Describing halo nuclei using effective field theory

arXiv:1001.1511, Nucl. Phys. A865 (2011), 17; Phys. Lett. B723 (2013), 196;
Nucl. Phys. A913 (2013), 105; arXiv:1311.6822; arXiv:1401.4482

Daniel Phillips
Ohio University

Work done in collaboration with: B. Acharya, K. Nollett, and X. Zhang (Ohio); H.-W. Hammer (TU, Darmstadt); C. Ji (TRIUMF)

Research supported by the US Department of Energy

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- 21st International Conference on Few-body Problems in Physics: Chicago, May 18-22, 2015. Organized by Argonne National Laboratory and Ohio University.

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Halo nuclei

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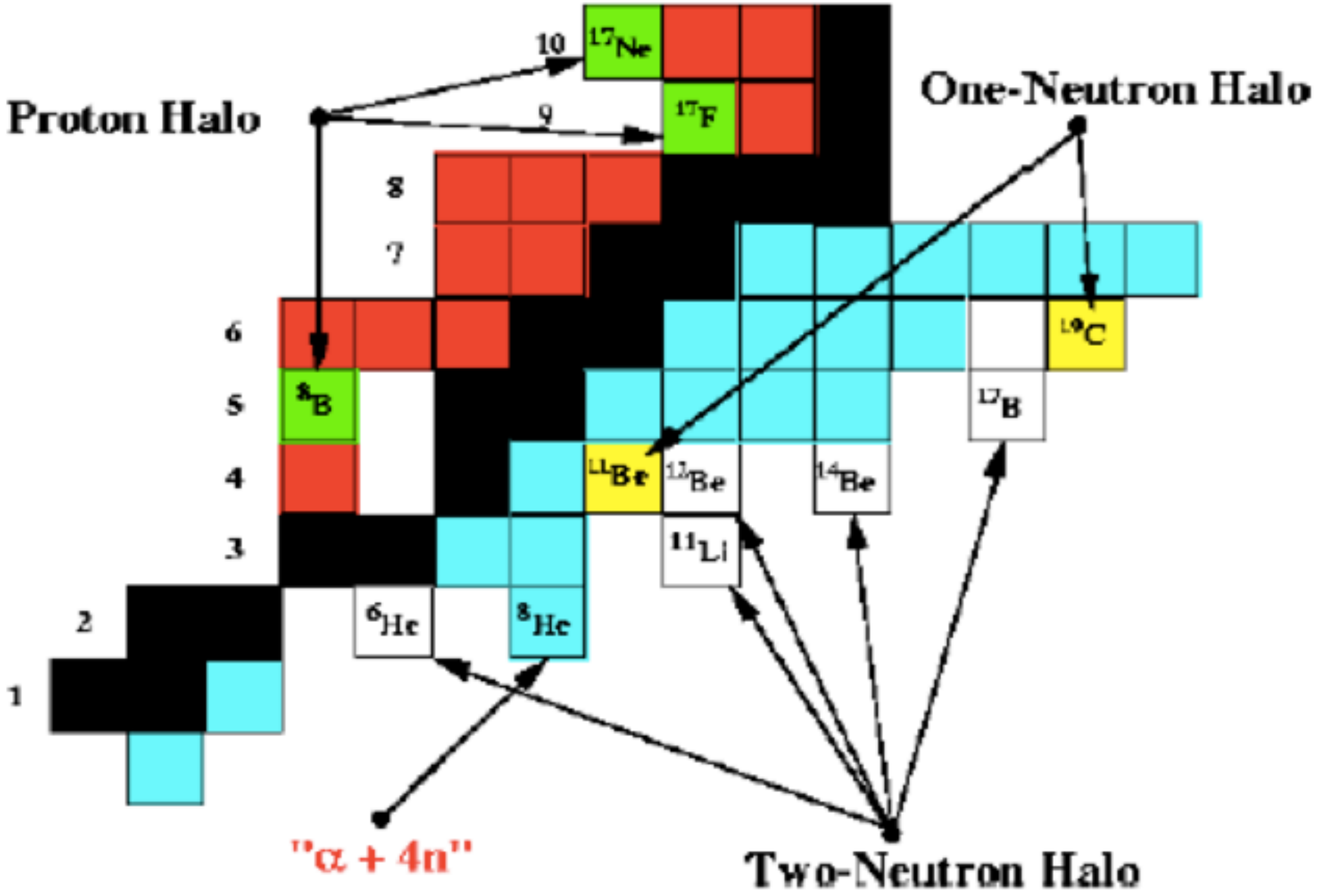

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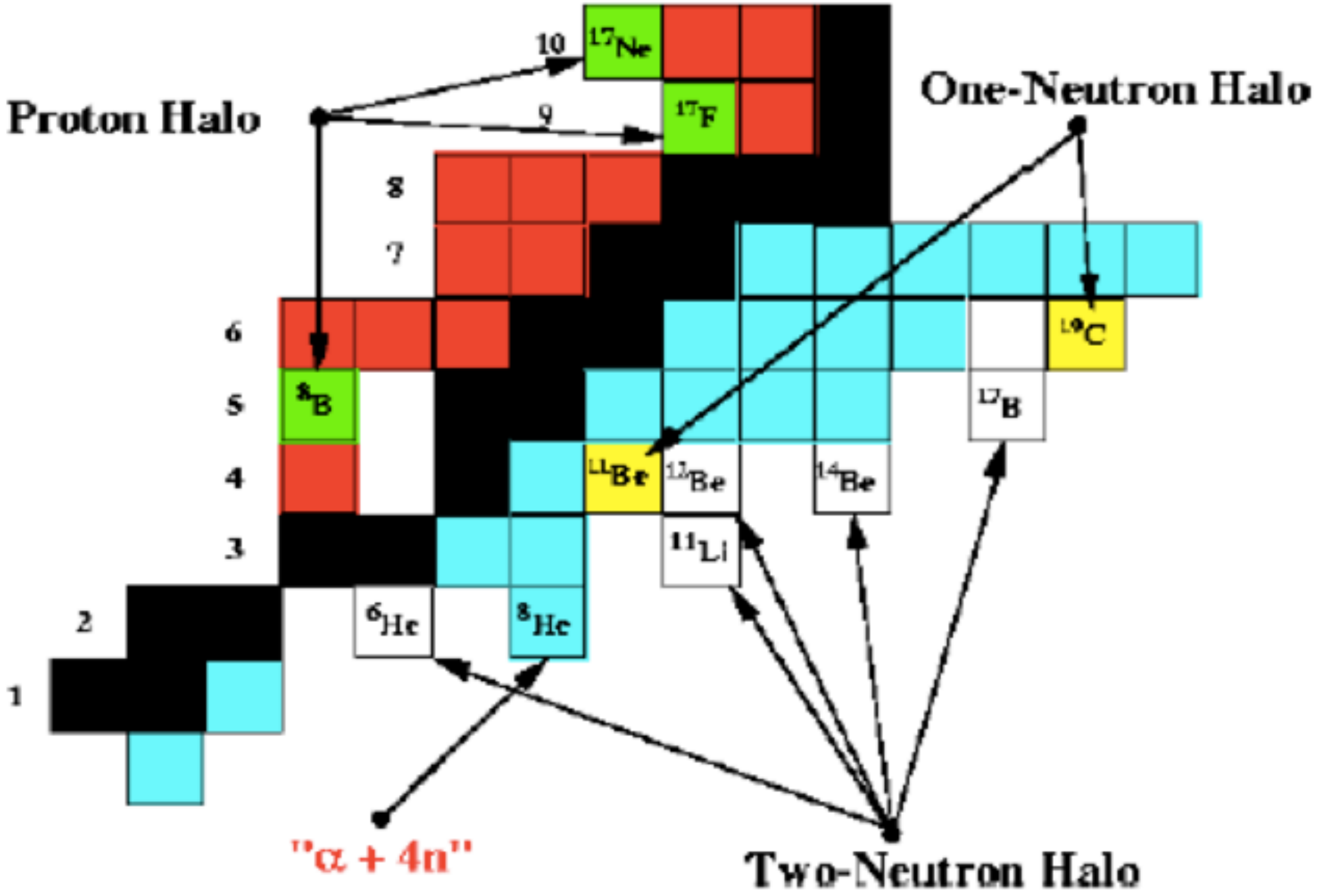

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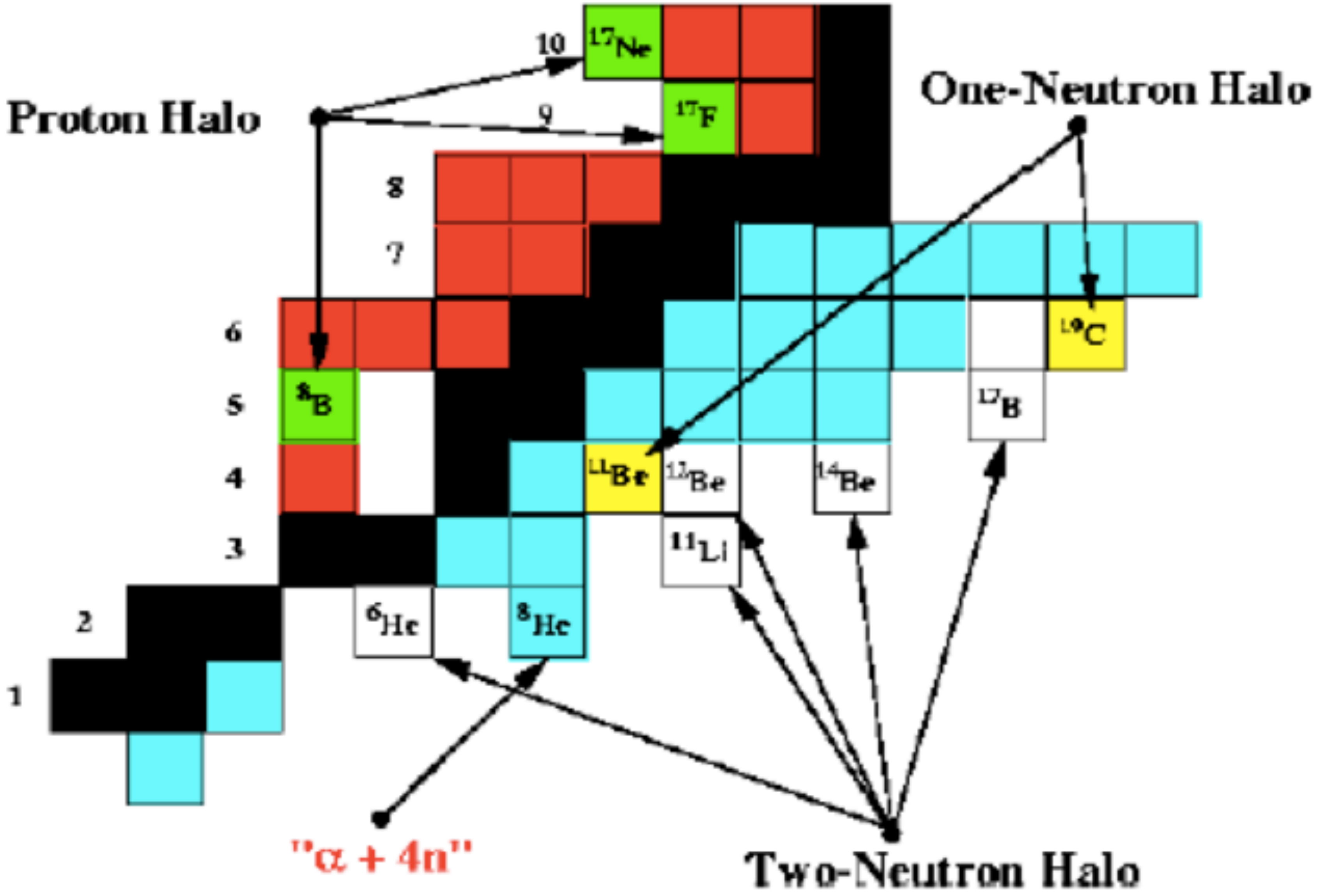

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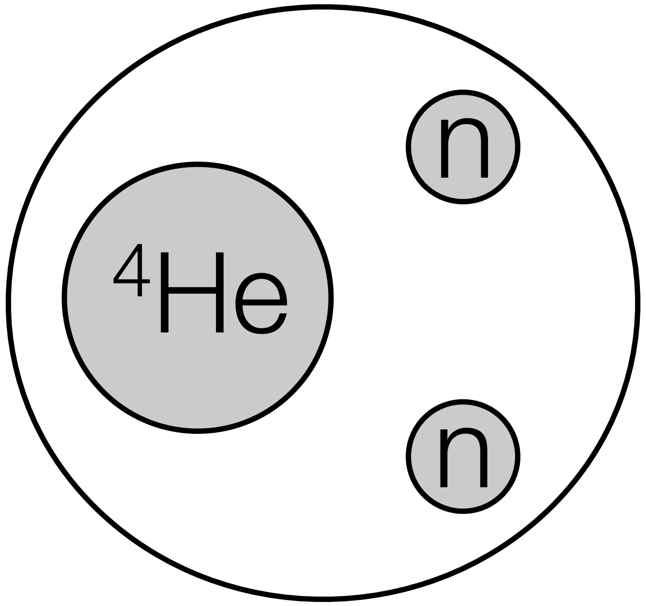
- Halo nuclei are characterized by small nucleon binding energies, large interaction cross sections, large radii, large E1 transition strengths.

Halo EFT

^{6}He cartoon



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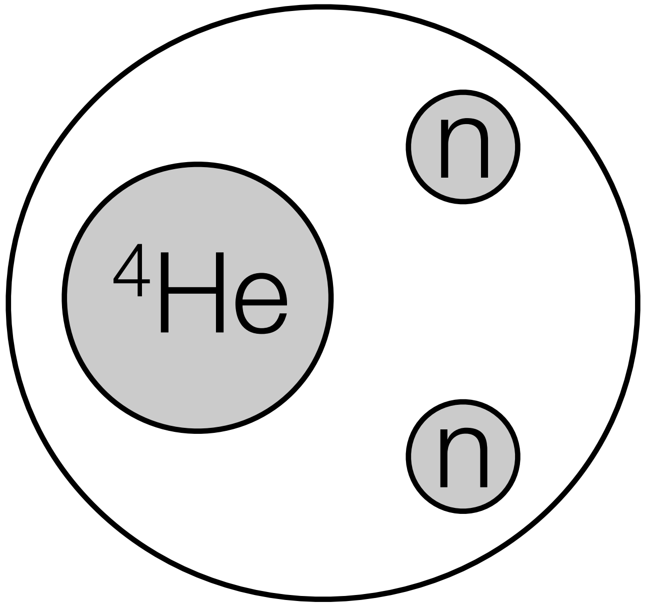
$\lambda \gg R_{core}$; $\lambda \lesssim R_{halo}$



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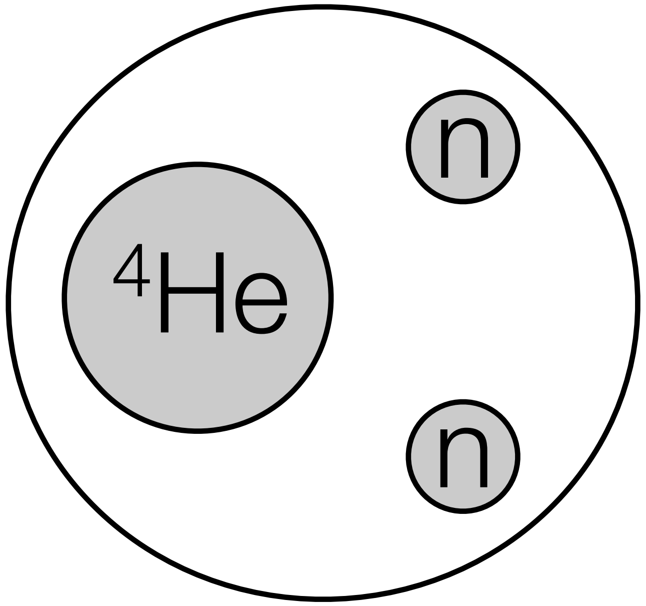

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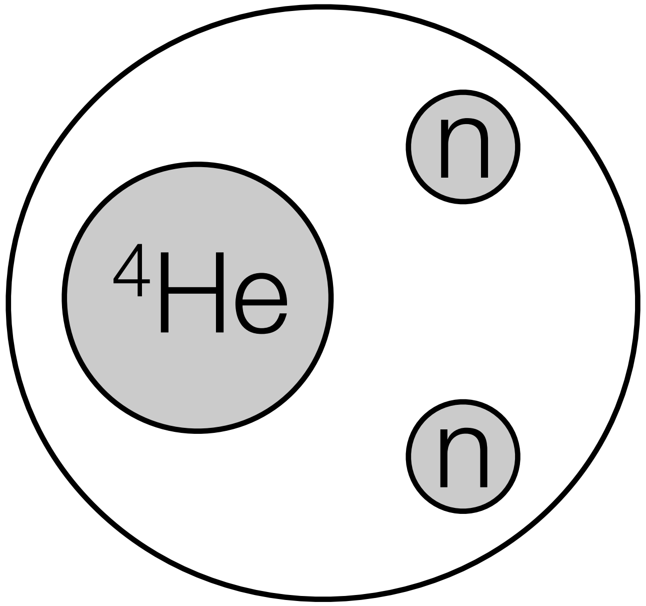

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- By this definition the deuteron is the lightest halo nucleus, and the pionless EFT for few-nucleon systems is a specific case of halo EFT

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- Halo EFT: generalities

- Pionless EFT for few-nucleon systems

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Role of P waves
The need for good input

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Sensitivity to p-^{7}Be scattering parameters

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Matter radius-binding energy connection

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An EFT for shallow bound states

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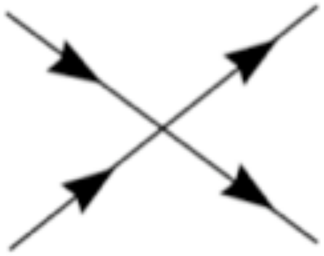

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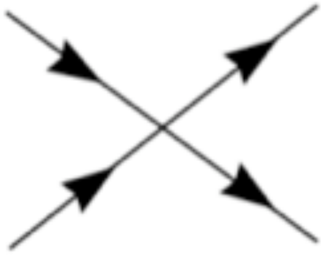

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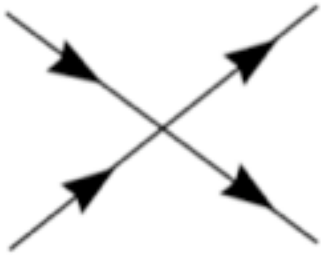

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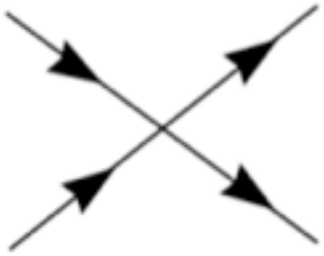

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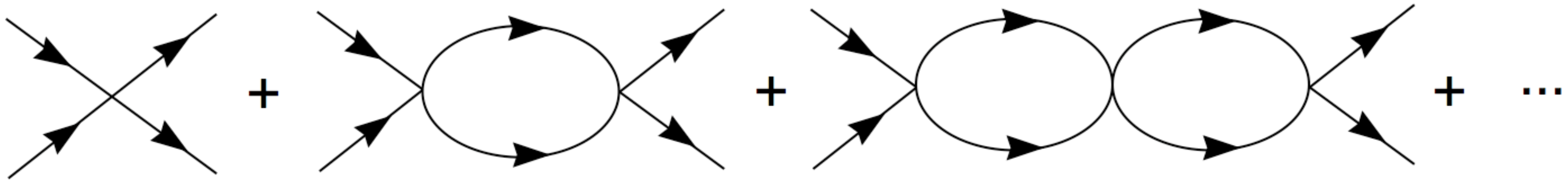
Shallow bound state: $B=1/(Ma^2) \ll 1/(MR^2)$

- If $|a| \gg R$ then quantum corrections matter

$$t=\frac{4\pi a}{M}\frac{1}{1+iak}$$



- Results then "universal": apply to all quantum systems with $R \ll |a|$

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- Physics is “universal”: few-N, Cold atoms (Efimov physics!), Halos, X(3872)

Example 1: Coulomb dissociation of Carbon-19

Acharya, Phillips. NPA 913, 103 (2013)

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- No discussion of nodes, details of n-core interaction, spectroscopic factors
- Compute part of matrix element for $r \sim R_{halo}$ with "zero-range" wave functions. Part for $r \sim R_{core}$ enters at higher order, and is parameterized via contact operators.

Lagrangian: shallow S- and P-states

$$
\begin{aligned}
\mathcal{L} \quad = \quad & c^\dagger \left(i\partial_t + \frac{\nabla^2}{2M} \right) c + n^\dagger \left(i\partial_t + \frac{\nabla^2}{2m} \right) n \\
& + \sigma^\dagger \left[\eta_0 \left(i\partial_t + \frac{\nabla^2}{2M_{nc}} \right) + \Delta_0 \right] \sigma + \pi_j^\dagger \left[\eta_1 \left(i\partial_t + \frac{\nabla^2}{2M_{nc}} \right) + \Delta_1 \right] \pi_j \\
& - g_0 \left[\sigma n^\dagger c^\dagger + \sigma^\dagger n c \right] - \frac{g_1}{2} \left[\pi_j^\dagger (n \; i\overset{\leftrightarrow}{\nabla}_j \; c) + (c^\dagger \; i\overset{\leftrightarrow}{\nabla}_j \; n^\dagger) \pi_j \right] \\
& - \frac{g_1}{2} \frac{M - m}{M_{nc}} \left[\pi_j^\dagger \; i\overset{\rightarrow}{\nabla}_j \; (nc) - i\overset{\leftrightarrow}{\nabla}_j \; (n^\dagger c^\dagger) \pi_j \right] + \ldots \, ,
\end{aligned}
$$

- c, n: “core”, “neutron” fields. c: boson, n: fermion.
- σ, π_j: S-wave and P-wave fields
- Minimal substitution generates leading EM couplings
- Short-range EM operators parameterize interior part of matrix element

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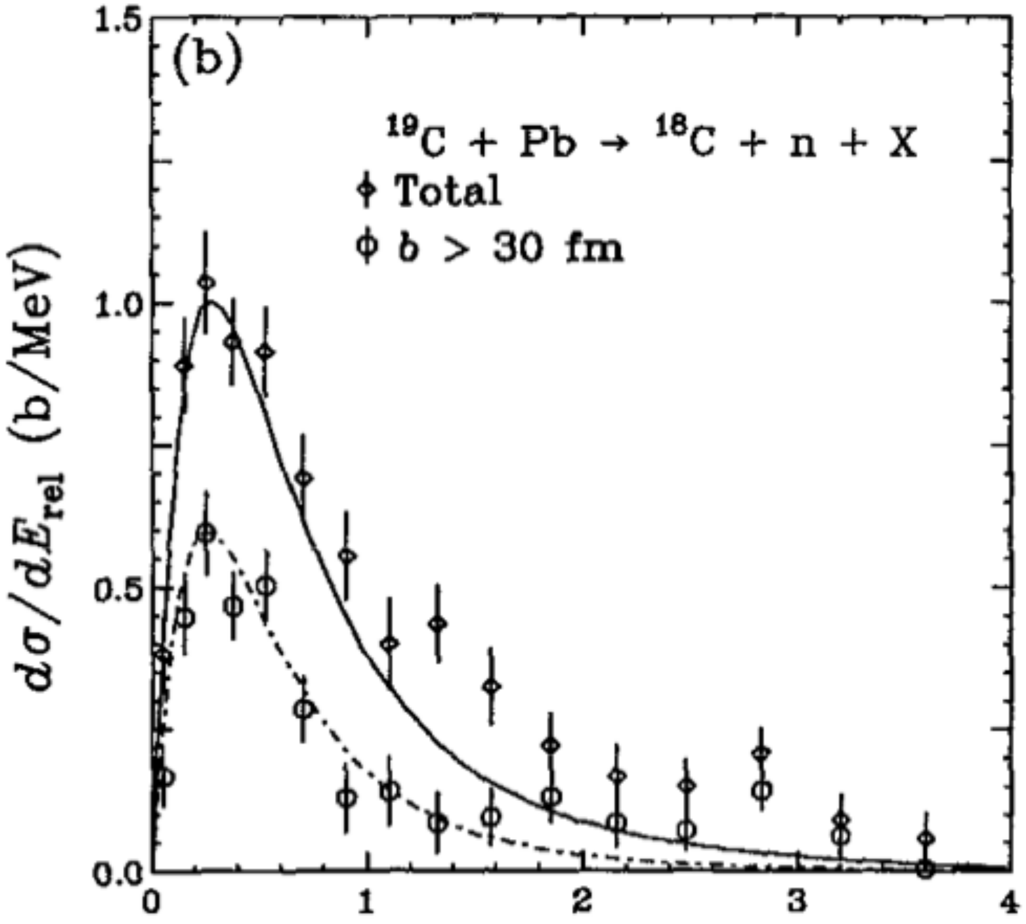
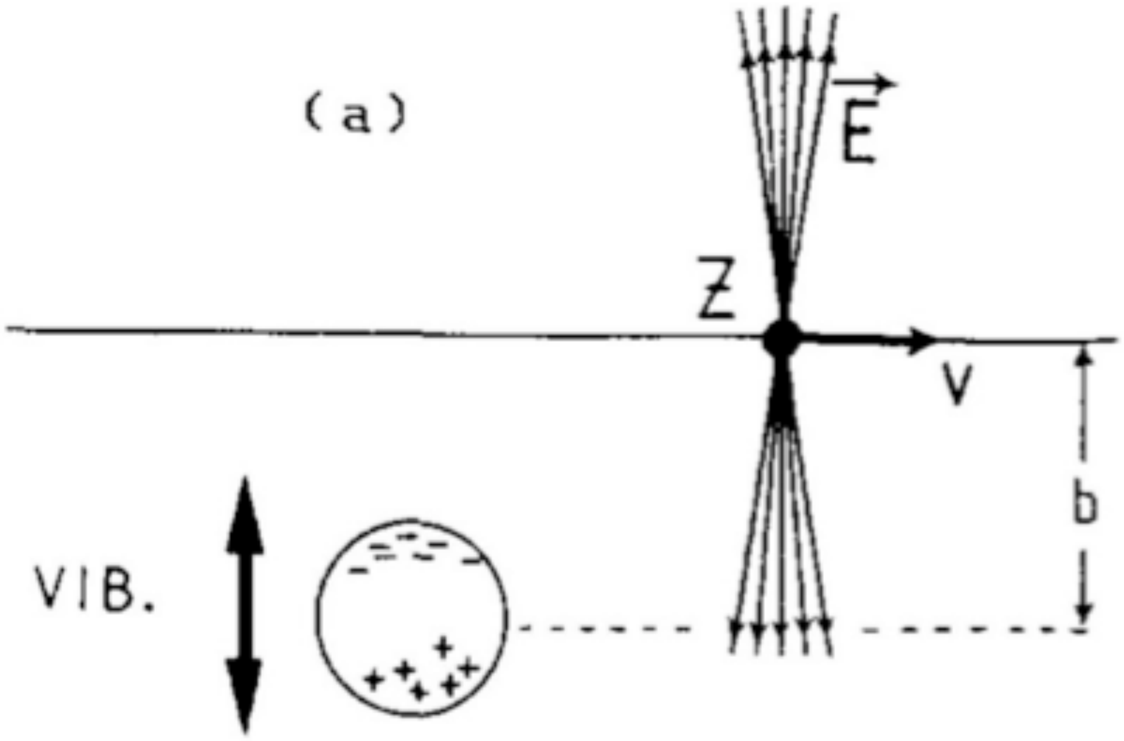


Nakamura et al. (2003)

Coulomb dissociation of halo nuclei

Bertulani, arXiv:0908.4307

- Coulomb dissociation: collide halo nucleus (we hope peripherally) with a high-Z nucleus
- Do with different Z, different nuclear sizes, different energies to test systematics



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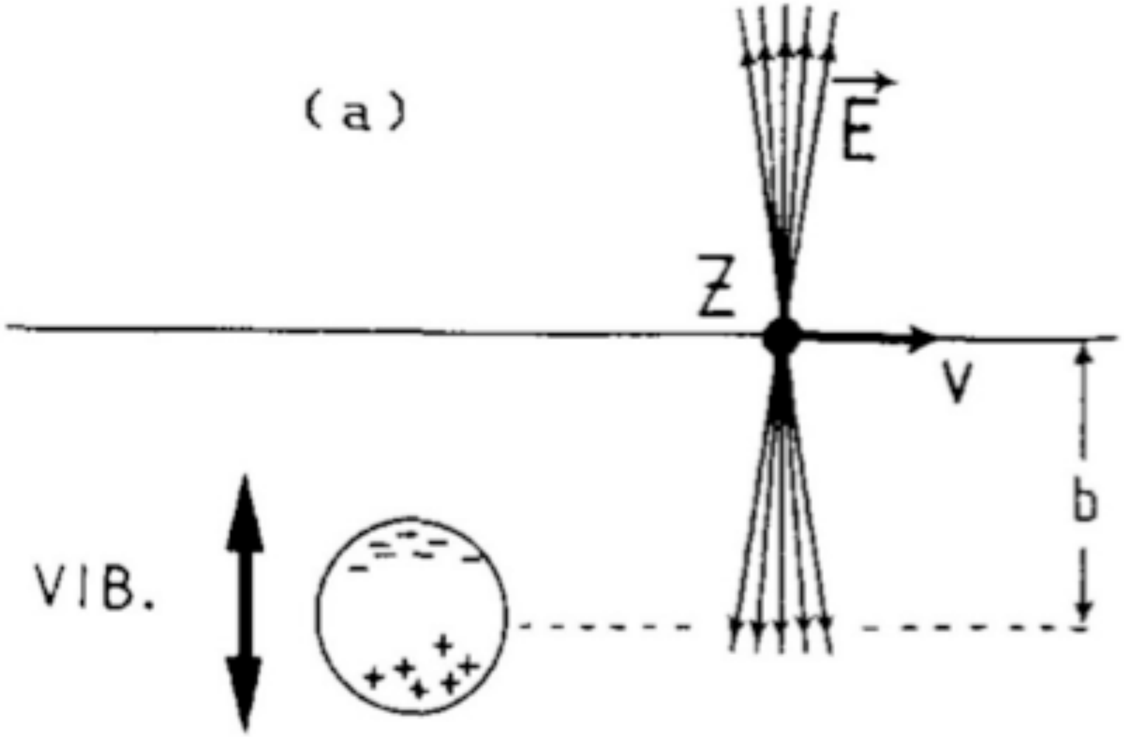

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$$\frac{d\sigma_C}{2\pi b db} = \sum_{\pi L} \int \frac{dE_\gamma}{E_\gamma} n_{\pi L}(E_\gamma, b) \sigma_\gamma^{\pi L}(E_\gamma)$$

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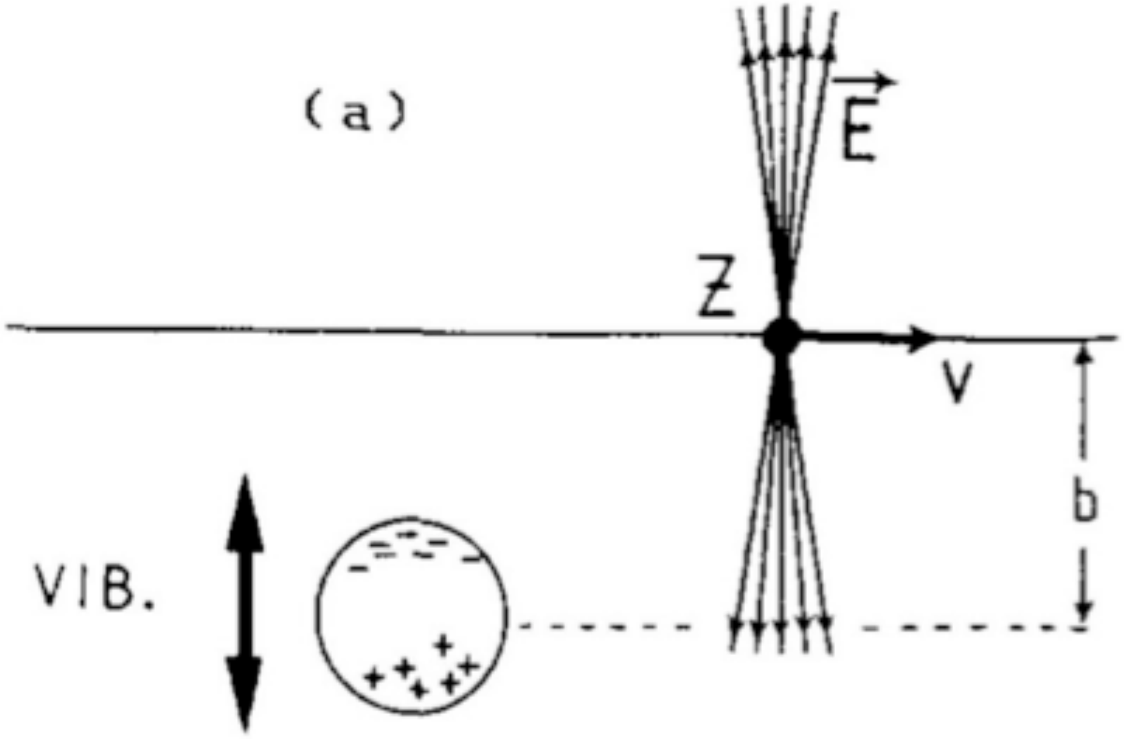

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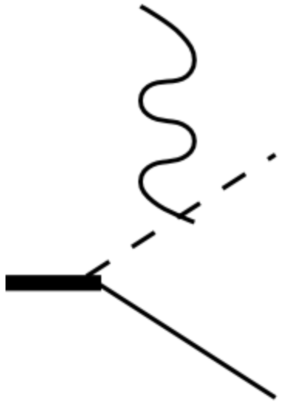
  - $\sigma_\gamma^{\pi L}(E_\gamma)$ can then be extracted: it's the (total) cross section for dissociation of the nucleus due to the impact of photons of multipolarity πL.

Predicting dissociation

c.f. Singh et al., NPA 802, 82 (2008)

- Leading order: no FSI, $r_0=0 \Rightarrow \gamma_0$ is only free parameter=0.16 fm^{-1}

$$\mathcal{M} = \frac{eQ_c g_0 2m_R}{\gamma_0^2 + \left(\mathbf{p}' - \frac{m}{M_{nc}}\mathbf{k}\right)^2}$$



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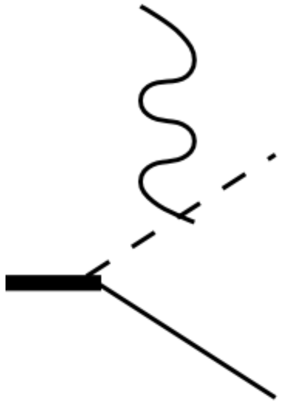

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$$\frac{dB(E1)}{e^2 dE} = \frac{12 m_R}{\pi^2} Z_{eff}^2 \frac{\gamma_0}{1 - r_0\gamma_0} \frac{p^3}{(\gamma_0^2 + p^2)^4}$$

Z_{eff}=6/19

- Corresponds to $u_0(r)=A_0 \exp(-\gamma_0 r)$; $A_0^2 = \frac{2\gamma_0}{1 - r_0\gamma_0}$

Universal E1 strength formula for S-wave halos

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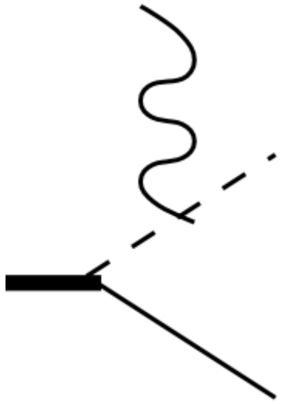

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Universal E1 strength formula for S-wave halos

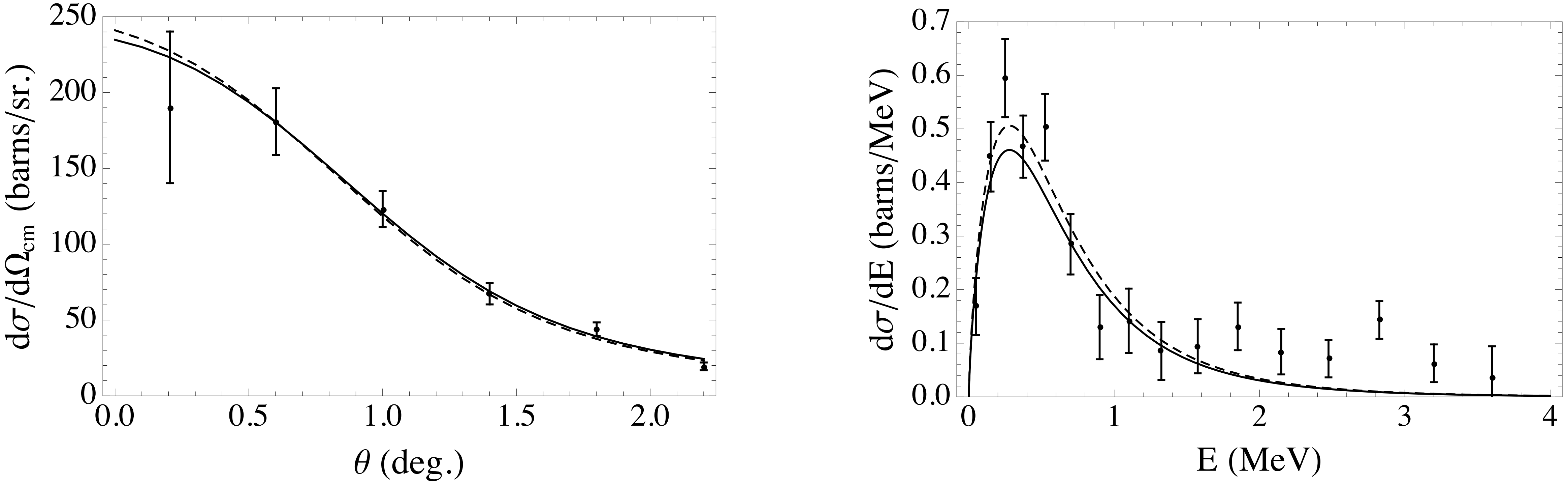
- Final-state interactions suppressed by $(R_{core}/R_{halo})^3$

- First gauge-invariant contact operator: $L_{E1}\sigma^\dagger \mathbf{E} \cdot (n \overset{\leftrightarrow}{\nabla} c) + \mathrm{h.c.}$

Results

Data: Nakamura et al., 1999, 2003
Analysis: Acharya, Phillips. NPA, 2013

- Integrate this E1 strength for transition to a core + neutron state, per unit energy per unit solid angle, as function of energy of the outgoing nc pair over differential photon numbers and over angle, respectively.



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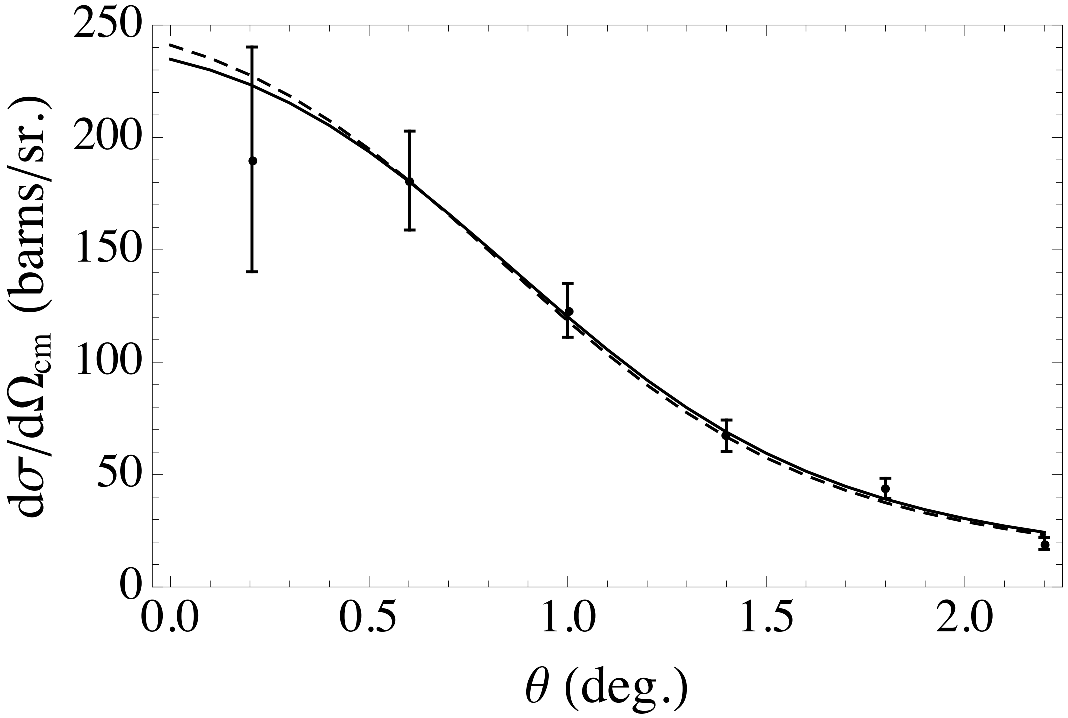

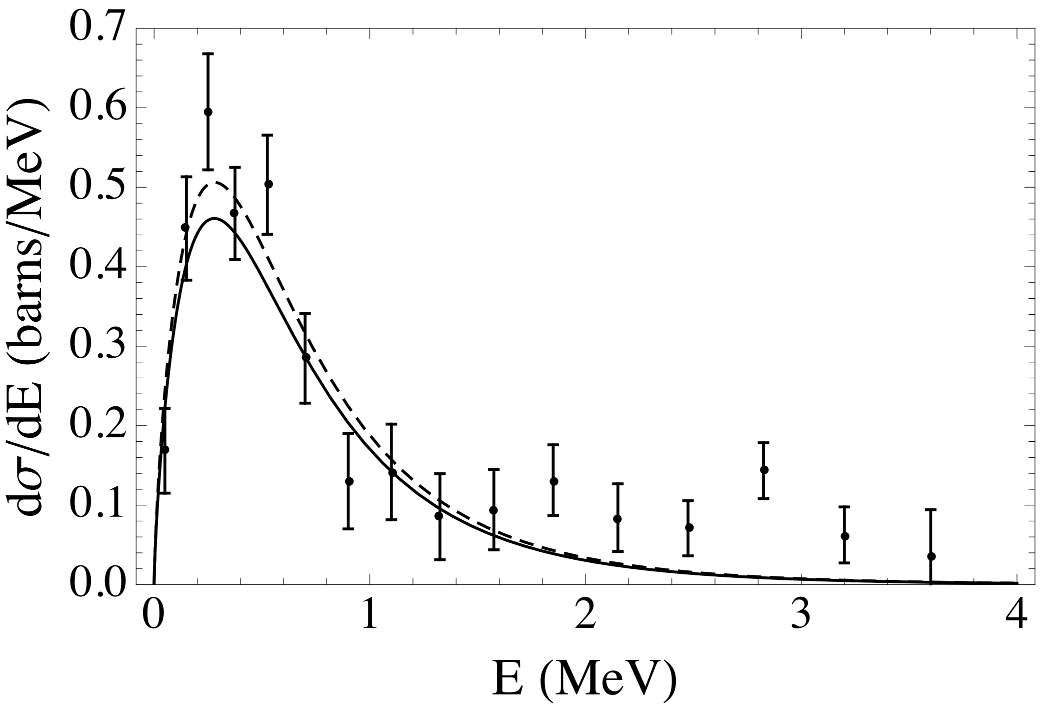

- $\gamma_0 \equiv a$ determines peak position and fall off of angular distribution

- r_0 fixed from fitting height of peak

Results

Data: Nakamura et al., 1999, 2003
Analysis: Acharya, Phillips. NPA, 2013

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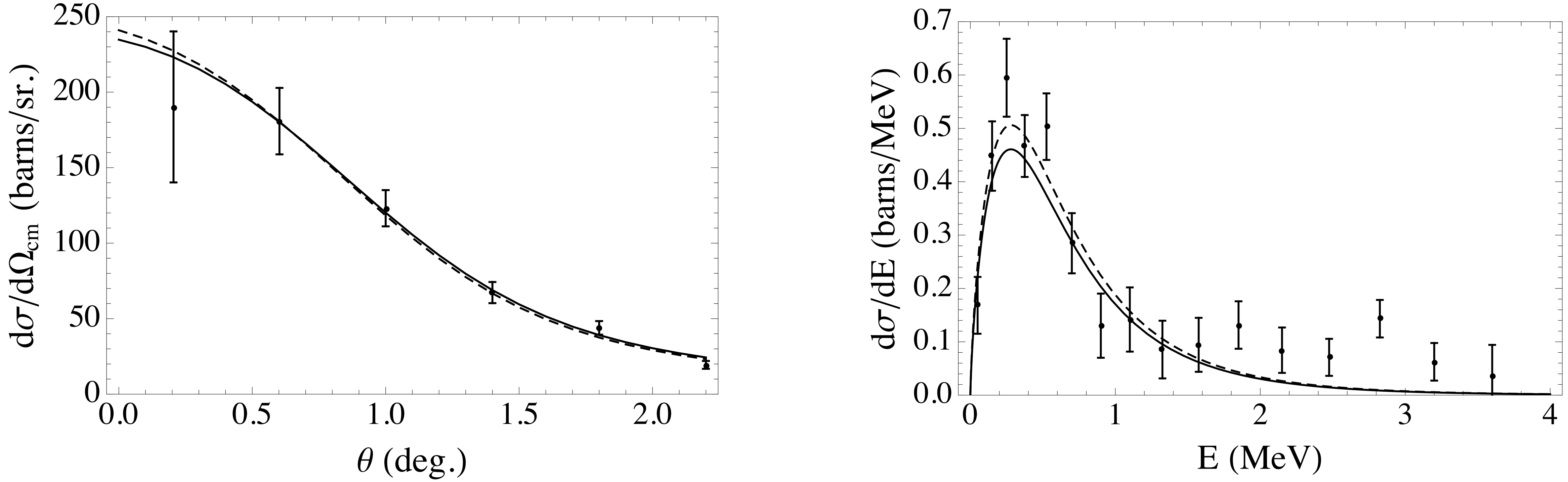
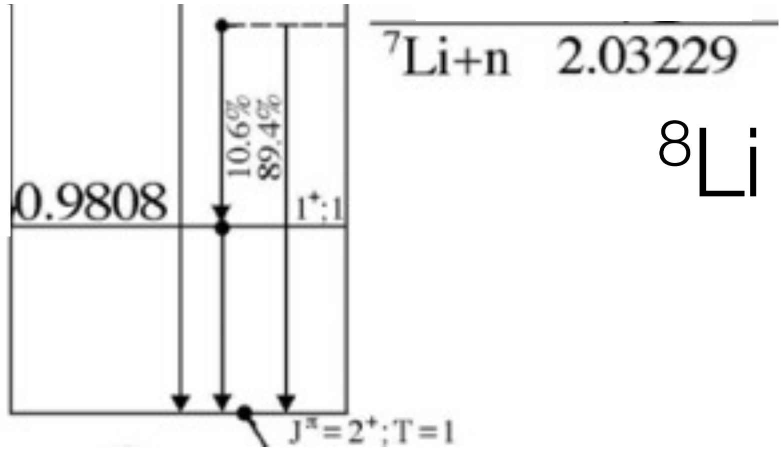


$$
\begin{aligned}
a &= (7.75 \pm 0.35(\mathrm{stat.}) \pm 0.3(\mathrm{EFT}))\ \mathrm{fm}; \\
r_0 &= (2.6^{+0.6}_{-0.9}(\mathrm{stat.}) \pm 0.1(\mathrm{EFT}))\ \mathrm{fm}.
\end{aligned}
$$

Determine S-wave ^{18}C-n scattering parameters⇔ANCs from dissociation data.

Turning things around: $^{7}\text{Li} + n \rightarrow {}^{8}\text{Li} + \gamma_{E1}$

- Interesting mainly because it is isospin mirror of $^{7}\text{Be} + p \rightarrow {}^{8}\text{B} + \gamma_{E1}$
- ^{7}Li has spin-3/2: S-wave n scattering in $^{5}S_{2}$ and $^{3}S_{1}$
- ^{8}Li ground state is 2^{+}: both $^{5}P_{2}$ and $^{3}P_{2}$ components



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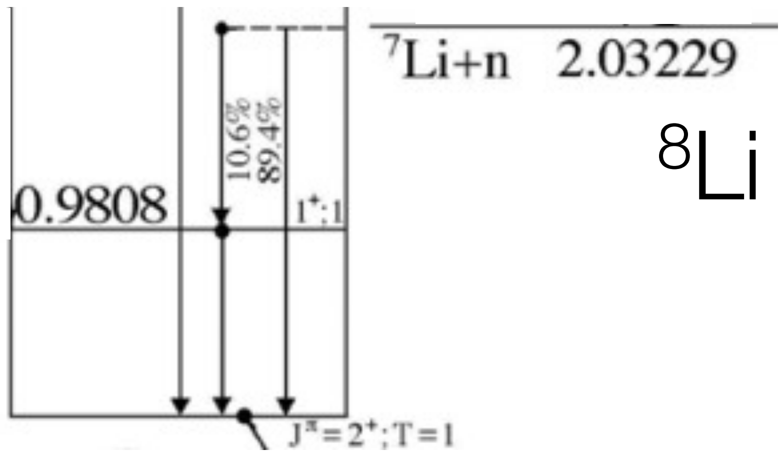

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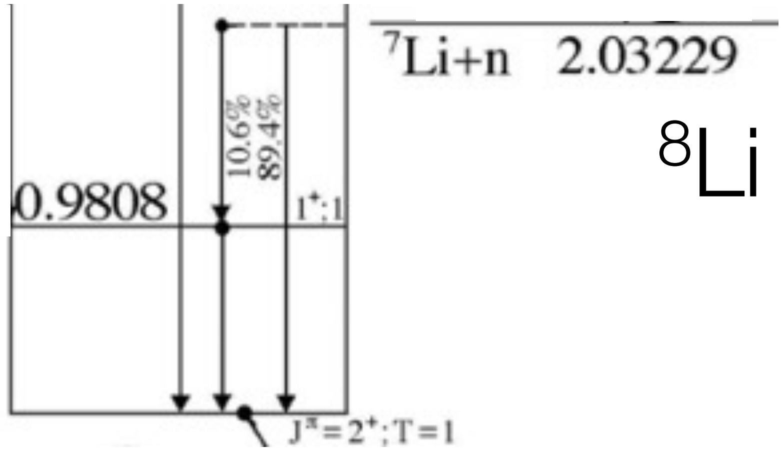

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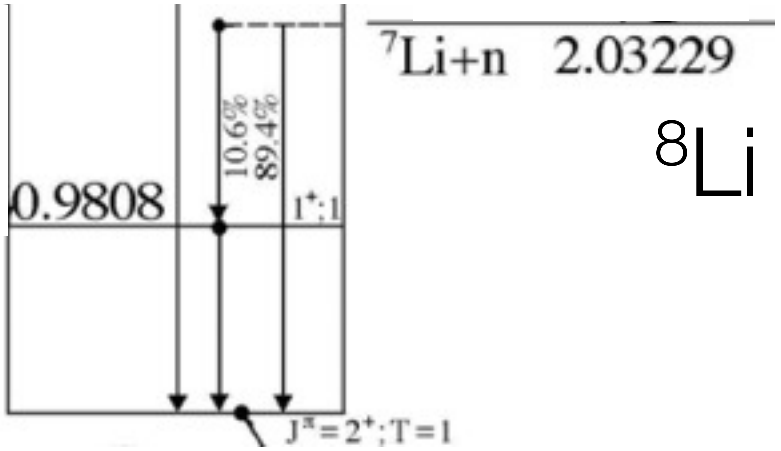

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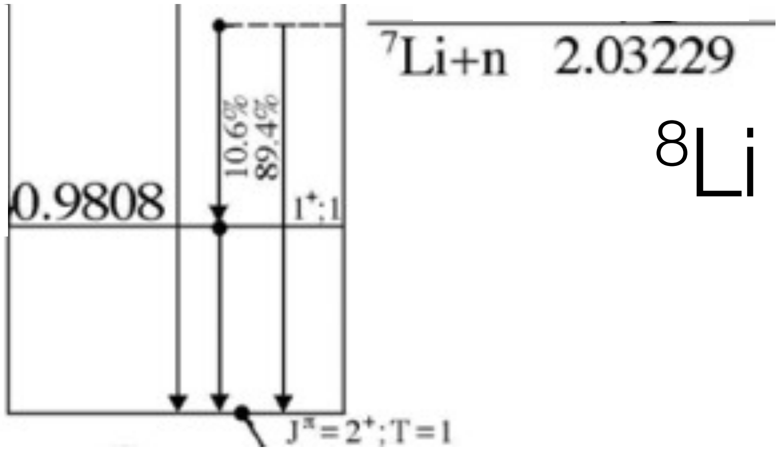

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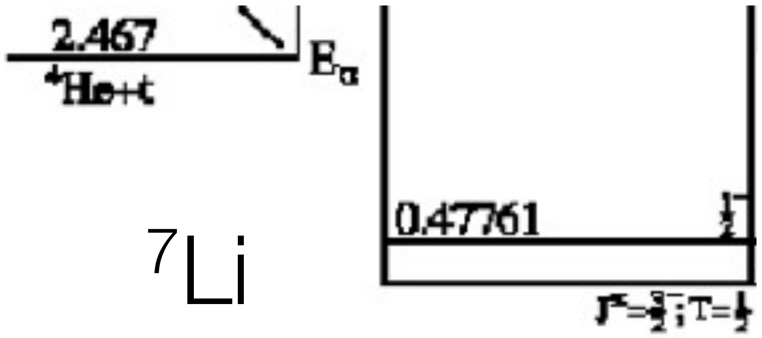

- ^{8}Li first excited state: 1^+, bound by 1.05 MeV

- $1/2^-$ in ^{7}Li can also play a role in structure of ^{8}Li (“core excitation”)

Fixing ^{8}Li parameters

Zhang, Nollett, Phillips, arXiv:1311.6822
c.f. Rupak, Higa, PRL 106, 222501 (2011);
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- Input at LO: B_1=2.03 MeV; B_1^*=1.05 MeV⇒γ_1=58 MeV; γ_1^*=42 MeV;

  $a_{S=2}$=-3.63(5) fm, $a_{S=1}$=0.87(7) fm

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- Need to also fix **2+2** P-wave ANCs already at LO. ANCs for first excited state of ^{8}Li (1^+) too.

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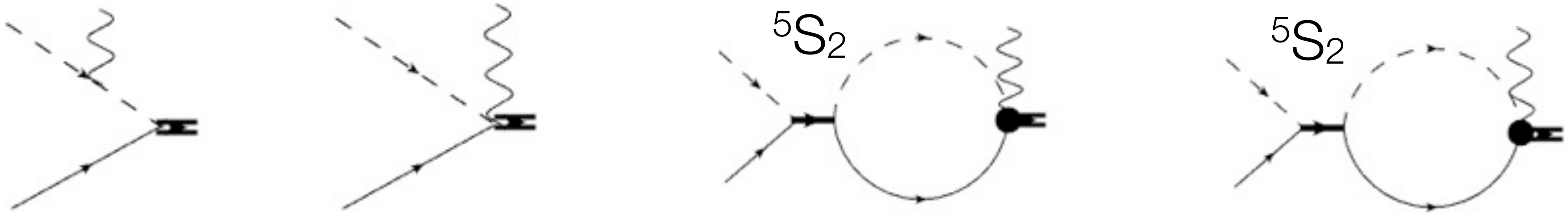
	$A_{(3P2)}$	$A_{(5P2)}$	$A_{(3P2^*)}$	$A_{(3P1)}^*$	$A_{(5P1)}^*$
Nollett	-0.283(12)	-0.591(12)	-0.384(6)	0.220(6)	0.197(5)
Trache	-0.284(23)	-0.593(23)		0.187(16)	0.217(13)

Results for $^{7}\mathrm{Li} + n \rightarrow {}^{8}\mathrm{Li} + \gamma_{E1}$

Data: Barker (1996), cf. Nagai et al. (2005)

Analysis: Zhang, Nollett, Phillips, arXiv:1311.6822

LO



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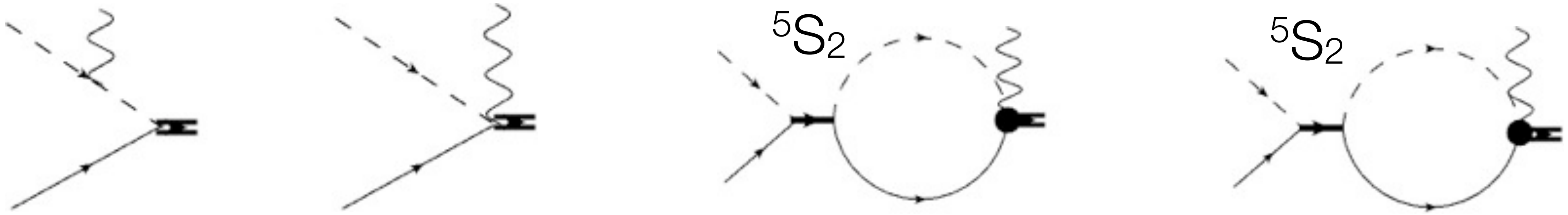

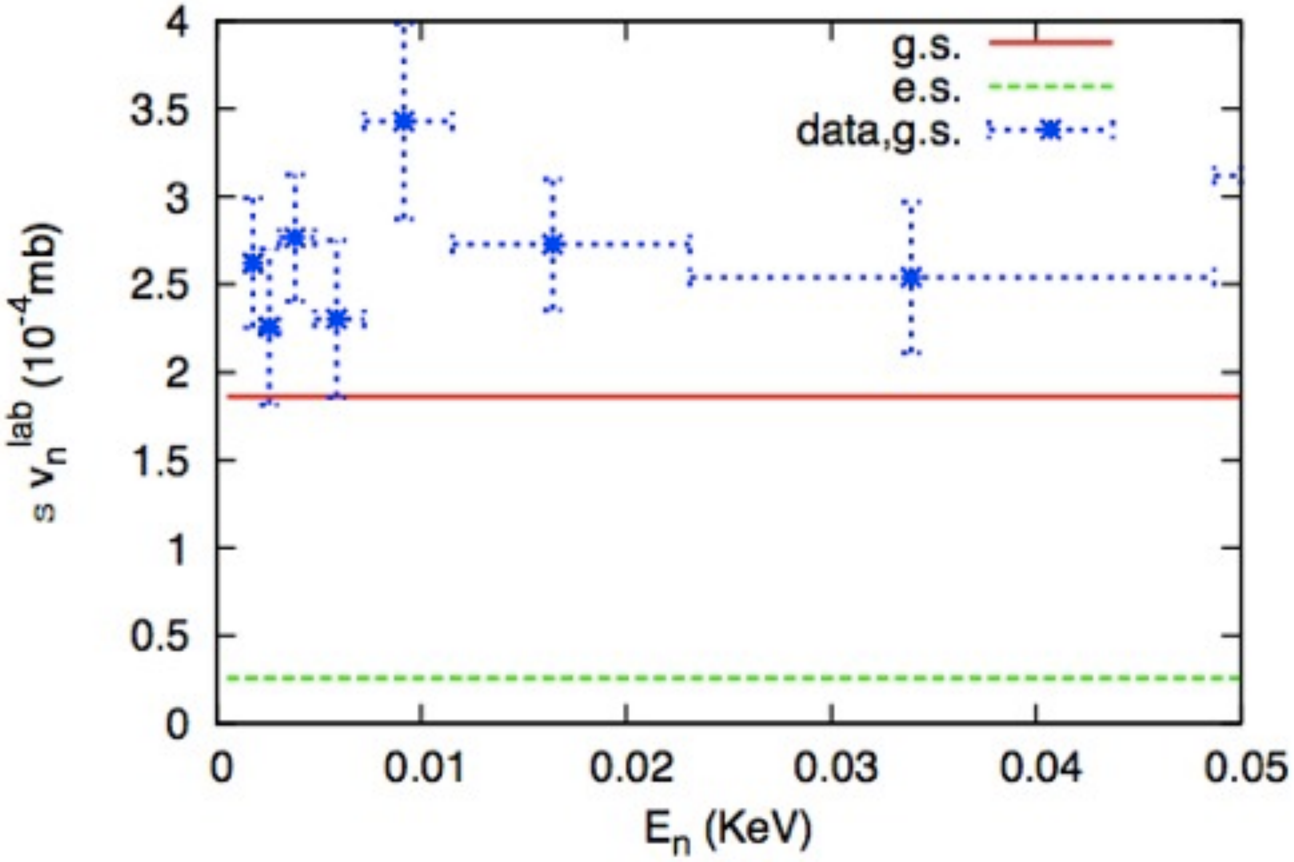

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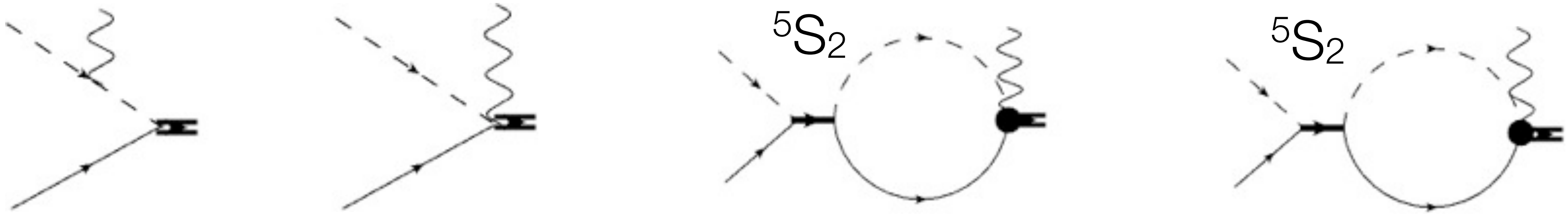

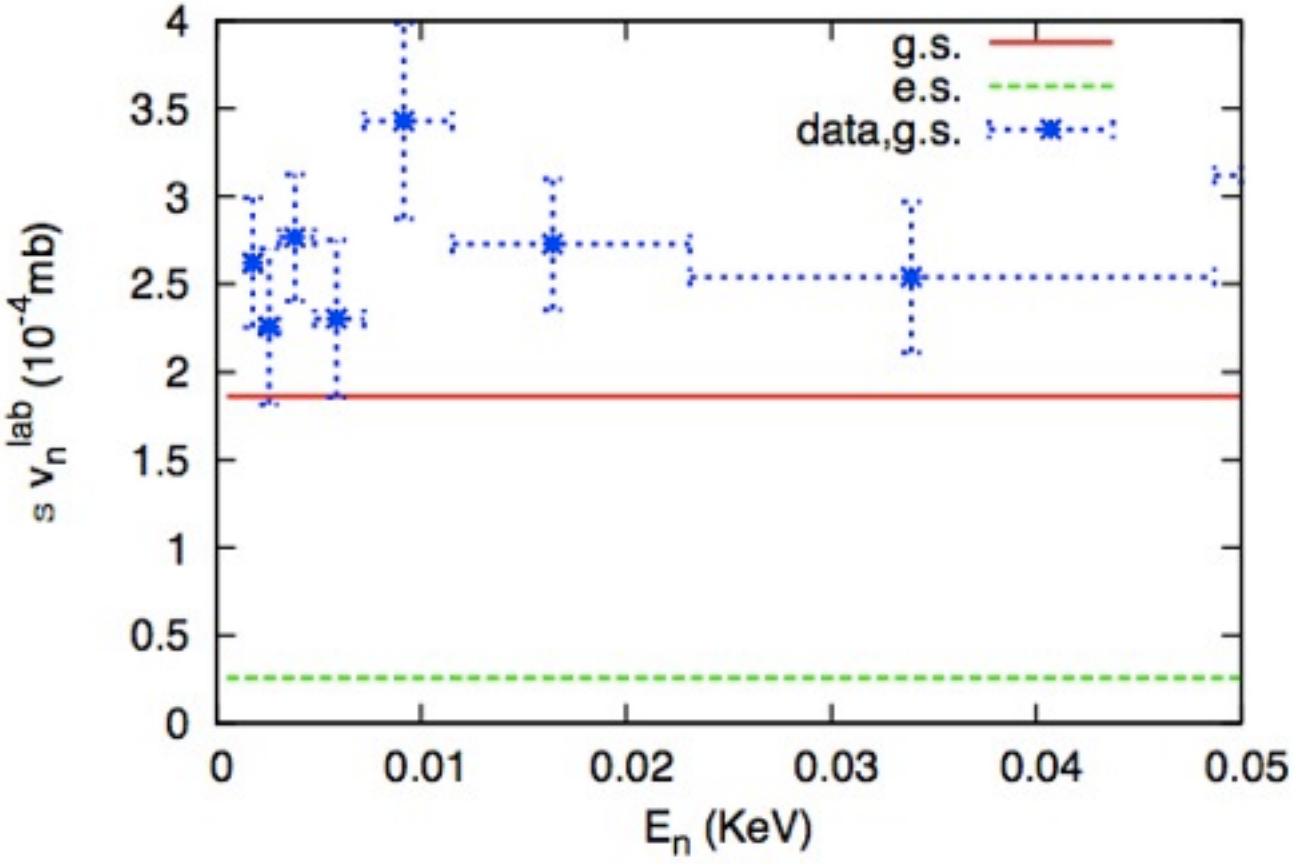

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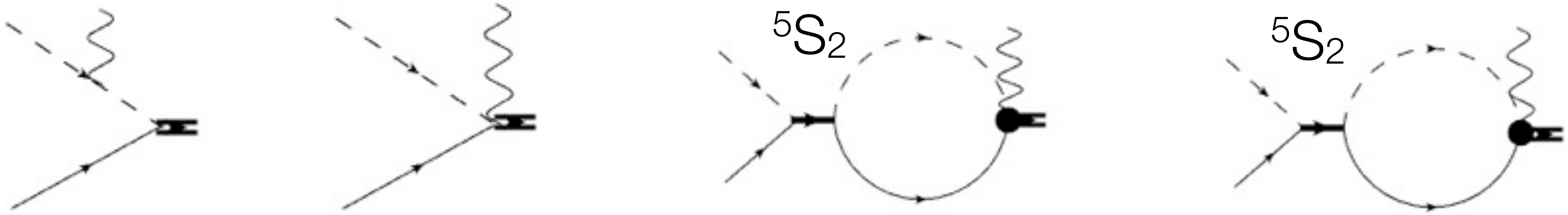

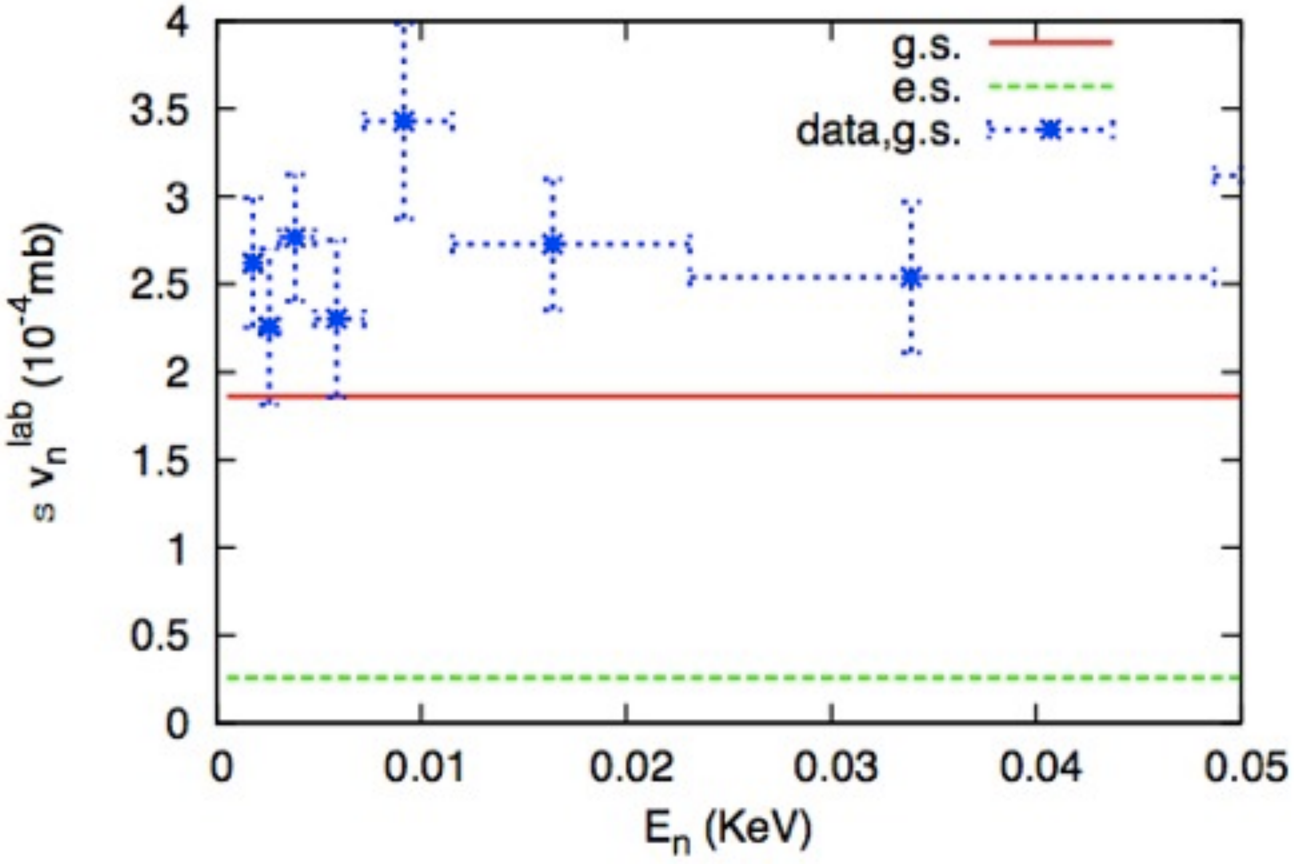

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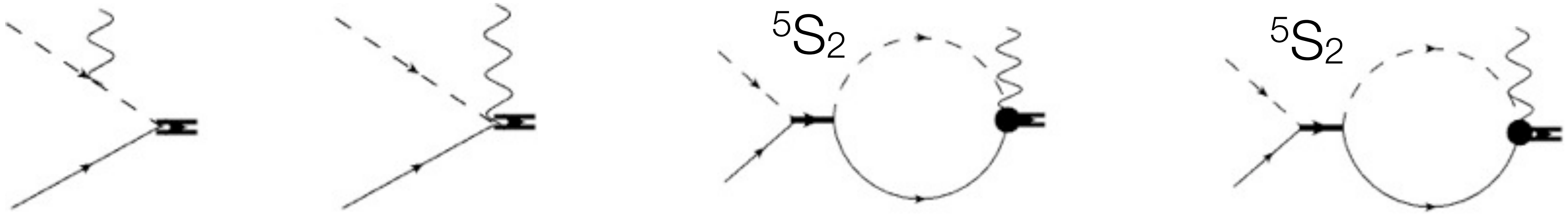

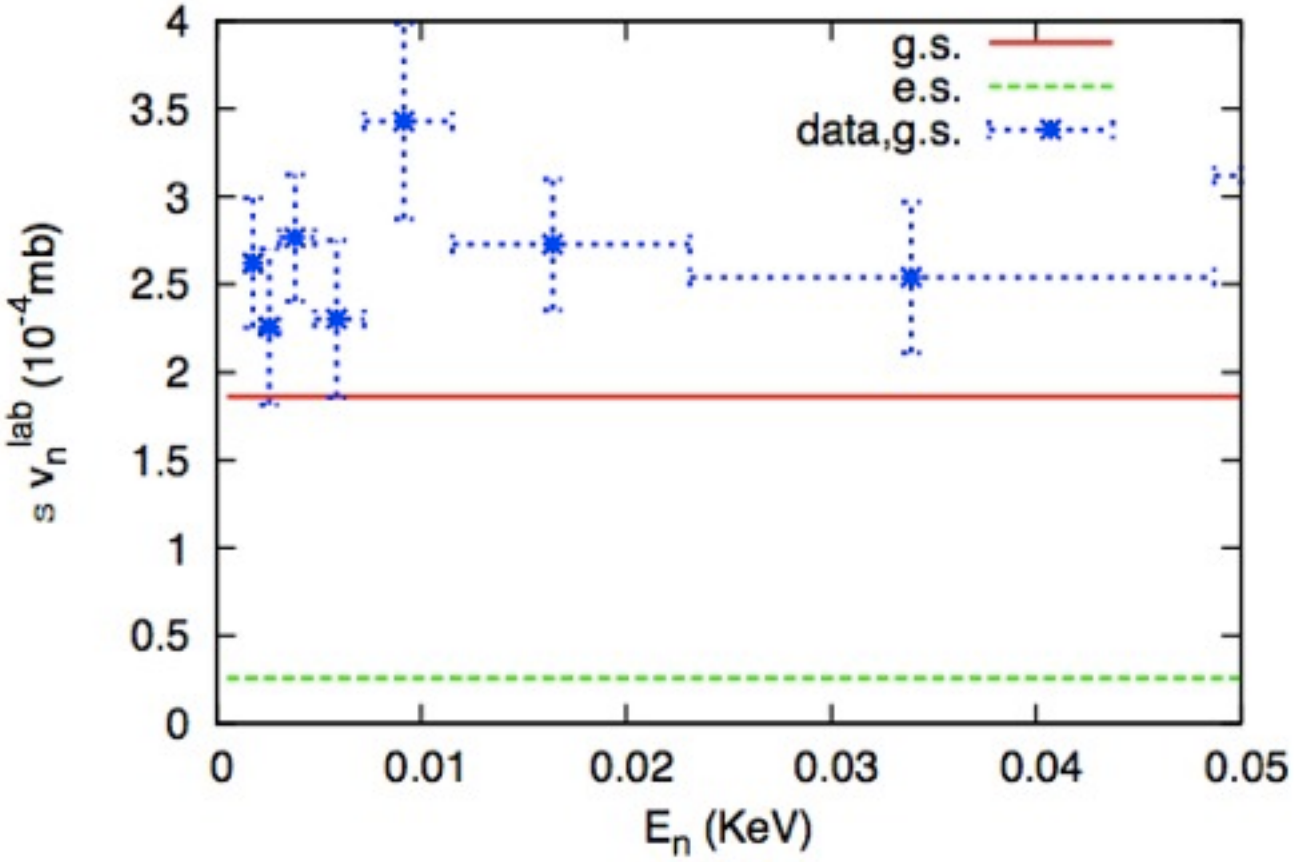

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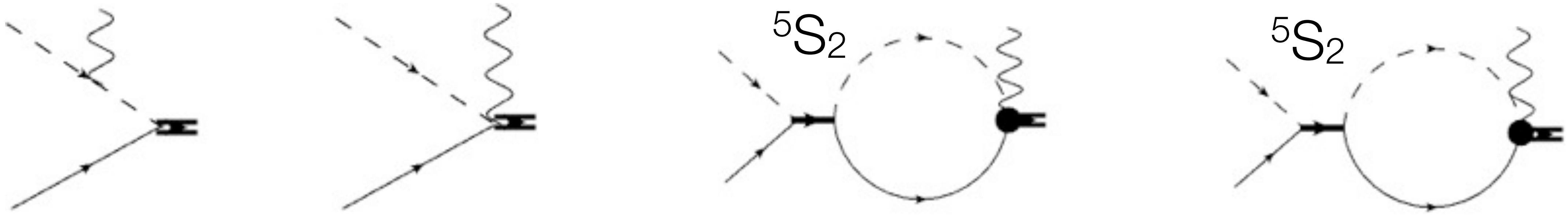

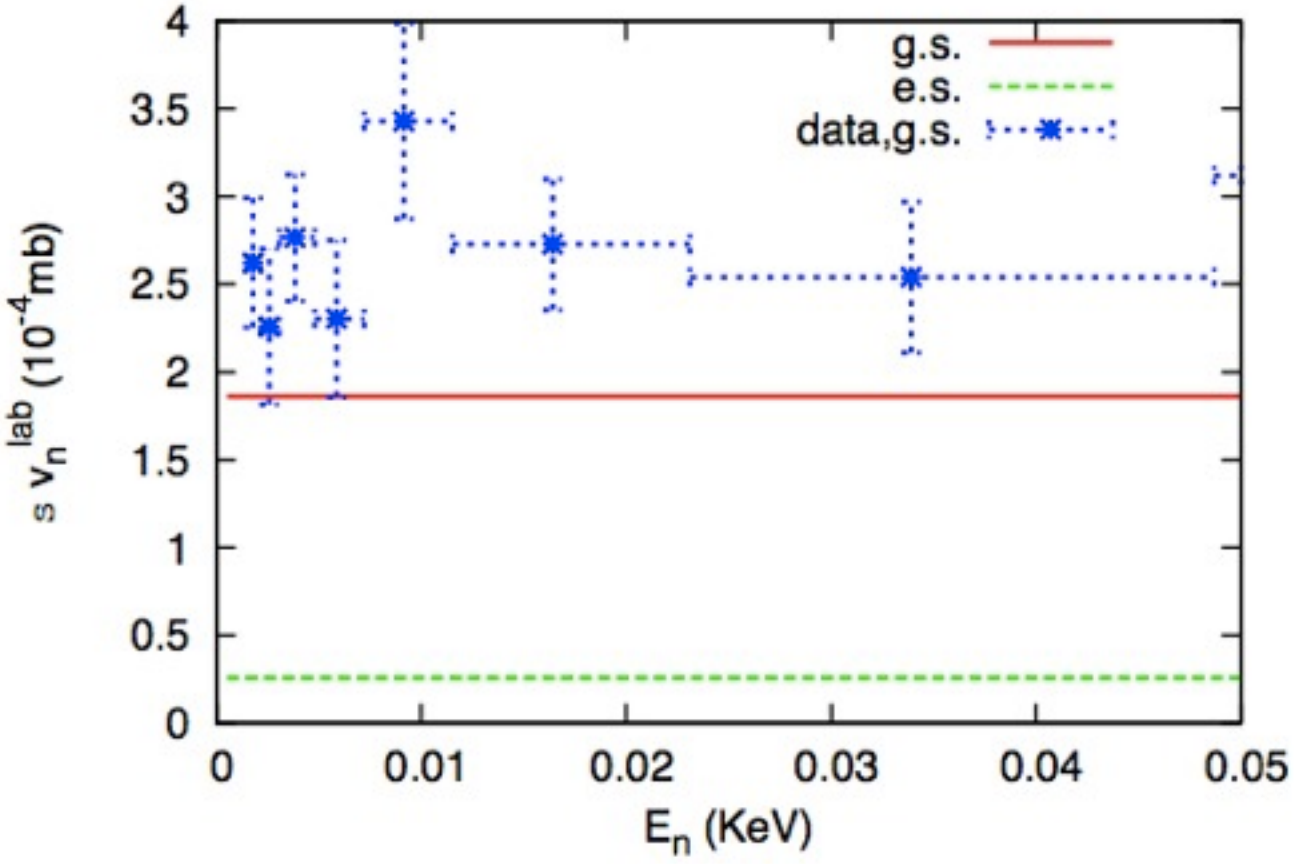

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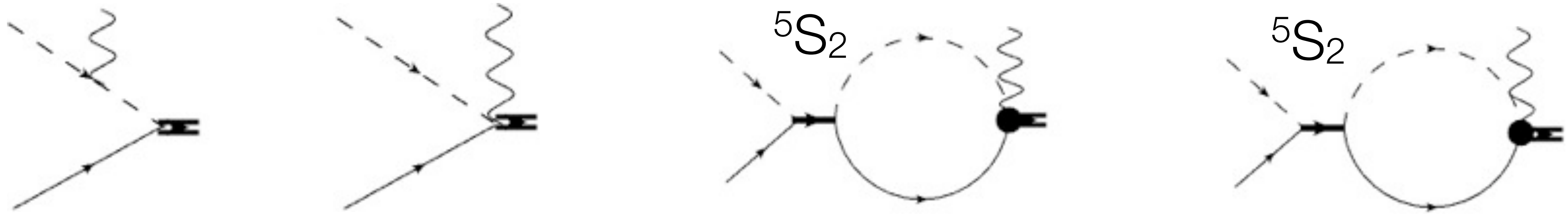

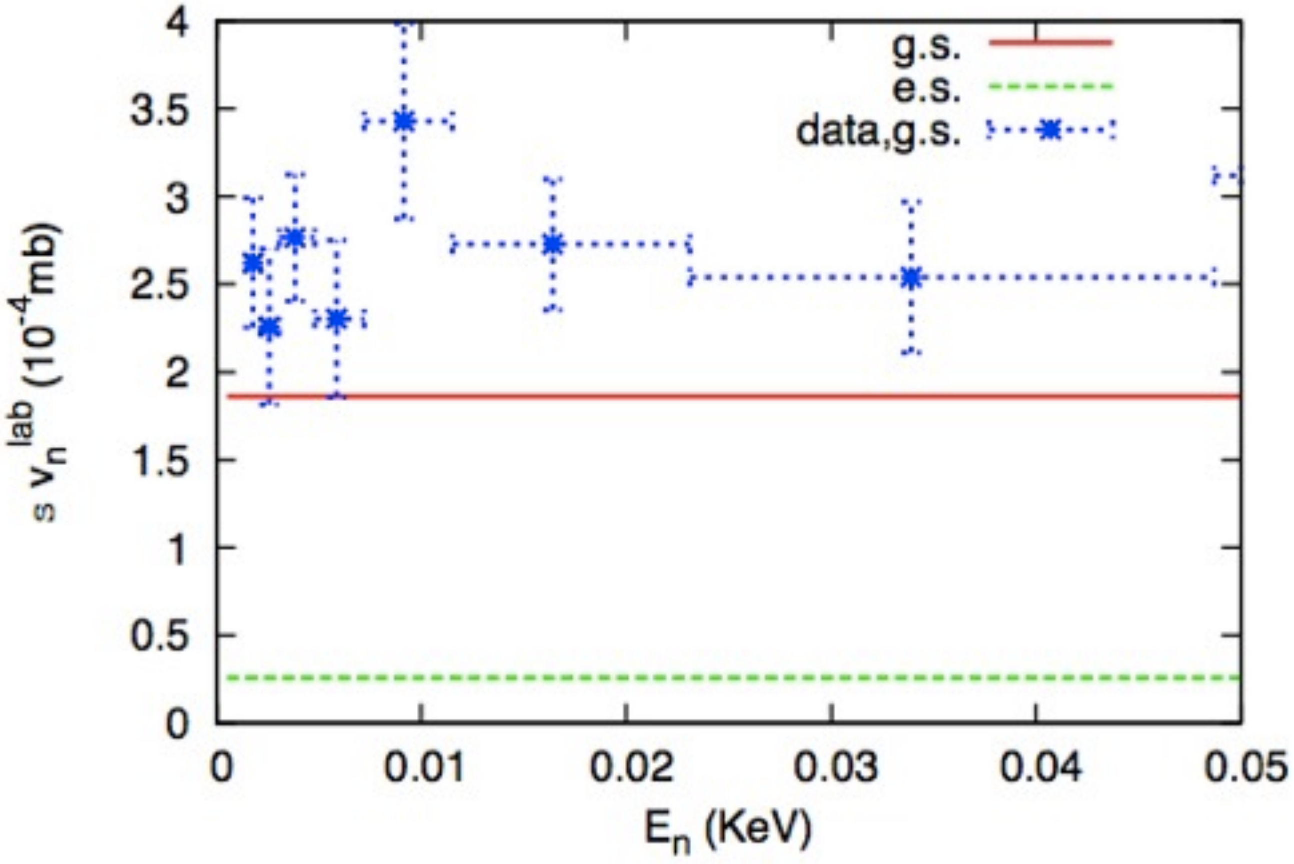

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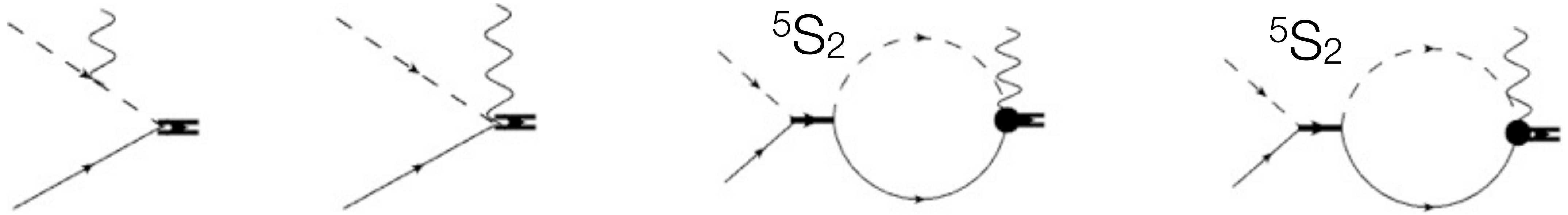

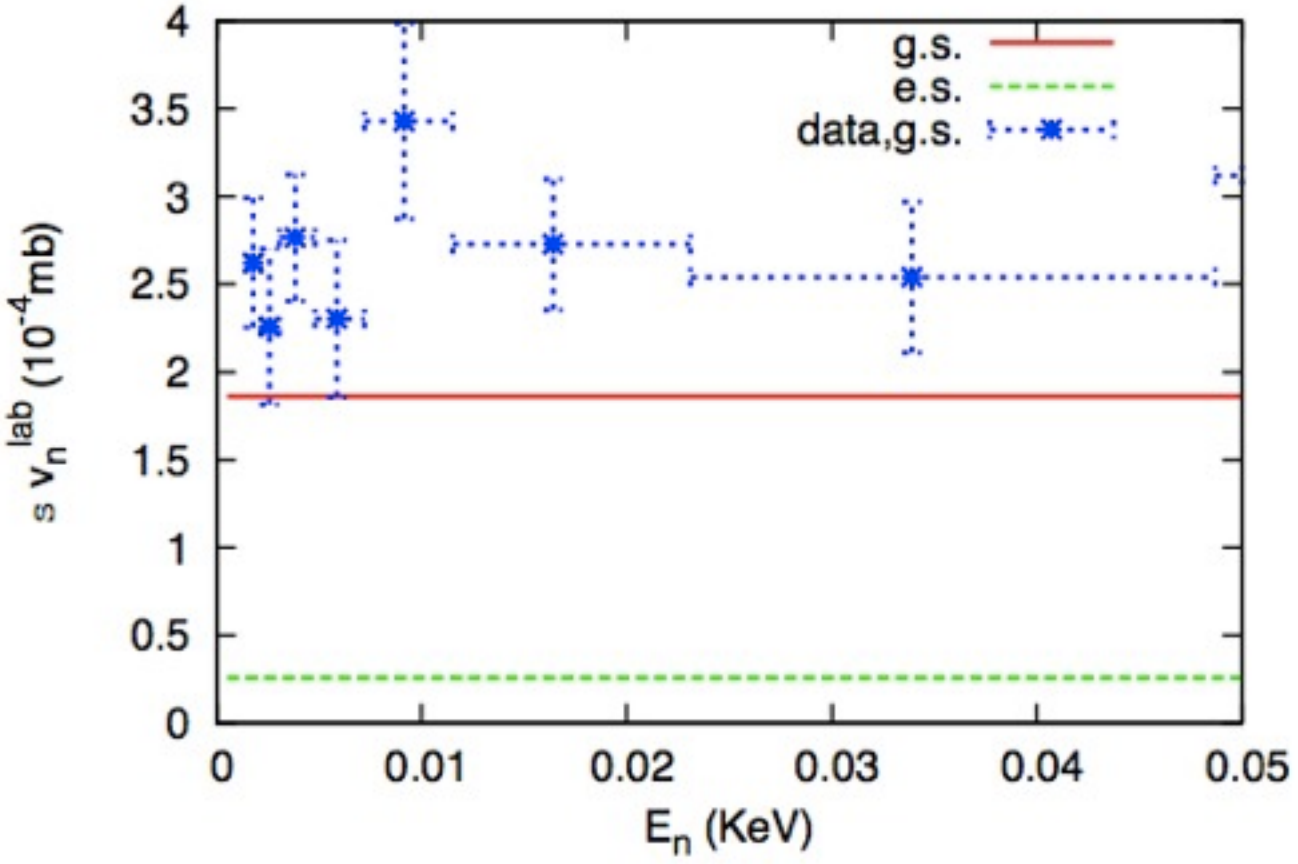

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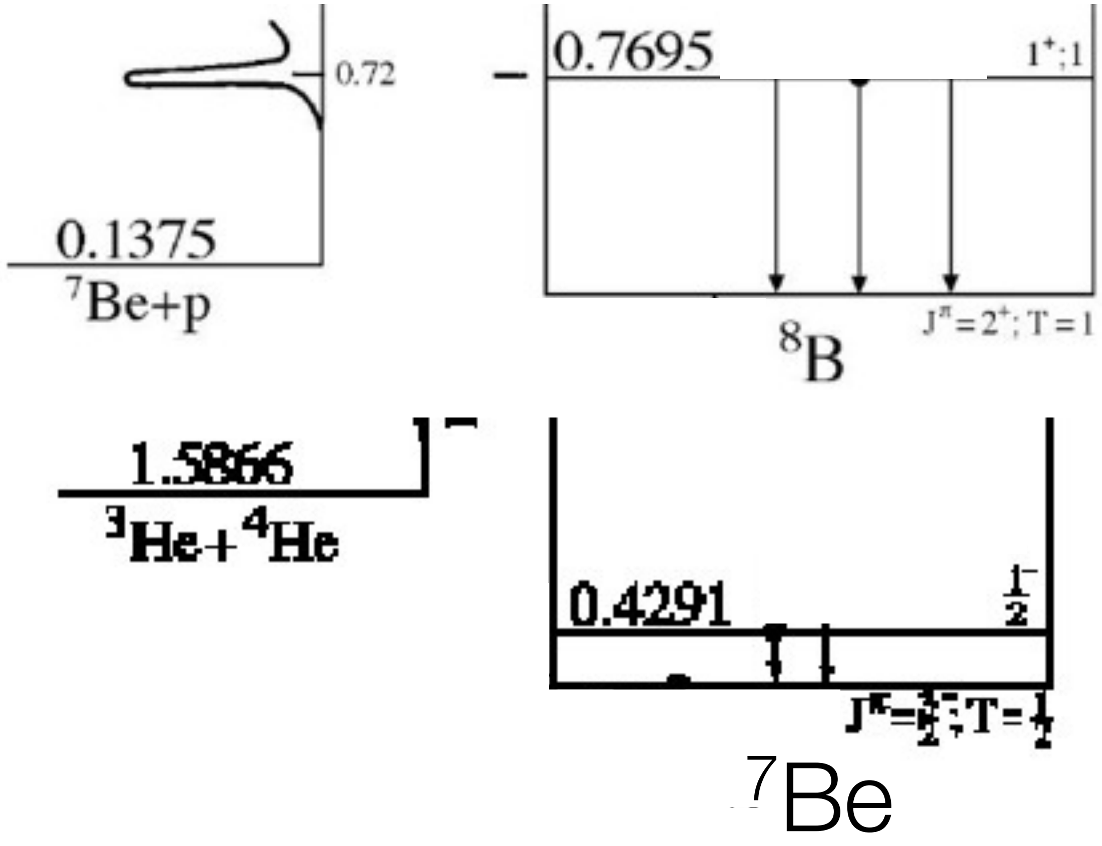
NLO effects include $a_{S=1}$, ^{7}Li* in loop, change in ANCs, and CT for E1 capture

The charged case: ^{7}Be + p→^{8}B + γ_{E1}

- Reaction important to prediction of solar-neutrino flux, esp. at upper energies
- Isospin mirror of ^{7}Li + n→^{8}Li + γ_{E1}
- Now γ_1=15 MeV cf. Λ≈70 MeV



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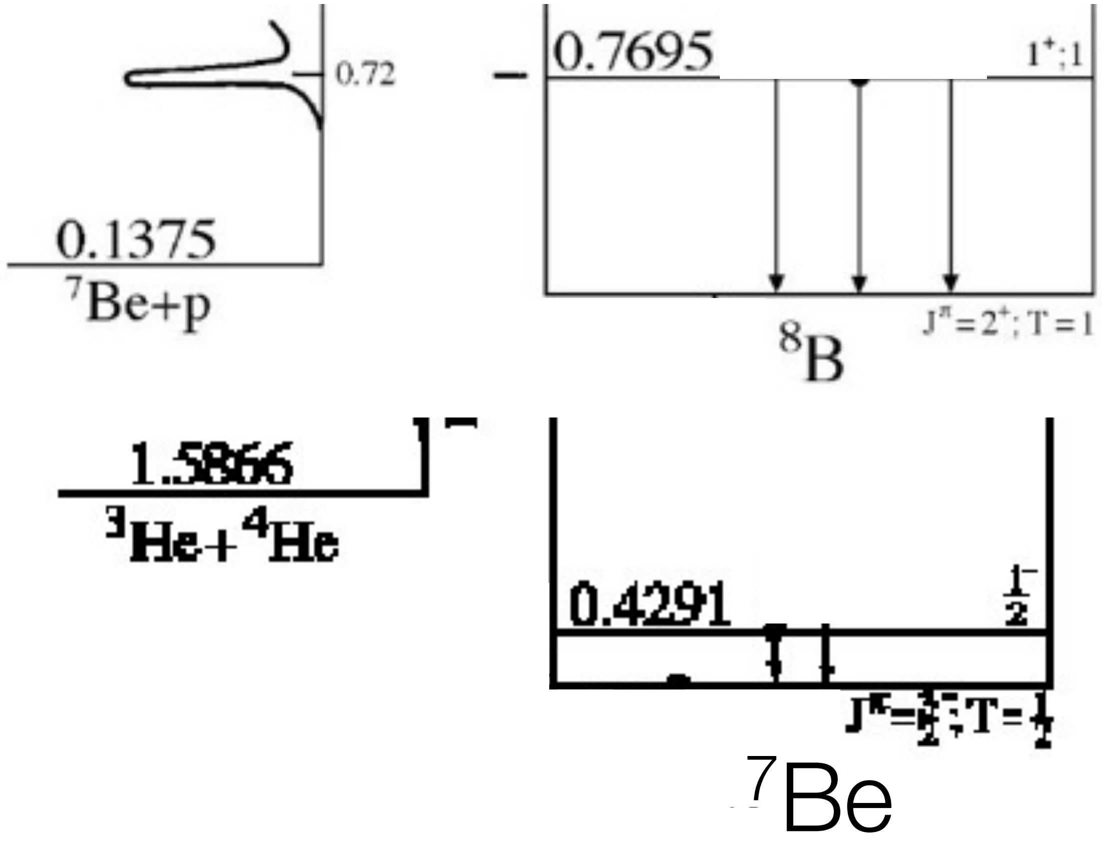

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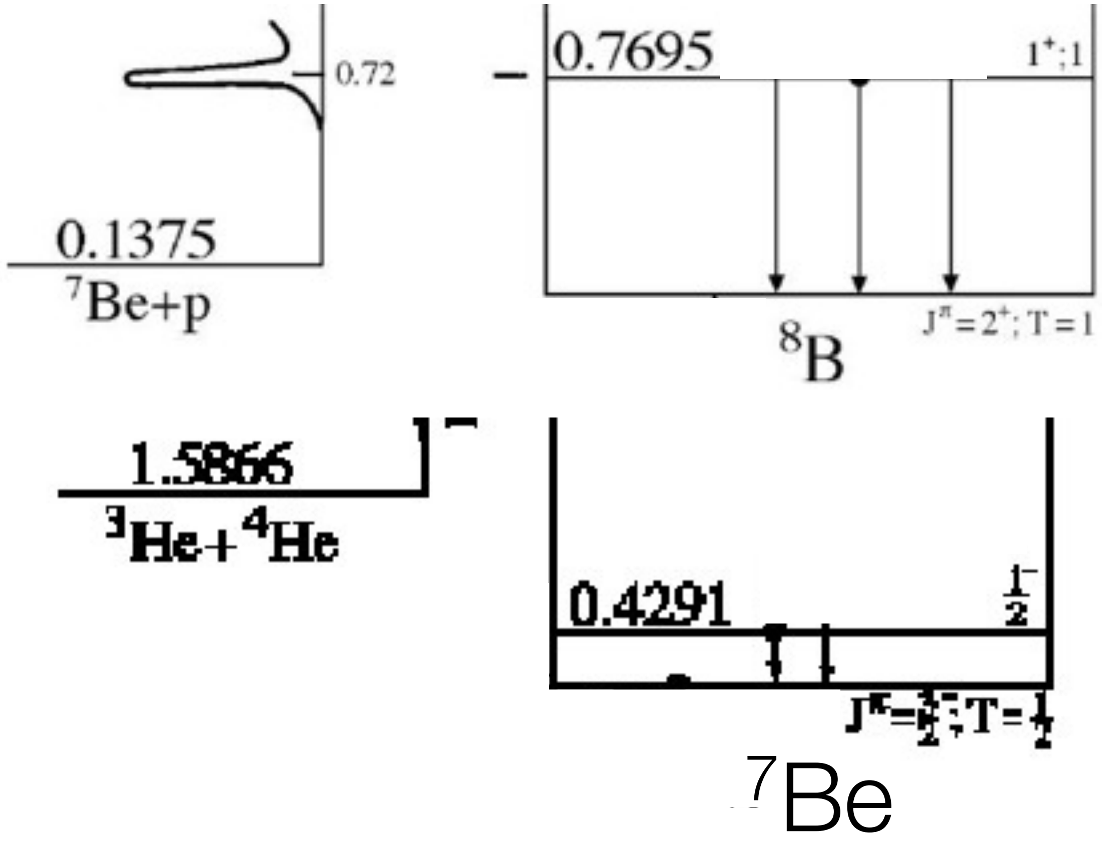

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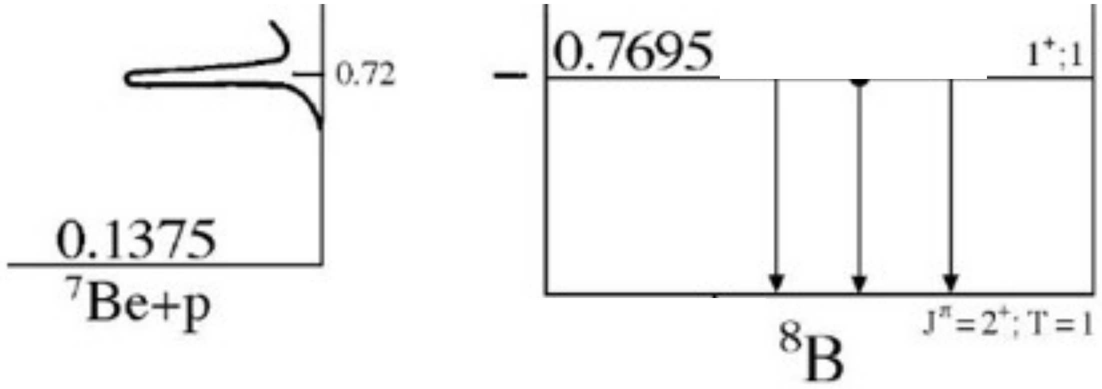

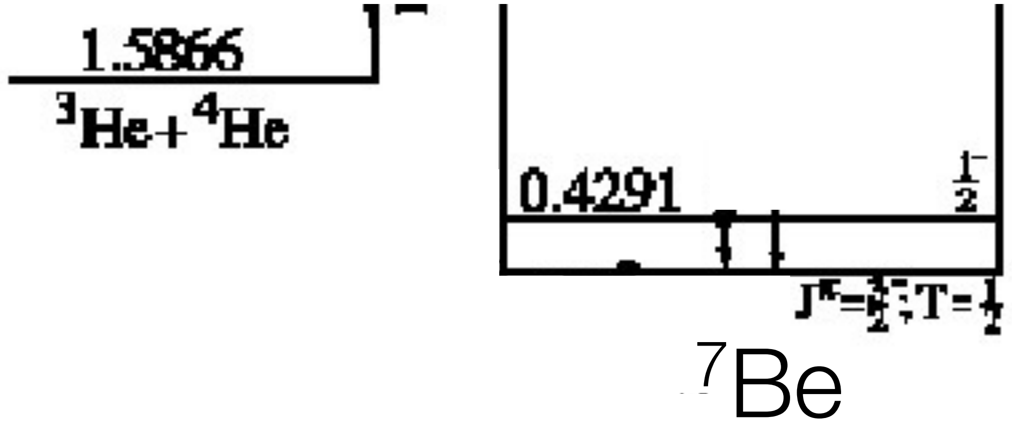

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- LO calculation: ISI in S=2 & S=1 into P-wave bound state. Scattering wave functions are now linear combinations of Coulomb wave functions F_0 and G_0. Bound state wave function=the appropriate Whittaker function.

Proton capture details

Zhang, Nollett, Phillips, arXiv:1401.4482

cf.Ryberg, Forssen, Hammer, Platter (2013)

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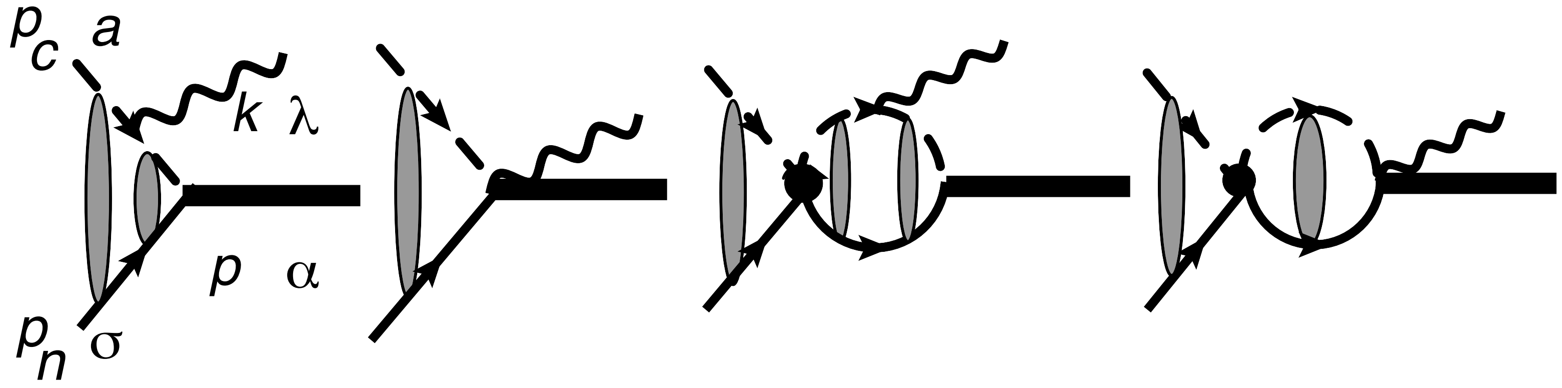

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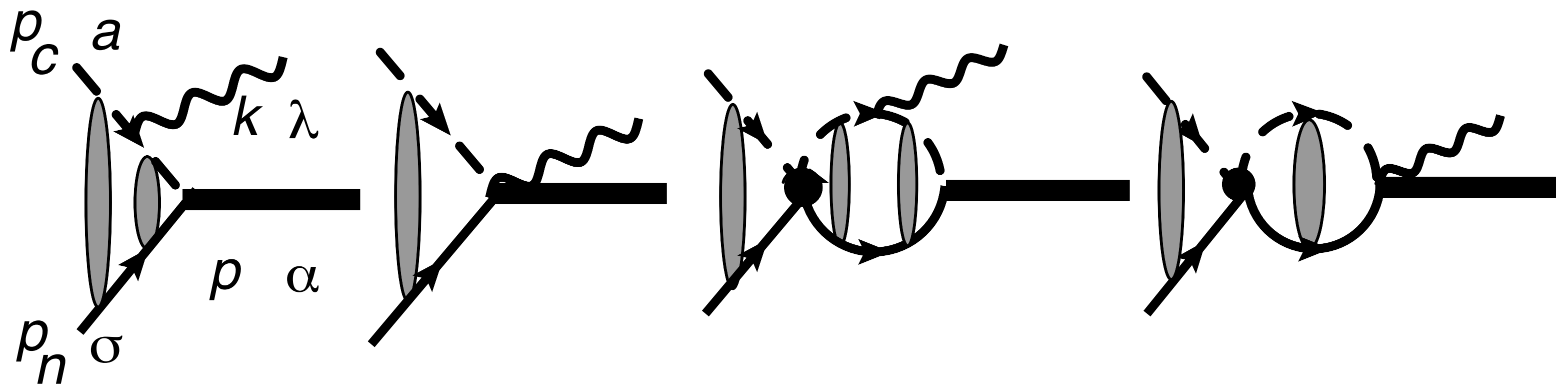

$$S(E)=\frac{e^{2\pi\eta}}{e^{2\pi\eta}-1}\frac{Z_{eff}^2}{M_R^2}\frac{\pi}{24}\omega k_c\left(\gamma^2+k^2\right)^2\ \ \frac{5}{3}\left[{C^{\mathrm{LO}}_{(^3P_2)}}^2\left(\mid\mathcal{S}(^3S_1)\mid^2+2\mid\mathcal{D}\mid^2\right)+{C^{\mathrm{LO}}_{(^5P_2)}}^2\left(\mid\mathcal{S}(^5S_2)\mid^2+2\mid\mathcal{D}\mid^2\right)\right],$$

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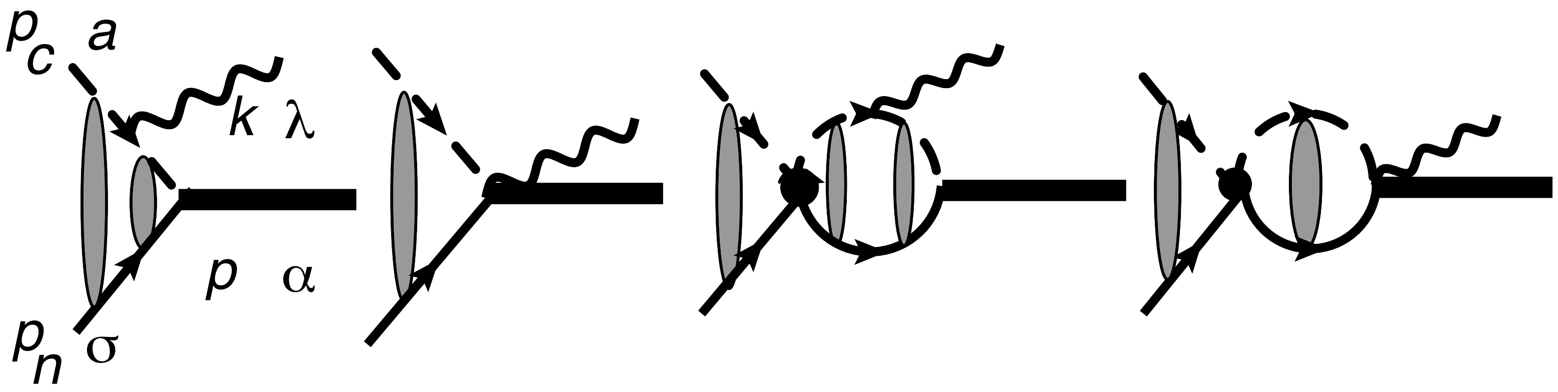

$$S(E)=\frac{e^{2\pi\eta}}{e^{2\pi\eta}-1}\frac{Z_{eff}^2}{M_R^2}\frac{\pi}{24}\omega k_c\left(\gamma^2+k^2\right)^2\;\;\frac{5}{3}\left[{C^{\mathrm{LO}}_{(^3P_2)}}^2\left(\mid\mathcal{S}(^3S_1)\mid^2+2\mid\mathcal{D}\mid^2\right)+{C^{\mathrm{LO}}_{(^5P_2)}}^2\left(\mid\mathcal{S}(^5S_2)\mid^2+2\mid\mathcal{D}\mid^2\right)\right]\;,$$

	$A_{(3P2)}$ (fm$^{-1/2}$)	$A_{(5P2)}$ (fm$^{-1/2}$)	$a_{(S=1)}$ (fm)	$a_{(S=2)}$ (fm)
Nollett	-0.315(19)	-0.662(19)		
Navratil	-0.294	-0.650	-5.2	-15.3
Tabacaru	-0.294(45)	-0.615(45)		
Angulo			25(9)	-7(3)

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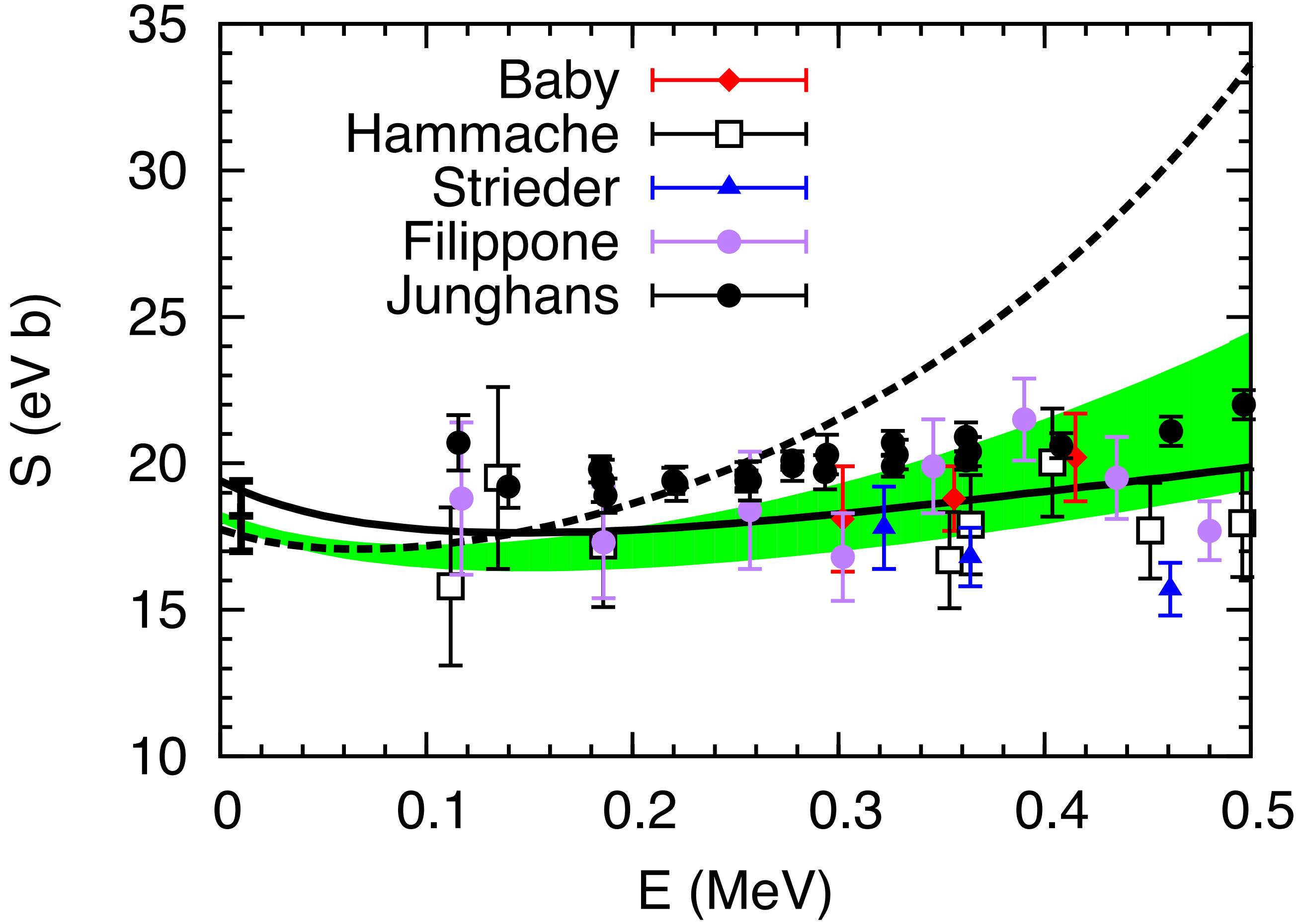

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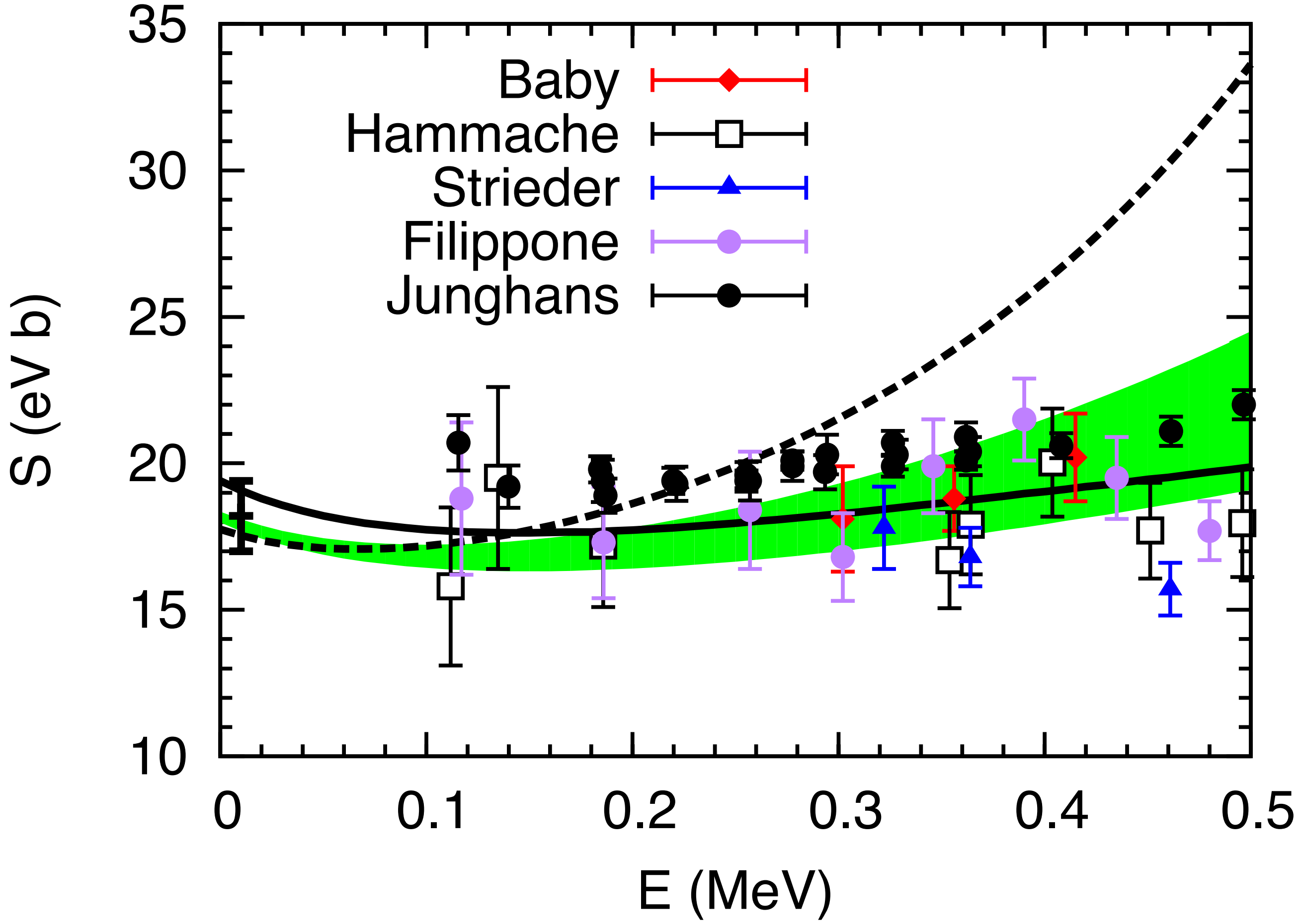

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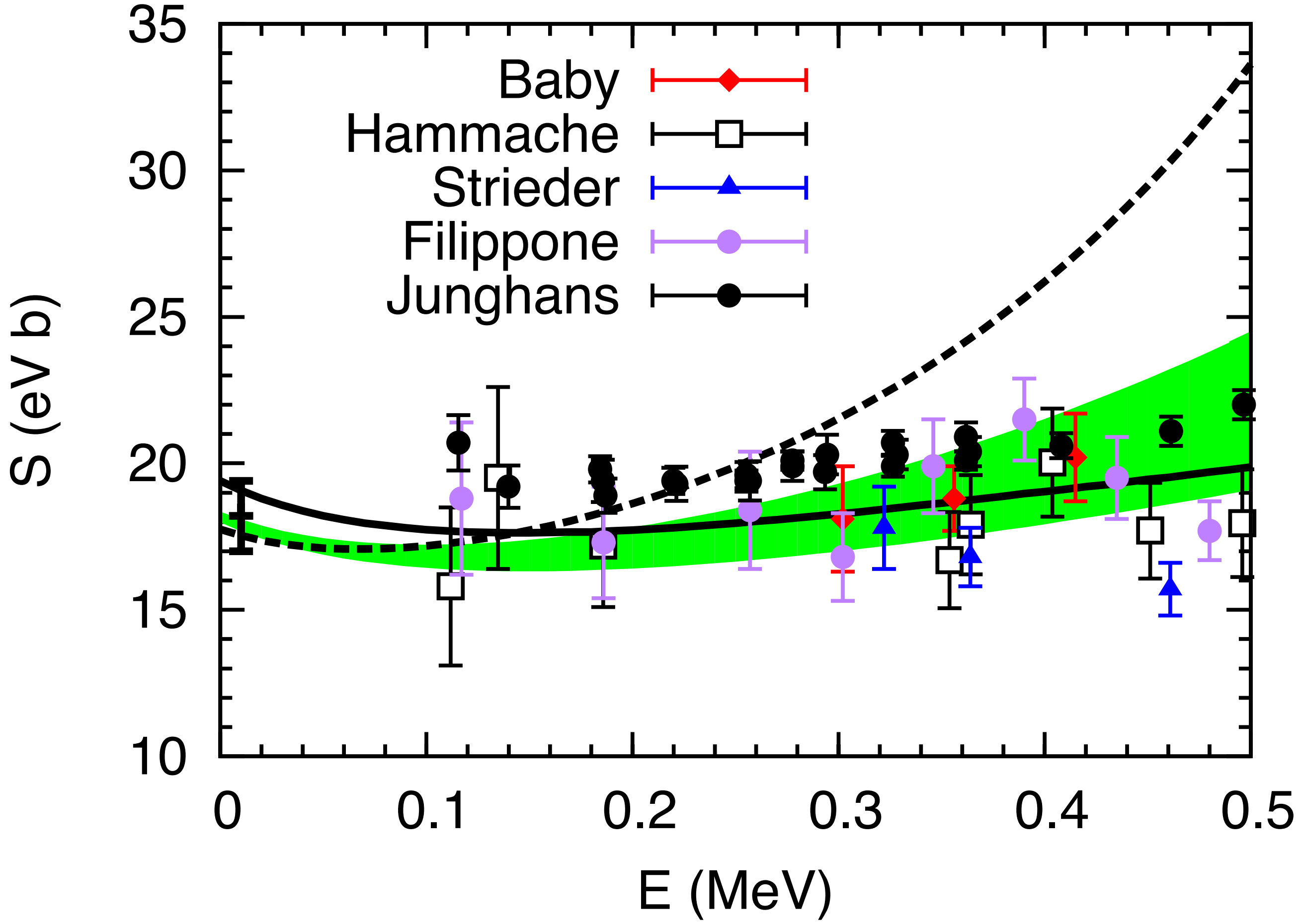

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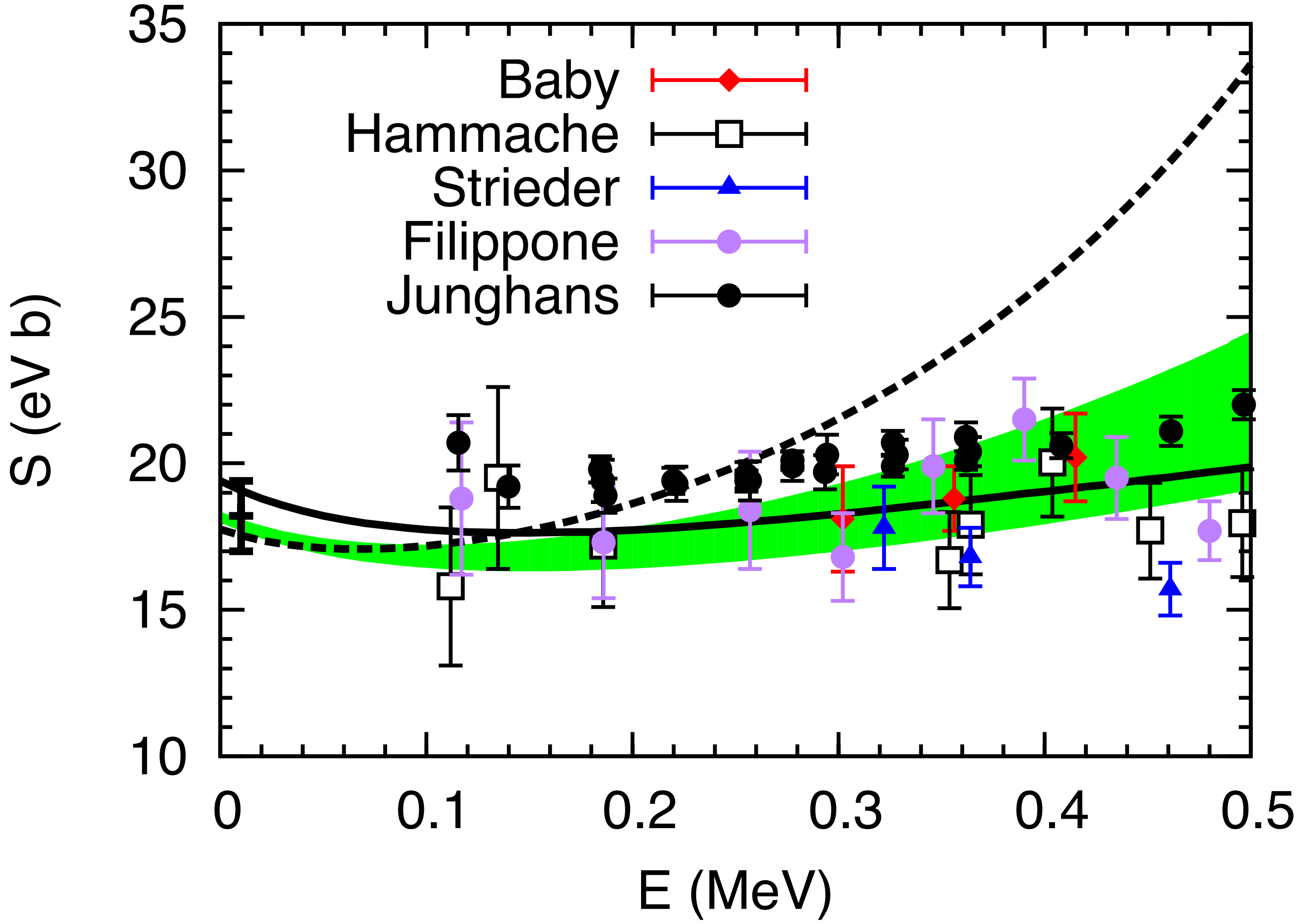

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- Approved TRIUMF experiment on p + ^{7}Be elastic scattering

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Ozawa et al. (2001)

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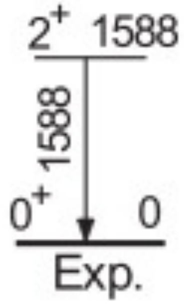

Stanoiu et al. (2008)

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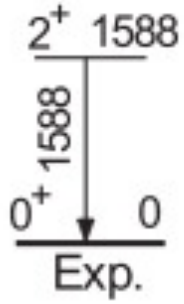

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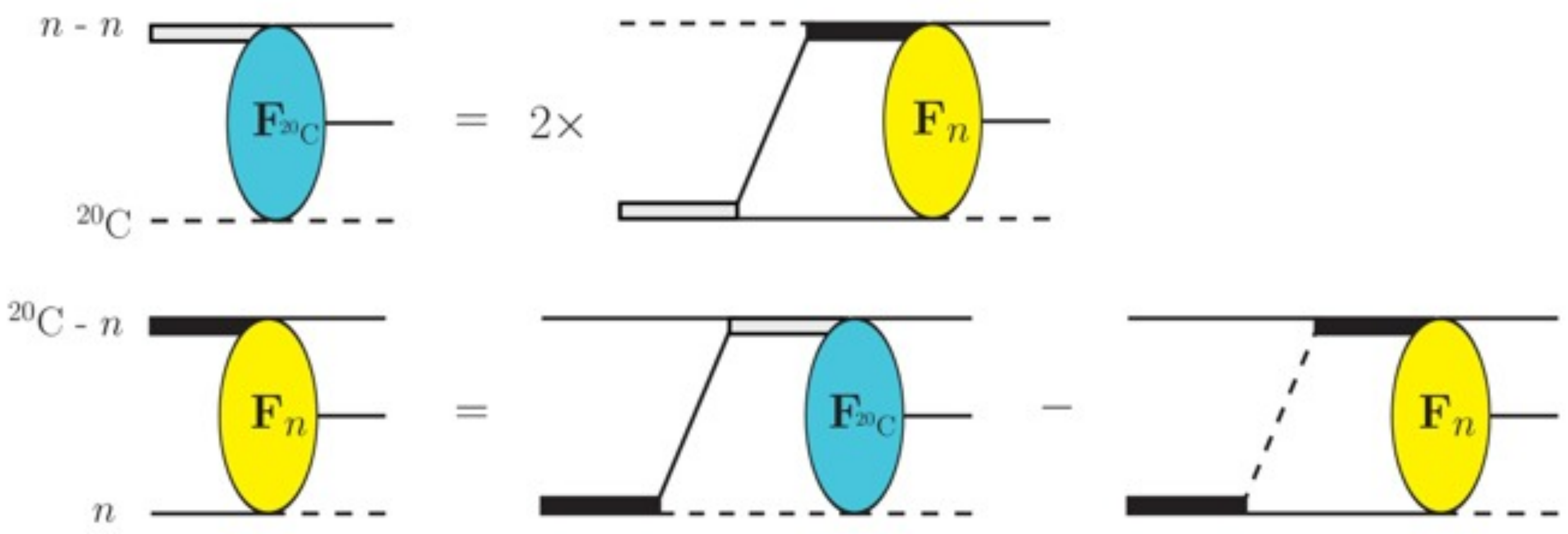
Kobayashi et al. (2012)

Halo EFT for ^{22}C

- Working hypothesis: ^{22}C is an s-wave 2n Borromean halo with a ^{20}C core
- Halo EFT: ^{20}C-n and n-n contact interactions at leading order



Canham,
Hammer (2011)

- ^{21}C-n contact interaction to stabilize three-body system

→Efimov/Thomas effects

- Inputs: $E_{nn}=1/(m\ a_{nn}^2)$=120 keV, E_{nc}, B (=S_{2n})
- Output: everything. At LO accuracy

Universality and matter radii of 2n halos

- Define: $f\left(\frac{E_{nn}}{B}, \frac{E_{nc}}{B}; A\right) \equiv mB\langle r_0^2\rangle$
- “Unitary limit”, $E_{nn}=E_{nc}=0$: f becomes a number depending solely on A

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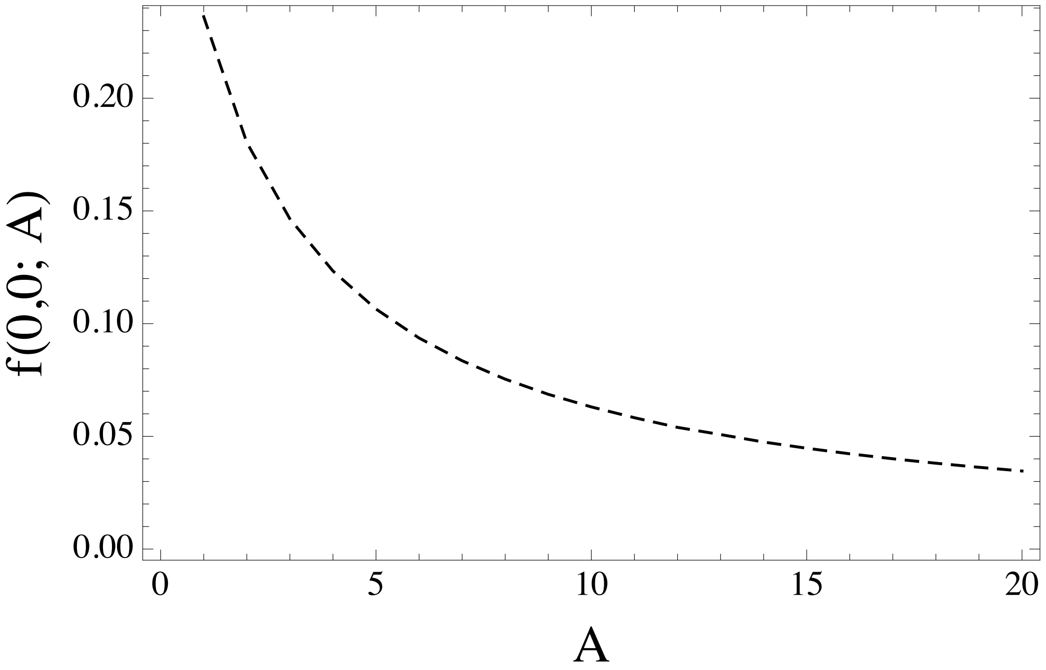

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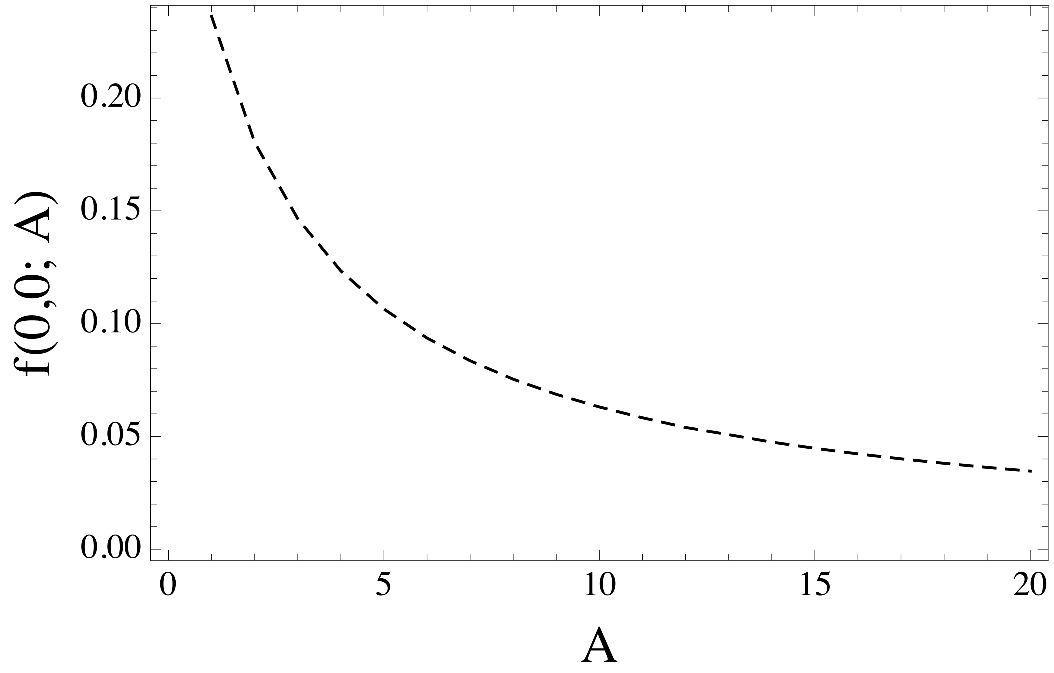

- Fix A=20, plot f as a function of E_{nn} and E_{nc}

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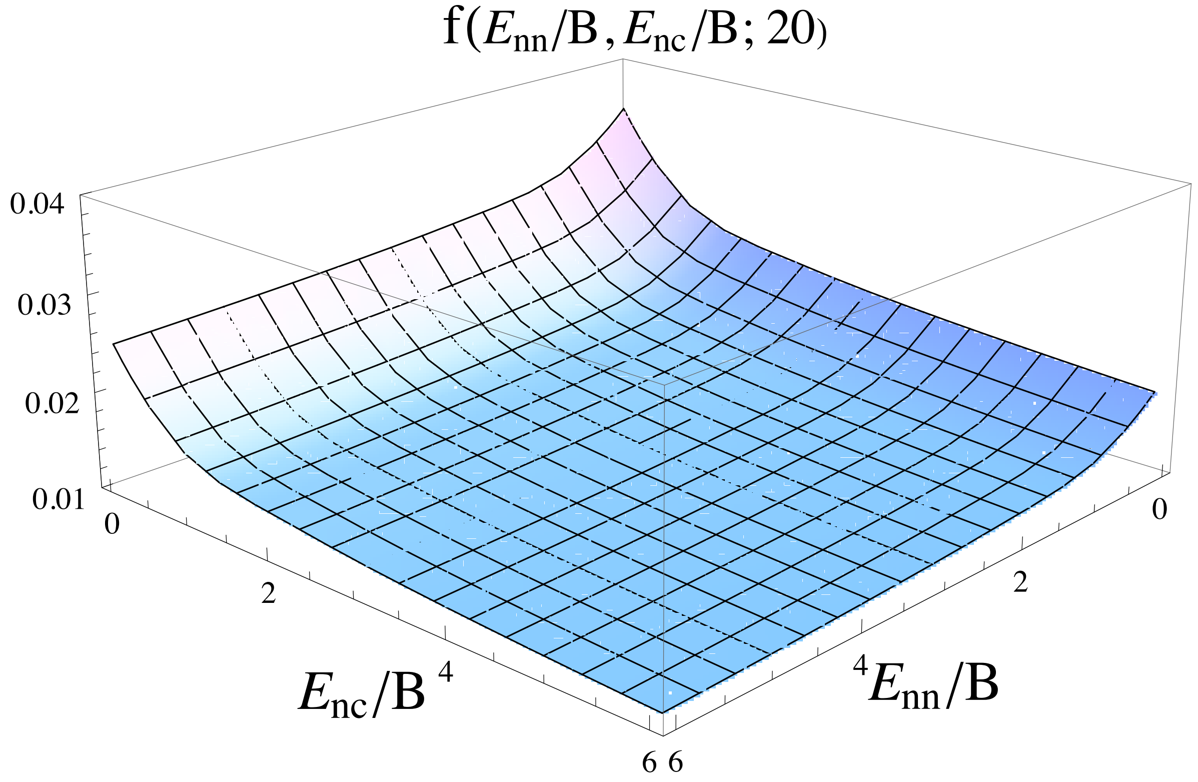
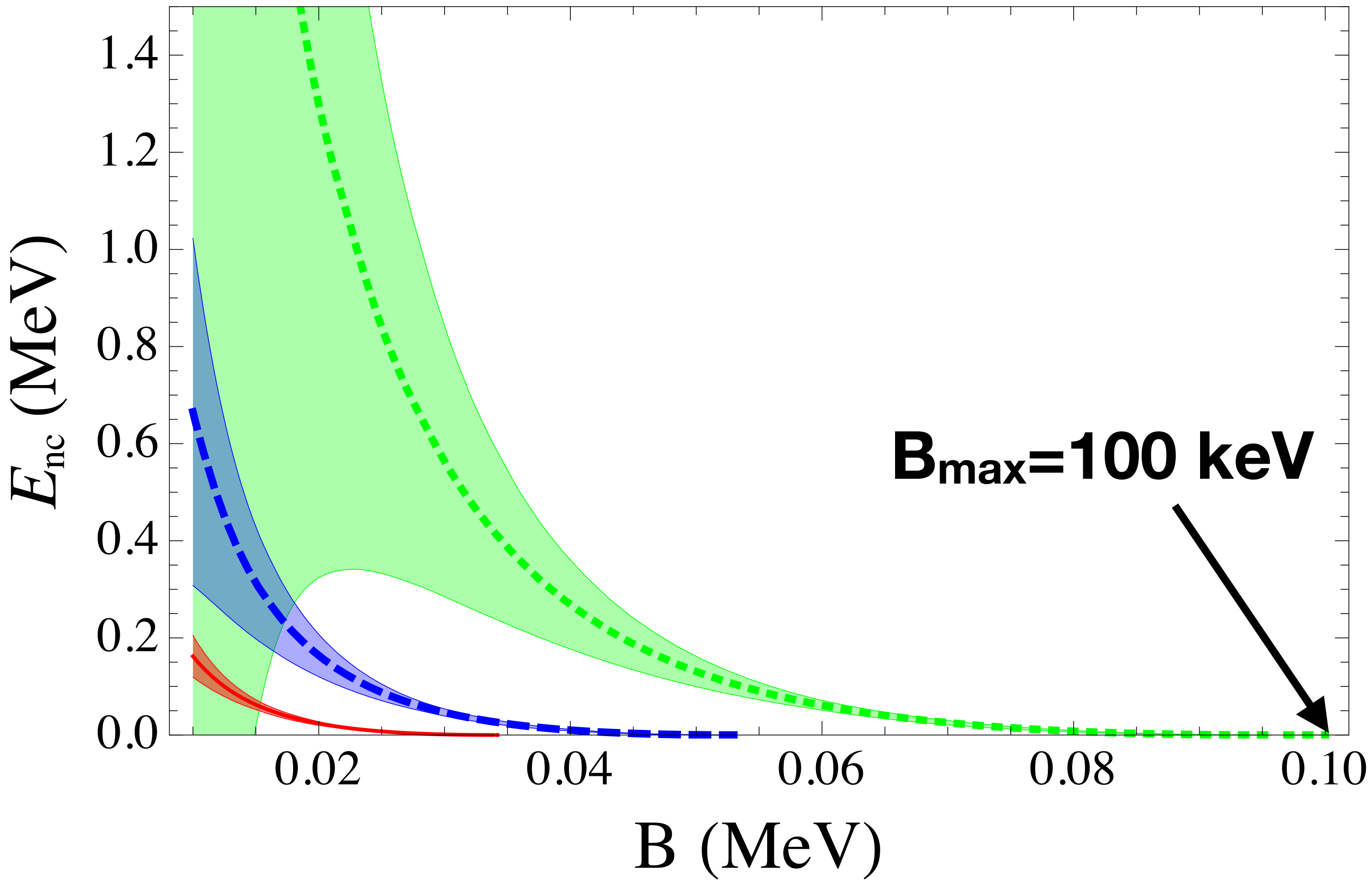


Implications for ^{21}C and ^{22}C

- Include finite size of ^{20}C

- Consider uncertainty due to NLO effects:

Relative size ~ largest of $(mE_{nn})^{1/2}/\Lambda_0$; $(2mE_{nc})^{1/2}/\Lambda_0$; $(2mB)^{1/2}/\Lambda_0$



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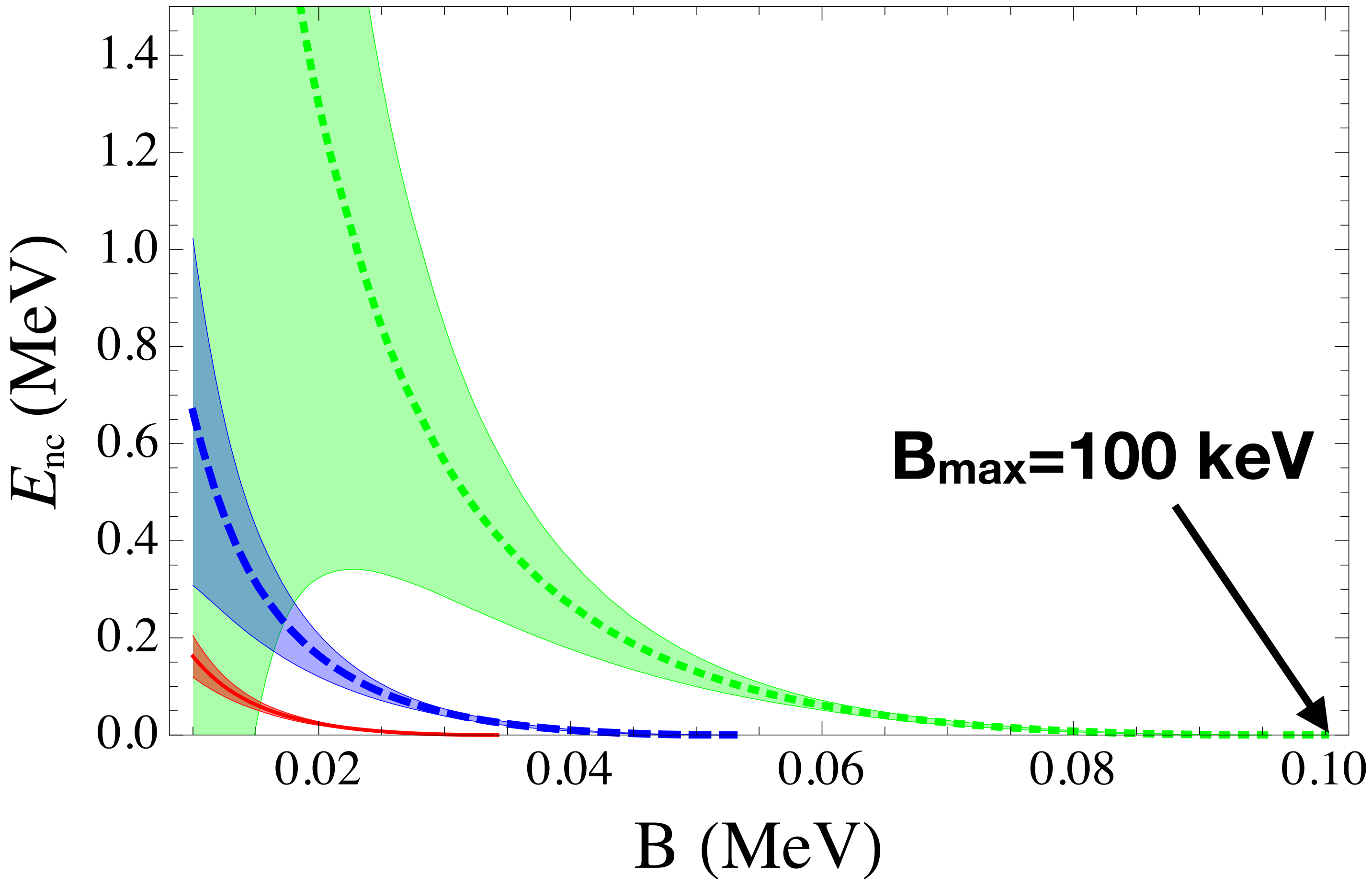

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Mazumdar et al. (2000), Frederico et al. (2012)

Conclusions: photoreactions and one-neutron halos

- Halo EFT describes reactions involving halo nuclei in a systematic expansion in R_{core}/R_{halo}
- Discussion of interior structure (e.g. nodes) of the wave function unnecessary
- Reveals correlations between low-energy observables; complementary and supplementary to *ab initio* calculations
- Connect structure & reactions, *ab initio* theory and different experiments
- ^{19}C: one-neutron halo, shallow S-wave state. Coulomb dissociation can determine a and r_0 rather accurately. Test?
- ^8Li: too many parameters to fit to data. Use *ab initio* input. Successful LO description of a number of capture observables.

Outlook: two-neutron and one-proton halos

- Predictions for ^{22}C radii in terms of input parameters: R_{core}/R_{halo} expansion
- Mosby et al. constraint on ^{21}C + Tanaka et al. $\langle r^2\rangle^{1/2} \Rightarrow {}^{22}$C bound by < 20 keV
- Computation of ^{6}He

Ji, Elster, DP (2014)

- Other 2n halos with s- and p-wave interactions: ^{11}Li, ^{12}Be, ^{62}Ca?

Hagen, Hagen, Hammer, Platter, PRL (2013)

- Coulomb dissociation of 2n halos

Nakamura et al., Experiment on ^{22}C, data taken
Acharya, Hagen, Hammer, DP, in progress

- Charge radius of 2n halos. Signatures of Efimovian physics?

Hagen, Hammer, Platter (2013)

- Proton halos: big E1 strength, e.g. ^{7}Be + p→^{8}B + γ. Good LO description of S(E). Sensitivity to $a_{(S=2)}$, $a_{(S=1)}$. NLO coming
- Also ^{16}O + p→^{17}F* + γ. Radii measurbale: e.g. ^{8}B. Test of halo picture.

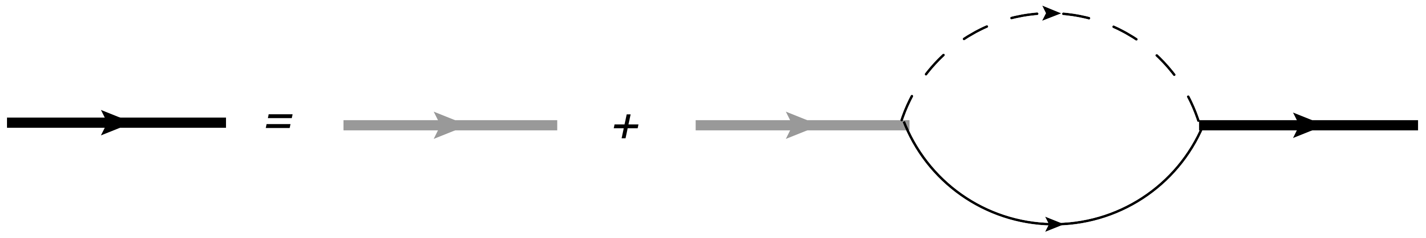
Ryberg, Forssen, Hammer, Platter (2013)

FOR USE IF NEEDED....

Dressing the S-wave state

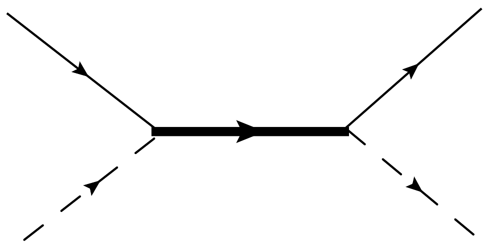
Kaplan, Savage, Wise; van Kolck; Gegelia;
Birse, Richardson, McGovern

- σnc coupling g_0 of order R_{halo}, nc loop of order $1/R_{\text{halo}}$. Therefore need to sum all bubbles:



$$D_\sigma(p) = \frac{1}{\Delta_0 + \eta_0[p_0 - \mathbf{p}^2/(2M_{nc})] - \Sigma_\sigma(p)}$$

$$\Sigma_\sigma(p) = -\frac{g_0^2 m_R}{2\pi}\left[\mu + i\sqrt{2m_R\left(p_0 - \frac{\mathbf{p}^2}{2M_{nc}} + i\eta\right)}\right] \qquad \text{(PDS)}$$



$$t = \frac{2\pi}{m_R}\frac{1}{\frac{1}{a_0} - \frac{1}{2}r_0 k^2 + ik}$$

$$D_\sigma(p) = \frac{2\pi\gamma_0}{m_R^2 g_0^2}\frac{1}{1 - r_0\gamma_0}\frac{1}{p_0 - \frac{\mathbf{p}^2}{2M_{nc}} + B_0} + \text{regular}$$

Counting in S waves:
$a \sim R_{\text{halo}} \sim 1/\gamma_0$; $r_0 \sim R_{\text{core}}$.
$r_0=0$ at LO.

Enter P-wave states: $\gamma_{E1} + {}^{11}\mathrm{Be} \rightarrow {}^{10}\mathrm{Be} + n$

Typel & Baur, Phys. Rev. Lett. 93, 142502 (2004); Nucl. Phys. A759, 247 (2005); Eur. Phys. J. A 38, 355 (2008)

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$$k^3 \cot \delta_1 = -1/2\, r_1 (k^2 + \gamma_1^2) \Rightarrow \delta_1 \sim R_{core}/R_{halo} \text{ if } k \sim 1/R_{halo} \sim \gamma_1.$$

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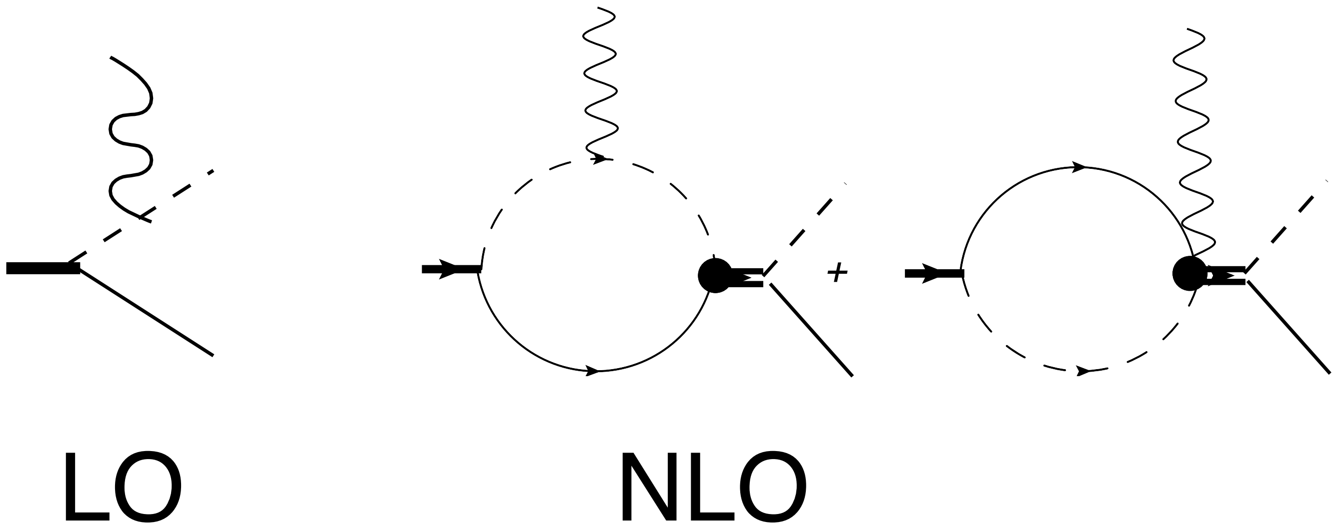

- Need both γ_1 and $r_1 \equiv A_1$ at NLO in this observable. A_0 also becomes a free parameter at NLO: fit it to Coulomb dissociation data

Coulomb dissociation: formulae

c.f. Rupak & Higa arXiv:1101.0207

- Straightforward computation of diagrams yields:

$$\frac{d\mathrm{B(E1)}}{dE} = e^2 Z_{eff}^2 \frac{m_R}{2\pi^2} A_0^2 \left(\frac{p'^3[2p'^3\cot(\delta^{(1/2)}(p')) + \gamma_0^3 + 3\gamma_0 p'^2]^2}{[p'^6 + p'^6\cot^2(\delta^{(1/2)}(p'))](p'^2+\gamma_0^2)^4} + \frac{8p'^3}{(p'^2+\gamma_0^2)^4} \right)$$

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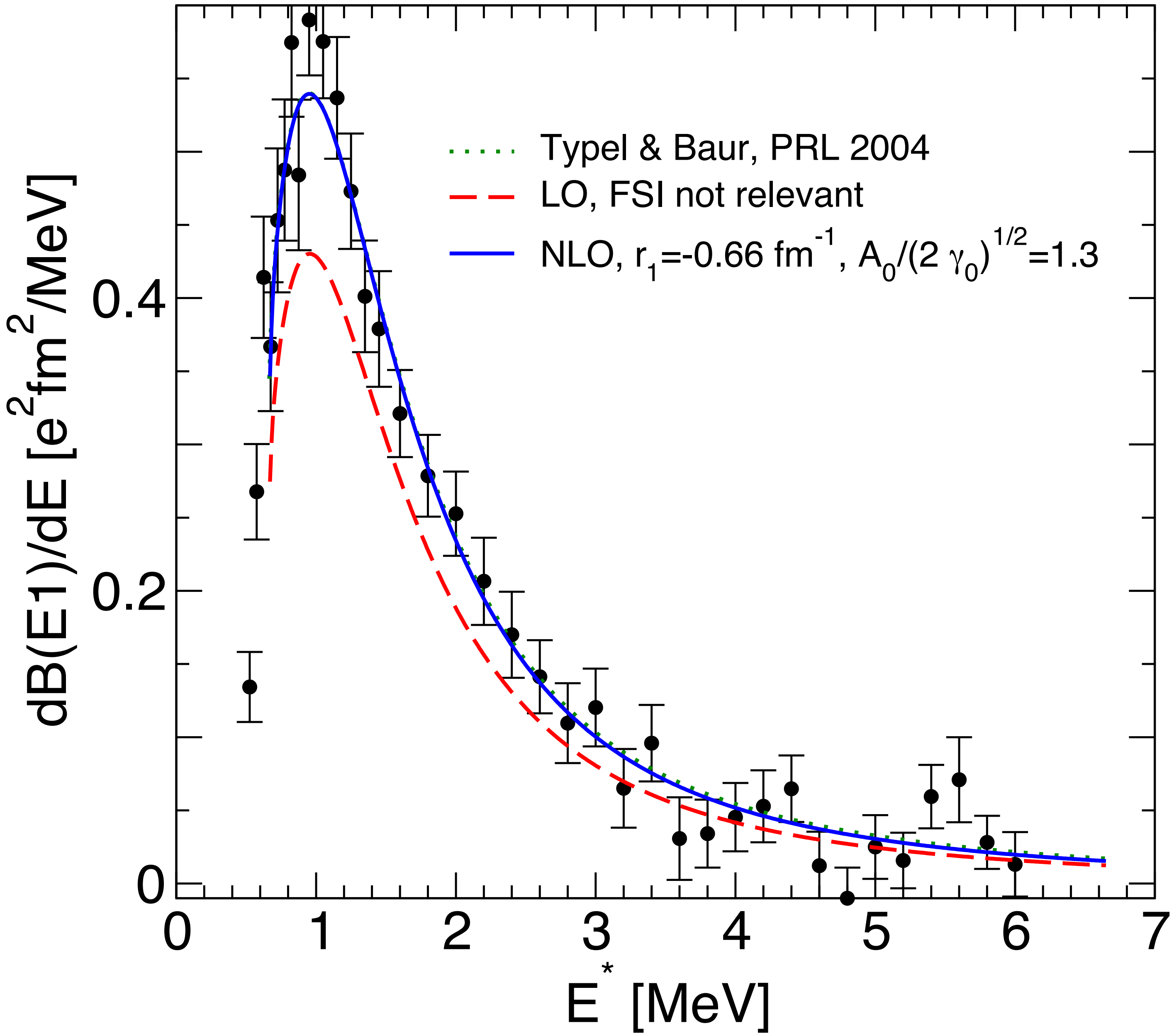
- Higher-order corrections to phase shift at NNLO. Appearance of S-to-$^2P_{1/2}$ E1 counterterm also at that order.

Coulomb dissociation: result

Data: Palit et al., 2003
Analysis: Hammer, Phillips. NPA, 2011



- Reasonable convergence
- Information on value of r_0 through fitting of A_0:

r_0=2.7 fm

Need P-wave effective range

- Here value of r_1 used to fit B(E1:1/2$^+$→1/2$^-$) works.

r_1=-0.66 fm^{-1}

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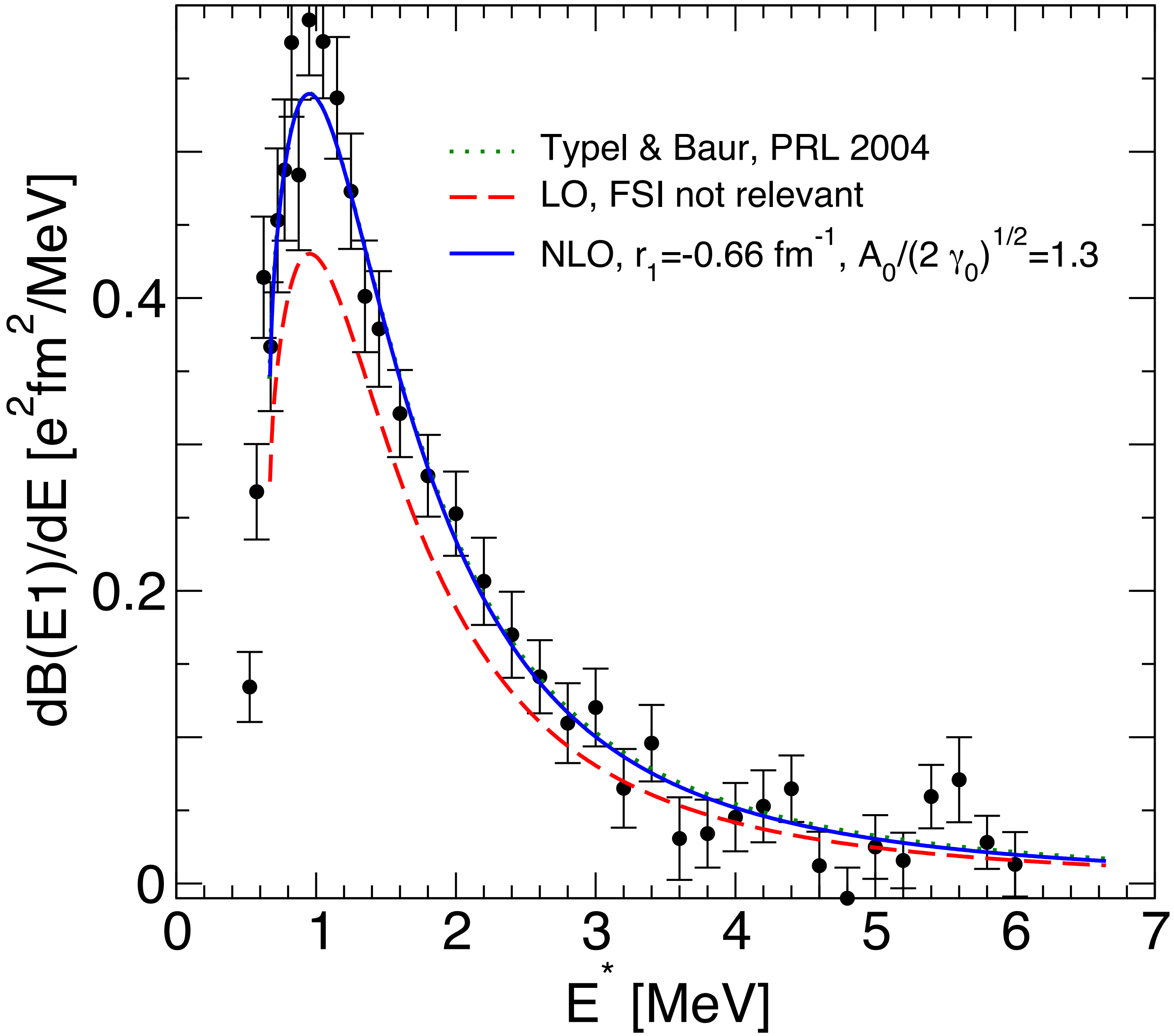

- Reasonable convergence
- Information on value of r_0 through fitting of A_0:

r_0=2.7 fm

Need P-wave effective range

- Here value of r_1 used to fit B(E1:1/2$^+$→1/2$^-$) works.

r_1=-0.66 fm^{-1}

EFT gives **universal relations** between different observables

Coulomb dissociation: result

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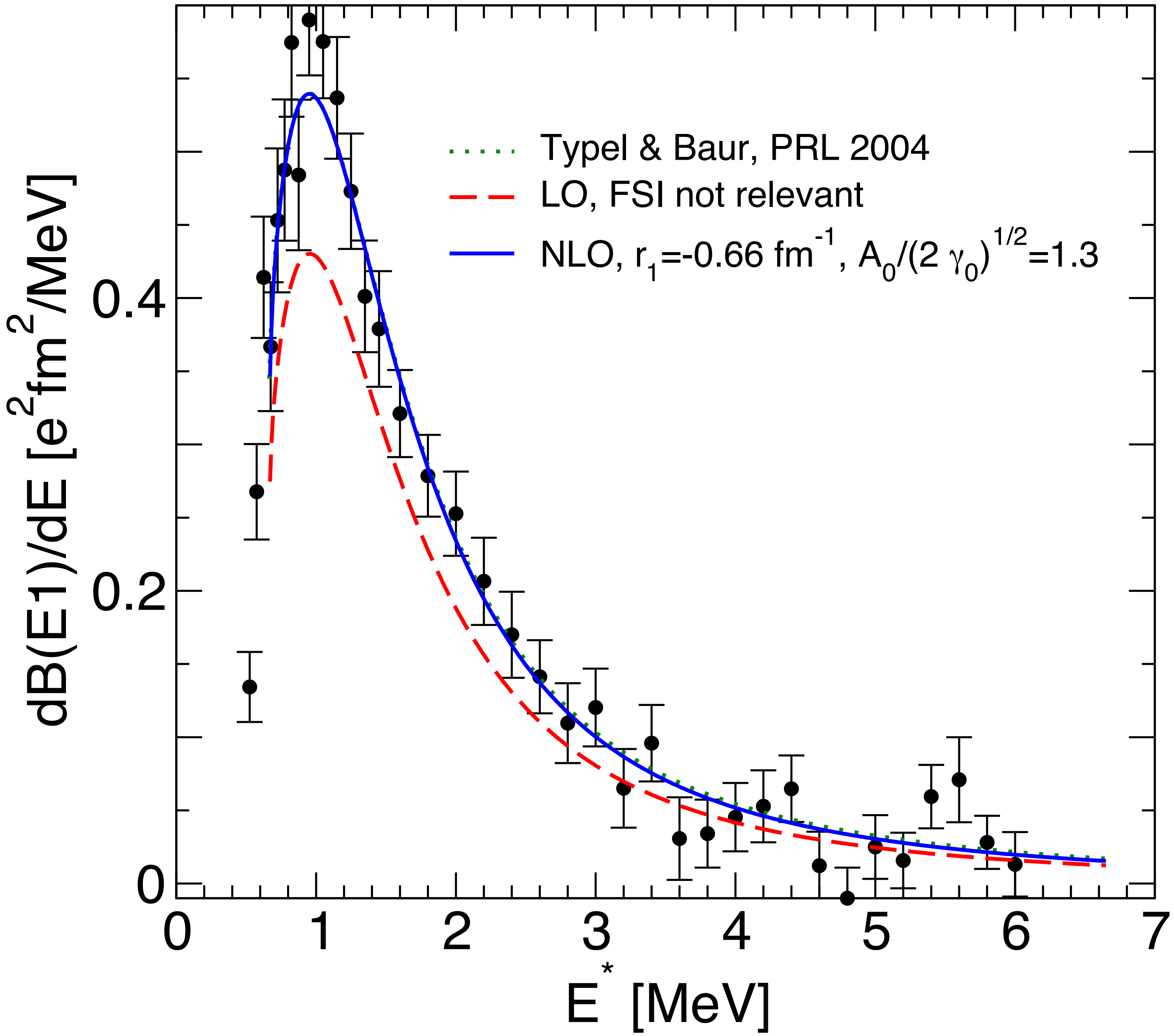

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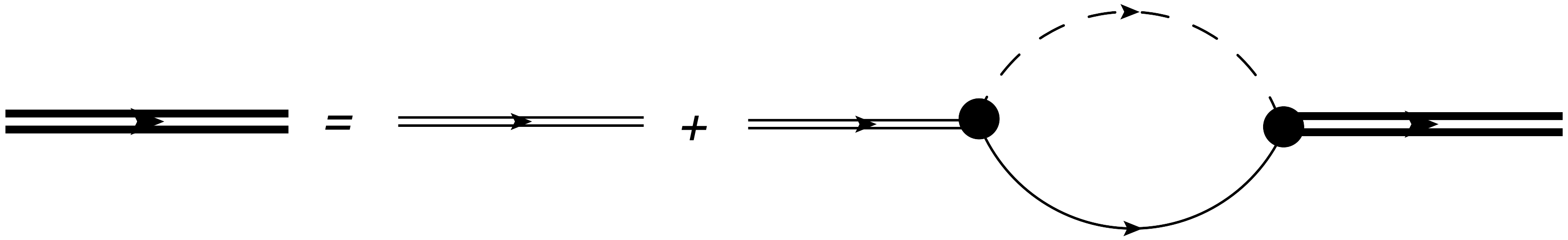
Other ANC/r_1 measurements? Tests of universal relations?

Dressing the P-wave state

Bertulani, Hammer, van Kolck (2002); Bedaque, Hammer, van Kolck (2003)

- Proceed similarly for P-wave state as for S-wave state



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$$\Sigma_\pi(p) = -\frac{m_R g_1^2 k^2}{6\pi}\left[\frac{3}{2}\mu + ik\right]$$

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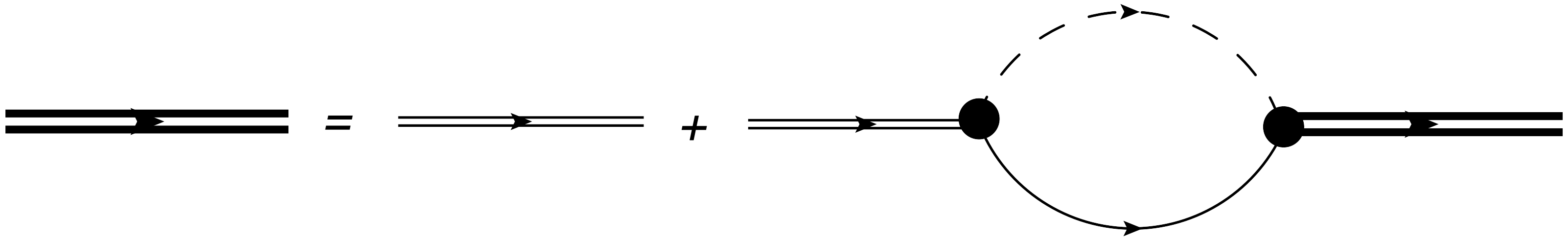

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$$D_\pi(p) = -\frac{3\pi}{m_R^2 g_1^2}\frac{2}{r_1 + 3\gamma_1}\frac{i}{p_0 - \mathbf{p}^2/(2M_{nc}) + B_1}$$ + regular

Counting in P waves:
$R_{halo} \sim 1/\gamma_1$; $r_1 \sim R_{core}$.
$A_1 \longleftrightarrow r_1$ at LO

ANCs from an integral relation

Nollett, Wiringa, PRC (2011)

- VMC calculation using AV18 + UIX

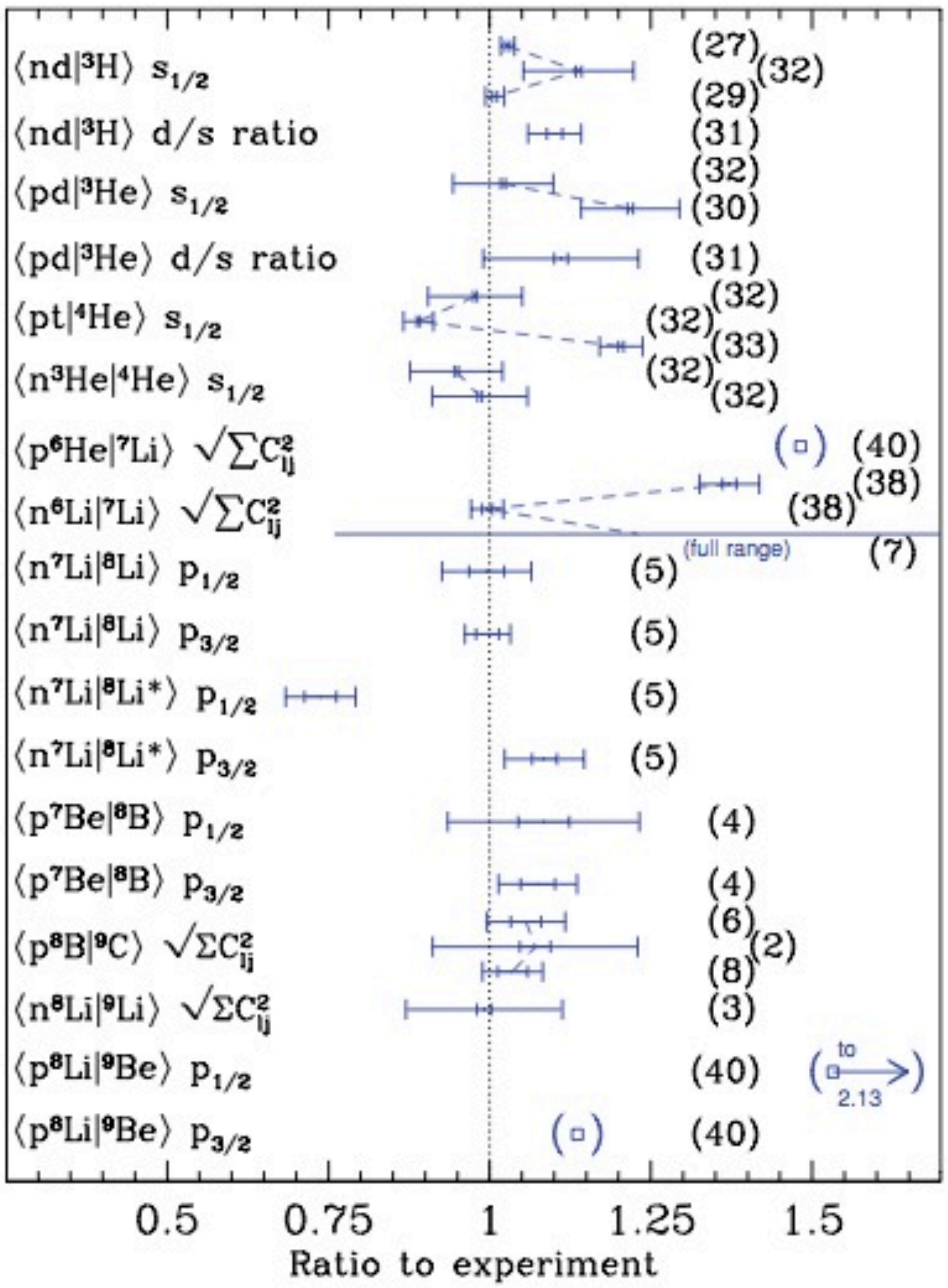
- Integral relation:

$$C_{lj} = \frac{2\mu}{k\hbar^2 w}\mathcal{A}\int \frac{M_{-\eta m}(2kr_{\rm cc})}{r_{\rm cc}}\Psi^{\dagger}_{A-1}\chi^{\dagger}Y_l^{\dagger}(\hat{\mathbf{r}}_{\rm cc}) \times (U_{\rm rel} - V_C)\Psi_A\, d\mathbf{R}.$$

facilitates extraction using MC sampling

- Also results for resonant states

Nollett, PRC (2012)



Matter radii of 2n s-wave halos

- Wave function: $\Psi_c(p,q) =$

- One-body form factors:

$$\mathcal{F}_x(k^2) = \int_0^\infty \mathrm{d}p\; p^2 \int_0^\infty \mathrm{d}q\; q^2 \int_{-1}^{1} \mathrm{d}\left(\hat{q}\cdot\hat{k}\right) \Psi_x(p,q)\;\Psi_x(p,|\vec{q}-\vec{k}|).$$

x=c,n

Canham, Hammer (2011)

- Radii: $\mathcal{F}_x(k^2) = 1 - \frac{1}{6}\langle r_x^2\rangle k^2 + O(k^4)$

- Matter radius:

$$\langle r_0^2\rangle = \frac{2(A+1)^2}{(A+2)^3}\langle r_n^2\rangle + \frac{4A}{(A+2)^3}\langle r_c^2\rangle$$

- So matter radius can be computed straightforwardly, for a given E_{nc} and B

Efimov states in ^{22}C

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Mazumdar et al. (2000), Frederico et al. (2012)

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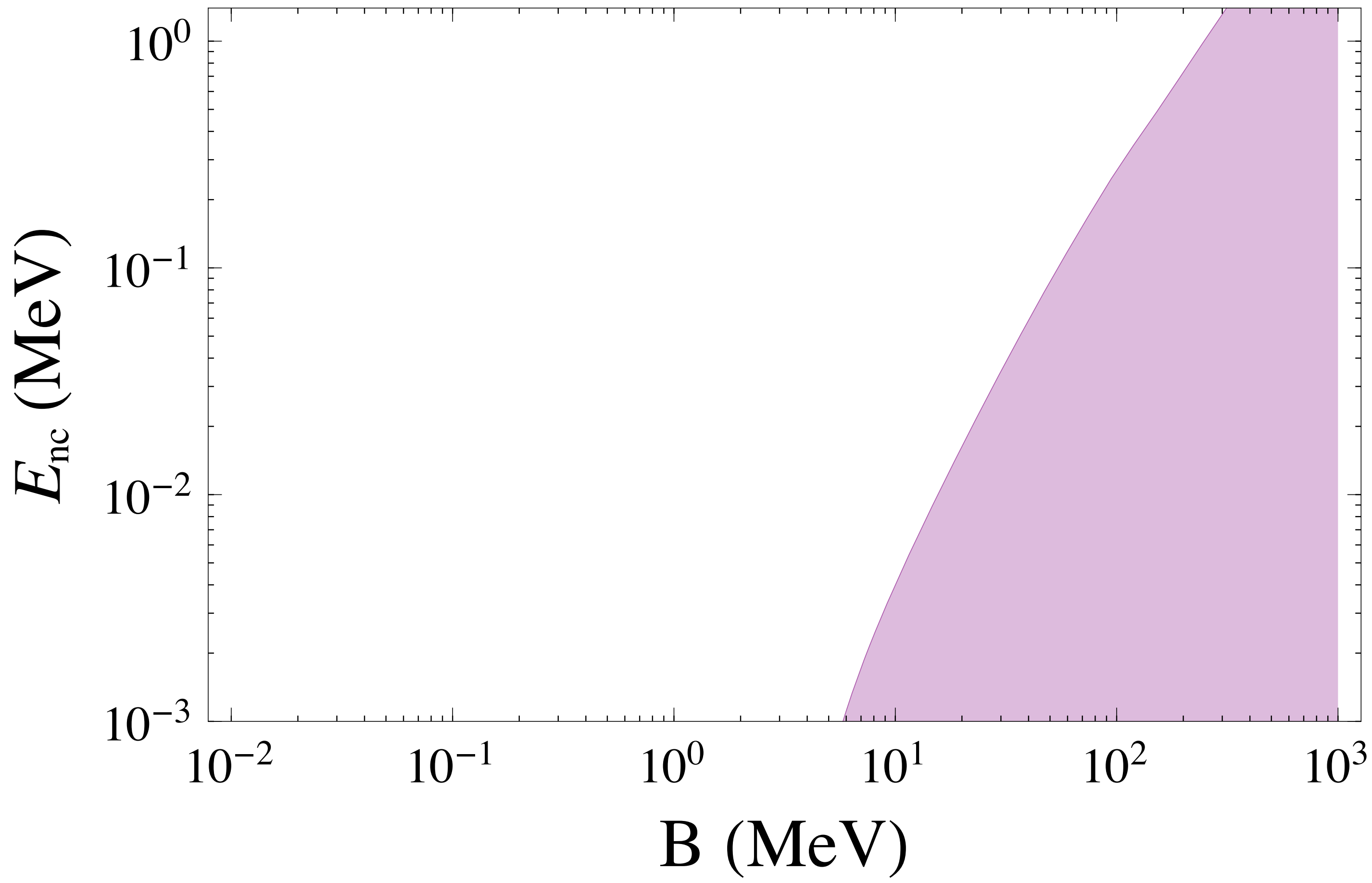

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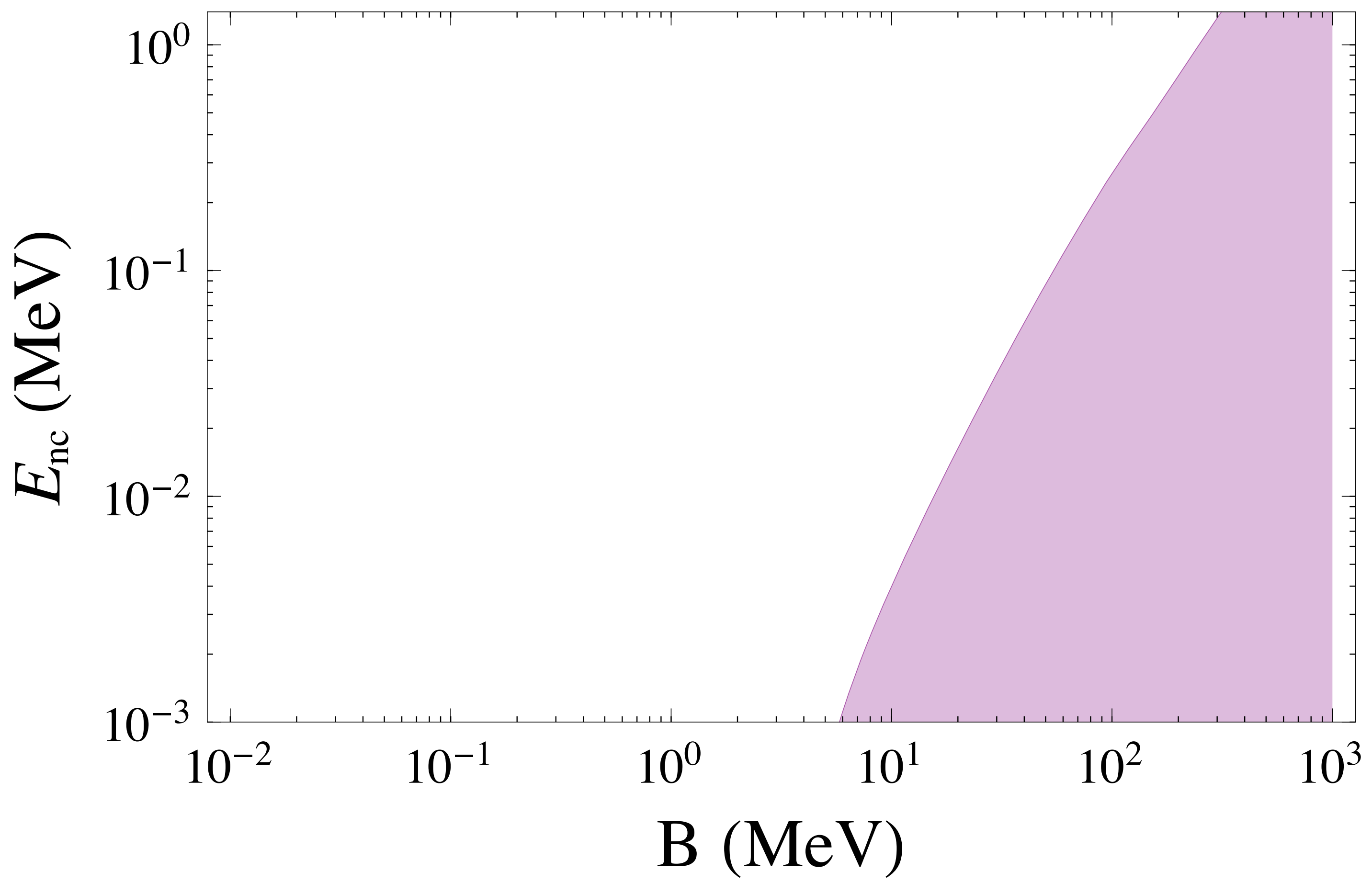

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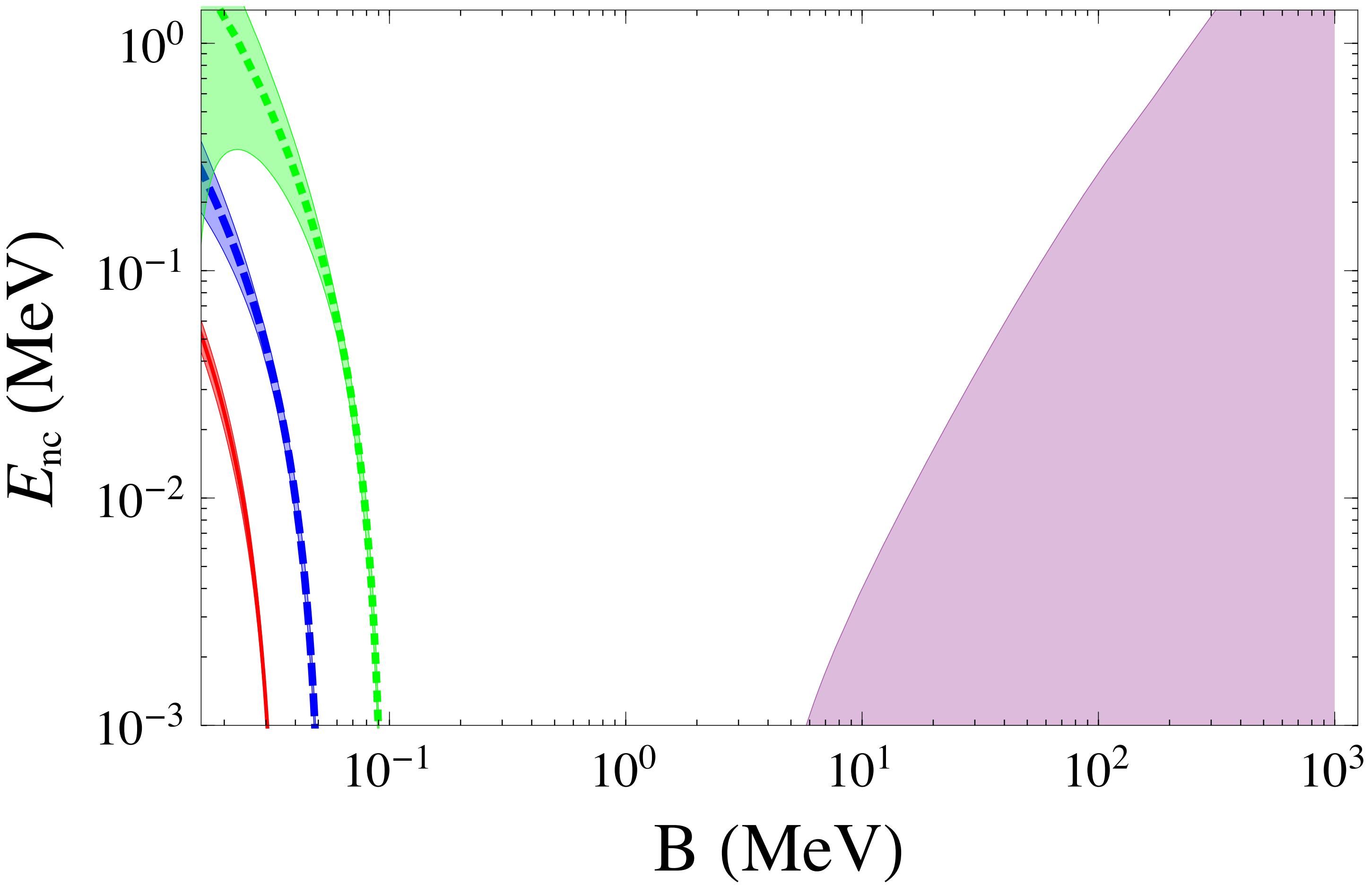

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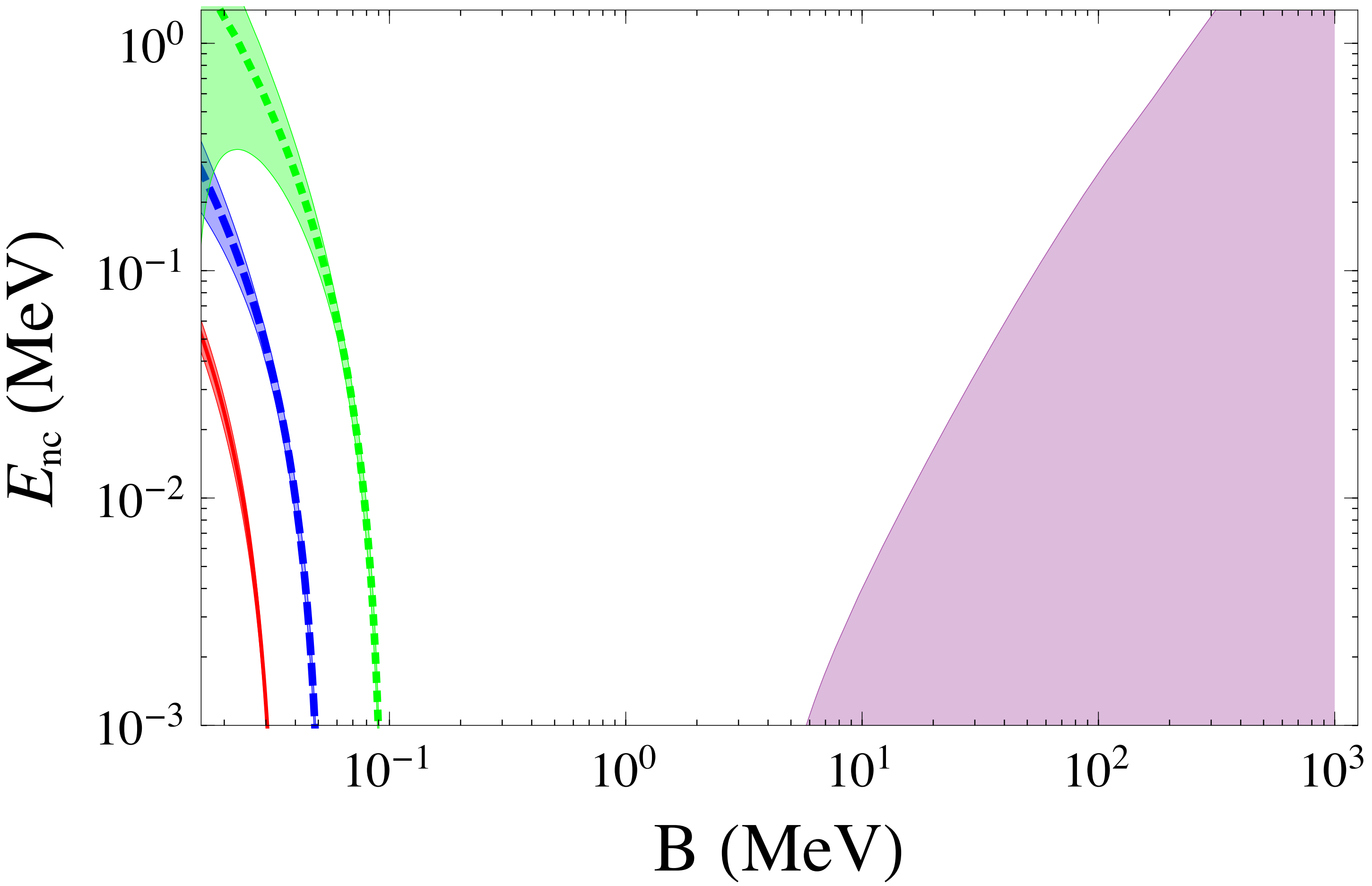


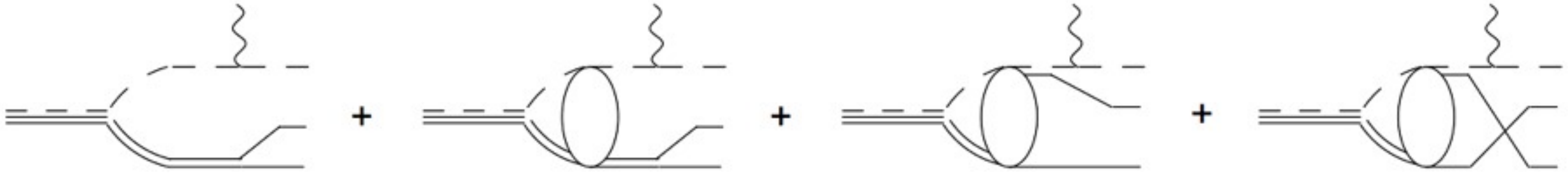
- But ground state of ^{22}C bound by at most 100 keV

- No Efimov state in ^{22}C unless ^{21}C resonance occurs within 1 keV of threshold

Coulomb dissociation of two-neutron halos

Acharya, Hagen, Hammer, Phillips, in progress

- Extend treatment of 1n halos to 2n halos
- Calculation “without FSI”, for example



- Agrees with NEWSR. Formulae with FSI coded, results in progress
- Current application is to ^{22}C

Nakamura et al., data being analyzed

- Differential distributions contain information on sub-system interactions
- Could apply to ^{6}He, ^{11}Li, etc.

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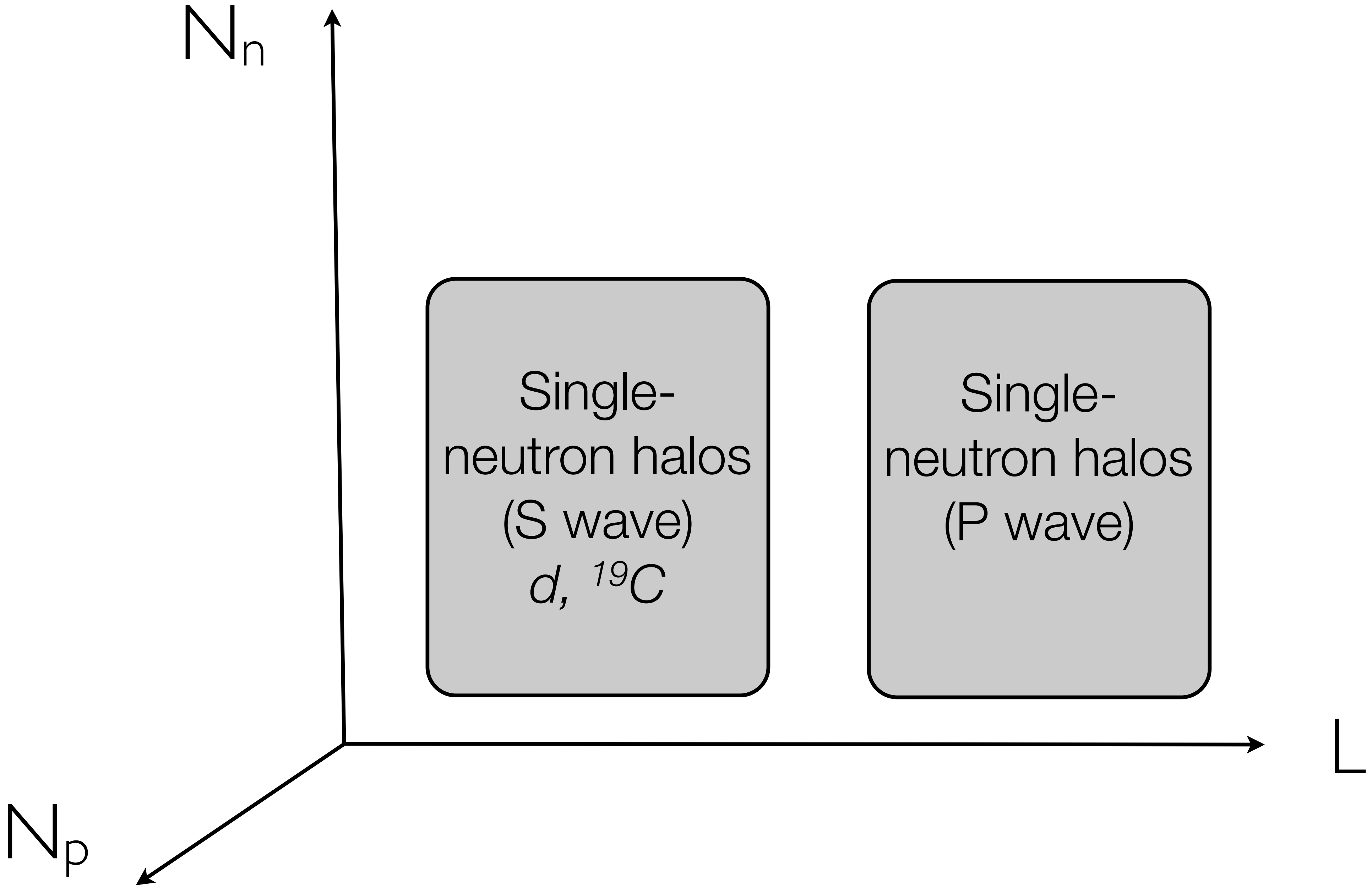
It does:

- Connect structure and reactions, including in multi-nucleon halos
- Allow collection/comparison of information/data from different theories/ experiments in one calculation
- Treat same physics as cluster models, in a systematically improvable way
- Provide information on inter-dependencies of low-energy observables, including along the core + n, core + 2n, core + 3n, etc. chain

The multi-dimensional Halo EFT space



Single-neutron halos (S wave)
d, ^{19}C

Single-neutron halos (P wave)

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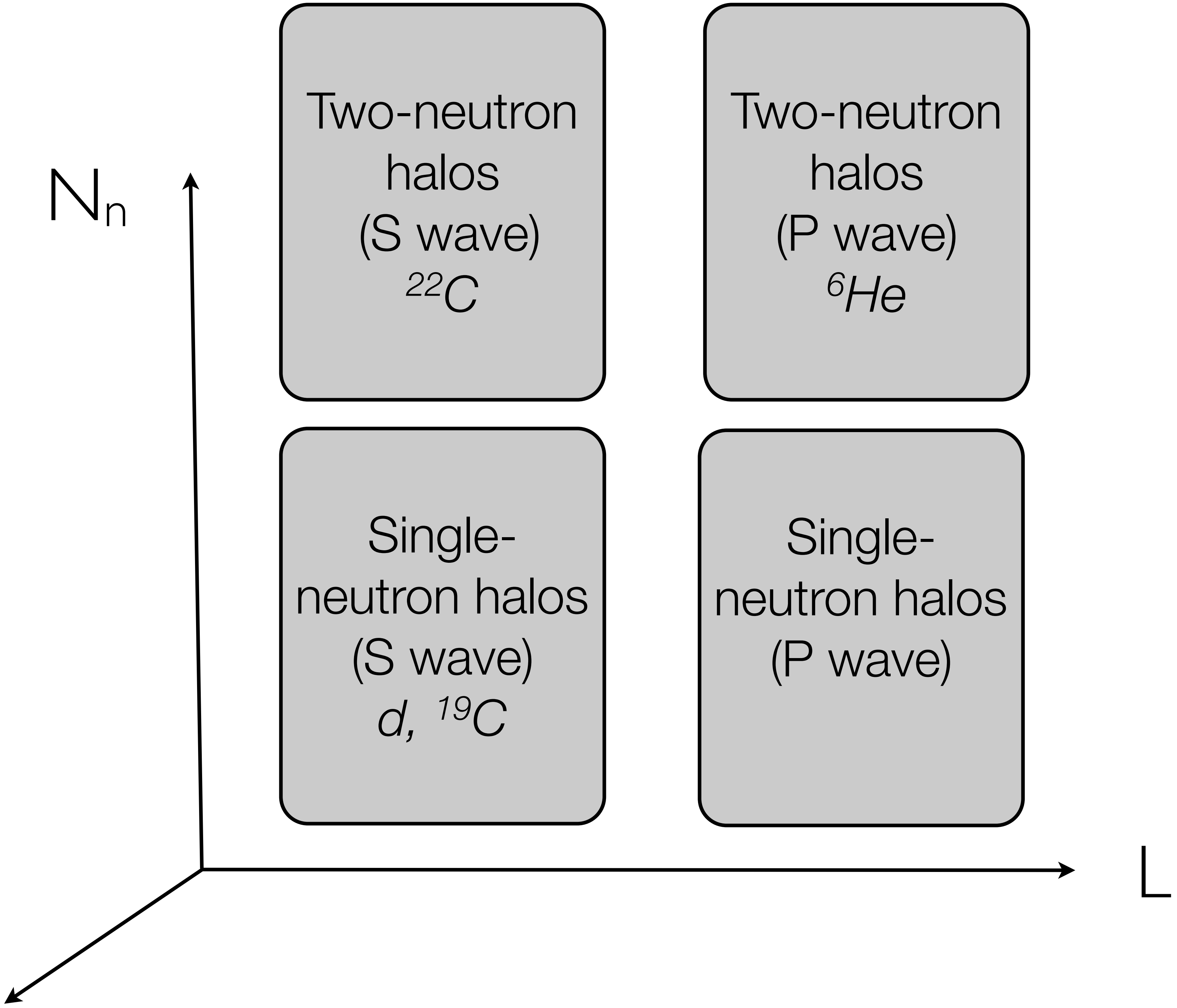

Two-neutron halos (S wave) ^{22}C

Two-neutron halos (P wave) ^{6}He

Single-neutron halos (S wave) *d*, ^{19}C

Single-neutron halos (P wave)

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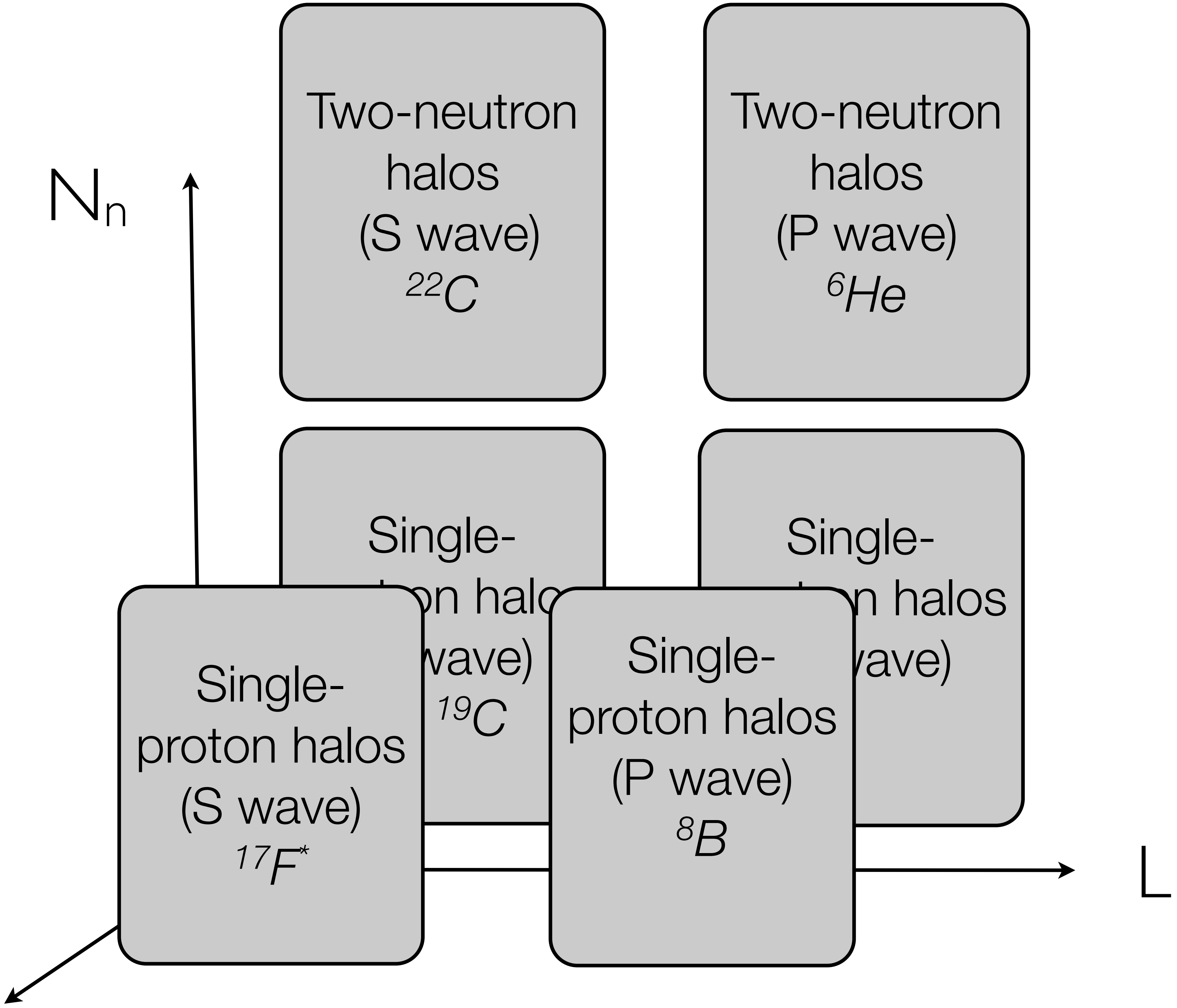

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^{6}He

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on halo
wave)
^{19}C

Single-
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ave)

Single-
proton halos
(S wave)
$^{17}F^{*}$

Single-
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^{8}B

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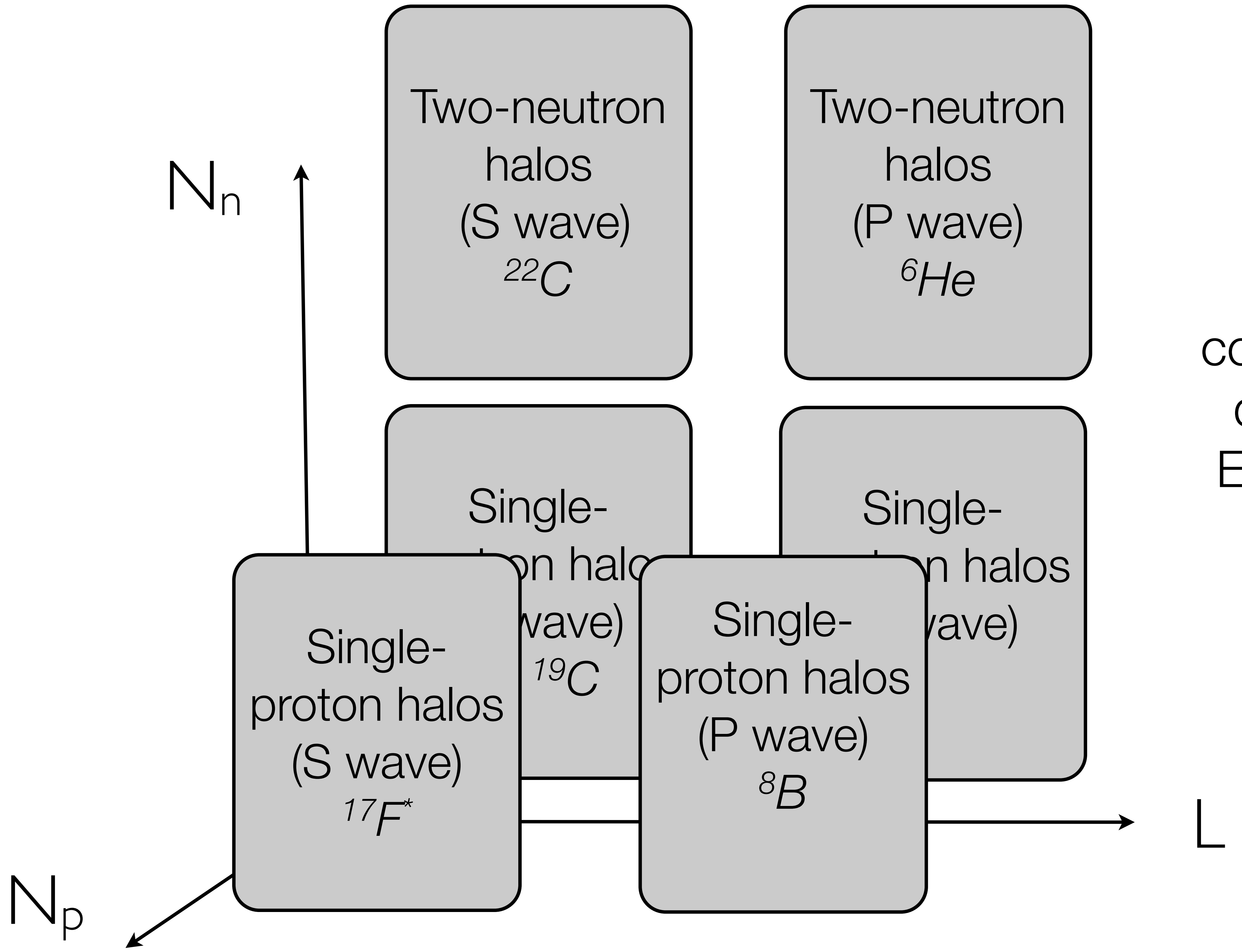

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Single- on halo vave) ^{19}C

Single- n halos ave)

Single-proton halos (S wave) $^{17}F^{*}$

Single-proton halos (P wave) ^{8}B

N_n

N_p

L

Plus complementary direction: N_α
E.g. ^{9}Be, $^{12}C^{*}$, etc.