SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

Sixth Asia-Pacific Conference on Few-Body Problems in Physics APFB 2014

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TRANSVERSE SPIN EFFECTS IN DIHADRON SIDIS

Hrayr Matevosyan

Collaborators:
A.W. Thomas, A. Kotzinian and W. Bentz

CoEPP
ARC Centre of Excellence for Particle Physics at the Terascale

THE UNIVERSITY of ADELAIDE

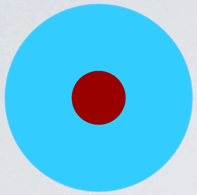
SUB CRUCE LUMEN

OUTLOOK

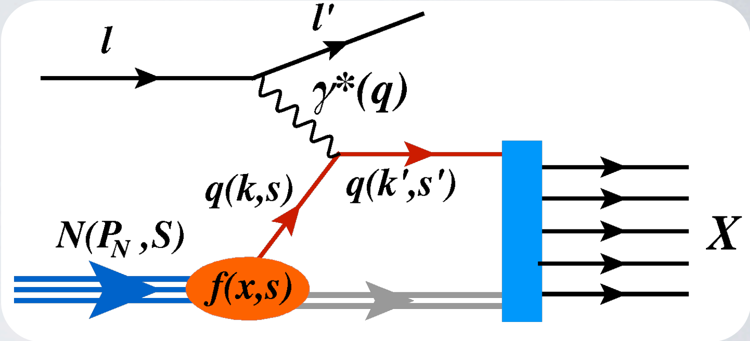
- **Introduction and Motivation.**
- **SIDIS with Transversely Polarized Target:**
  - ***Transversity PDF I: From Collins Effect with One Hadron Production.***
  - ***Transversity PDF II: From Interference DiFF in Two Hadron Production.***
  - ***Sivers PDF from Two Hadron Production.***
- **Conclusions.**

NUCLEON PARTON DISTRIBUTION FUNCTIONS

- *Unpolarized* quark in *Unpolarized* nucleon.

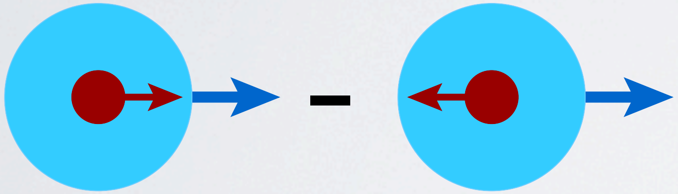


$$f_1^q(x, Q^2)$$



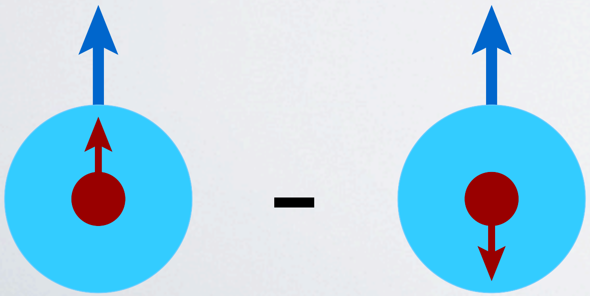
- ***The momentum and the spin of the partons are correlated with the polarization of the nucleon!***

- *Longitudinally* polarized quark in *Longitudinally* polarized nucleon.



$$g_{1L}^q(x, Q^2)$$

- *Transversely* polarized quark in *Transversely* polarized nucleon.



$$h_{1T}^q(x, Q^2)$$

Chiral-odd: Suppressed in Inclusive DIS

PDFS WITH TRANSVERSE MOMENTUM DEPENDENCE

- ***The transverse momentum (TM) of the parton can couple with both its own spin and the spin of the nucleon:***
  - ***TMD PDFs***

N/q	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}^\perp$	$h_1 \, h_{1T}^\perp$

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✦ Survive after TM integration!

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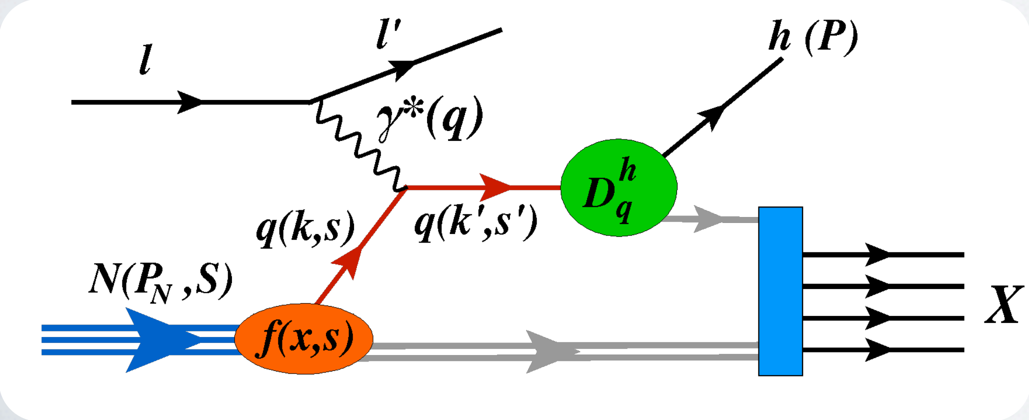

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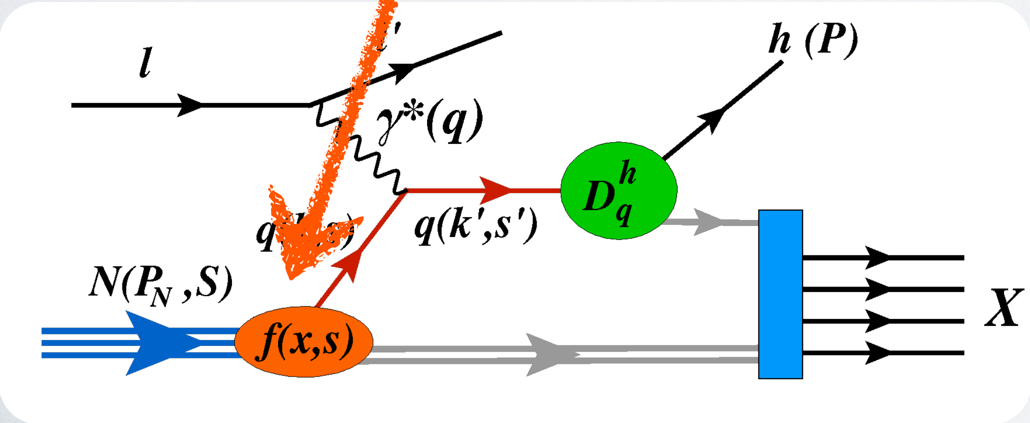

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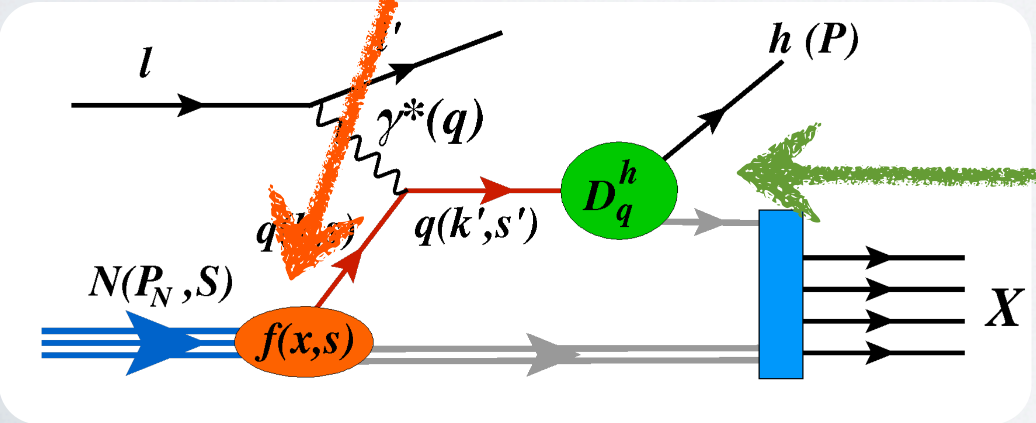
✦ Survive after TM integration!

- ***Accessible in SIDIS Process***

- ***NEED TMD FFs***

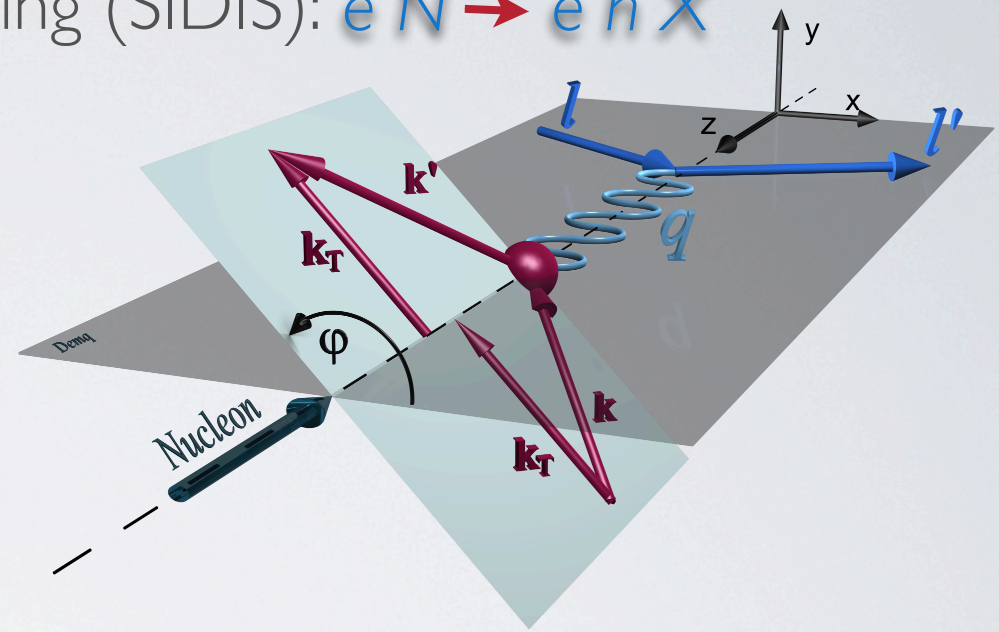


q/h	U
U	D_1
L	
T	$H_1^\perp$

EXPLORING HADRON STRUCTURE

A. Kotzinian, Nucl. Phys. B441, 234 (1995).

- Semi-inclusive deep inelastic scattering (SIDIS): $e\,N \rightarrow e\,h\,X$
- Cross-section factorizes: $P_T^2 \ll Q^2$



Distribution

$$\frac{d\sigma^{lN\rightarrow l'hX}}{dx dQ^2 dz d^2 P_T} = \sum_q f_1^q(x, k_T^2, Q^2) \otimes d\sigma^{lq\rightarrow lq} \otimes D_q^h(z, P_\perp^2, Q^2)$$

Fragmentation

- **Measure TM dependence.**
- Flavour Decomposition - Different Hadron FFs - different weights!

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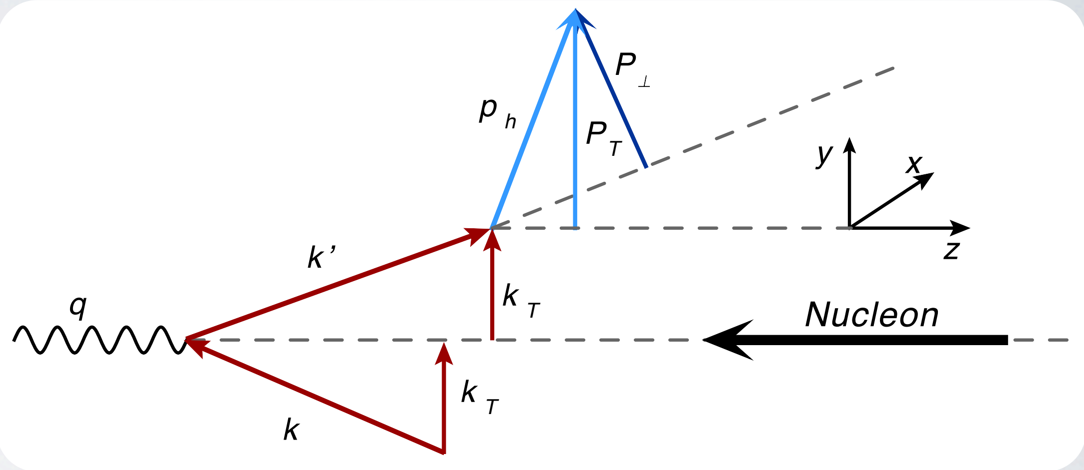
$$\mathbf{P_T} = \mathbf{P}_\perp + z\mathbf{k_T}$$



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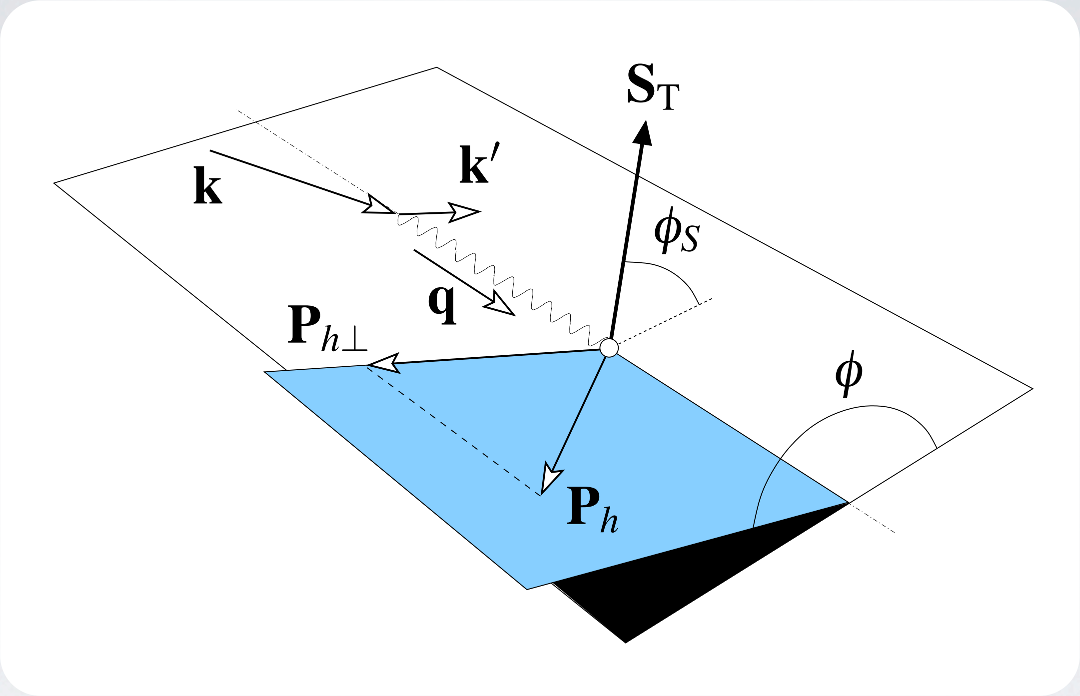
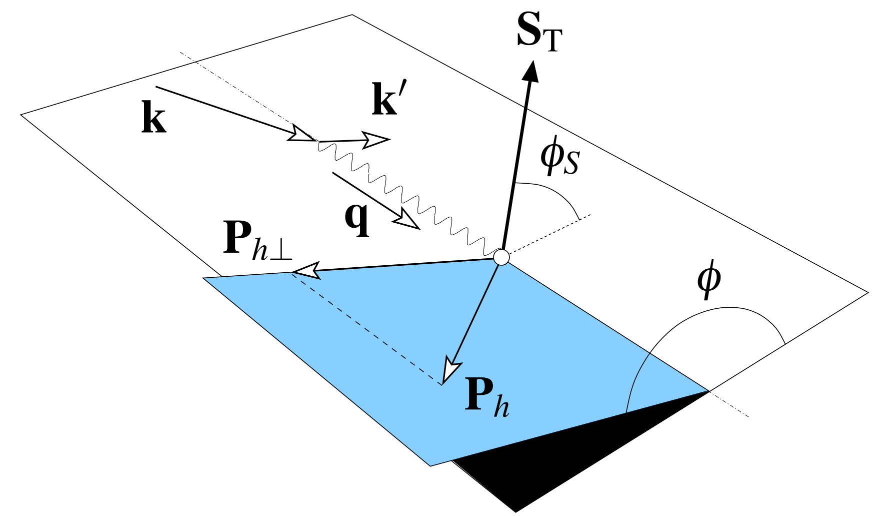


TRANSVERSITY I: ONE HADRON SIDIS AND COLLINS FRAGMENTATION FUNCTION

SIDIS POLARIZED CROSS-SECTION

A. Bacchetta, JHEP08, 023 (2008).

- **For polarized SIDIS cross-section there are *18 terms* in leading twist expansion:**



$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_S\,d\phi_h\,dP_{h\perp}^2} \sim F_{UU,T} + \varepsilon F_{UU,L} + \ldots$$

Collins Effect

$$+ |\boldsymbol{S}_\perp| \left[\sin(\phi_h - \phi_S) \left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\, F_{UT,L}^{\sin(\phi_h-\phi_S)} \right) + \varepsilon\, \sin(\phi_h+\phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)} + .. \right]$$

- **Extract the specific harmonics:**

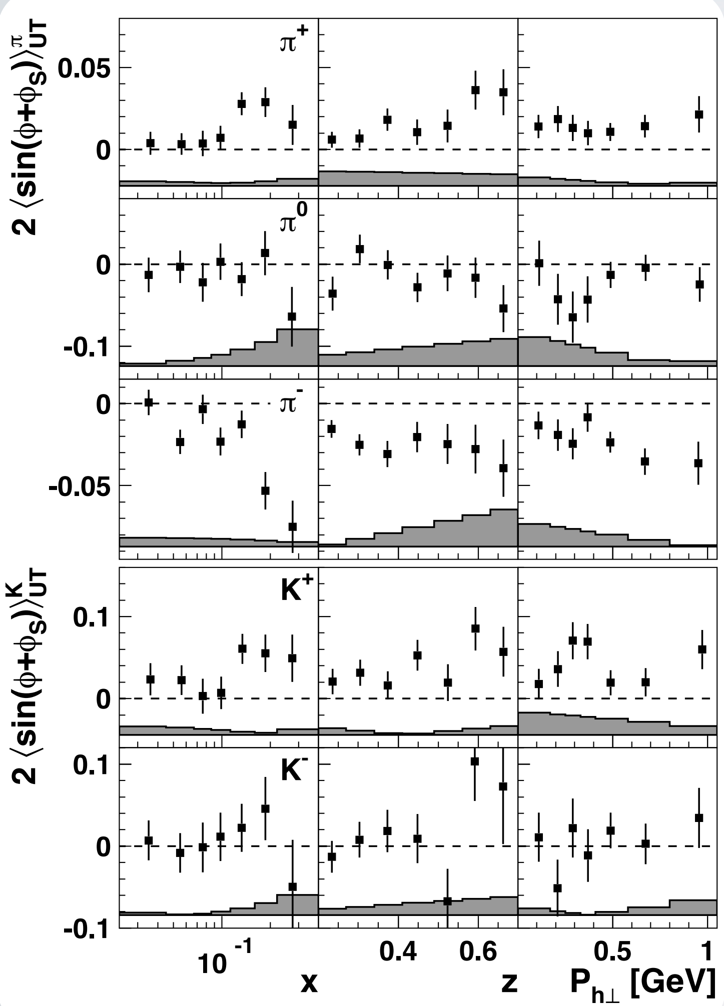
$$F_{UU,T} \sim \mathcal{C}[f_1\ D_1] \qquad F_{UT}^{\sin(\phi_h+\phi_S)} \sim \mathcal{C}[h_1\ H_1^\perp]$$

- ***NEED Collins Fragmentation Function to access Transversity PDF from SIDIS!***
- ***Convolutions*** of Collins FFs measured in e^+e^- annihilation.

EMPIRICAL EXTRACTIONS OF TRANSVERSITY

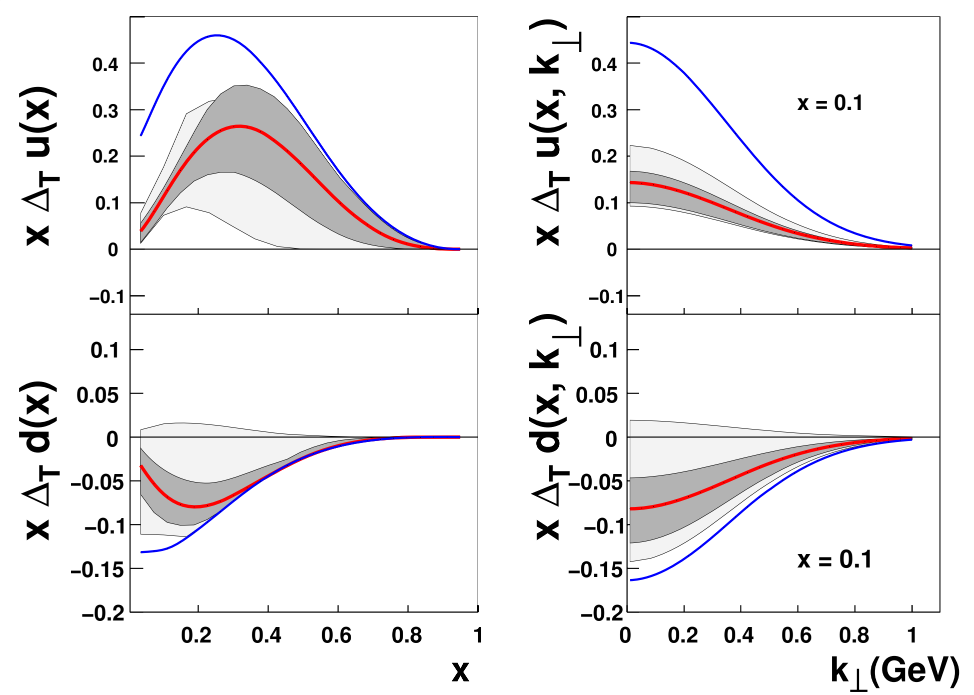
- ***SIDIS at HERMES***
  PLB693 (2010) 11-16.

$$\langle \sin(\phi + \phi_S) \rangle^h_{UT} \sim \frac{\mathcal{C}[h_1^q \ H_{1q}^{\perp h/q}]}{\mathcal{C}[f_1^q \ D_1^{h/q}]}$$

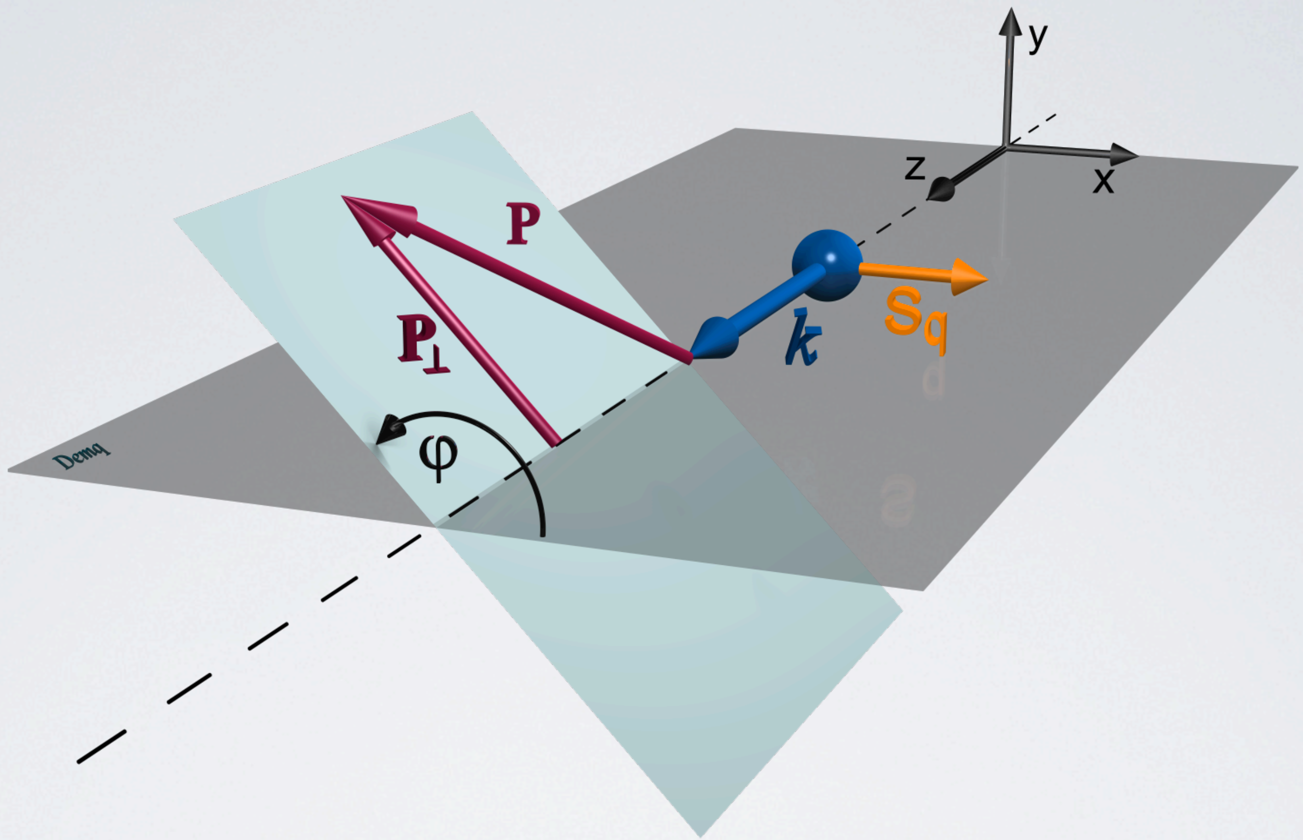


- Opposite sign for the charged pions.
- Large positive signal for K^+.
- Consistent with 0 for π^0 and K^-.

- ***Fits to HERMES, COMPASS and BELLE:*** **NPB (Proc. Suppl.) 191 (2009) 98-107.**



- **Large Uncertainties!**
- **Simplistic Approximations !**

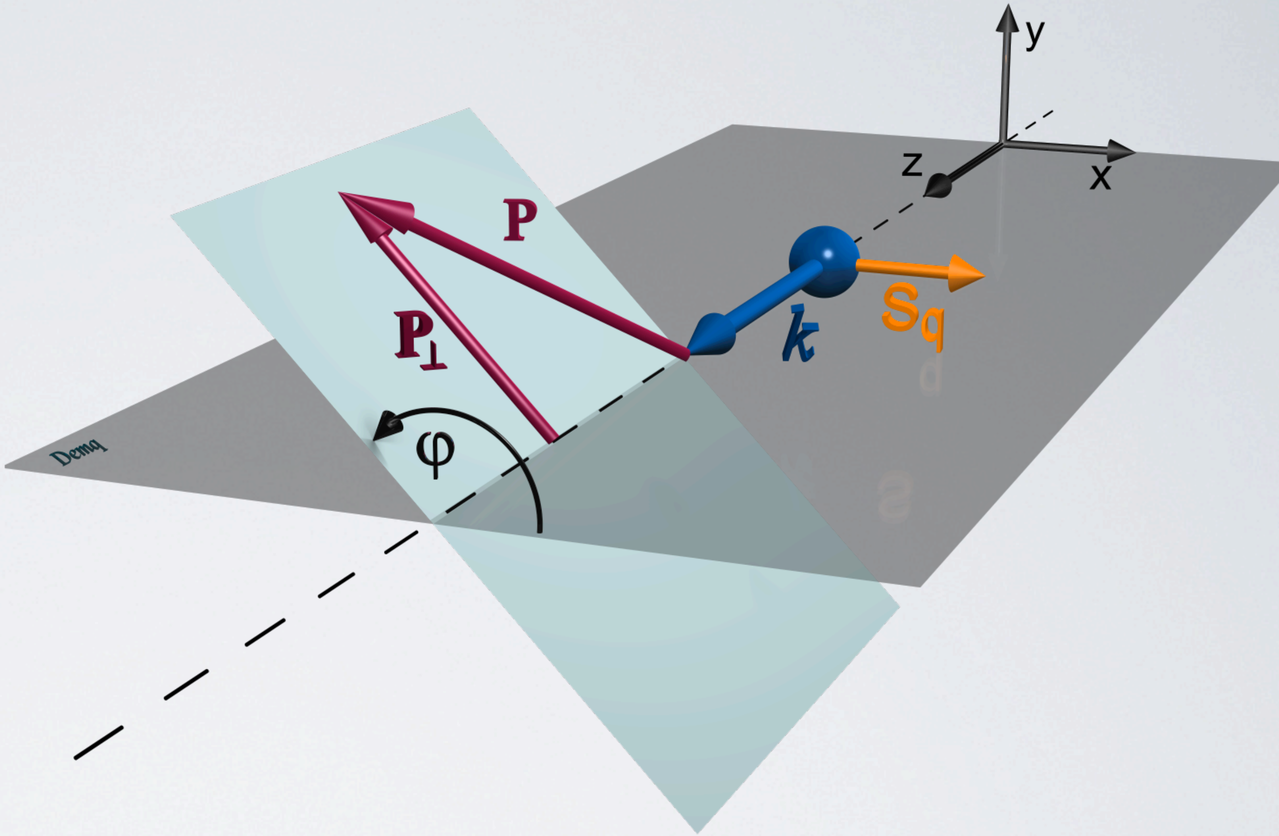


TRANSVERSELY POLARIZED QUARK FRAGMENTATION:
COLLINS EFFECT

COLLINS FRAGMENTATION FUNCTION

- **Collins Effect:**

  Azimuthal Modulation of Transversely Polarized Quark' Fragmentation Function.



Unpolarized

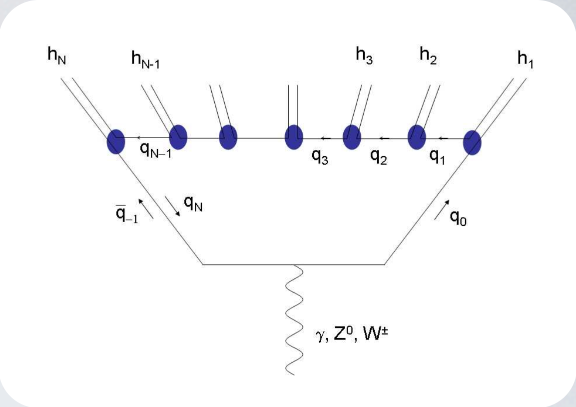
$$D_{h/q^\uparrow}(z, P_\perp^2, \varphi) = D_1^{h/q}(z, P_\perp^2) - H_1^{\perp h/q}(z, P_\perp^2)\frac{P_\perp S_q}{z m_h}\sin(\varphi)$$

Collins

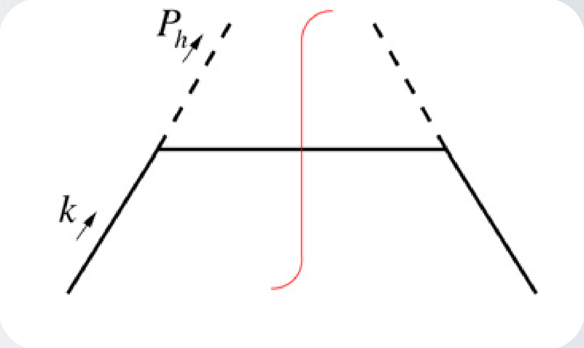
- **Chiral-ODD: Needs to be coupled with another chiral-odd quantity to be observed.**

MODELS FOR FRAGMENTATION

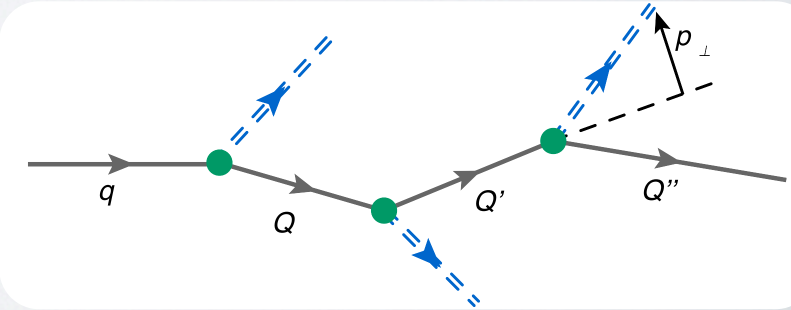
- *Lund String Model*
  - *Very Successful* implementation in *JETSET, PYTHIA*.
  - Highly Tunable - Limited Predictive Power.
  - No Spin Effects - Formal developments by X. Artru et al but no quantitative results!



- *Spectator Model*
  - Quark model calculations with empirical form factors.
  - No unfavored fragmentations.
  - Need to tune parameters for small z dependence.



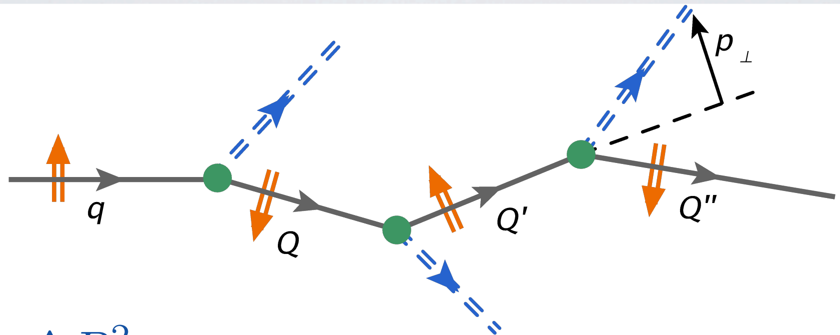
- *NJL-jet Model*
  - *Multi-hadron* emission framework with effective quark model input.
  - *Monte-Carlo framework* allows flexibility in including the transverse momentum, spin effects, two-hadron correlations, etc.



COLLINS FRAGMENTATION FUNCTION FROM NJL-JET

H.M.,Bentz, Thomas, PRD.86:034025, 2012.

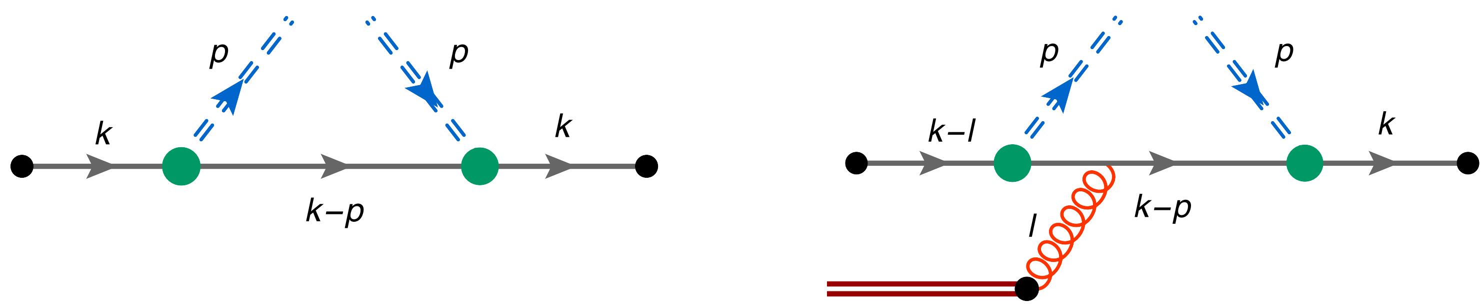
- **Extend the NJL-jet Model to Include the Quark's Spins.**



$$D_{h/q^\uparrow}(z, P_\perp^2, \varphi)\,\Delta z\,\frac{\Delta P_\perp^2}{2}\,\Delta\varphi = \left\langle N_{q^\uparrow}^h(z, z+\Delta z; P_\perp^2, P_\perp^2 + \Delta P^2; \varphi, \varphi + \Delta\varphi)\right\rangle$$

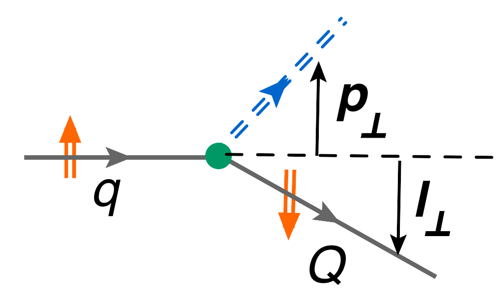
- **Model Calculated Elementary Collins Function as Input**

A. Bacchetta et. al., PLB659, 234 (2008).



- ***Spin flip probability:*** $\mathcal{P}_{SF}$

$$\boxed{\mathcal{P}_{SF} = \mathcal{C}\,\frac{l_y^2 + (M_2 - (1-z)M_1)^2}{l_\perp^2 + (M_2 - (1-z)M_1)^2}}$$

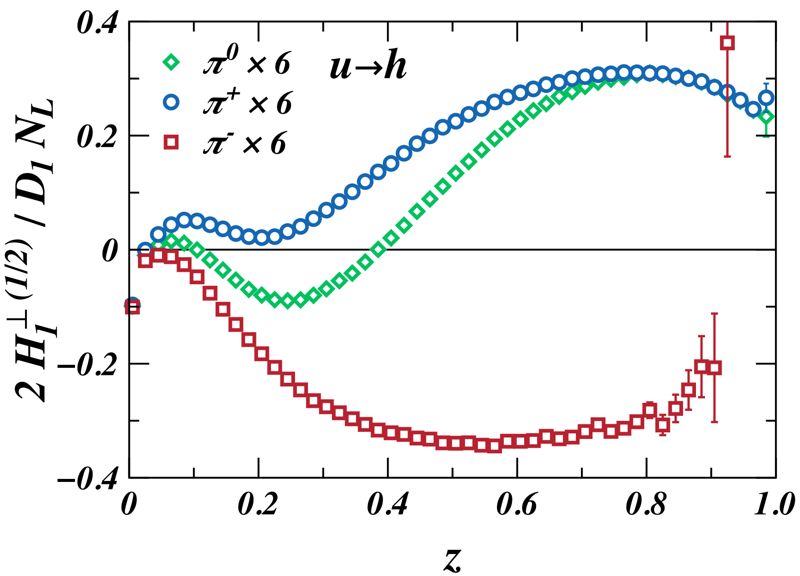


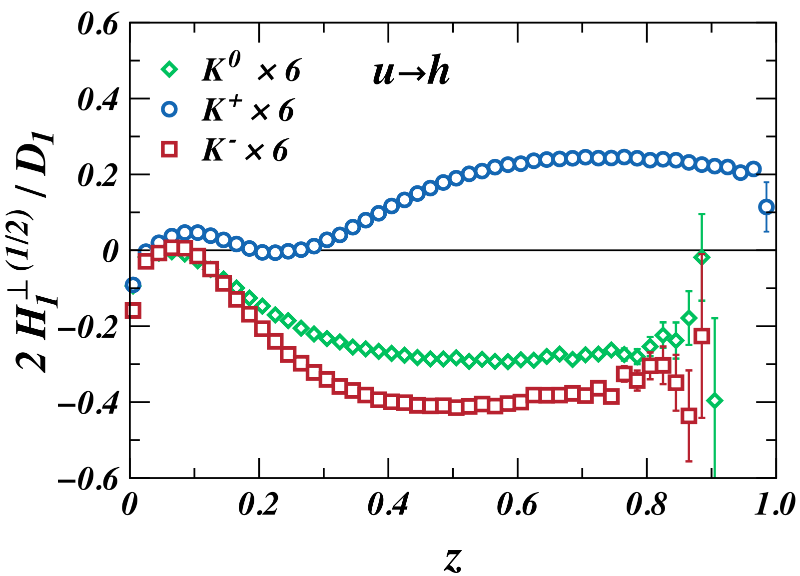
COLLINS EFFECT - MK2

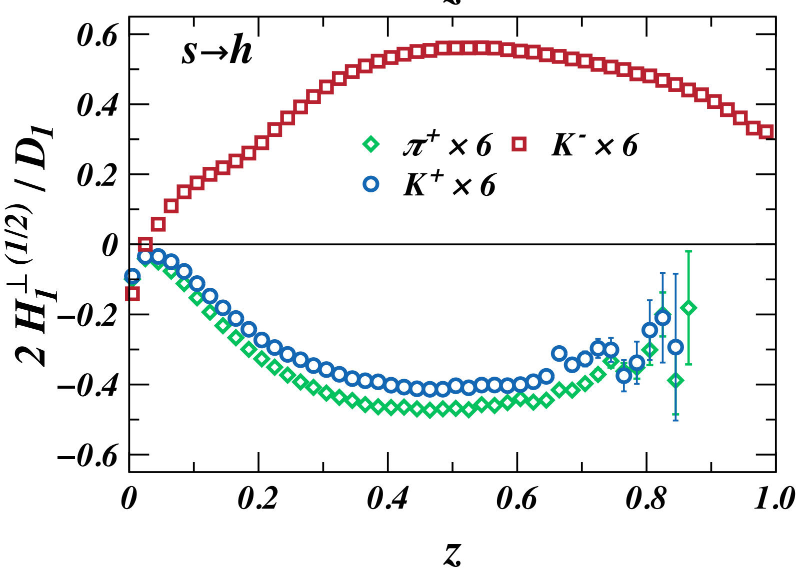
MK2 Model Assumptions:

H.M., Kotzinian, Thomas, PLB731 (2014) 208-216.

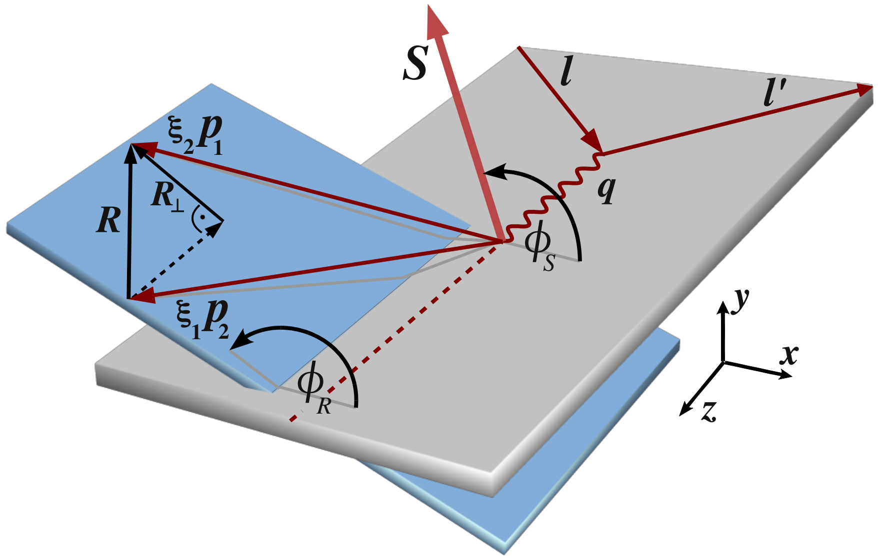
1. Allow for Collins Effect only in a SINGLE emission vertex.
2. Use constant values for $\mathcal{P}_{SF} = 1$ and N_L=6.







- ✦ **Opposite sign** and **similar size** for favored and unfavored Collins FF ratios!
- ✦ **Similar indications** from SIDIS data (if assuming u-dominance in cross-section).
- ✦ **TM conservation** and **quark spin flip** - are the underlying mechanisms.

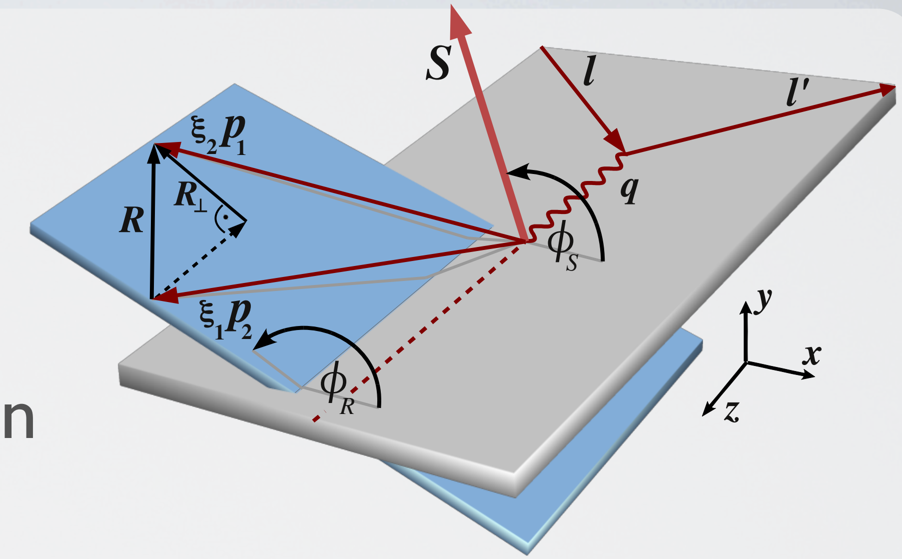


TRANSVERSITY II:
DIHADRON WAY

ACCESS TO TRANSVERSITY PDF FROM DiFF

M. Radici, et al: PRD 65, 074031 (2002).

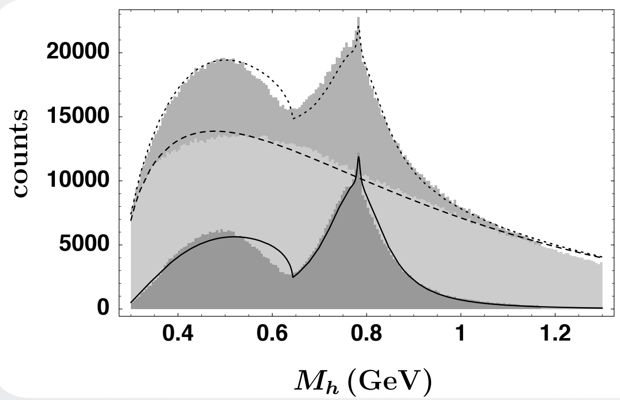
- In two hadron production from polarized target the cross section factorizes *collinearly* - no TMD!
- Allows clean access to *transversity*.
- *Unpolarized* and *Interference* Dihadron FFs are needed!

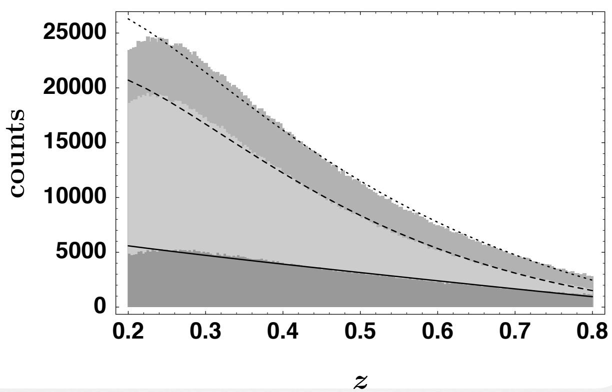


$$\frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \propto \sin(\phi_R + \phi_S) \frac{\sum_q e_q^2 \; h_1^q(x)/x \; H_1^{\sphericalangle q}(z, M_h^2)}{\sum_q e_q^2 \; f_1^q(x)/x \; D_1^q(z, M_h^2)}$$

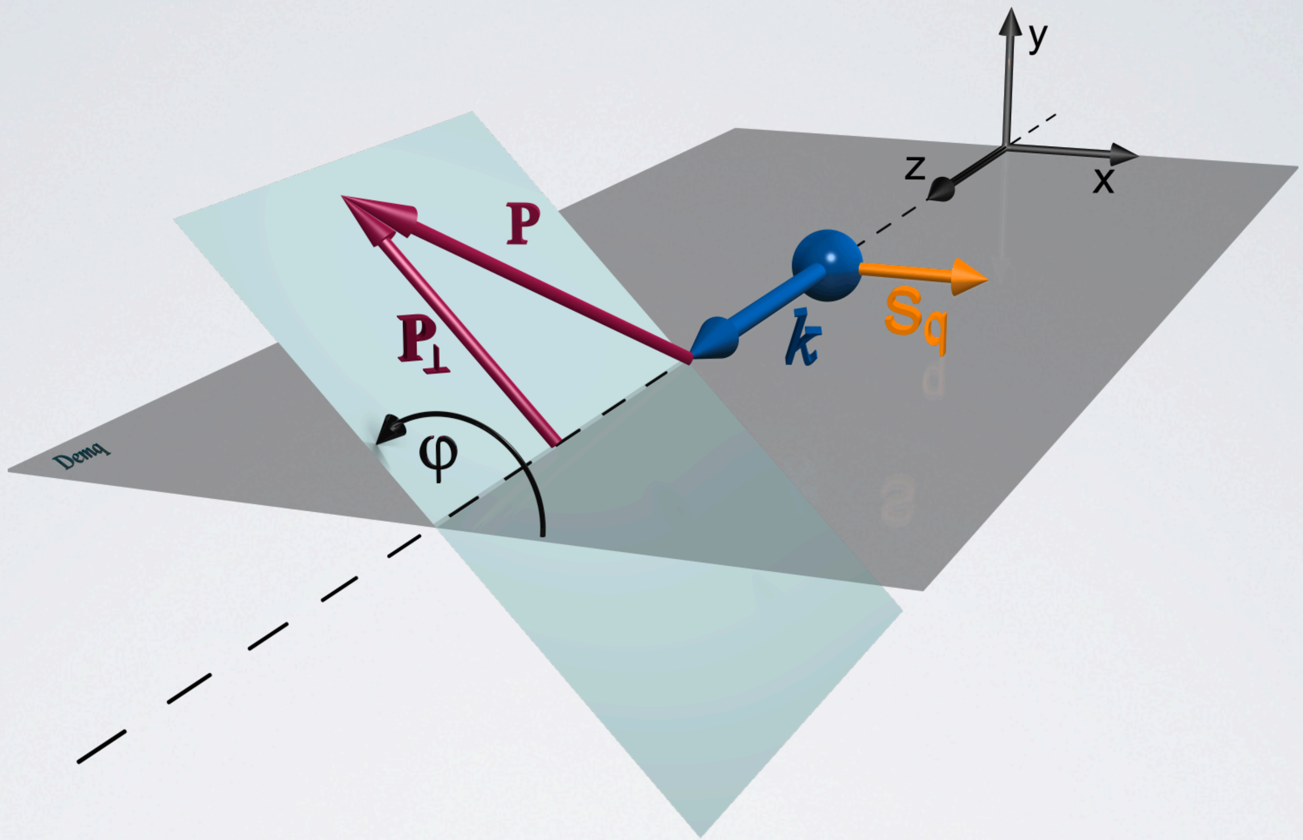
- Empirical Model for D_1^q have been fitted to PYTHIA simulations.

A. Bacchetta and M. Radici, PRD 74, 114007 (2006).





Experiments: BELLE, HERMES, COMPASS.

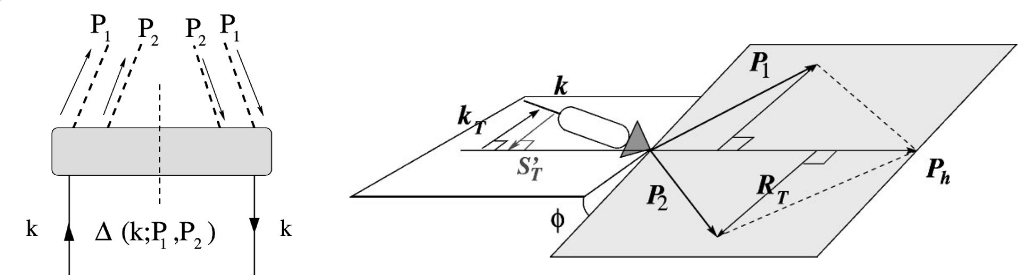


TWO-HADRON FRAGMENTATION OF A TRANSVERSELY POLARIZED QUARK:

INTERFERENCE DiFF

TWO-HADRON FRAGMENTATION

A. Bianconi, et al: PRD 62, 034008 (2000). M. Radici, et al: PRD 65, 074031 (2002).



- Kinematic Variables:

$$P_1 = \left[\xi P_h^- , \frac{M_1^2 + \vec{R}_T^2}{2\,\xi P_h^-}, \vec{R}_T \right], \qquad k = \left[\frac{P_h^-}{z}, z\frac{k^2 + \vec{k}_T^2}{2 P_h^-}, \vec{k}_T \right] \qquad z \equiv z_h = z_1 + z_2$$

$$P_2 = \left[(1-\xi) P_h^- , \frac{M_2^2 + \vec{R}_T^2}{2(1-\xi) P_h^-}, -\vec{R}_T \right] \qquad \mathbf{R} = \frac{\mathbf{P}_1 - \mathbf{P}_2}{2} \qquad \xi = \frac{z_1}{z_1 + z_2}$$

✦ ***Note different definition for the TM!***

- The relevant terms of the quark correlator at leading order for a **Transversely Polarized Quark:**

Unpolarized

$$\Delta^{[\gamma^-]} = D_1(z_h, \xi, k_T^2, R_T^2, \mathbf{k}_T \cdot \mathbf{R}_T)$$

Interference

$$\Delta^{[i\sigma^{i-}\gamma_5]} = \frac{\epsilon_T^{ij} R_{Tj}}{M_1 + M_2} H_1^{\sphericalangle}(z_h, \xi, k_T^2, R_T^2, \mathbf{k}_T \cdot \mathbf{R}_T) + \frac{\epsilon_T^{ij} k_{Tj}}{M_1 + M_2} H_1^{\perp}(z_h, \xi, k_T^2, R_T^2, \mathbf{k}_T \cdot \mathbf{R}_T)$$

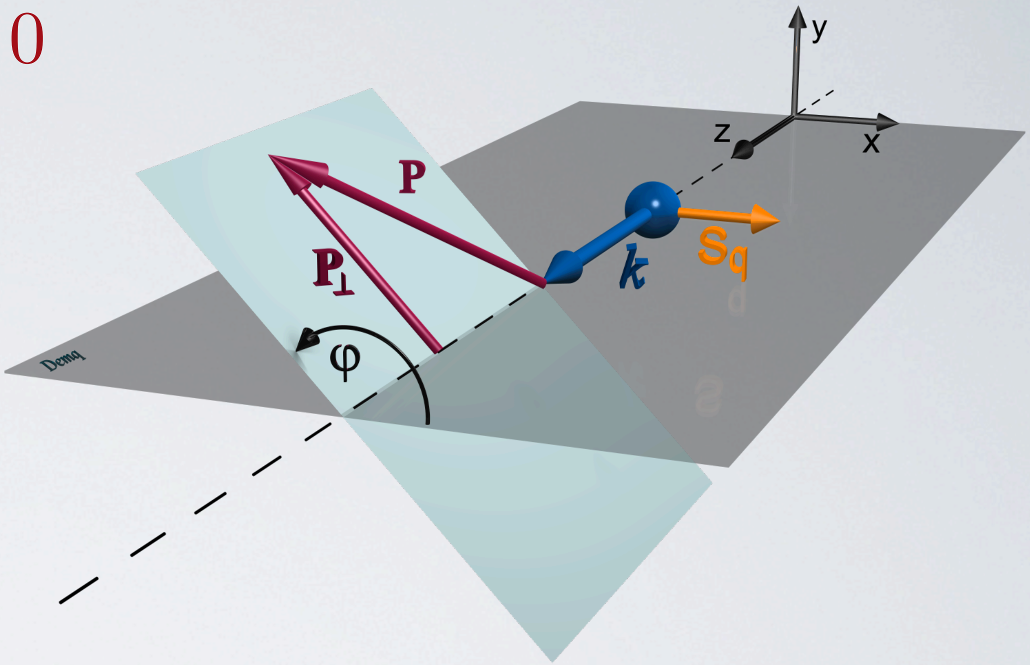
- **IFFS are Chiral-ODD: Need to be coupled with another chiral-odd quantity to be observed (e.g. transversity).**

TWO-HADRON FRAGMENTATION

- **Transformation to frame $\mathbf{k}_T = 0$**

$$k = (k^-, k^+, \mathbf{0})$$
$$\mathbf{k}_T = -\mathbf{P}_T / z_h$$

$$\mathbf{P}_T = \mathbf{P}_{h_1}^{\perp} + \mathbf{P}_{h_2}^{\perp}$$
$$\mathbf{R} = (\mathbf{P}_{h_1}^{\perp} - \mathbf{P}_{h_2}^{\perp})/2$$



- **Integrate over one or other momentum:**

$$D_{q^\uparrow}^{h_1 h_2}(\varphi_R) = D_{1,q}^{h_1 h_2} + \sin(\varphi_R - \varphi_S)\mathcal{F}[H_1^{\sphericalangle}, H_1^{\perp}]$$
$$D_{q^\uparrow}^{h_1 h_2}(\varphi_T) = D_{1,q}^{h_1 h_2} + \sin(\varphi_T - \varphi_S)\mathcal{F}'[H_1^{\sphericalangle}, H_1^{\perp}]$$

- **The IFF surviving after $\mathbf{k}_T$ integration is redefined as**

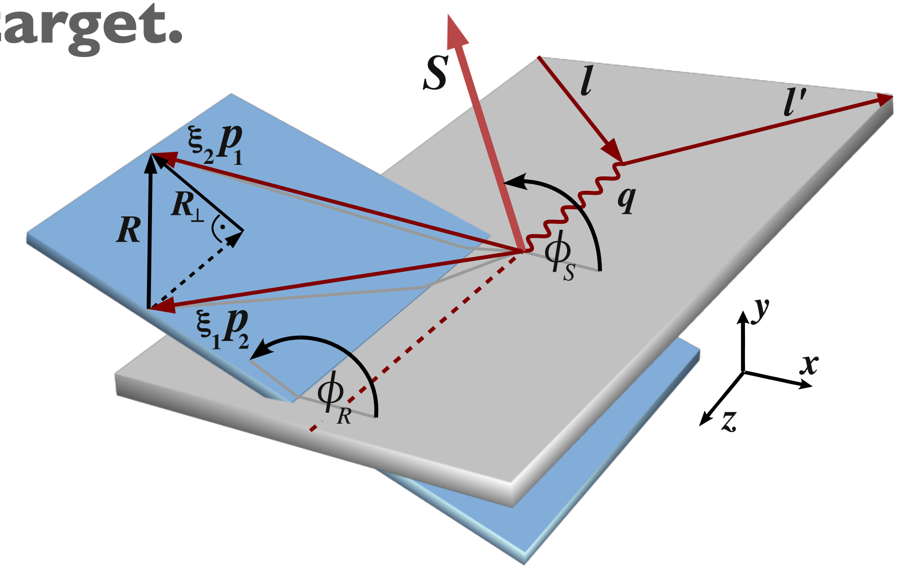
A. Bacchetta, M. Radici: PRD 69, 074026 (2004).

$$H_1^{\sphericalangle}(z_h, \xi, M_h^2) \equiv \int d^2\mathbf{k}_T \left[H_1^{\sphericalangle' e}(z_h, \xi, M_h^2, k_T^2, \mathbf{k}_T \cdot \mathbf{R}_T) + \frac{k_T^2}{2M_h^2} H_1^{\perp o}(z_h, \xi, k_T^2, R_T^2, \mathbf{k}_T \cdot \mathbf{R}_T) \right]$$

RECENT COMPASS RESULTS

COMPASS Collaboration, arXiv:1401.7873 (2014).

- **SIDIS with transversely polarized target.**



- **Collins single spin asymmetry:**

$$A_{Coll} = \frac{\sum_q e_q^2 \; h_1^q \otimes H_1^{\perp h/q}}{\sum_q e_q^2 \; f_1^q \otimes D_1^{h/q}}$$

- **Two hadron single spin asymmetry:**

$$A_{UT}^{\sin\phi_{RS}} = \frac{|\boldsymbol{p}_1 - \boldsymbol{p}_2|}{2M_{h^+h^-}} \frac{\sum_q e_q^2 \cdot h_1^q(x) \cdot H_{1,q}^{\sphericalangle}(z,\, M_{h^+h^-}^2,\, \cos\theta)}{\sum_q e_q^2 \cdot f_1^q(x) \cdot D_{1,q}(z,\, M_{h^+h^-}^2,\, \cos\theta)}$$

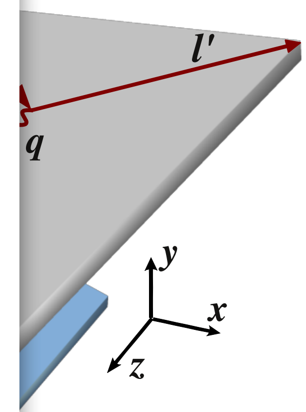
- **Note the choice of the vector**

$$\boldsymbol{R}_{Artru} = \frac{z_2 \boldsymbol{P}_1 - z_1 \boldsymbol{P}_2}{z_1 + z_2}$$

RECENT COMPASS RESULTS

COMPASS Collaboration, arXiv:1401.7873 (2014).

✦ **SIDIS wi**



✦ **Collins s**

A_{Col}

✦ **Two had**

$A_{UT}^{\sin\phi_{R}}$

$\cos\theta)$

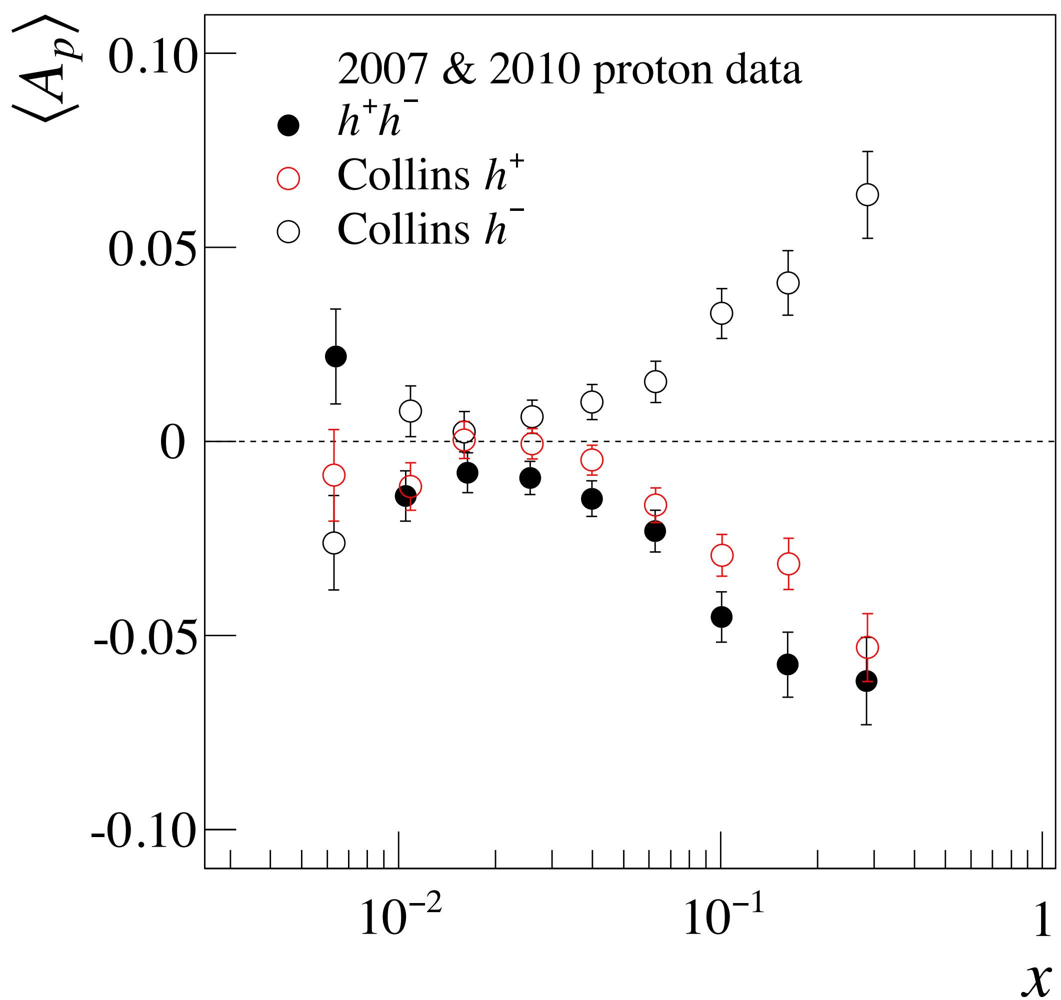
$\cos\theta)$

✦ **Note the**

$R_{Artru} =$

$z_1 + z_2$

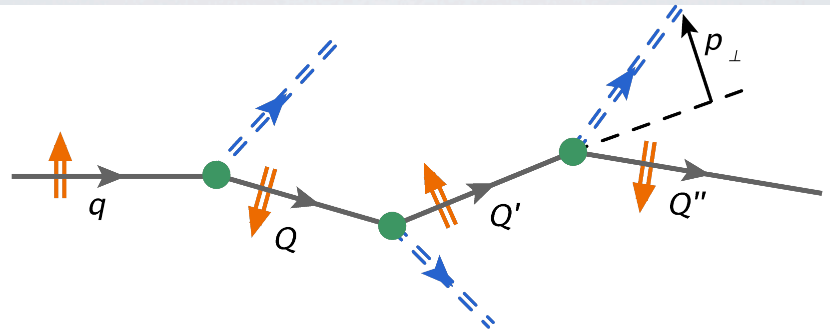
Mirror Symmetry for Collins Asymmetry and Correlations with IFF Asymmetry



POLARIZED QUARK DIFF IN QUARK-JET.

H.M., Kotzinian, Thomas, PLB731 (2014) 208-216.

- ***Use the NJL-jet Model including Collins effect (Mk 2) to study DiFFs.***



$$D_{q^\uparrow}^{h_1 h_2}(z, M_h^2, \varphi_R)\ \Delta z\ \Delta M_h^2\ \Delta\varphi_R = \left\langle N_{q^\uparrow}^{h_1 h_2}(z, z+\Delta z; M_h^2, M_h^2+\Delta M_h^2; \varphi_R, \varphi_R+\Delta\varphi_R)\right\rangle .$$

- ***Choose a constant Spin flip probability:*** $\mathcal{P}_{SF}$

- ***Simple model to start with:***

Only pions and extreme ansatz for the Collins term in elementary function.

$$\boxed{d_{h/q^\uparrow}(z, \mathbf{p}_\perp) = d_1^{h/q}(z, p_\perp^2)(1 - 0.9\sin\varphi)}$$

POLARIZED QUARK DIFF IN QUARK-JET.

H.M., Kotzinian, Thomas, PLB731 (2014) 208-216.

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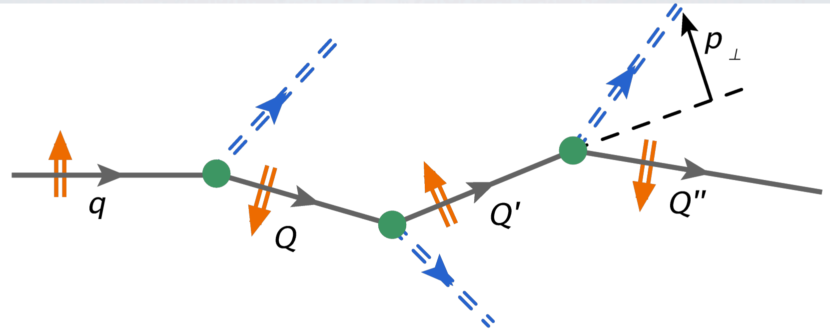

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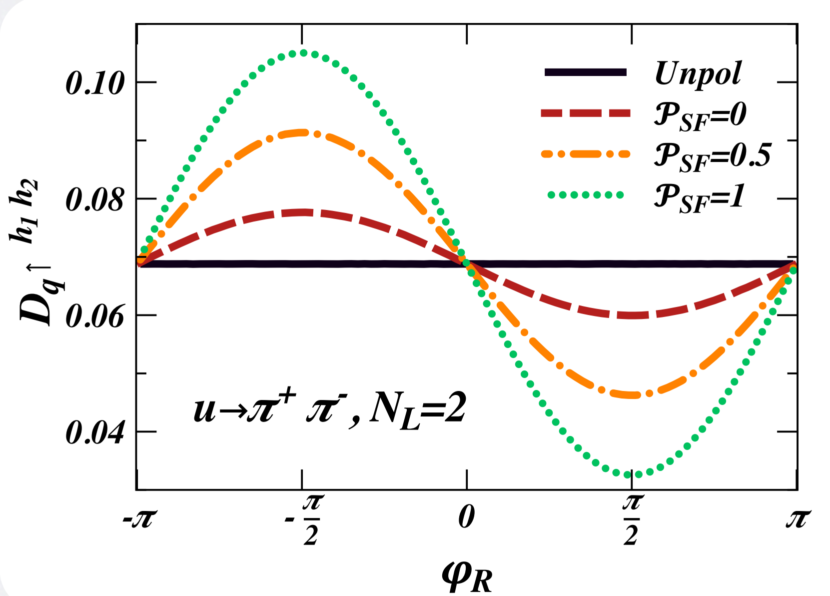
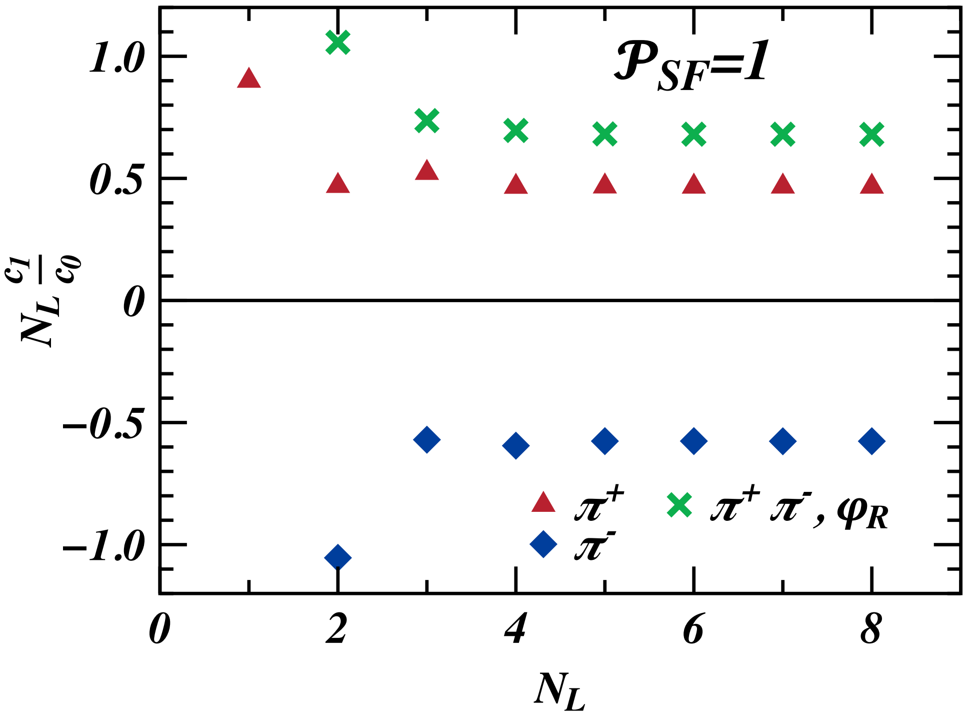


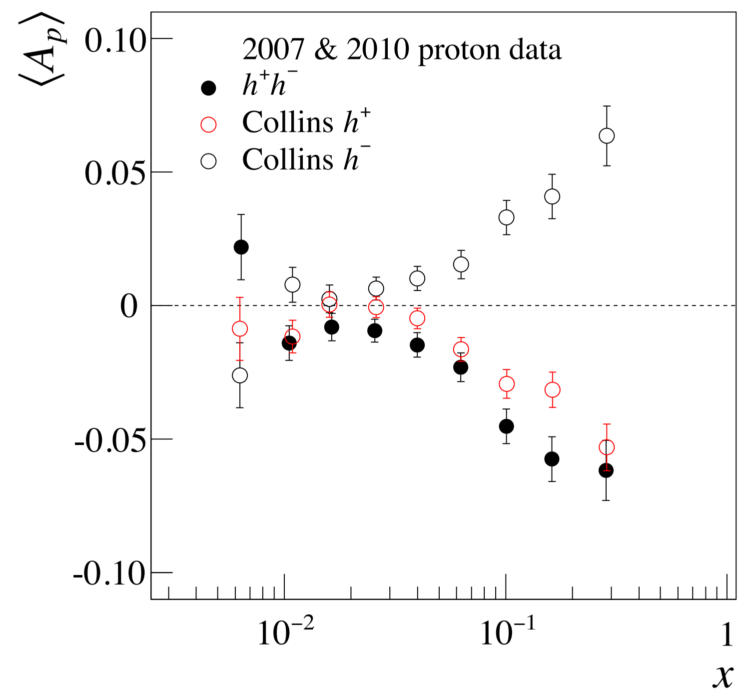
✦ ***Non-zero*** $\sin(\varphi_R - \varphi_S)$ ***modulations for all*** $\mathcal{P}_{SF}$ ***!***

INTEGRATED ANALYZING POWERS

NJL-Jet
PLB 731, 208 (2014).



COMPASS
arXiv:1401.7873 (2014).



$$D_{q\uparrow} = c_0 - \sin(\phi) c_1$$

✦ **NJL-Jet results for analysing powers have the same sign and the relative size for Collins Function of positive and negative pions, and the IFF mods as indicated by the COMPASS measurements!**

INTEGRATED ANALYZING POWERS

Single Hadron

$$\boxed{N_h \propto \sigma_{UU}(1 + \sin(\phi_C) A_C G)}$$

$$\phi_C = \phi_h - \phi_{S'}$$
$$= \phi_h + \phi_S - \pi$$

$$A_{Coll} = \frac{\sum_q e_q^2\, h_1^q \otimes H_1^{\perp h/q}}{\sum_q e_q^2\, f_1^q \otimes D_1^{h/q}}$$

DiHadron

$$\boxed{N_{h^+h^-} \propto \sigma_{UU}(1 + \sin(\phi_{RS}) A_{UT}^{\sin\phi_{RS}} F)}$$

$$\phi_{RS} = \phi_R - \phi_{S'}$$
$$= \phi_R + \phi_S - \pi$$

$$A_{UT}^{\sin\phi_{RS}} \propto \frac{\sum_q e_q^2 \cdot h_1^q \cdot H_{1,q}^{\sphericalangle}}{\sum_q e_q^2 \cdot f_1^q \cdot D_{1,q}}$$

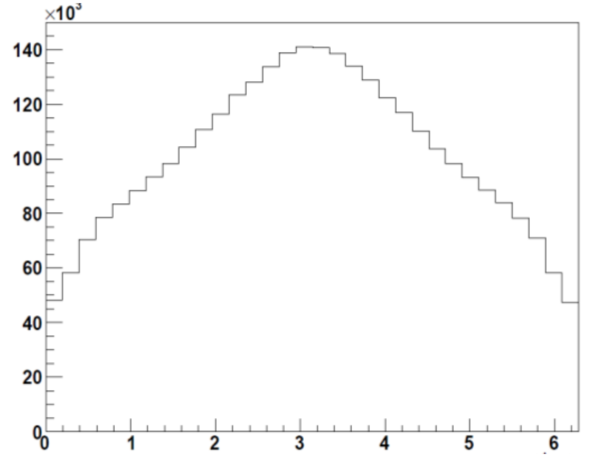
NJL-Jet Fits

$$\boxed{D_{q\uparrow} = c_0 - \sin(\phi) c_1}$$

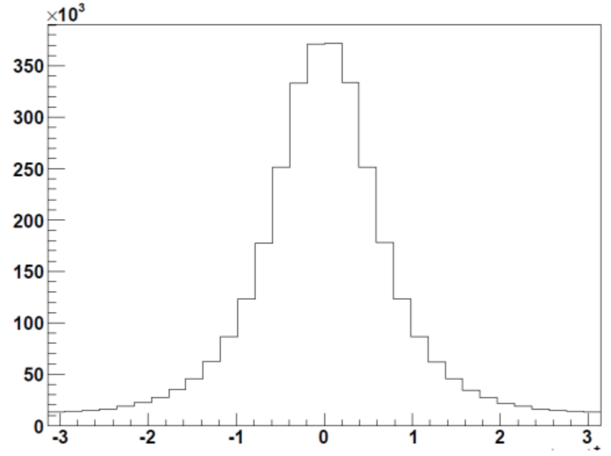
ANGULAR CORRELATIONS: $u \to \pi^+\pi^-$

Angular Correlations @ COMPASS

F. Bradamante - COMO 2013.

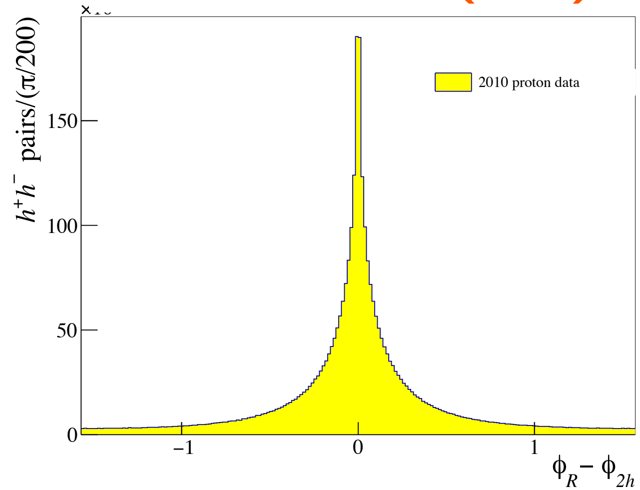


$\phi_{h+} - \phi_{h-}$

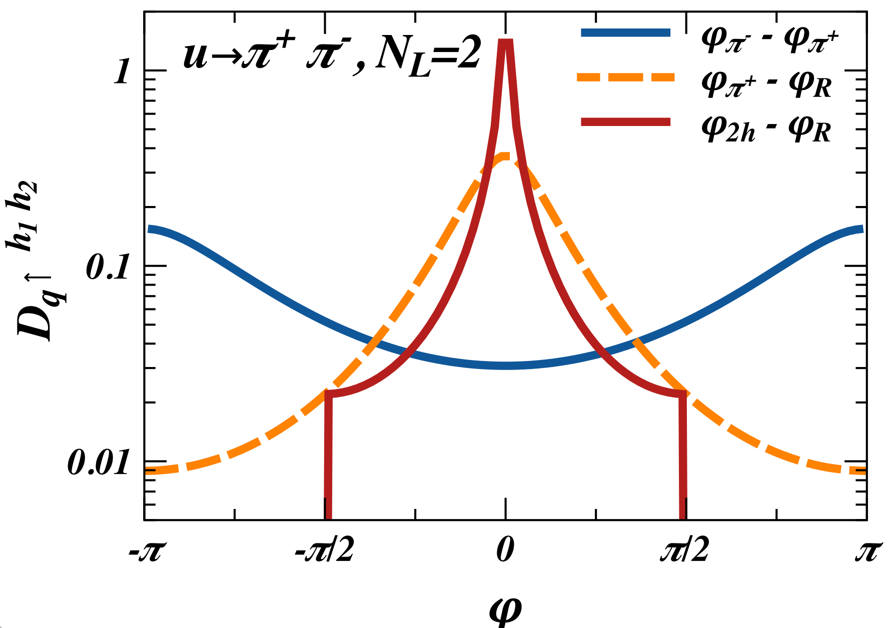


$\phi_{h+} - \phi_R$

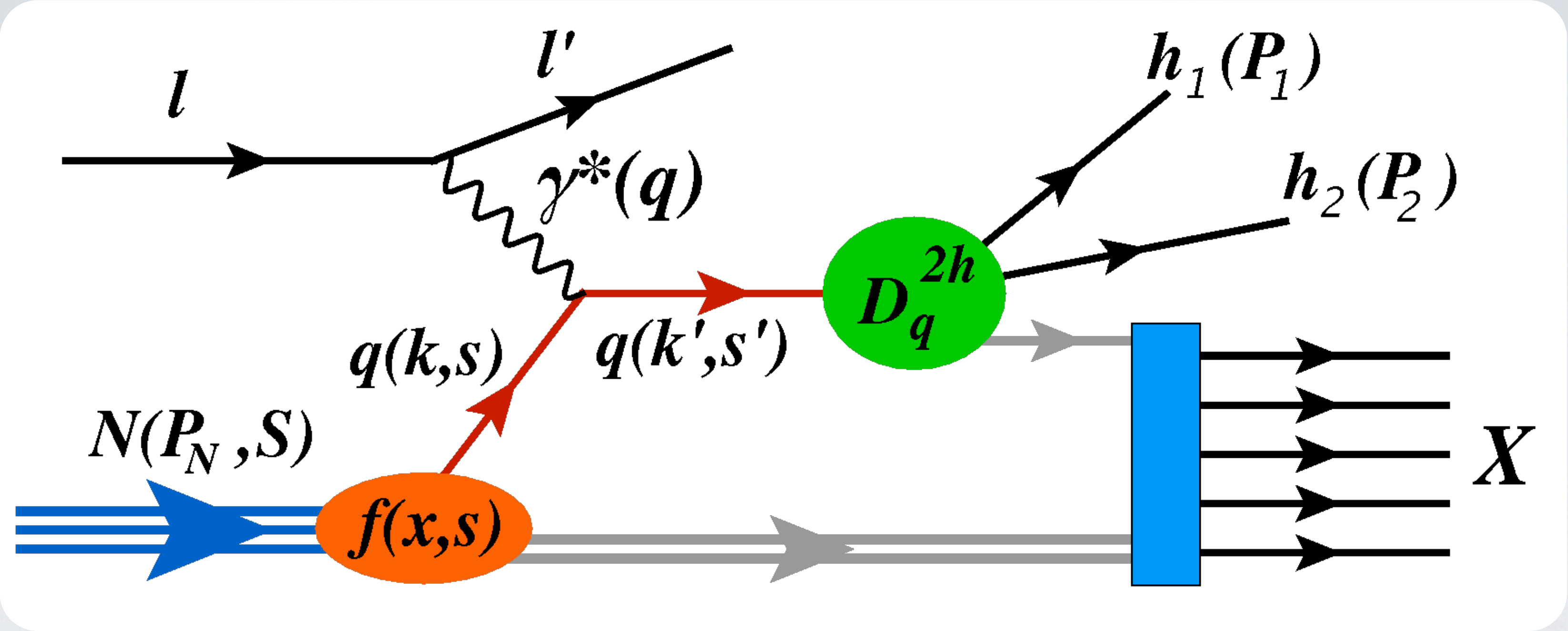
arXiv:1401.7873 (2014).



NJL-Jet Results



- We define: $\mathbf{P}_{2h} = (\widehat{\mathbf{P}}_1 - \widehat{\mathbf{P}}_2)/2$
- **COMPASS claim:** ϕ_{h+} **-** ϕ_{h-} **correlation is a consequence of** ***mirror symmetry*** **of** ***Collins effect*** **for** ***h^+*** **and** ***h^-*.**
- **We showed that there is** ***NO Spin Dependence*** **and** ***simply TM Conservation explains this corr. !***



Sivers Effect in Two Hadron SIDIS

SIVERS PDF

D. Sivers: PRD 41, 83 (1990).

- *Sivers Effect* describes the correlation of the *unpolarized quark's TM* with the *transverse spin* of the nucleon

N/q	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}^\perp$	$h_1\, h_{1T}^\perp$

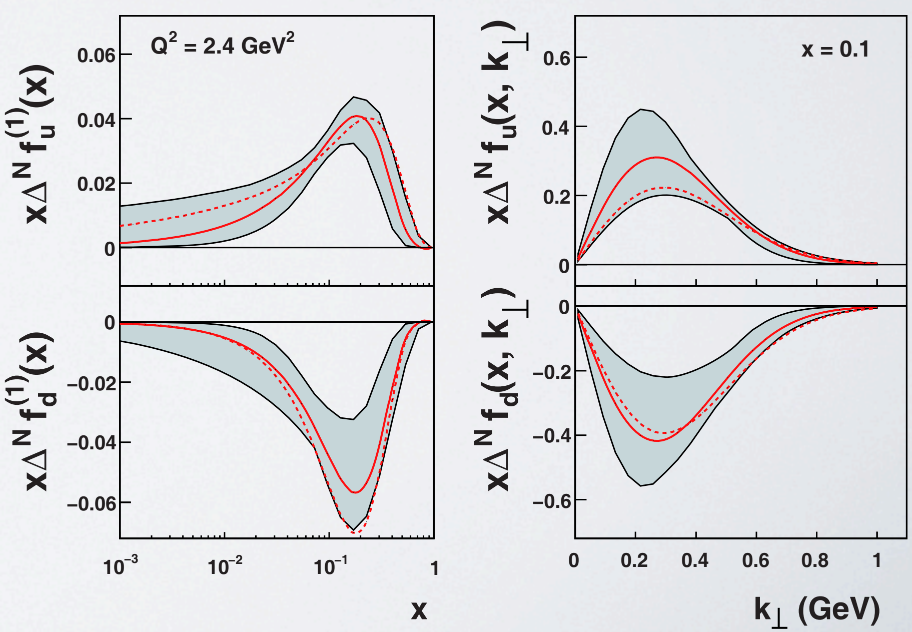
$$f_\uparrow^q(x,\boldsymbol{k}_T) = f_1^q(x,k_T) + \frac{[\boldsymbol{S}\times\boldsymbol{k}_T]_3}{M} f_{1T}^{\perp q}(x,k_T)$$

- **Accessible in Polarized SIDIS:**

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} \sim \mathcal{C}[f_{1T}^{\perp q}\ D_1]$$

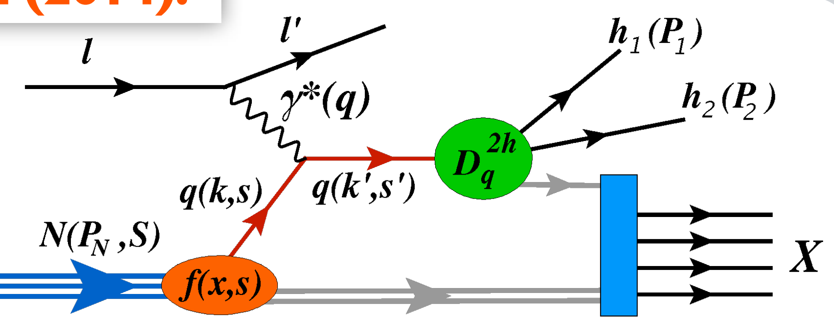
- ***Fits to HERMES and COMPASS***
  EPJ A39 (2009) 89-100.

- **Still Large Uncertainties/ Many Assumptions!**



TWO-HADRON SIDIS

A. Kotzinian, H.M., A.W. Thomas: arXiv:1403.5562 (2014).



- Cross Section in terms of **Total and Relative TM**

$$\frac{d\sigma^{h_1 h_2}}{dz_1\, dz_2\, d^2\boldsymbol{R}\, d^2\ \boldsymbol{T}} = C(x,Q^2)\,(\sigma_U + \sigma_S)$$

$$\sigma_U = \sum_q e_q^2 \int d^2\boldsymbol{k}_T\ f_1^q\ D_q^{h_1 h_2} \qquad \sigma_S = \sum_q e_q^2 \int d^2\boldsymbol{k}_T \frac{[\boldsymbol{S}_T \times \boldsymbol{k}_T]_3}{M} f_{1T}^{\perp q}\ D_{1q}^{h_1,h_2}$$

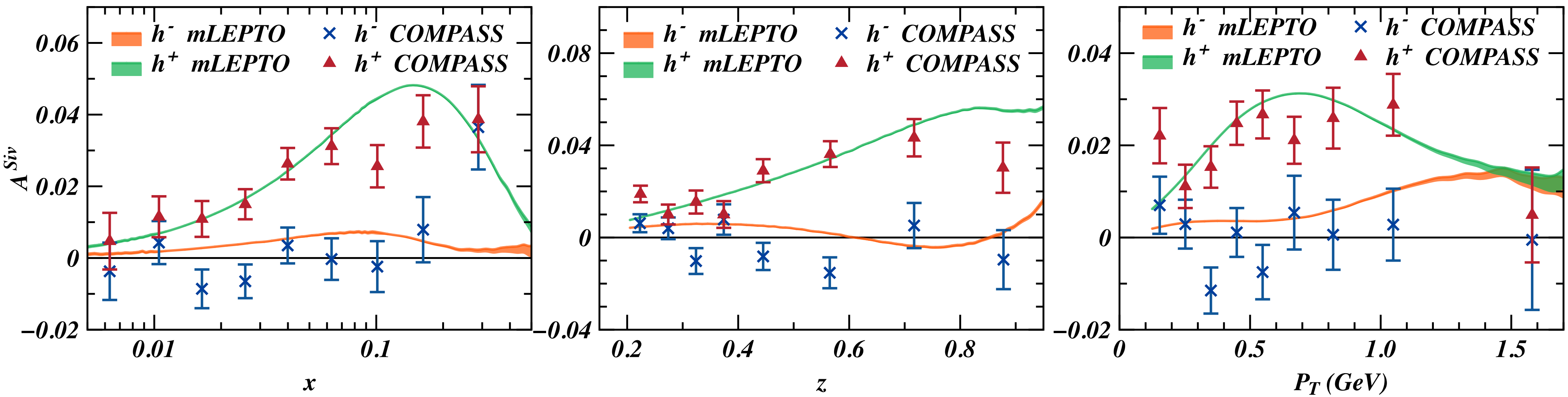
- The Sivers term:

$$\sigma_S = S_T\left(\sigma_T \frac{T}{M}\sin(\phi_T - \phi_S) + \sigma_R \frac{R}{M}\sin(\phi_R - \phi_S)\right)$$

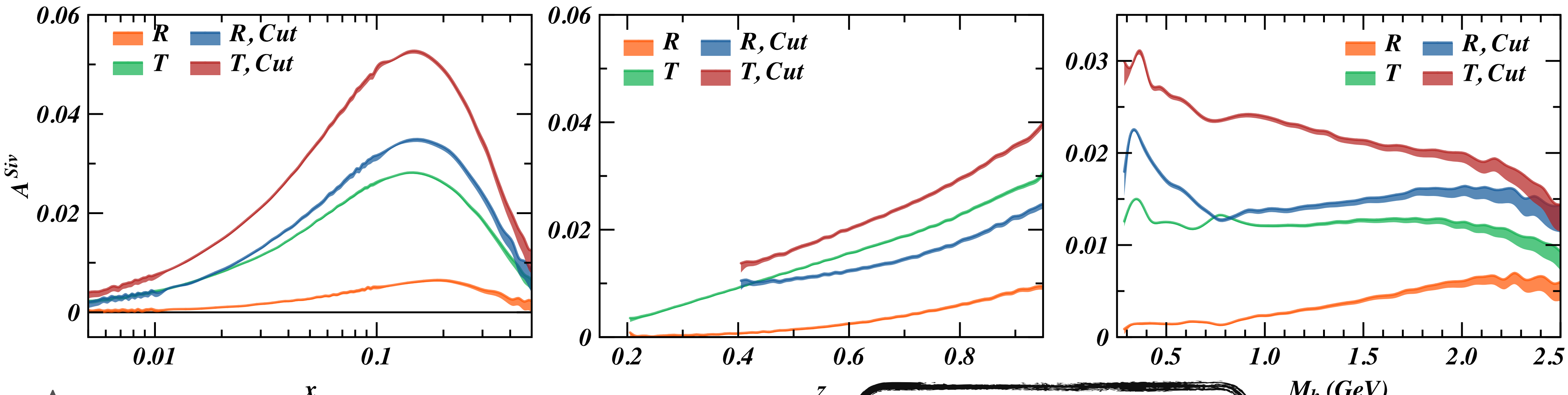
- **Non-vanishing σ_R is new!**

DIHADRON SIVERS USING **MLEPTO** MC

✦COMPASS Results in **One** Hadron Sivers and **mLEPTO** MC



✦ **mLEPTO** Predictions for DiHadron Sivers in COMPASS kinematics



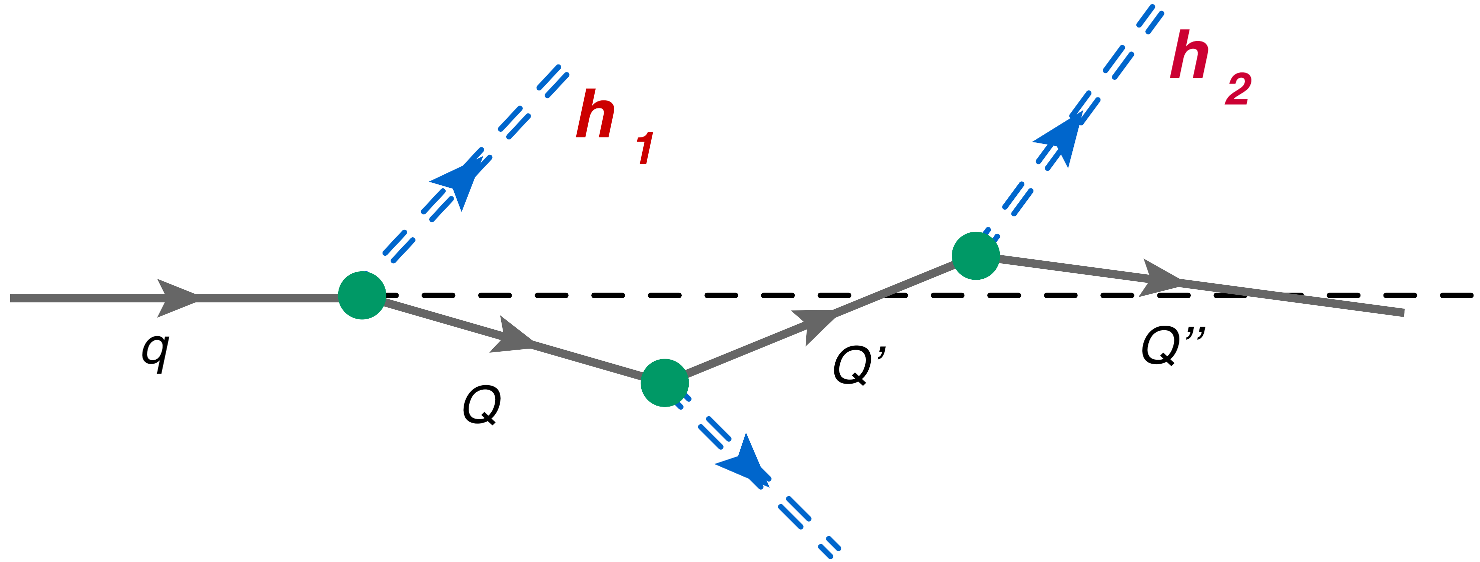
✦ **Asymmetric Cuts** on hadron pair momenta enhances the signal !

$$z_1 > 0.3$$
$$P_{1T} > 0.3 \text{ GeV}$$

CONCLUSIONS

- ***SIDIS*** process allows us to explore the spin and momentum correlations of partons inside of the nucleon.
- ***NJL-Jet MC*** helps us to test and understand important aspects of various processes using a specific underlying quark model:
  - ***Quark spin flip*** and ***TM conservation*** generate the opposite sign of ***favored*** and ***unfavored*** Collins FFs.
  - The role of the ***Collins*** mechanism in ***IFFs***.
- Measurements of ***Sivers Effect*** in ***Two-Hadron SIDIS*** will provide ***a new input*** for extracting ***Sivers PDF*** and better understanding of the ***underlying physics.***
- Further developments of the model are underway:
  - Including vector mesons in polarized fragmentations.
  - Exploring the target fragmentation.

BACKUP SLIDES

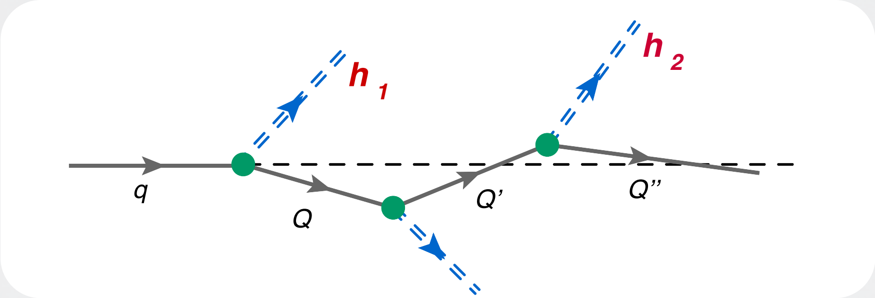


TWO HADRON FRAGMENTATION OF AN UNPOLARIZED QUARK:

UNPOLARIZED DiFF

UNPOLARIZED DIHADRON FRAGMENTATIONS

H.M. Thomas, Bentz, PRD.88:094022, 2013.



- The probability density for observing two hadrons:

$$P_1 = (z_1 k^-, P_1^+, \boldsymbol{P}_{1,\perp}),\ \ P_1^2 = M_{h1}^2$$

$$P_2 = (z_2 k^-, P_2^+, \boldsymbol{P}_{2,\perp}),\ \ P_2^2 = M_{h2}^2$$

- The corresponding number density:

$$\boxed{D_q^{h_1 h_2}(z, M_h^2)\ \Delta z\ \Delta M_h^2 = \langle N_q^{h_1 h_2}(z, z+\Delta z; M_h^2, M_h^2 + \Delta M_h^2)\rangle}$$

$$z = z_1 + z_2 \quad M_h^2 = (P_1 + P_2)^2$$

- Kinematic Constraint.

$$\boxed{z_1 z_2 M_h^2 - (z_1 + z_2)(z_2 M_{h1}^2 + z_1 M_{h2}^2) \geq 0}$$

- In MC simulations record all the pairs in every decay chain.

THE TREATMENT OF VM DECAYS: COMPARISON TO PYTHIA.

✦ PYTHIA MODELING OF VM DECAYS

- 2-body decay amplitude: non-relativistic Breit-Wigner:

$$\mathcal{P}(m)dm \propto \frac{1}{(m-m_0)^2+\Gamma^2/4}dm$$

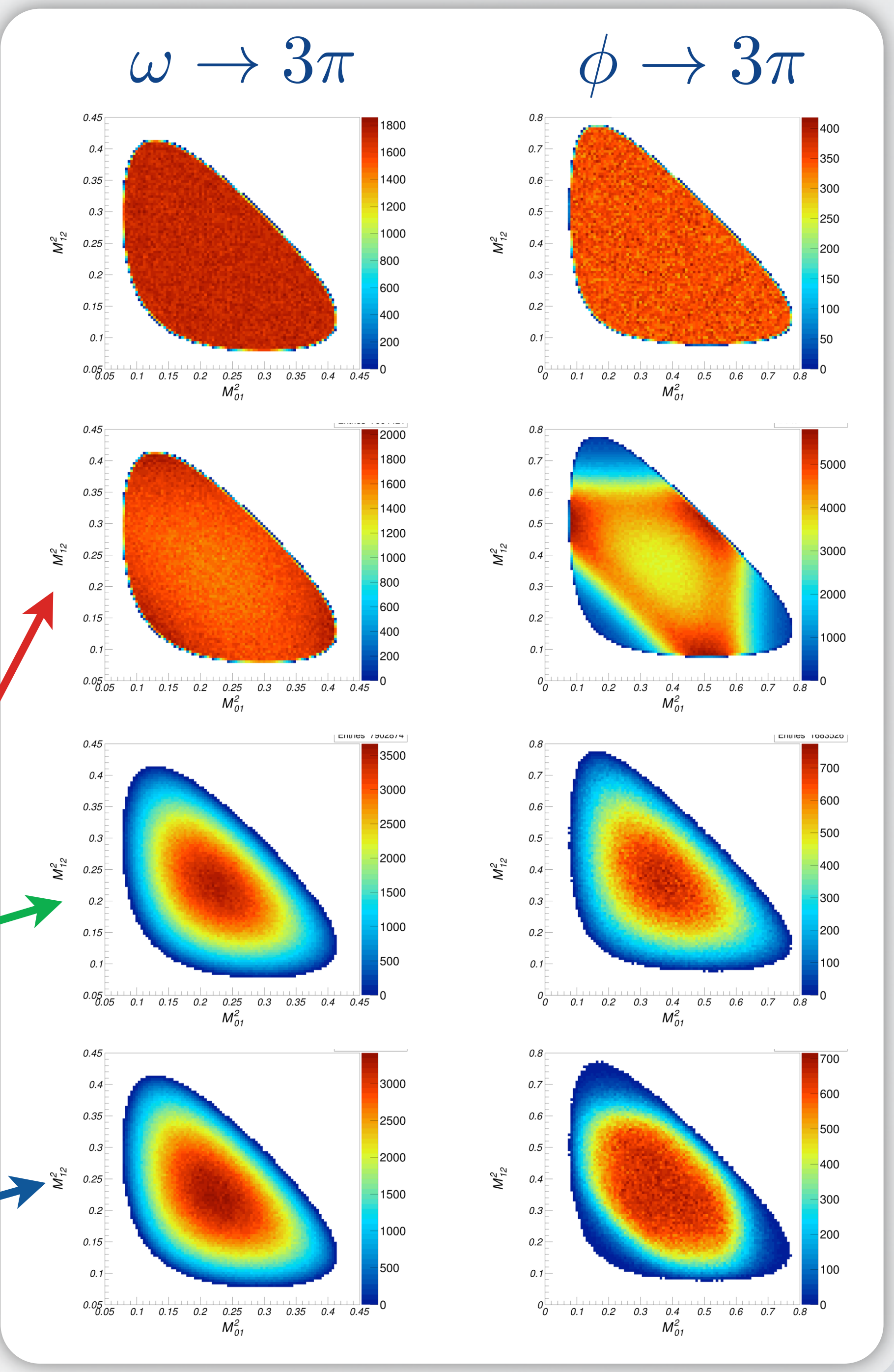
- Constant decay width of VM.

$$\Gamma_V(s) = \frac{m_V^2}{s}\Gamma_V\left(\frac{q(s)}{q(m_V^2)}\right)^3$$

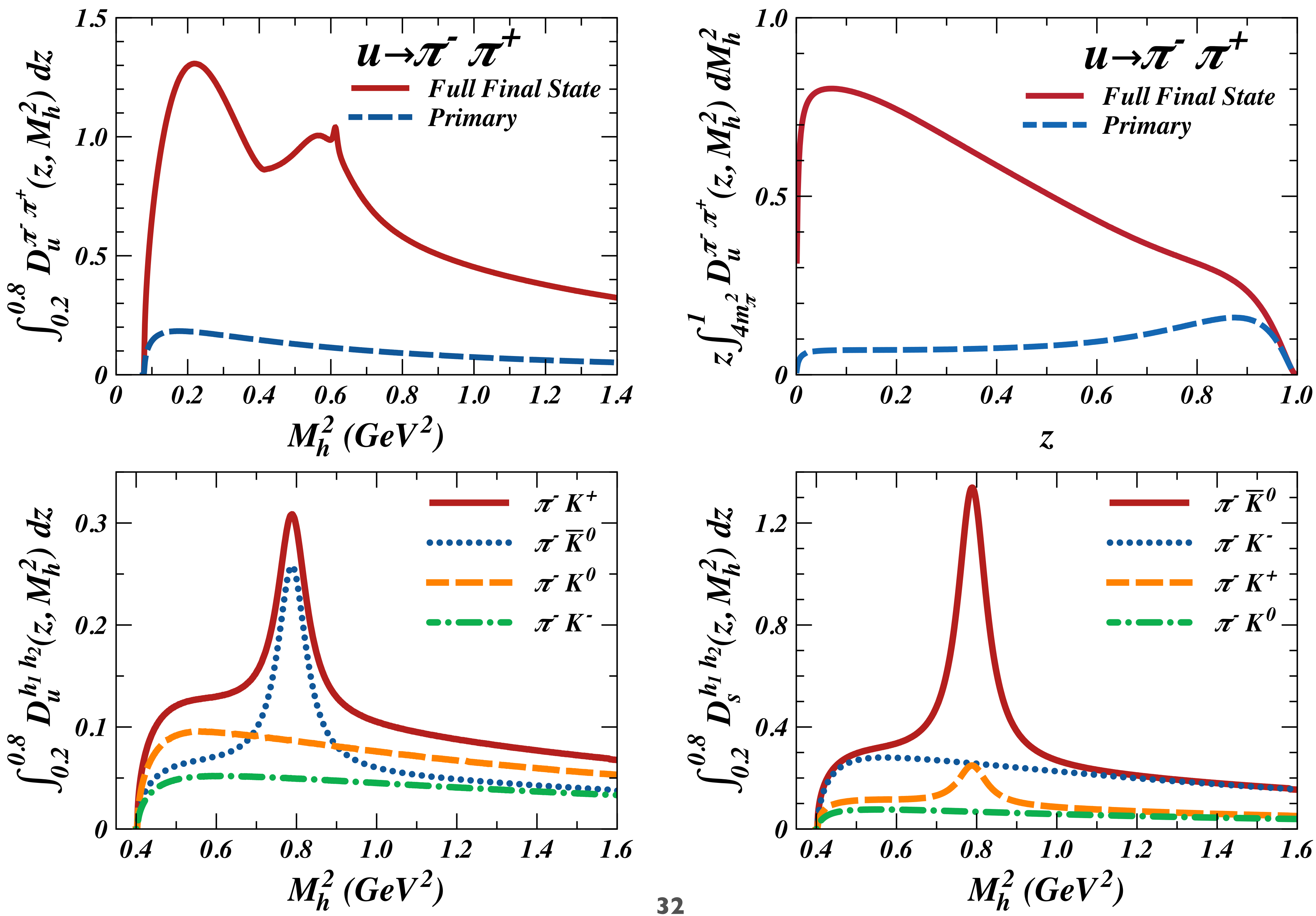
✦ Comparison of 3-body decay amplitudes:

- Point-like coupling (PYTHIA).
- "Isobar" model (HERWIG, NJL-jet). Relativistic BW and E dependent widths.

$$M = \varepsilon_{\mu\alpha\beta\gamma}\epsilon^{\mu}p_1^{\alpha}p_2^{\beta}p_3^{\gamma} \sum_{i=0,\pm}\frac{g_{V\rho_i\pi}\;g_{\rho_i\pi\pi}}{D_{\rho_i}(v_i^2)}$$



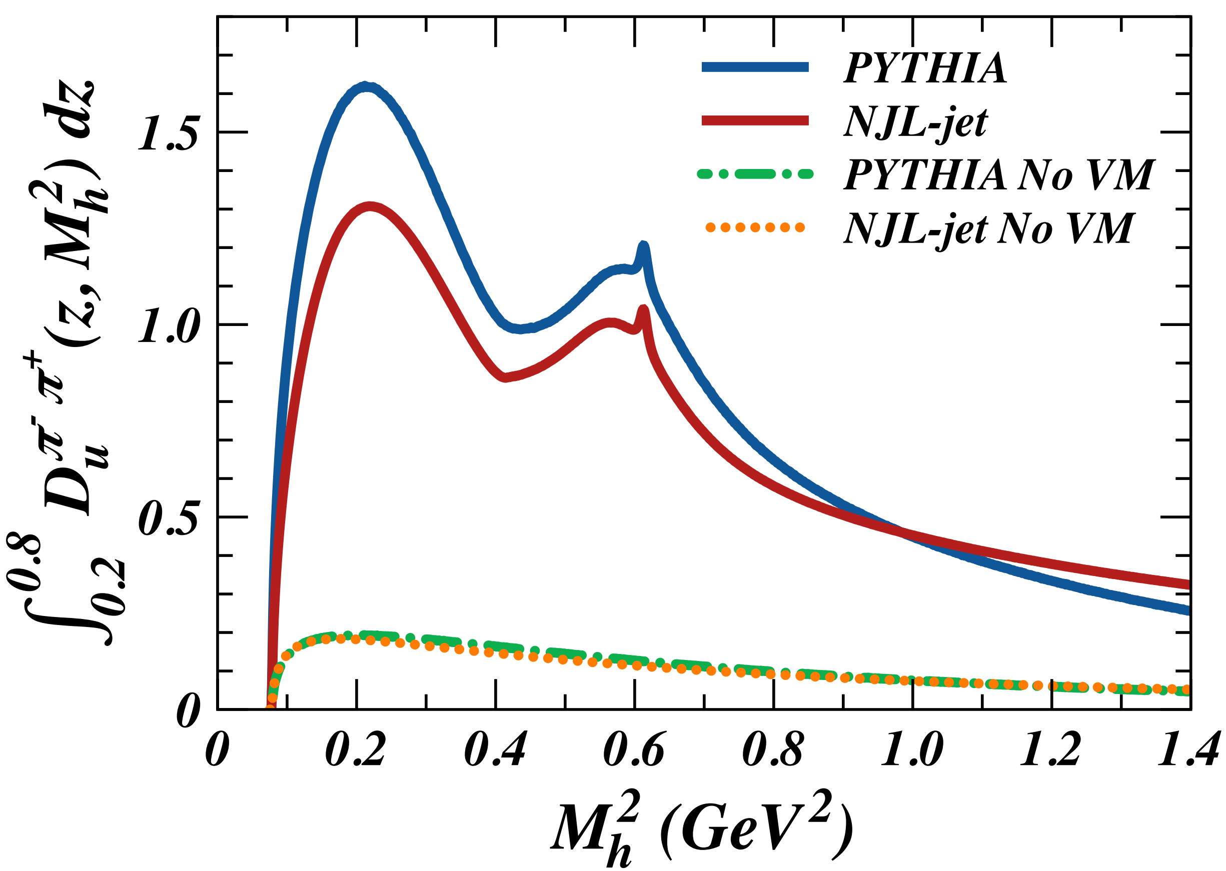
RESULTS FOR DFFS $N_{Links} = 8$



PYTHIA SIMULATIONS

- Setup hard process with back to back $q\ \bar{q}$ along **z** axis.
- ***Only Hadronize.*** Allow the same resonance decays as NJL.
- Assign hadrons with positive p_z to q fragmentation.

$$E_q = 10\ \mathrm{GeV}$$



Remarkable Agreement!

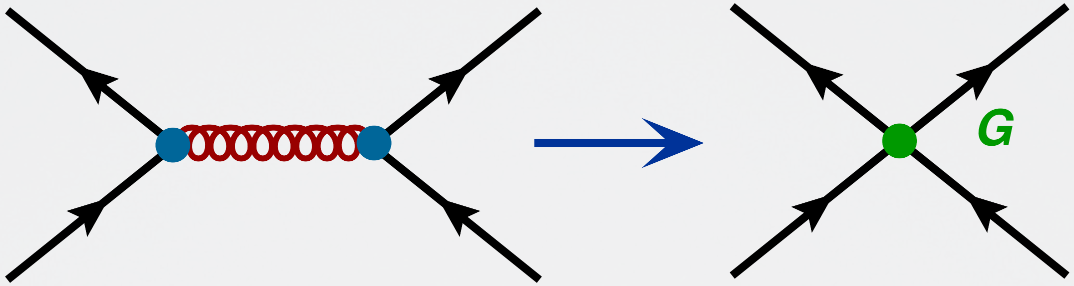
Though at vastly different Scales.

NAMBU--JONA-LASINIO MODEL

Effective Quark model of QCD

- Effective Quark Lagrangian

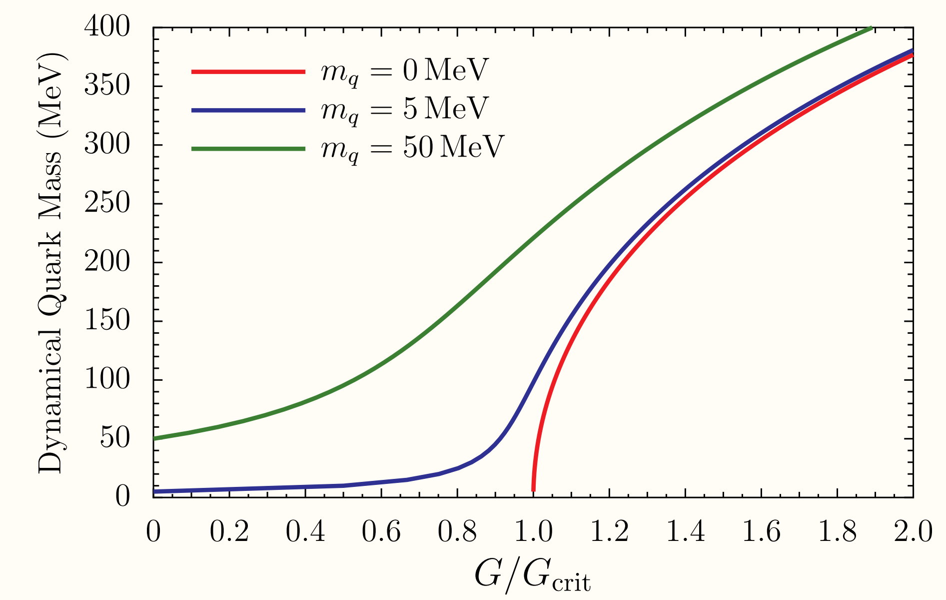
$$\mathcal{L}_{NJL} = \overline{\psi}_q(i\partial\!\!\!/ - m_q)\psi_q + G(\overline{\psi}_q\Gamma\psi_q)^2$$



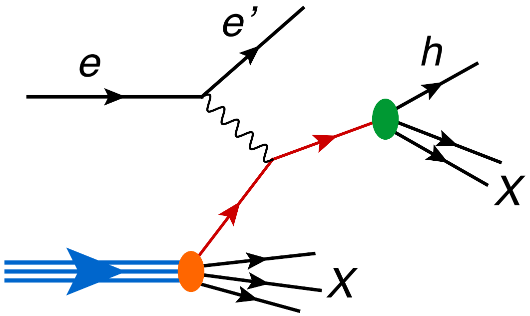
- Low energy chiral effective theory of QCD.

- Covariant, has the same flavor symmetries as QCD.

- Dynamically Generated Quark Mass from GAP Eqn.

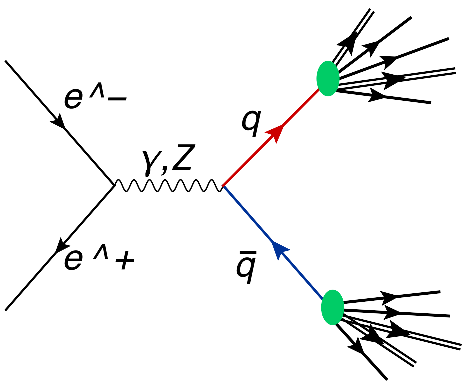


COLLINEAR ***FACTORIZATION*** AND ***UNIVERSALITY***



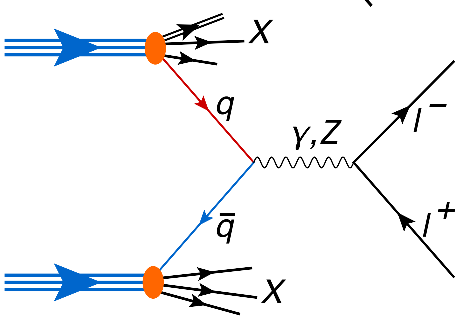
- SEMI INCLUSIVE DIS (SIDIS)

$$\sigma^{eP\to ehX} = \sum_q f_q^P \otimes \sigma^{eq\to eq} \otimes D_q^h$$



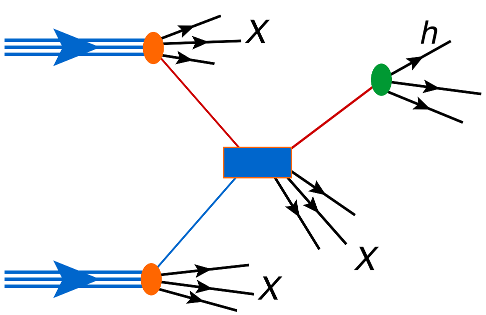
- e^+e^-

$$\sigma^{e^+e^-\to hX} = \sum_q \sigma^{e^+e^-\to q\bar{q}} \otimes (D_q^h + D_{\bar{q}}^h)$$



- DRELL-YAN (DY)

$$\sigma^{PP\to l^+l^-X} = \sum_{q,q'} f_q^P \otimes f_{\bar{q}}^P \otimes \sigma^{q\bar{q}\to l^+l^-}$$



- Hadron Production

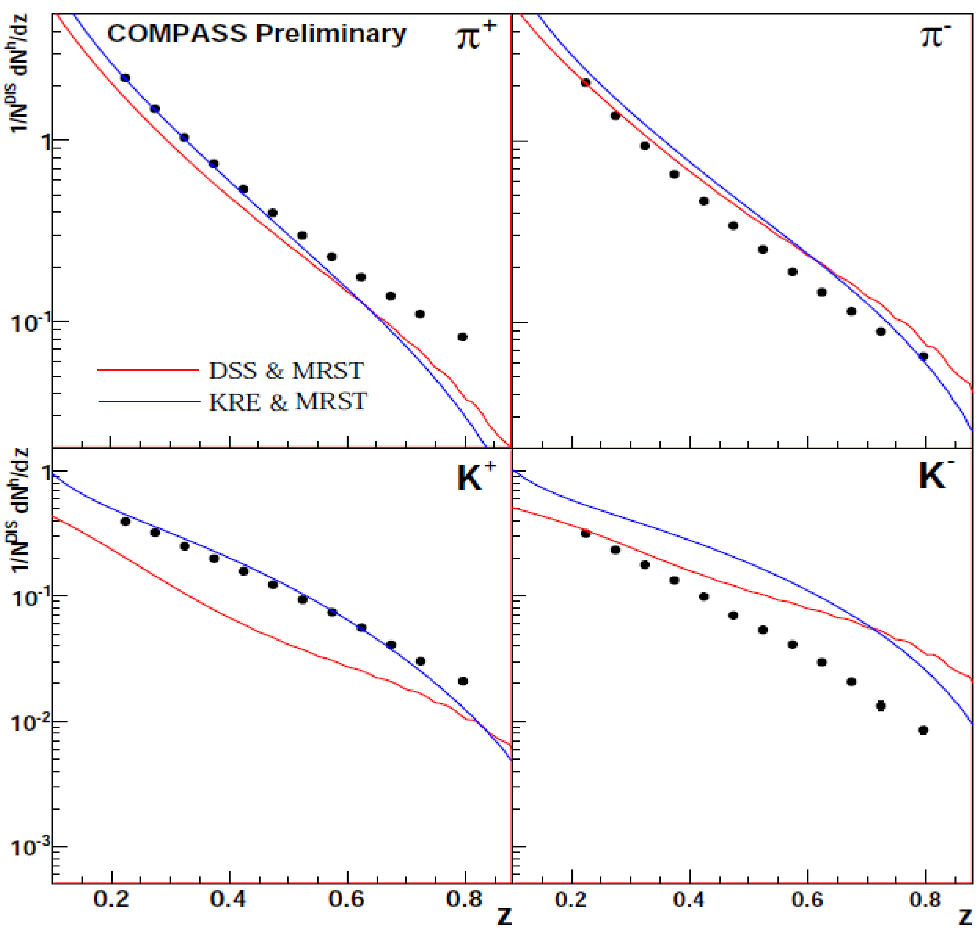
$$\sigma^{PP\to hX} = \sum_{q,q'} f_q^P \otimes f_{q'}^P \otimes \sigma^{qq'\to qq'} \otimes D_q^h$$

Unfavored FFs NOT well known!

Hadron Multiplicities

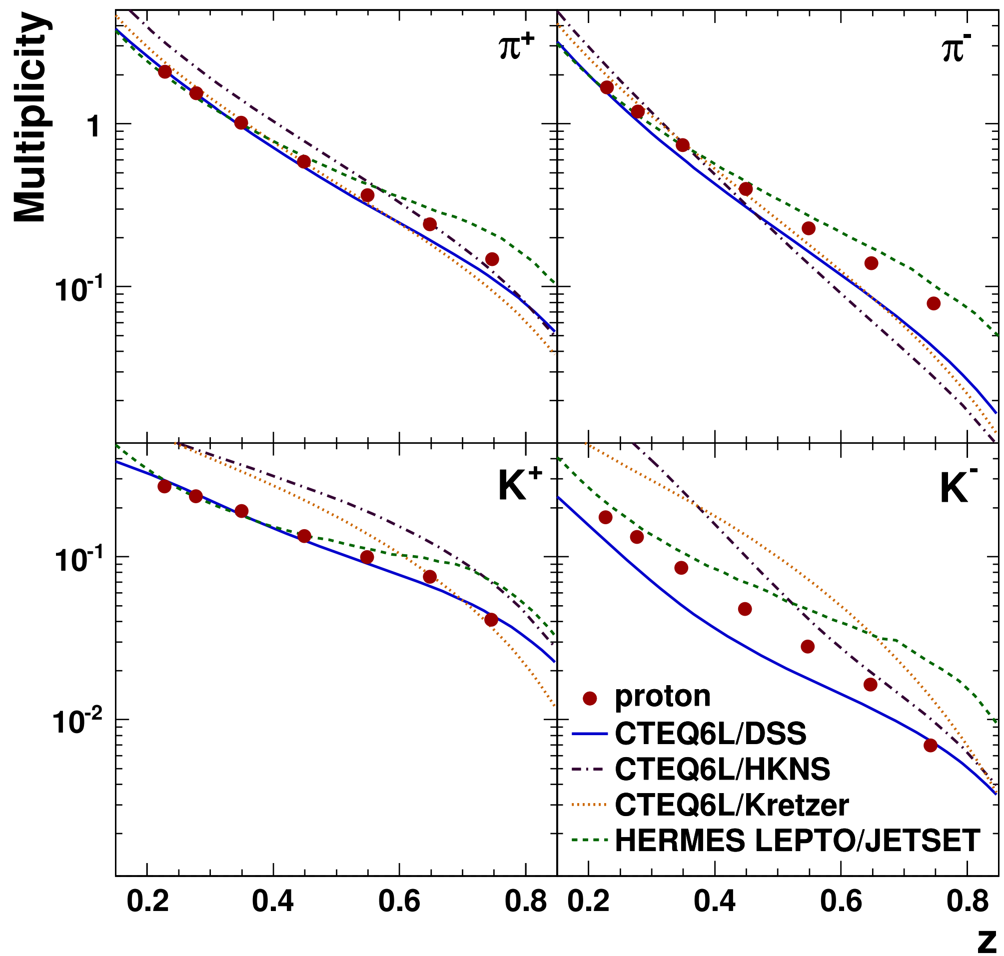
- Preliminary from COMPASS

Talk by C.Franco at CIPANP 2012.



- Also results from HERMES

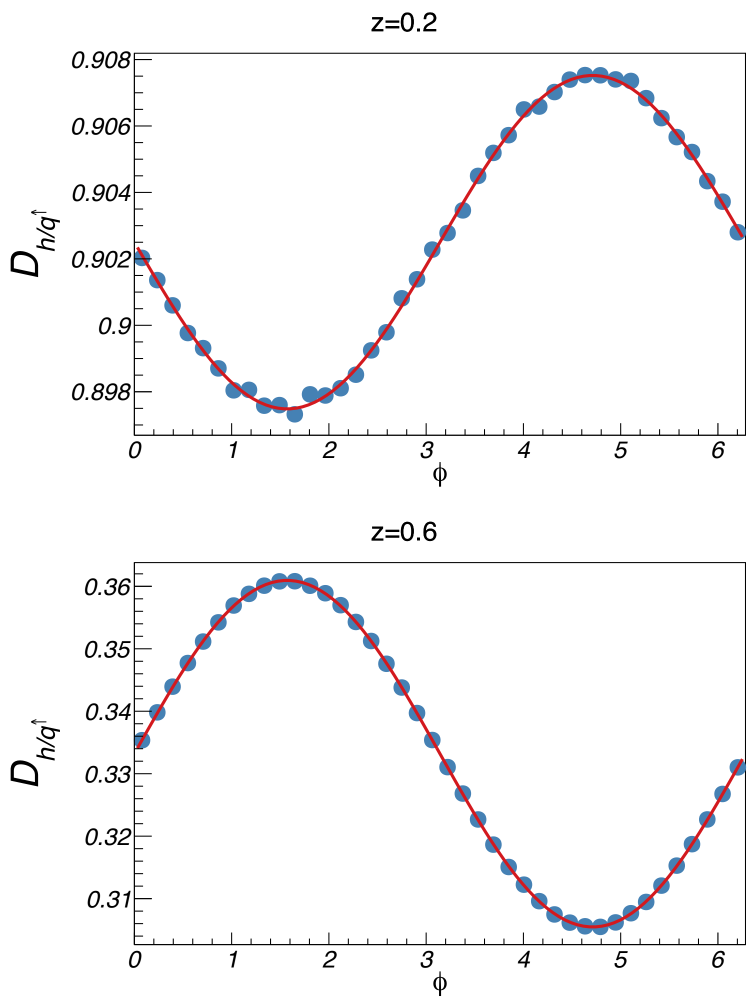
Phys. Rev. D 87, 074029 (2013)



INTEGRATED POLARIZED FRAGMENTATIONS

- Integrate Polarized Fragmentations over $P_\perp^2$

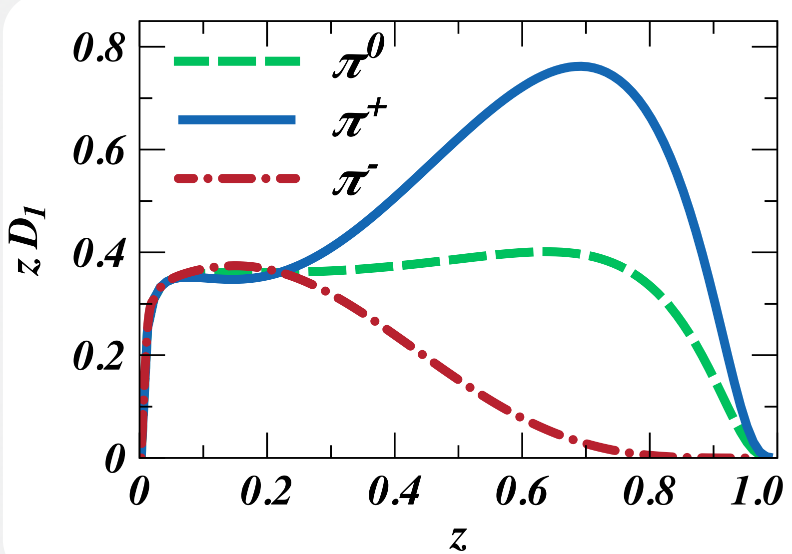
$$D_{h/q^\uparrow}(z,\varphi) \equiv \int_0^\infty dP_\perp^2\ D_{h/q^\uparrow}(z,P_\perp^2,\varphi) = \frac{1}{2\pi}\left[D_1^{h/q}(z)\ - 2H_{1(h/q)}^{\perp(1/2)}(z) S_q \sin(\varphi)\right]$$



$$D_1^{h/q}(z) \equiv \pi \int_0^\infty dP_\perp^2\ D_1^{h/q}(z,P_\perp^2)$$

$$H_{1(h/q)}^{\perp(1/2)}(z) \equiv \pi \int_0^\infty dP_\perp^2 \frac{P_\perp}{2zm_h} H_1^{\perp h/q}(z,P_\perp^2)$$

- Fit with form: $F(c_0,c_1) = c_0 - c_1 \sin(\varphi)$



THE EFFECT OF VECTOR MESONS (VM)

- A naive assumption: VMs should have modest contribution due to relatively small production probability $P(\pi^+)/P(\rho^+) \approx 1.7$
- ***But***: Combinatorial factors enhance VM contribution significantly!
- Let's consider only two hadron emission

Direct: $u \to d + \pi^+ \to u + \pi^- + \pi^+$

VM: $u \to d + \pi^+ \to u + \underbrace{\rho^-}_{\to \pi^- \pi^0} + \pi^+$

...

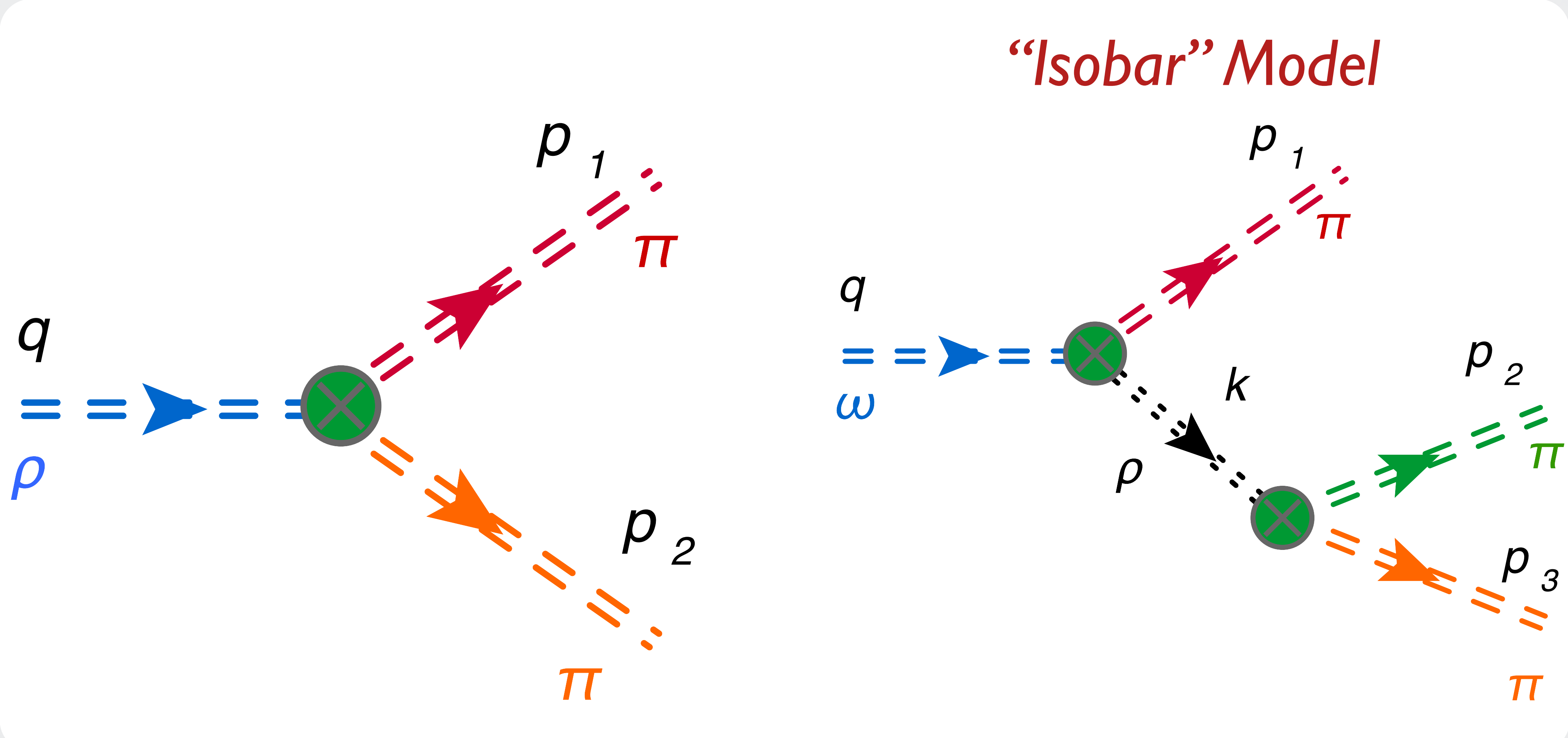
$u \to u + \rho^0 \to u + \underbrace{\rho^0}_{\to \pi^+ \pi^-} + \rho^0 \to \pi^+ \pi^-$

$$P_{Dir}(\pi^+\pi^-)/P_{VM}(\pi^+\pi^-) \approx \frac{1}{4}$$

2- AND 3-BODY DECAYS

The M_h^2 spectrum of pseudoscalars is strongly affected by VM decays.

- We include only the 2-body decays ρ, K^*.
- Both 2- and 3-body decays of ω, ϕ .



2- AND 3-BODY DECAYS

The M_h^2 spectrum of pseudoscalars is strongly affected by VM decays.

- We include only the 2-body decays ρ, K^*.
- Both 2- and 3-body decays of ω, ϕ .

Achasov et al. (SND), PRD 68, 052006, (2003).

- 2-body decay amplitude:

$$M(p_1, p_2) = \frac{g_V^{h_1 h_2} \epsilon^\mu (p_{2\mu} - p_{1\mu})}{D_V(q^2)}$$

- Resonance propagator:

$$D_V(s) = m_V^2 - s - i\sqrt{s}\Gamma_V(s)$$

$$\Gamma_V(s) = \frac{m_V^2}{s}\Gamma_V\left(\frac{q(s)}{q(m_V^2)}\right)^3$$

Relative Momentum of daughters in their CM frame.

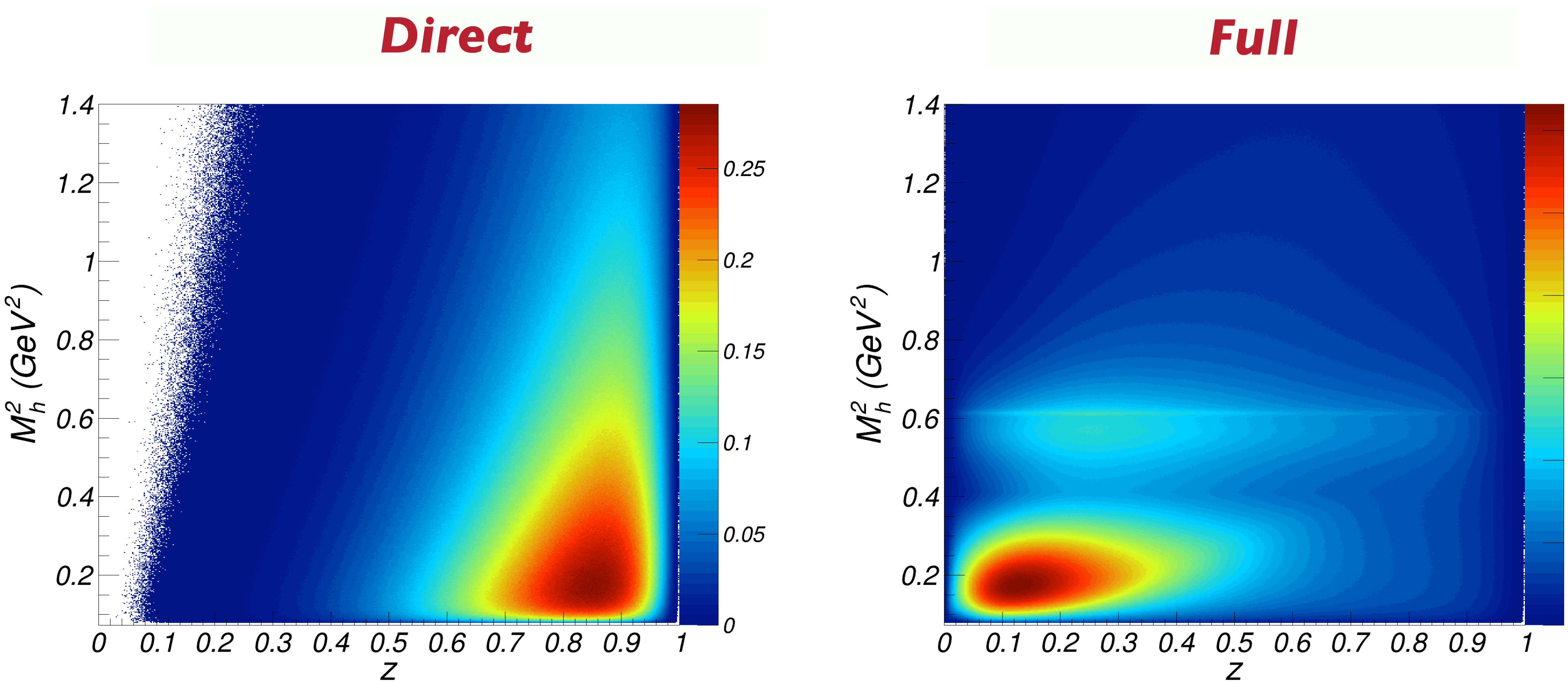
- 3-body decay amplitude (ignore small width):

$$M(p_1, p_2, p_3) = \varepsilon_{\mu\alpha\beta\gamma}\epsilon^\mu p_1^\alpha p_2^\beta p_3^\gamma \sum_{i=0,\pm} \frac{g_{V\rho_i\pi}\ g_{\rho_i\pi\pi}}{D_{\rho_i}(v_i^2)}$$

- Simulate 2- and 3-body phase space in LC.

RESULTS FOR PION DFF $u \to \pi^- \pi^+$

$$N_{Links} = 2$$



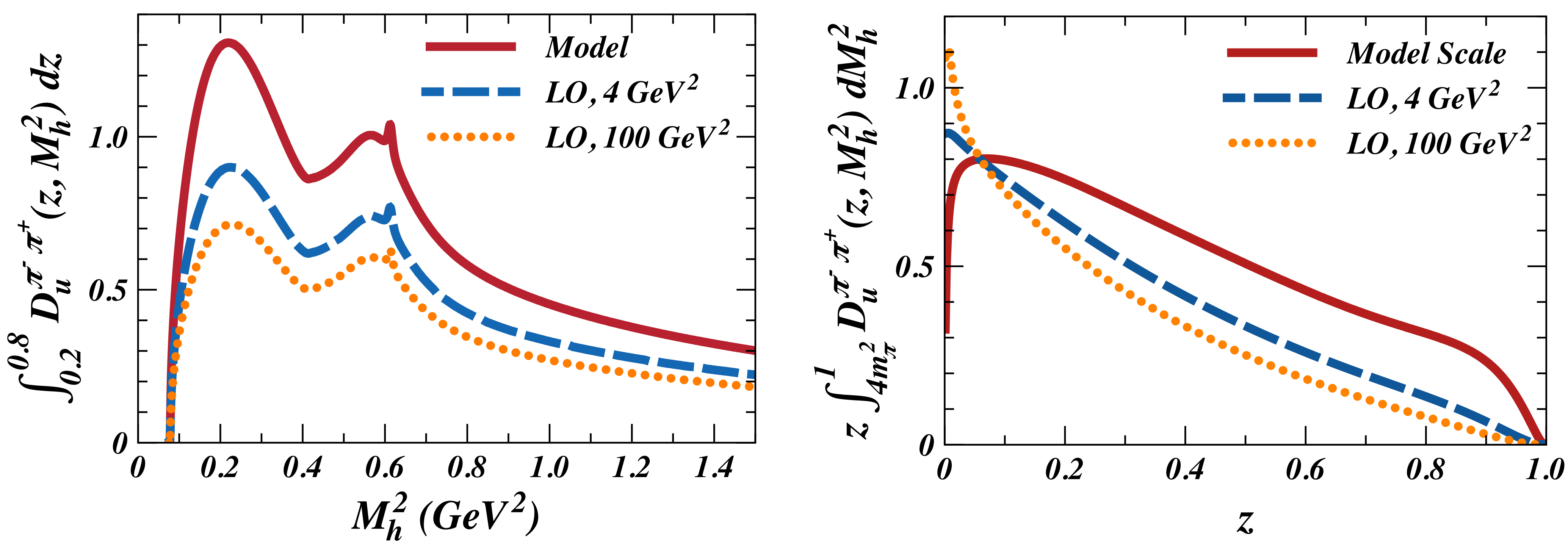
EVOLUTION OF DFF

Bacchetta et. al., Phys.Rev. D79, 034029 (2009).

At leading order:

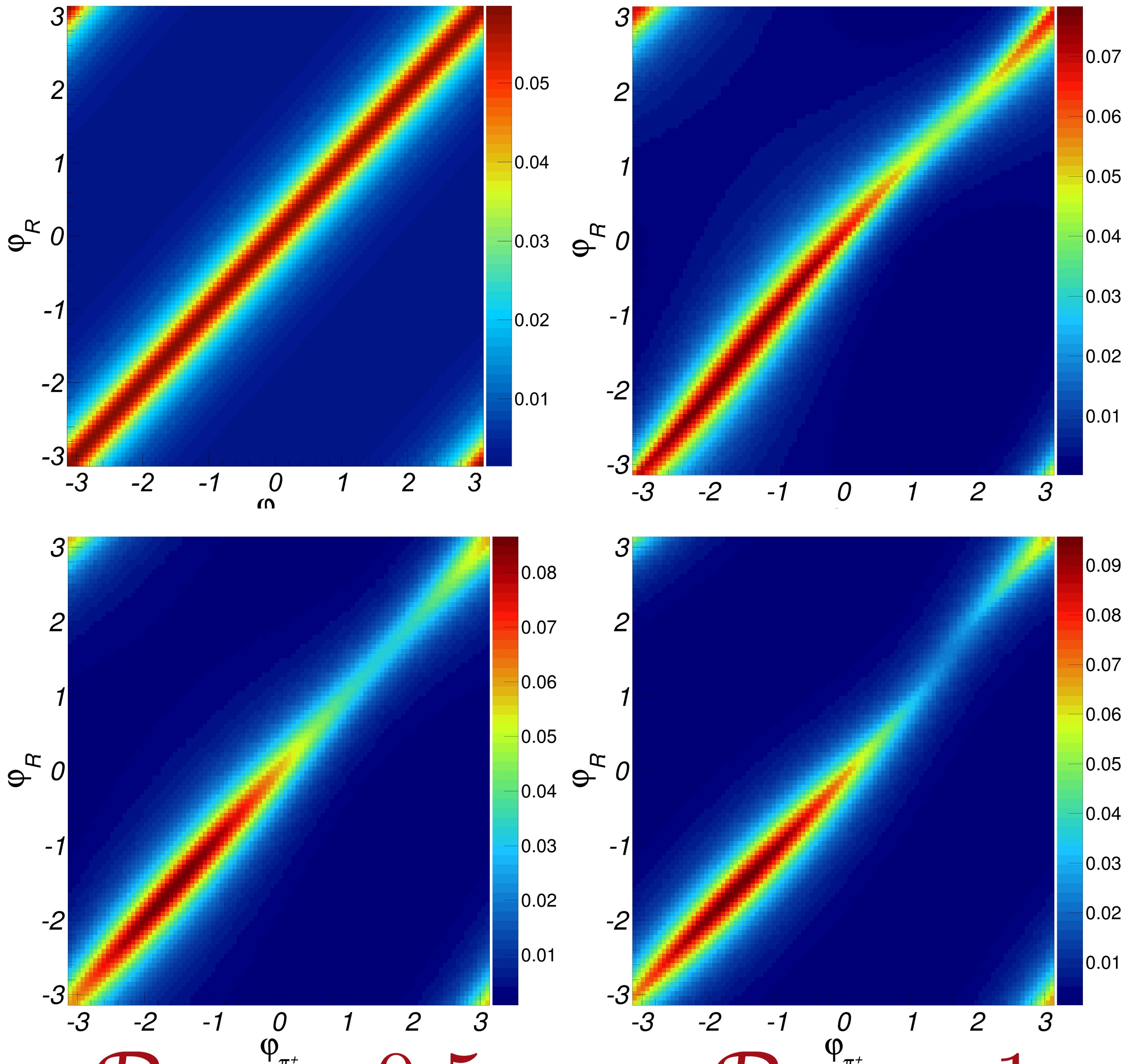
$$\frac{d}{d\log Q^2} D_{1,q}(z, M_h^2, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_z^1 \frac{du}{u} D_{1,q'}\left(\frac{z}{u}, M_h^2, Q^2\right) P_{q'q}(u)$$

$$u \rightarrow \pi^- \pi^+$$



ANGULAR CORRELATIONS: $u \to \pi^+\pi^-$

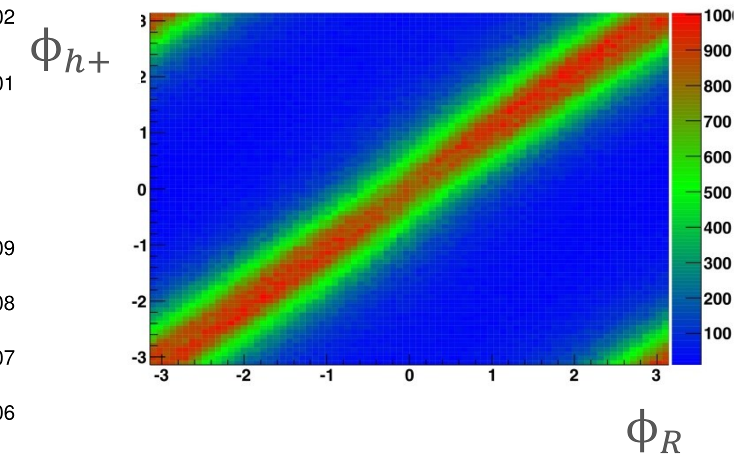
Unpolarized $\qquad \mathcal{P}_{SF} = 0$



$\mathcal{P}_{SF} = 0.5 \qquad \mathcal{P}_{SF} = 1$

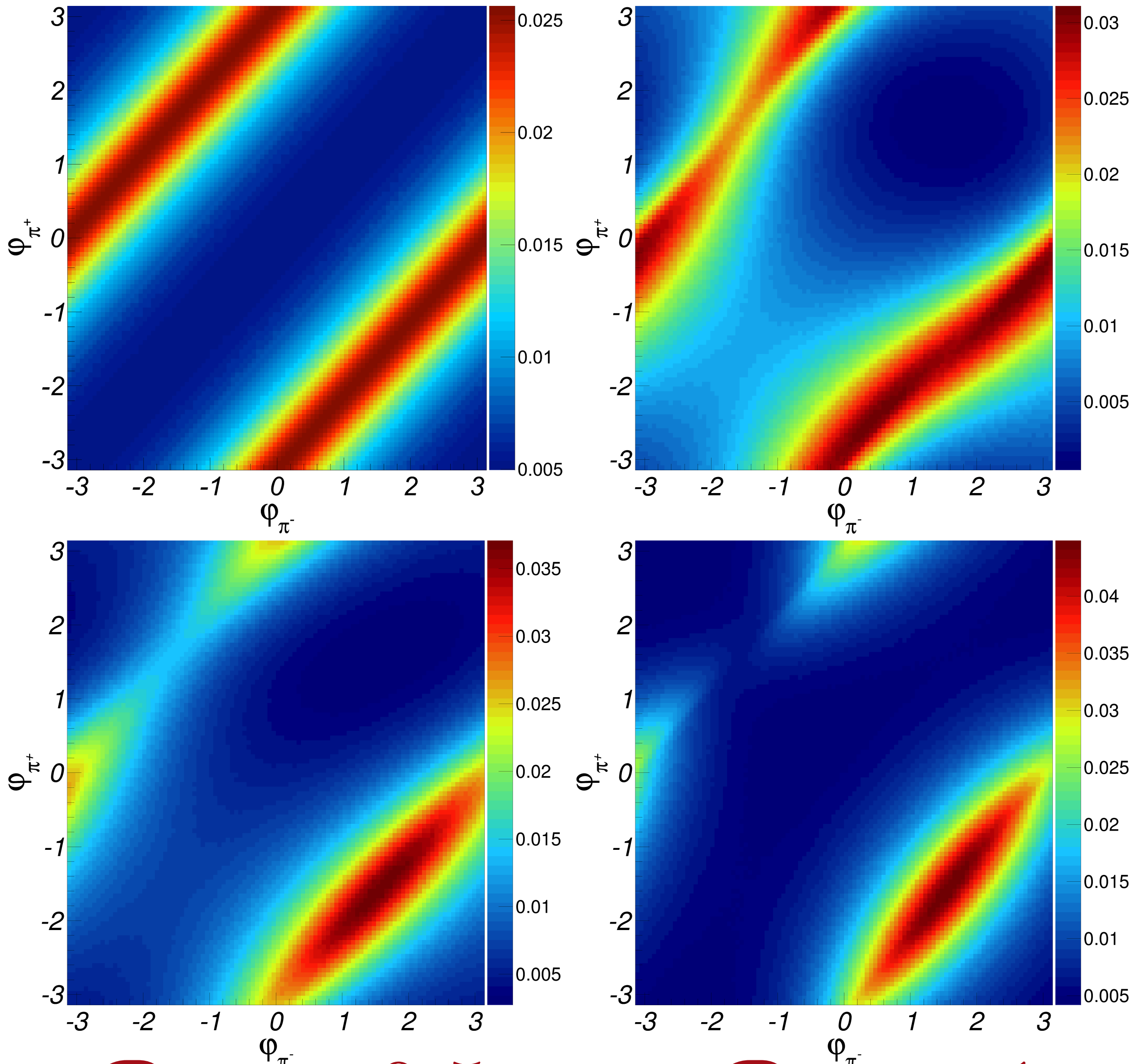
COMPASS Preliminary:

F. Bradamante - COMO 2013.



ANGULAR CORRELATIONS: $u \to \pi^+\pi^-$

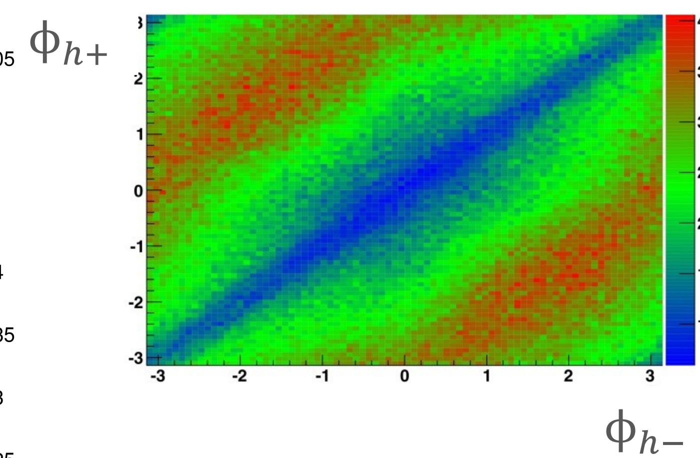
Unpolarized $\quad\quad\quad \mathcal{P}_{SF} = 0$



$\mathcal{P}_{SF} = 0.5 \quad\quad\quad \mathcal{P}_{SF} = 1$

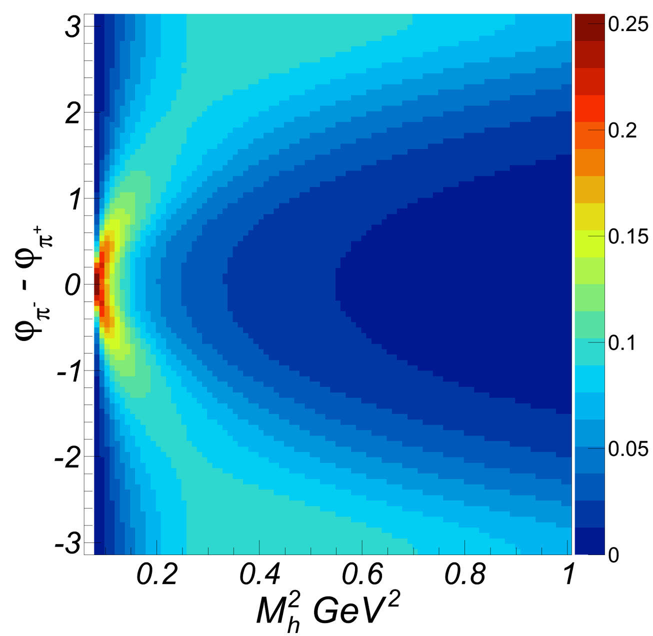
COMPASS Preliminary:

F. Bradamante - COMO 2013.



ANGULAR CORRELATIONS: $u \to \pi^+\pi^-$

$$M_h^2 = \frac{(z_1+z_2)(z_2 M_{h1}^2 + z_1 M_{h2}^2) + (z_2\mathbf{P}_{1\perp} - z_1\mathbf{P}_{2\perp})^2}{z_1 z_2}$$



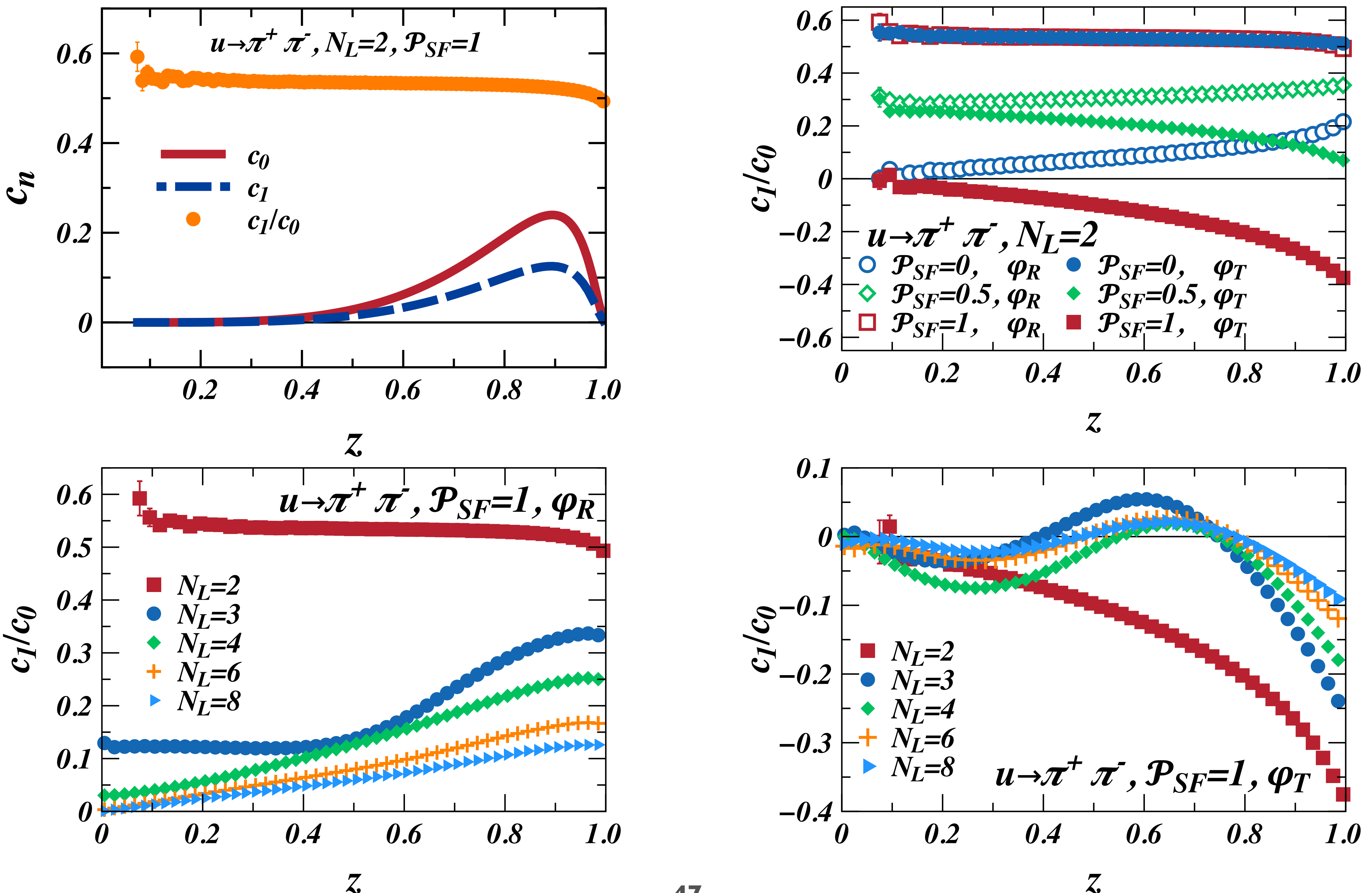
IMPROVED MODEL

- ✦**Use the *spectator model* for Collins function:**

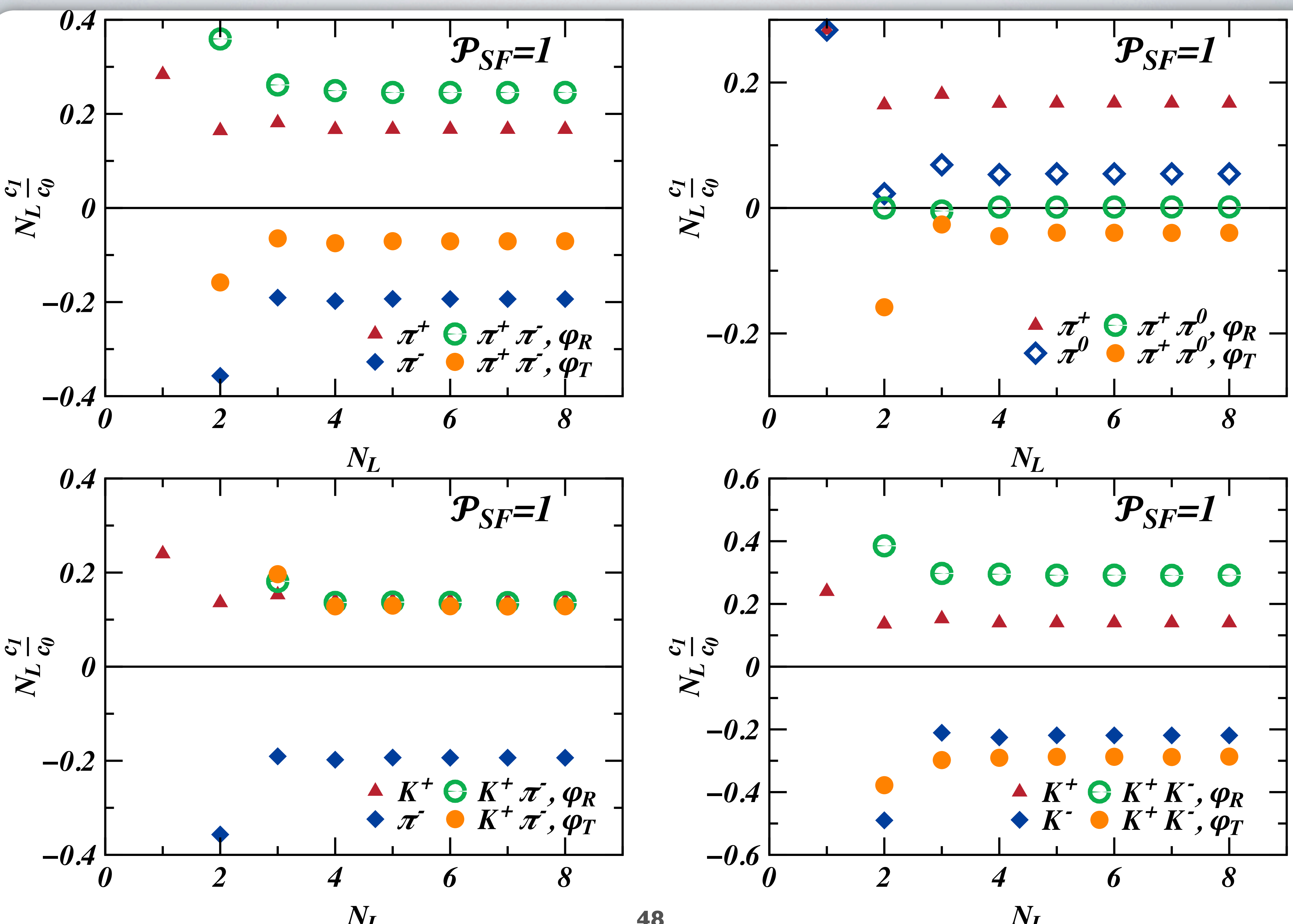
  No singularities at vanishing transverse momentum.

- ✦**Include both *pion and kaon* channels.**

ANALYZING POWERS

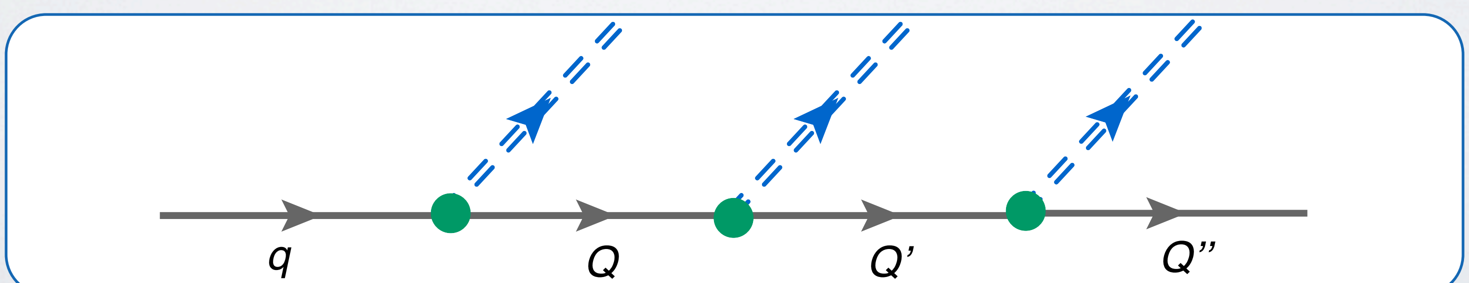


IMPROVED MODEL RESULTS



MONTE-CARLO (MC) APPROACH

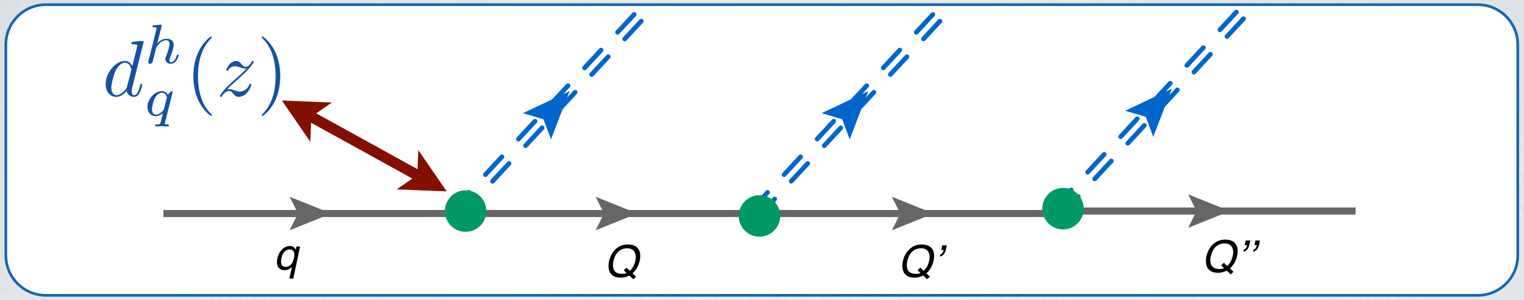
- ✦ Using the ***probabilistic*** interpretation of fragmentation funcs. to include the effect of ***multiple*** hadron emissions.



INTEGRATED FRAGMENTATIONS FROM MC

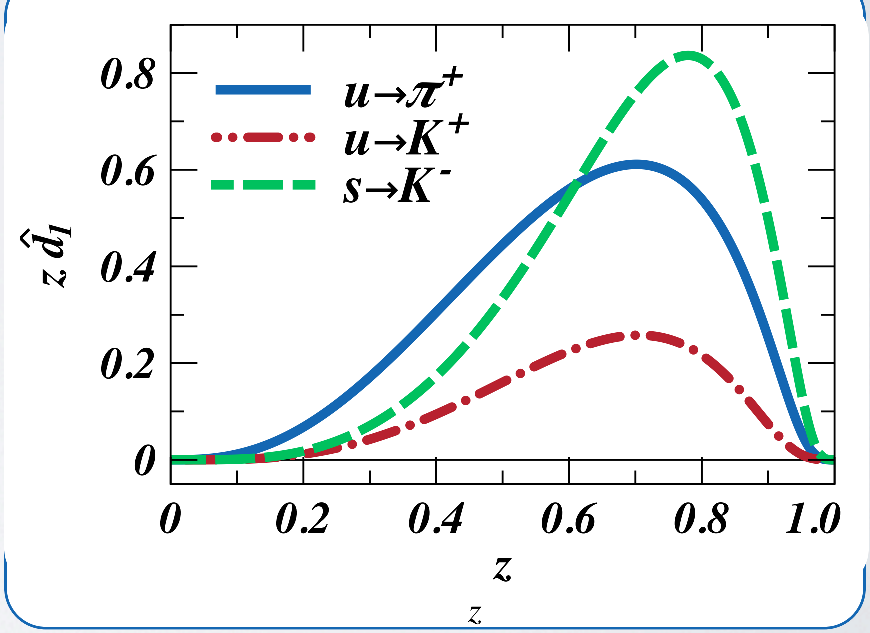
H.M., Thomas, Bentz, PRD. 83:07400; PRD.83:114010, 2011.

- Input: One hadron emission probability



- Sample the emitted hadron type and z according to input splitting.

- *CONSERVE*: Momentum and Quark Flavor in each step.



- Repeat for decay chains with the same initial quark.

$$D_q^h(z)\Delta z = \langle N_q^h(z, z+\Delta z)\rangle \equiv \frac{\sum_{N_{Sims}} N_q^h(z, z+\Delta z)}{N_{Sims}}$$

INTEGRATED FRAGMENTATIONS FROM MC

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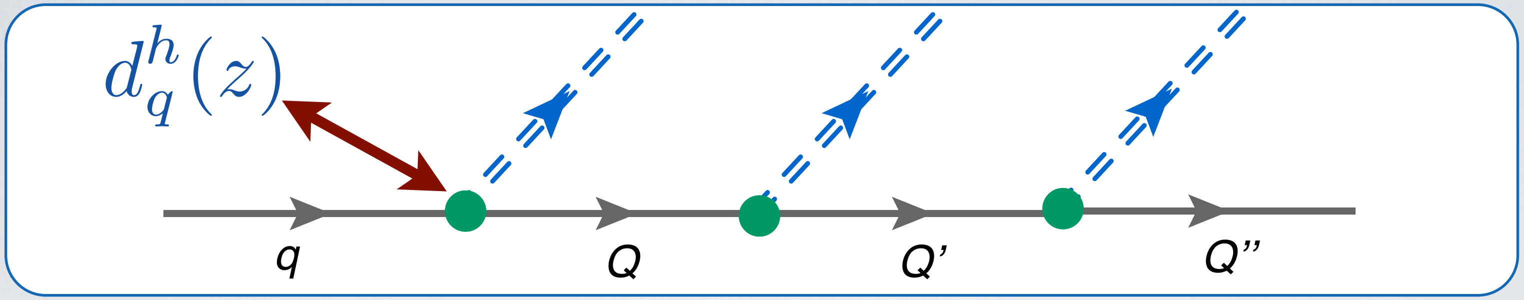

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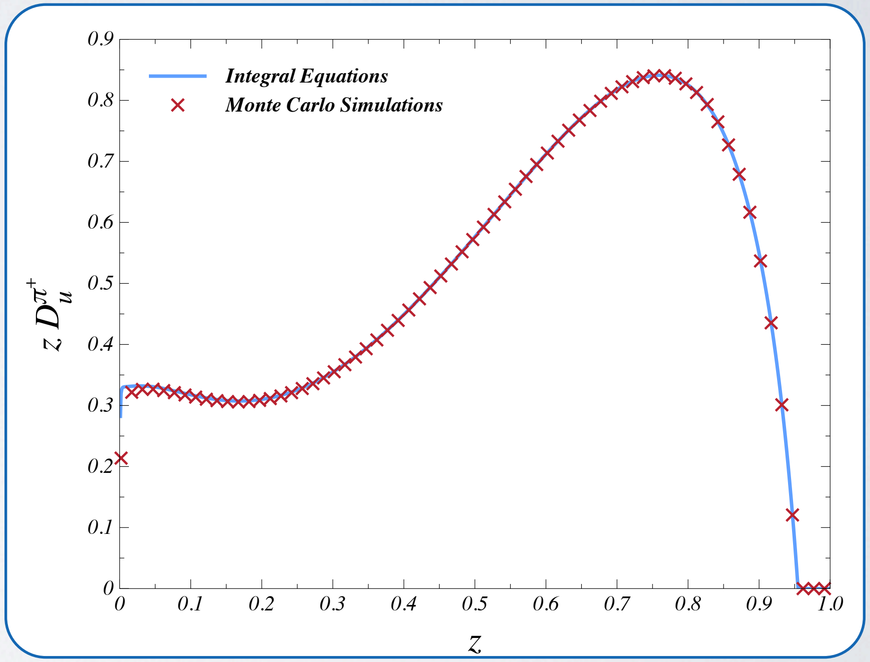
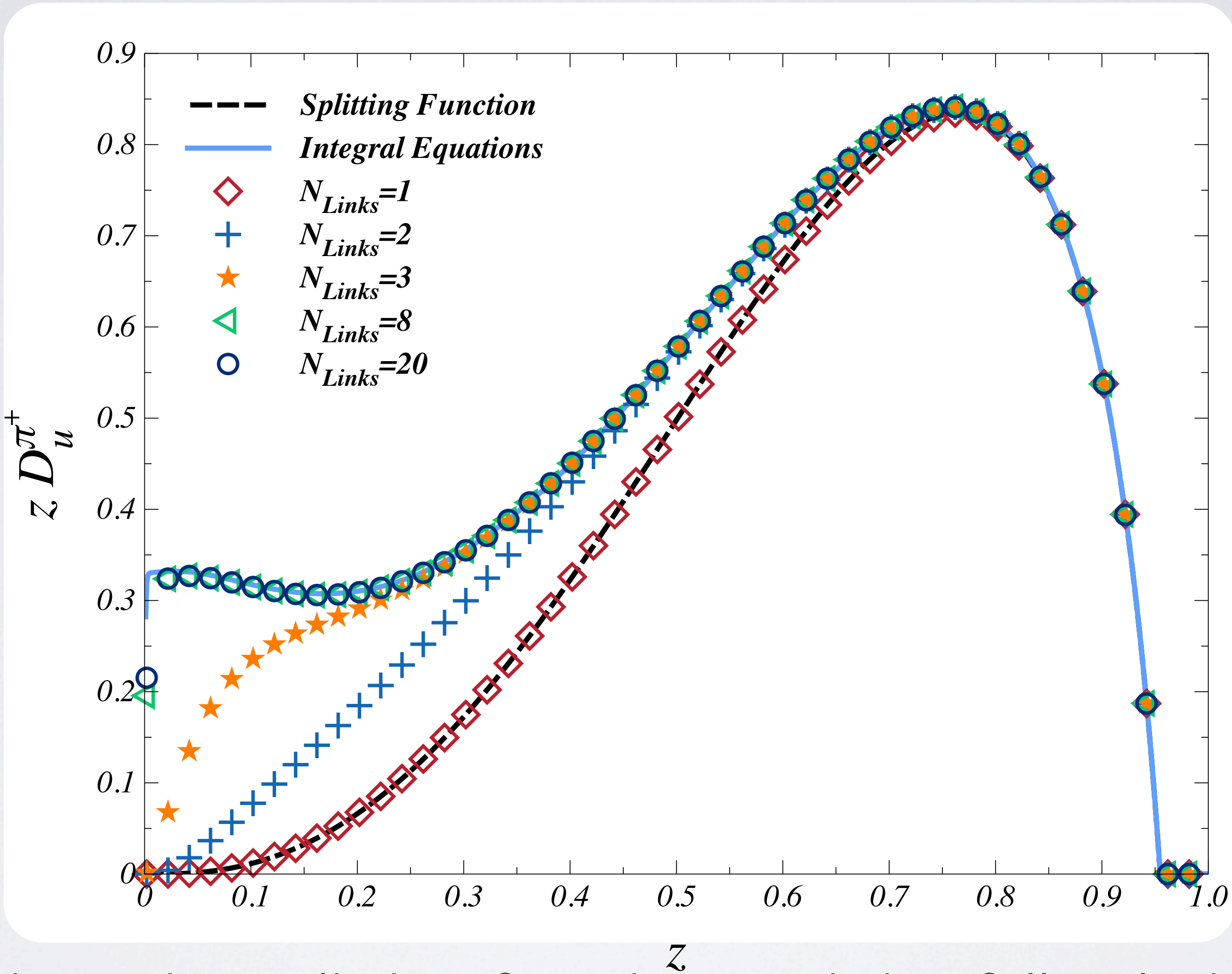


$$D_q^h(z)\Delta z = \langle N_q^h(z, z+\Delta z)\rangle \equiv \frac{\sum_{N_{Sims}} N_q^h(z, z+\Delta z)}{N_{Sims}}$$

DEPENDENCE ON CHAIN CUTOFF

- Restrict the number of emitted hadrons, N_{Links} in MC.



- We reproduce the splitting function and the full solution perfectly.

- The low z region is saturated with just a few emissions.

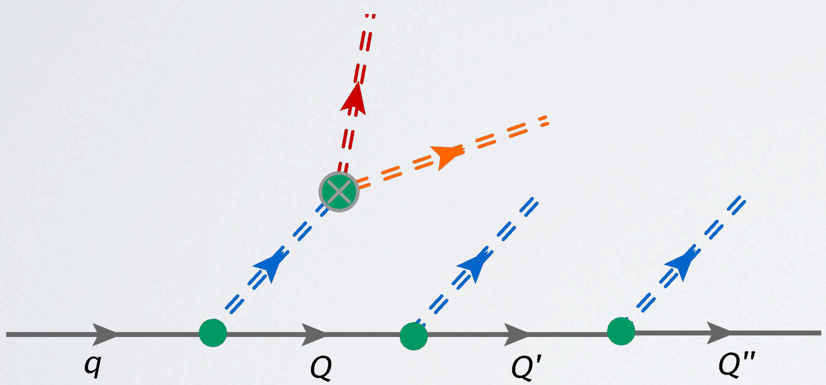
MORE CHANNELS

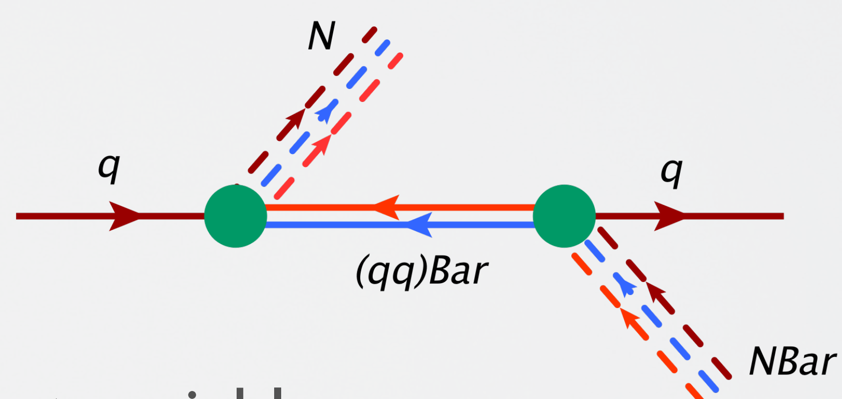
H.M., Thomas, Bentz, PRD. 83:074003, 2011

- Calculate quark splittings to vector mesons, Nucleon Anti-Nucleon: $d_q^h(z)$

$$h = \rho^0, \rho^\pm, K^{*0}, \overline{K}^{*0}, K^{*\pm}, \phi, N, \bar{N}$$

- Add the decay of the resonances:



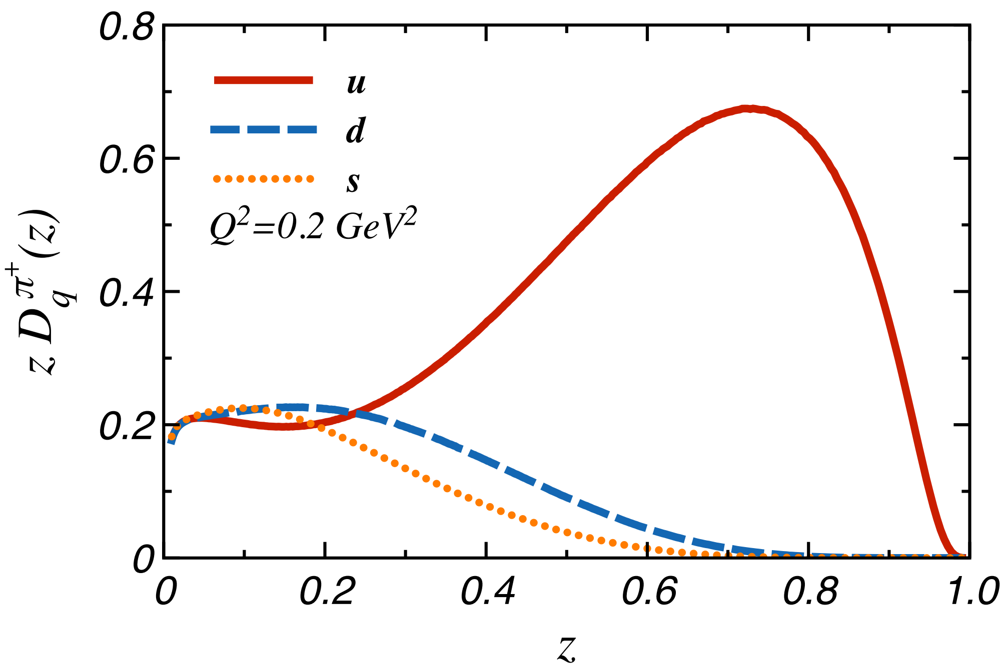


- Decay cross-section in light-front variables:

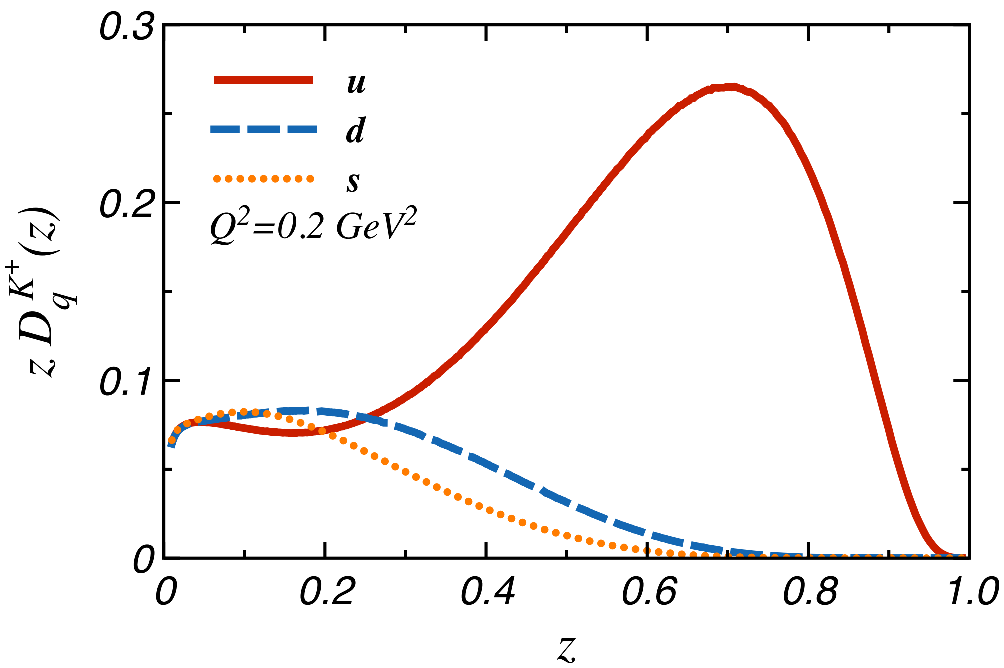
$$dP^{h\to h_1,h_2}(z_1) = \begin{cases} \frac{C_h^{h_1 h_2}}{8\pi}\, dz_1 & \text{if } z_1 z_2\, m_h^2 - z_2 m_{h1}^2 - z_1 m_{h2}^2 \geq 0;\ z_1 + z_2 = 1, \\ 0 & \text{otherwise.} \end{cases}$$

SOLUTIONS OF THE INTEGRAL EQUATIONS

H.M., Thomas, Bentz, PRD. 83:074003, 2011



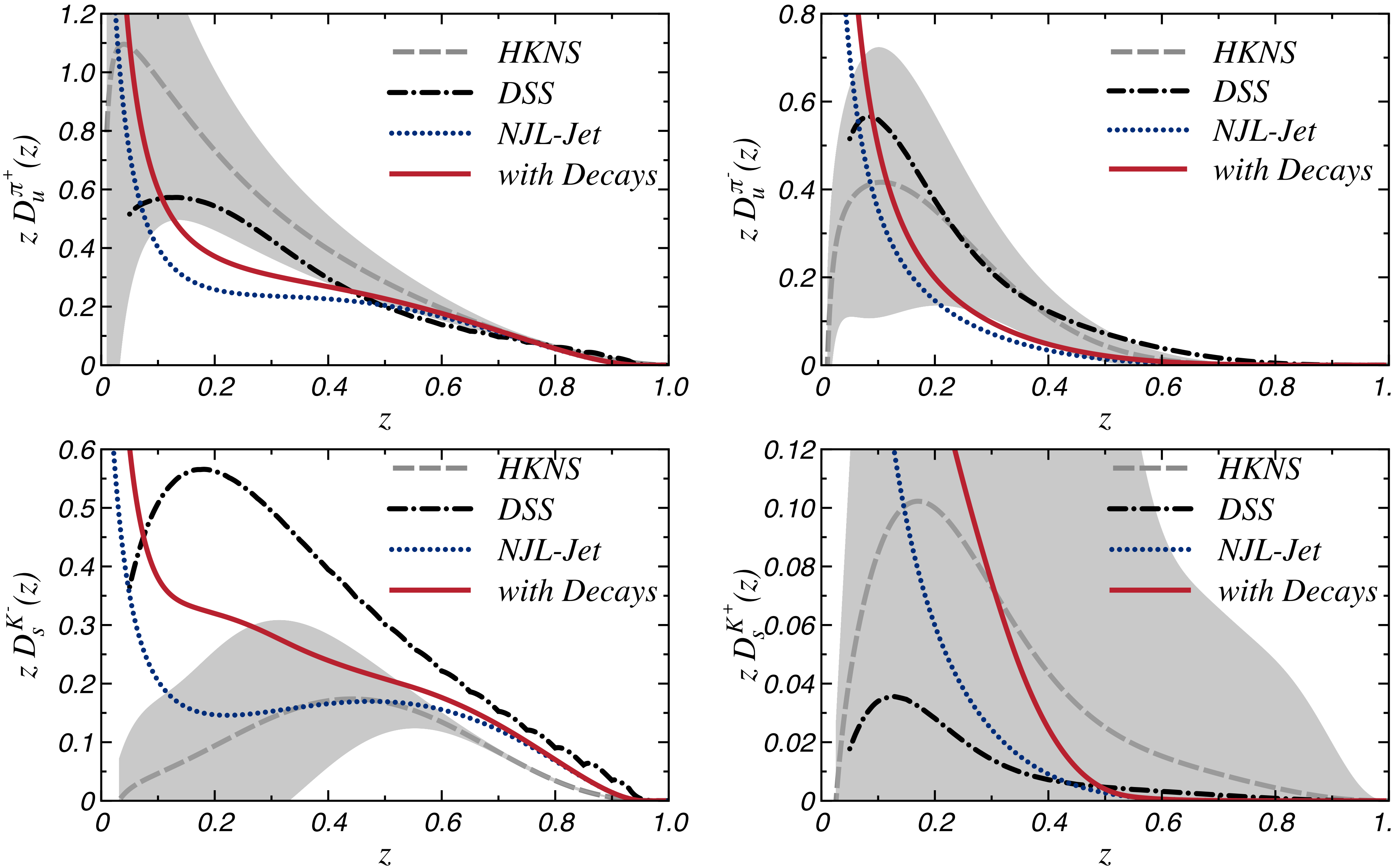
π^+



K^+

Results with VM decays: $Q^2 = 4\ \mathrm{GeV}^2$

Favored **Unfavored**

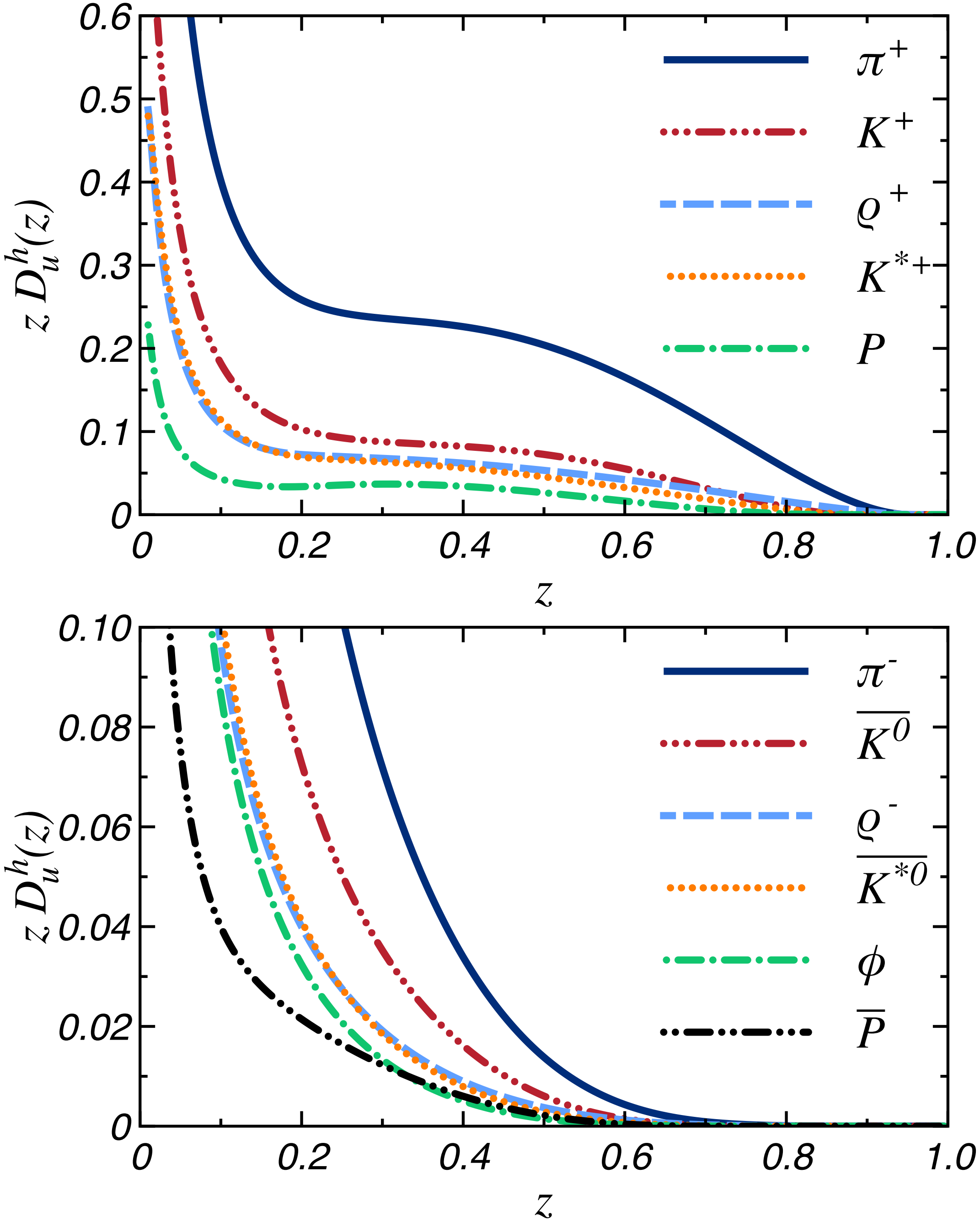


Results: Fragmentations to All Hadrons

$Q^2 = 4\ \mathrm{GeV}^2$

Favored

Unfavored



TMDS FROM SIDIS $e\,N \rightarrow e\,h\,X$

- Cross-section factorizes: $P_T^2 \ll Q^2$

Fragmentation

$$\frac{d\sigma^{lN\to l'hX}}{dx\,dQ^2\,dz\,d^2P_T} = \sum_q f_1^q(x, k_T^2, Q^2) \otimes d\sigma^{lq\to lq} \otimes D_q^h(z, p_\perp^2, Q^2)$$

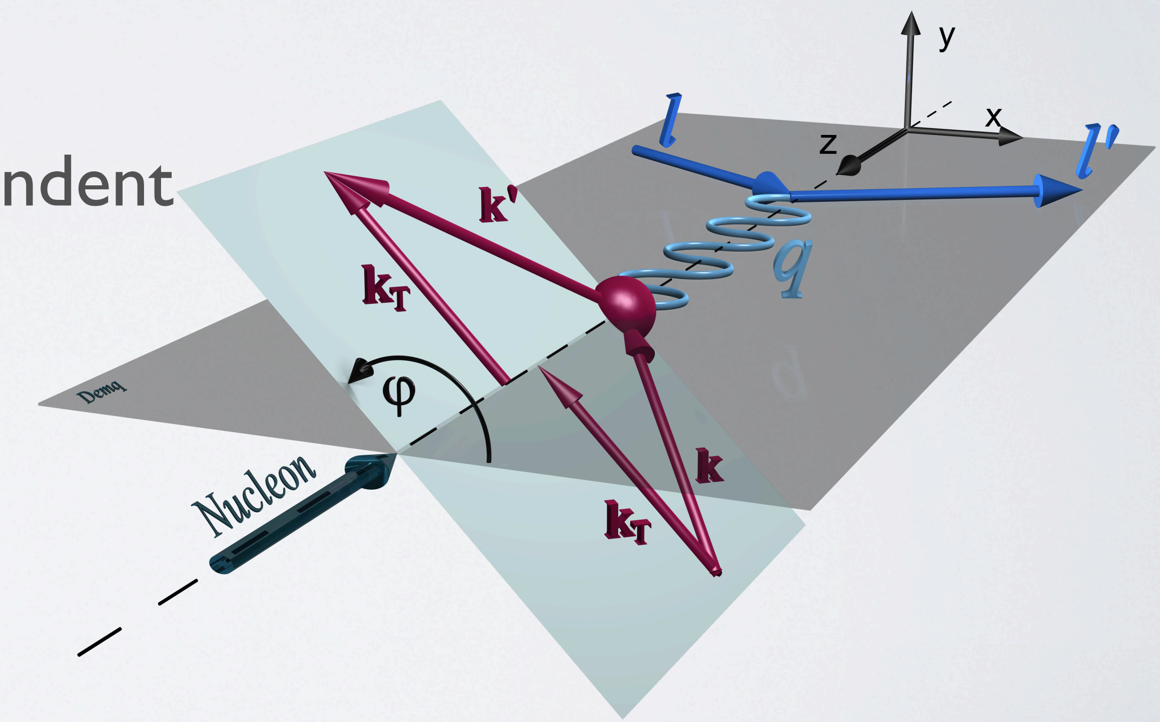
Distribution

$$\mathbf{P_T} = \mathbf{P}_\perp + z\mathbf{k_T}$$

- Transverse Momentum Dependent (TMD) PDFs and FFs:

$$\int d^2\mathbf{k}_\perp f(x, k_\perp^2) = f(x)$$

$$\int d^2\mathbf{P}_\perp D(z, P_\perp^2) = D(z)$$

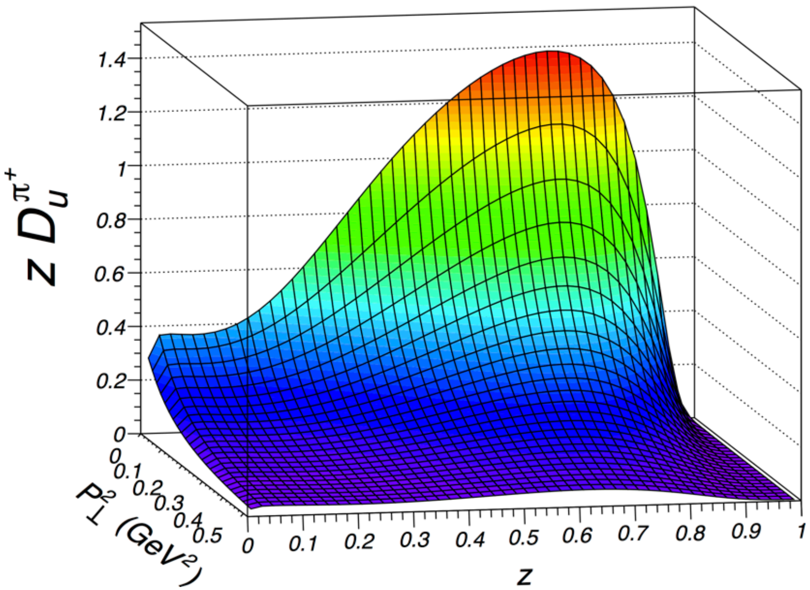


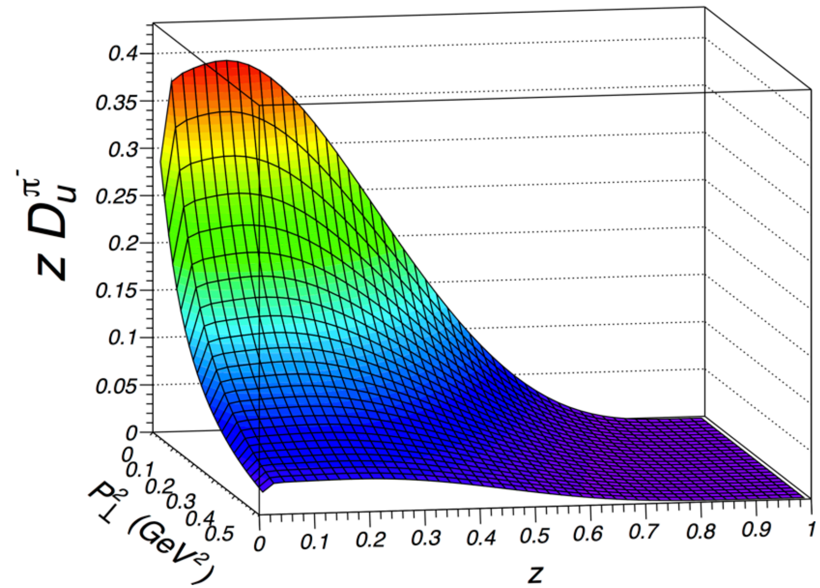
- Access to nucleon's transverse structure.
- NJL provides microscopic description of TMD PDFs and FFs!

TMD FRAGMENTATION FUNCTIONS

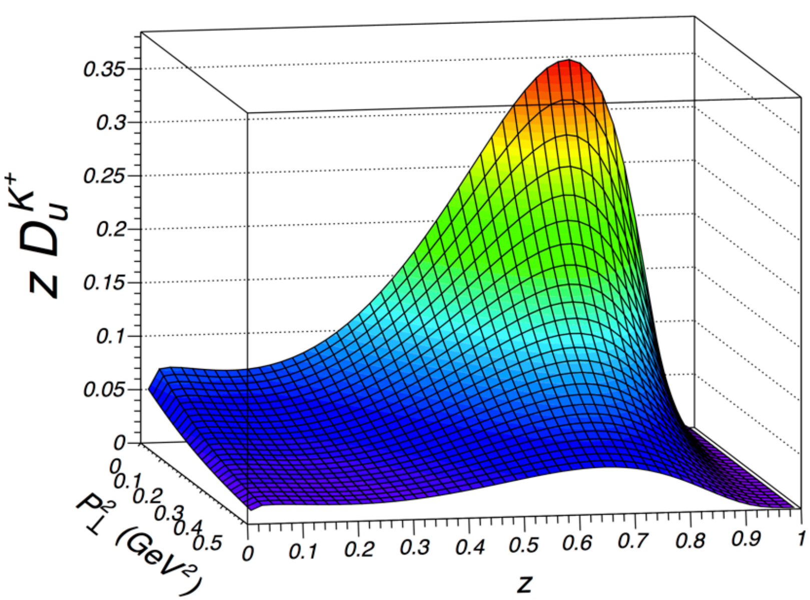
FAVORED

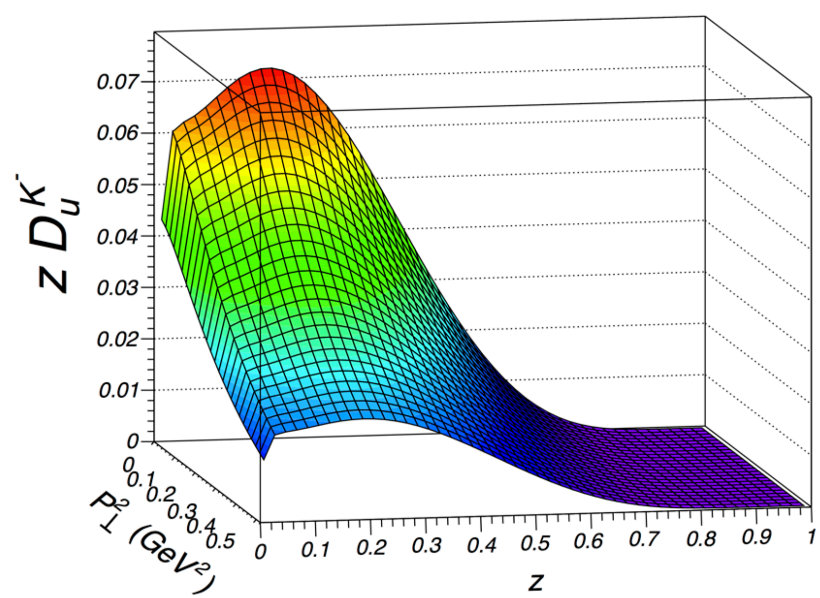
- UNFAVORED





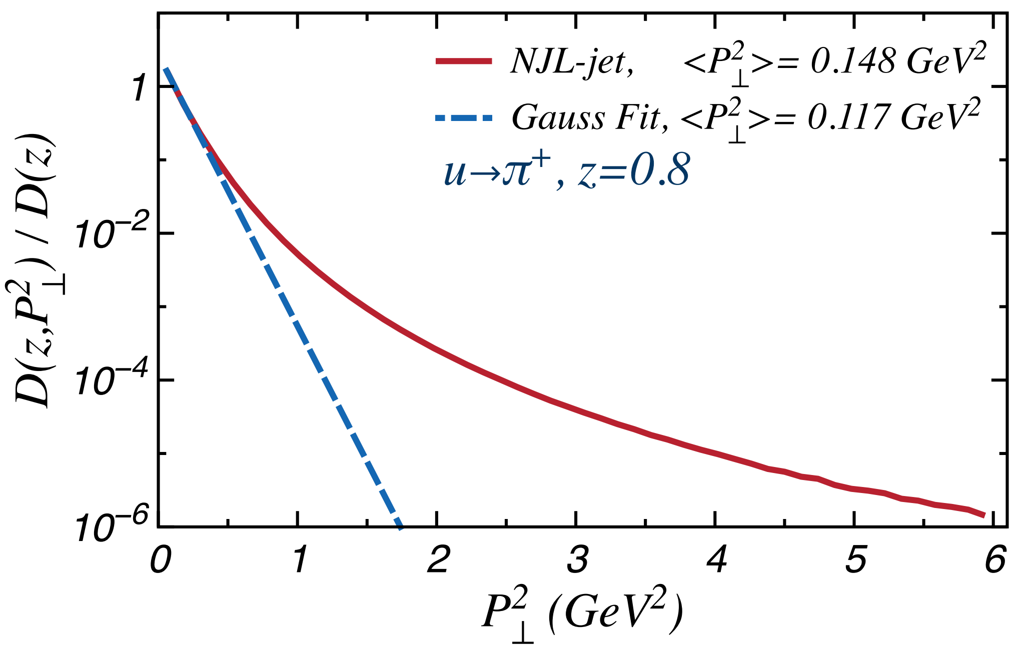
π

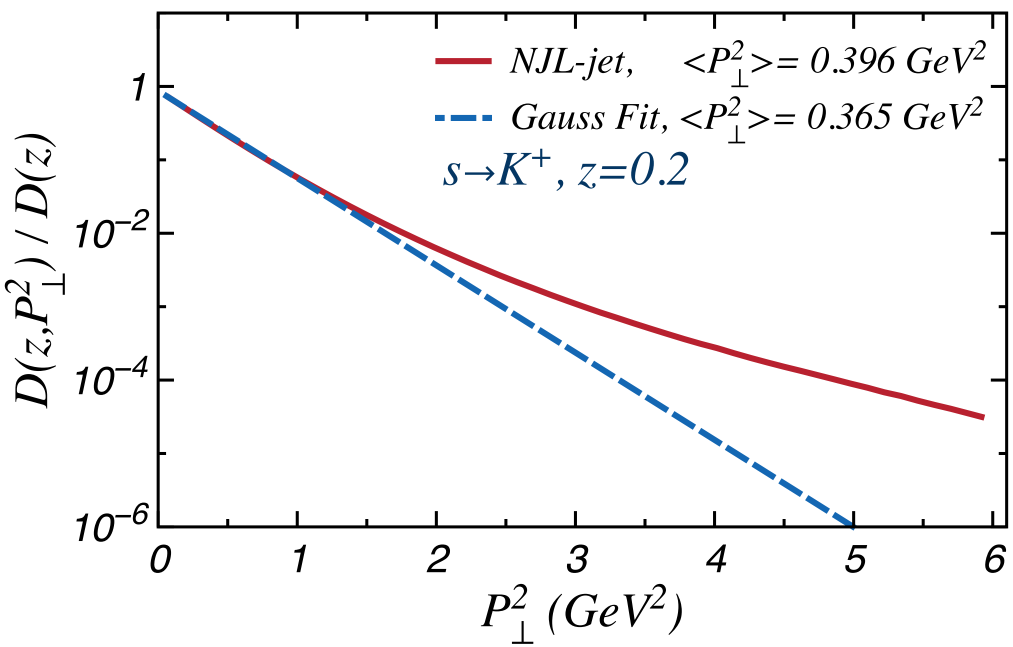




K

COMPARISON WITH GAUSSIAN ANSATZ

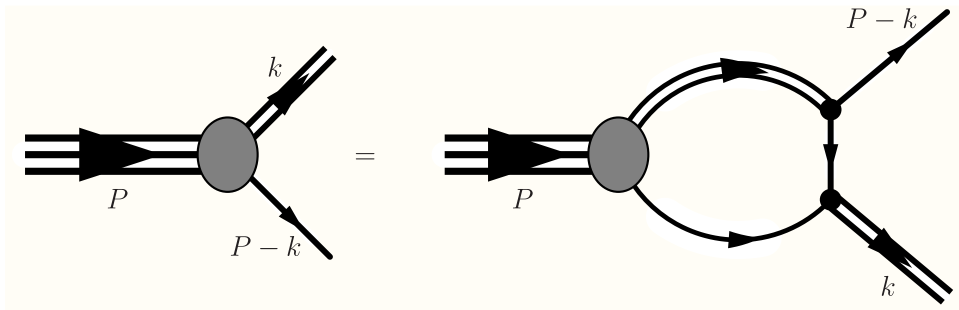




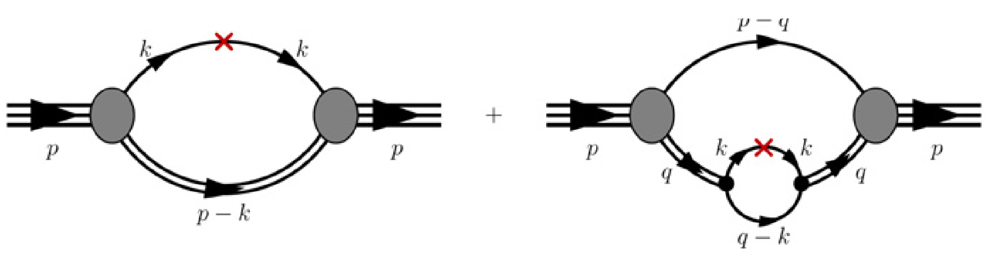
- Average TM: $\langle P_\perp^2 \rangle \equiv \dfrac{\int d^2\mathbf{P}_\perp \; P_\perp^2 D(z, P_\perp^2)}{\int d^2\mathbf{P}_\perp \; D(z, P_\perp^2)}$
- Gaussian ansatz assumes: $D(z, P_\perp^2) = D(z) \dfrac{e^{-P_\perp^2/\langle P_\perp^2 \rangle}}{\pi \langle P_\perp^2 \rangle}$

NJL: NUCLEON PDFS

- **Quark-diquark description of Nucleon using relativistic**



- **PDFs from Feynman diagrams**



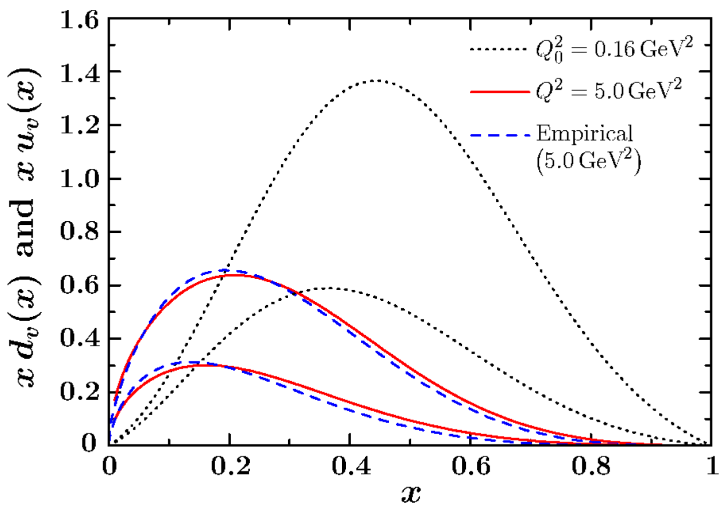
$$\mathcal{Q}(x, \boldsymbol{k_T}) = p^+ \int \frac{d\xi^- d\boldsymbol{\xi}_T}{(2\pi)^3}\, e^{ix\,p^+\,\xi^-}\, e^{-i\,\boldsymbol{k_T}\cdot\boldsymbol{\xi_T}} \langle N, S\,|\bar{\psi}_q(0)\,\gamma^+\,\mathcal{W}(\xi)\,\psi_q(\xi^-, \xi_T)|\,N, S\rangle\Big|_{\xi^+=0}$$

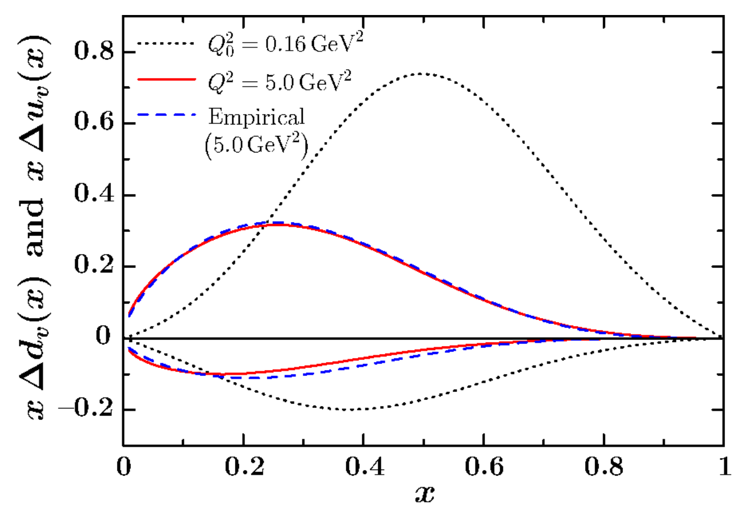
$$\mathcal{Q}(x, \boldsymbol{k_T}) = q(x, k_T^2) - \frac{\varepsilon^{-+ij}\,k_T^i\,S_T^j}{M}\,q_{1T}^{\perp}(x, k_T^2)$$

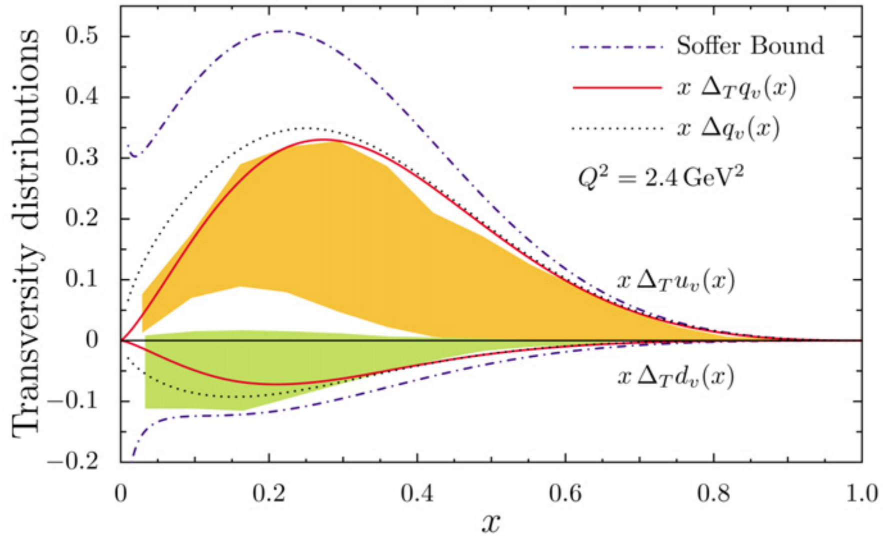
NJL: INTEGRATED NUCLEON PDFS

I. C. Cloet, W. Bentz, and A. W. Thomas, PLB 621, 246 (2005).

A good description of both unpolarized and polarized PDFs.



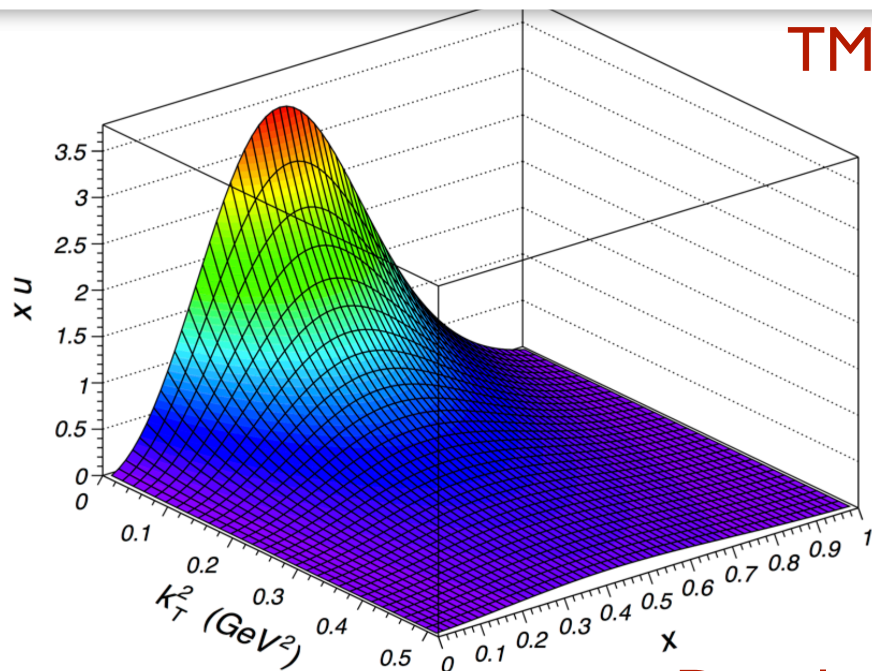


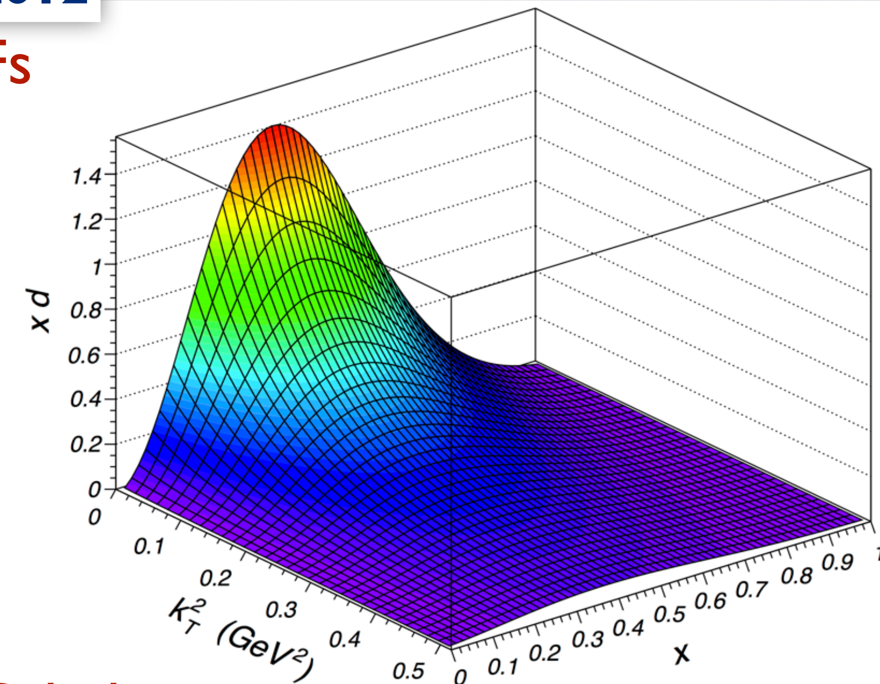


NJL: NUCLEON PDFS - TMD RESULTS

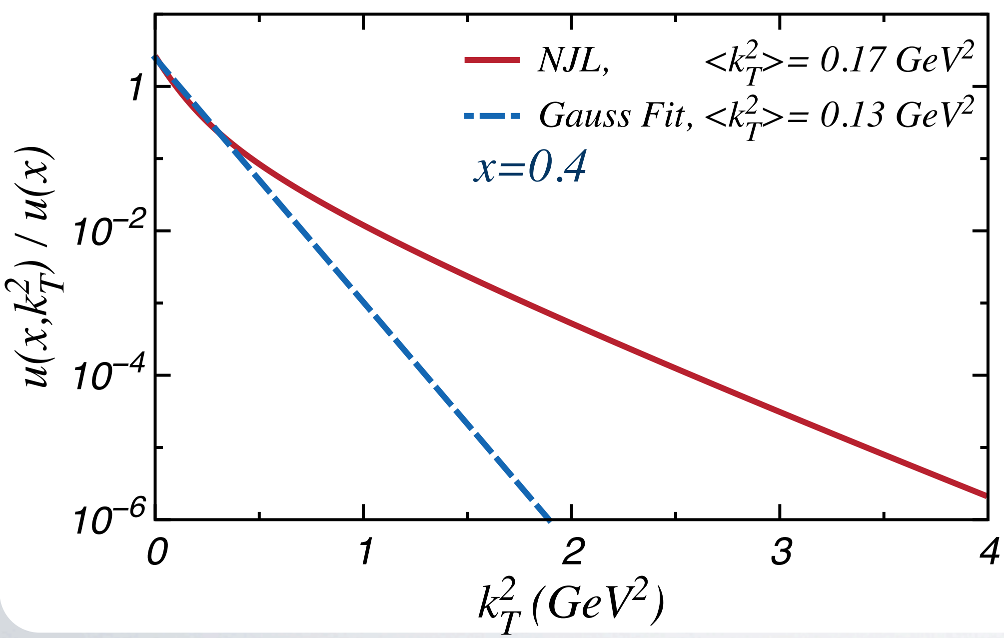
H.M., Bentz, Cloet, Thomas, PRD.85:014021, 2012

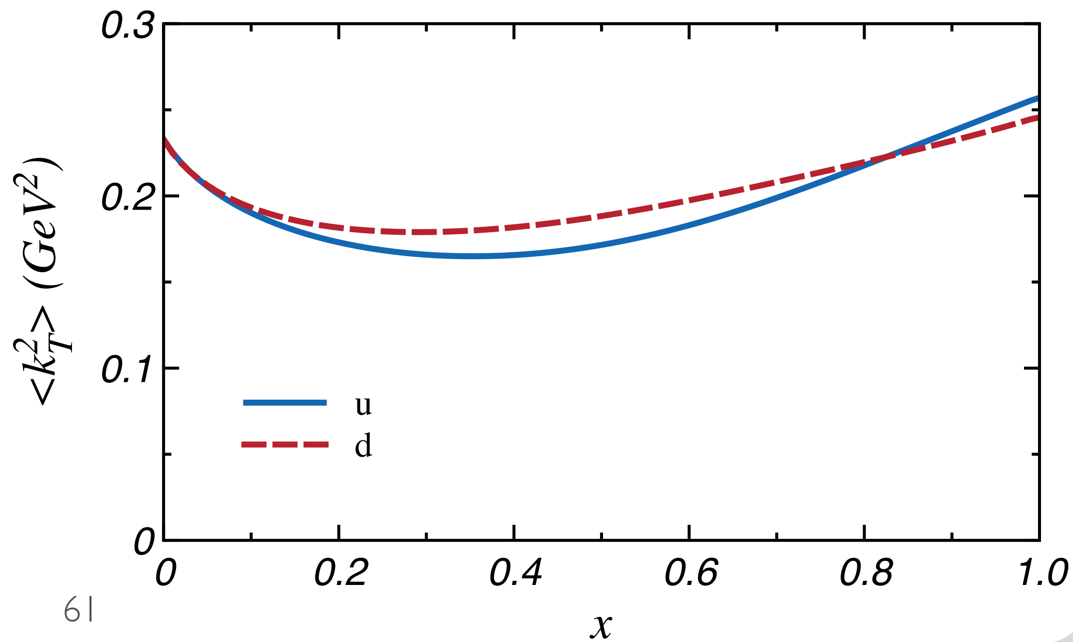
TMD PDFs





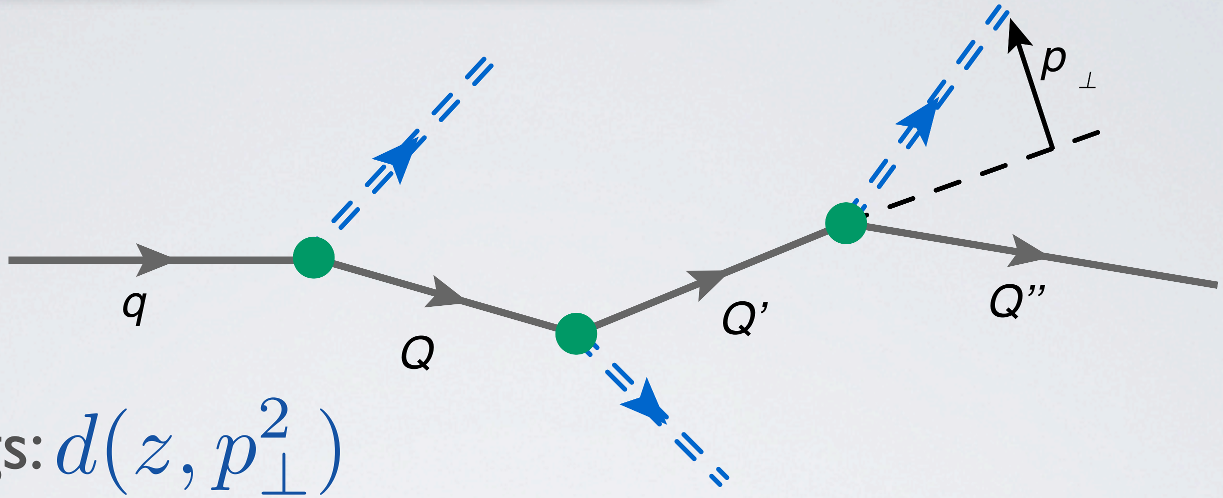
Details of TMD behavior





INCLUDING THE TRANSVERSE MOMENTUM

H.M., Bentz, Cloet, Thomas, PRD.85:014021, 2012

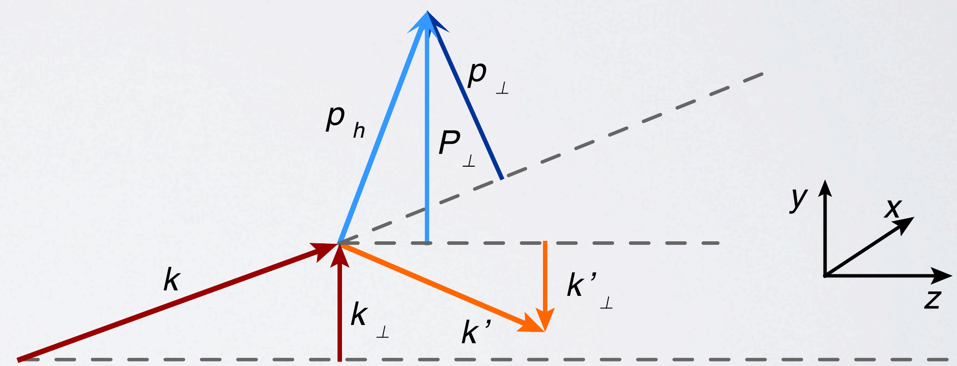


- TMD splittings: $d(z, p_\perp^2)$

- Conserve transverse momenta at each link.

$$\boxed{\mathbf{P}_\perp = \mathbf{p}_\perp + z^h \mathbf{k}_\perp}$$

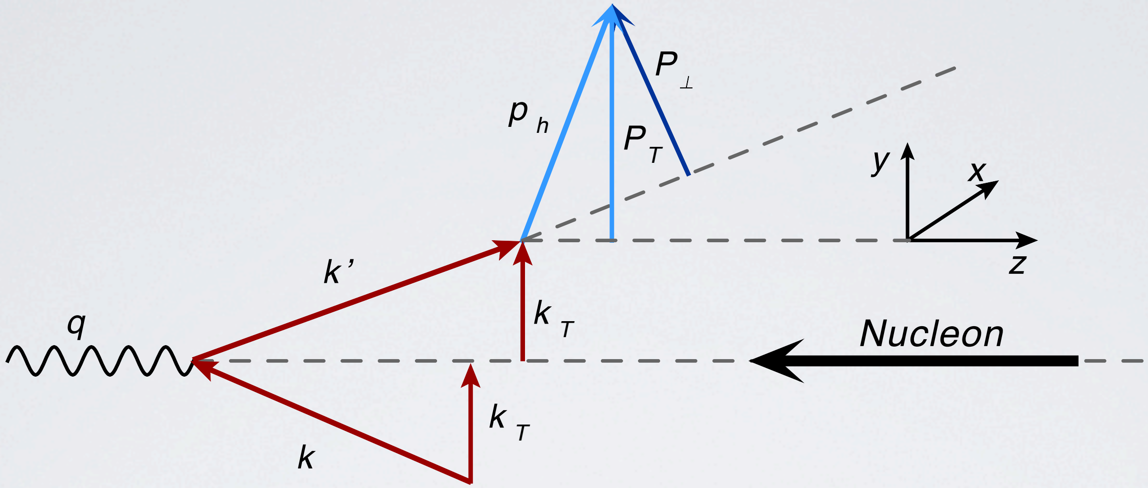
$$\boxed{\mathbf{k}_\perp = \mathbf{P}_\perp + \mathbf{k}'_\perp}$$



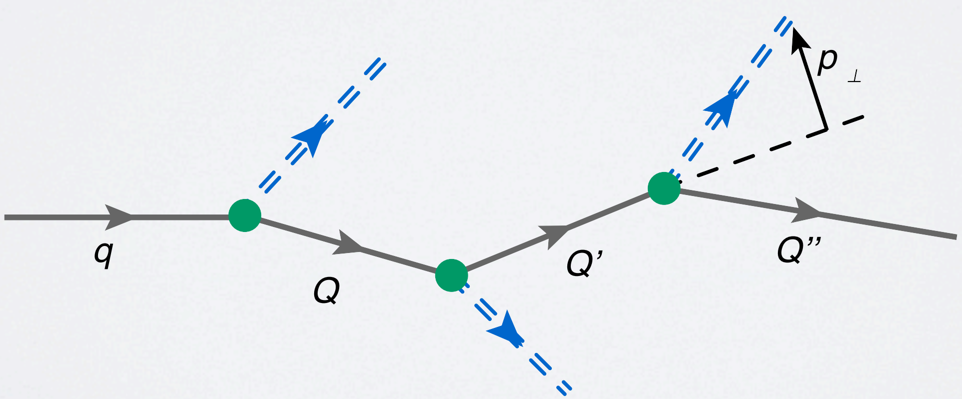
- Calculate the Number Density

$$D_q^h(z, P_\perp^2)\Delta z\ \pi \Delta P_\perp^2 = \frac{\sum_{N_{Sims}} N_q^h(z, z+\Delta z, P_\perp^2, P_\perp^2 + \Delta P_\perp^2)}{N_{Sims}}.$$

THE TRANSVERSE MOMENTA OF HADRONS IN SIDIS



- Use TMD quark distribution functions from the NJL model .
- Use NJL-Jet hadronization model.



- Evaluate the cross-section using MC simulation.

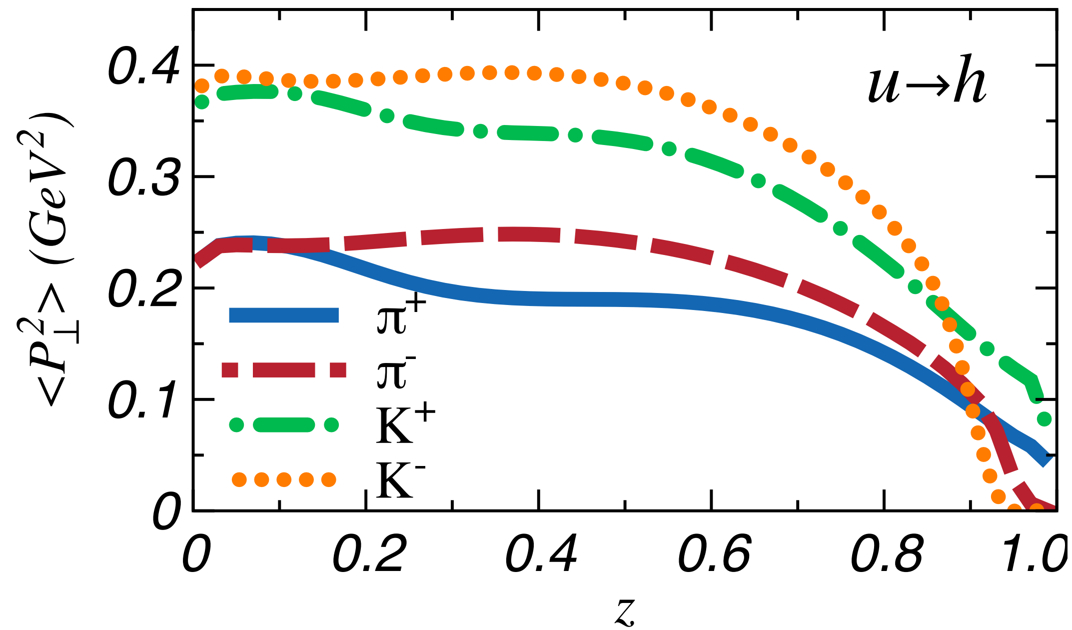
AVERAGE TRANSVERSE MOMENTA VS Z

FRAGMENTATION

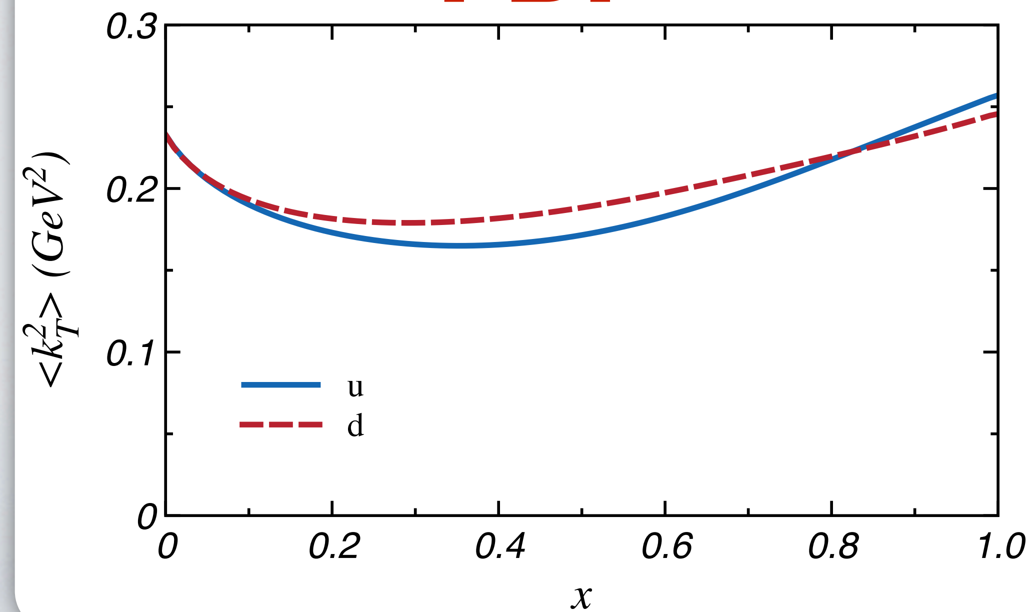
$$\langle P_\perp^2 \rangle_{unf} > \langle P_\perp^2 \rangle_f$$

✦ Indications from HERMES data:

A. Signori, et al: JHEP 1311, 194 (2013)



PDF



SIDIS