin{pmatrix} K_a^a(\vec{x},E) & K_b^a(\vec{x},E) \\ K_a^b(\vec{x},E) & K_b^b(\vec{x},E) \end{pmatrix}\begin{pmatrix} \tilde{\psi}_a^a(\vec{y},E) & \tilde{\psi}_b^a(\vec{y},E) \\ \tilde{\psi}_a^b(\vec{y},E) & \tilde{\psi}_b^b(\vec{y},E) \end{pmatrix}$$

Energy independent potential in Schrödinger equation.

Interactions in SU(3) limit and dibaryon

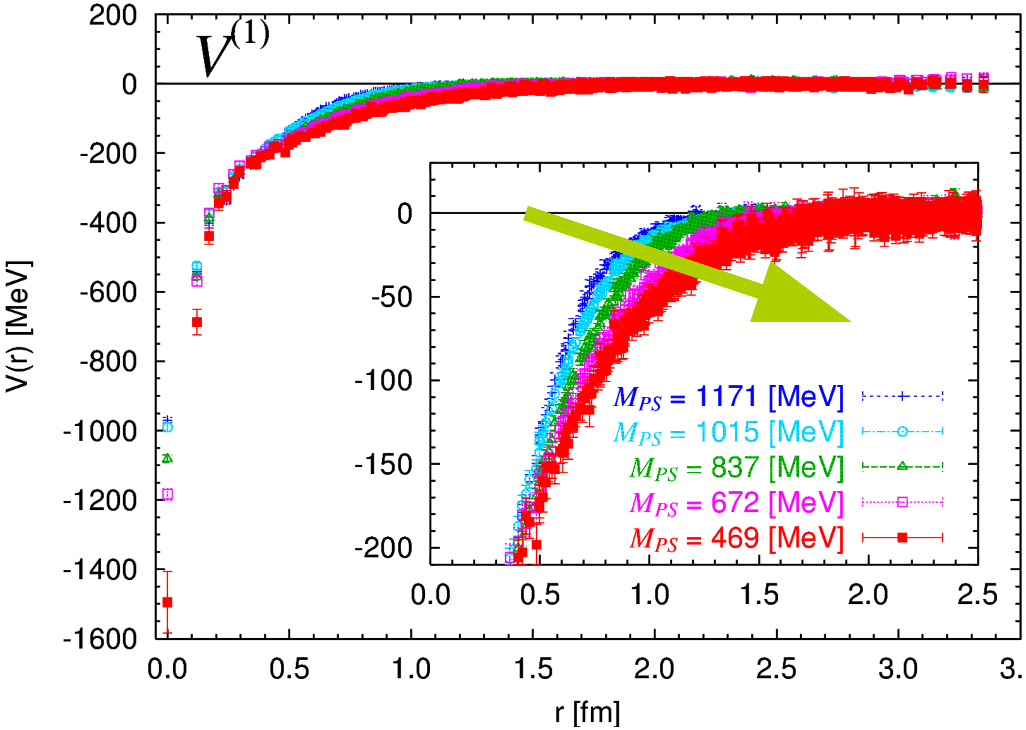
Setup for SU(3) limit situation

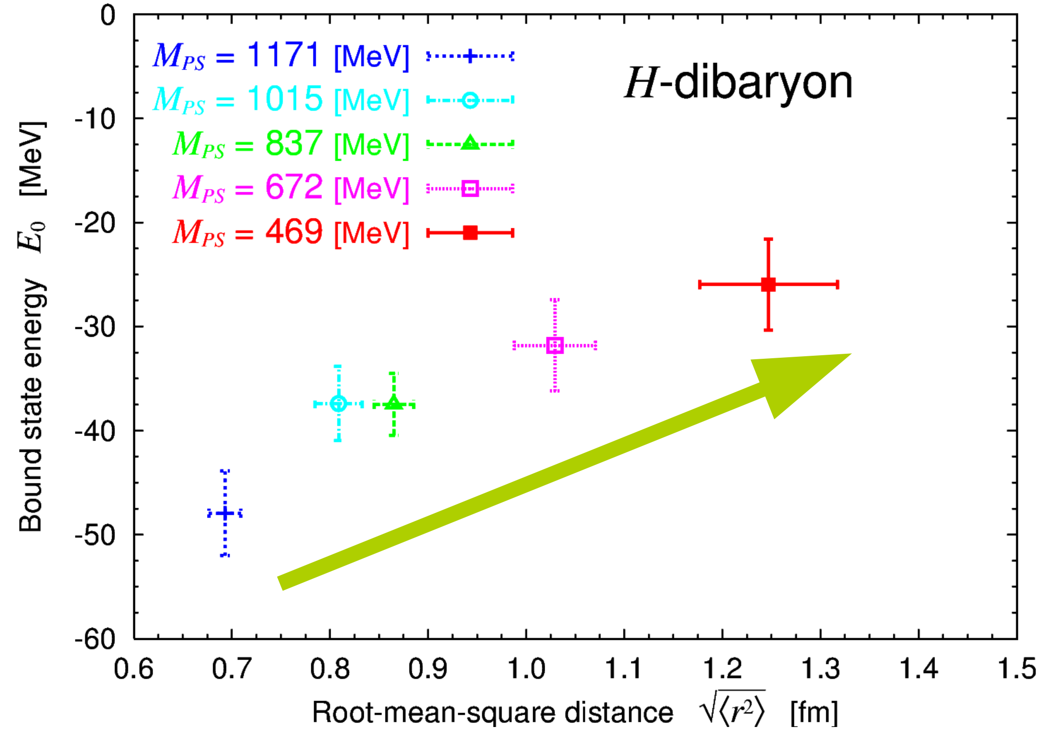
- 3 flavor gauge configurations generated by DDHMC/PHMC code.
  - We appreciate PACS-CS collab for giving us their code sets.
  - RG improved gauge action & O(a) improved clover quark action
  - $\beta = 1.83$, $a^{-1} = 1.631$ [GeV], 32^3x32 lattice, $L = 3.872$ [fm].
  - Five values of κ_{uds} are considered.
- Flat wall source is considered to produce S-wave B-B state.
- Numerical simulations are curried out at T2K-Tsukuba.

κ_{uds}	# of confs	M_ps [MeV]	M_b [MeV]
0.13660	420	1170.9(7)	2274(2)
0.13710	360	1015.2(6)	2031(2)
0.13760	480	836.8(5)	1749(1)
0.13800	360	672.3(6)	1484(2)
0.13840	720	468.6(7)	1161(2)

T.Inoue (HAL QCD collab.) PRL106 (2011) 162002

Looking for H-dibaryon in SU(3) limit

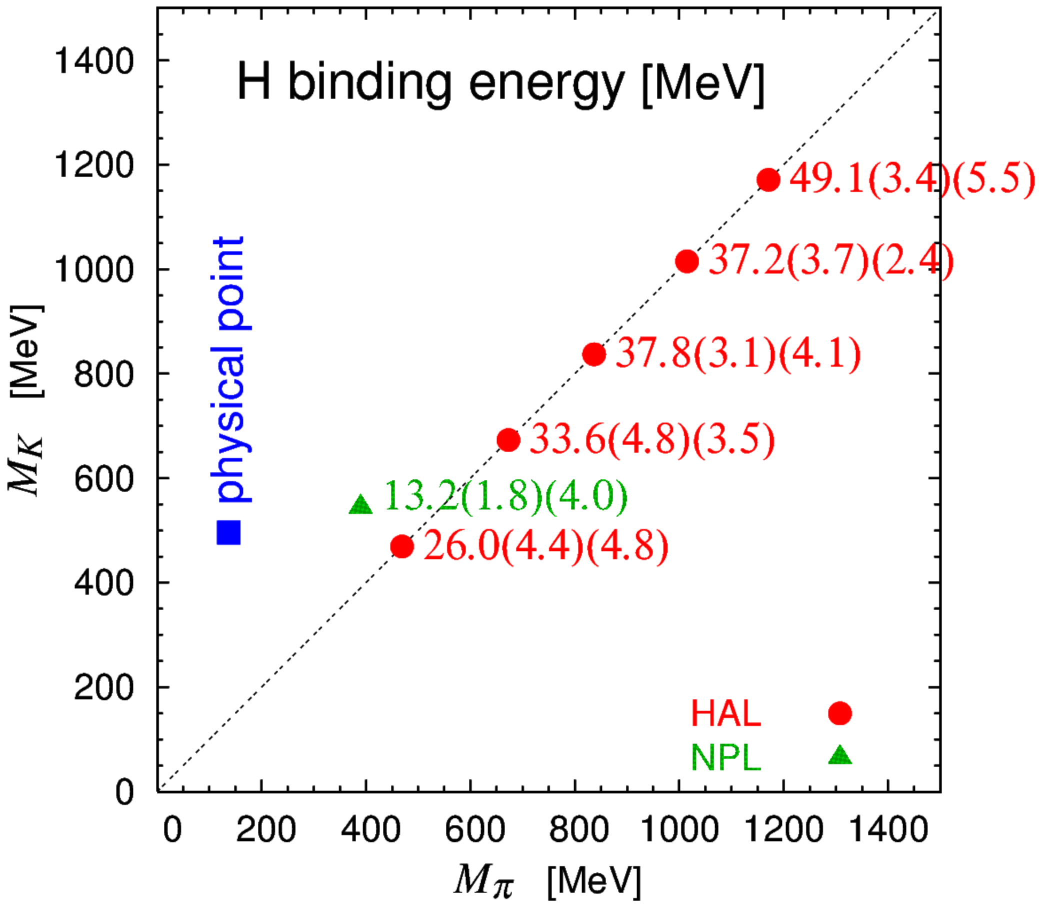




Growth of kinetic energy of baryon pair could be quicker than enhancement of attraction.

- Potential in flavor singlet channel is getting more attractive as decreasing quark masses
- Ground state energies in all different quark masses are below the free BB threshold.
- There is a 6q bound state in this mass range with SU(3) symmetry.

Recent results for H-particle in Lattice QCD



- Summary of binding energies of H-dibaryon from recent LQCD calc.
- S. R. Beane et al [NPLQCD colla.] Phys. Rev. Lett. 106, 162001 (2011), arXive: 1109.2889[hep-lat].
- The results of ours and NPLQCD look consistent
- Note that all results are still far away from physical point.
- No deeply bound H-dibaryon from experiment

What happen at physical point?

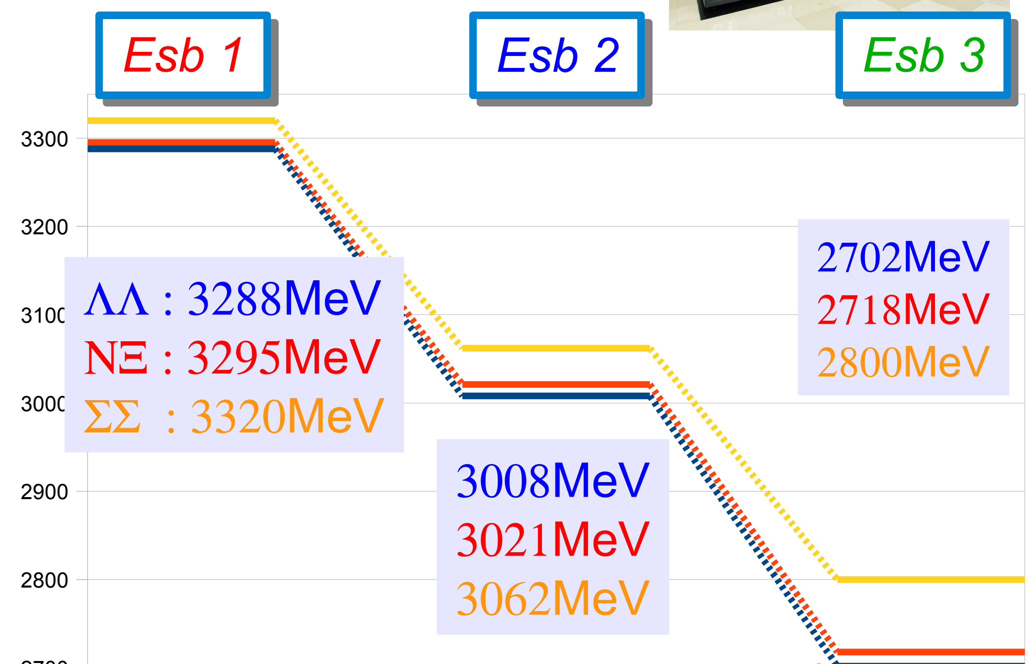
Simulation at SU(3) broken point by our method is necessary.
We have to depart from the SU(3) line.

Numerical setup

- **2+1 flavor** gauge configurations by PACS-CS collaboration.
  - RG improved gauge action & O(a) improved Wilson quark action
  - β = 1.90, a^{-1} = 2.176 [*GeV*], 32^3x64 lattice, L = 2.902 [*fm*].
  - κ_s = 0.13640 is fixed, κ_{ud} = 0.13700, 0.13727 and 0.13754 are chosen.
- **Flat wall source** is considered to produce S-wave B-B state.
- The KEK computer system A resources are used.

In unit of MeV	*Esb 1*	*Esb 2*	*Esb 3*
π	701±1	570±2	411±2
K	789±1	713±2	635±2
m_π/m_K	0.89	0.80	0.65
N	1585±5	1411±12	1215±12
Λ	1644±5	1504±10	1351± 8
Σ	1660±4	1531±11	1400±10
Ξ	1710±5	1610± 9	1503± 7

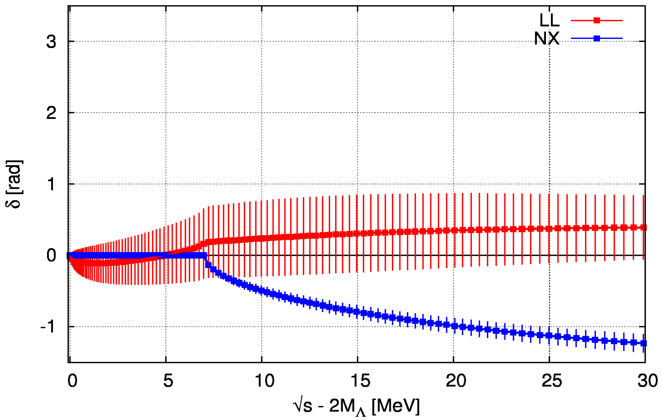
u,d quark masses lighter



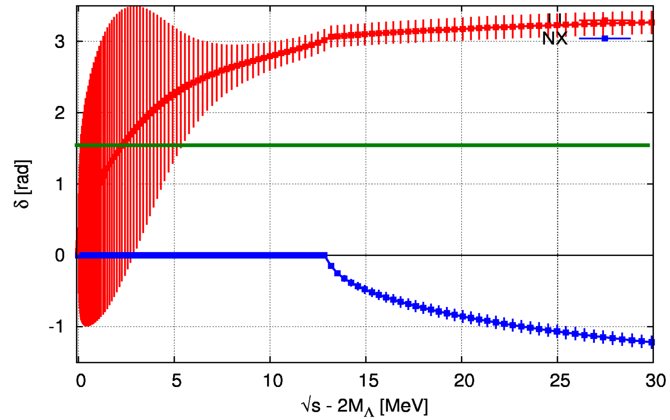
SU(3) breaking effects becomes larger

$\Lambda\Lambda$ and $N\Xi$ phase shifts

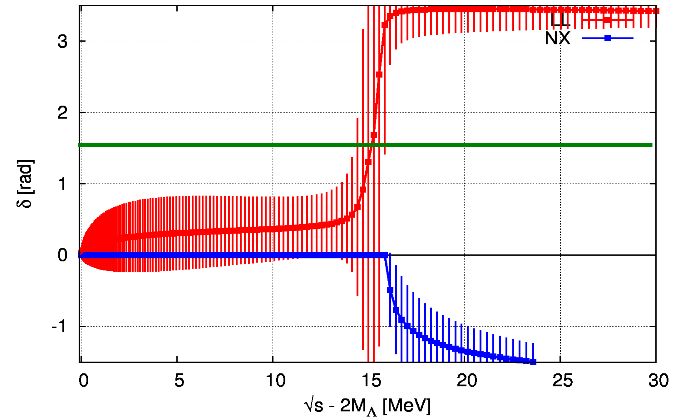
Esb1 : $m\pi$= 701 MeV



Esb2 : $m\pi$= 570 MeV



Esb3 : $m\pi$= 411 MeV



Preliminary!

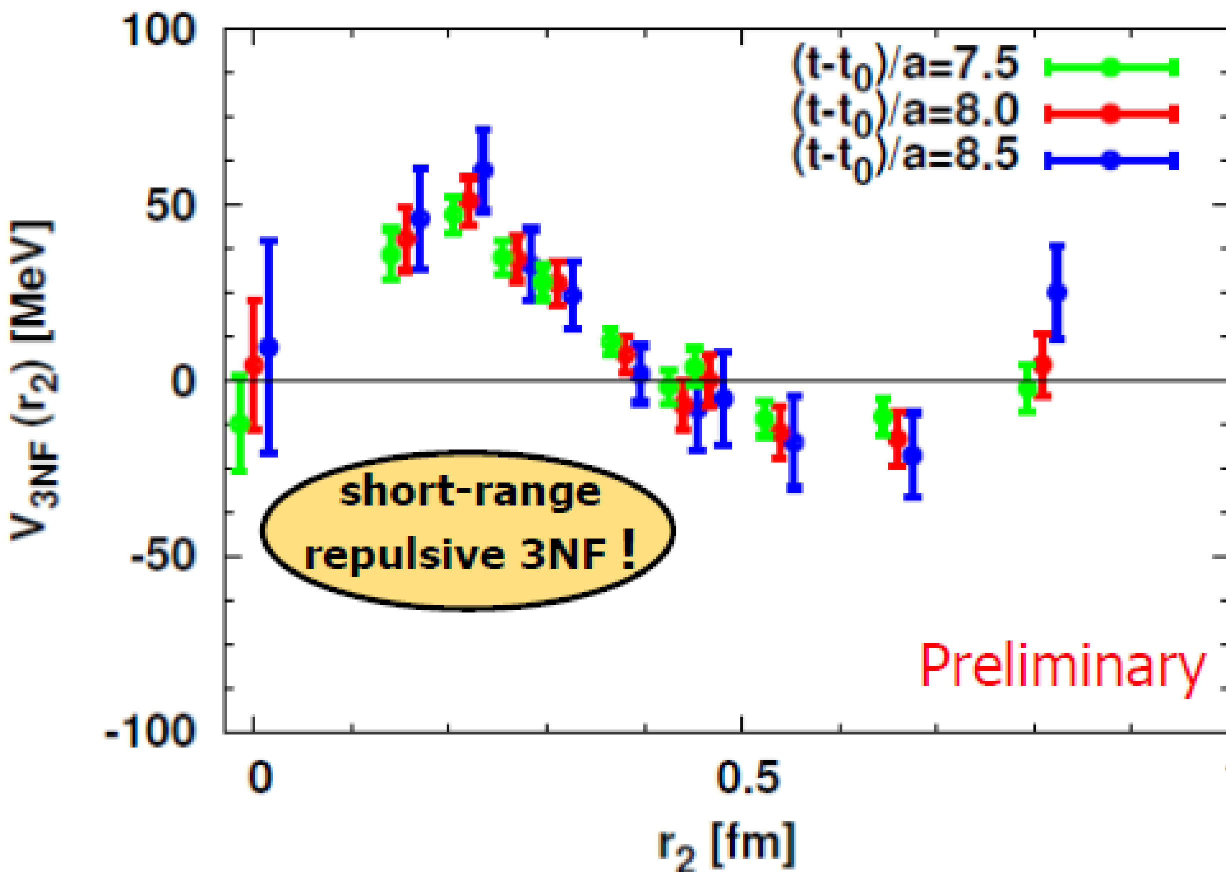
- Esb1:
  - Bound H-dibaryon
- Esb2:
  - H-dibaryon is near the $\Lambda\Lambda$ threshold
- Esb3:
  - The H-dibaryon resonance energy is close to $N\Xi$ threshold..

- We can see the clear resonance shape in $\Lambda\Lambda$ phase shifts for Esb2 and 3.
- The "binding energy" of H-dibaryon from $N\Xi$ threshold becomes smaller as decreasing of quark masses.

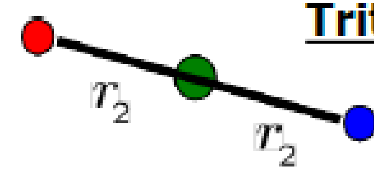
Three nucleon force on the tattice

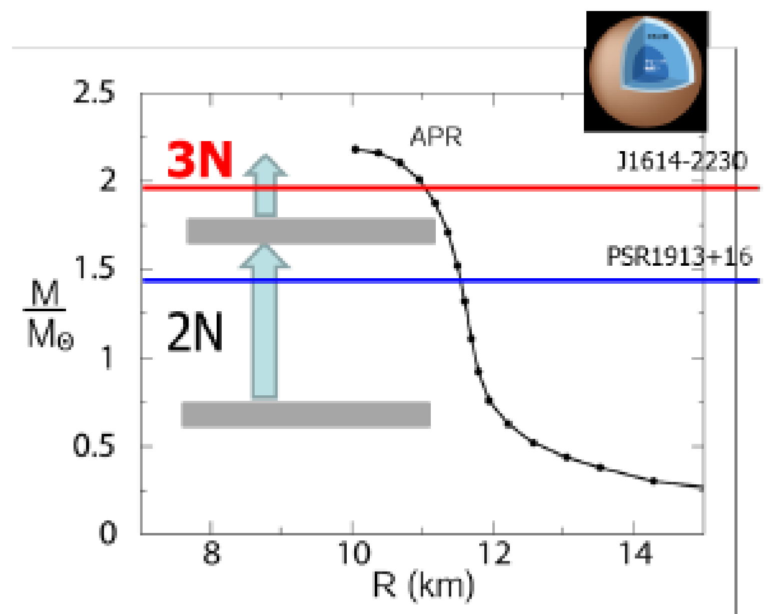
T.D. et al. (HAL QCD Coll.) PTP127(2012)723

+ t-dep method updates etc.



Triton channel



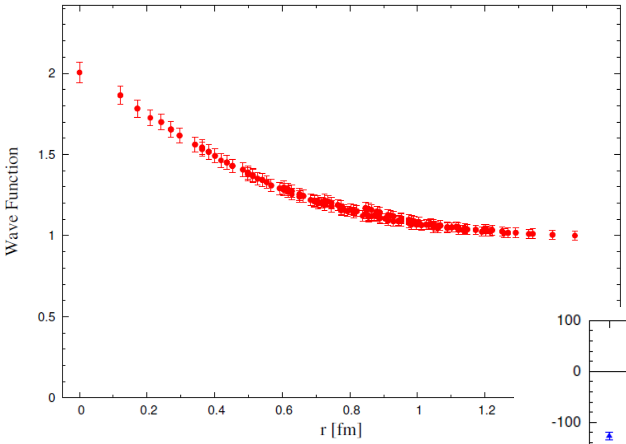


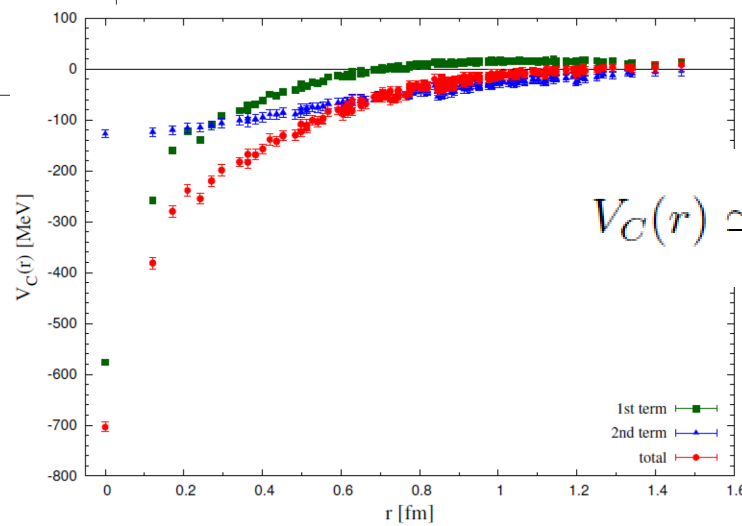
Nf=2 clover (CP-PACS), 1/a=1.27GeV,
L=2.5fm, m_π=1.1GeV, m_N=2.1GeV

Spin-2 NΩ Dibaryon from Lattice QCD

E. Faisal et al submitted to NPA

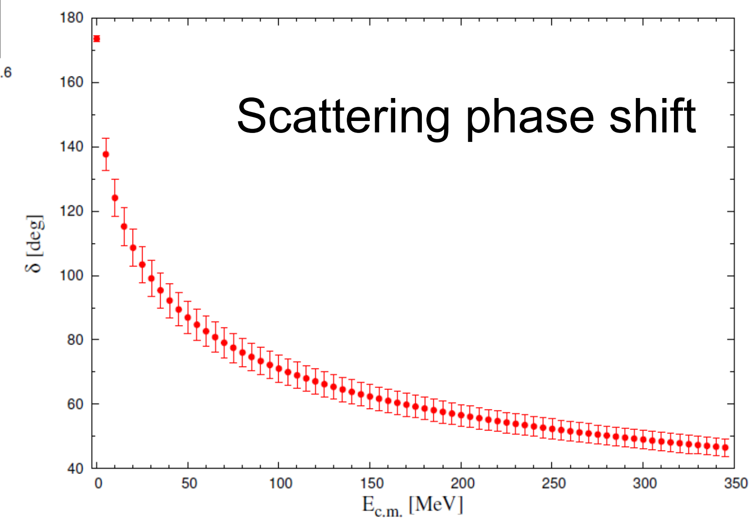
NBS wave function for spin-2 NΩ system





$$V_C(r) \simeq \frac{1}{2\mu}\nabla^2 R(r,t)/R(r,t) - \frac{\partial}{\partial t}\log R(r,t),$$

Strongly attractive potential



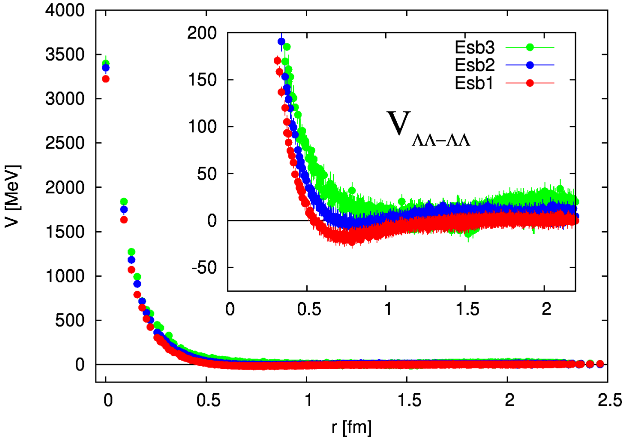
Scattering phase shift

$$B_{N\Omega} = 18.9(5.0)(^{+12.1}_{-1.8}) \text{ MeV},$$
$$a_{N\Omega} = -1.28(0.13)(^{+0.14}_{-0.15}) \text{ fm},$$
$$(r_e)_{N\Omega} = 0.499(0.026)(^{+0.029}_{-0.048}) \text{ fm}.$$

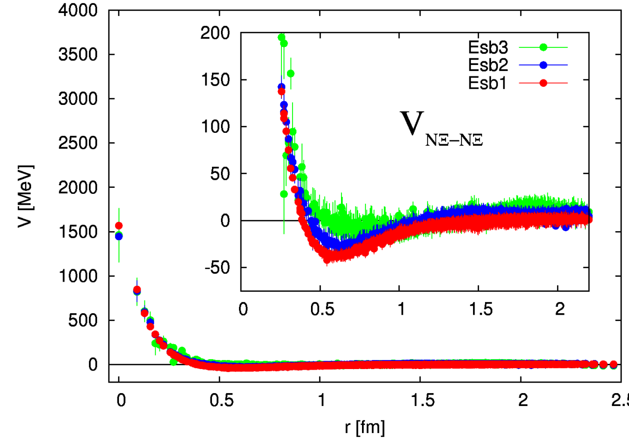
$\Lambda\Lambda$, $N\Xi$, $\Sigma\Sigma$ (I=0) $^1S_0$ channel

- **Esb1 : m_π= 701 MeV**
- **Esb2 : m_π= 570 MeV**
- **Esb3 : m_π= 411 MeV**

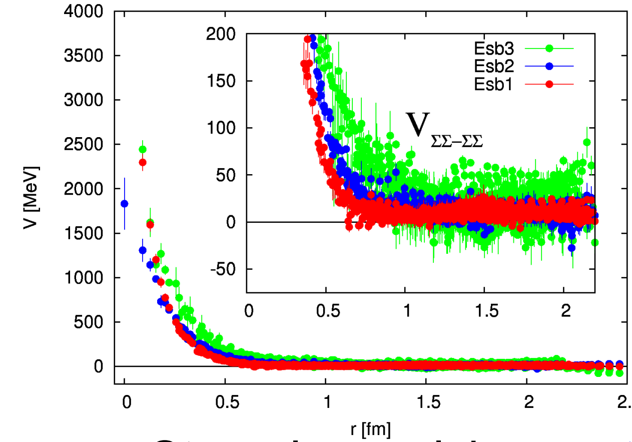
Diagonal elements



shallow attractive pocket



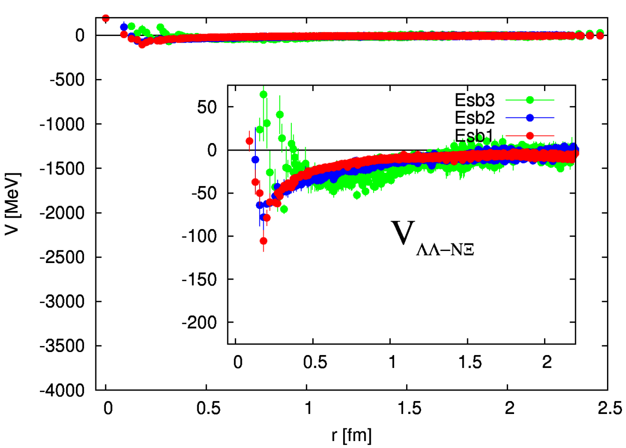
Deeper attractive pocket

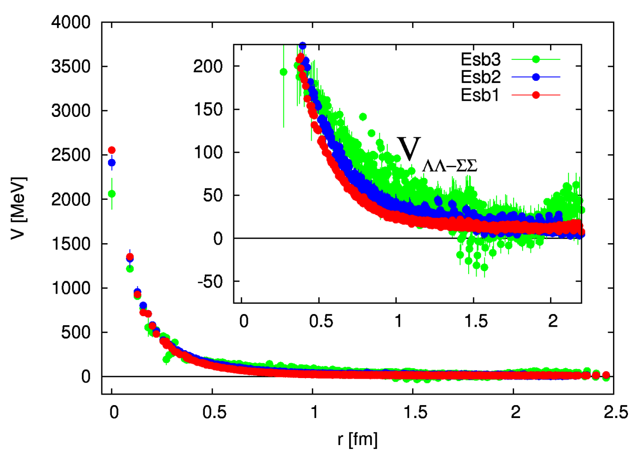


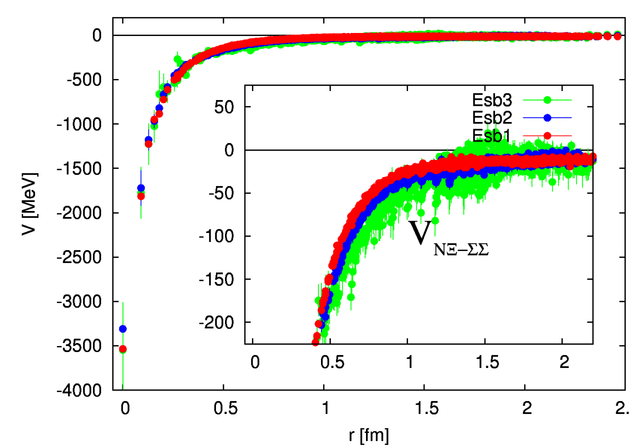
Strongly repulsive

All channels have repulsive core

Off-diagonal elements







In this channel, our group found the "H-dibaryon" in the SU(3) limit.

Comparison of potential matrices

Transformation of potentials
from the particle basis to the SU(3) irreducible representation (irrep) basis.

SU(3) Clebsh-Gordan coefficients

$$\begin{pmatrix} |1\rangle \\ |8\rangle \\ |27\rangle \end{pmatrix} = U \begin{pmatrix} |\Lambda\Lambda\rangle \\ |N\Xi\rangle \\ |\Sigma\Sigma\rangle \end{pmatrix}, \quad U \begin{pmatrix} V^{\Lambda\Lambda} & V^{\Lambda\Lambda}_{N\Xi} & V^{\Lambda\Lambda}_{\Sigma\Sigma} \\ V^{N\Xi}_{\Lambda\Lambda} & V^{N\Xi} & V^{N\Xi}_{\Sigma\Sigma} \\ V^{\Sigma\Sigma}_{\Lambda\Lambda} & V^{\Sigma\Sigma}_{N\Xi} & V^{\Sigma\Sigma} \end{pmatrix} U^t \to \begin{pmatrix} V_1 & & \\ & V_8 & \\ & & V_{27} \end{pmatrix}$$

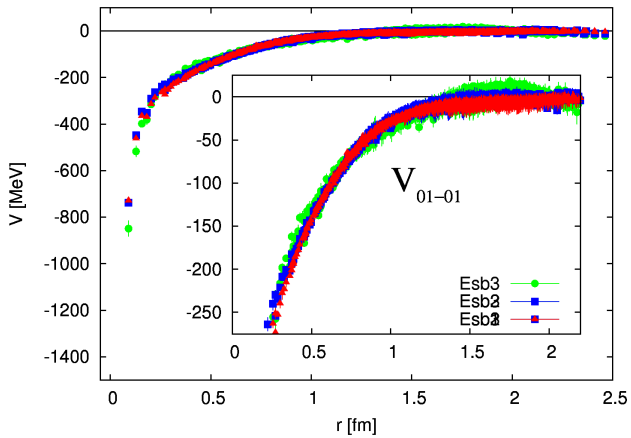
In the SU(3) irreducible representation basis,
the potential matrix should be diagonal in the SU(3) symmetric configuration.

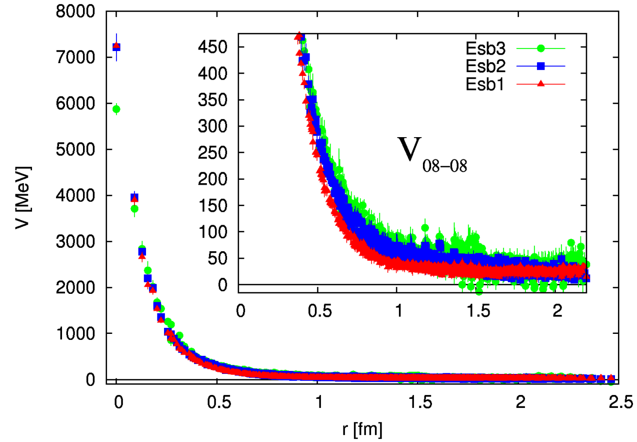
Off-diagonal part of the potential matrix in the SU(3) irrep basis
would be an effectual measure of the SU(3) breaking effect.

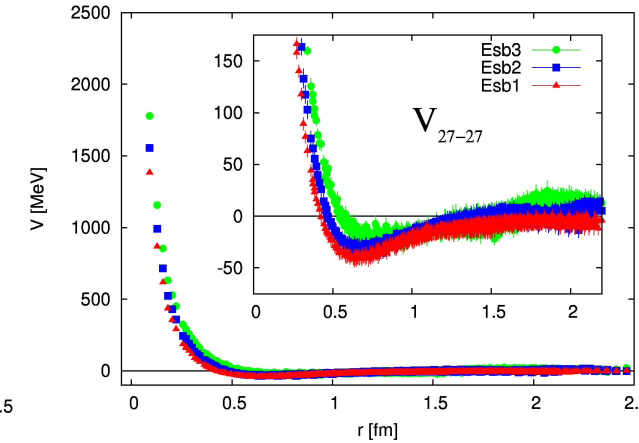
We will see how the SU(3) symmetry of potential will be broken
by changing the u,d quark masses lighter.

1, 8_s, 27 (I=0) $^1S_0$ channel

Esb1 : mπ= 701 MeV
Esb2 : mπ= 570 MeV
Esb3 : mπ= 411 MeV

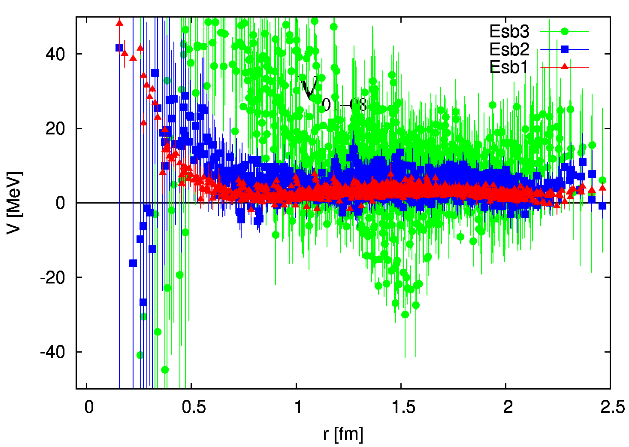


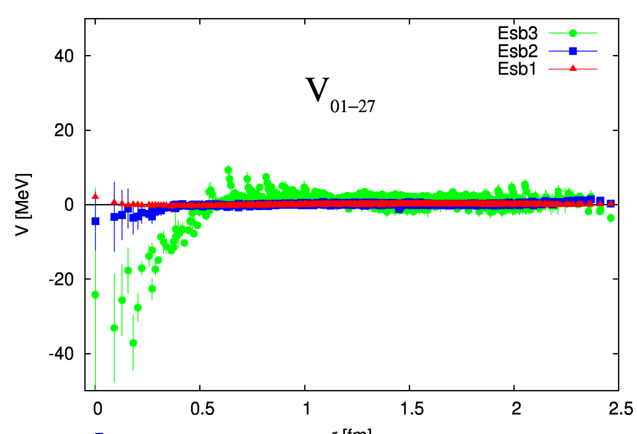


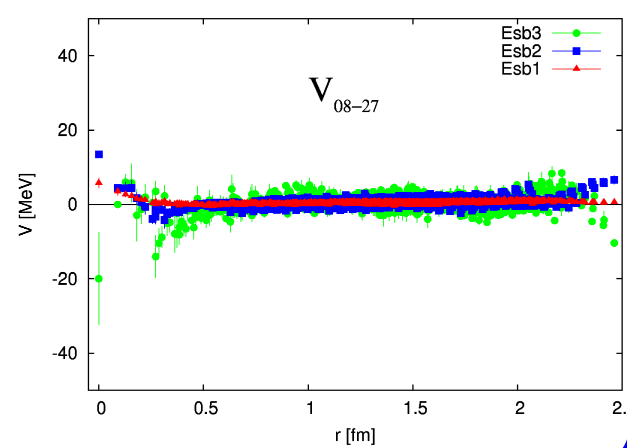


Strongly attractive
H-dibaryon channel

Pauli blocking effect







Mixture of singlet and octet
Is relatively larger than the others

27 plet does not mix so much to the other representations