The $\Lambda(1405)$ is an anti-kaon–nucleon molecule

Jonathan Hall, Waseem Kamleh, Derek Leinweber, Ben Menadue,
Ben Owen, Tony Thomas, Ross Young

SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

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- Even though it contains a heavy strange quark and has odd parity its mass is lower than any other excited spin-$1/2$ baryon.
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- Before the existence of quarks was confirmed, Dalitz and co-workers speculated that it might be a molecular state of an anti-kaon bound to a nucleon.
- For almost 50 years the structure of the Λ(1405) resonance has been a subject of debate.

The $\Lambda(1405)$

- Here we'll see how a new lattice QCD simulation showing
  - The $\Lambda(1405)$ strange magnetic form factor vanishes, together with
  - A Hamiltonian effective field theory analysis of the lattice QCD energy levels,

  unambiguously establishes that the structure is dominated by a bound anti-kaon–nucleon component.

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- In both cases the strange quark is confined within a spin-0 kaon and has no preferred spin orientation.
- To conserve parity, the kaon has zero orbital angular momentum.
- Thus, the strange quark does not contribute to the magnetic form factor of the $\Lambda(1405)$ when it is in a $\overline{K}N$ molecule.

Outline

Techniques for exciting the $\Lambda(1405)$ in Lattice QCD

Quark-sector contributions to the electric form factor of the $\Lambda(1405)$

Quark-sector contributions to the magnetic form factor of the $\Lambda(1405)$

Hamiltonian effective field theory model: m_0, $\pi\Sigma$, $\overline{K}N$, $K\Xi$ and $\eta\Lambda$.

Conclusion

The $\Lambda(1405)$ and Lattice QCD

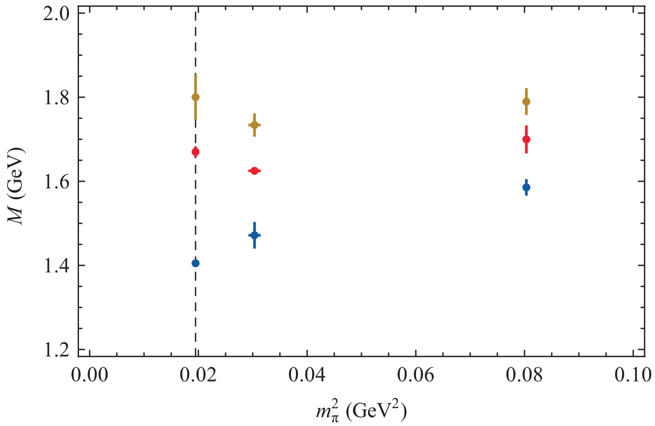
Our recent work has successfully isolated three low-lying odd-parity spin-$1/2$ states.

B. Menadue, W. Kamleh, D. B. Leinweber, M. S. Mahbub, Phys. Rev. Lett. **108**, 112001 (2012)

- An extrapolation of the trend of the lowest state reproduces the mass of the $\Lambda(1405)$.
- Subsequent studies have confirmed these results.

G. P. Engel, C. B. Lang, A. Schäfer, Phys. Rev. D **87**, 034502 (2013)

The Λ(1405) and Lattice QCD



Simulation Details

We are using the PACS-CS $(2+1)$-flavour ensembles, available through the ILDG.

S. Aoki *et al* (PACS-CS Collaboration), Phys. Rev. D **79**, 034503 (2009)

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    - Changes in the strange quark contributions are environmental effects.

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- The strange quark κ_s is held fixed as the light quark masses vary.
  - Changes in the strange quark contributions are environmental effects.
- We consider both the Sommer and PACS-CS schemes to set the scale.

The $\Lambda(1405)$ and Lattice QCD

The variational analysis is necessary to isolate the $\Lambda(1405)$.

Variational Analysis

By using multiple operators, we can isolate and analyse individual energy eigenstates:

- Construct the correlation matrix

$$G_{ij}(\mathbf{p}; t) = \sum_{\mathbf{x}} \mathrm{e}^{-\mathrm{i}\,\mathbf{p}\cdot\mathbf{x}}\, \mathrm{tr}\left(\Gamma \, \langle\Omega|\, \chi_i(x)\, \overline{\chi}_j(0)\, |\Omega\rangle \right),$$

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  for some set $\{\, \chi_i \,\}$ operators that couple to the states of interest.
- We seek the linear combinations of the operators $\{\, \chi_i \,\}$ that perfectly isolate individual energy eigenstates, α, at momentum $\mathbf{p}$:

$$\phi^\alpha = v_i^\alpha(\mathbf{p})\, \chi_i\,, \qquad \overline{\phi}^\alpha = u_i^\alpha(\mathbf{p})\, \overline{\chi}_i\,.$$

Variational Analysis

- When successful, only state α participates in the correlation function, and one can write recurrence relations

$$G(\mathbf{p}; t + \delta t)\, \mathbf{u}^{\alpha}(\mathbf{p}) = \mathrm{e}^{-E_\alpha(\mathbf{p})\, \delta t}\, G(\mathbf{p}; t)\, \mathbf{u}^{\alpha}(\mathbf{p})$$

$$\mathbf{v}^{\alpha \mathsf{T}}(\mathbf{p})\, G(\mathbf{p}; t + \delta t) = \mathrm{e}^{-E_\alpha(\mathbf{p})\, \delta t}\, \mathbf{v}^{\alpha \mathsf{T}}(\mathbf{p})\, G(\mathbf{p}; t)$$

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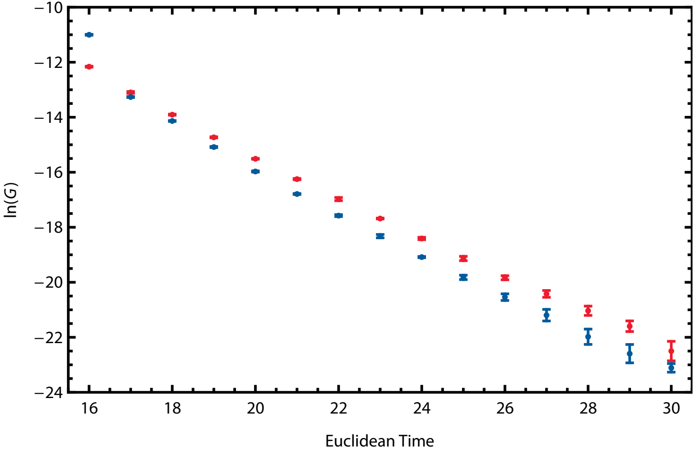
- Solve for the left, $\mathbf{v}^{\alpha}(\mathbf{p})$, and right, $\mathbf{u}^{\alpha}(\mathbf{p})$, generalised eigenvectors of $G(\mathbf{p}; t + \delta t)$ and $G(\mathbf{p}; t)$:

Eigenstate-Projected Correlation Functions

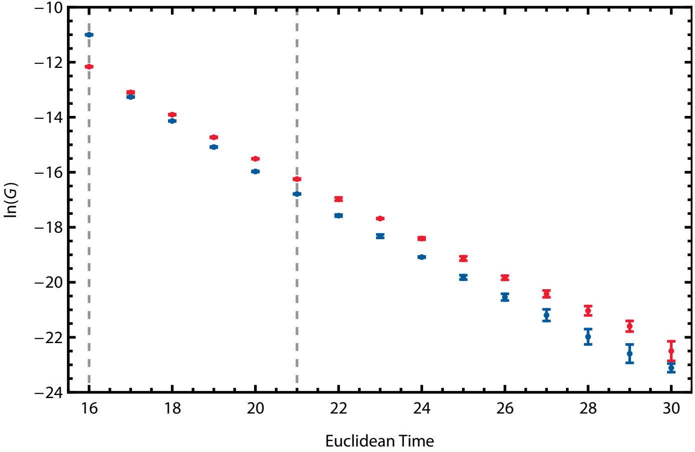
- Using these optimal operators, eigenstate-projected correlation functions are obtained

$$
\begin{aligned}
G^{\alpha}(\mathbf{p};t) &= \sum_{\mathbf{x}} \mathrm{e}^{-\mathrm{i}\,\mathbf{p}\cdot\mathbf{x}} \, \langle\Omega|\phi^{\alpha}(x)\,\overline{\phi}^{\alpha}(0)|\Omega\rangle \\
&= \sum_{\mathbf{x}} \mathrm{e}^{-\mathrm{i}\,\mathbf{p}\cdot\mathbf{x}} \, \langle\Omega|v_i^{\alpha}(\mathbf{p})\,\chi_i(x)\,\overline{\chi}_j(0)\,u_j^{\alpha}(\mathbf{p})|\Omega\rangle \\
&= \mathbf{v}^{\alpha\mathsf{T}}(\mathbf{p})\,G(\mathbf{p};t)\,\mathbf{u}^{\alpha}(\mathbf{p})
\end{aligned}
$$

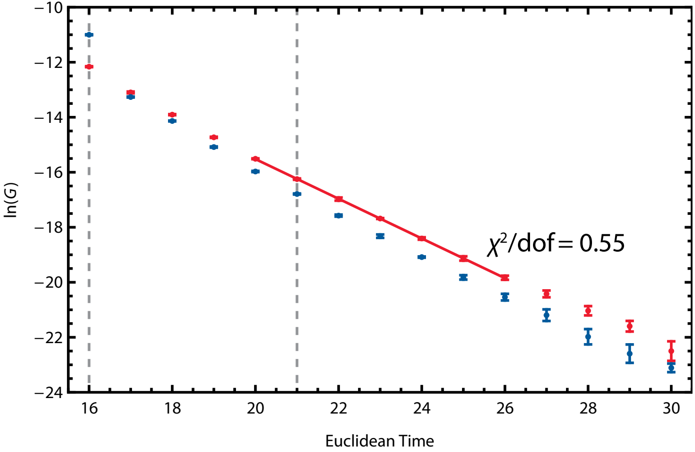
The importance of eigenstate isolation (red)



Probing with the electromagnetic current



Only the projected correlator has acceptable χ^2/dof



Operators Used in $\Lambda(1405)$ Analysis

We consider local three-quark operators with the correct quantum numbers for the Λ channel, including

- Flavour-octet operators

$$\chi_1^8 = \frac{1}{\sqrt{6}}\varepsilon^{abc}\left(2(u^a C\gamma_5 d^b)s^c + (u^a C\gamma_5 s^b)d^c - (d^a C\gamma_5 s^b)u^c\right)$$
$$\chi_2^8 = \frac{1}{\sqrt{6}}\varepsilon^{abc}\left(2(u^a C d^b)\gamma_5 s^c + (u^a C s^b)\gamma_5 d^c - (d^a C s^b)\gamma_5 u^c\right)$$

- Flavour-singlet operator

$$\chi^1 = 2\varepsilon^{abc}\left((u^a C\gamma_5 d^b)s^c - (u^a C\gamma_5 s^b)d^c + (d^a C\gamma_5 s^b)u^c\right)$$

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- These results use 16 and 100 sweeps.
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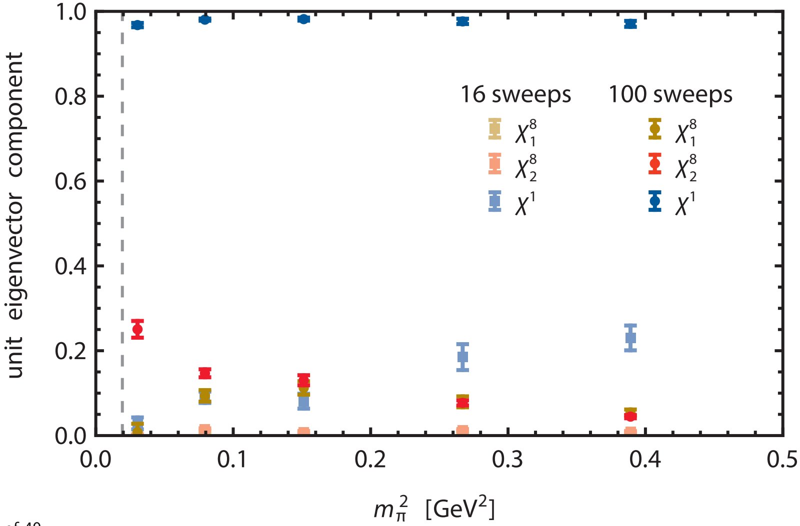
- These results use 16 and 100 sweeps.
  - Gives a 6×6 matrix.
- Also considered 35 and 100 sweeps.
  - Results are consistent with larger statistical uncertainties.

Flavour structure of the $\Lambda(1405)$



Extracting Form Factors from Lattice QCD

- To extract the form factors for a state α, we need to calculate the three-point correlation function

$$G^{\mu}_{\alpha}(\mathbf{p}', \mathbf{p}; t_2, t_1) = \sum_{\mathbf{x}_1, \mathbf{x}_2} \mathrm{e}^{-\mathrm{i}\,\mathbf{p}'\cdot\mathbf{x}_2} \mathrm{e}^{\mathrm{i}(\mathbf{p}'-\mathbf{p})\cdot\mathbf{x}_1} \langle\Omega|\phi^{\alpha}(x_2)\, j^{\mu}(x_1)\, \overline{\phi}^{\alpha}(0)|\Omega\rangle$$

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- This takes the form

$$\mathrm{e}^{-E_{\alpha}(\mathbf{p}')(t_2-t_1)}\mathrm{e}^{-E_{\alpha}(\mathbf{p})t_1} \sum_{s,\, s'} \langle\Omega|\phi^{\alpha}|p', s'\rangle \langle p', s'|j^{\mu}|p, s\rangle \langle p, s|\overline{\phi}^{\alpha}|\Omega\rangle$$

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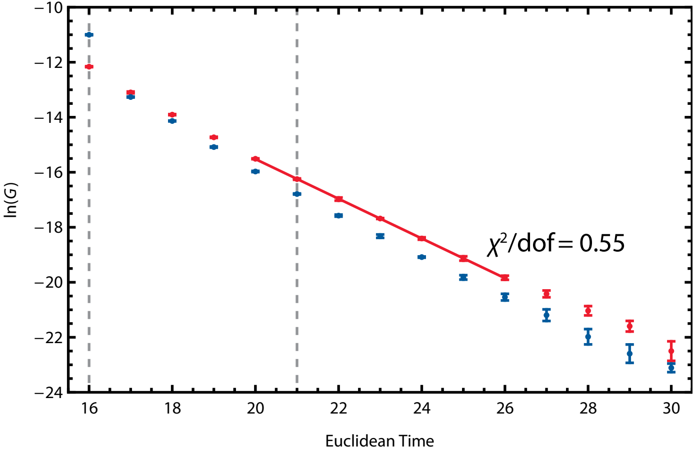

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- $\langle p', s' | j^{\mu} | p, s \rangle$ encodes the form factors of the interaction.

Current Matrix Elements for Spin-1/2 Baryons

The current matrix element for spin-1/2 baryons has the form

$$\langle p', s' | j^\mu | p, s \rangle = \left(\frac{m_\alpha^2}{E_\alpha(\mathbf{p}) E_\alpha(\mathbf{p}')} \right)^{1/2} \times$$

$$\times \, \overline{u}(\mathbf{p}') \, \left(F_1(q^2) \, \gamma^\mu + \mathrm{i} \, F_2(q^2) \, \sigma^{\mu\nu} \frac{q^\nu}{2m_\alpha} \right) u(\mathbf{p})$$

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- The Dirac and Pauli form factors are related to the Sachs form factors through

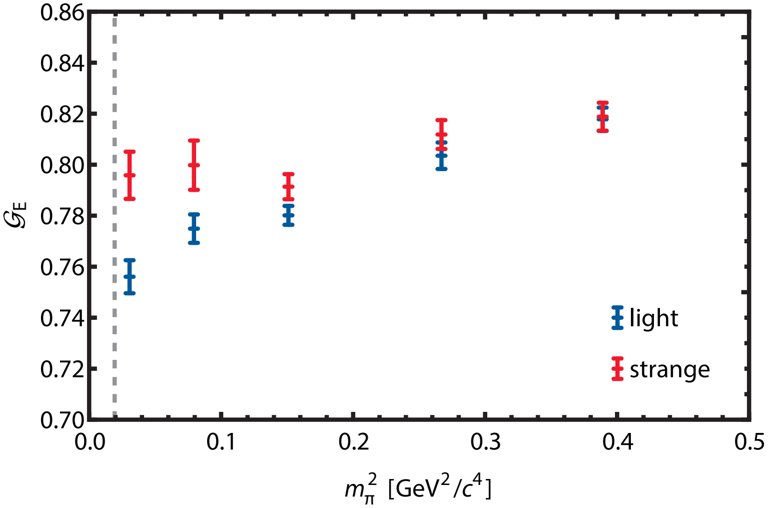
$$\mathcal{G}_\mathrm{E}(q^2) = F_1(q^2) - \frac{q^2}{(2m^\alpha)^2} F_2(q^2)$$
$$\mathcal{G}_\mathrm{M}(q^2) = F_1(q^2) + F_2(q^2)$$

Current Matrix Elements for Spin-1/2 Baryons

The light- and strange-quark sector contributions can be isolated.

- Eg. The strange sector is isolated by setting $q_u = q_d = 0$.
- q_s is set to unity such that we report results for single quarks of unit charge.
- Symmetry in the u-d sector provides $\mathcal{G}^u(Q^2) = \mathcal{G}^d(Q^2) \equiv \mathcal{G}^\ell(Q^2)$ for $q_u = q_d = 1$.

$\mathcal{G}_E$ for the $\Lambda(1405)$ at $Q^2 \sim 0.15\,\mathrm{GeV}^2$



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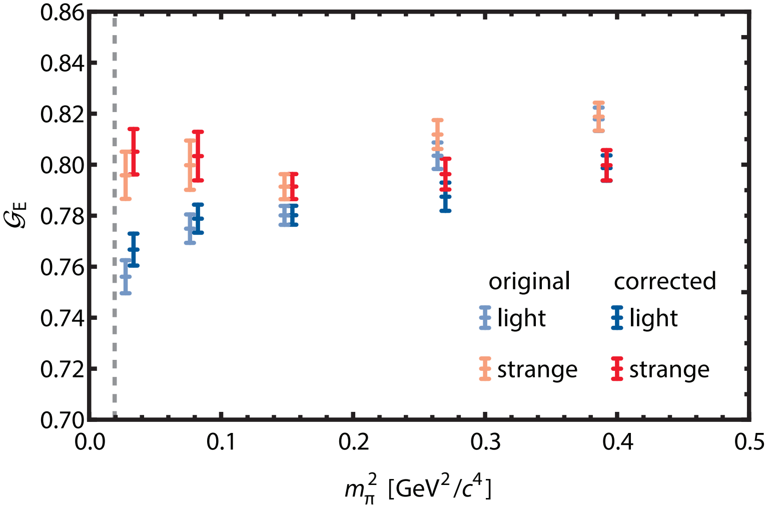
- Evaluate $\mathcal{G}_{\mathrm{E}}$ at a common Q^2 (we select $0.16\,\mathrm{GeV}^2$)

$\mathcal{G}_E$ for the $\Lambda(1405)$ at $Q^2 = 0.16\,\mathrm{GeV}^2$



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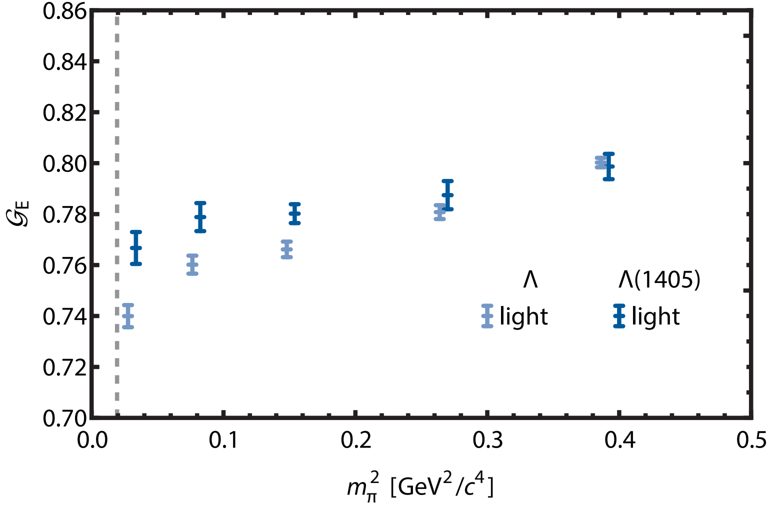

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    - The strange quark may be distributed further out by the $\overline{\mathrm{K}}$ and thus have a smaller form factor.

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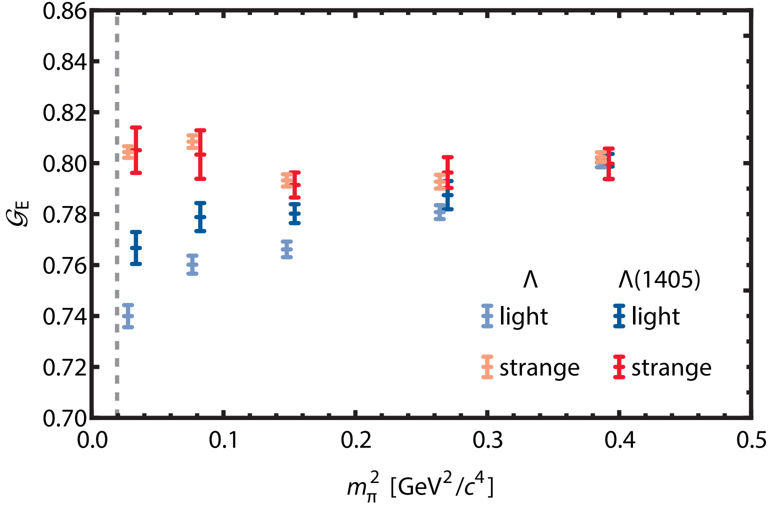
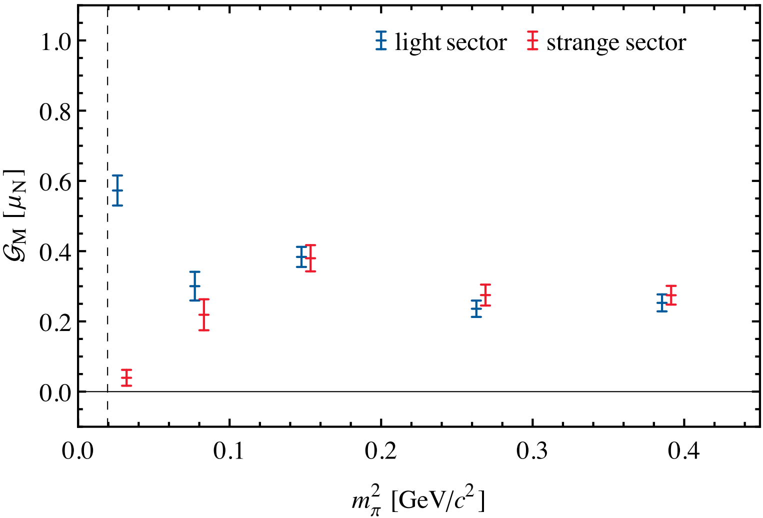


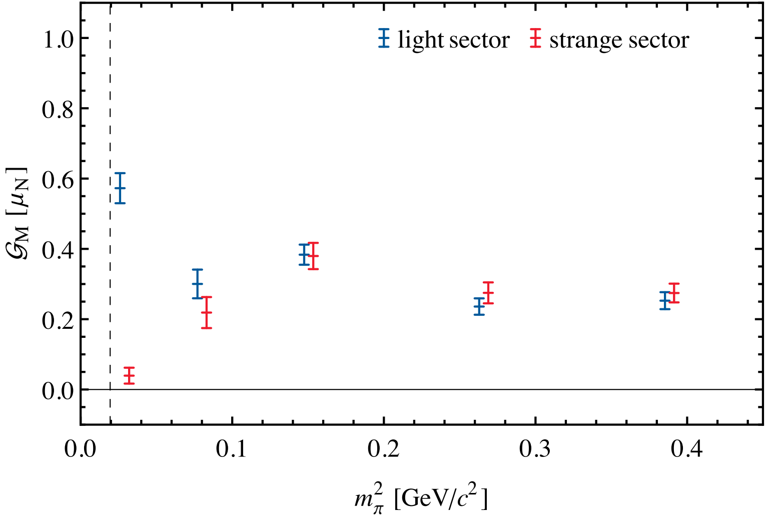
$\mathcal{G}_M$ for the $\Lambda(1405)$ at $Q^2 \sim 0.15\,\mathrm{GeV}^2$



$\Lambda(1405)$ magnetic form factor observations

- $SU(3)$-flavour symmetry is manifest for $m_\ell \sim m_s$. All three quark flavours play a similar role.
- $\mathcal{G}_M^\ell \equiv \mathcal{G}_M^u \equiv \mathcal{G}_M^d \simeq \mathcal{G}_M^s$ for the heaviest three masses.

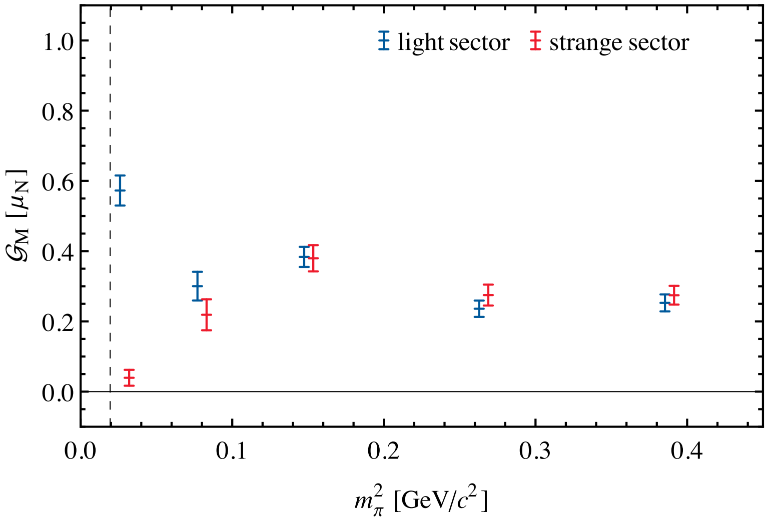
Λ(1405) magnetic form factor observations



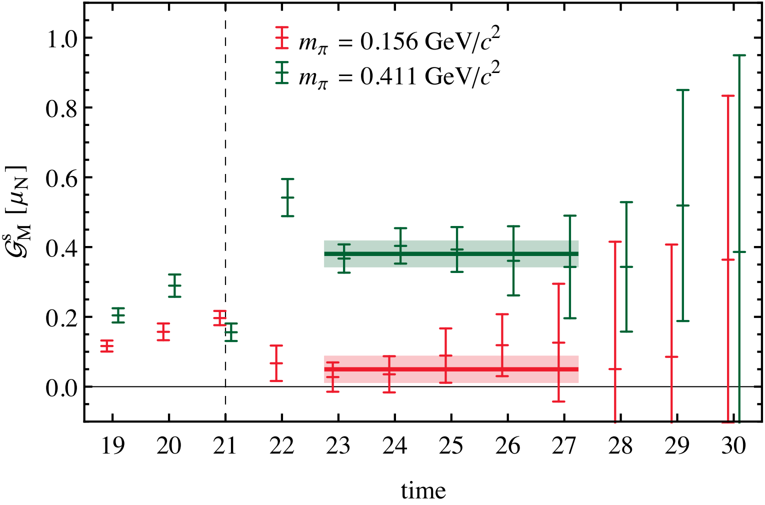
$\Lambda(1405)$ magnetic form factor observations

- The internal structure of the $\Lambda(1405)$ reorganises at the lightest quark mass.
- The strange quark contribution to the magnetic form factor of the $\Lambda(1405)$ drops by an order of magnitude and approaches zero.

Λ(1405) magnetic form factor observations



Correlation function ratio providing $\mathcal{G}_M^s(Q^2)$



$\Lambda(1405)$ magnetic form factor observations

- As the simulation parameters describing the strange quark are held fixed, this is a remarkable environmental effect of unprecedented strength.
- We observe an important rearrangement of the quark structure within the $\Lambda(1405)$ consistent with the dominance of a molecular $\overline{K}N$ bound state.

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- Working on a cubic volume of extent L on each side, it is convenient to define the momentum magnitudes

$$k_n = \sqrt{n_x^2 + n_y^2 + n_z^2}\,\frac{2\pi}{L}\,,$$

with $n_i = 0, 1, 2, \ldots$ and integer $n = n_x^2 + n_y^2 + n_z^2$.

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  with $n_i = 0, 1, 2, \ldots$ and integer $n = n_x^2 + n_y^2 + n_z^2$.
- The non-interacting Hamiltonian H_0 has diagonal entries corresponding to the relativistic non-interacting meson-baryon energies available on the finite periodic volume at total three-momentum zero.

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$$k_n = \sqrt{n_x^2 + n_y^2 + n_z^2}\,\frac{2\pi}{L}\,,$$

  with $n_i = 0, 1, 2, \ldots$ and integer $n = n_x^2 + n_y^2 + n_z^2$.
- The non-interacting Hamiltonian H_0 has diagonal entries corresponding to the relativistic non-interacting meson-baryon energies available on the finite periodic volume at total three-momentum zero.
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- It also includes a single-particle state with bare mass, $m_0 + \alpha_0\, m_\pi^2$.

Hamiltonian model, H_0

Denoting each meson-baryon energy by $\omega_{MB}(k_n) = \omega_M(k_n) + \omega_B(k_n)$, with $\omega_A(k_n) \equiv \sqrt{k_n^2 + m_A^2}$, the non-interacting Hamiltonian takes the form

$$H_0 = \begin{pmatrix} m_0 + \alpha_0\, m_\pi^2 & & 0 & & & 0 & & \cdots \\ & \omega_{\pi\Sigma}(k_0) & & & & & & \\ 0 & & \ddots & & & 0 & & \cdots \\ & & & \omega_{\eta\Lambda}(k_0) & & & & \\ & & & & \omega_{\pi\Sigma}(k_1) & & & \\ 0 & & 0 & & & \ddots & & \cdots \\ & & & & & & \omega_{\eta\Lambda}(k_1) & \\ \vdots & & \vdots & & & \vdots & & \ddots \end{pmatrix} .$$

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- Each entry represents the S-wave interaction energy of the $\Lambda(1405)$ with one of the four channels at a certain value for k_n.

$$H_I = \begin{pmatrix} 0 & g_{\pi\Sigma}(k_0) & \cdots & g_{\eta\Lambda}(k_0) & g_{\pi\Sigma}(k_1) & \cdots & g_{\eta\Lambda}(k_1)\cdots \\ g_{\pi\Sigma}(k_0) & 0 & \cdots & & & & \\ \vdots & \vdots & 0 & & & & \\ & & & \ddots & & & \\ g_{\eta\Lambda}(k_0) & & & & & & \\ g_{\pi\Sigma}(k_1) & & & & & & \\ \vdots & & & & & & \\ g_{\eta\Lambda}(k_1) & & & & & & \\ \vdots & & & & & & \end{pmatrix} .$$

Eigenvalue Equation Form

- The eigenvalue equation corresponding to our Hamiltonian model is

$$\lambda = m_0 + \alpha_0\, m_\pi^2 - \sum_{M,B} \sum_{n=0}^{\infty} \frac{g_{MB}^2(k_n)}{\omega_{MB}(k_n) - \lambda}\,.$$

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- Reference to chiral effective field theory provides the form of $g_{MB}(k_n)$.

Hamiltonian model, H_I

- The form of the interaction is derived from chiral effective field theory.

$$g_{MB}(k_n) = \left(\frac{\kappa_{MB}}{16\pi^2 f_\pi^2} \frac{C_3(n)}{4\pi} \left(\frac{2\pi}{L}\right)^3 \omega_M(k_n)\, u^2(k_n) \right)^{1/2} .$$

- κ_{MB} denotes the $SU(3)$-flavour singlet couplings

$$\kappa_{\pi\Sigma} = 3\xi_0, \qquad \kappa_{\bar{K}N} = 2\xi_0, \qquad \kappa_{K\Xi} = 2\xi_0, \qquad \kappa_{\eta\Lambda} = \xi_0,$$

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Hamiltonian model solution and fit

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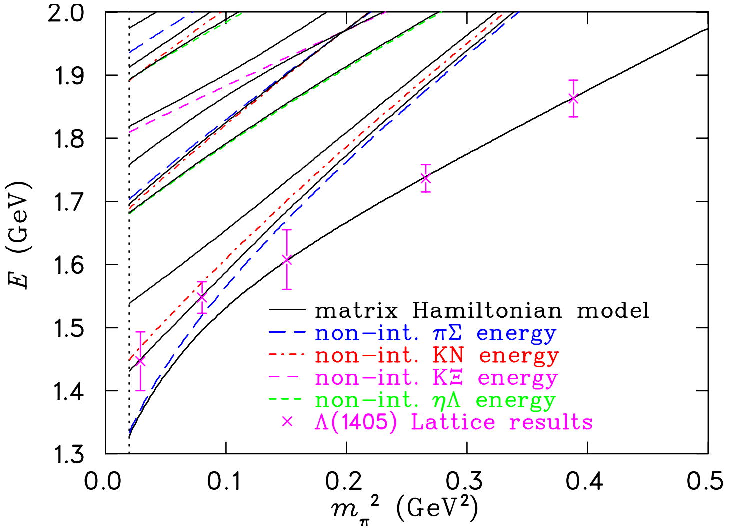
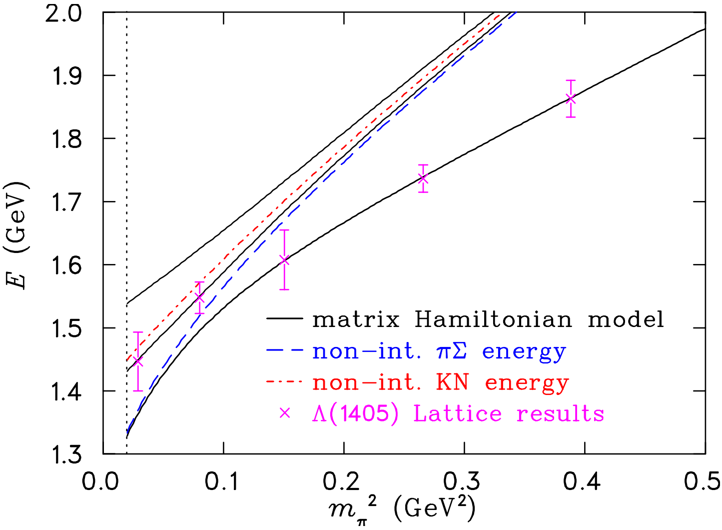
- The bare mass parameters m_0 and α_0 are determined by a fit to the lattice QCD results.

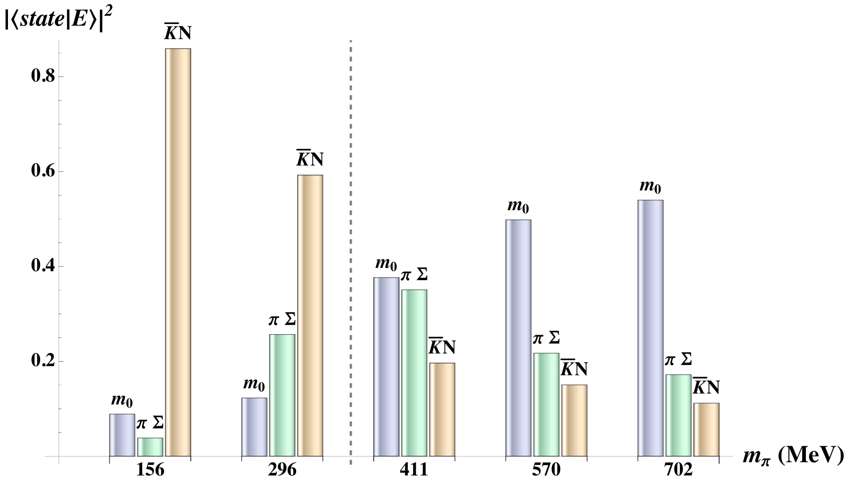
Hamiltonian model fit



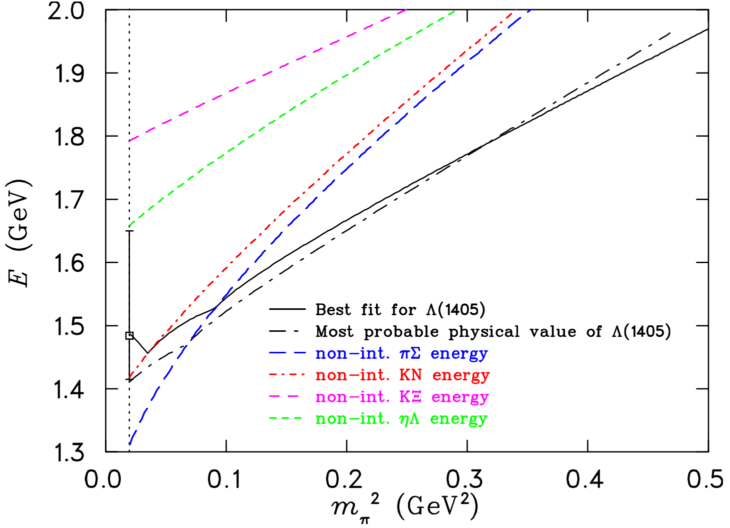
Avoided Level Crossing



Energy eigenstate, $|E\rangle$, basis $|state\rangle$ composition

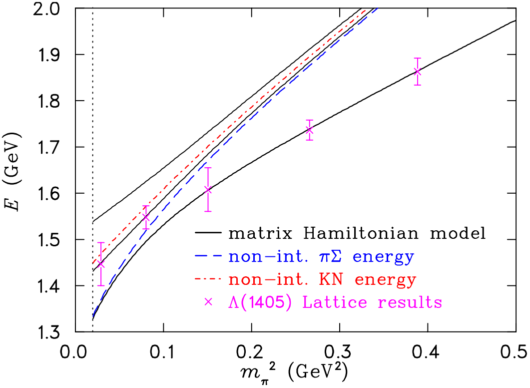


Infinite-volume reconstruction of the Λ(1405) energy

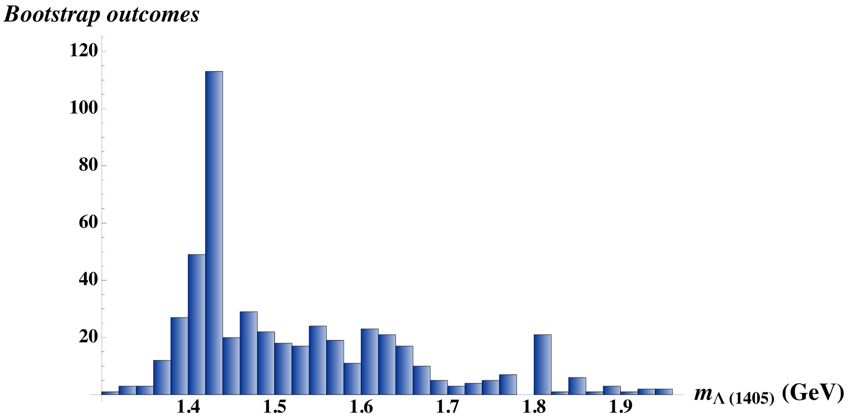


Bootstrap uncertainty at the physical pion mass

- Bootstraps are calculated by altering the value of each lattice data point by a Gaussian-distributed random number, weighted by the uncertainty.



$\Lambda(1405)$ mass distribution at the physical pion mass



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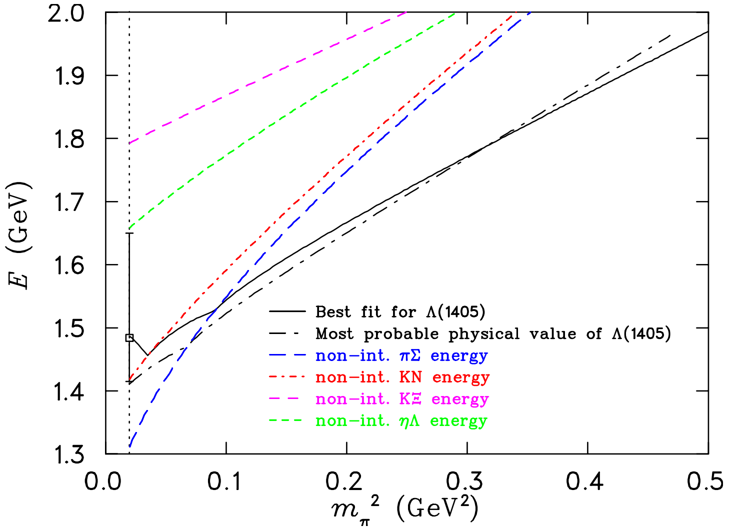

Conclusions

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