Probing Nucleon Resonances on the Lattice

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Outline

Accessing states on the Lattice

- Quantities of interest are correlators

$$\mathcal{G}(\vec{p},t) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle\Omega|\chi(x)\bar{\chi}(0)|\Omega\rangle$$

- Inserting complete set of states, $\mathbf{I} = \sum |\alpha\rangle\langle\alpha|$

$$\mathcal{G}(\vec{0},t) \sim \sum_{\alpha} e^{-M_\alpha\, t}\, \langle\Omega|\, \chi\, |\alpha\rangle\langle\alpha|\, \bar{\chi}\, |\Omega\rangle$$

The terms $\langle\Omega|\chi|\alpha\rangle$ describe the coupling strength Z^α of operator χ to state α

- Result is a sum over exponentials of increasing mass

$$\mathcal{G}(t) \sim \sum_{\alpha=0}^{\infty} e^{-M_\alpha t} |Z^\alpha|^2$$

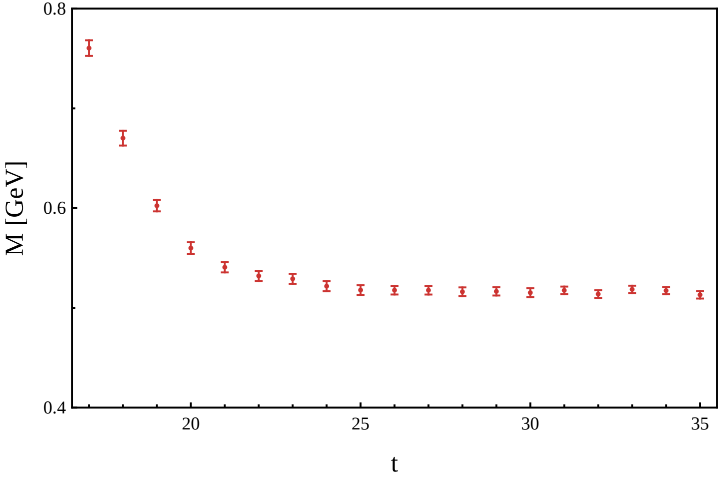
- Consider the effective mass

$$M_{\text{eff}} \equiv \log\left(\frac{\mathcal{G}(t)}{\mathcal{G}(t+1)}\right)$$

- At large times ($t \to \infty$), the ground state will dominate

$$M_{\text{eff}} \to M_0$$

Effective Mass for the pion



Correlation Matrix methods

- Ideally we want interpolators $\bar{\phi}_\alpha$ such that $\langle\beta|\bar{\phi}_\alpha|\Omega\rangle \propto \delta^{\alpha\beta}$
- Seek a linear combination of operators $\bar{\chi}_j$ to produce $\bar{\phi}_\alpha$

$$\bar{\phi}_\alpha = \sum_{j=1}^{N} \bar{\chi}_j u_j^\alpha$$

- Begin with a matrix of cross-correlators

$$\mathcal{G}_{ij}(t) = \sum_{\vec{x}} \langle\Omega|\chi_i \bar{\chi}_j|\Omega\rangle$$

- multiply on the right by u_j^α

$$\mathcal{G}_{ij}(t) u_j^\alpha = \sum_{\vec{x}} \langle\Omega|\chi_i \bar{\chi}_j u_j^\alpha|\Omega\rangle$$

- Knowledge of the time dependence provides the recurrence relation

$$\mathcal{G}_{ij}(t+\delta t)u_j^\alpha = e^{-M_\alpha\,\delta t}\mathcal{G}_{ij}(t)u_j^\alpha$$

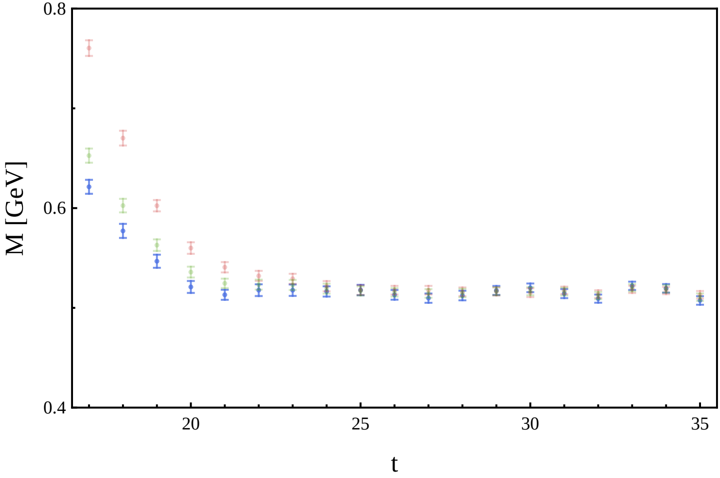
- Multiplying from the left by $\mathcal{G}^{-1}$ provides an eigenvalue equation for eigenvectors

$$[\,\mathcal{G}^{-1}(t)\,\mathcal{G}(t+\delta t)\,]_{ij}u_j^\alpha = e^{-M_\alpha\,\delta t}u_j^\alpha$$

- Having solved for u_j^α, we can project the matrix of correlators to produce correlators for the state α

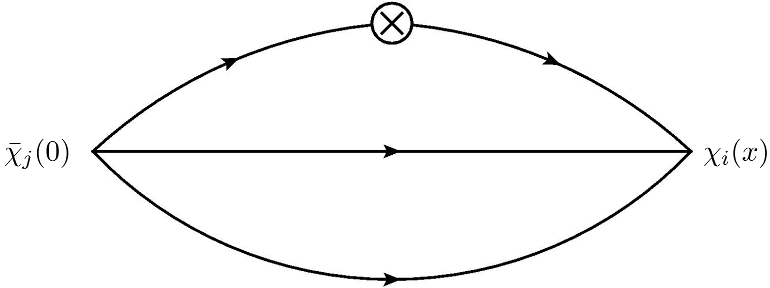
$$\mathcal{G}^\alpha(t) = v_i^\alpha \mathcal{G}_{ij}(t)u_j^\alpha$$

Pion effective mass with Correlation Matrix approach



Probing hadron structure on the Lattice

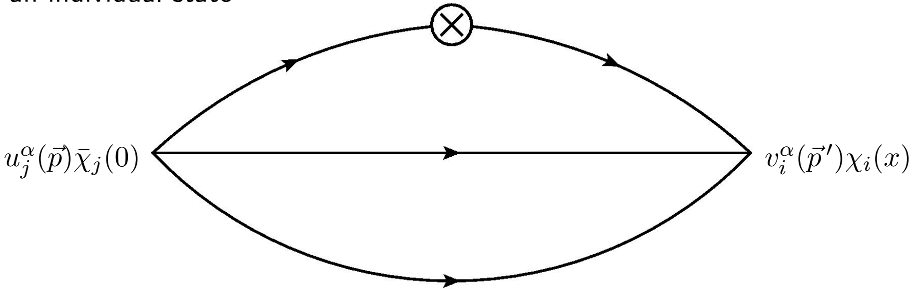
To explore the structure a state, we must probe it with some external current



Once again, the correlator will have contributions from a tower of states

Probing hadron structure on the Lattice

We can use our optimised operators, determined via the two-point correlation function, to project out the three-point correlation function for an individual state



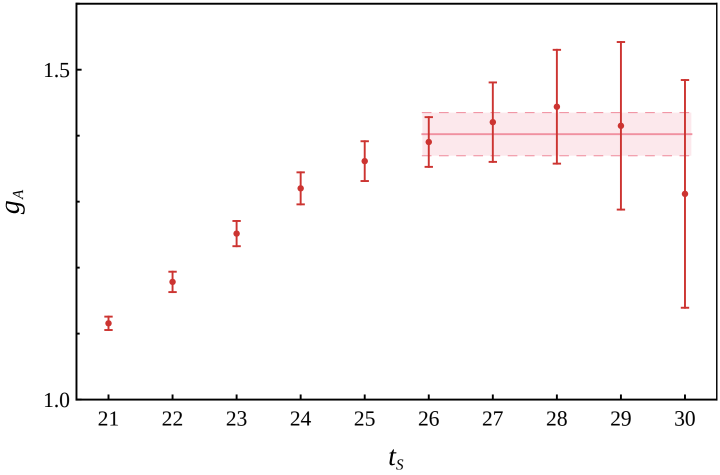
One needs to take care of the source and sink momenta as the optimised operators are momentum dependent

Nucleon Axial Charge

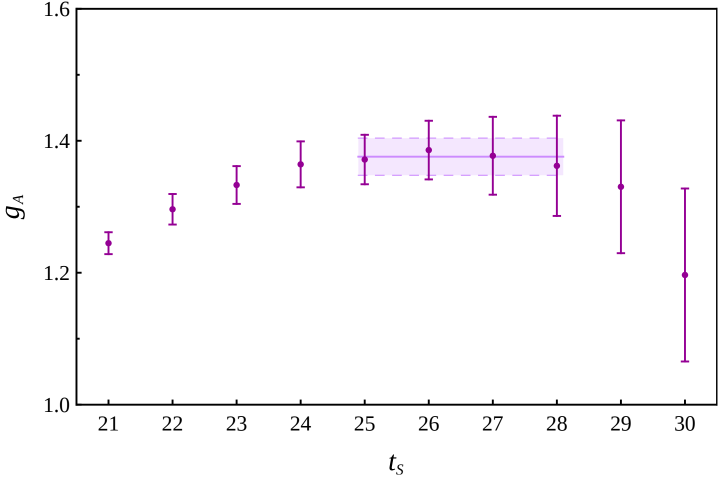
- Nucleon axial charge has been quantity of significant interest on the lattice
- Despite relative simplicity, lattice determinations have been consistently low
- Finite volume effects are known to play a role, but are not the complete answer
- Excited state contamination has suggested as possible issue
- Can use correlation matrix methods to eliminate excited state contaminations

B. J. Owen et al., Phys. Lett. B ***723****, 217-223 (2013)*

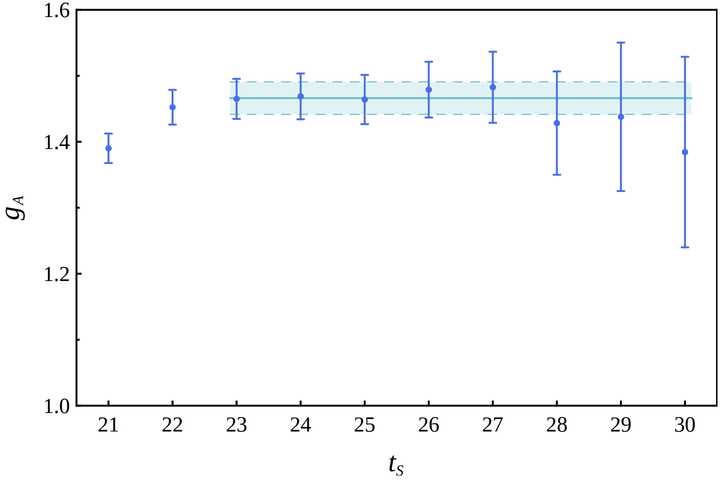
Axial charge of the nucleon – standard approach
Using single operator for source and sink



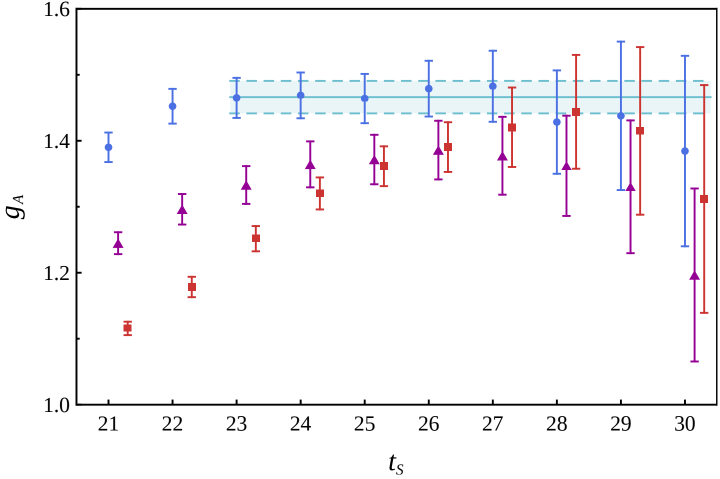
Axial charge of the nucleon – standard approach
Using a different operator



Axial charge of the nucleon – Correlation matrix approach

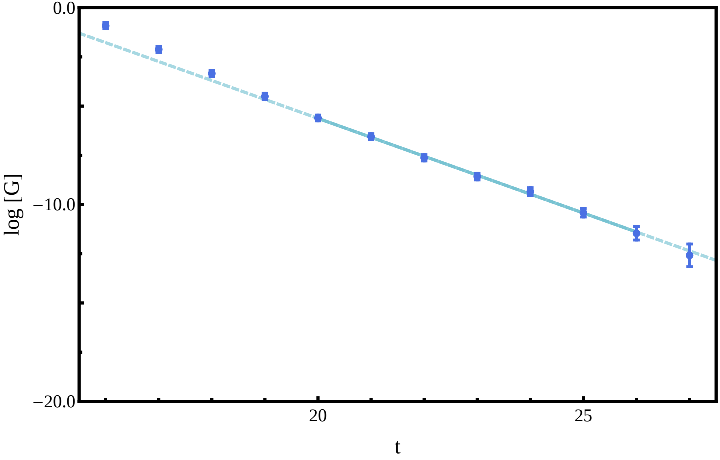


Axial charge of the nucleon – Comparison between methods

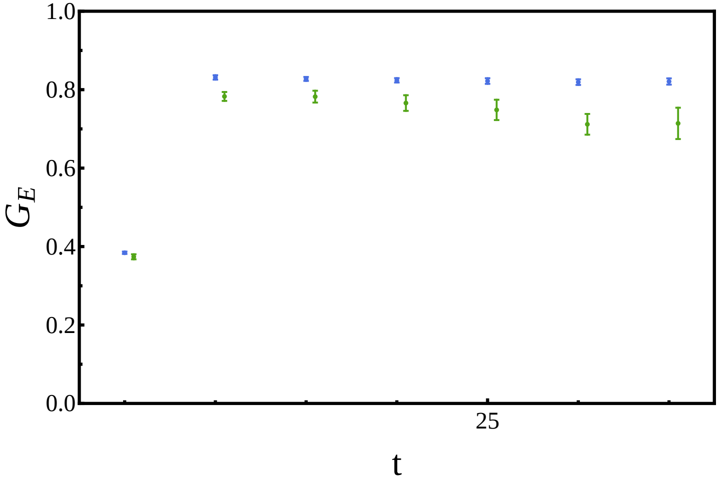


Projected Correlator for the first nucleon excitation

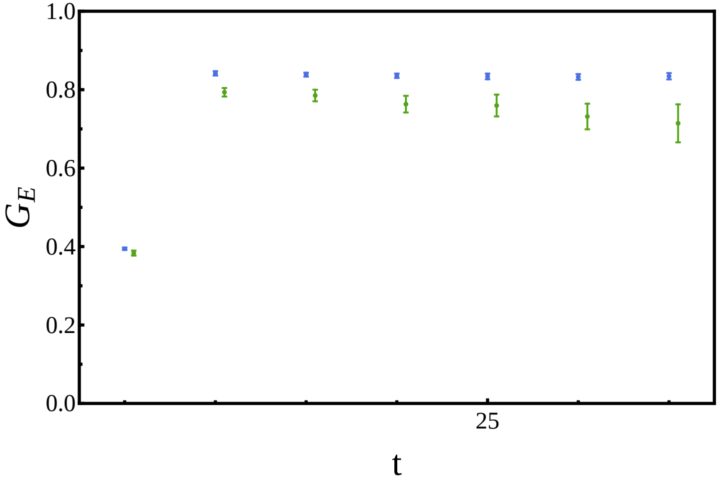
Correlation Matrix approach is a method for studing excited states



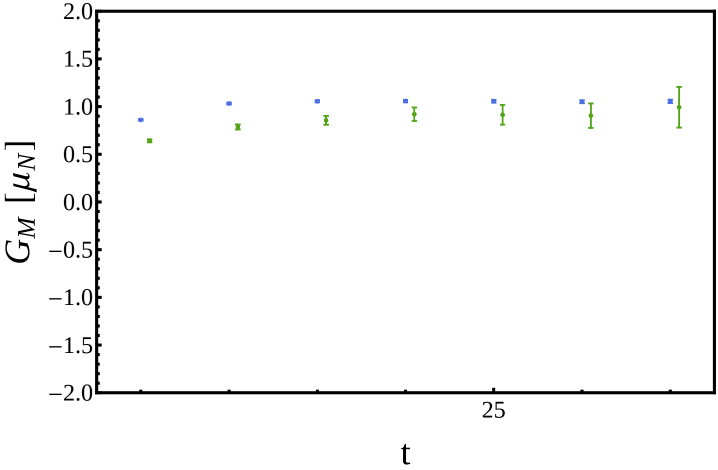
The Sachs Electric form factor - Quark Sector comparison (u sector in the proton)



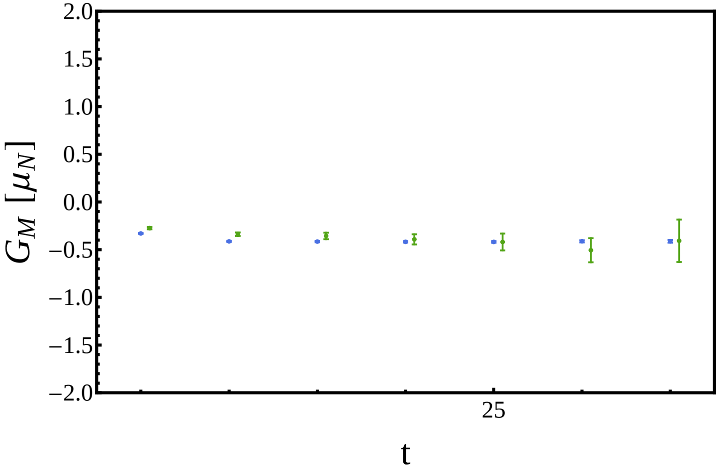
The Sachs Electric form factor - Quark Sector comparison (d sector in the proton)



The Sachs Magnetic form factor - Quark Sector comparison (u sector in the proton)

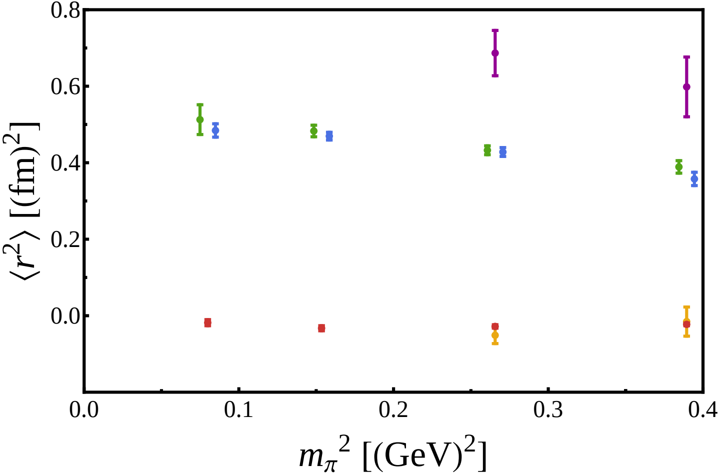


The Sachs Magnetic form factor - Quark Sector comparison (d sector in the proton)

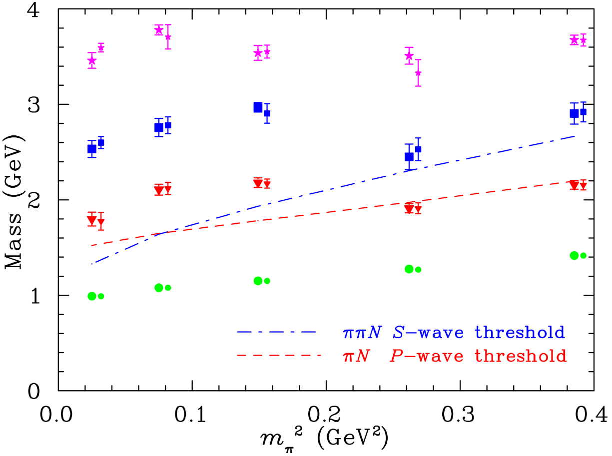


Charge radii for the nucleon, Δ^+ and first radial excitation of the nucleon (proton and neutron)

Use dipole Ansatz to calculate charge-square radii

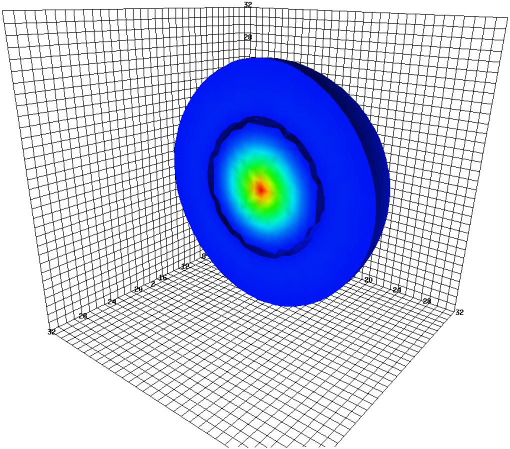


Nucleon spectrum



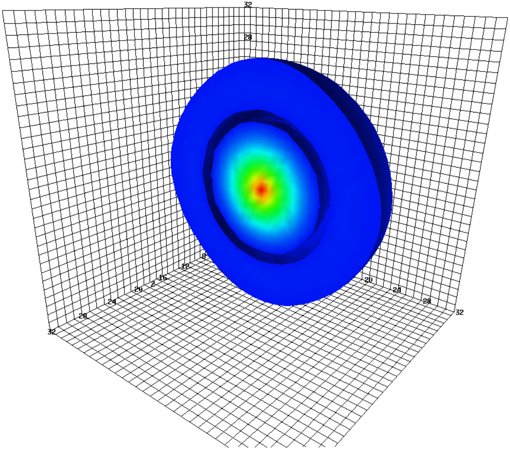
M. S. Mahbub et al., Phys. Lett. B ***707****, 389-313 (2012)*
D. S. Roberts et al., arXiv: 1311.6626 [hep-lat]

Wave Function of 1^{st} Nucleon Resonance - heaviest mass



D. S. Roberts et al., arXiv: 1311.6626 [hep-lat]

Wave Function of 1^{st} Nucleon Resonance - 2^{nd} heaviest mass



D. S. Roberts et al., arXiv: 1311.6626 [hep-lat]

Comparison of radii

- One can use wave function to calculate $\langle r \rangle$
- Gauge dependent
- Consider ratio of $\langle r \rangle$ between states as comparison between methods

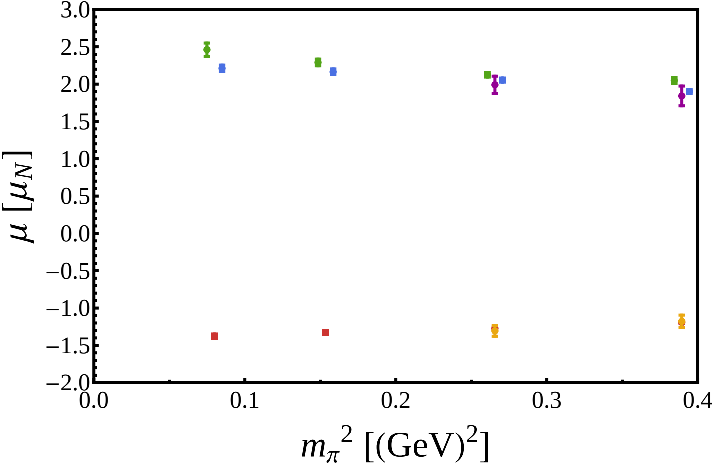
FF: $\frac{\langle r \rangle_1}{\langle r \rangle_0} = 1.26, \quad 1.32$

WF: $\frac{\langle r \rangle_1}{\langle r \rangle_0} = 1.16, \quad 1.08$

- Consistency, noting that WF method suffers significantly from finite volume effects which will tend to suppress the value

Magnetic moments for the nucleon, Δ^+ and first radial excitation of the nucleon (proton and neutron)

Use dipole Ansatz to calculate magnetic moment



Conclusion

- Demonstrated how use correlation matrix methods to construct "ideal" operators
- Seen how correlation techniques can improve ground state overlap
- Clear plateau observed for Form Factors of the 1^{st} nucleon excitation
- Consistency with wave function results
- Magnetic moment is consistent with simple quark model expectation for s-wave excitation
- Further investigation required to determine dominant contribution to resonance at heavier masses
- In process of extending analysis down to light masses