



# Transverse Charge and Spin Structures of the Nucleon

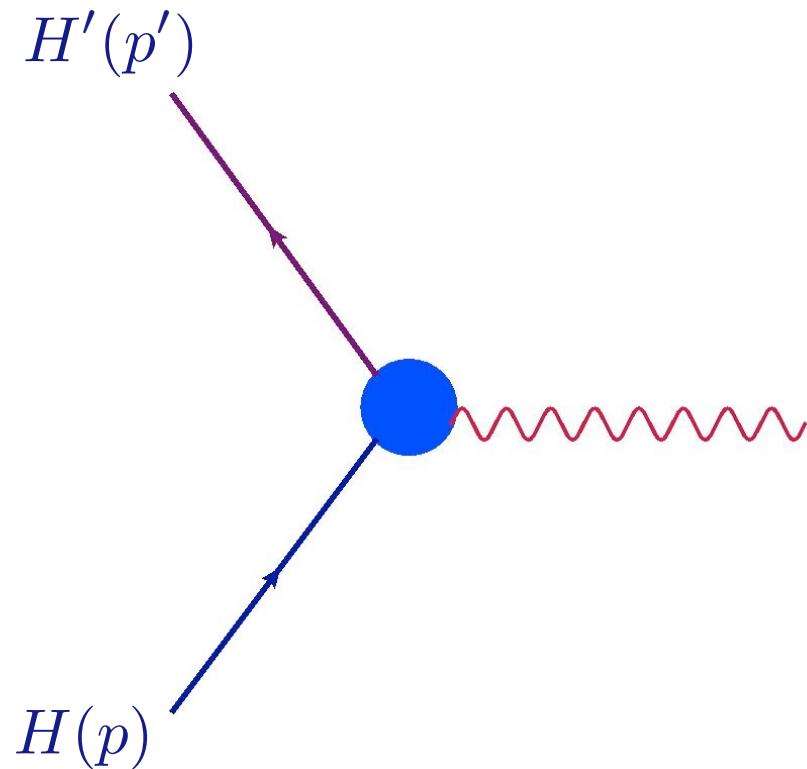
---

Hyun-Chul Kim

Department of Physics  
Inha University

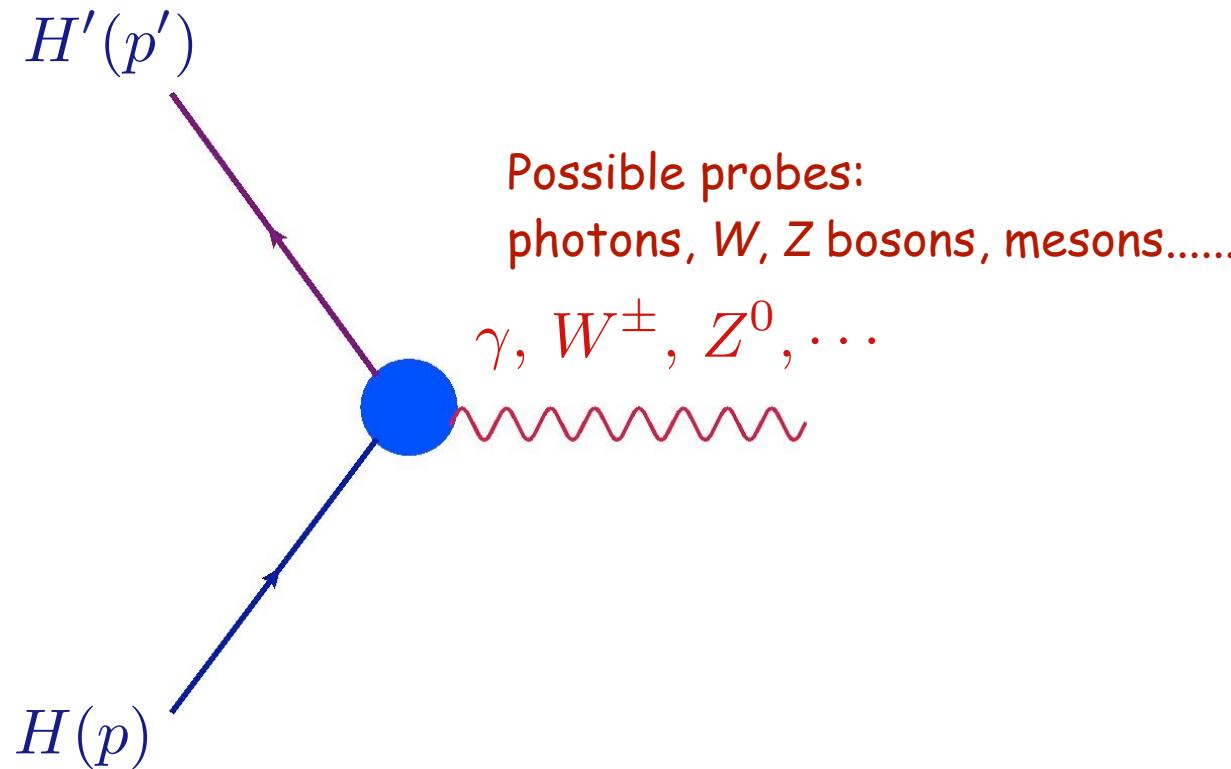
# Structure of hadrons

Traditional way of studying structures of hadrons



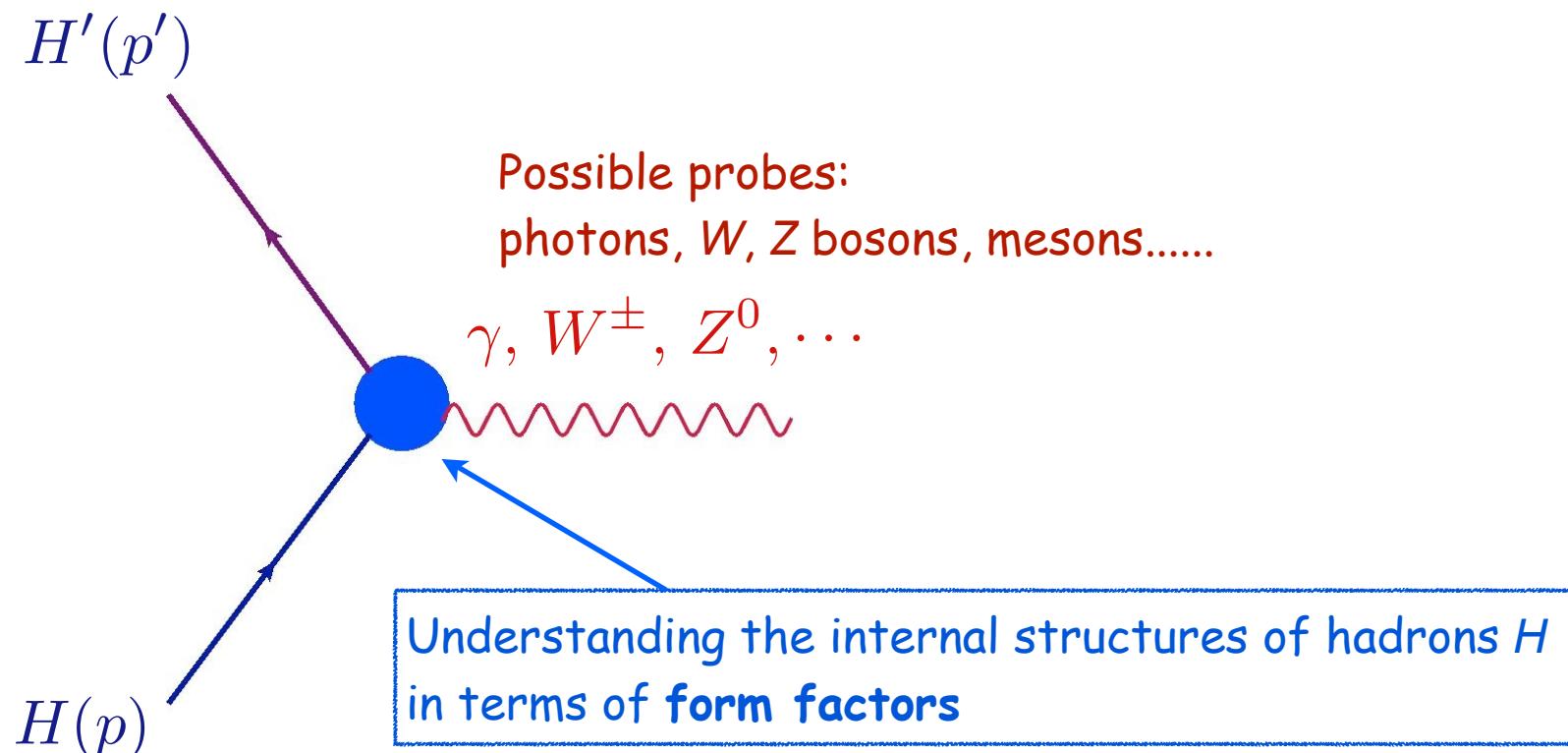
# Structure of hadrons

Traditional way of studying structures of hadrons



# Structure of hadrons

# Traditional way of studying structures of hadrons



# Structure of hadrons

1. **Scalar form factors:** Sigma pion-nucleon term

Quark contribution to the nucleon mass

$$\langle h(p') | \bar{\psi}(0) \psi(0) | h(p) \rangle \sim \Sigma_{\pi N}(t)$$

2. **Vector form factors:** Electromagnetic & weak properties

Charge, EM radii, EM quark distributions in the nucleon

$$\langle N(p') | \bar{\psi}(0) \gamma_\mu \lambda^a \psi(0) | N(p) \rangle \sim G_E(t), G_M(t), G_E^s(t), G_M^s(t)$$

3. **Axial-vector form factors:** Weak properties, spin content of the nucleon, pion-N couplings (PCAC).....

$$\langle N(p') | \bar{\psi}(0) \gamma_\mu \gamma_5 \lambda^a \psi(0) | N(p) \rangle \sim g_A(t), g_A^0(t), G_A^s(t), g_{\pi NN} \dots$$

# Structure of hadrons

## 4. Energy-momentum tensor (gravitational) form factors:

Mass of the nucleon, orbital angular momentum,  
D1 term (pressure, shear force)

$$\langle N(p') | T_{\mu\nu} | N(p) \rangle \sim M_2(t), J(t), d_1(t)$$

## 5. Tensor form factors: Transverse spin structure of the nucleon

$$\langle N(p') | \bar{\psi}(0) \sigma_{\mu\nu} \lambda^a \psi(0) | N(p) \rangle \sim H_T(t), E_T(t), \tilde{H}_T(t)$$



As equally important as vector & axial-vector form factors  
but No probes into these EMT and tensor form factors!

# Structure of hadrons

**Modern approach:** Generalized parton distributions make it possible to get access to these EMT & tensor form factors.

Form factors as Mellin moments of the GPDs

In the present talk, I would like to concentrate on the EM & tensor form factors of the nucleon and their transverse charge & spin structures.

# Chiral quark–soliton model

## Merits of the chiral quark-soliton model

- Fully relativistic field theoretic model.
- Related to QCD via the Instanton vacuum.
- Renormalization scale is naturally given.  
 $1/\rho \approx 600 \text{ MeV}$
- All relevant parameters were fixed already.

$$\mathcal{Z}_{\chi\text{QSM}} = \int \mathcal{D}U \exp(-S_{\text{eff}})$$

$$S_{\text{eff}} = -N_c \text{Tr} \ln D(U)$$

# Chiral quark–soliton model

## Merits of the chiral quark–soliton model

- Fully relativistic field theoretic model.
- Related to QCD via the Instanton vacuum.
- Renormalization scale is naturally given.  
 $1/\rho \approx 600 \text{ MeV}$
- All relevant parameters were fixed already.

$$\mathcal{Z}_{\chi\text{QSM}} = \int \mathcal{D}U \exp(-S_{\text{eff}})$$

$$S_{\text{eff}} = -N_c \text{Tr} \ln D(U) \rightarrow D(U) = \partial_4 + H(U) + \hat{m}$$

# Chiral quark–soliton model

## Merits of the chiral quark–soliton model

- Fully relativistic field theoretic model.
- Related to QCD via the Instanton vacuum.
- Renormalization scale is naturally given.  
 $1/\rho \approx 600 \text{ MeV}$
- All relevant parameters were fixed already.

$$\mathcal{Z}_{\chi\text{QSM}} = \int \mathcal{D}U \exp(-S_{\text{eff}}) \quad H(U) = -i\gamma_4\gamma_i\partial_i + \gamma_4 M U^{\gamma_5}$$
$$S_{\text{eff}} = -N_c \text{Tr} \ln D(U) \quad D(U) = \partial_4 + H(U) + \hat{m}$$

# Chiral quark–soliton model

## Merits of the chiral quark–soliton model

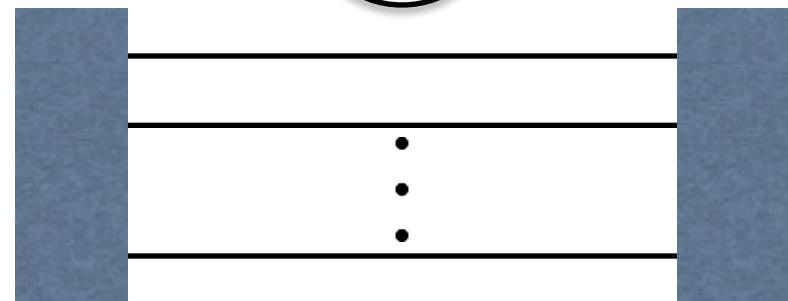
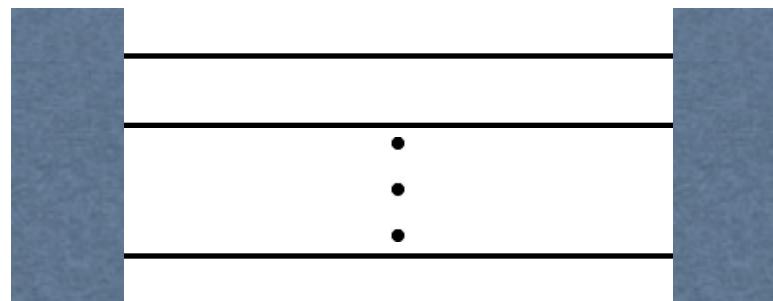
- Fully relativistic field theoretic model.
- Related to QCD via the Instanton vacuum.
- Renormalization scale is naturally given.  
 $1/\rho \approx 600 \text{ MeV}$
- All relevant parameters were fixed already.

$$\mathcal{Z}_{\chi\text{QSM}} = \int \mathcal{D}U \exp(-S_{\text{eff}})$$
$$H(U) = -i\gamma_4\gamma_i\partial_i + \gamma_4 M U^{\gamma_5}$$
$$S_{\text{eff}} = -N_c \text{Tr} \ln D(U)$$
$$D(U) = \partial_4 + H(U) + \hat{m}$$
$$\hat{m} = \text{diag}(m_u, m_d, m_s)\gamma_4$$

# Chiral quark–soliton model

## Classical solitons

$$\langle J_N(\vec{x}, T) J_N^\dagger(\vec{y}, -T) \rangle_0 \sim \Pi_N(T) \sim e^{-[(N_c E_{\text{val}} + E_{\text{sea}})T]}$$

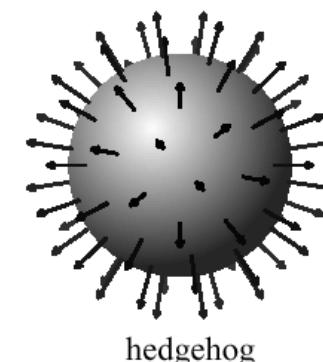


Vacuum  
Polarization

$$\frac{\delta}{\delta U} (N_c E_{\text{val}} + E_{\text{sea}}) = 0 \rightarrow M_{\text{cl}} = N_c E_{\text{val}}(U_c) + E_{\text{sea}}(U_c)$$

Hedgehog Ansatz:

$$U_{\text{SU}(2)} = \exp [i\gamma_5 \mathbf{n} \cdot \boldsymbol{\tau} P(r)]$$



# Chiral quark–soliton model

## Collective (Zero-mode) quantization

$$U_0 = \begin{bmatrix} e^{i\vec{n}\cdot\vec{\tau} P(r)} & 0 \\ 0 & 1 \end{bmatrix}$$

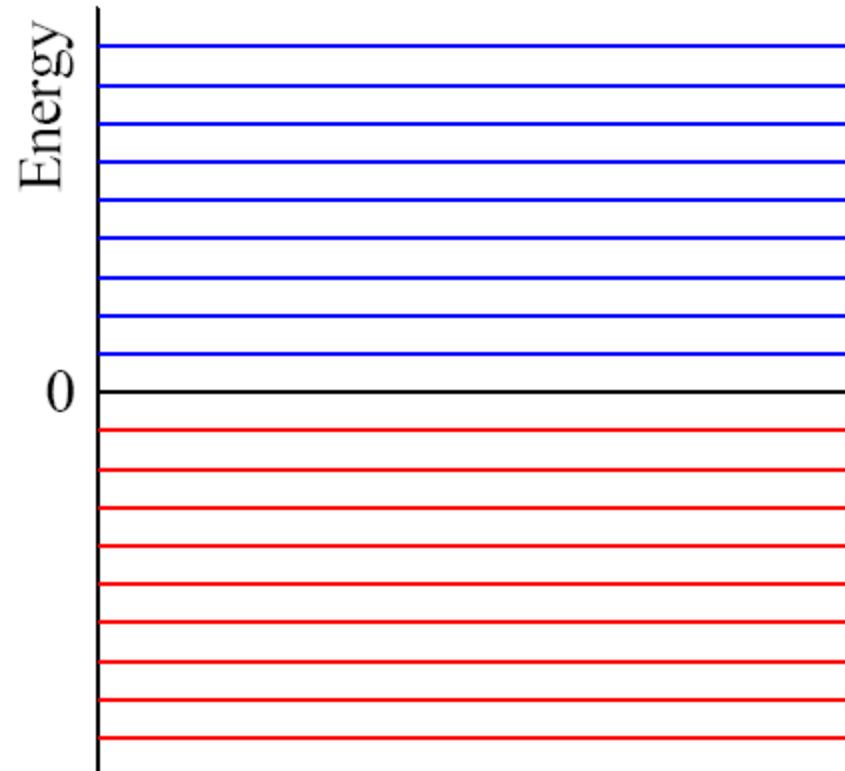
$$\textcolor{red}{U}(\boldsymbol{x}, t) = R(t) U_c(\boldsymbol{x} - \mathbf{Z}(t)) R^\dagger(t)$$

$$\int D\textcolor{red}{U}[\cdots] \rightarrow \int D\mathbf{A} D\mathbf{Z}[\cdots]$$

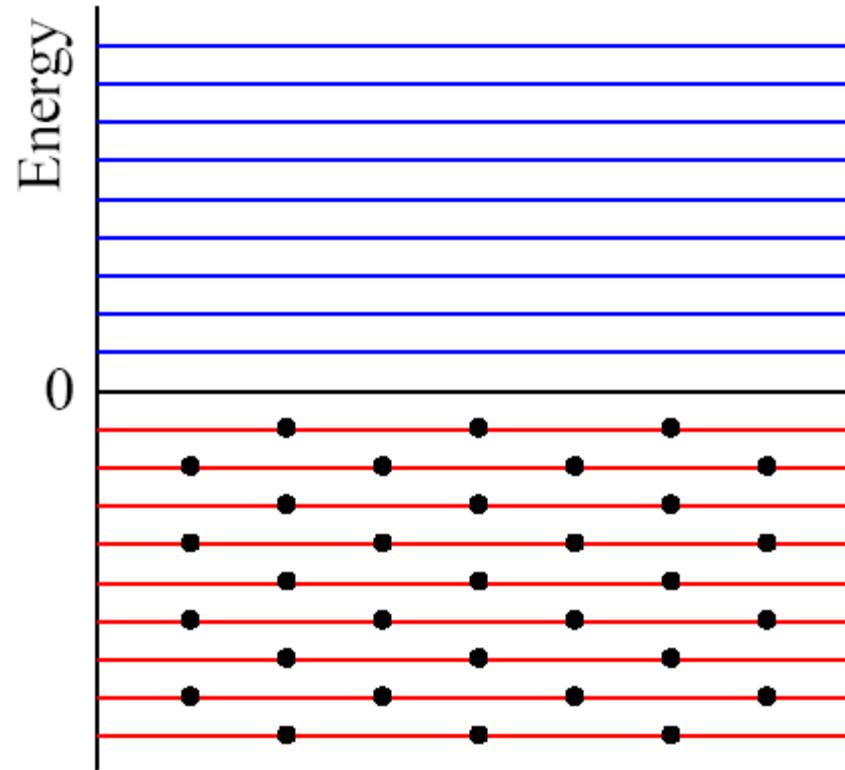
$$\mathcal{L} = -M_{sol} + \frac{\textcolor{blue}{I}_1}{2} \sum_{i=1}^3 \Omega_i^2 + \frac{\textcolor{green}{I}_2}{2} \sum_{i=4}^7 \Omega_i^2 + \frac{\textcolor{red}{N}_c}{2\sqrt{3}} \Omega_8$$

# Chiral quark-soliton picture

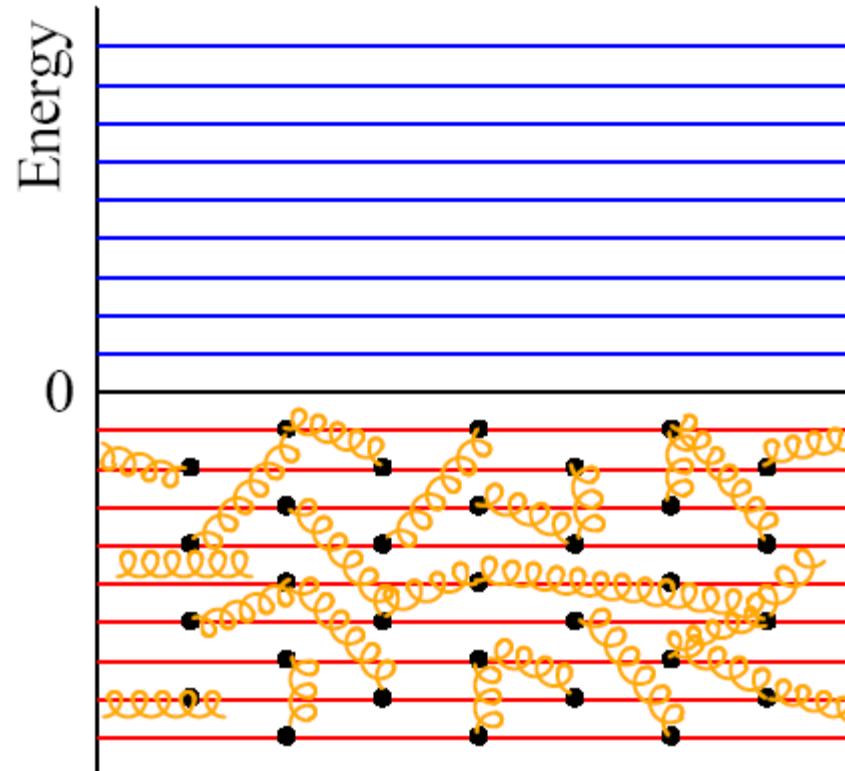
# Chiral quark-soliton picture



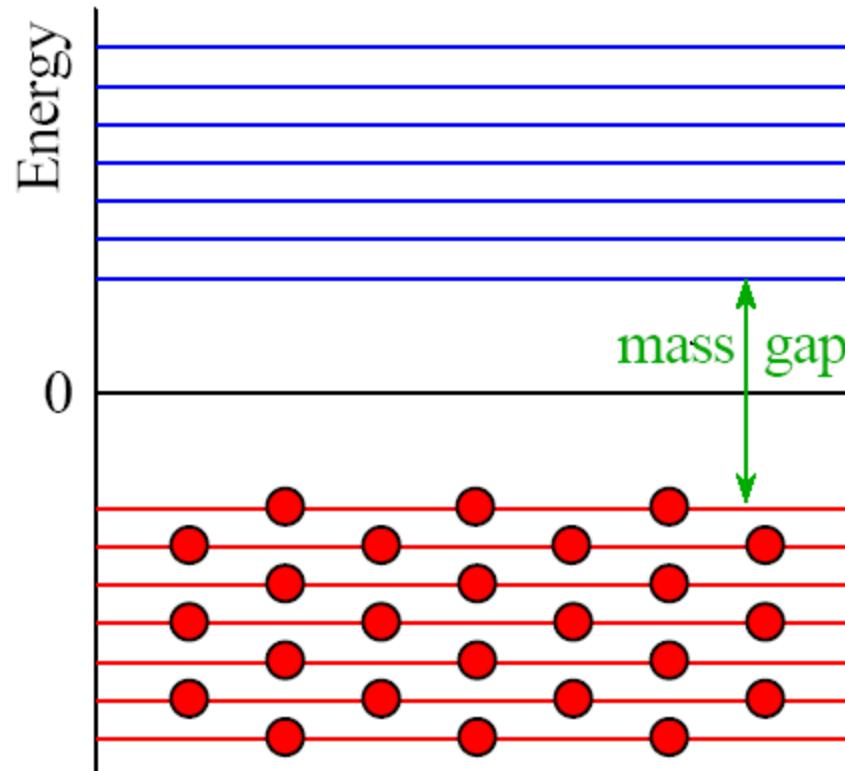
# Chiral quark-soliton picture



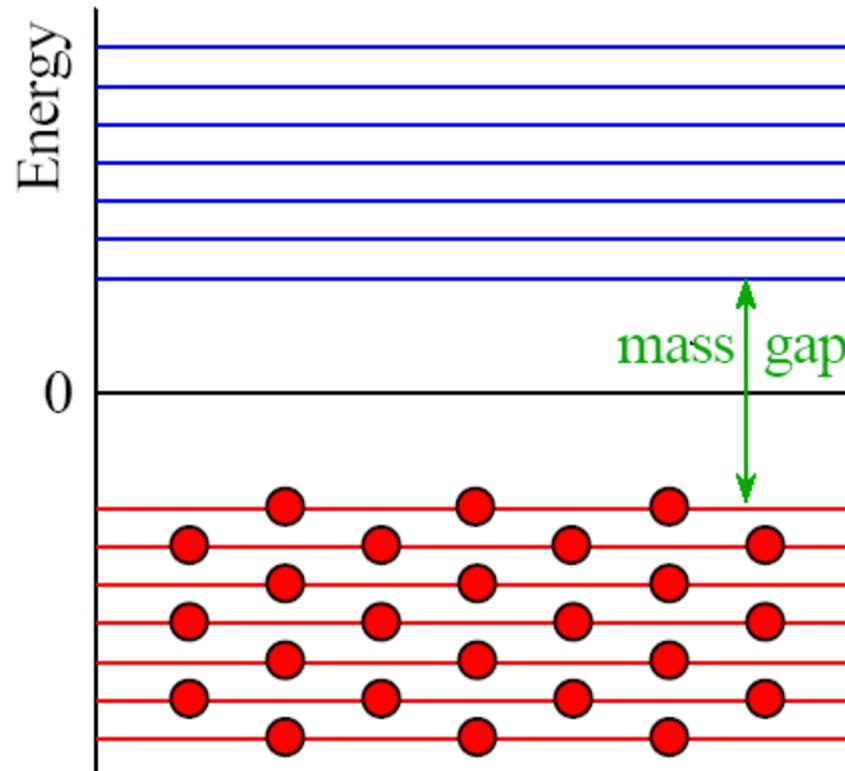
# Chiral quark-soliton picture



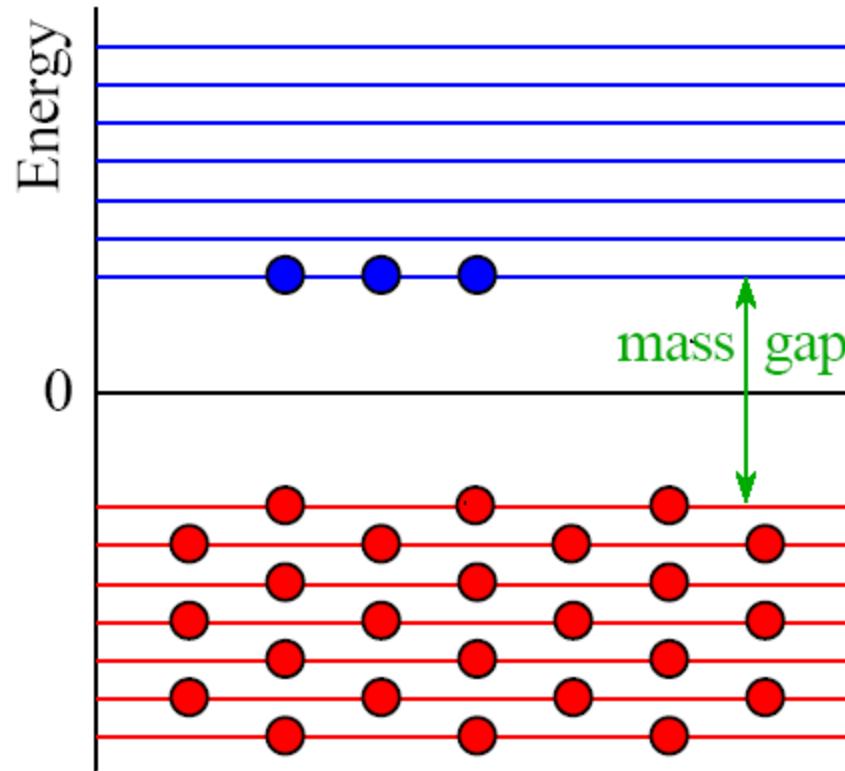
# Chiral quark-soliton picture



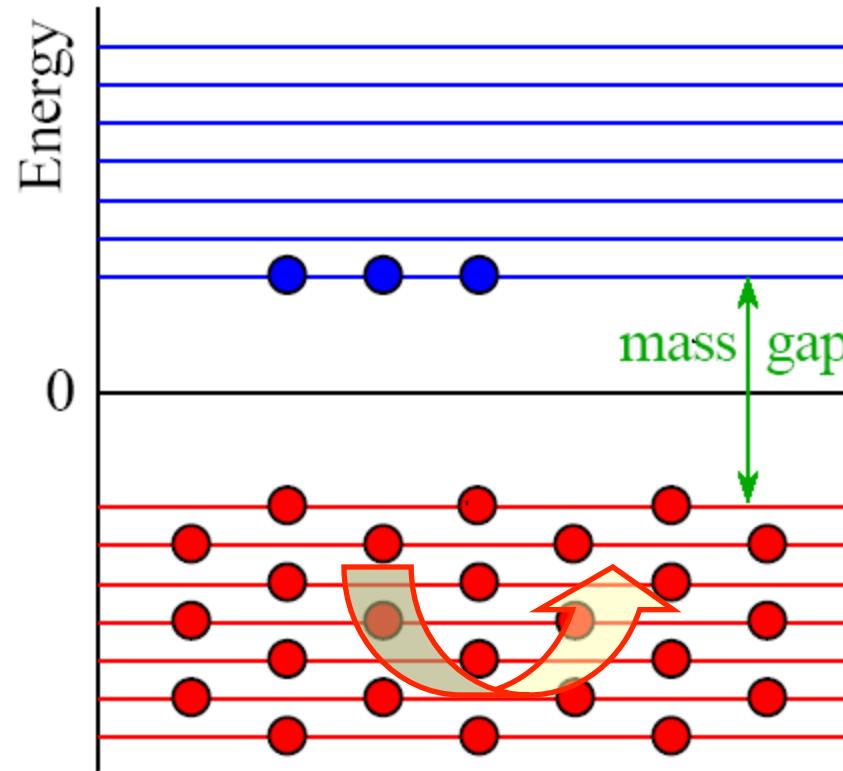
# Chiral quark-soliton picture



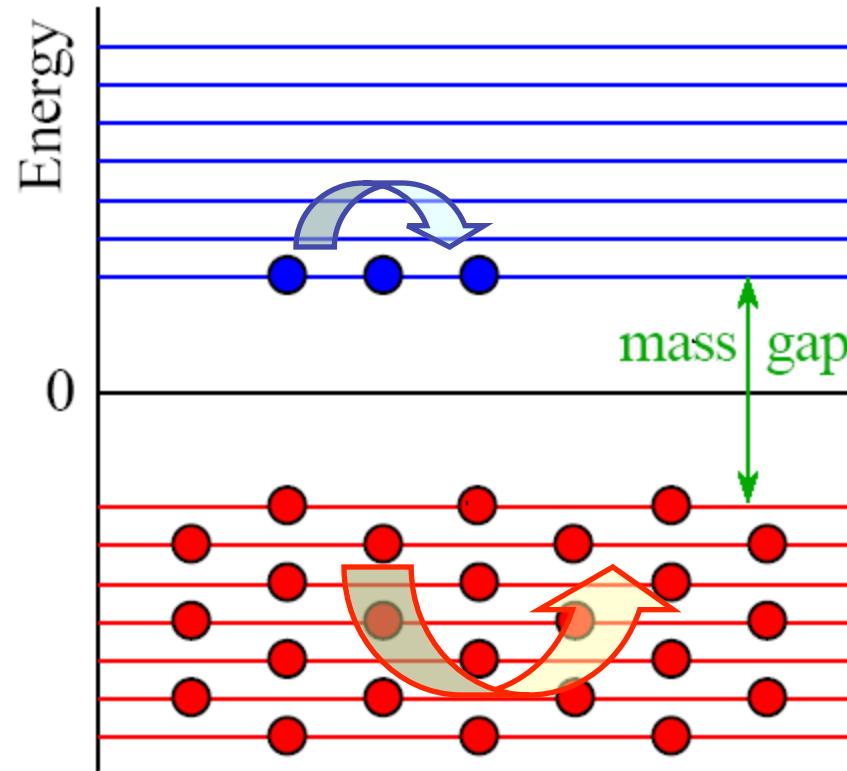
# Chiral quark-soliton picture



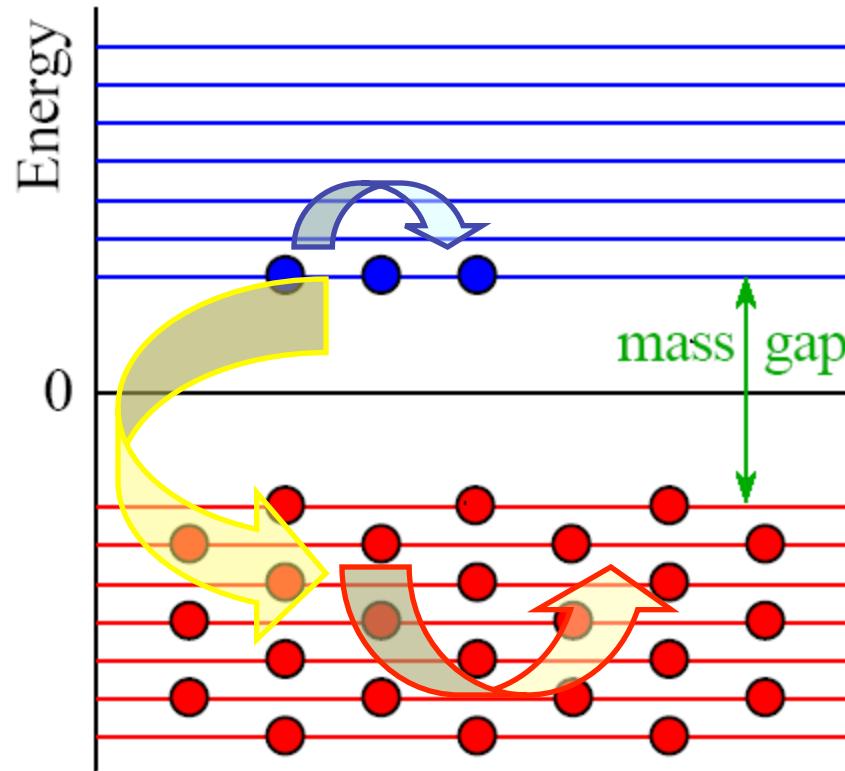
# Chiral quark-soliton picture



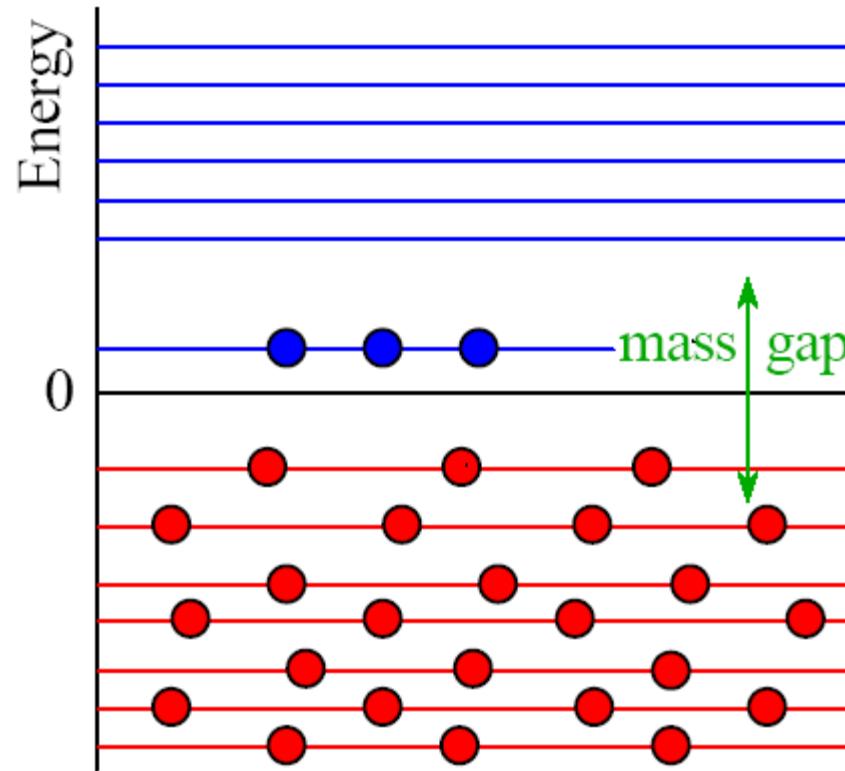
# Chiral quark-soliton picture



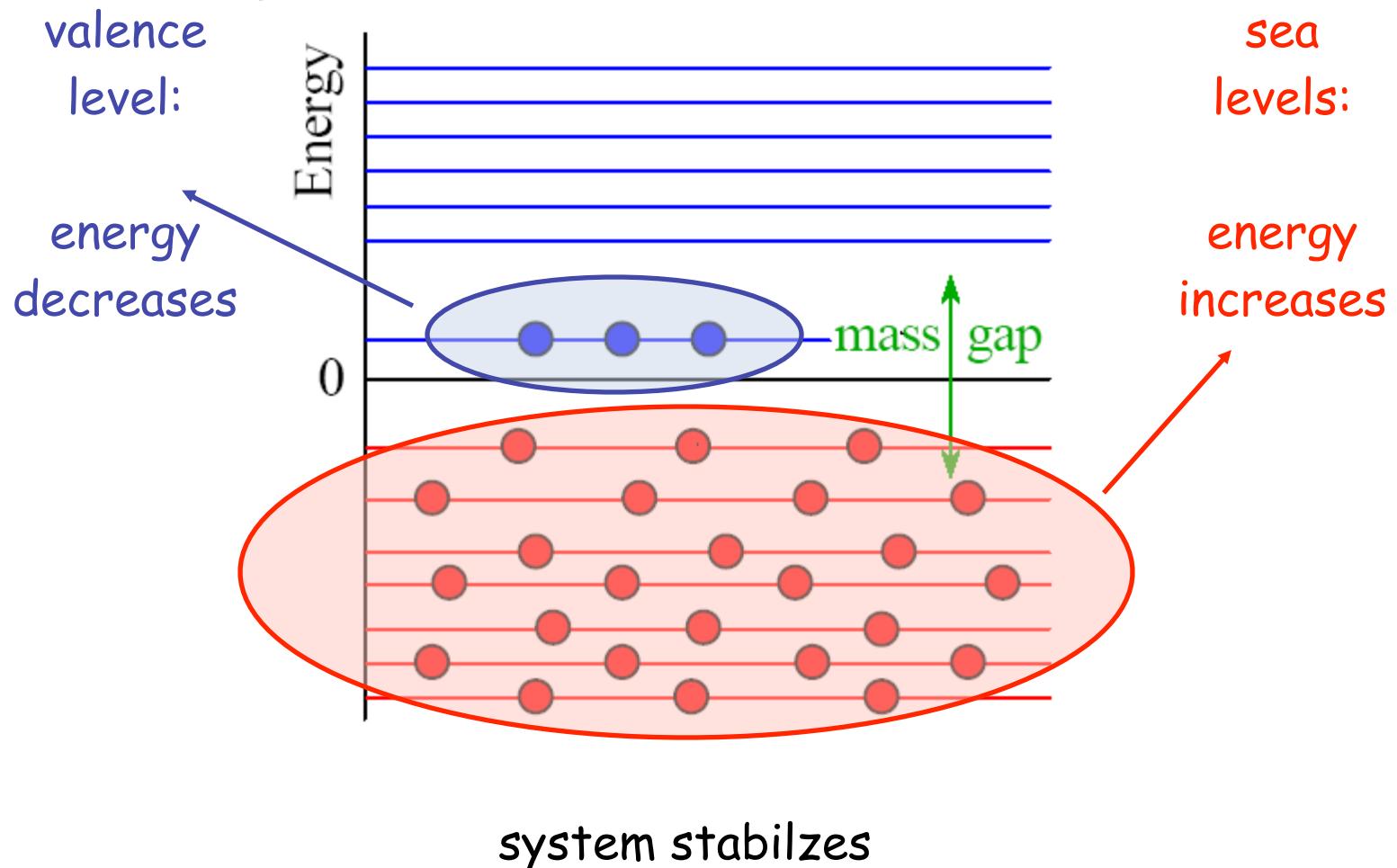
# Chiral quark-soliton picture



# Chiral quark-soliton picture

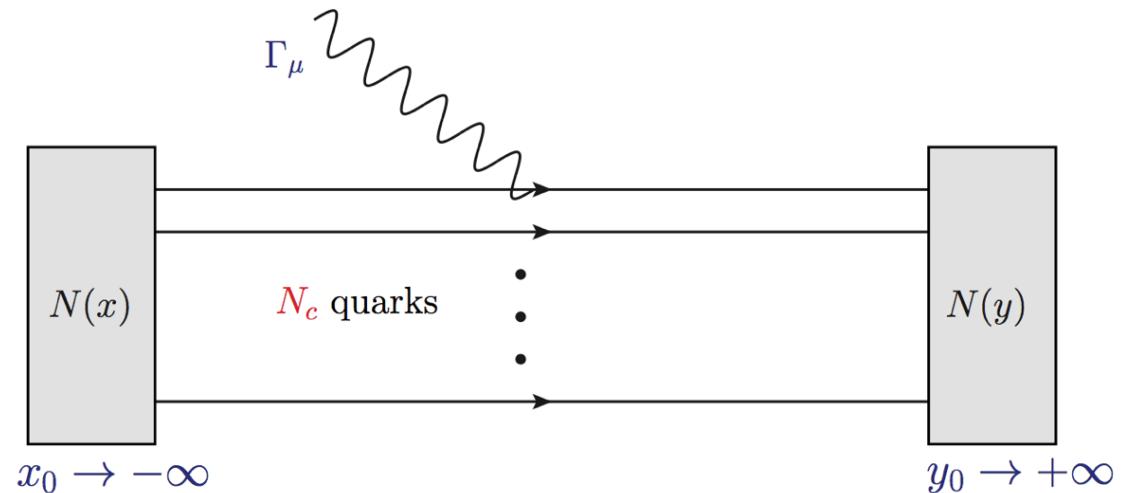


# Chiral quark-soliton picture

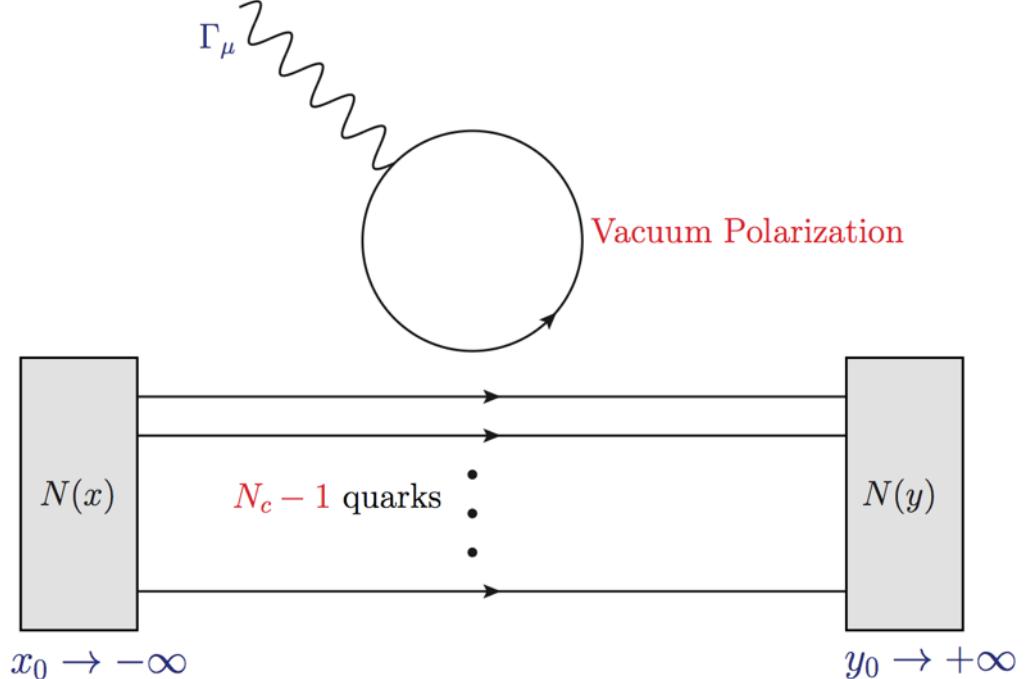


# Observables

Valence part

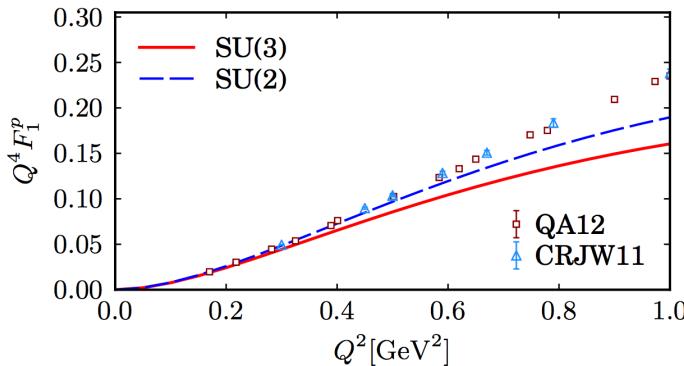


Sea part

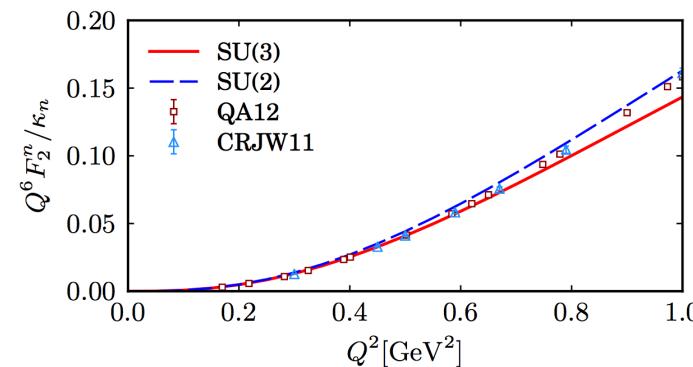
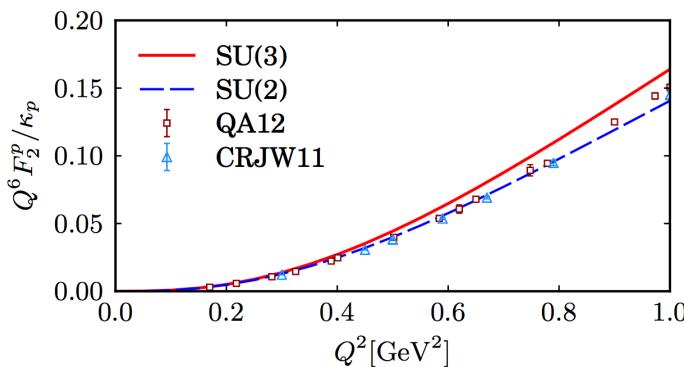
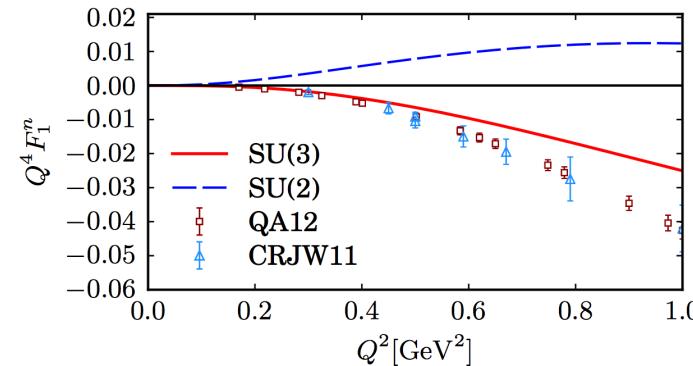


# Dirac & Pauli Form factors

Proton

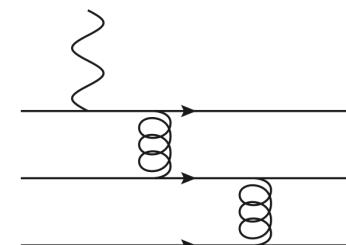


Neutron

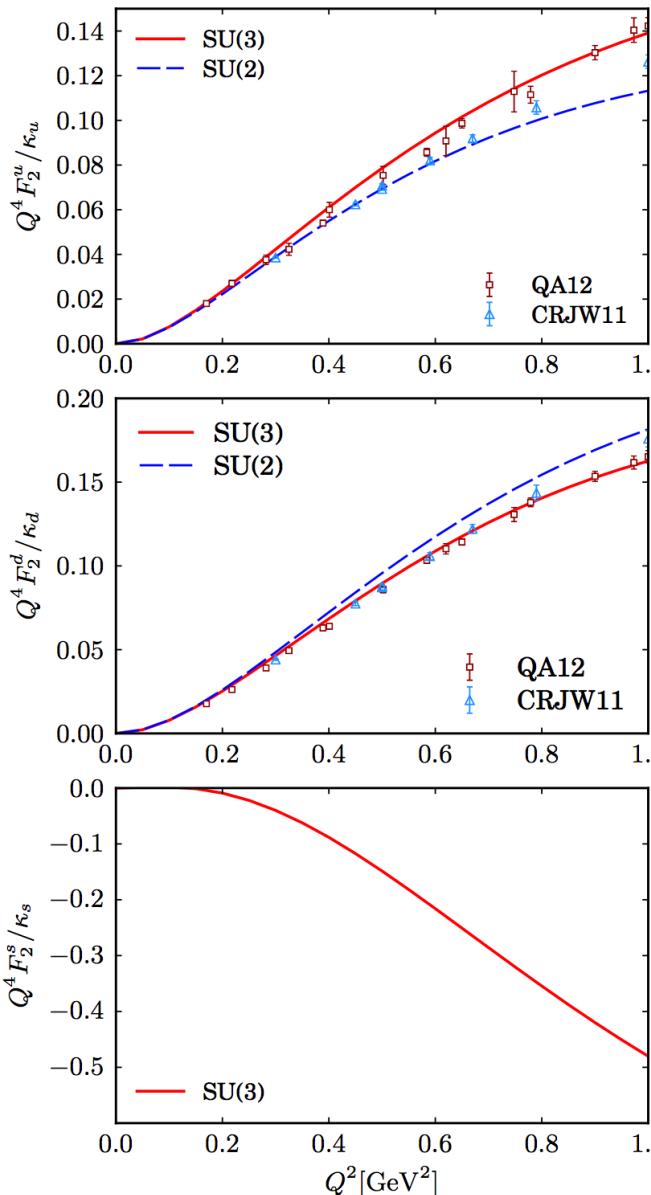
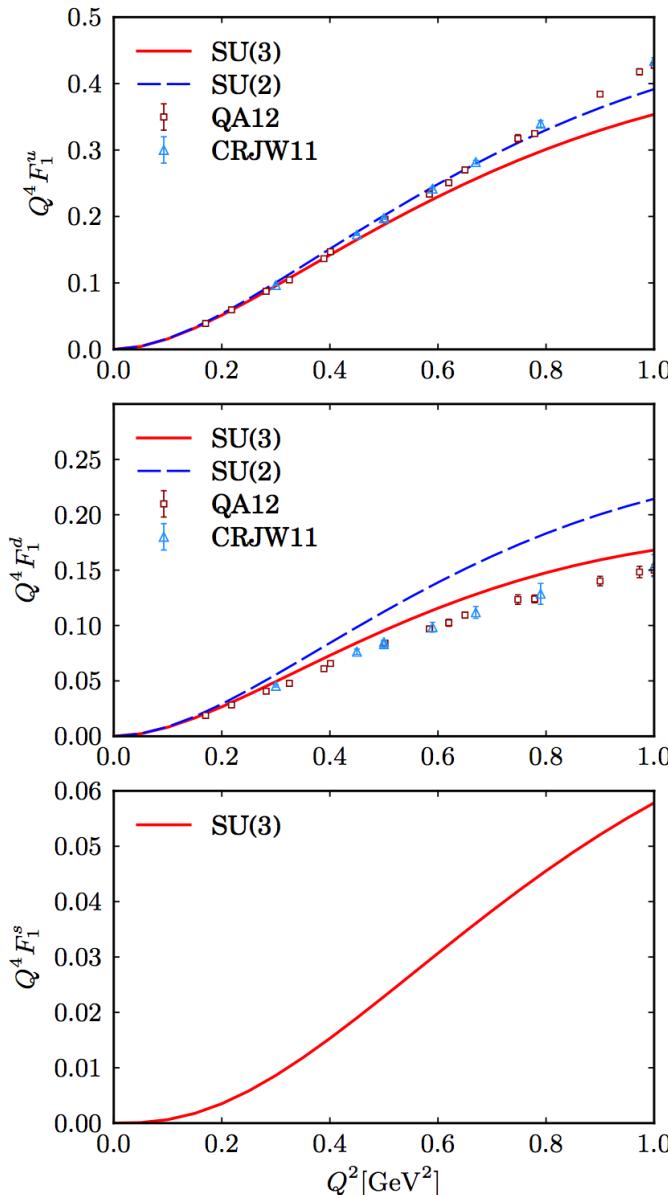


$$F_1(Q^2) \sim \frac{1}{Q^4}, \quad Q^2 \rightarrow \infty \quad F_2(Q^2) \sim \frac{1}{Q^6}, \quad Q^2 \rightarrow \infty$$

S. J. Brodsky and G. R. Farrar, PRD 11, 1309 (1975).



# Dirac & Pauli Form factors



Up quark FFs

Down quark FFs

Strange quark FFs

# Transverse charge densities

Why transverse charge densities?

For the pion, Son's Talk on Tuesday.

$$\begin{aligned} & \langle P', S' | \bar{\psi}(\mathbf{0}) \gamma_\mu \hat{Q} \psi(\mathbf{0}) | P, S \rangle \\ &= \bar{u}(p', s') \left( \gamma_\mu F_1(t) + i \frac{\sigma^{\mu\nu} \Delta_n u}{2M_N} F_2(t) \right) u(p, s) \end{aligned}$$

# Transverse charge densities

Why transverse charge densities?

Electromagnetic form factors:

$$\begin{aligned} & \langle P', S' | \bar{\psi}(0) \gamma_\mu \hat{Q} \psi(0) | P, S \rangle \\ &= \bar{u}(p', s') \left( \gamma_\mu F_1(t) + i \frac{\sigma^{\mu\nu} \Delta_n u}{2M_N} F_2(t) \right) u(p, s) \end{aligned}$$

For the pion, Son's Talk on Tuesday.

# Transverse charge densities

Why transverse charge densities?

Electromagnetic form factors:

For the pion, Son's Talk on Tuesday.

$$\begin{aligned} & \langle P', S' | \bar{\psi}(0) \gamma_\mu \hat{Q} \psi(0) | P, S \rangle \\ &= \bar{u}(p', s') \left( \gamma_\mu F_1(t) + i \frac{\sigma^{\mu\nu} \Delta_n u}{2M_N} F_2(t) \right) u(p, s) \end{aligned}$$

GPDs

$$\begin{aligned} & \int \frac{dx^-}{4\pi} \langle P', S' | \bar{q}(-\frac{x^-}{2}, \mathbf{0}_\perp) \gamma^+ q(\frac{x^-}{2}, \mathbf{0}_\perp) | P, S \rangle \\ &= \frac{1}{2\bar{p}^+} \bar{u}(p', s') \left( \gamma^+ H_q(x, \xi, t) + i \frac{\sigma^{+\nu} \Delta_\nu}{2M_N} E_q(x, \xi, t) \right) u(p, s) \end{aligned}$$

# Transverse charge densities

Why transverse charge densities?

Electromagnetic form factors:

For the pion, Son's Talk on Tuesday.

$$\begin{aligned} & \langle P', S' | \bar{\psi}(\mathbf{0}) \gamma_\mu \hat{Q} \psi(\mathbf{0}) | P, S \rangle \\ &= \bar{u}(p', s') \left( \gamma_\mu F_1(t) + i \frac{\sigma^{\mu\nu} \Delta_n u}{2M_N} F_2(t) \right) u(p, s) \end{aligned}$$

GPDs

$$\begin{aligned} & \int \frac{dx^-}{4\pi} \langle P', S' | \bar{q}(-\frac{x^-}{2}, \mathbf{0}_\perp) \gamma^+ q(\frac{x^-}{2}, \mathbf{0}_\perp) | P, S \rangle \\ &= \frac{1}{2\bar{p}^+} \bar{u}(p', s') \left( \gamma^+ H_q(x, \xi, t) + i \frac{\sigma^{+\nu} \Delta_\nu}{2M_N} E_q(x, \xi, t) \right) u(p, s) \end{aligned}$$

$$F_1(t) = \sum_q e_q \int dx H_q(x, 0, t),$$

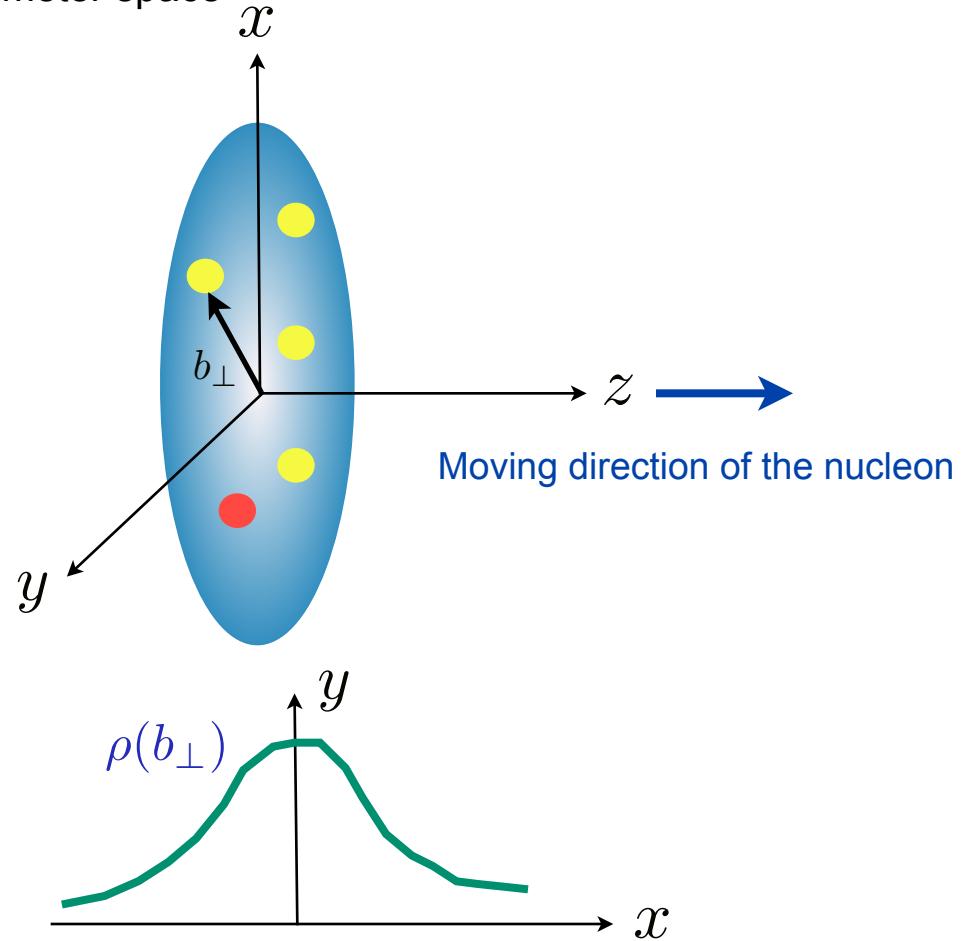
$$F_2(t) = \sum_q e_q \int dx E_q(x, 0, t)$$

# Transverse charge densities

Why transverse charge densities?

2-D Fourier transform of the GPDs in impact-parameter space

$$q(x, \mathbf{b}) = \int \frac{d^2 q}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{b}} H_q(x, -\mathbf{q}^2)$$



# Transverse charge densities

## Why transverse charge densities?

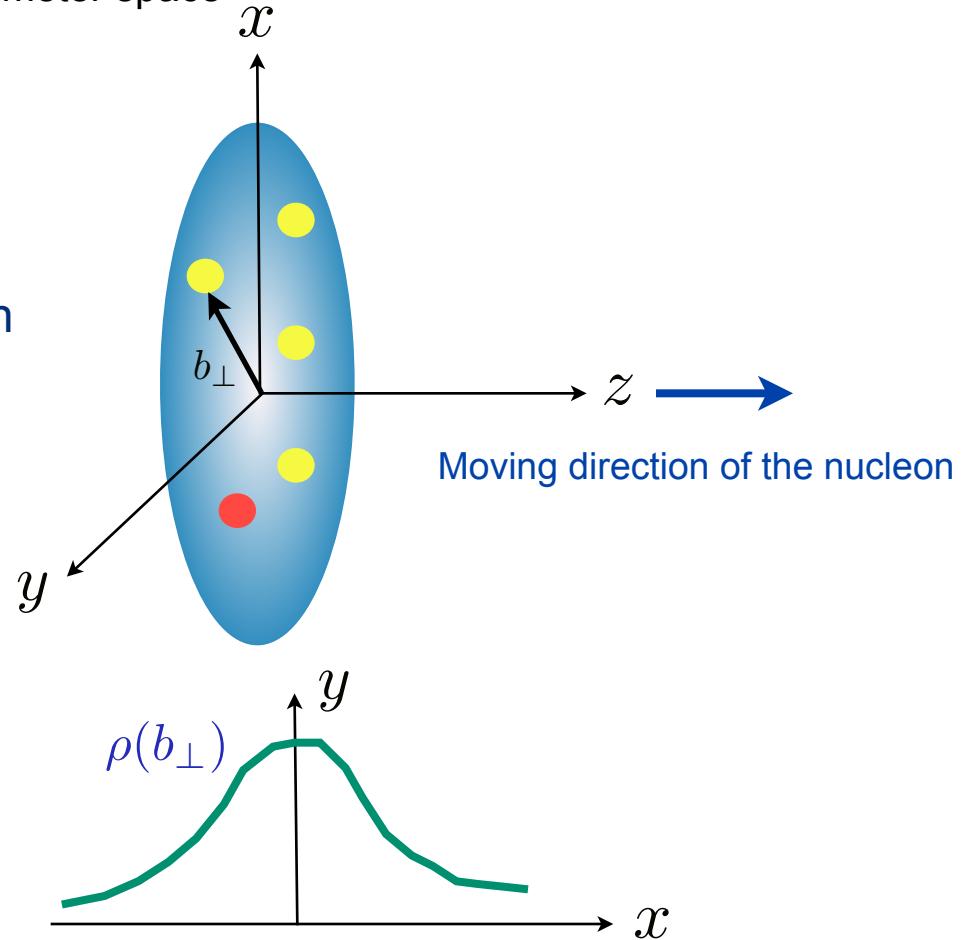
2-D Fourier transform of the GPDs in impact-parameter space

$$q(x, \mathbf{b}) = \int \frac{d^2 q}{(2\pi)^2} e^{i\mathbf{q} \cdot \mathbf{b}} H_q(x, -\mathbf{q}^2)$$

→ It can be interpreted as the probability distribution of a quark in the transverse plane.

M. Burkardt, PRD **62**, 071503 (2000); Int. J. Mod. Phys. A **18**, 173 (2003).

$$\begin{aligned}\rho(\mathbf{b}) &:= \sum_q e_q \int dx q(x, \mathbf{b}) \\ &= \int \frac{d^2 q}{(2\pi)^2} F_1(Q^2) e^{i\mathbf{q} \cdot \mathbf{b}}\end{aligned}$$



# Transverse charge densities

## Inside an unpolarized nucleon

M. Burkardt, PRD **62**, 071503 (2000); Int. J. Mod. Phys. A **18**, 173 (2003).

G.A. Miller, PRL **99**, 112001 (2007)

$$\rho_{\text{ch}}^{\chi}(b) = \int_0^{\infty} \frac{dQ}{2\pi} Q J_0(Qb) F_1^{\chi}(Q^2)$$

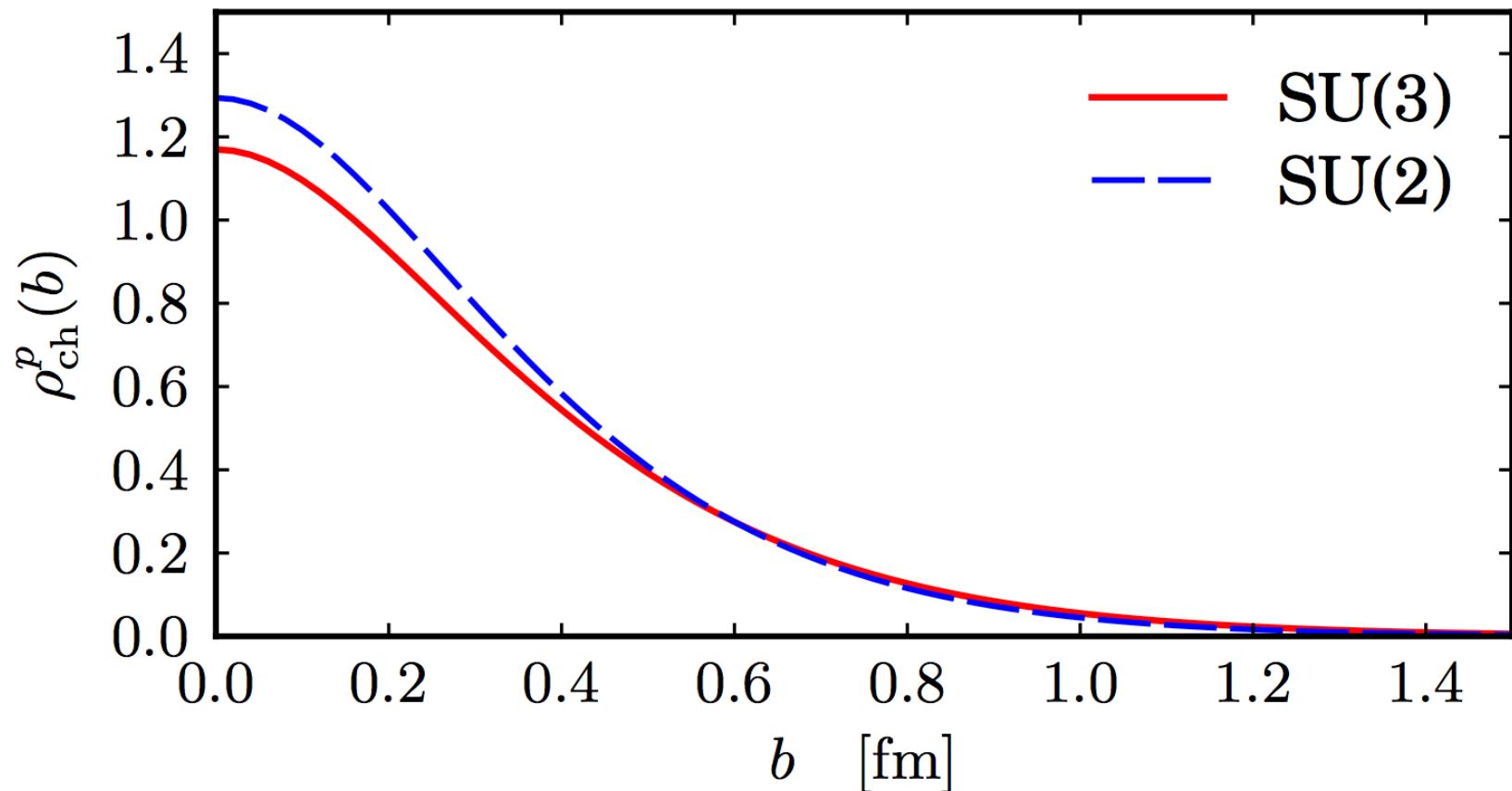
## Inside a polarized nucleon

Carlson and Vanderhaeghen, PRL **100**, 032004

$$\rho_T^{\chi}(b) = \rho_{\text{ch}}^{\chi}(b) - \sin(\phi_b - \phi_S) \frac{1}{2M_N} \int_0^{\infty} \frac{dQ}{2\pi} Q^2 J_1(Qb) F_2^{\chi}(Q^2)$$

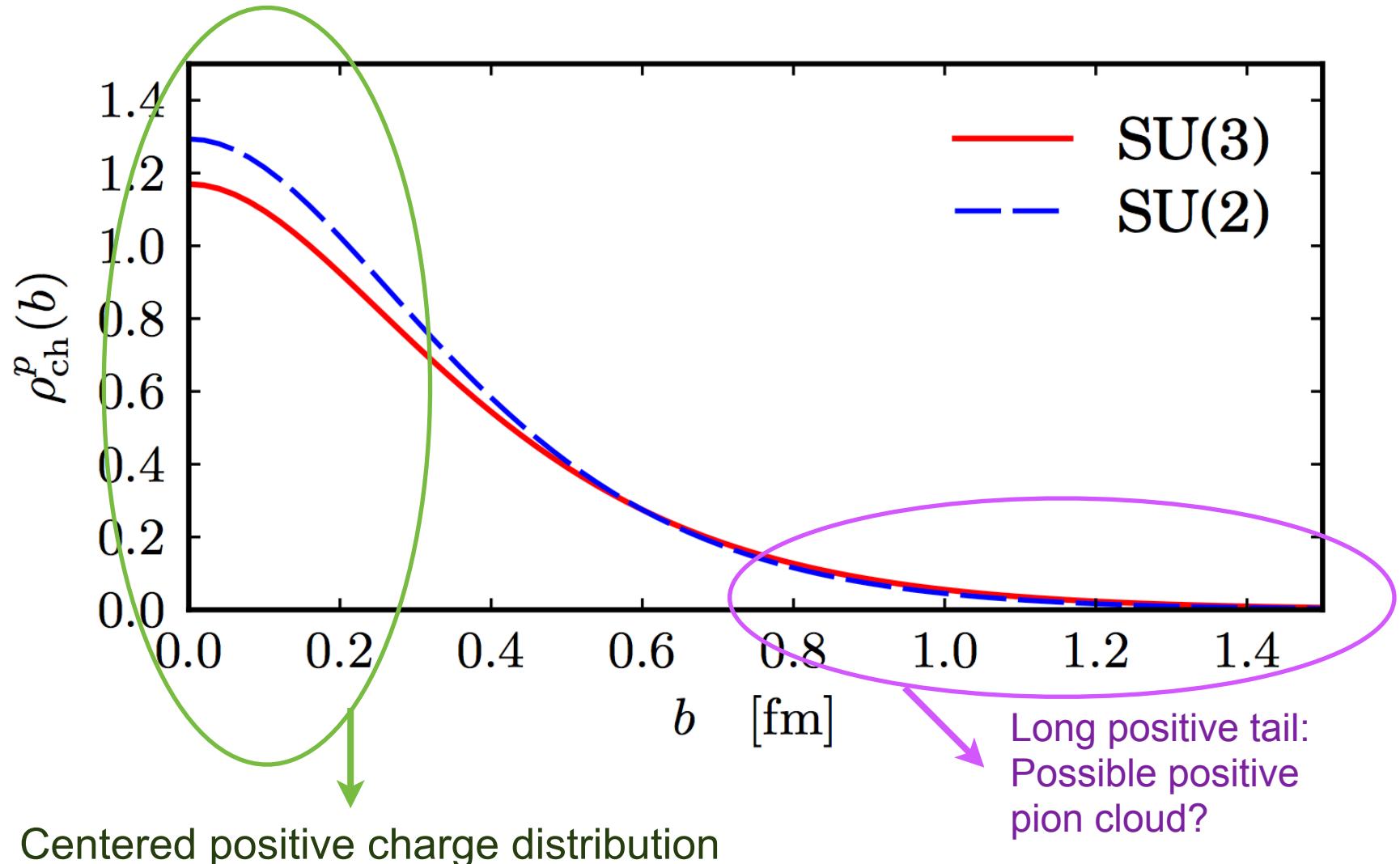
# Results

Transverse charge densities inside an **unpolarized** proton



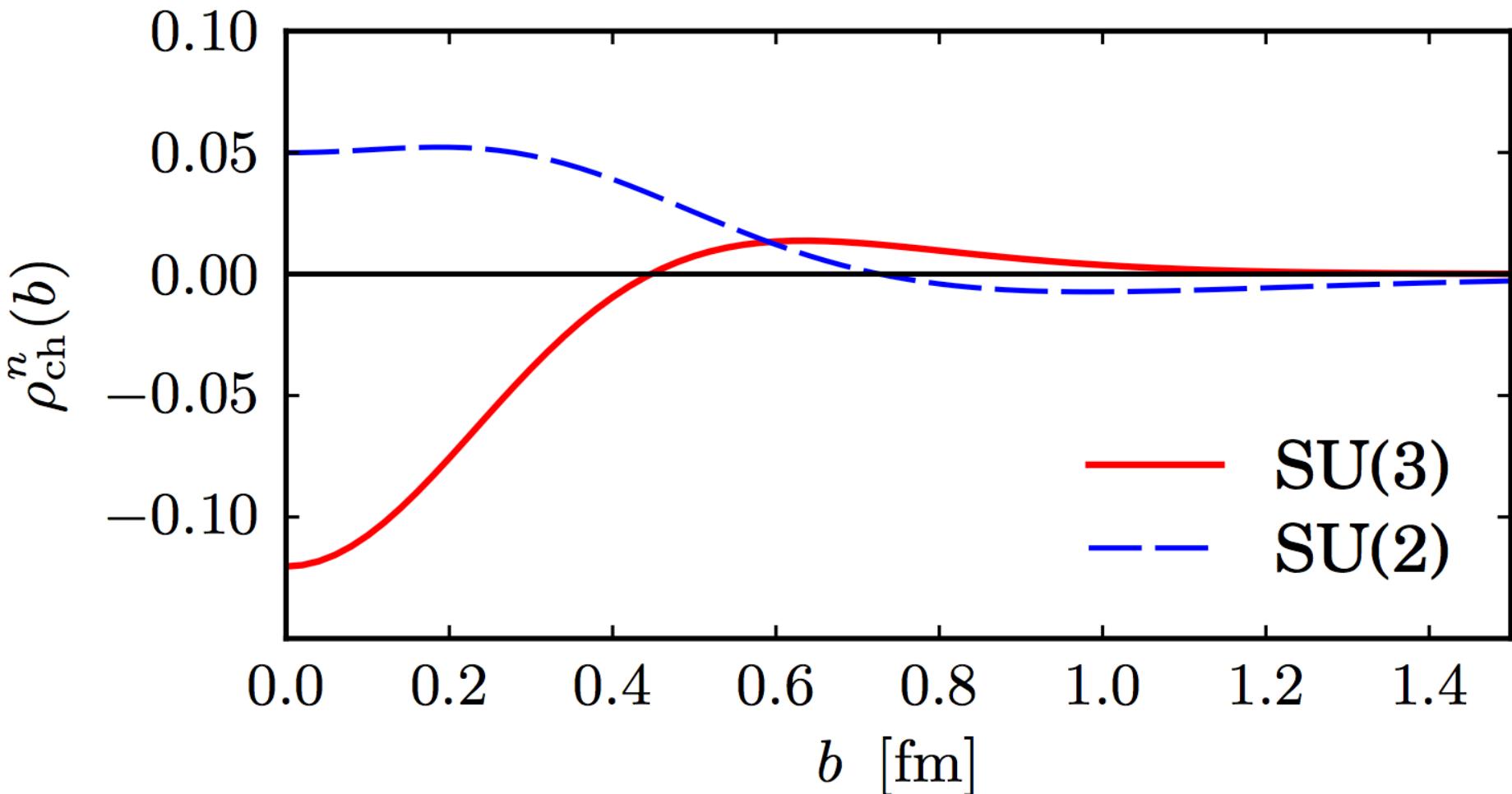
# Results

Transverse charge densities inside an **unpolarized** proton



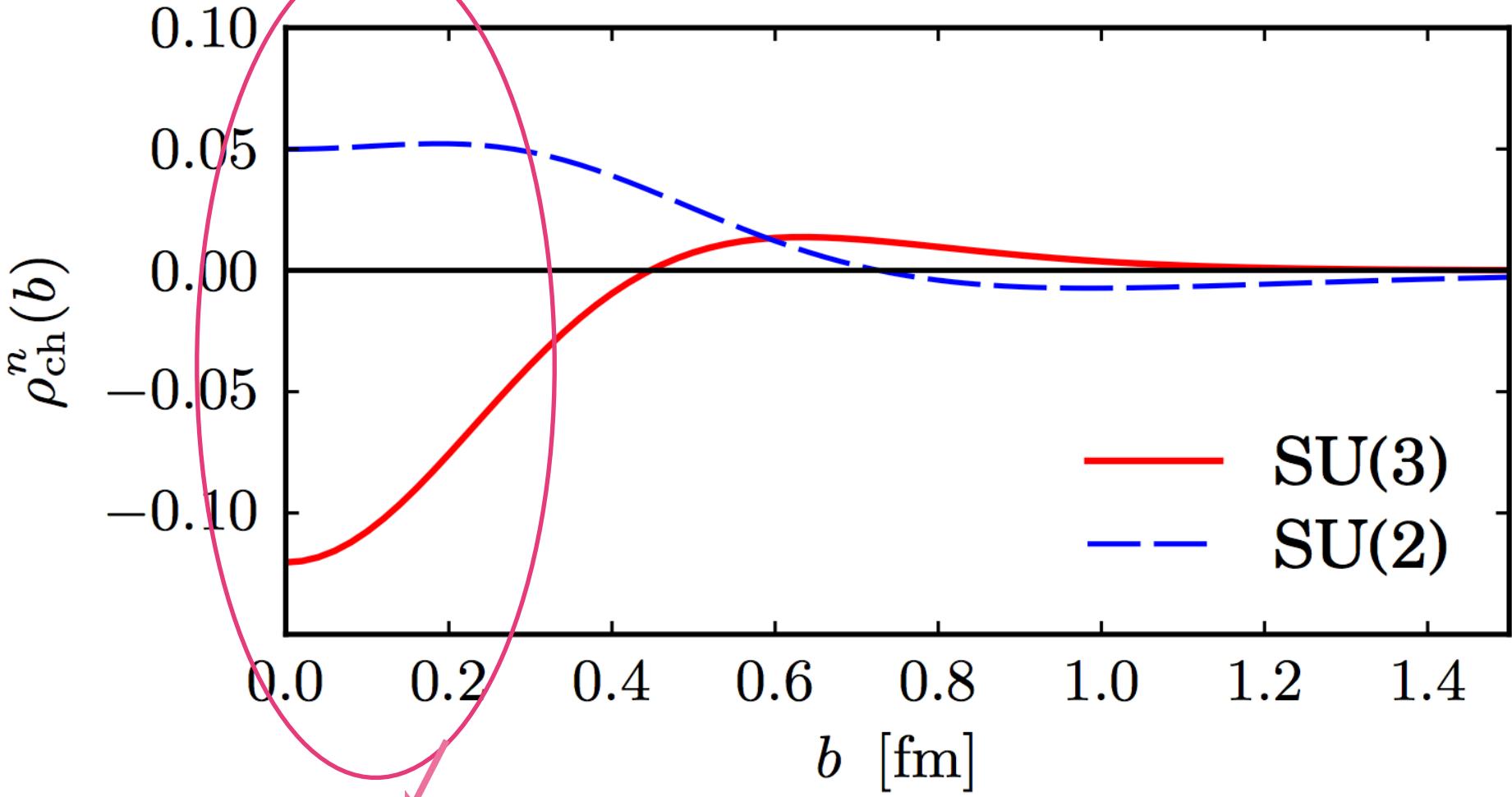
# Results

Transverse charge densities inside an **unpolarized** neutron



# Results

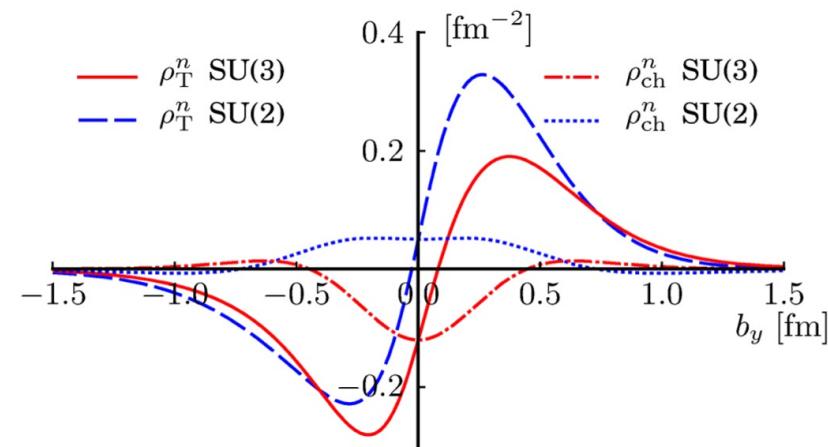
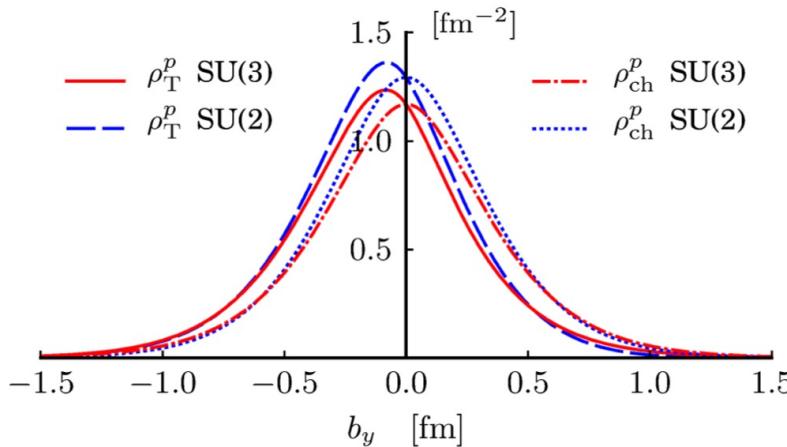
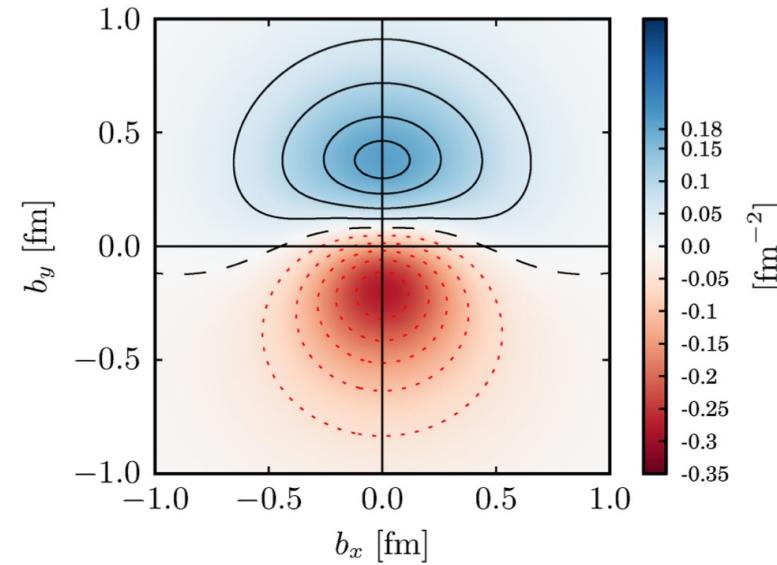
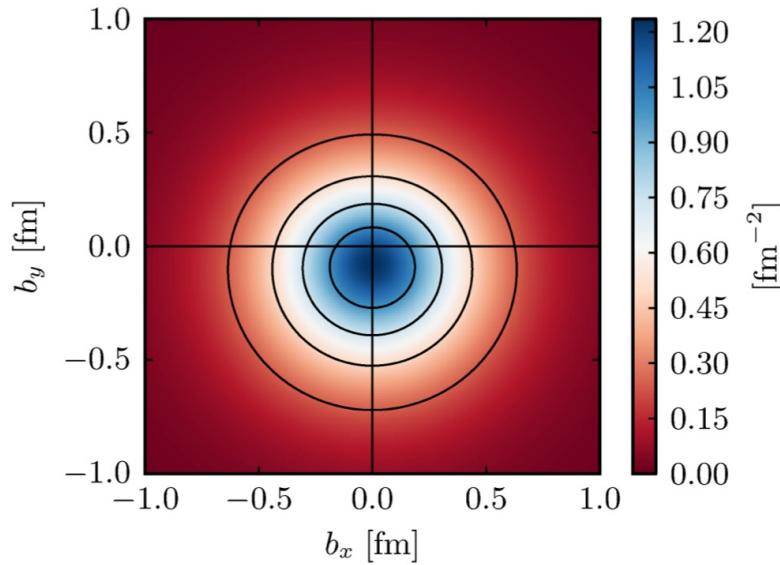
Transverse charge densities inside an **unpolarized** neutron



Surprisingly, negative charge distribution in the center of the neutron!

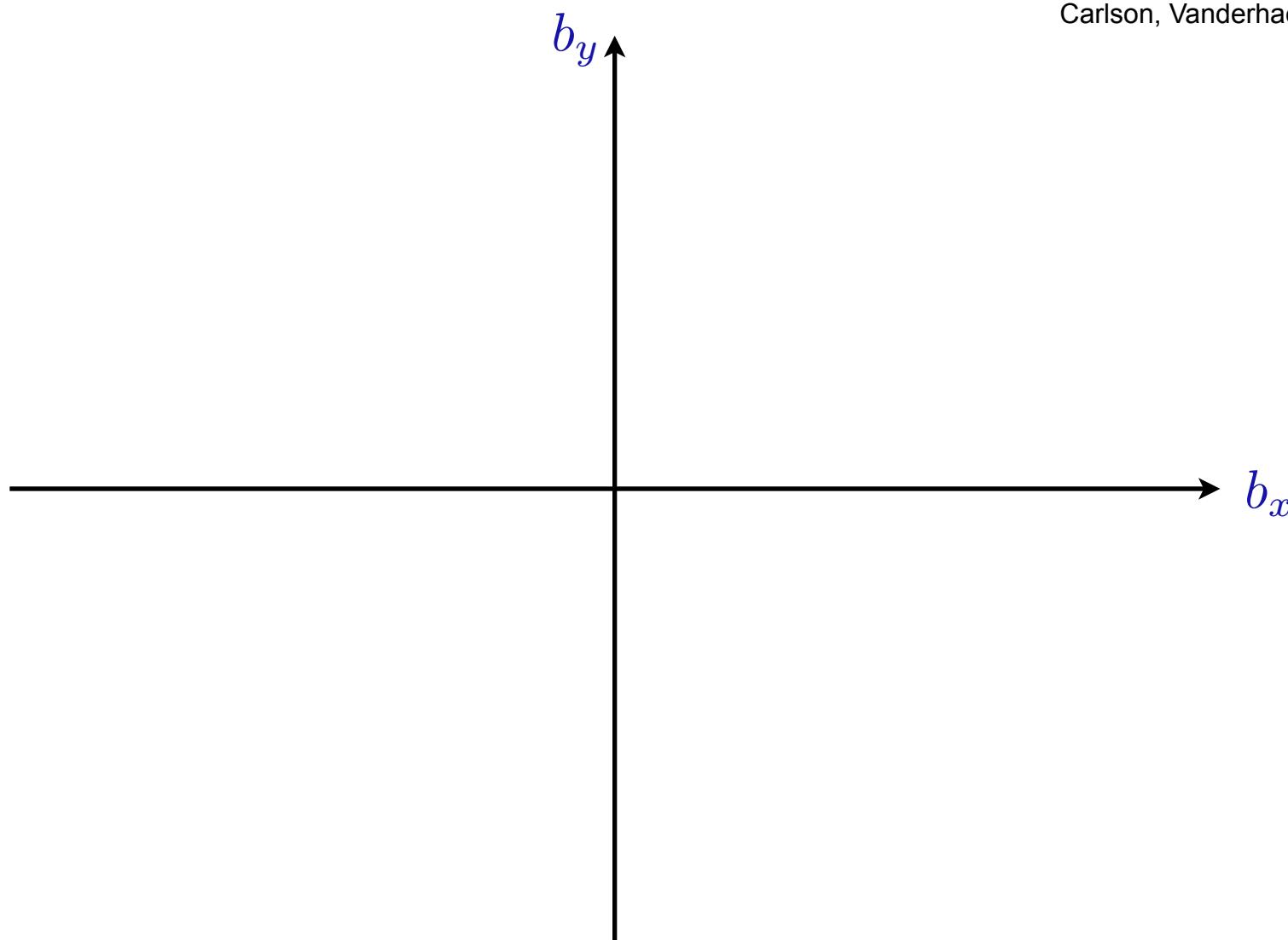
# Results

Transverse charge densities inside an **polarized** nucleon



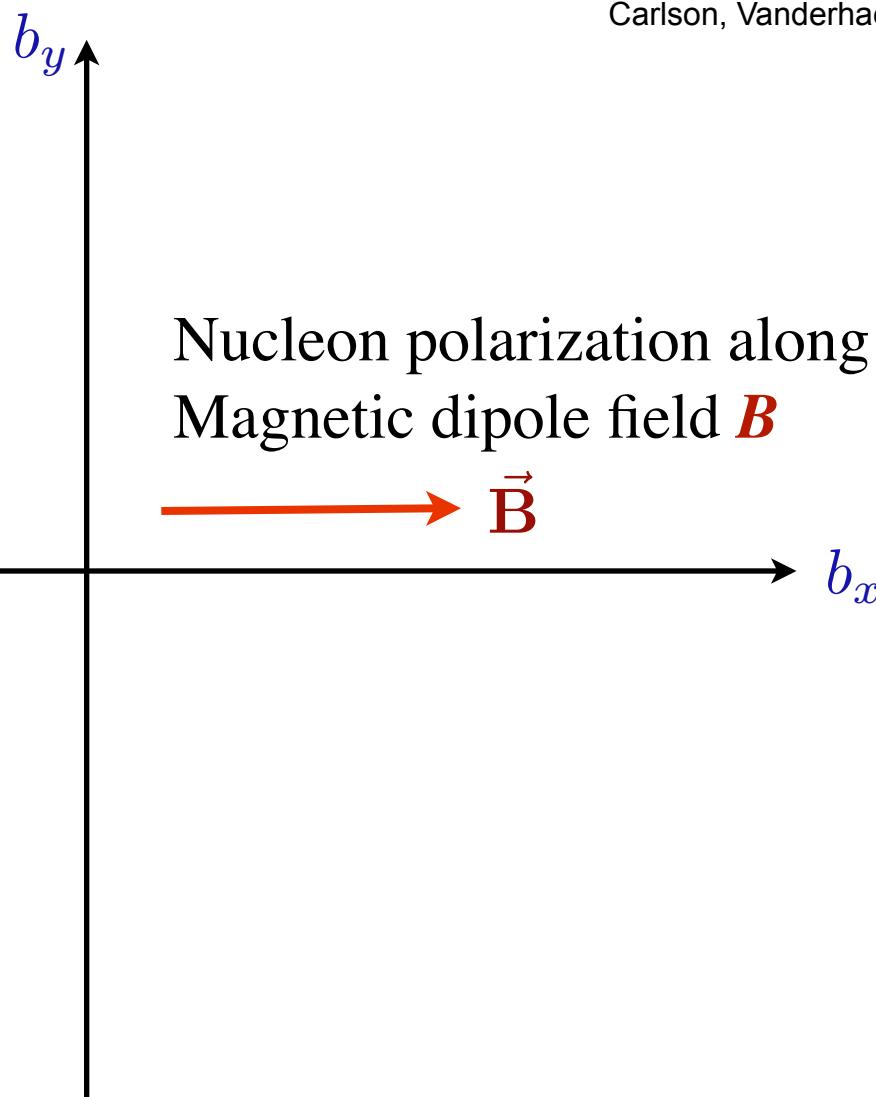
# Discussion

Carlson, Vanderhaeghen, PRL **100**, 032004



# Discussion

Carlson, Vanderhaeghen, PRL **100**, 032004



# Discussion

Carlson, Vanderhaeghen, PRL **100**, 032004



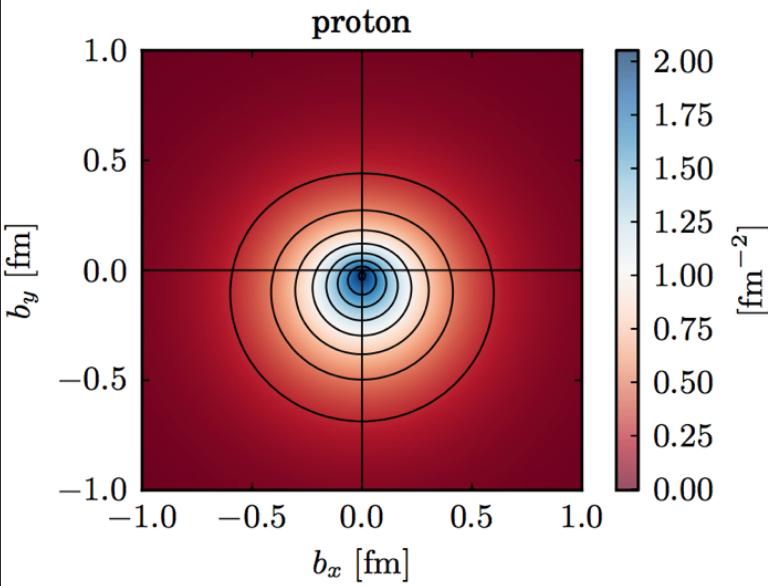
Nucleon polarization along the  $\textcolor{red}{x}$  axis:  
Magnetic dipole field  $\vec{B}$

$$\longrightarrow \vec{B}$$

$$\vec{E}' = -\gamma(\vec{v} \times \vec{B})$$

Induced electric dipole field along the negative  $\textcolor{blue}{y}$  axis: Relativistic effects

# Discussion



Carlson, Vanderhaeghen, PRL **100**, 032004

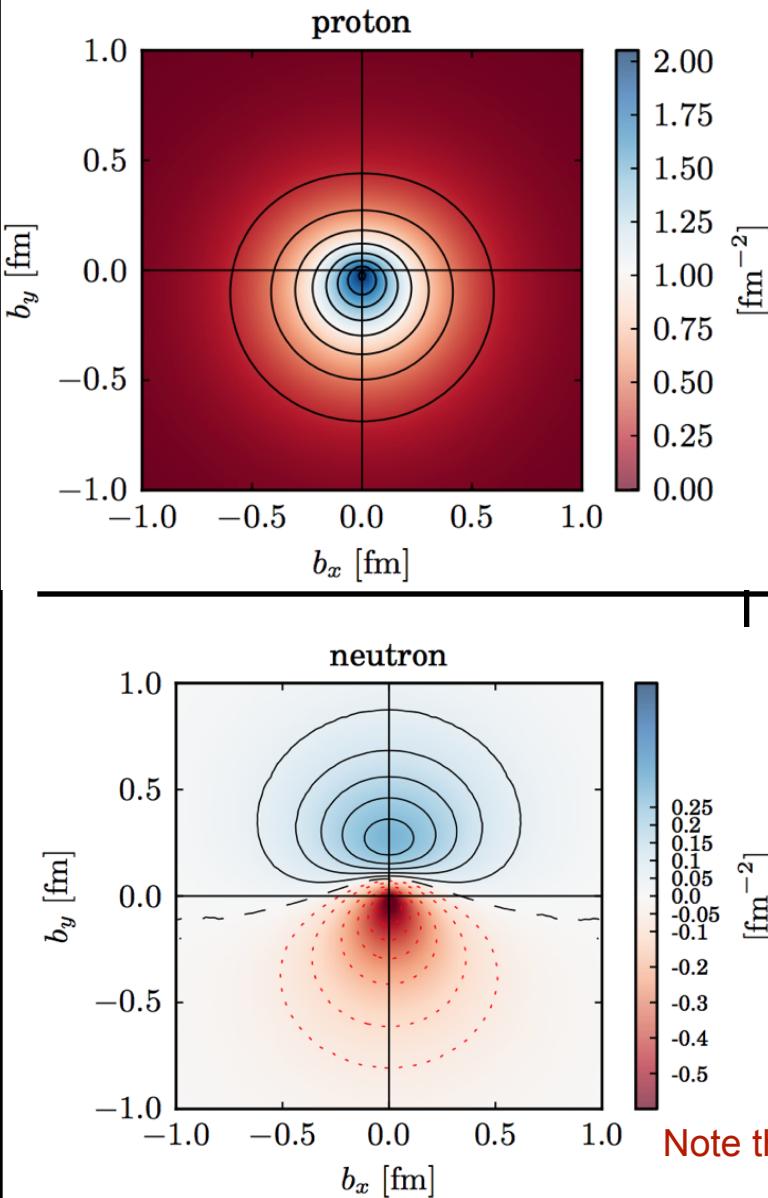
Nucleon polarization along the  $\textcolor{red}{x}$  axis:  
Magnetic dipole field  $\vec{B}$

$$\longrightarrow \vec{B} \quad b_x$$

$$\vec{E}' = -\gamma(\vec{v} \times \vec{B})$$

Induced electric dipole field along the negative  $\textcolor{blue}{y}$  axis: Relativistic effects

# Discussion



Carlson, Vanderhaeghen, PRL **100**, 032004

Nucleon polarization along the  $x$  axis:  
Magnetic dipole field  $\vec{B}$

$$\xrightarrow{\quad \quad \quad} \vec{B} \quad \quad \quad b_x$$

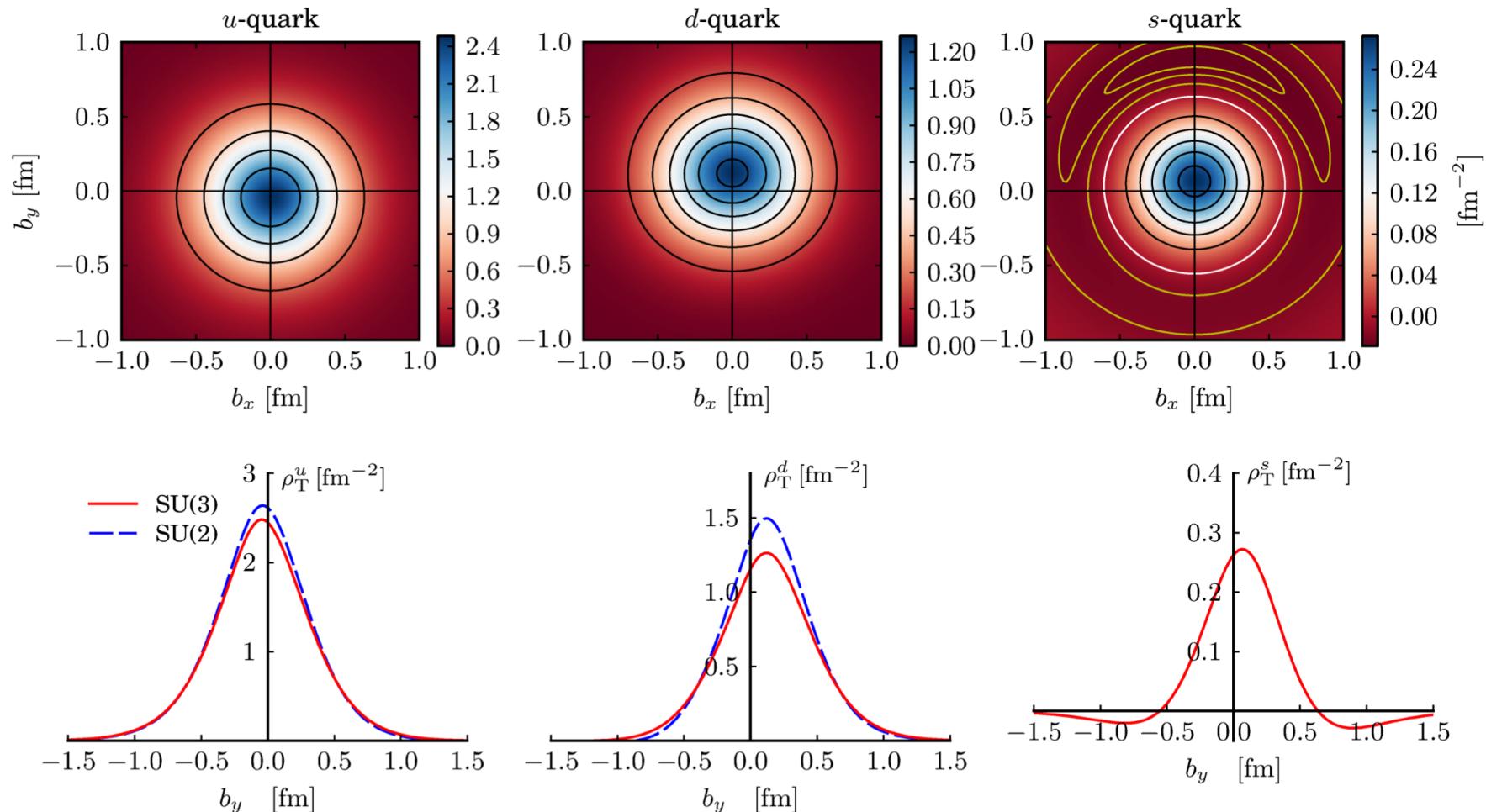
$$\vec{E}' = -\gamma(\vec{v} \times \vec{B})$$

Induced electric dipole field along the  
negative  $y$  axis: Relativistic effects

Note that the neutron anomalous magnetic moment is negative!

# Results

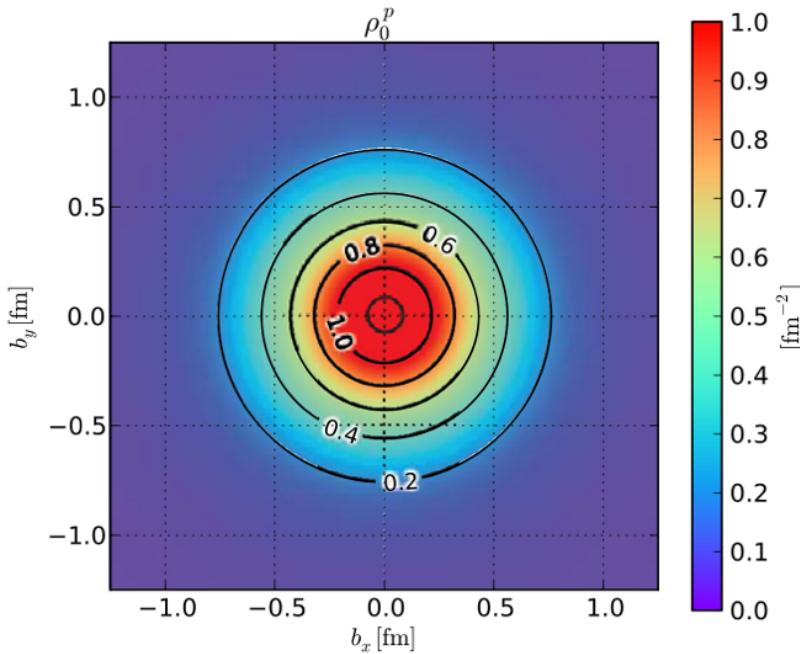
Flavor-decomposed Transverse charge densities inside a **polarized** nucleon



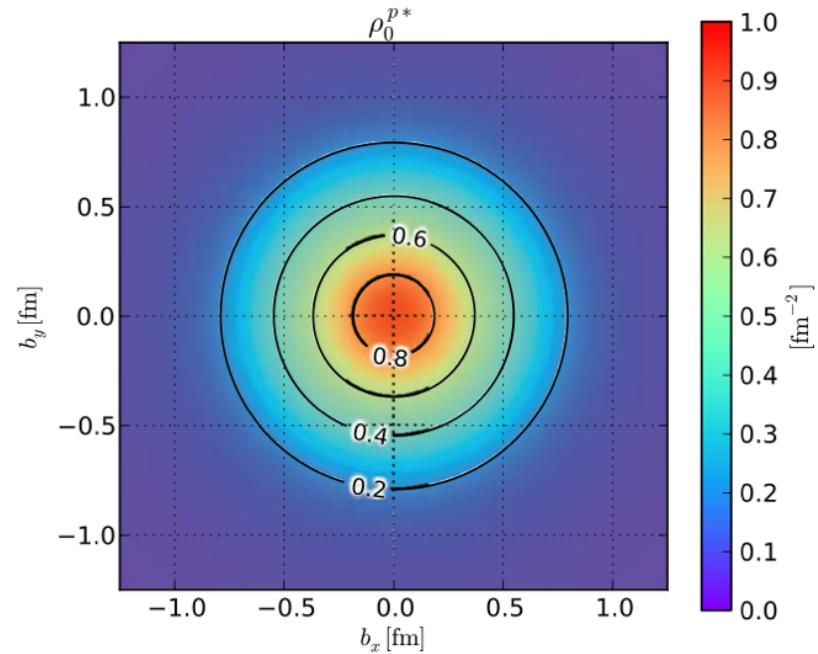
# Results

Transverse charge densities inside an **unpolarized proton in nuclear matter**

Free-space Skyrme Model



Medium-Modified Skyrme Model



The proton bulges out in nuclear matter!

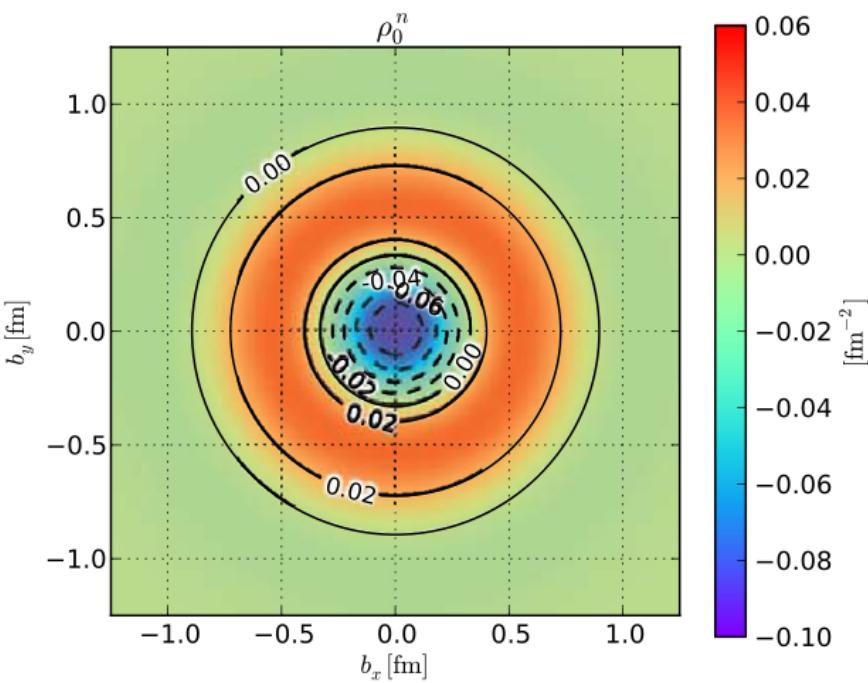
Yakhshiev's Talk on Tuesday for details.

U. Yakhshiev and HChK, PLB, (2013)

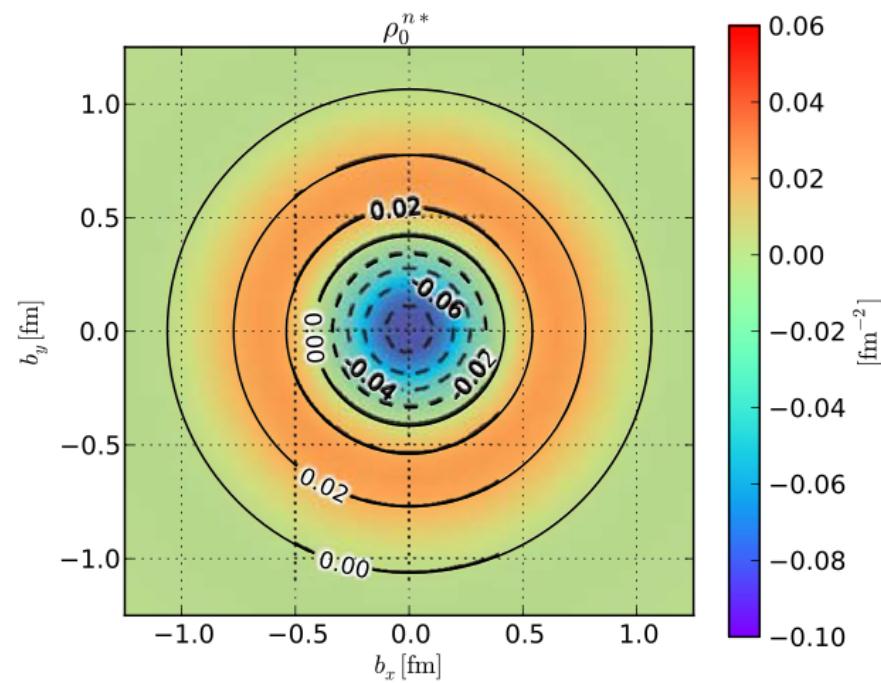
# Results

Transverse charge densities inside an **unpolarized neutron in nuclear matter**

Free-space Skyrme Model



Medium-Modified Skyrme Model



The neutron also bulges out in nuclear matter!

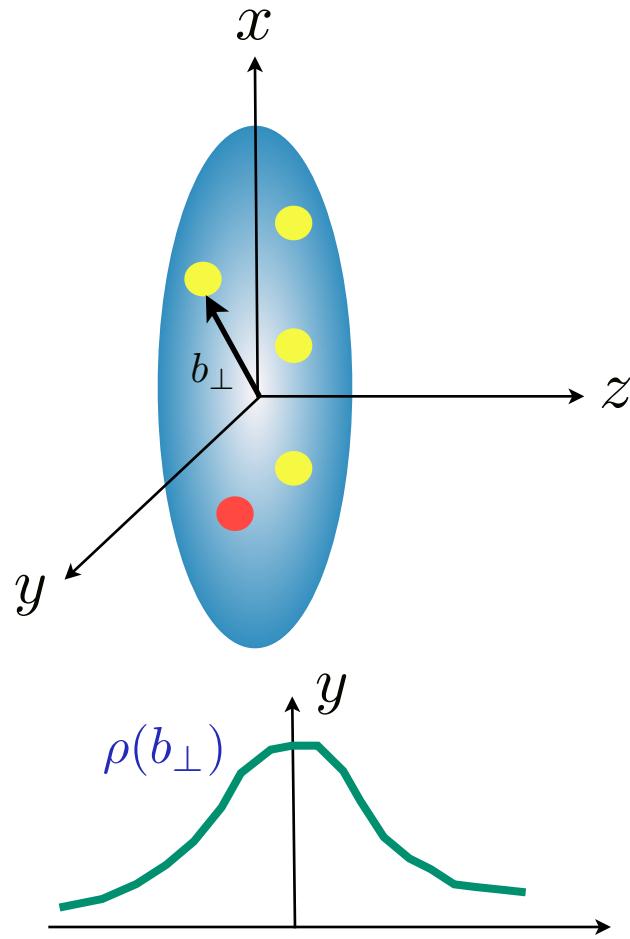
Yakhshiev's Talk on Tuesday for details.

U. Yakhshiev and HChK, PLB, (2013)

# The spin structure of the Nucleon

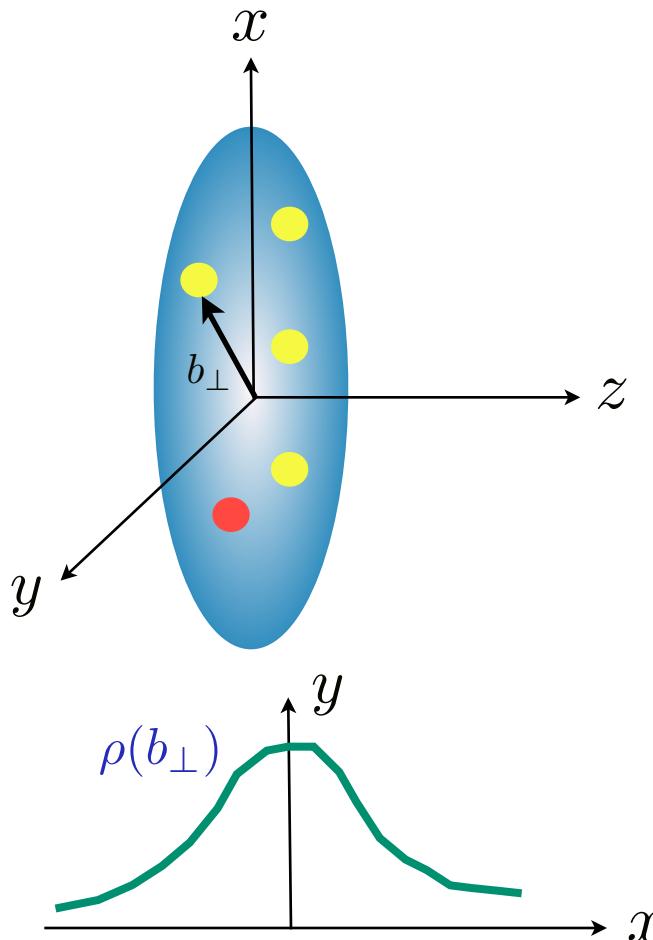
# The spin structure of the Nucleon

Axial & **Tensor** Form factors, Axial-vector charges, **Tensor** charges

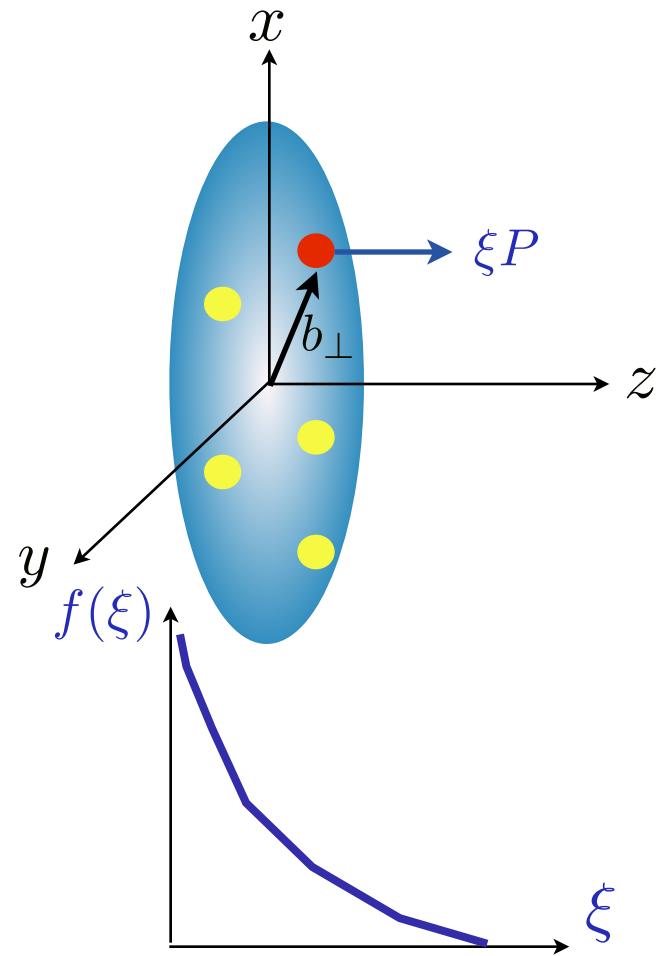


# The spin structure of the Nucleon

Axial & **Tensor** Form factors, Axial-vector charges, **Tensor** charges

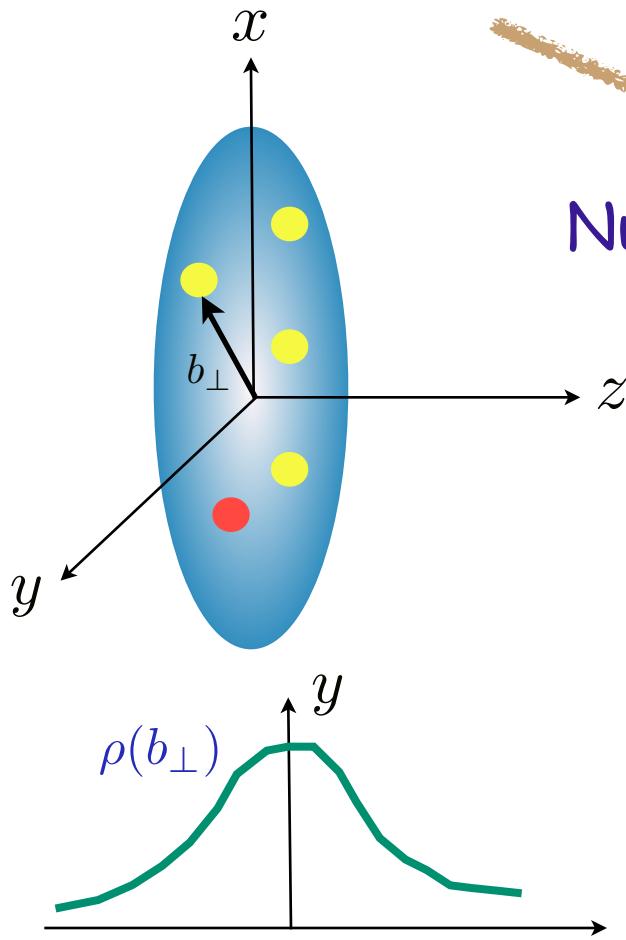


Structure functions



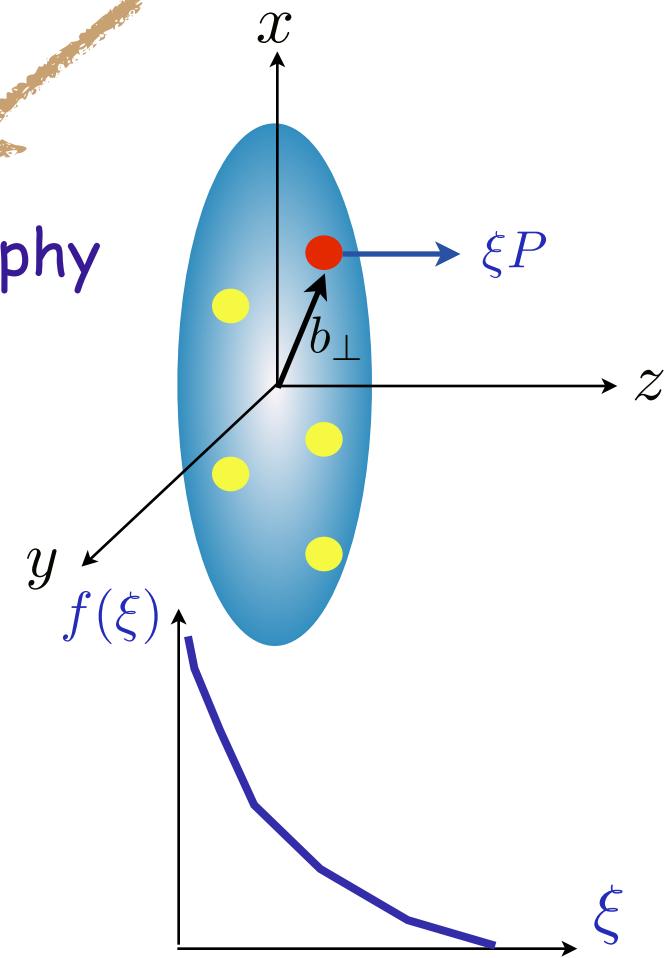
# The spin structure of the Nucleon

Axial & **Tensor** Form factors, Axial-vector charges, **Tensor** charges



Nucleon Tomography  
(GPDs)

Structure functions



Spin Structure

# Tensor form factors

$$\langle N_{s'}(p') | \bar{\psi}(0) i\sigma^{\mu\nu} \lambda^\chi \psi(0) | N_s(p) \rangle = \bar{u}_{s'}(p') \left[ H_T^\chi(Q^2) i\sigma^{\mu\nu} + E_T^\chi(Q^2) \frac{\gamma^\mu q^\nu - q^\mu \gamma^\nu}{2M} + \tilde{H}_T^\chi(Q^2) \frac{(n^\mu q^\nu - q^\mu n^\nu)}{2M^2} \right] u_s(p)$$

$$\int_{-1}^1 dx H_T^\chi(x, \xi, t) = H_T^\chi(q^2),$$

$$\int_{-1}^1 dx E_T^\chi(x, \xi, t) = E_T^\chi(q^2), \quad H_T^0(0) = g_T^0 = \delta u + \delta d + \delta s$$

$$H_T^3(0) = g_T^3 = \delta u - \delta d$$

$$\int_{-1}^1 dx \tilde{H}_T^\chi(x, \xi, t) = \tilde{H}_T^\chi(q^2), \quad H_T^8(0) = g_T^8 = \frac{1}{\sqrt{3}}(\delta u + \delta d - 2\delta s)$$

# Tensor form factors

$$\langle N_{s'}(p') | \bar{\psi}(0) i\sigma^{\mu\nu} \lambda^\chi \psi(0) | N_s(p) \rangle = \bar{u}_{s'}(p') \left[ H_T^\chi(Q^2) i\sigma^{\mu\nu} + E_T^\chi(Q^2) \frac{\gamma^\mu q^\nu - q^\mu \gamma^\nu}{2M} + \tilde{H}_T^\chi(Q^2) \frac{(n^\mu q^\nu - q^\mu n^\nu)}{2M^2} \right] u_s(p)$$

$$\boxed{\int_{-1}^1 dx H_T^\chi(x, \xi, t) = H_T^\chi(q^2)},$$

$$\int_{-1}^1 dx E_T^\chi(x, \xi, t) = E_T^\chi(q^2),$$

$$\int_{-1}^1 dx \tilde{H}_T^\chi(x, \xi, t) = \tilde{H}_T^\chi(q^2),$$

$$H_T^0(0) = g_T^0 = \delta u + \delta d + \delta s$$

$$H_T^3(0) = g_T^3 = \delta u - \delta d$$

$$H_T^8(0) = g_T^8 = \frac{1}{\sqrt{3}}(\delta u + \delta d - 2\delta s)$$

# Tensor form factors

$$\langle N_{s'}(p') | \bar{\psi}(0) i\sigma^{\mu\nu} \lambda^\chi \psi(0) | N_s(p) \rangle = \bar{u}_{s'}(p') \left[ H_T^\chi(Q^2) i\sigma^{\mu\nu} + E_T^\chi(Q^2) \frac{\gamma^\mu q^\nu - q^\mu \gamma^\nu}{2M} + \tilde{H}_T^\chi(Q^2) \frac{(n^\mu q^\nu - q^\mu n^\nu)}{2M^2} \right] u_s(p)$$

$$\int_{-1}^1 dx H_T^\chi(x, \xi, t) = H_T^\chi(q^2),$$

$$\int_{-1}^1 dx E_T^\chi(x, \xi, t) = E_T^\chi(q^2),$$

$$\int_{-1}^1 dx \tilde{H}_T^\chi(x, \xi, t) = \tilde{H}_T^\chi(q^2),$$

$$H_T^0(0) = g_T^0 = \delta u + \delta d + \delta s$$

$$H_T^3(0) = g_T^3 = \delta u - \delta d$$

$$H_T^8(0) = g_T^8 = \frac{1}{\sqrt{3}}(\delta u + \delta d - 2\delta s)$$

$$H_T^{*\chi}(Q^2) = \frac{2M}{\mathbf{q}^2} \int \frac{d\Omega}{4\pi} \langle N_{\frac{1}{2}}(p') | \psi^\dagger \gamma^k q^k \lambda^\chi \psi | N_{\frac{1}{2}}(p) \rangle$$

$$\kappa_T^\chi = -H_T^\chi(0) - H_T^{*\chi}(0)$$

Together with the anomalous magnetic moment, this will allow us to describe the **transverse spin quark densities** inside the nucleon.

# Tensor form factors

Tensor charges and anomalous tensor magnetic moments are **scale-dependent**.

$$\delta q(\mu^2) = \left( \frac{\alpha_S(\mu^2)}{\alpha_S(\mu_i^2)} \right)^{4/27} \left[ 1 - \frac{337}{486\pi} (\alpha_S(\mu_i^2) - \alpha_S(\mu^2)) \right] \delta q(\mu_i^2),$$

$$\alpha_S^{NLO}(\mu^2) = \frac{4\pi}{9 \ln(\mu^2/\Lambda_{\text{QCD}}^2)} \left[ 1 - \frac{64}{81} \frac{\ln \ln(\mu^2/\Lambda_{\text{QCD}}^2)}{\ln(\mu^2/\Lambda_{\text{QCD}}^2)} \right]$$

$$\Lambda_{\text{QCD}} = 0.248 \text{ GeV}$$

M. Gluck, E. Reya, and A. Vogt, Z.Phys. C 67, 433(1995).

# Results

	$g_T^0$	$g_T^3$	$g_T^8$	$g_A^0$	$g_A^3$	$g_A^8$	$\Delta u$	$\delta u$	$\Delta d$	$\delta d$	$\Delta s$	$\delta s$
$\chi$ QSM SU(3)	0.76	1.40	0.45	0.45	1.18	0.35	0.84	1.08	-0.34	-0.32	-0.05	-0.01
$\chi$ QSM SU(2)	0.75	1.44	--	0.45	1.21	--	0.82	1.08	-0.37	-0.32	--	--
NRQM	1	5/3	--	1	5/3	--	$\frac{4}{3}$	$\frac{4}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	--	--

# Results

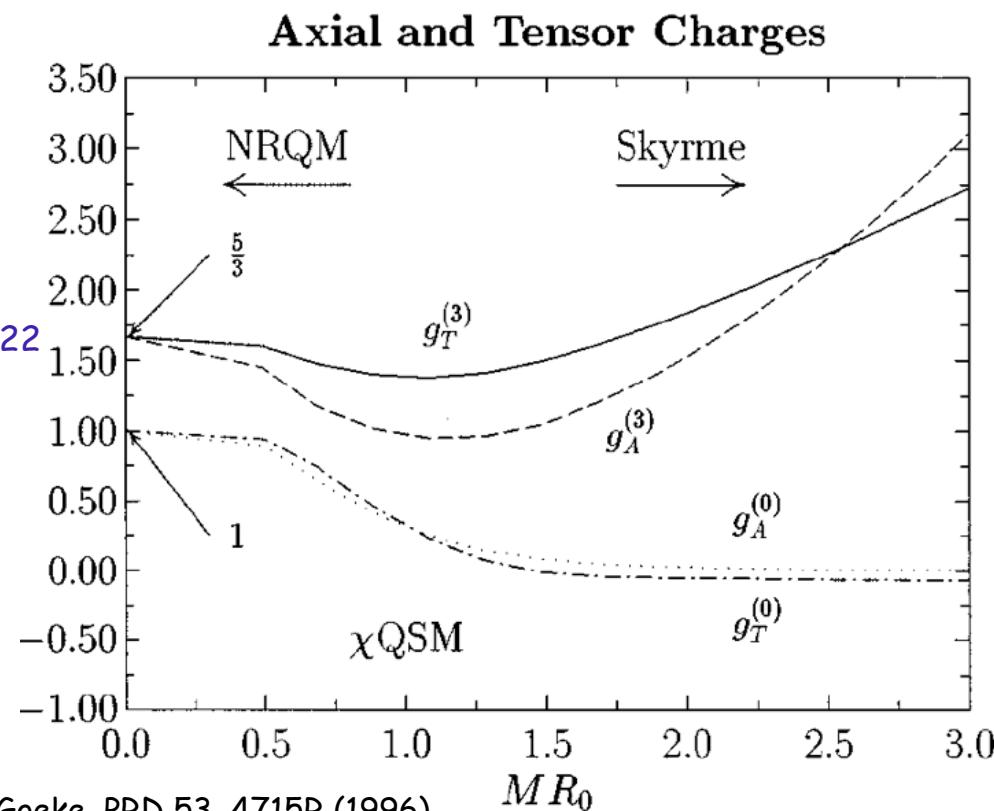
	$g_T^0$	$g_T^3$	$g_T^8$	$g_A^0$	$g_A^3$	$g_A^8$	$\Delta u$	$\delta u$	$\Delta d$	$\delta d$	$\Delta s$	$\delta s$
$\chi$ QSM SU(3)	0.76	1.40	0.45	0.45	1.18	0.35	0.84	1.08	-0.34	-0.32	-0.05	-0.01
$\chi$ QSM SU(2)	0.75	1.44	--	0.45	1.21	--	0.82	1.08	-0.37	-0.32	--	--
NRQM	1	5/3	--	1	5/3	--	$\frac{4}{3}$	$\frac{4}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	--	--

# Results

	$g_T^0$	$g_T^3$	$g_T^8$	$g_A^0$	$g_A^3$	$g_A^8$	$\Delta u$	$\delta u$	$\Delta d$	$\delta d$	$\Delta s$	$\delta s$
$\chi$ QSM SU(3)	0.76	1.40	0.45	0.45	1.18	0.35	0.84	1.08	-0.34	-0.32	-0.05	-0.01
$\chi$ QSM SU(2)	0.75	1.44	--	0.45	1.21	--	0.82	1.08	-0.37	-0.32	--	--
NRQM	1	5/3	--	1	5/3	--	$\frac{4}{3}$	$\frac{4}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	--	--

$$\boxed{\begin{array}{ll} g_A^3 \sim (MR_0)^2 & g_T^3 \sim MR_0 \\ g_A^0 \sim \frac{1}{(MR_0)^4} & g_T^0 \sim \frac{1}{MR_0} \end{array}}$$

T. Ledwig, A. Silva, HChK, Phys. Rev. D 82 (2010) 034022



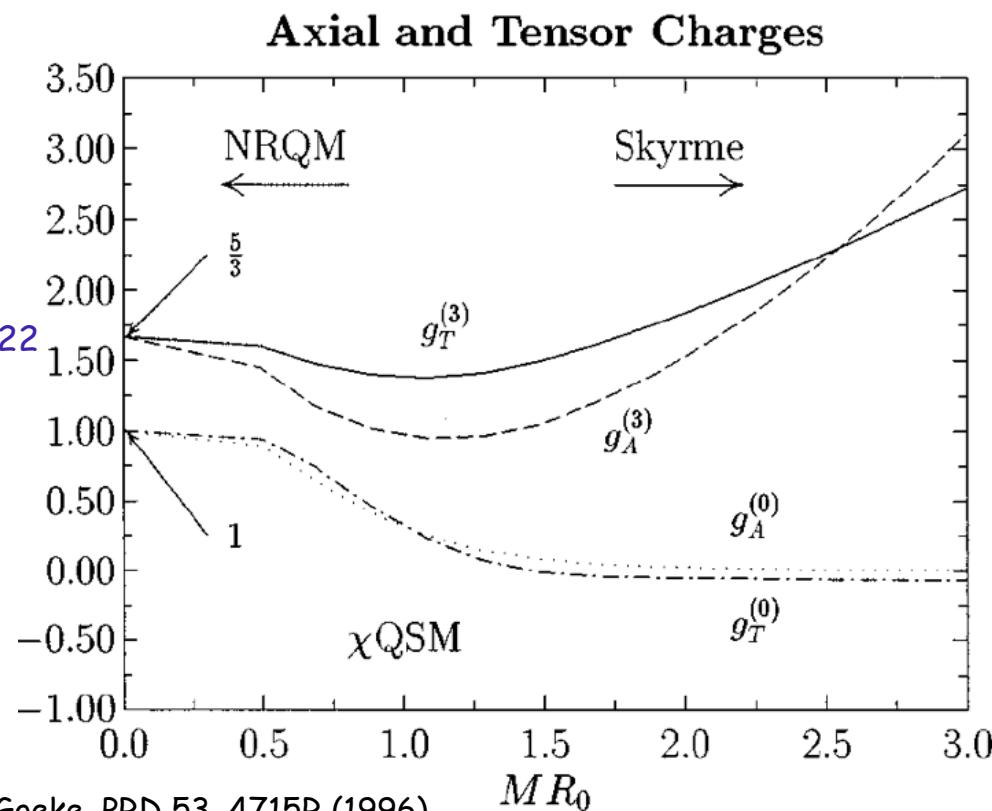
# Results

	$g_T^0$	$g_T^3$	$g_T^8$	$g_A^0$	$g_A^3$	$g_A^8$	$\Delta u$	$\delta u$	$\Delta d$	$\delta d$	$\Delta s$	$\delta s$
$\chi$ QSM SU(3)	0.76	1.40	0.45	0.45	1.18	0.35	0.84	1.08	-0.34	-0.32	-0.05	-0.01
$\chi$ QSM SU(2)	0.75	1.44	--	0.45	1.21	--	0.82	1.08	-0.37	-0.32	--	--
NRQM	1	5/3	--	1	5/3	--	$\frac{4}{3}$	$\frac{4}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	--	--

$$\boxed{\begin{array}{ll} g_A^3 \sim (MR_0)^2 & g_T^3 \sim MR_0 \\ g_A^0 \sim \frac{1}{(MR_0)^4} & g_T^0 \sim \frac{1}{MR_0} \end{array}}$$

T. Ledwig, A. Silva, HChK, Phys. Rev. D 82 (2010) 034022

$$g_T^\chi > g_A^\chi$$



# Results

Proton	This work	SU(2)	Lattice	SIDIS	NR
$ \delta d/\delta u $	0.30	0.36	0.25	$0.42^{+0.0003}_{-0.20}$	0.25

# Results

Proton	This work	SU(2)	Lattice	SIDIS	NR
$ \delta d/\delta u $	0.30	0.36	0.25	$0.42^{+0.0003}_{-0.20}$	0.25

SIDIS [16] ( $0.80 \text{ GeV}^2$ ):  $\delta u = 0.54^{+0.09}_{-0.22}, \quad \delta d = -0.231^{+0.09}_{-0.16},$   
SIDIS [16] ( $0.36 \text{ GeV}^2$ ):  $\delta u = 0.60^{+0.10}_{-0.24}, \quad \delta d = -0.26^{+0.1}_{-0.18},$   
Lattice [21] ( $4.00 \text{ GeV}^2$ ):  $\delta u = 0.86 \pm 0.13, \quad \delta d = -0.21 \pm 0.005,$   
Lattice [21] ( $0.36 \text{ GeV}^2$ ):  $\delta u = 1.05 \pm 0.16, \quad \delta d = -0.26 \pm 0.01,$   
 $\chi\text{QSM}$  ( $0.36 \text{ GeV}^2$ ):  $\delta u = 1.08, \quad \delta d = -0.32,$

[16] M. Anselmino et al. Nucl. Phys. B, Proc. Suppl. 191, 98 (2009)

[21] M. Goeckeler et al., PLB 627, 113 (2005)

# Results

$$\mu^2 = 0.36 \text{ GeV}^2$$

	Present work SU(3)	Present work SU(2)	Lattice
$\kappa_T^u$	3.56	3.72	3.00 (3.70)
$\kappa_T^d$	1.83	1.83	1.90 (2.35)
$\kappa_T^s$	$0.2 \sim -0.2$		
$\kappa_T^u / \kappa_T^d$	1.95	2.02	1.58

# Results

$$\mu^2 = 0.36 \text{ GeV}^2$$

	Present work SU(3)	Present work SU(2)	Lattice
$\kappa_T^u$	3.56	3.72	3.00 (3.70)
$\kappa_T^d$	1.83	1.83	1.90 (2.35)
$\kappa_T^s$	$0.2 \sim -0.2$		
$\kappa_T^u / \kappa_T^d$	1.95	2.02	1.58

The present results are comparable with the lattice data!

M. Goeckeler et al. [QCDSF Coll. and UKQCD Coll.]  
PRL 98, 222001 (2007)

# Transverse spin density

$$\begin{aligned}\rho(\mathbf{b}, \mathbf{S}, \mathbf{s}) = & \frac{1}{2} \left[ H(b^2) - S^i \epsilon^{ij} b^j \frac{1}{M_N} \frac{\partial E(b^2)}{\partial b^2} - s^i \epsilon^{ij} b^j \frac{1}{M_N} \frac{\partial \kappa_T(b^2)}{\partial b^2} \right. \\ & + s^i S^i \left\{ H_T(b^2) - \frac{1}{4M_N^2} \nabla^2 \tilde{H}_T(b^2) \right\} \\ & \left. + s^i (2b^i b^j - b^2 \delta^{ij}) S^j \frac{1}{M_N^2} \left( \frac{\partial}{\partial b^2} \right)^2 \tilde{H}_T(b^2) \right],\end{aligned}$$

# Transverse spin density

$$\rho(\mathbf{b}, \mathbf{S}, \mathbf{s}) = \frac{1}{2} \left[ H(b^2) - S^i \epsilon^{ij} b^j \frac{1}{M_N} \frac{\partial E(b^2)}{\partial b^2} - s^i \epsilon^{ij} b^j \frac{1}{M_N} \frac{\partial \kappa_T(b^2)}{\partial b^2} \right]$$

$$[\mathbf{S}, \mathbf{s}] = [(1, 0), (0, 0)], \quad [\mathbf{S}, \mathbf{s}] = [(0, 0), (1, 0)]$$

# Transverse spin density

$$\rho(\mathbf{b}, \mathbf{S}, \mathbf{s}) = \frac{1}{2} \left[ H(b^2) - S^i \epsilon^{ij} b^j \frac{1}{M_N} \frac{\partial E(b^2)}{\partial b^2} - s^i \epsilon^{ij} b^j \frac{1}{M_N} \frac{\partial \kappa_T(b^2)}{\partial b^2} \right]$$

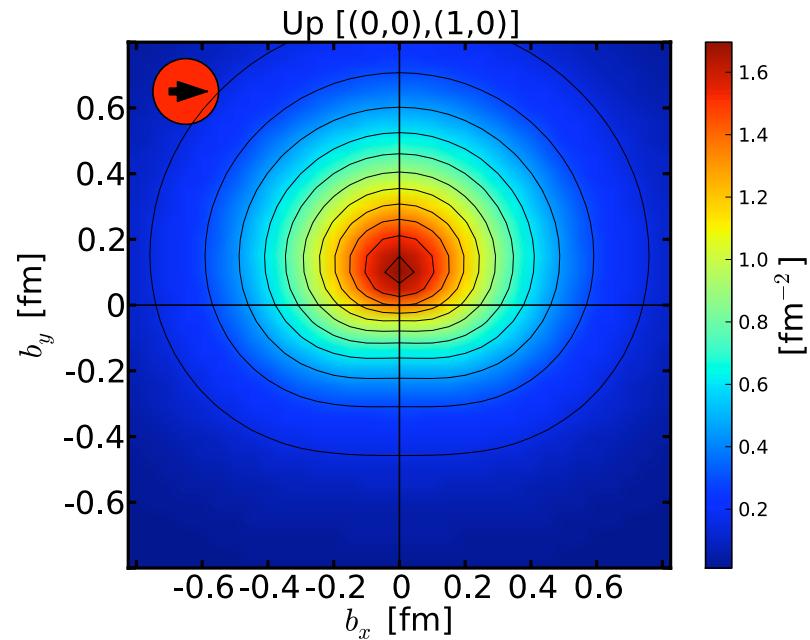
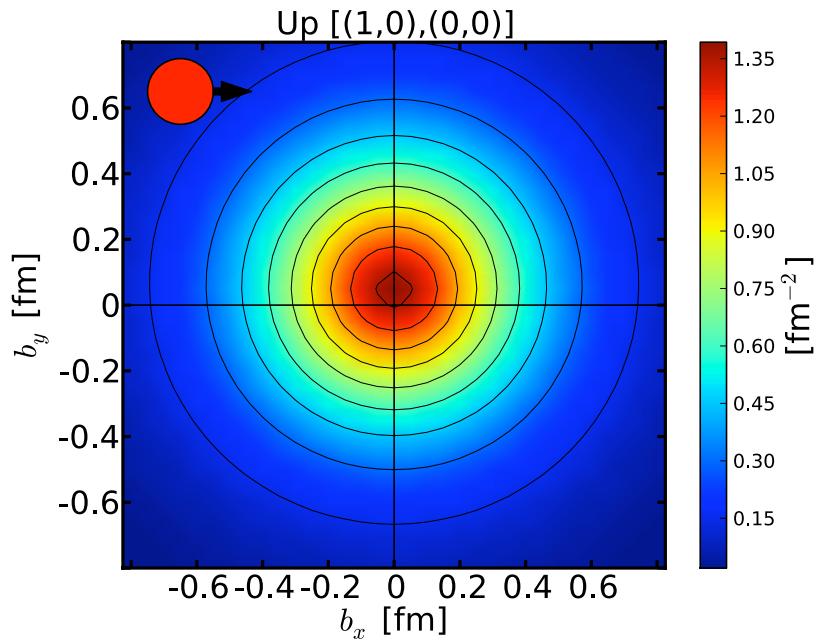
$$[\mathbf{S}, \mathbf{s}] = [(1, 0), (0, 0)], \quad [\mathbf{S}, \mathbf{s}] = [(0, 0), (1, 0)]$$

$$\mathcal{F}^\chi(b^2) = \int_0^\infty \frac{dQ}{2\pi} Q J_0(bQ) F^\chi(Q^2)$$

$$H(b^2) = F_1(b^2), \quad E(b^2) = F_2(b^2)$$

# Results

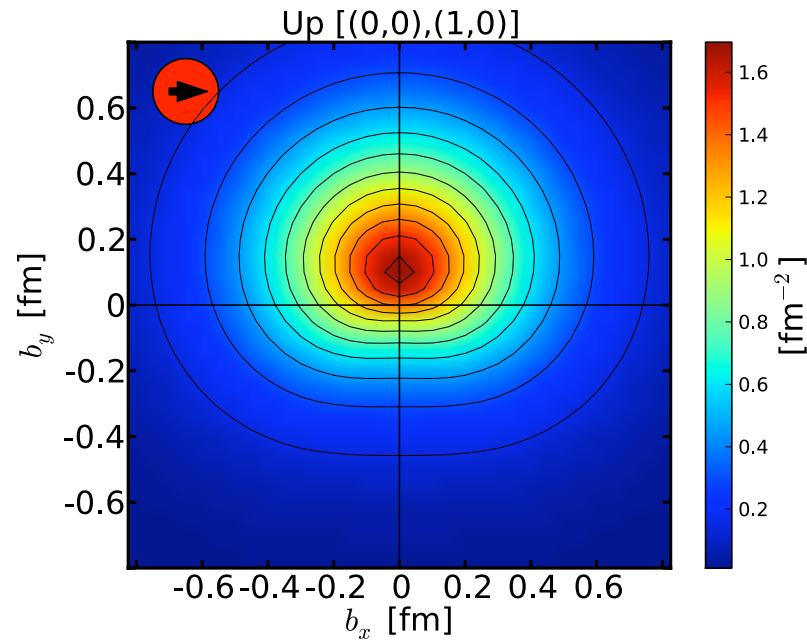
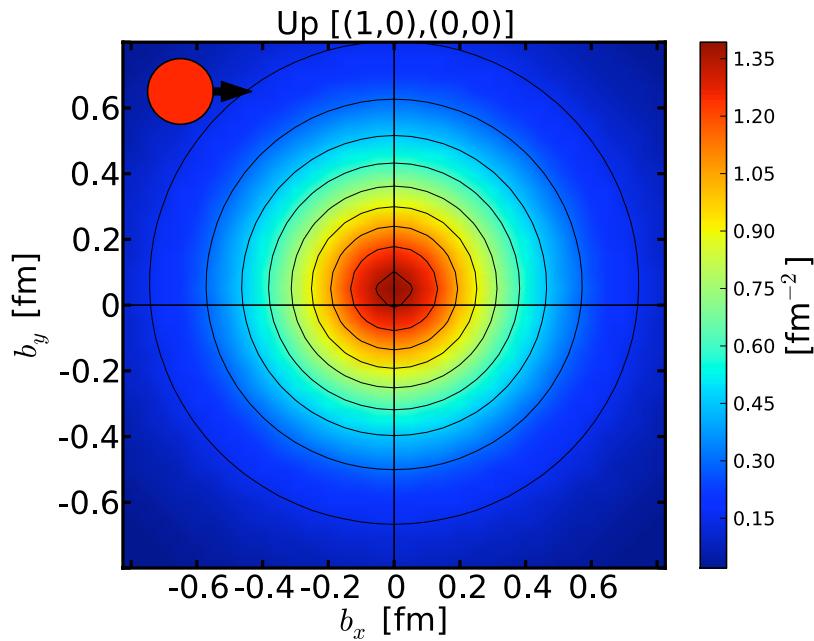
## Up quark transverse spin density inside a nucleon



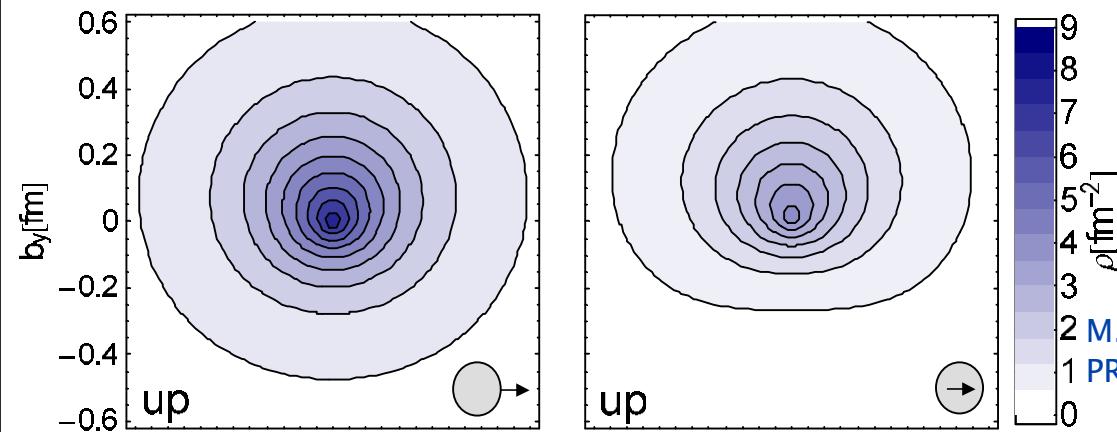
T. Ledwig & H-Ch.K, PRD 85, 034041 (2012)

# Results

## Up quark transverse spin density inside a nucleon



T. Ledwig & H-Ch.K, PRD 85, 034041 (2012)

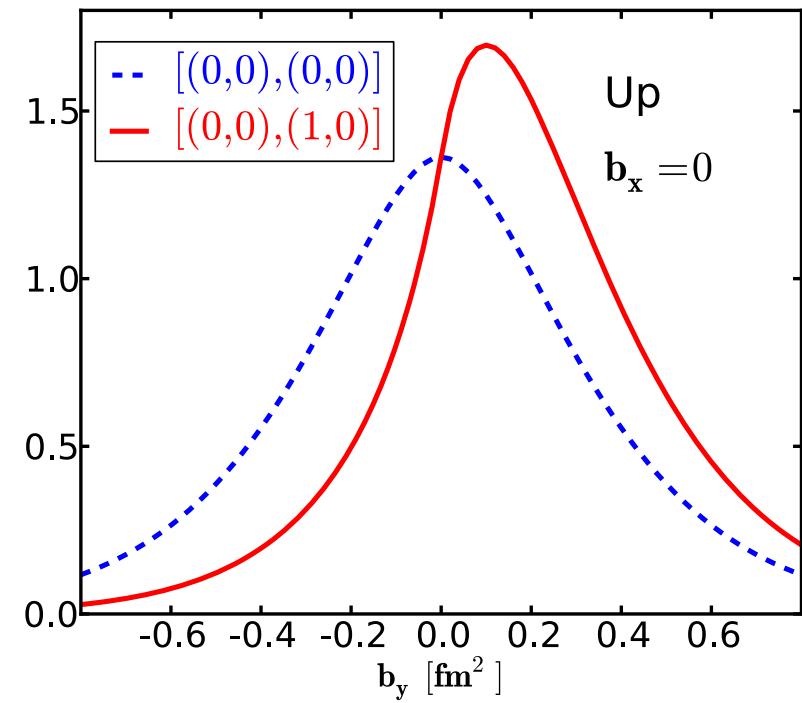
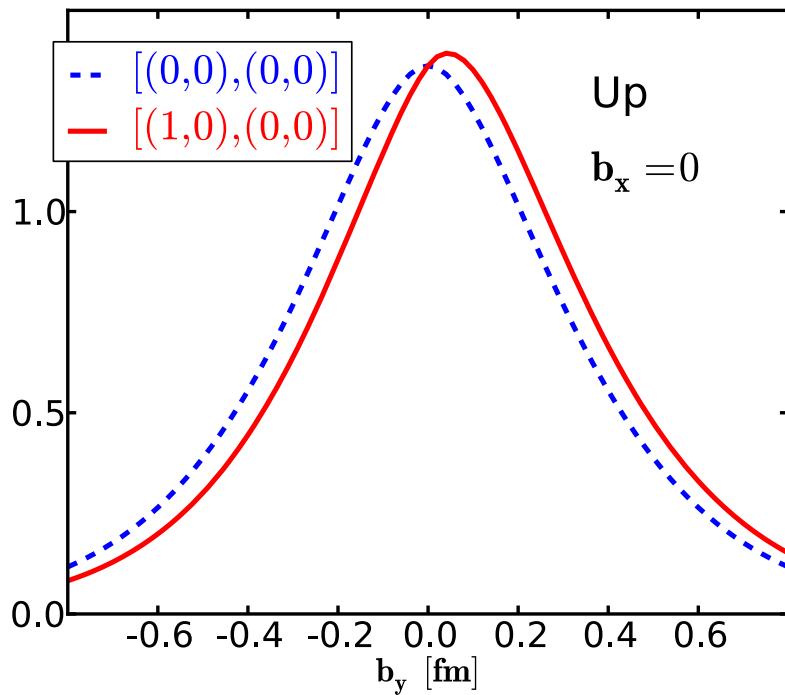


Lattice results

M. Goeckeler et al. [QCDSF Coll. and UKQCD Coll.]  
PRL 98, 222001 (2007)

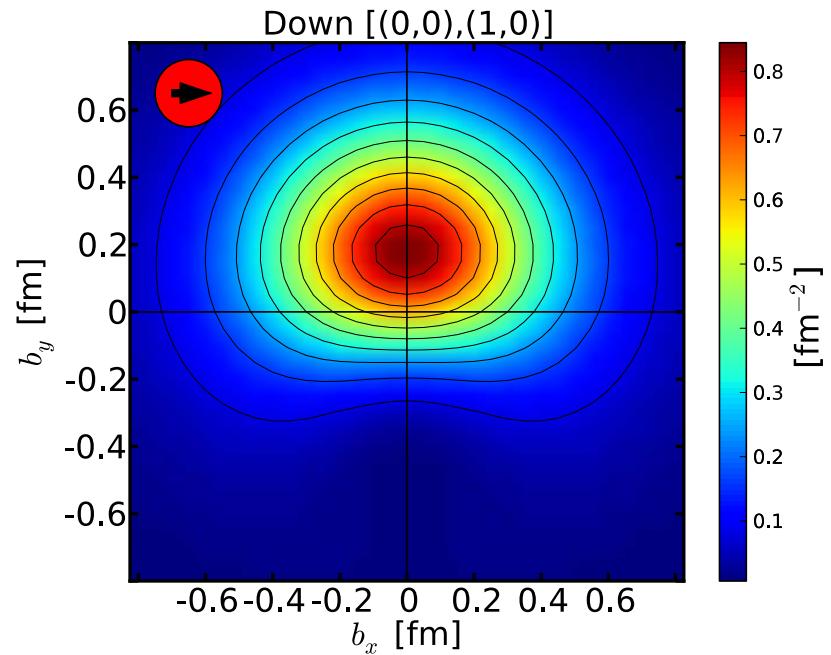
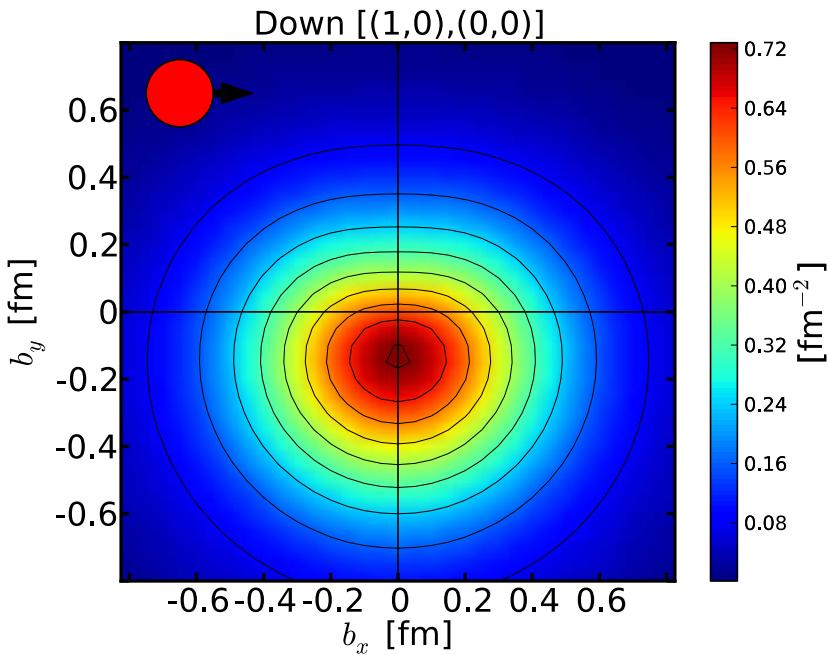
# Results

## Up quark transverse spin density inside a nucleon



# Results

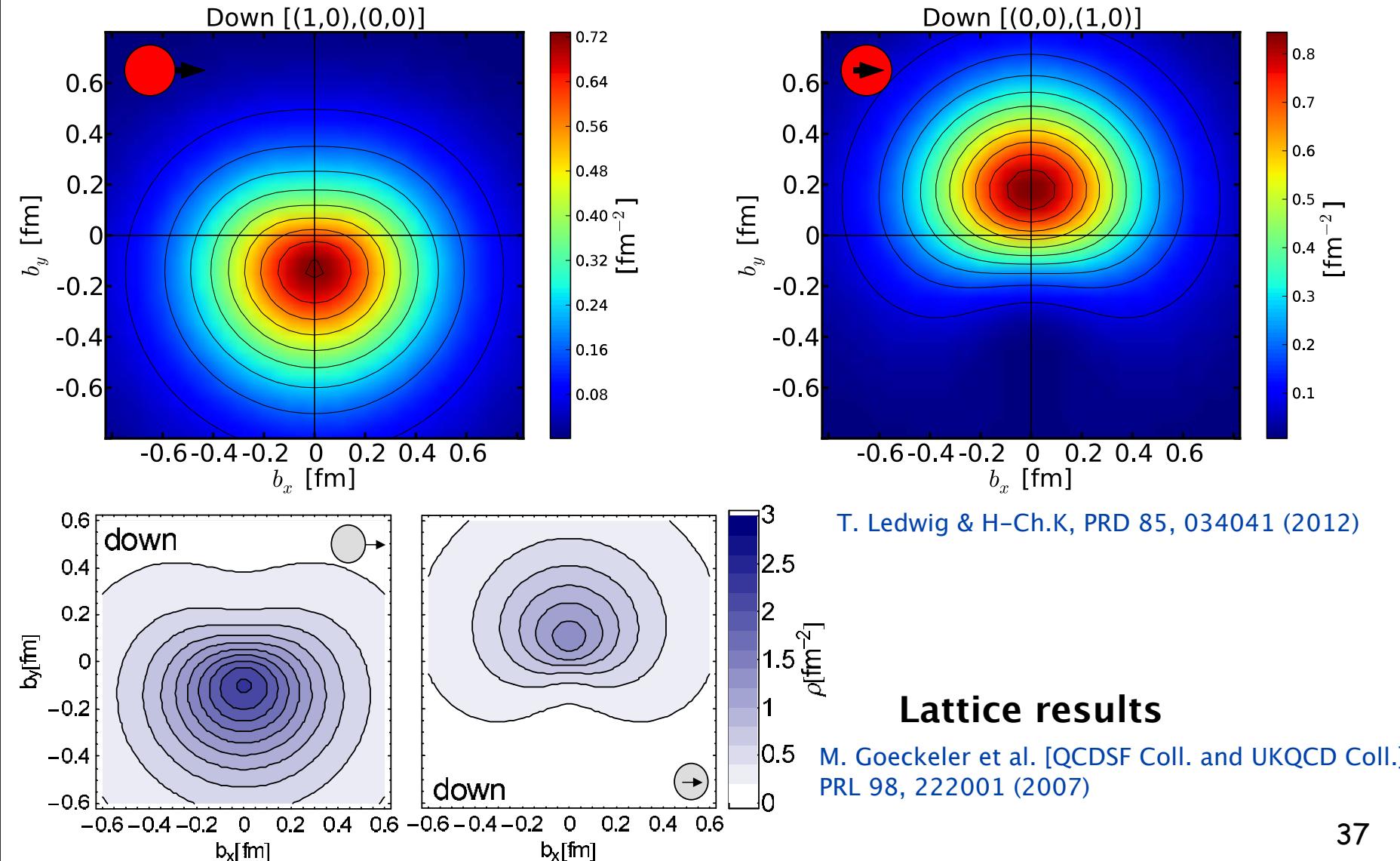
Down quark transverse spin density inside a nucleon



T. Ledwig & H-Ch.K, PRD 85, 034041 (2012)

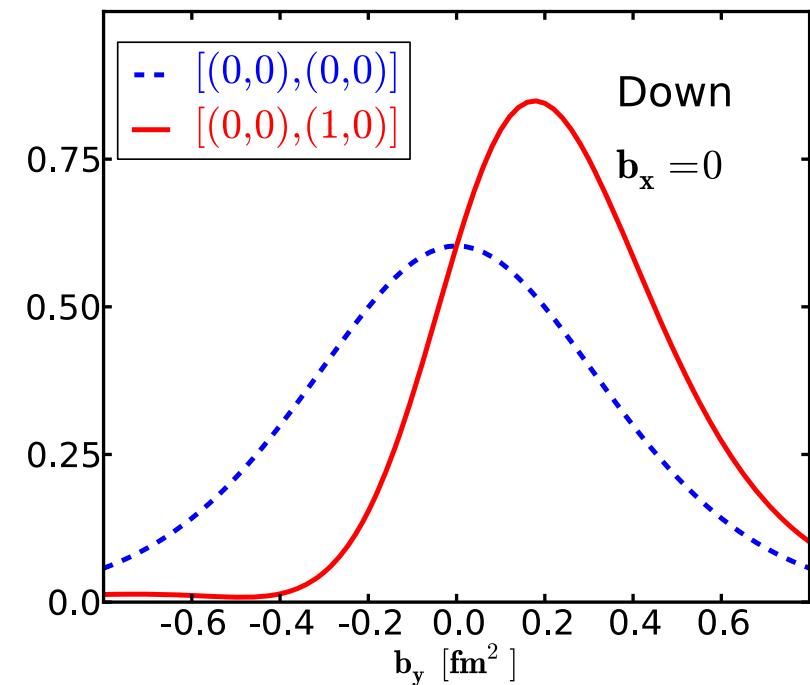
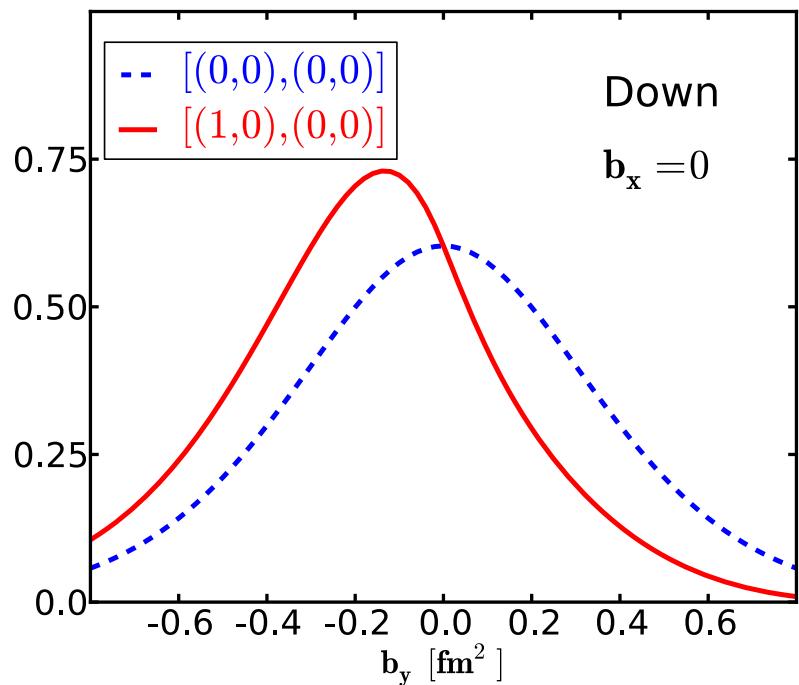
# Results

# Down quark transverse spin density inside a nucleon



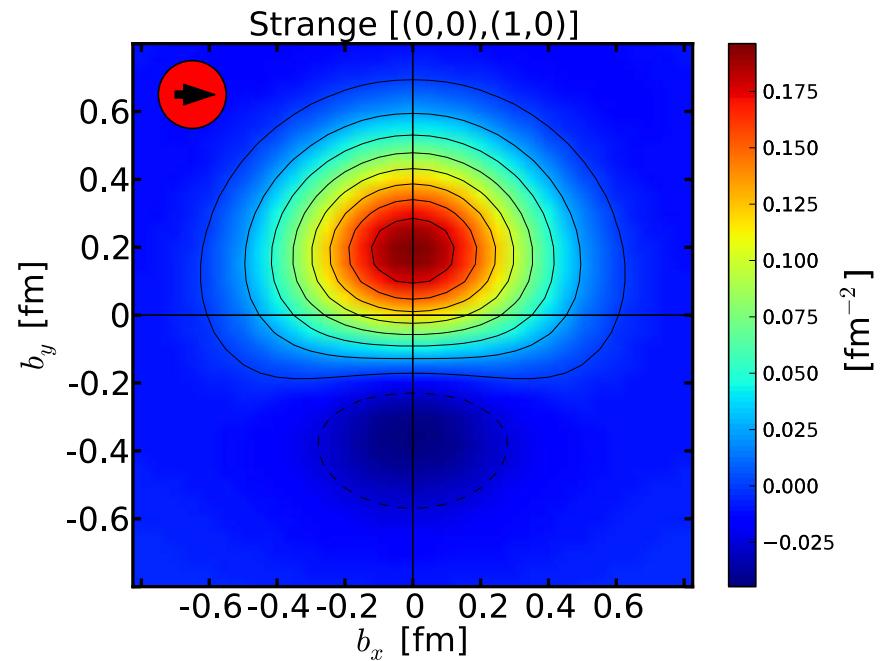
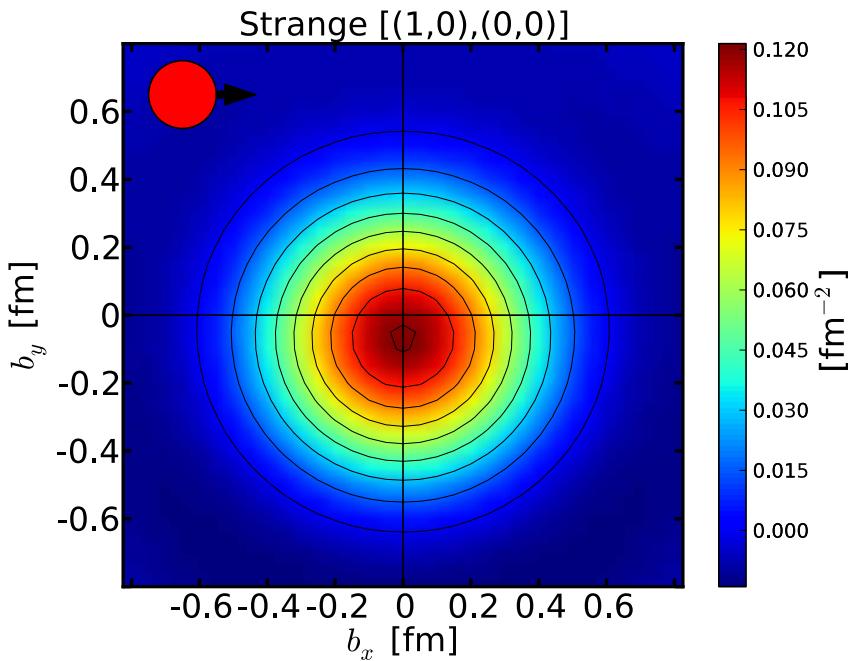
# Results

## Down quark transverse spin density inside a nucleon



# Results

## Strange quark transverse spin density inside a nucleon

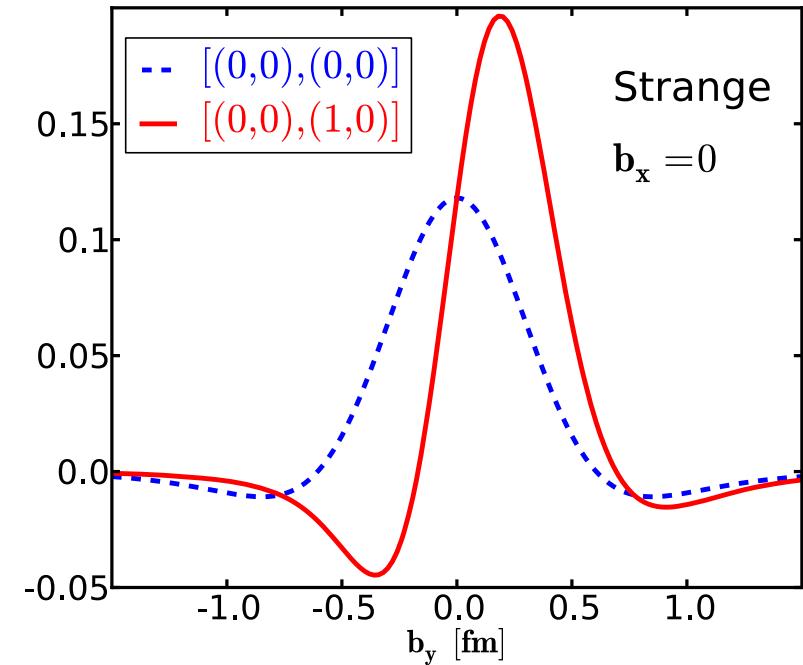
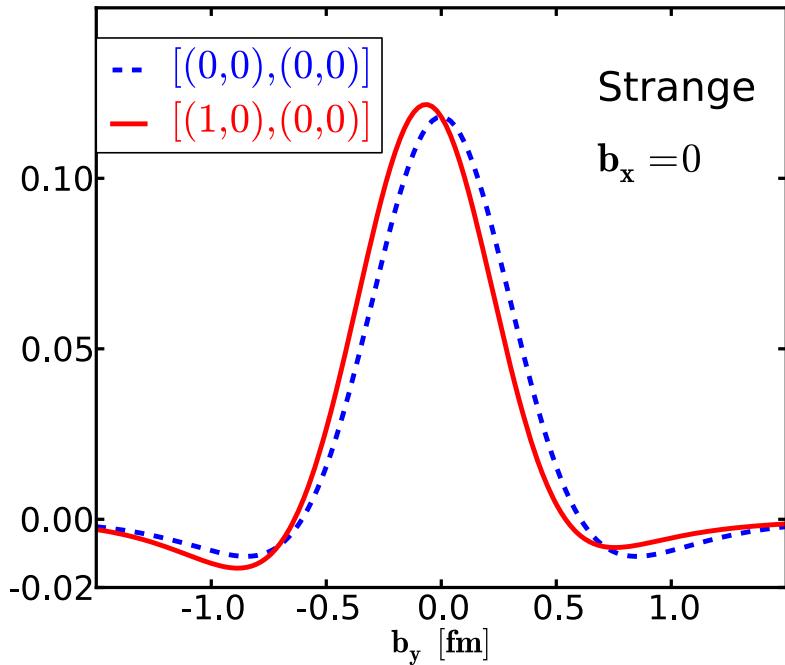


T. Ledwig & H-Ch.K, PRD 85, 034041 (2012)

This is the **first result** of the strange quark transverse spin density inside a nucleon

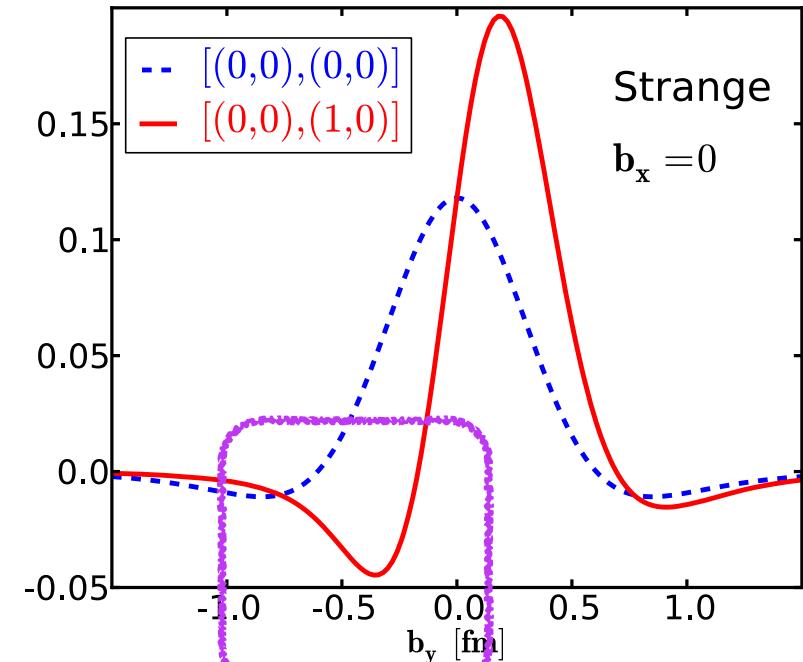
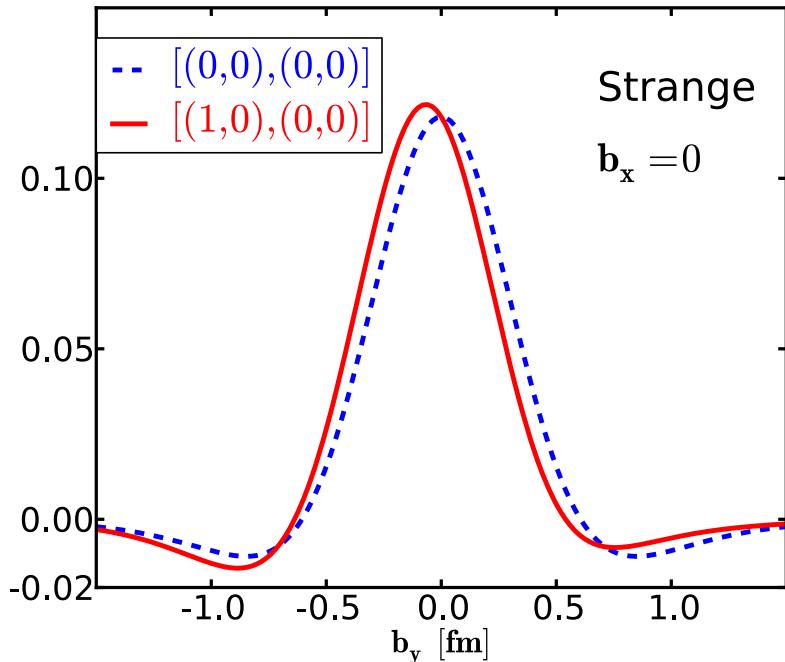
# Results

## Strange quark transverse spin density inside a nucleon



# Results

## Strange quark transverse spin density inside a nucleon



Polarized to the negative direction in the  $b$  plane.

# Summary

- We have reviewed recent investigations on the charge and spin structures of the nucleon, based on the chiral quark-soliton model.
- We have derived the EM and tensor form factors of the nucleon, from which we have obtained their transverse charge & spin densities. The results are compared with the lattice and "experimental" data.
- The first strange anomalous tensor magnetic moment was obtained, though it is compatible with zero.
- The strange quark transverse spin density was first announced in this work.
- We also extended the investigation to nuclear matter case.

*Though this be madness,  
yet there is method in it.*

Hamlet Act 2, Scene 2

Thank you very much!

# Chiral quark–soliton model

$$S_{\text{eff}} = -N_c \text{Tr} \ln(i\partial + iMU^{\gamma_5} + i\hat{m})$$

**Nucleon consisting of Nc quarks**

$$\Pi_N = \langle 0 | J_N(0, T/2) J_N^\dagger(0, -T/2) | 0 \rangle$$

$$J_N(\vec{x}, t) = \frac{1}{N_c!} \varepsilon^{\beta_1 \cdots \beta_{N_c}} \Gamma_{JJ_3 Y' TT_3 Y}^{\{f\}} \psi_{\beta_1 f_1}(\vec{x}, t) \cdots \psi_{\beta_{N_c} f_{N_c}}(\vec{x}, t)$$

$$\lim_{T \rightarrow \infty} \Pi_N(T) \simeq e^{-M_N T}$$

$$\Pi_N(\vec{x}, t) = \Gamma_N^{\{f\}} \Gamma_N^{\{g\}*} \frac{1}{Z} \int dU \prod_{i=1}^{N_c} \left\langle 0, T/2 \middle| \frac{1}{D(U)} \middle| 0, -T/2 \right\rangle_{f,g} e^{-S_{\text{eff}}}.$$

$$\lim_{T \rightarrow \infty} \frac{1}{Z} \prod_{i=1}^{N_c} \left\langle 0, T/2 \middle| \frac{1}{D(U)} \middle| 0, -T/2 \right\rangle \sim e^{-(N_c E_{\text{val}}(U) + E_{\text{sea}}(U))T}$$

# Baryonic correlation functions

## Baryonic observables

$$\lim_{x_0 \rightarrow -\infty} \langle 0 | J_N(x) \Gamma_\mu(z) J_N^\dagger(y) | 0 \rangle = \lim_{\substack{x_0 \rightarrow -\infty \\ y_0 \rightarrow \infty}} \mathcal{K}_\mu$$

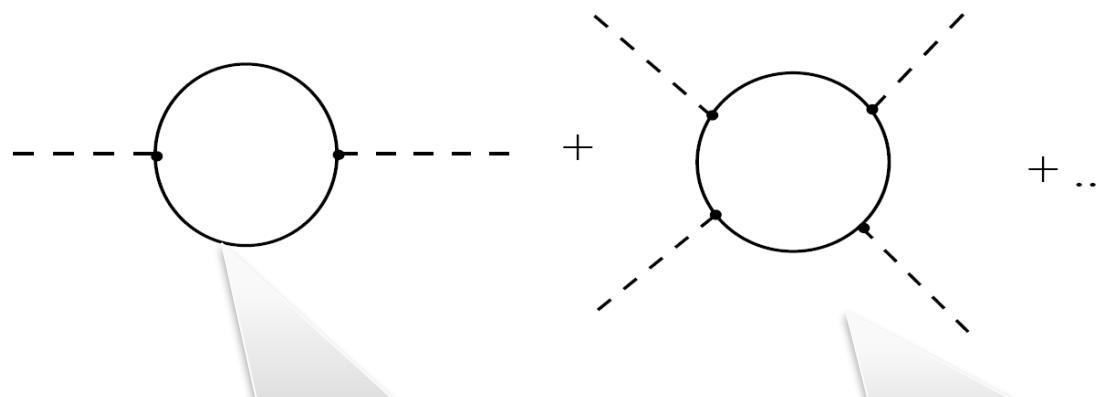
$$\begin{aligned} \mathcal{K}_\mu &= \frac{1}{\mathcal{Z}} \int D\psi D\psi^\dagger DU J_N \Gamma_\mu J_N^\dagger \\ &\times \exp \left[ \int d^4x \psi^\dagger (i\partial + iMU^{\gamma_5} + i\hat{m})] \psi \right] \end{aligned}$$

# Skyrme model as a limit of the XQSM

## Effective Chiral Lagrangian and LECs

$$S_{\text{eff}} = -N_c \text{Tr} \ln(i\partial + i\sqrt{M(i\partial)} U^{\gamma_5} \sqrt{M(i\partial)})$$

Derivative expansions: pion momentum as an expansion parameter



Weinberg term

Gasser-Leutwyler terms

# Effective chiral Lagrangian

## Weinberg Lagrangian

$$\mathcal{O}(p^2)$$

$$\text{Re}S_{\text{eff}}^{(2)}[\pi^a] - \text{Re}S_{\text{eff}}^{(2)}[0] = \int d^4x \mathcal{L}^{(2)}$$

$$\mathcal{L}^{(2)} = \frac{F_\pi^2}{4} \left\langle D^\mu U^\dagger D_\mu U \right\rangle + \frac{F_\pi^2}{4} \left\langle \mathcal{X}^\dagger U + \mathcal{X} U^\dagger \right\rangle$$

## Gasser-Leutwyler Lagrangian

$$\mathcal{O}(p^4)$$

$$\mathcal{L}^{(4)} = L_1 \langle L_\mu L_\mu \rangle^2 + L_2 \langle L_\mu L_\nu \rangle^2 + L_3 \langle L_\mu L_\mu L_\nu L_\nu \rangle$$

# Low-energy constants

## Gasser-Leutwyler Lagrangian

	$M_0(\text{MeV})$	$\Lambda(\text{MeV})$	$L_1(\times 10^{-3})$	$L_2(\times 10^{-3})$	$L_3(\times 10^{-3})$
local $\chi$ QM	350	1905.5	0.79	1.58	-3.17
DP	350	611.7	0.82	1.63	-3.09
Dipole	350	611.2	0.82	1.63	-2.97
Gaussian	350	627.4	0.81	1.62	-2.88
GL			$0.9 \pm 0.3$	$1.7 \pm 0.7$	$-4.4 \pm 2.5$
Bijnens			$0.6 \pm 0.2$	$1.2 \pm 0.4$	$-3.6 \pm 1.3$
Arriola			0.96	1.95	-5.21
VMD			1.1	2.2	-5.5
Holdom(1)			0.97	1.95	-4.20
Holdom(2)			0.90	1.80	-3.90
Bolokhov et al.			0.63	1.25	2.50
Alfaro et al.			0.45	0.9	-1.8

# Low-energy constants

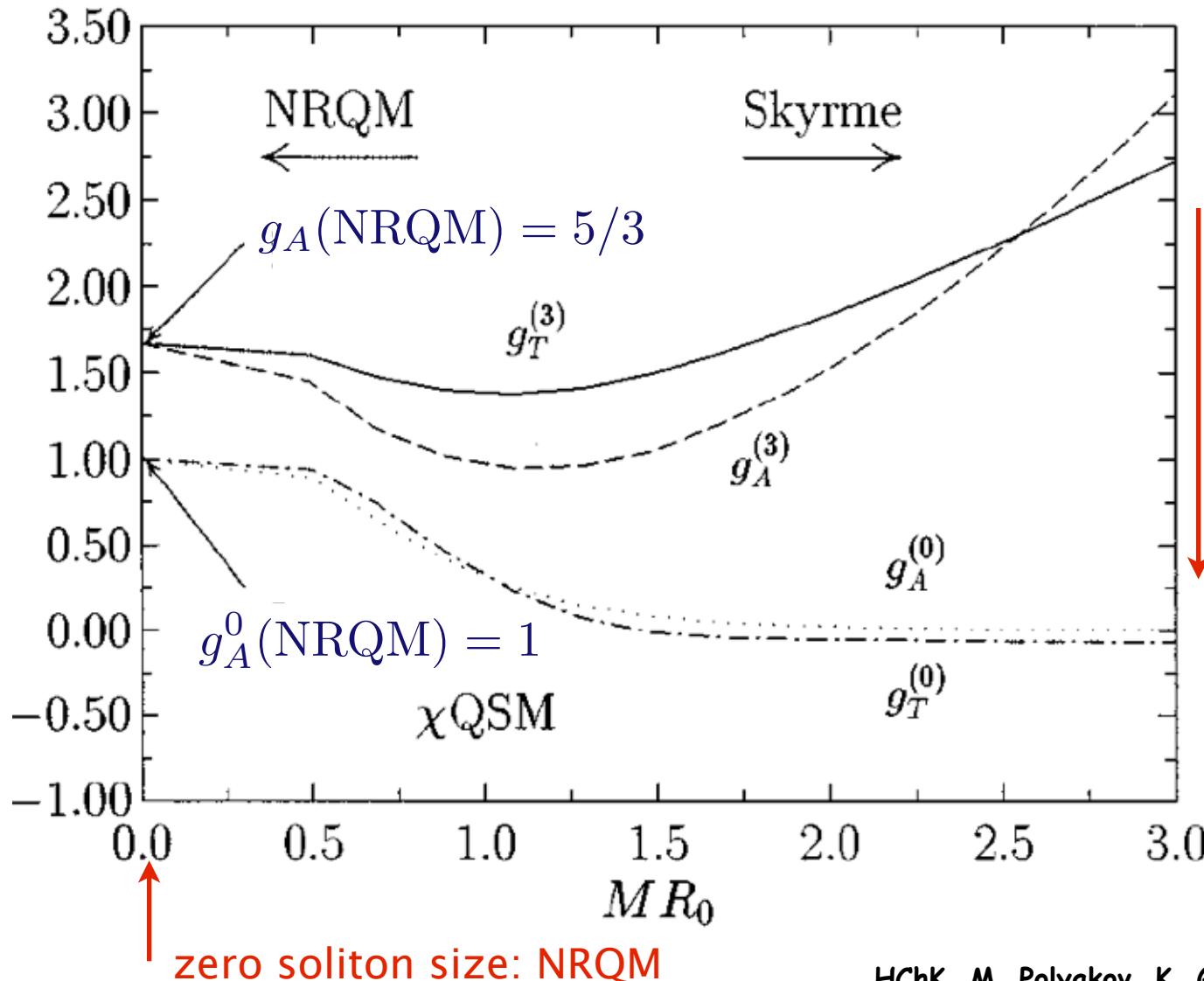
## Gasser-Leutwyler Lagrangian

	$M_0$ (MeV)	$\Lambda$ (MeV)	$L_1(\times 10^{-3})$	$L_2(\times 10^{-3})$	$L_3(\times 10^{-3})$
local $\chi$ QM	350	1905.5	0.79	1.58	-3.17
DP	350	611.7	0.82	1.63	-3.09
Dipole	350	611.2	0.82	1.63	-2.97
Gaussian	350	627.4	0.81	1.62	-2.88
GL			$0.9 \pm 0.3$	$1.7 \pm 0.7$	$-4.4 \pm 2.5$
Bijnens			$0.6 \pm 0.2$	$1.2 \pm 0.4$	$-3.6 \pm 1.3$
Arriola			0.96	1.95	-5.21
VMD			1.1	2.2	-5.5
Holdom(1)			0.97	1.95	-4.20
Holdom(2)			0.90	1.80	-3.90
Bolokhov et al.			0.63	1.25	2.50
Alfaro et al.			0.45	0.9	-1.8

# Limit to the Skyrme model

Example:

## Axial and Tensor Charges



# Medium-modified Skyrme model

Medium-modified effective chiral Lagrangian

$$\begin{aligned}\mathcal{L}^* = & \frac{F_\pi^2}{4} \text{Tr} \left( \frac{\partial U}{\partial t} \right) \left( \frac{\partial U^\dagger}{\partial t} \right) - \frac{F_\pi^2}{16} \alpha_p(\mathbf{r}) \text{Tr} (\nabla U) \cdot (\nabla U^\dagger) \\ & + \frac{1}{32e^2\gamma(\mathbf{r})} \text{Tr} [U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2 \\ & + \frac{F_\pi^2 m_\pi^2}{16} \alpha_s(\mathbf{r}) \text{Tr}(U + U^\dagger - 2)\end{aligned}$$

# Medium-modified Skyrme model

Medium-modified effective chiral Lagrangian

$$\begin{aligned}\mathcal{L}^* = & \frac{F_\pi^2}{4} \text{Tr} \left( \frac{\partial U}{\partial t} \right) \left( \frac{\partial U^\dagger}{\partial t} \right) - \frac{F_\pi^2}{16} \alpha_p(\mathbf{r}) \text{Tr} (\nabla U) \cdot (\nabla U^\dagger) \\ & + \frac{1}{32e^2\gamma(\mathbf{r})} \text{Tr} [U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2 \\ & + \frac{F_\pi^2 m_\pi^2}{16} \alpha_s(\mathbf{r}) \text{Tr}(U + U^\dagger - 2)\end{aligned}$$

$$\alpha_p(\mathbf{r}) = 1 - \chi_p(\mathbf{r})$$

$\chi_{p,s}$  : pion dipole susceptibility in medium

$$\alpha_s(\mathbf{r}) = 1 + \chi_s(\mathbf{r})/m_\pi^2$$

The parameters are fixed by pion-nucleus scattering data.

(See Ericson and Weise, “Pions in Nuclei”.)

# Medium-modified Skyrme model

Medium-modified effective chiral Lagrangian

$$\begin{aligned}\mathcal{L}^* = & \frac{F_\pi^2}{4} \text{Tr} \left( \frac{\partial U}{\partial t} \right) \left( \frac{\partial U^\dagger}{\partial t} \right) - \frac{F_\pi^2}{16} \alpha_p(\mathbf{r}) \text{Tr} (\nabla U) \cdot (\nabla U^\dagger) \\ & + \frac{1}{32e^2 \gamma(\mathbf{r})} \text{Tr} [U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2 \\ & + \frac{F_\pi^2 m_\pi^2}{16} \alpha_s(\mathbf{r}) \text{Tr}(U + U^\dagger - 2)\end{aligned}$$

$$\alpha_p(\mathbf{r}) = 1 - \chi_p(\mathbf{r})$$

$\chi_{p, s}$  : pion dipole susceptibility in medium

The parameters are fixed by pion-nucleus scattering data.

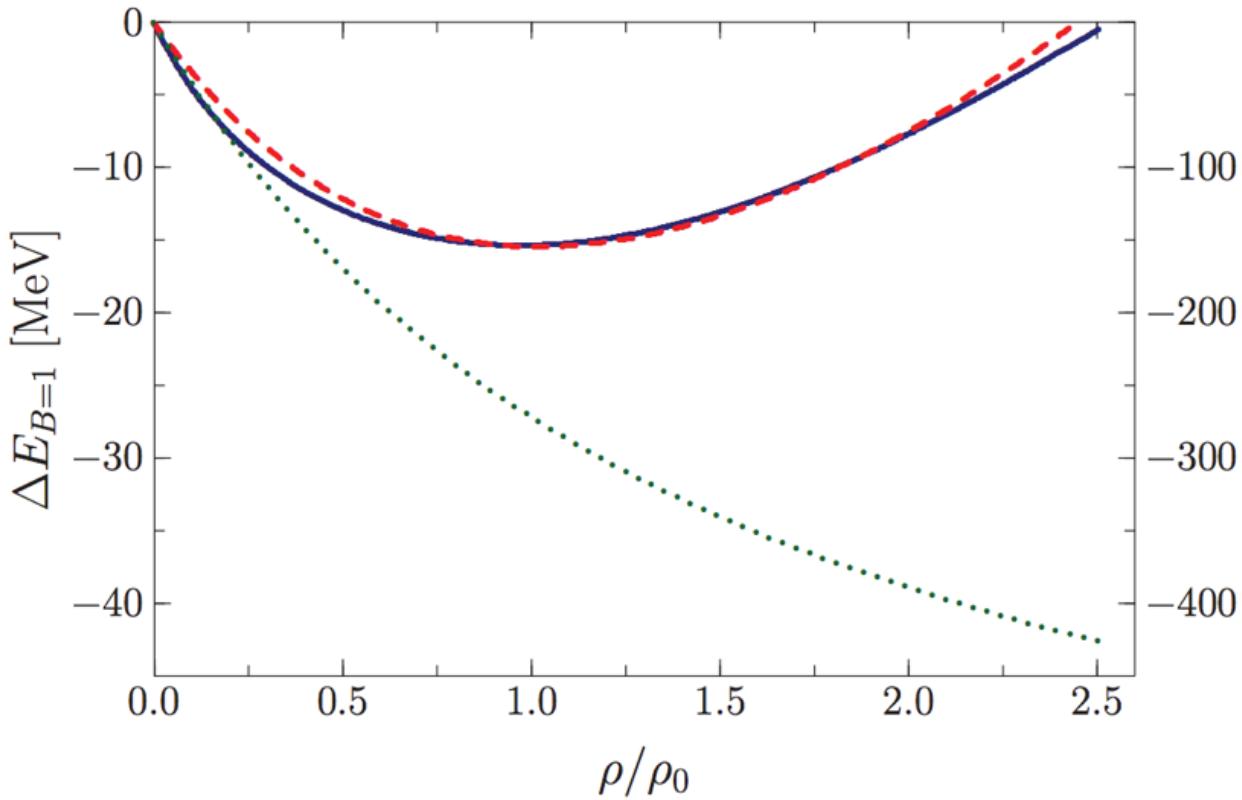
(See Ericson and Weise, “Pions in Nuclei”.)

$$\gamma(\mathbf{r}) = \exp \left( -\frac{\gamma_{\text{num}} \rho(\mathbf{r})}{1 + \gamma_{\text{den}} \rho(\mathbf{r})} \right)$$

Fitted to the volume term of the semi-empirical mass formula.

# Medium-modified Skyrme model

Binding Energy per nucleon

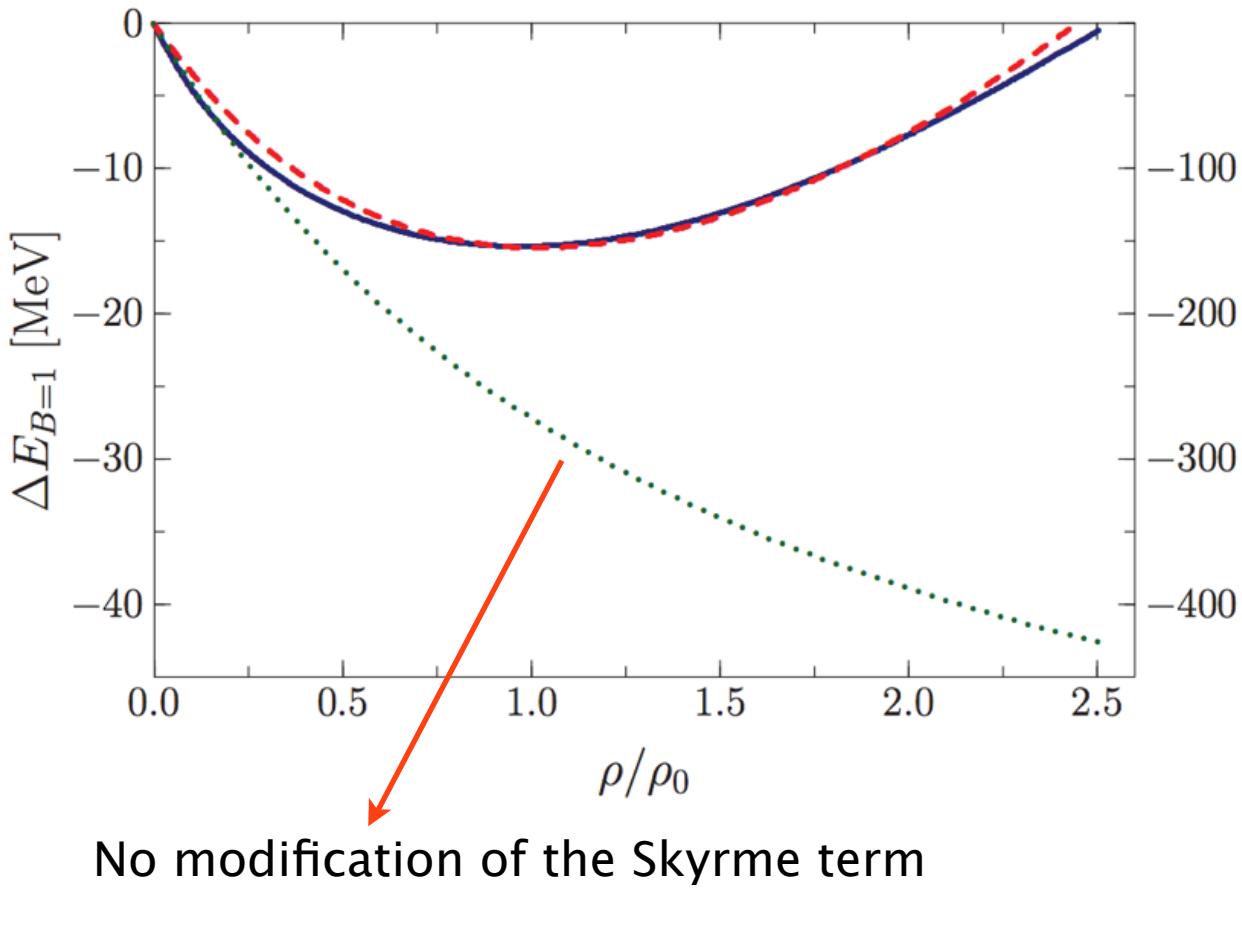


$$\gamma_{\text{num}} = 0.47 m_\pi^{-3}$$

$$\gamma_{\text{den}} = 0.17 m_\pi^{-3}$$

# Medium-modified Skyrme model

Binding Energy per nucleon

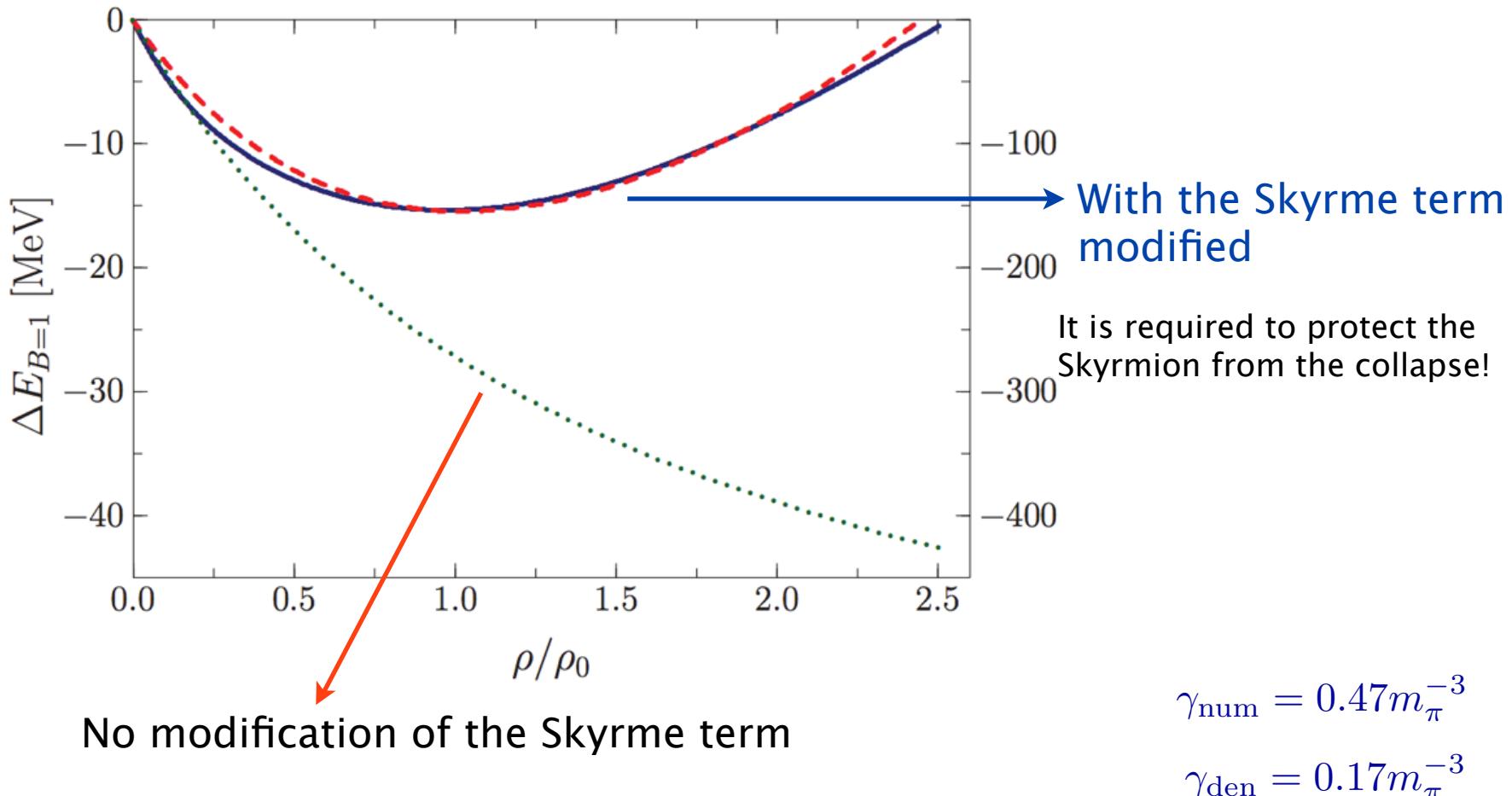


$$\gamma_{\text{num}} = 0.47 m_\pi^{-3}$$

$$\gamma_{\text{den}} = 0.17 m_\pi^{-3}$$

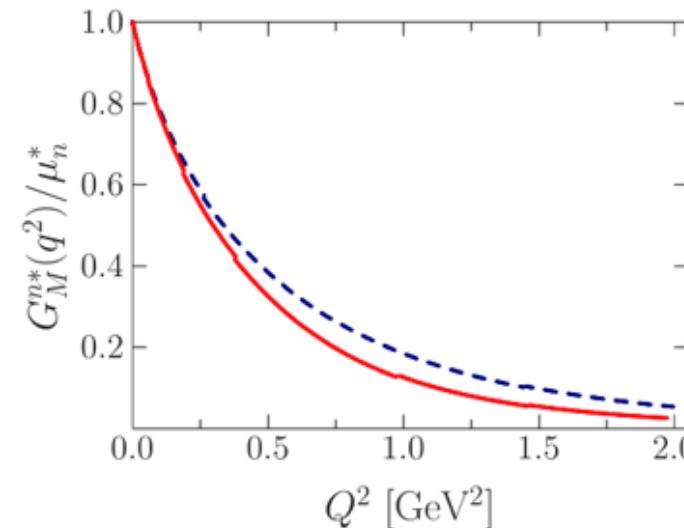
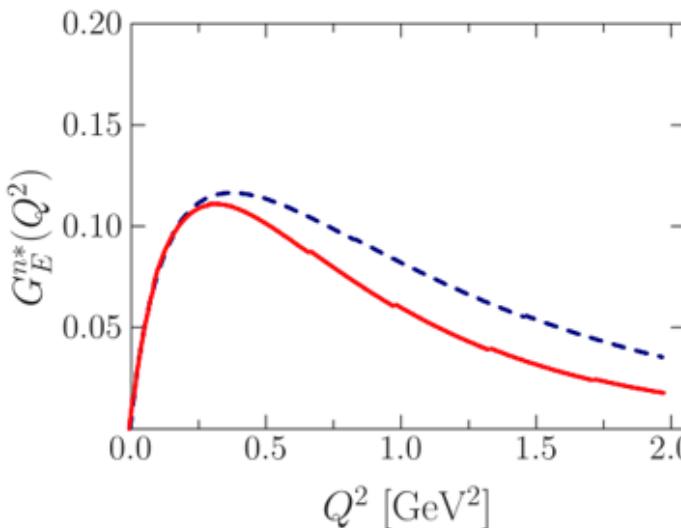
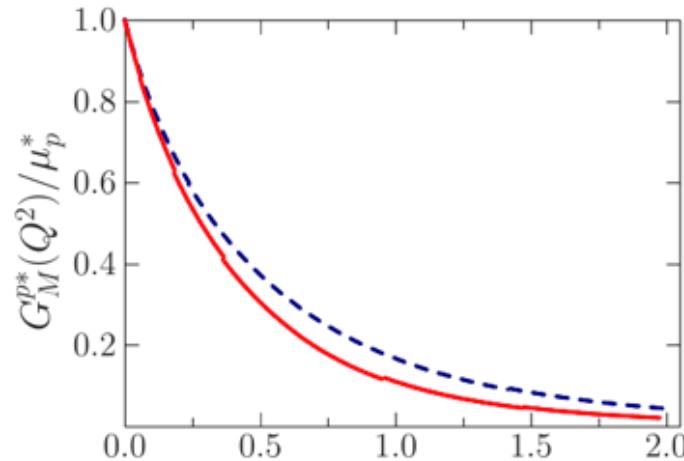
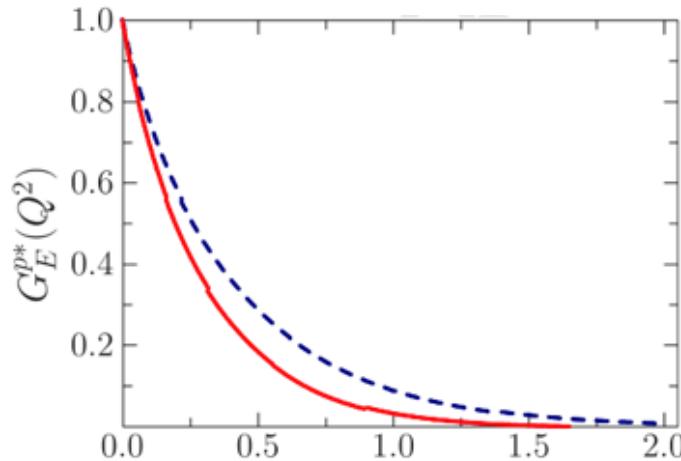
# Medium-modified Skyrme model

Binding Energy per nucleon

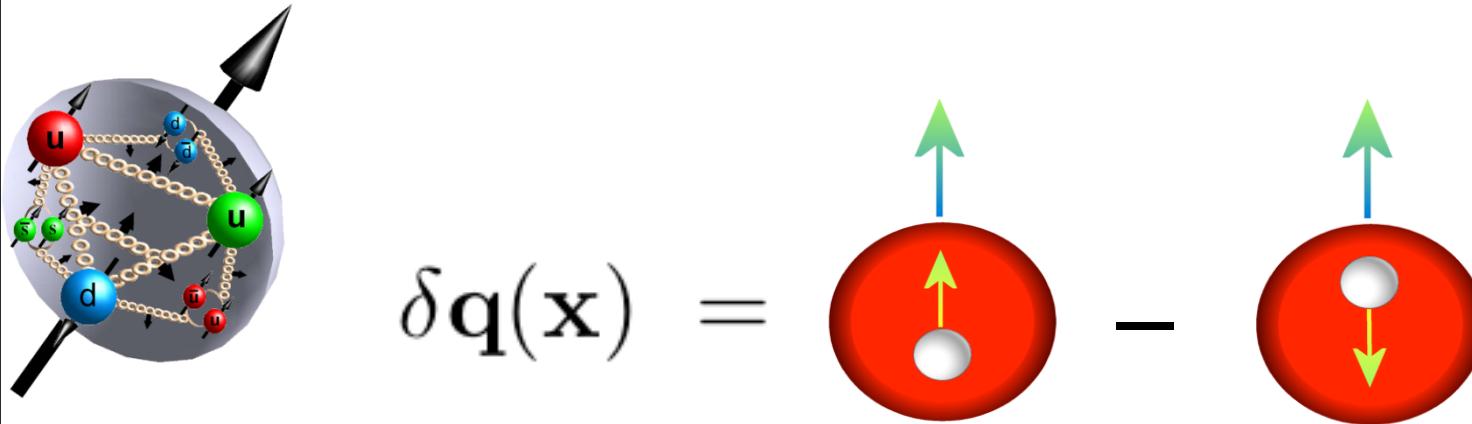


# Results

Electromagnetic form factors of the nucleon in nuclear matter



# Transversity: Tensor Charges



$$\langle N | \bar{\psi} \sigma_{\mu\nu} \lambda^\chi \psi | N \rangle \sim \text{Tensor charges}$$

- No explicit probe for the tensor charge! Difficult to be measured.
- Chiral-odd Parton Distribution Function can get accessed via the SSA of SIDIS (HERMES and COMPASS).

A. Airapetian et al. (HERMES Coll.), PRL 94, 012002 (2005).

E.S. Ageev et al. (COMPASS Coll.), NPB 765, 31 (2007).

CLAS & CLAS12 Coll.

ppbar Drell-Yan process (PAX Coll.): Technically too difficult for the moment (polarized antiproton: hep-ex/0505054).

# Transversity: Tensor Charges

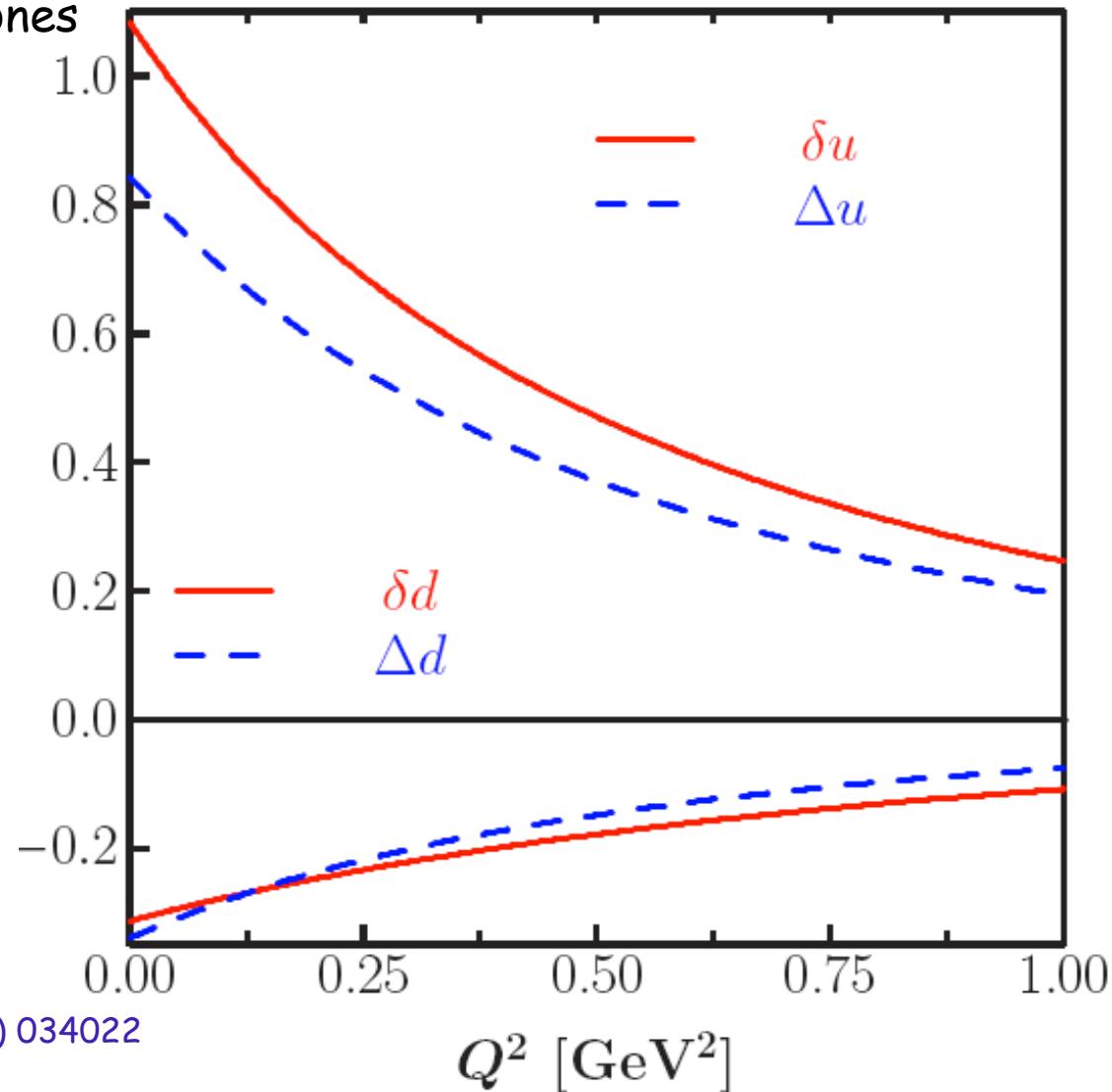
$$\delta u = 0.60_{-0.24}^{+0.10}, \quad \delta d = -0.26_{-0.18}^{+0.1} \text{ at } 0.36 \text{ GeV}^2$$

Based on SIDIS (HERMES) data:

M. Anselmino et al. Nucl. Phys. B, Proc. Suppl. 191, 98 (2009)

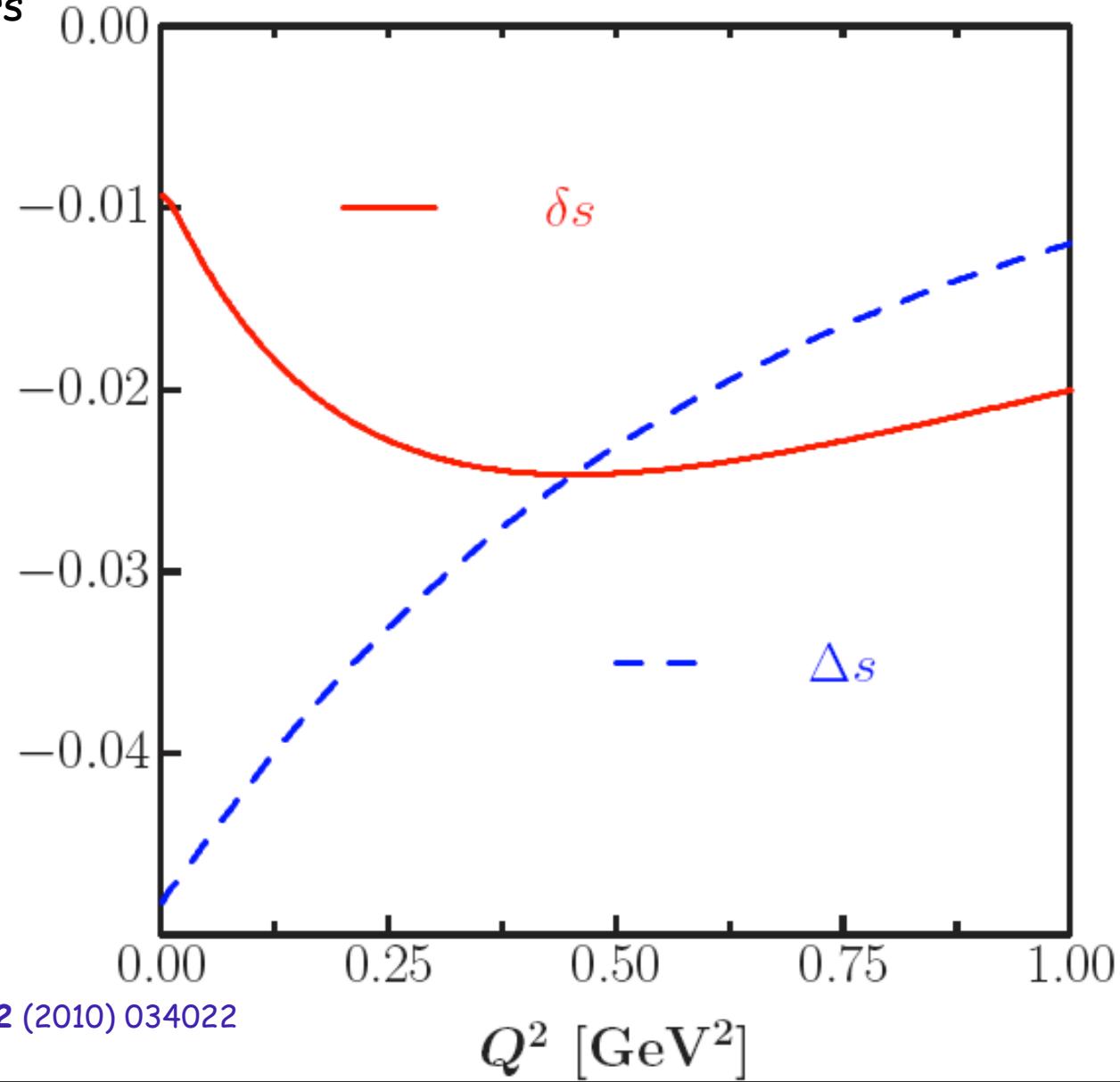
# Results

Up and down tensor form factors  
compared with the axial-vector ones



# Results

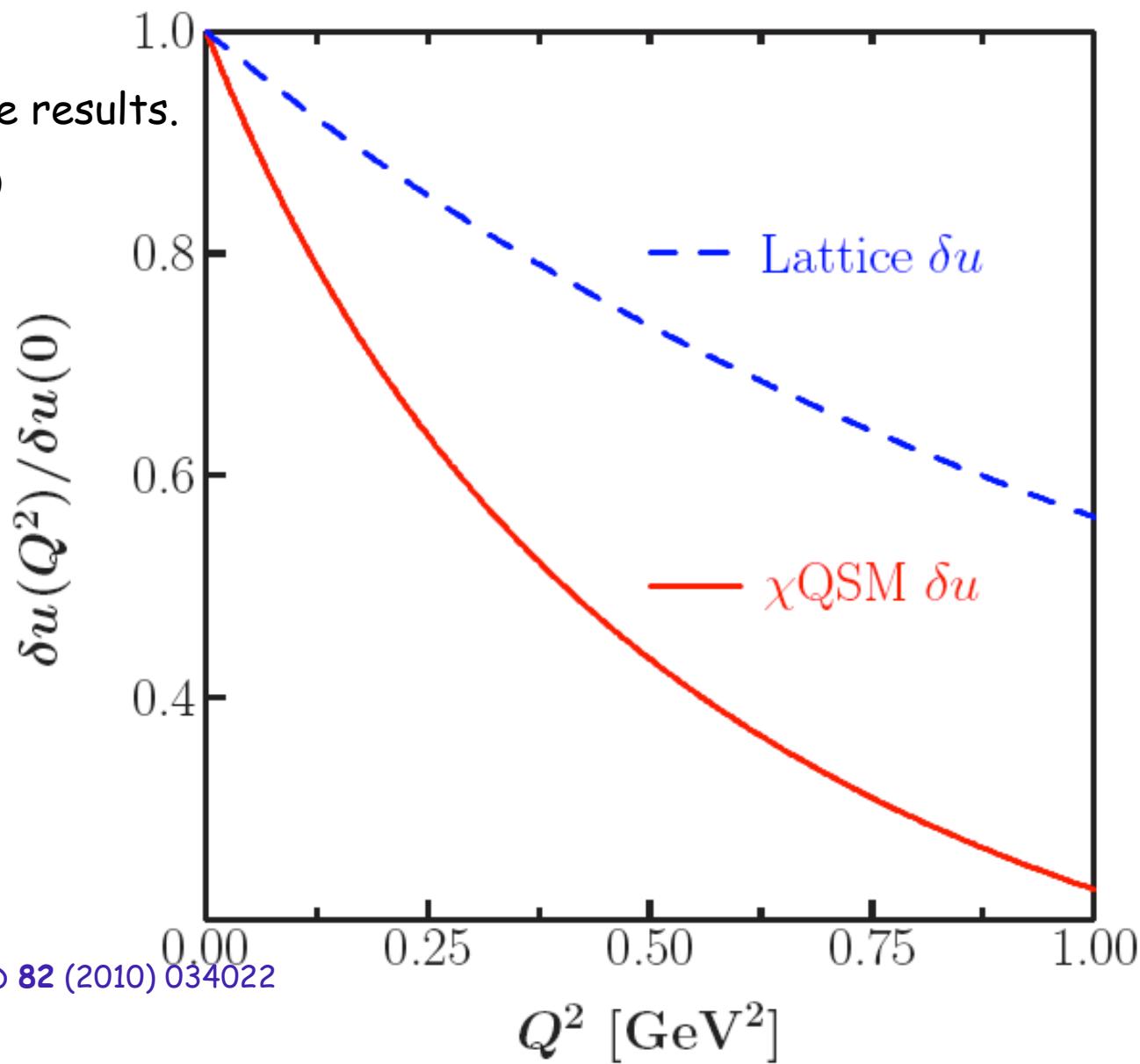
Strange tensor form factors  
compared with  
the axial-vector ones



# Results

Comparison with the lattice results.

M. Goeckeler et al., PLB 627, 113 (2005)

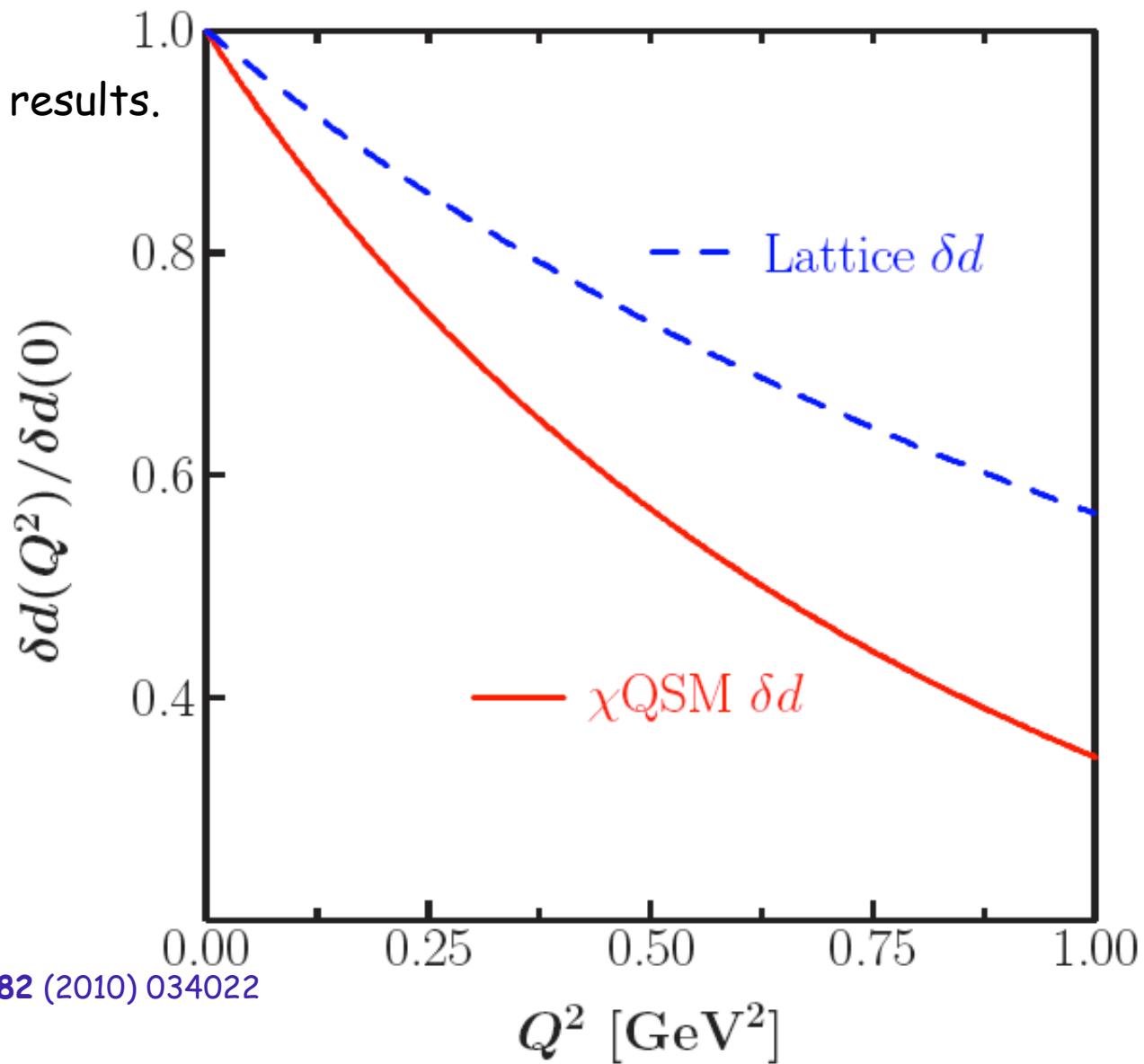


T. Ledwig, A. Silva, HChK, Phys. Rev. D 82 (2010) 034022

# Results

Comparison with the lattice results.

M. Goeckeler et al., PLB 627, 113 (2005)



T. Ledwig, A. Silva, HChK, Phys. Rev. D 82 (2010) 034022

# Results

	$p(uud)$	$n(ddu)$	$\Lambda(uds)$	$\Sigma^+(uus)$	$\Sigma^0(uds)$	$\Sigma^-(dds)$	$\Xi^0(uss)$	$\Xi^-(dss)$
$\delta u$	1.08	-0.32	-0.03	1.08	0.53	-0.02	-0.32	-0.02
$\delta d$	-0.32	1.08	-0.03	-0.02	0.53	1.08	-0.02	-0.32
$\delta s$	-0.01	-0.01	0.79	-0.29	-0.29	-0.29	1.06	1.06

## Isospin relations

$$\begin{aligned} \delta u_p &= \delta d_n, & \delta u_n &= \delta d_p, & \delta u_\Lambda &= \delta d_\Lambda, & \delta u_{\Sigma^+} &= \delta d_{\Sigma^-}, \\ \delta u_{\Sigma^0} &= \delta d_{\Sigma^0}, & \delta u_{\Sigma^-} &= \delta d_{\Sigma^+}, & \delta u_{\Xi^0} &= \delta d_{\Xi^-}, & \delta u_{\Xi^-} &= \delta d_{\Xi^0}, \\ \delta s_p &= \delta s_n, & \delta s_n &= \delta s_p, & \delta s_{\Sigma^\pm} &= \delta s_{\Sigma^0}, & \delta s_{\Xi^0} &= \delta s_{\Xi^-}, \end{aligned}$$

## SU(3) relations

$$\begin{aligned} \delta u_p &= \delta d_n = \delta u_{\Sigma^+} = \delta d_{\Sigma^-} = \delta s_{\Xi^0} = \delta s_{\Xi^-}, \\ \delta u_n &= \delta d_p = \delta u_{\Xi^0} = \delta d_{\Xi^-} = \delta s_{\Sigma^\pm} = \delta s_{\Sigma^0}. \end{aligned}$$

# Results

	$p(uud)$	$n(ddu)$	$\Lambda(uds)$	$\Sigma^+(uus)$	$\Sigma^0(uds)$	$\Sigma^-(dds)$	$\Xi^0(uss)$	$\Xi^-(dss)$
$\delta u$	1.08	-0.32	-0.03	1.08	0.53	-0.02	-0.32	-0.02
$\delta d$	-0.32	1.08	-0.03	-0.02	0.53	1.08	-0.02	-0.32
$\delta s$	-0.01	-0.01	0.79	-0.29	-0.29	-0.29	1.06	1.06

## Isospin relations

$$\begin{aligned} \delta u_p &= \delta d_n, & \delta u_n &= \delta d_p, & \delta u_\Lambda &= \delta d_\Lambda, & \delta u_{\Sigma^+} &= \delta d_{\Sigma^-}, \\ \delta u_{\Sigma^0} &= \delta d_{\Sigma^0}, & \delta u_{\Sigma^-} &= \delta d_{\Sigma^+}, & \delta u_{\Xi^0} &= \delta d_{\Xi^-}, & \delta u_{\Xi^-} &= \delta d_{\Xi^0}, \\ \delta s_p &= \delta s_n, & \delta s_{\Sigma^\pm} &= \delta s_{\Sigma^0}, & \delta s_{\Xi^0} &= \delta s_{\Xi^-}, \end{aligned}$$

## SU(3) relations

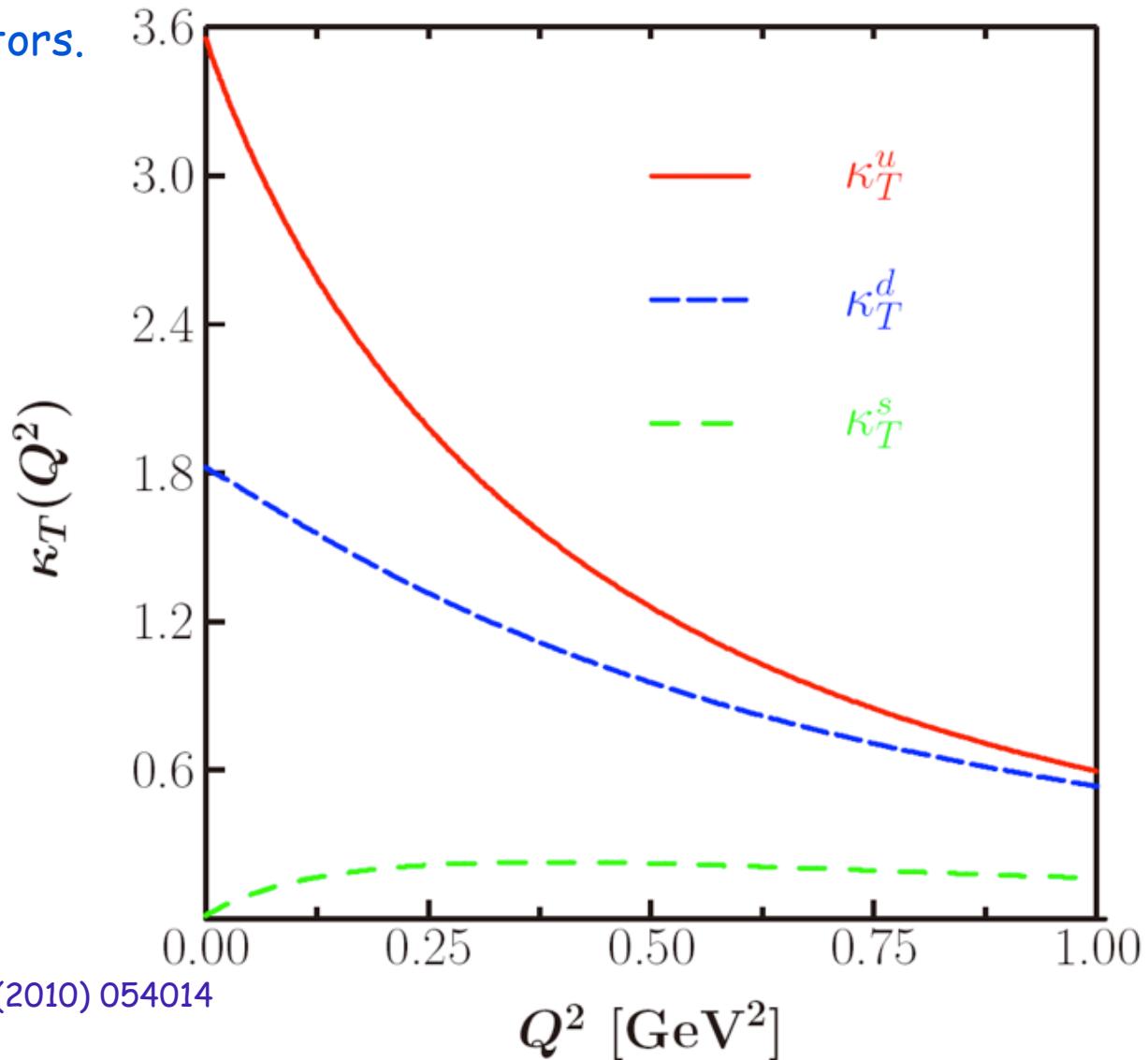


Effects of SU(3) symmetry breaking are almost negligible!

$$\begin{aligned} \delta u_p &= \delta d_n = \delta u_{\Sigma^+} = \delta d_{\Sigma^-} = \delta s_{\Xi^0} = \delta s_{\Xi^-}, \\ \delta u_n &= \delta d_p = \delta u_{\Xi^0} = \delta d_{\Xi^-} = \delta s_{\Sigma^\pm} = \delta s_{\Sigma^0}. \end{aligned}$$

# Results

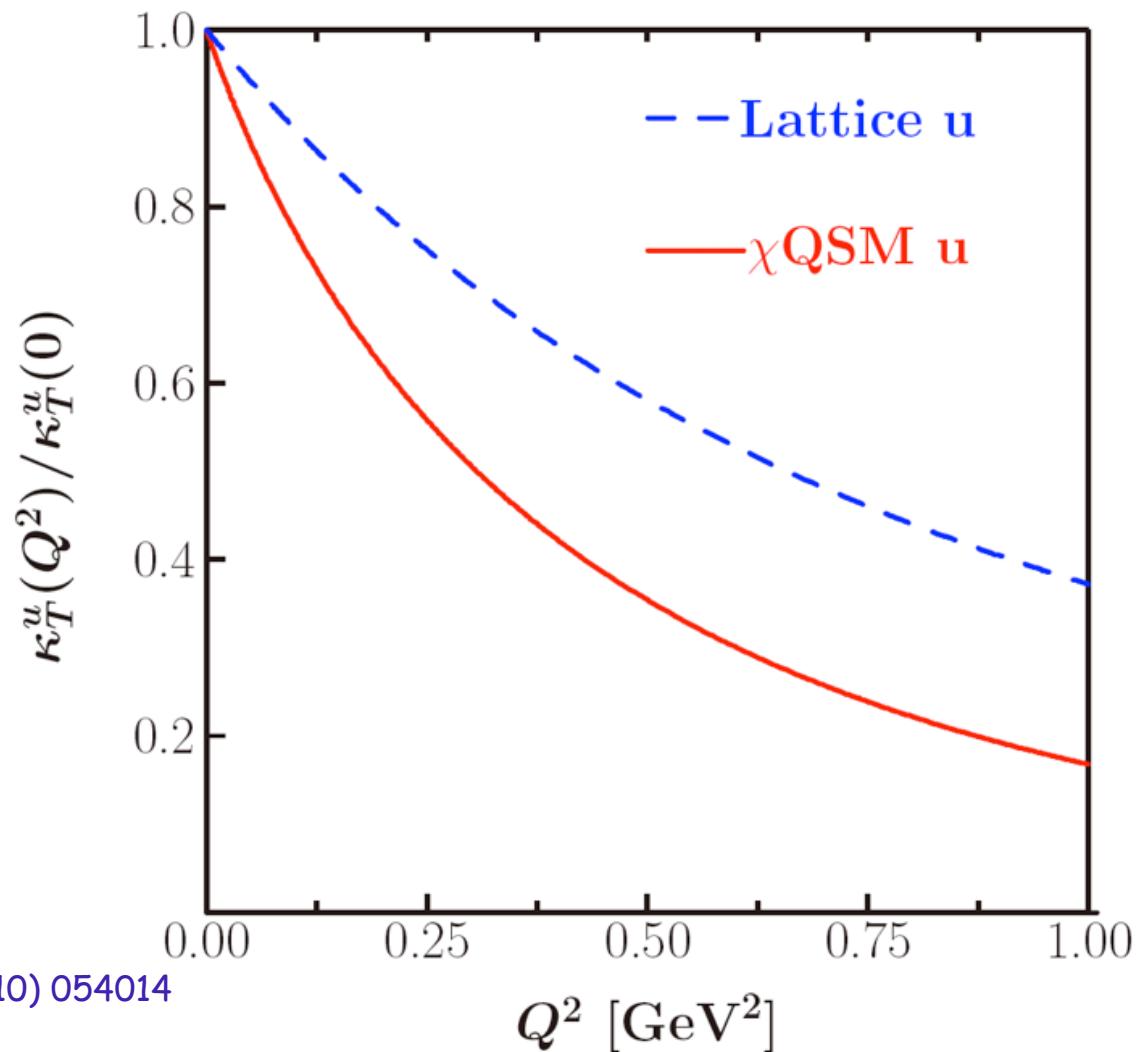
Flavor decomposition of the anomalous tensor magnetic form factors.



# Results

Up anomalous tensor magnetic form factors  
compared with the lattice one.

M. Goeckeler et al. [QCDSF Coll. and UKQCD Coll.]  
PRL 98, 222001 (2007)



# Results

Down anomalous tensor magnetic form factors  
compared with the lattice one.

M. Goeckeler et al. [QCDSF Coll. and UKQCD Coll.]  
PRL 98, 222001 (2007)

