Proton spin asymmetries and d_2 from SANE-HMS data

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For SANE Collaboration

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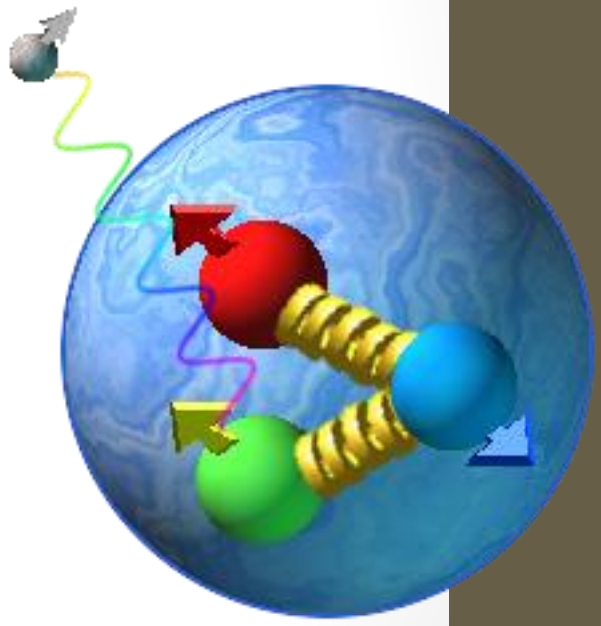
April 7th, 2014



Outline

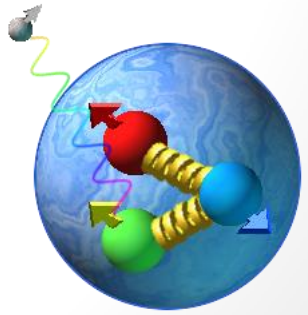
- Introduction to SANE
- Experimental Setup and SANE-HMS
- Analysis: Dilution Factor and Radiative Correction
- Result: HMS Asymmetries, Structure Functions and d_2 Matrix Element
- Summary

SANE - Spin Asymmetries on the Nucleon Experiment

Spin Asymmetries on the Nucleon Experiment, or SANE(TJNAF E07-003), is a measurement of the proton spin asymmetries

done in the Hall C of Thomas Jefferson National Accelerator Facility(Jefferson Lab), Virginia USA

during January-March 2009, excluding installation and commissioning periods.



Spin Structure Functions

Inclusive DIS cross section depends on four structure functions, two unpolarized (F_1, F_2) and two polarized (g_1, g_2). The spin structure functions g_1 and g_2 can be experimentally determined by measuring spin asymmetries:

$$A_{\parallel} = \frac{\sigma^{\downarrow\uparrow} - \sigma^{\uparrow\uparrow}}{\sigma^{\downarrow\uparrow} + \sigma^{\uparrow\uparrow}}, \quad A_{\perp} = \frac{\sigma^{\downarrow\rightarrow} - \sigma^{\uparrow\rightarrow}}{\sigma^{\downarrow\rightarrow} + \sigma^{\uparrow\rightarrow}}.$$

$$g_1(x,Q^2) = \frac{F_1(x,Q^2)}{d'}\left[A_{\parallel} + \tan(\theta/2)A_{\perp}\right],$$

$$g_2(x,Q^2) = \frac{yF_1(x,Q^2)}{2d'}\left[\frac{E + E'\cos(\theta)}{E'\sin(\theta)}A_{\perp} - A_{\parallel}\right]$$

Spin structure functions

When the spins of electron and nucleon are all polarized, we can see the dependence of scattering cross section on the spin structure functions $g_1(x, Q^2)$ and $g_2(x, Q^2)$.

$$g_1 = \frac{1}{2}\sum_i e_i^2[q_i^+ - q_i^-]$$

$$g_2 = g_2^{WW} + \overline{g_2}$$

$$g_2^{WW}(x, Q^2) = -g_1(x, Q^2) + \int_x^1 \frac{g_1(x', Q^2)}{x'} dx'$$

Purpose of SANE

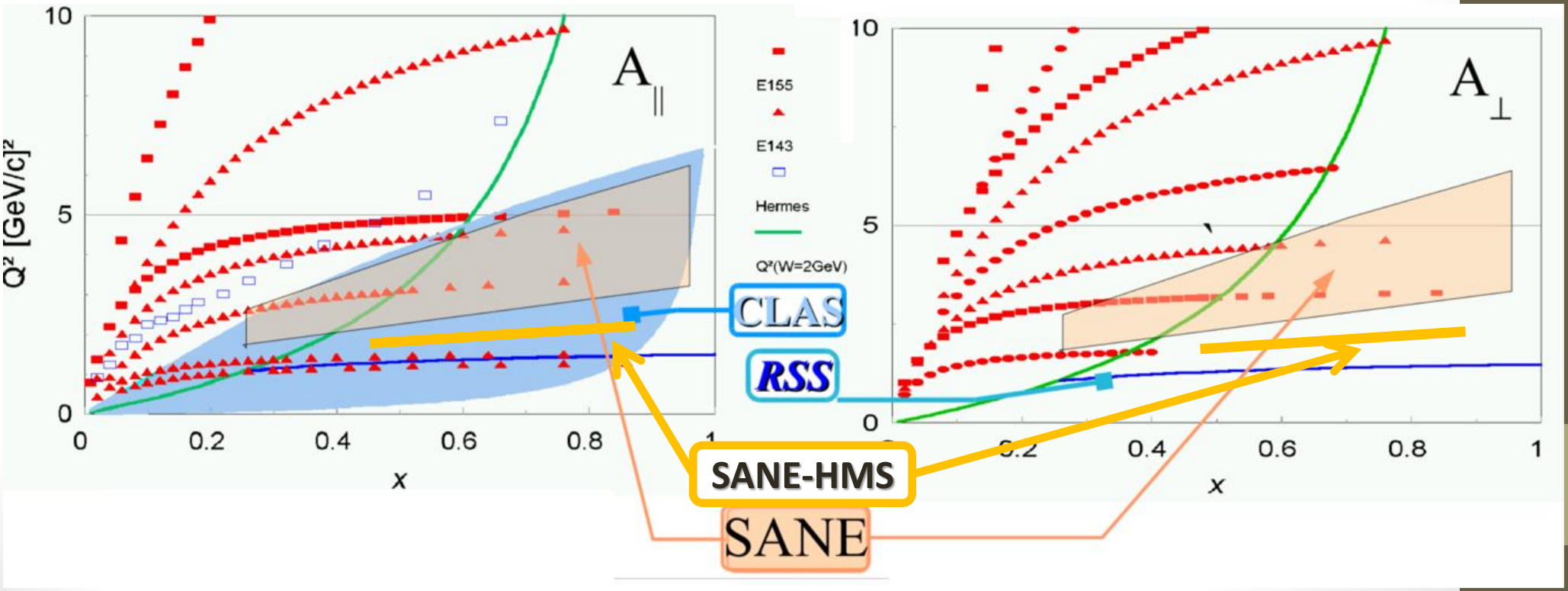
SANE's purpose is to learn everything possible about Proton Spin Asymmetries and Spin Structure Functions from Inclusive Polarized DIS:

- Resonances and Q^2 dependence of A_2
- SSF $g_2(x, Q^2)$
- Twist-3 effects (higher twist represents increasing interactions among partons)
- Comparing with lattice QCD, QCD sum rules
- Exploring high Bjorken x region

World Data and SANE Region

World data lacks big region, especially in the perpendicular asymmetry. SANE-BETA covers broad region of $0.3 \leq x \leq 0.8,\ 2.5GeV^2 \leq Q^2 \leq 6.5GeV^2$

SANE-HMS covers lower Q^2 region.



Jefferson Lab CEBAF

Polarized electrons can be accelerated up to $6\ GeV$ and the electron beam is continuous

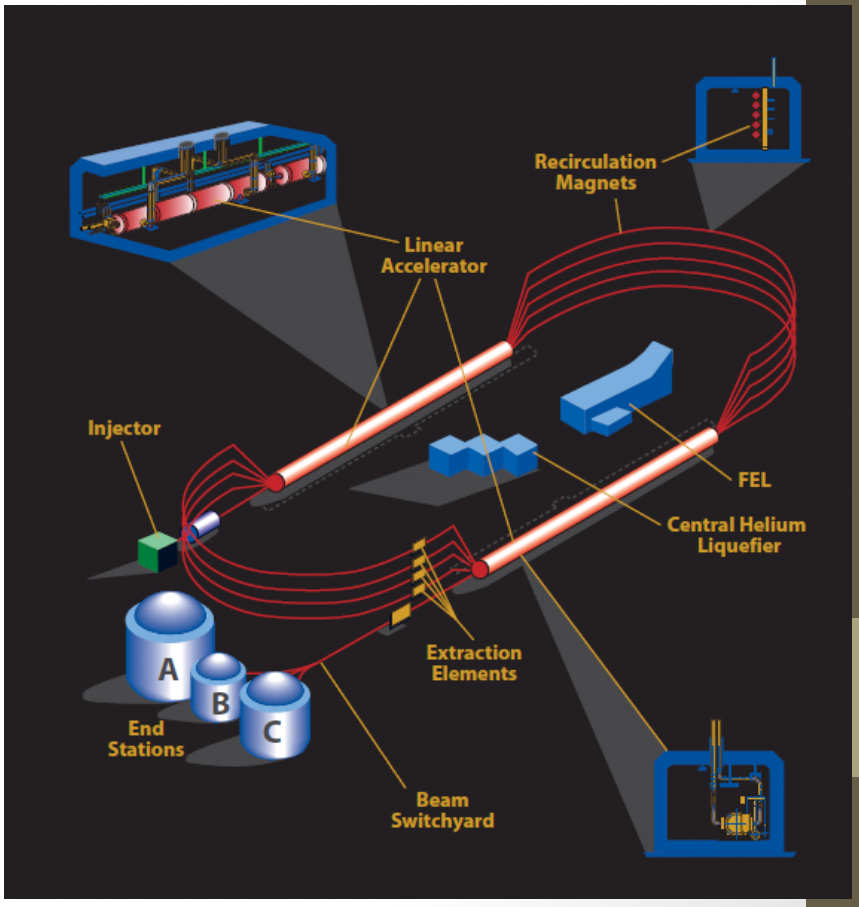
Two linear accererators and arcs

30 Hz electron helicity flip

by peudo-random basis

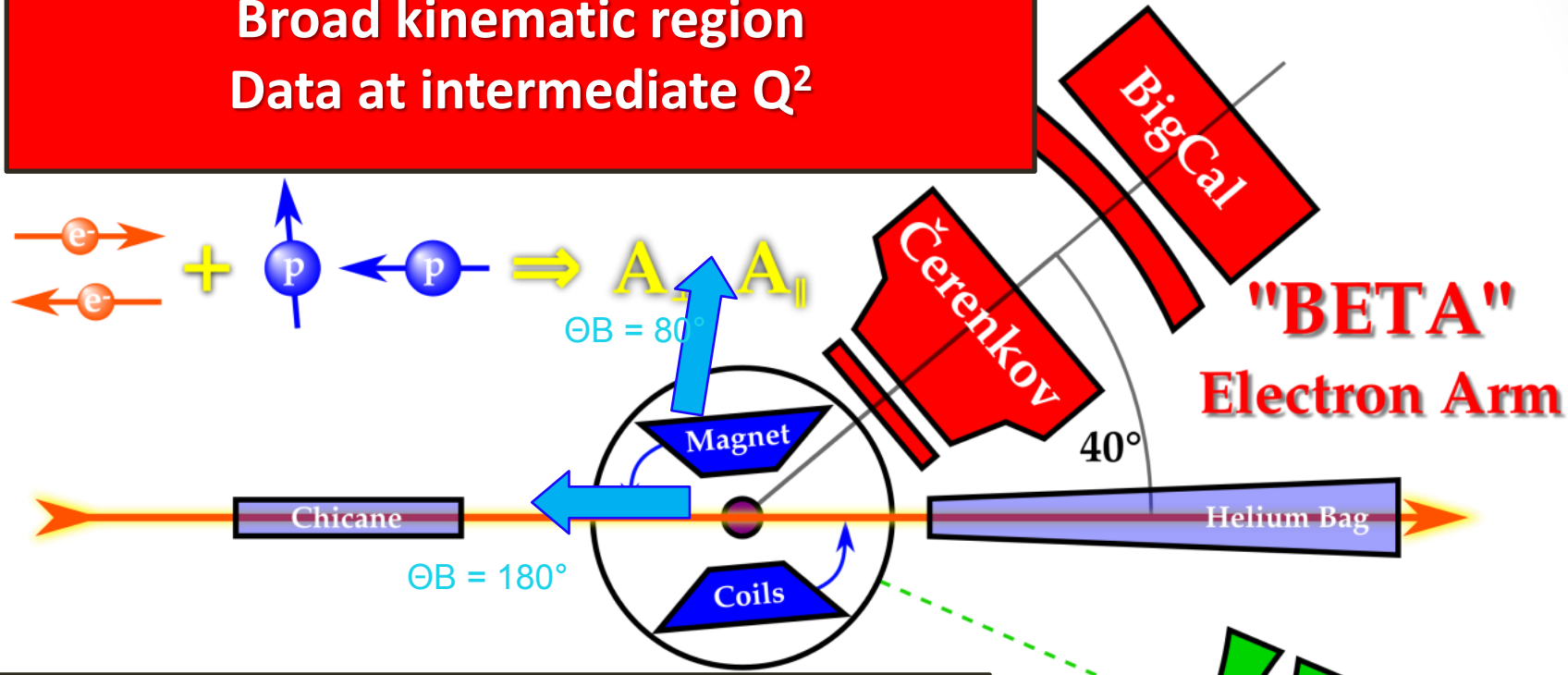
SANE used 4.7 and 5.9 GeV

beam of 90 nA



Experimental Setup

Big Electron Telescope Array:
Broad kinematic region
Data at intermediate Q^2

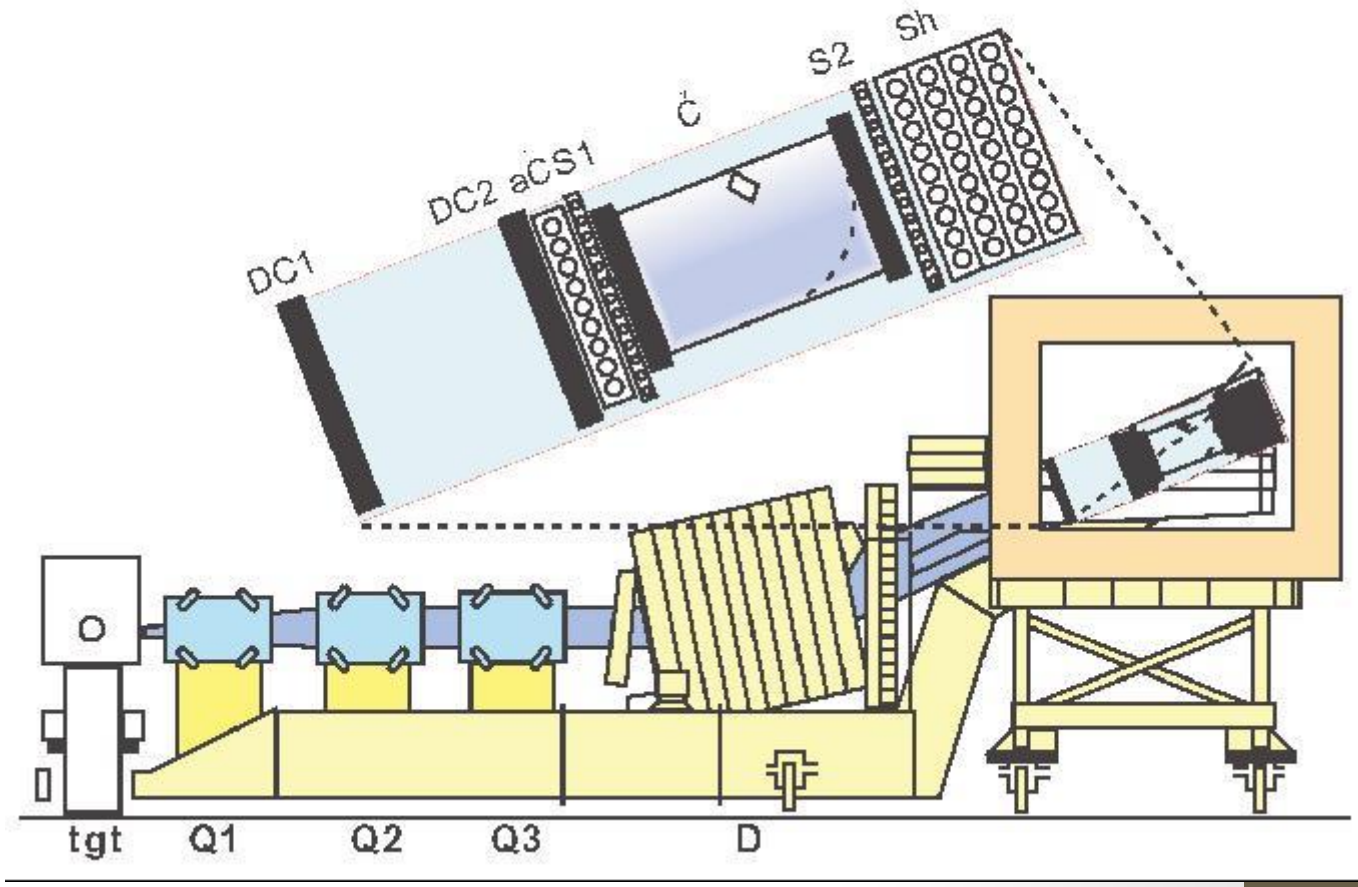


High Momentum Spectrometer:
Focused on small kinematic region
Low background
Data at lower Q^2 to extend RSS results

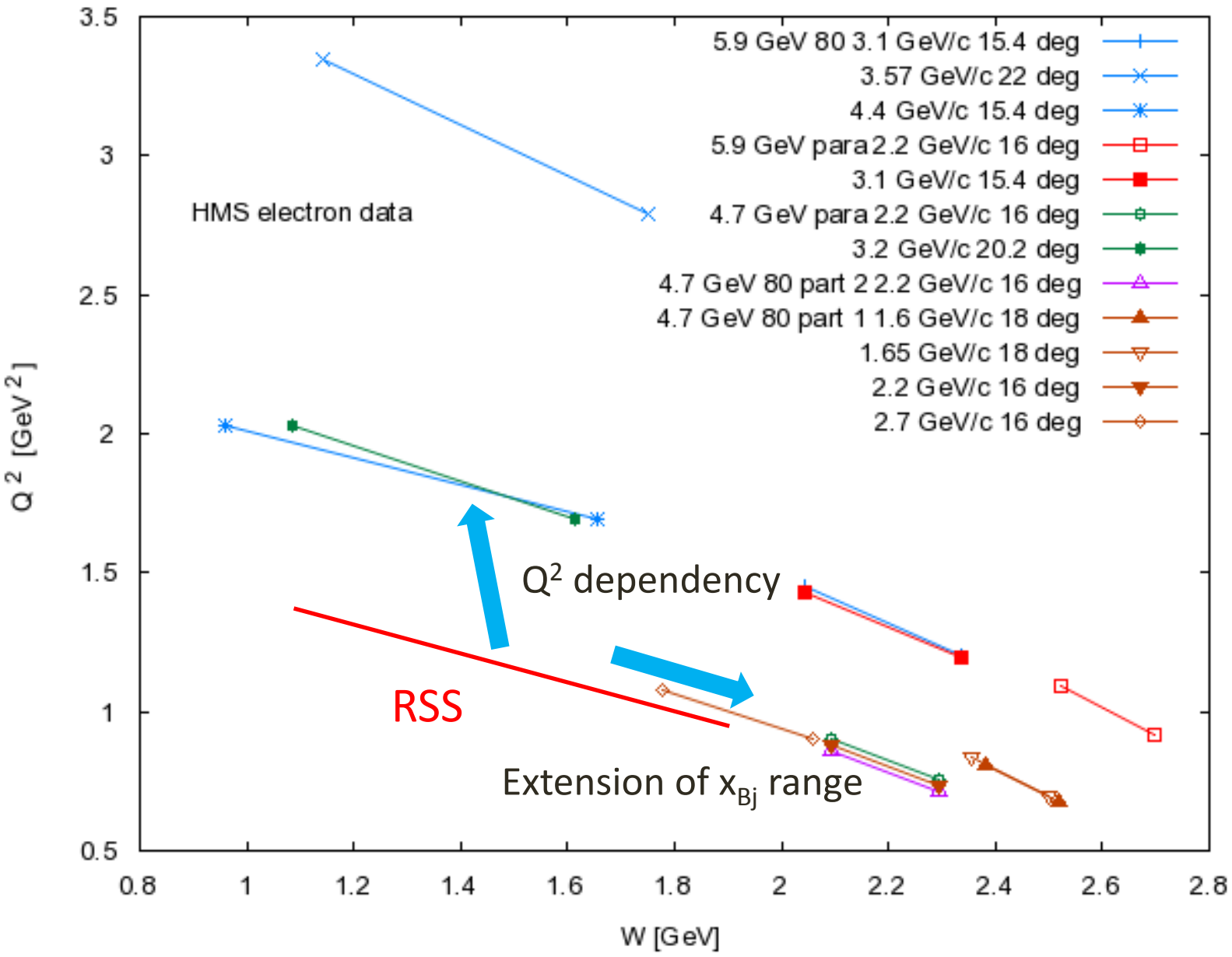
High Momentum Spectrometer

HMS has smaller angular acceptance than BETA, but with precise measurement.
HMS collected complementary data in SANE by varying central angle and momentum.

Resonance Spin Structure(RSS) experiment(2002) have produced meaningful results with limited HMS kinematics and data.
(1.085 GeV< W <1.910 GeV and $\langle Q^2 \rangle$ =1.3 GeV^2)
Phys. Rev. Lett. 105, 101601 (2010)
Phys. Rev. Lett. 98, 132003 (2007)
Phys. Rev. C 74, 035201 (2006)



HMS Coverage for SANE



Dilution Factor from Packing Fraction

The target and beam are not completely polarized. It contains also un-polarizable materials.

$$A = \frac{1}{P_b P_t f} \frac{d\sigma^{\downarrow\uparrow} - d\sigma^{\uparrow\uparrow}}{d\sigma^{\downarrow\uparrow} + d\sigma^{\uparrow\uparrow}}$$

Beam Polarization ~80%

Proton(target) Polarization ~70%

Dilution Factor

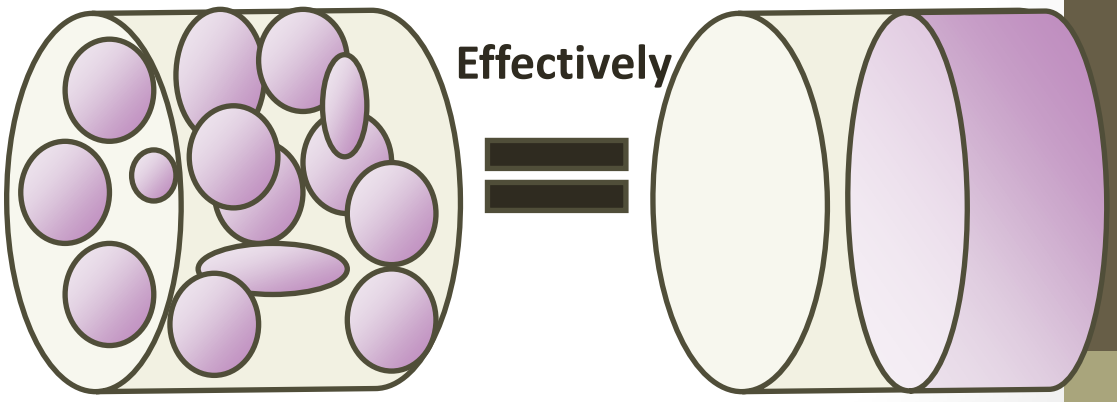
Dilution factor f is the ratio of free polarizable nucleons to the total amount of nucleons in the sample.

$$f = \frac{N_1\sigma_1}{N_1\sigma_1 + N_{14}\sigma_{14} + \sum N_A\sigma_A}$$

where $N_A = \frac{N_0 \varrho_A z_A}{M_A}$

Dilution Factor from Packing Fraction

Packing fraction is the relative volume ratio of ammonia to the target cell, or the fraction of the cell's length that would be filled with ammonia by cylindrical symmetry.

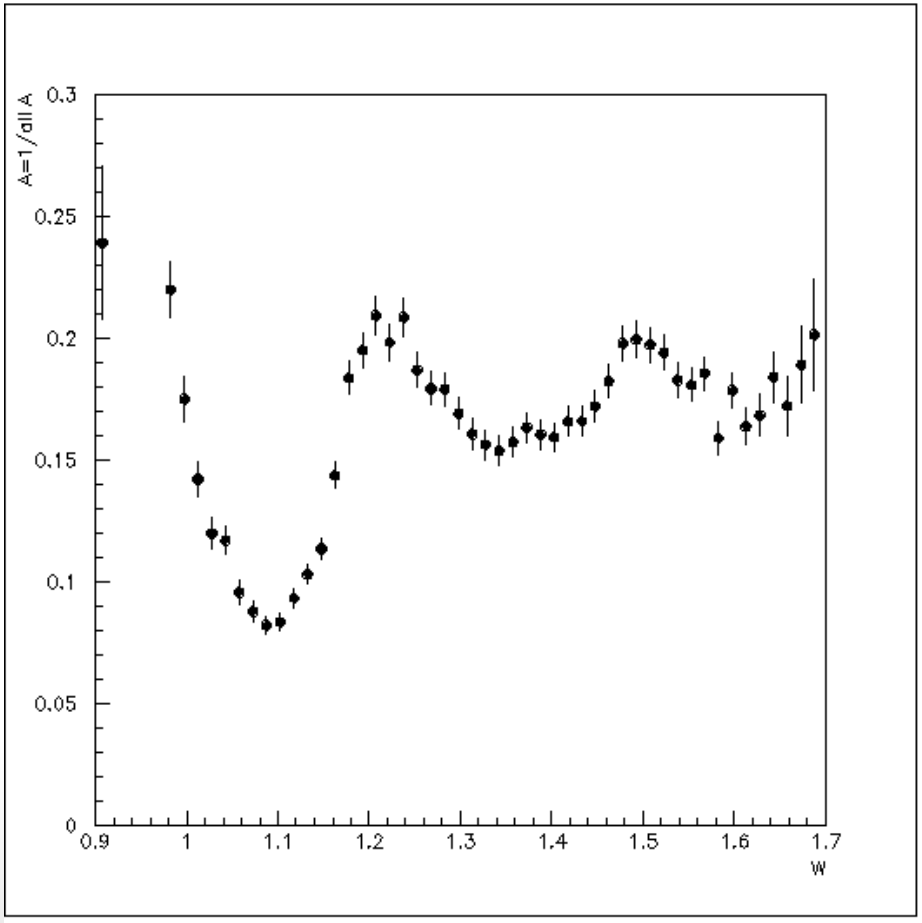


Packing fraction

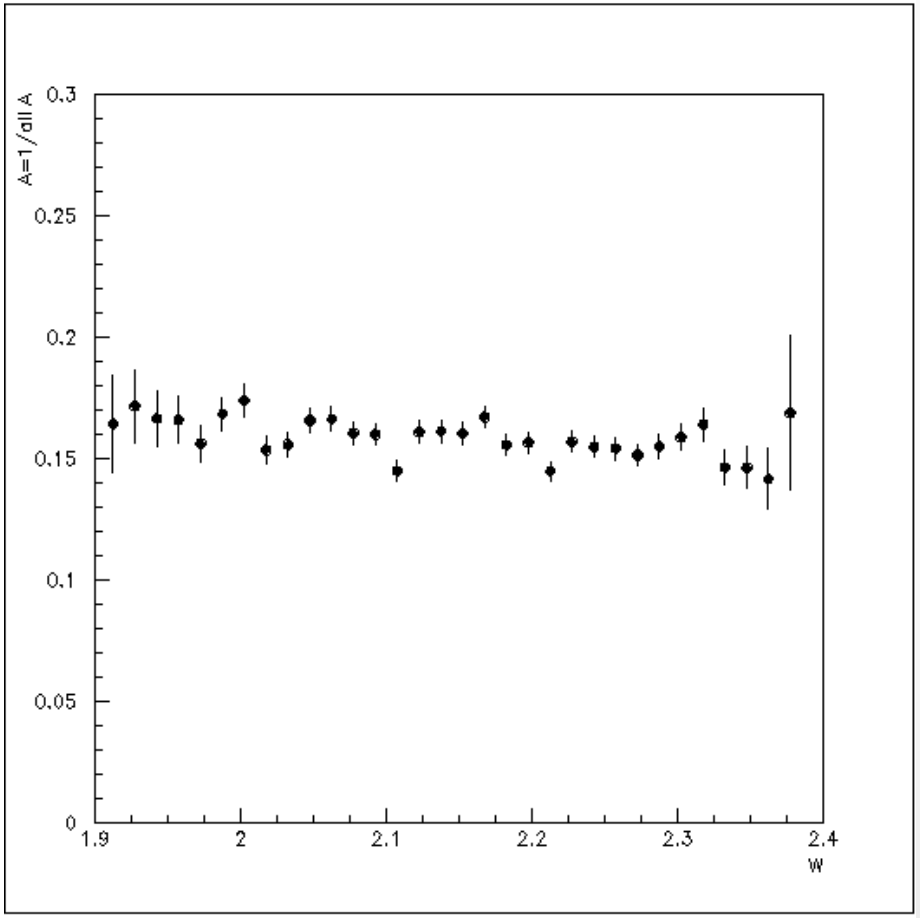
Material	PF(%)	Error(%p)
#2 JW 12/05 14NH3	61.0	4.32
14NH3 #13	60.7	4.22
#5 6-28-07 14NH3	59.6	4.47
#7 6-29-07 14NH3	59.6	4.47
#8 & 5 6-2x-07 14NH3	59.0	4.95
#5 & 6 6-2x-07 14NH3	56.9	4.74
#3 6/28/07 NIST irrad 14NH3	58.6	4.12
#2 6/28/07 NIST irrad 14NH3	58.8	4.34
#9 6-28-07 14NH3	61.9	4.59
#10 6-29-07 14NH3	59.4	4.44

Dilution factor

Dilution factor is calculated using MC, comparing cross sections of each materials in target cell. And packing fraction is the only necessary input for each target cell.



Dilution factor of resonance region (PF 60.9%)



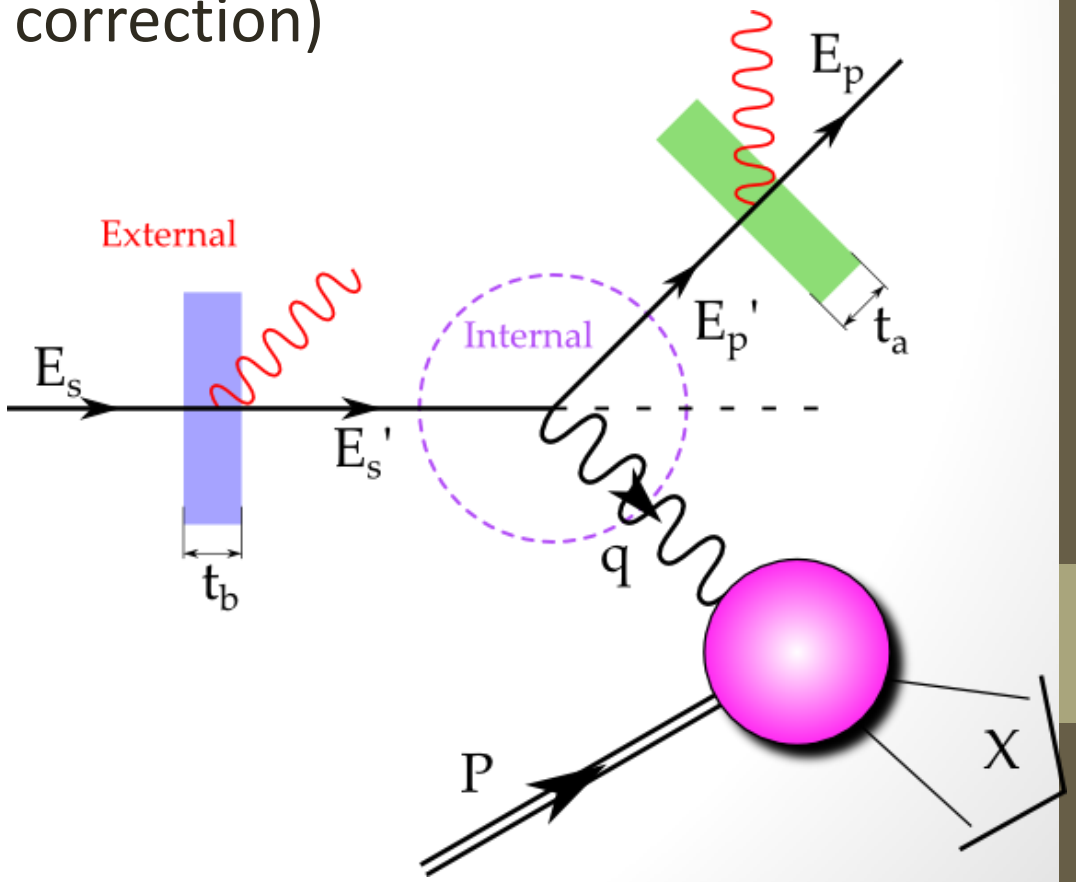
Dilution factor of DIS region (PF 57.2%)

Radiative Correction

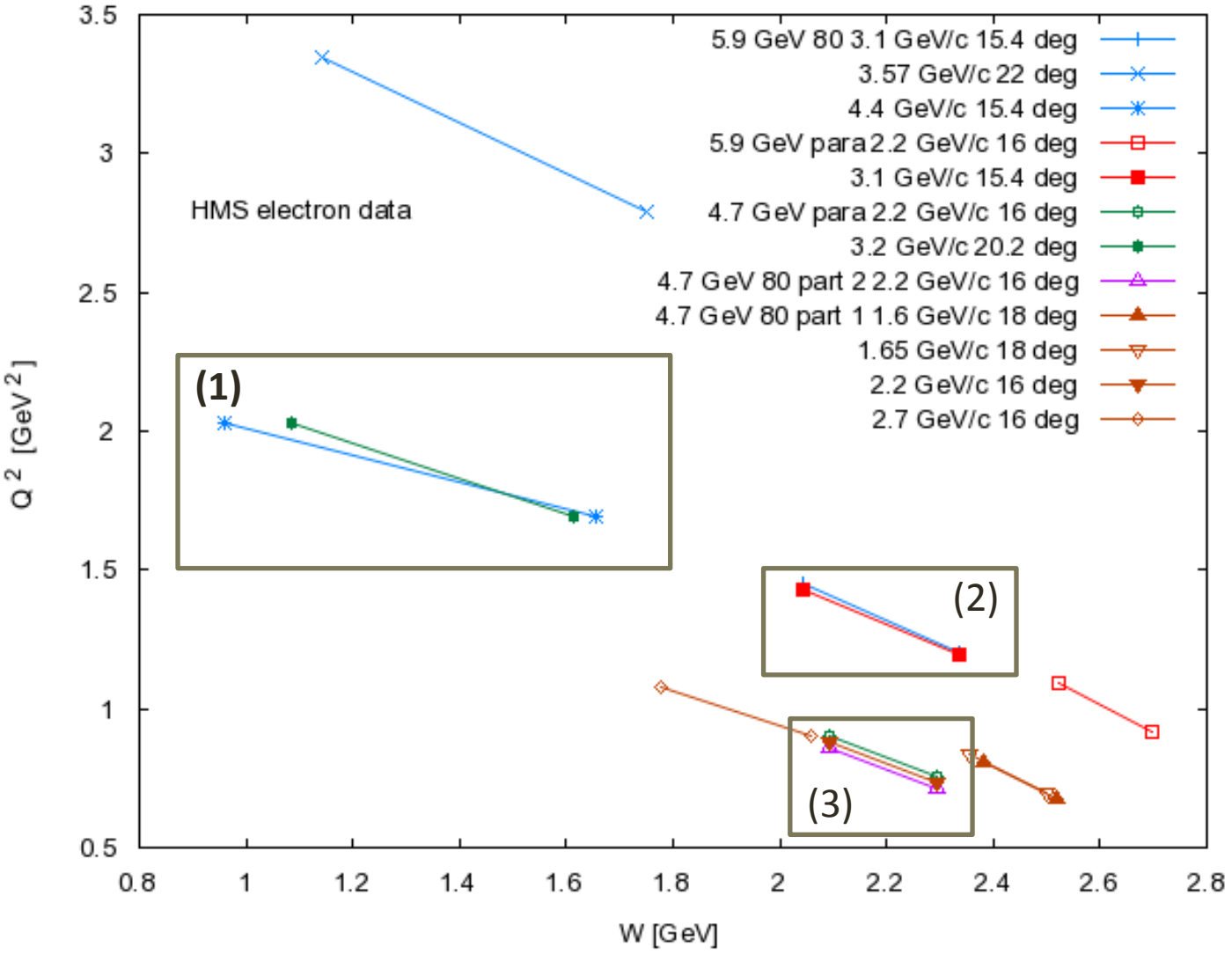
Following S. Stein et al., Phys. Rev. D 12, 1884 (1975), correction was mainly done by POLRAD 2.0

Initial fit parameters came from RSS, basically Breit-Wigner resonance and polynomial (with some correction) deep inelastic tail.

Newly corrected data was refitted to iterate.



HMS Asymmetries

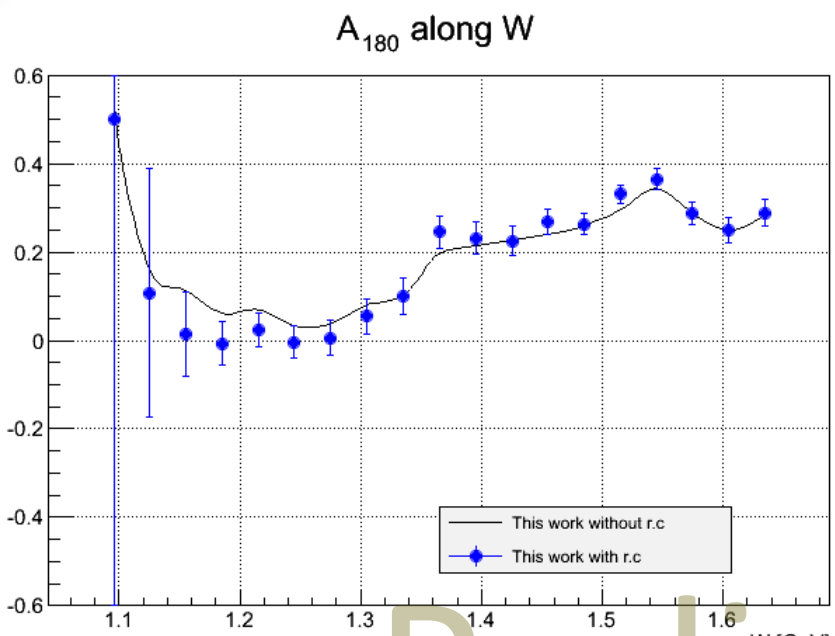


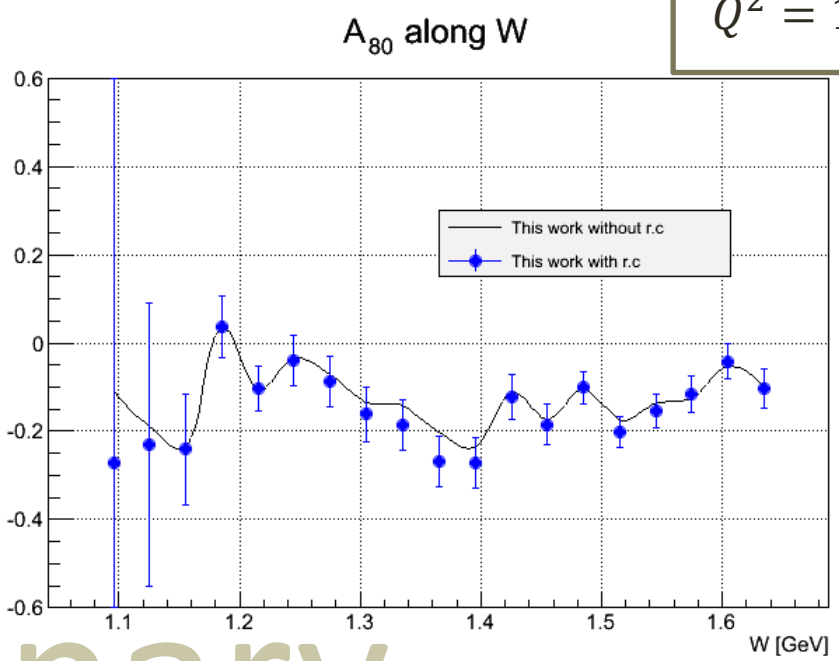
HMS Asymmetries

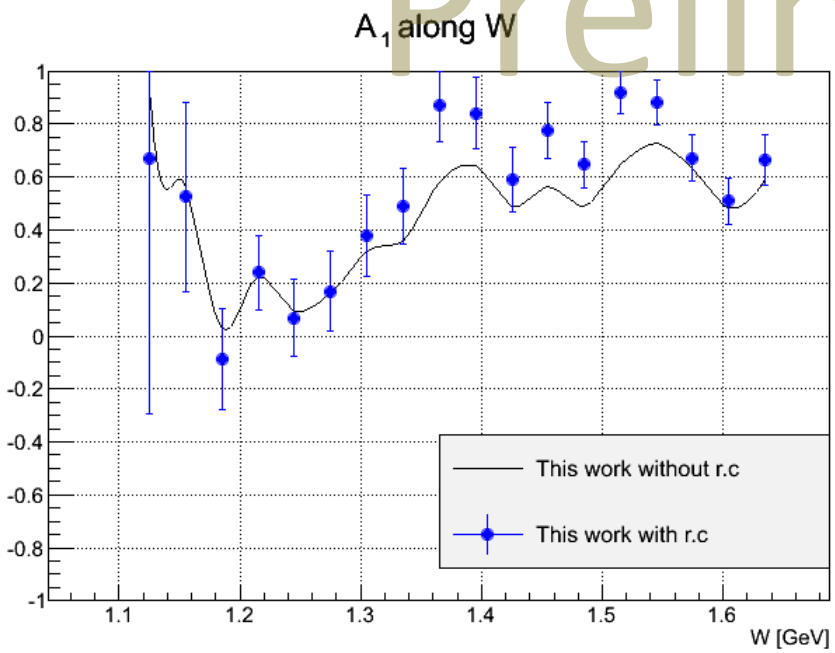
Setting	Beam energy (GeV)	HMS central momentum (GeV)	HMS angle from beamline (degree)	$< Q^2 > (GeV^2)$	$< W > (GeV)$
(1)	4.7 (par) / 5.9 (per)	3.2 (par) / 4.4 (per)	20.2 (par) / 15.4 (per)	1.863	1.353
(2)	5.9	3.1	15.4	1.313	2.196
(3)	4.7	2.2	16	0.806	2.196

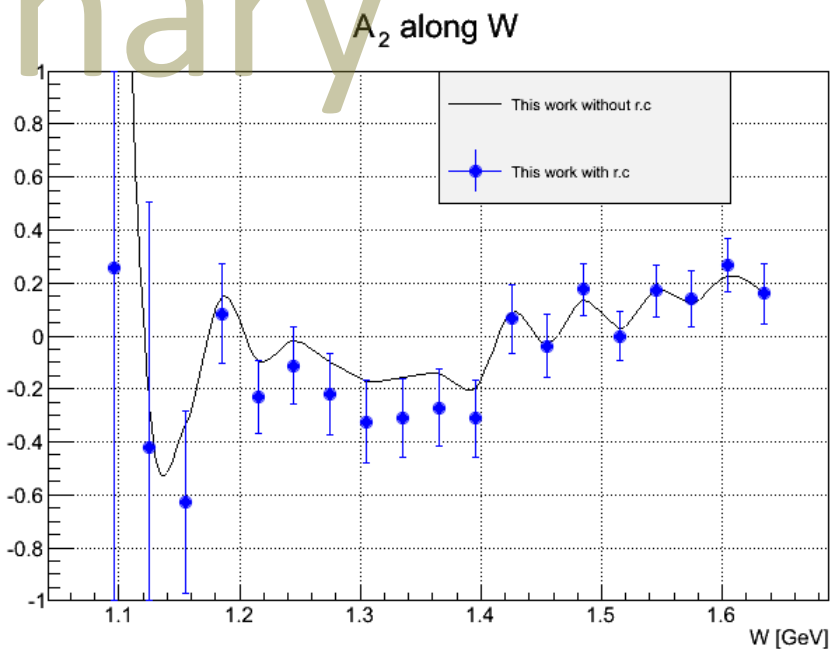
Asymmetries with(out) radiative correction

$Q^2 = 1.86\ GeV^2$









Asymmetries A_1 and A_2

A_1 and A_2 are virtual photoabsorption asymmetries.

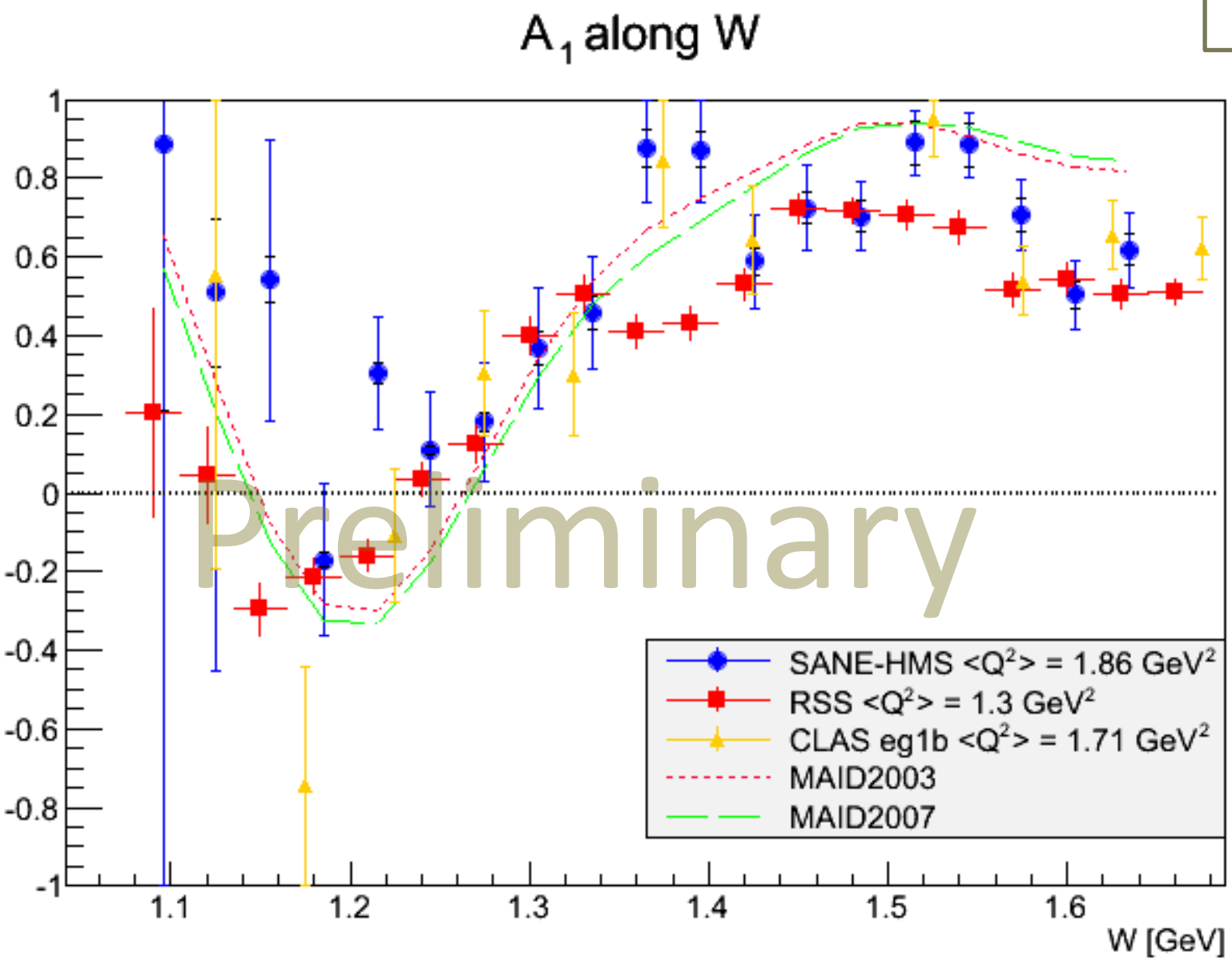
$$A_1 = \frac{\sigma_{1/2}^T - \sigma_{3/2}^T}{\sigma_{1/2}^T + \sigma_{3/2}^T} = \frac{\sigma_{TT}}{\sigma_T} = \frac{g_1 - \gamma^2 g_2}{F_1}$$

$$A_2 = \frac{2\sigma_{LT}}{\sigma_{1/2}^T + \sigma_{3/2}^T} = \frac{\sigma_{LT}}{\sigma_T} = \frac{\gamma(g_1 + g_2)}{F_1}$$

$\sigma_{1/2}^T$ and $\sigma_{3/2}^T$ are the virtual photon absorption transverse cross sections when total helicity of photon and nucleon is 1/2 and 3/2 respectively. σ_{LT} is the interference term between the transverse and longitudinal photon-nucleon amplitude.

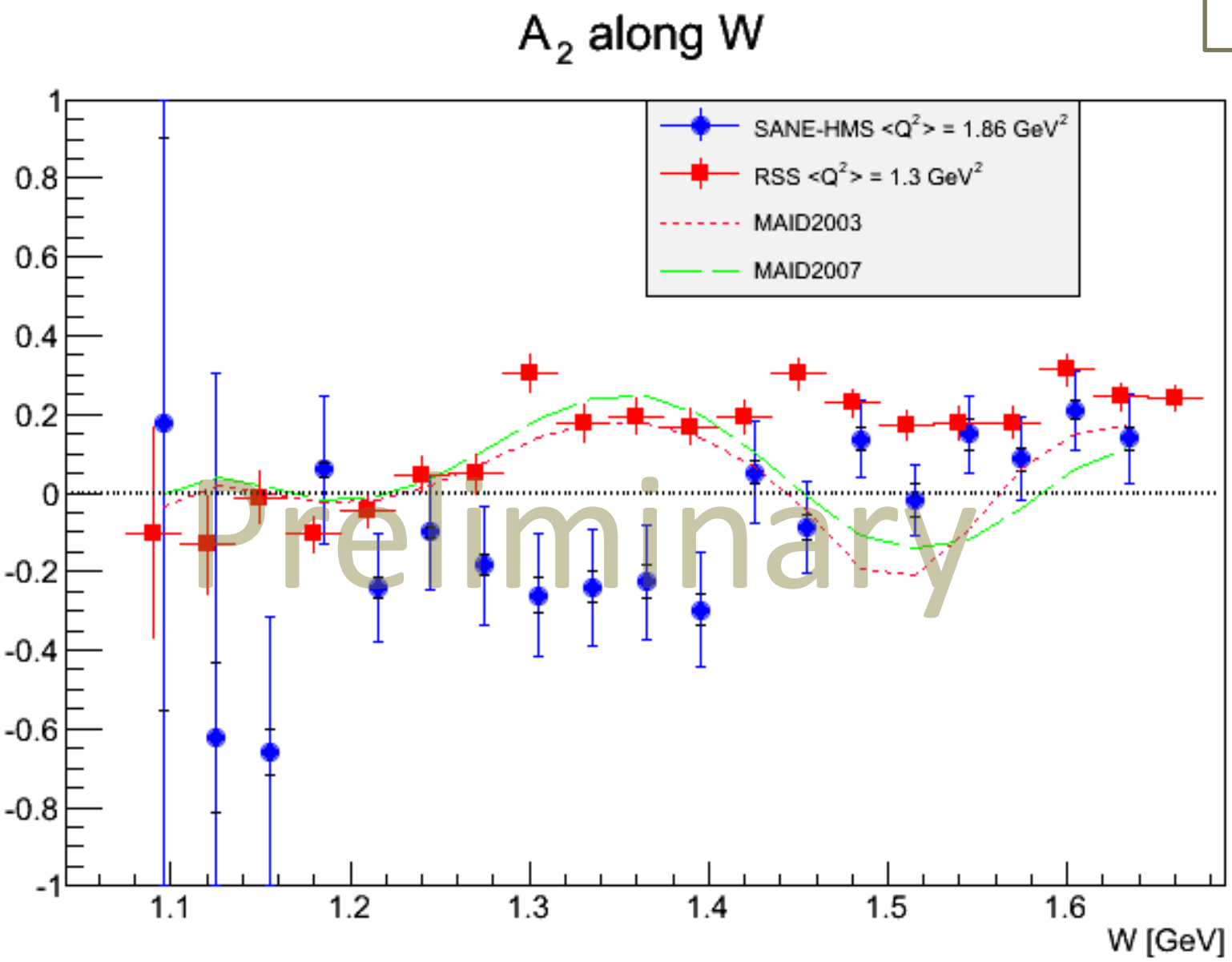
Asymmetry $A_1 = \frac{\sigma_{TT}}{\sigma_T}$

$Q^2 = 1.86\ GeV^2$



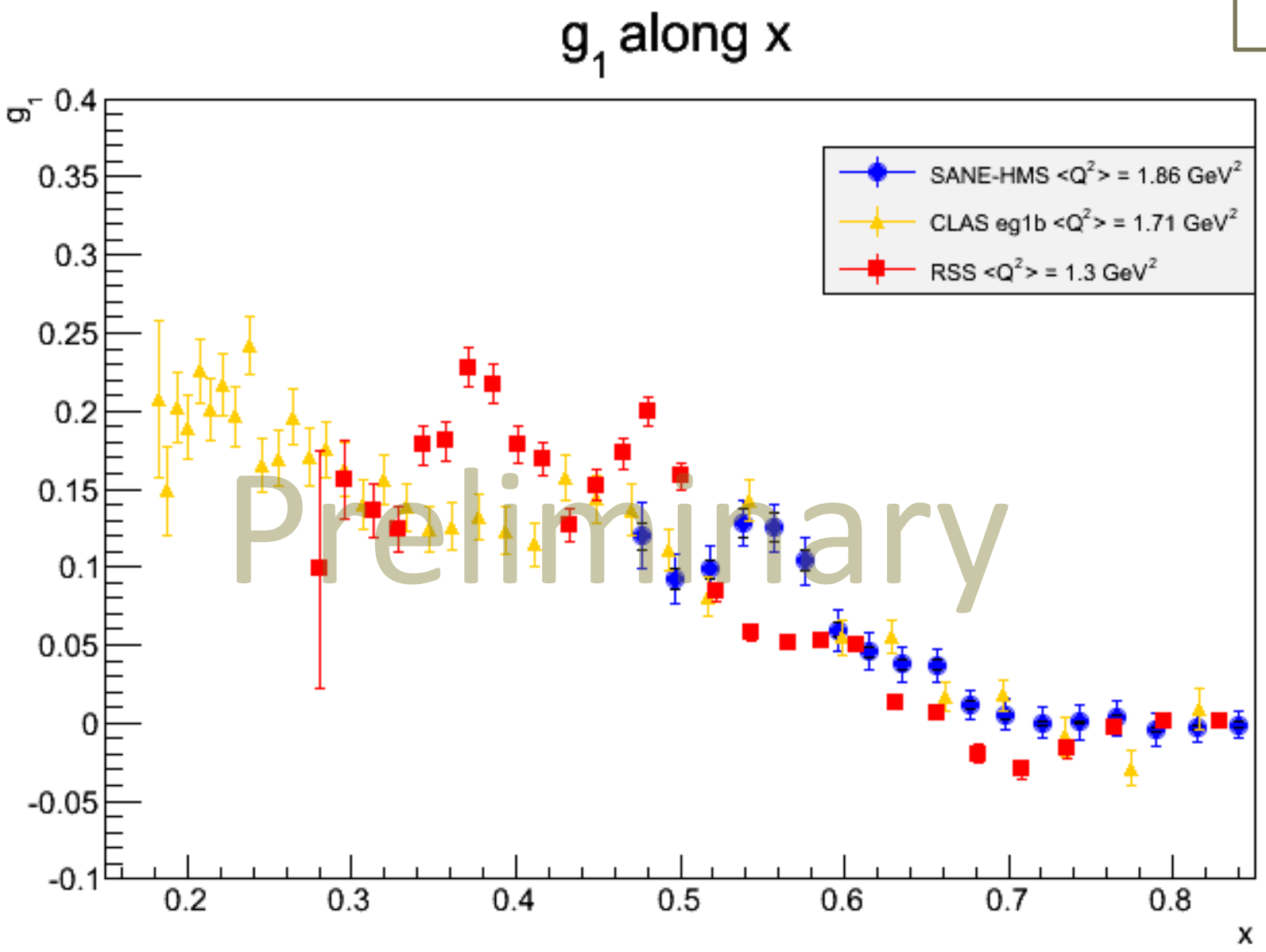
Asymmetry $A_2 = \frac{\sigma_{LT}}{\sigma_T}$

$Q^2 = 1.86\ GeV^2$



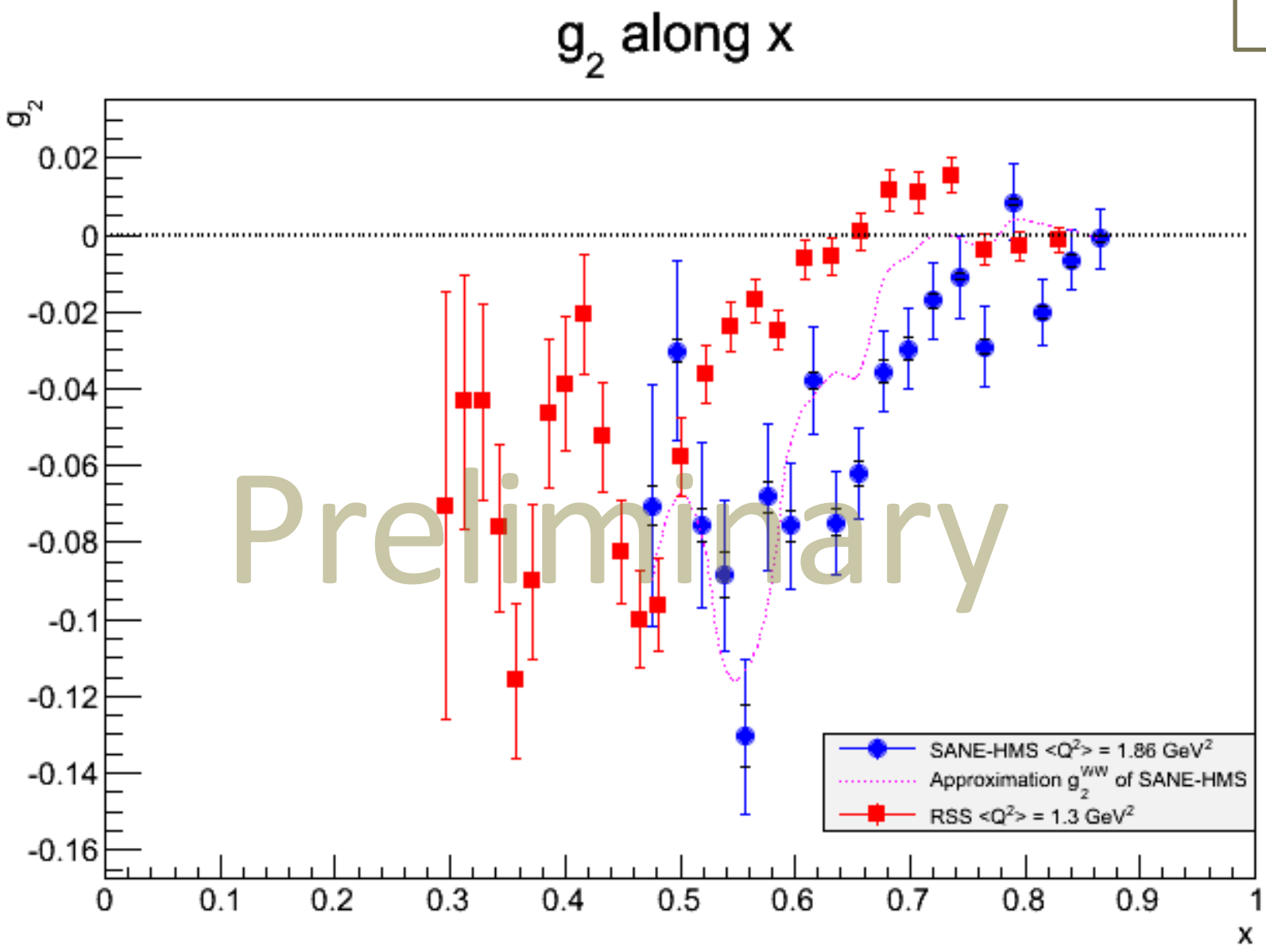
Structure Function g_1

$Q^2 = 1.86\ GeV^2$



Structure Function g_2

$Q^2 = 1.86\ GeV^2$



Preliminary

Twist-3 matrix element d_2

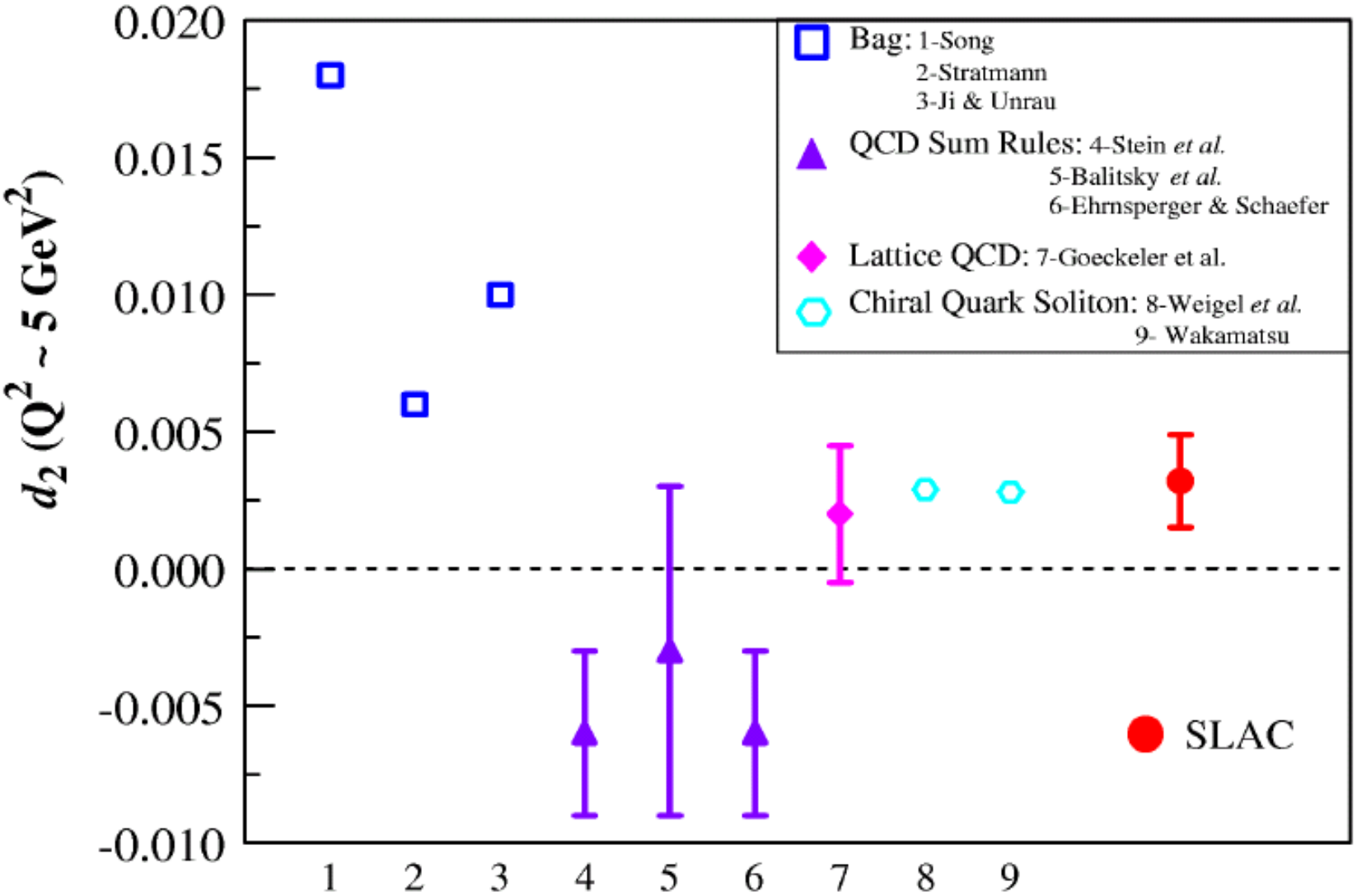
If we ignore higher twist effects, g_2 is determined by g_1, using Wandzura-Wilczek relation:

$$g_2{}^{WW}(x,Q^2) = -g_1(x,Q^2) + \int_x^1 g_1(y,Q^2)\frac{dy}{y}$$

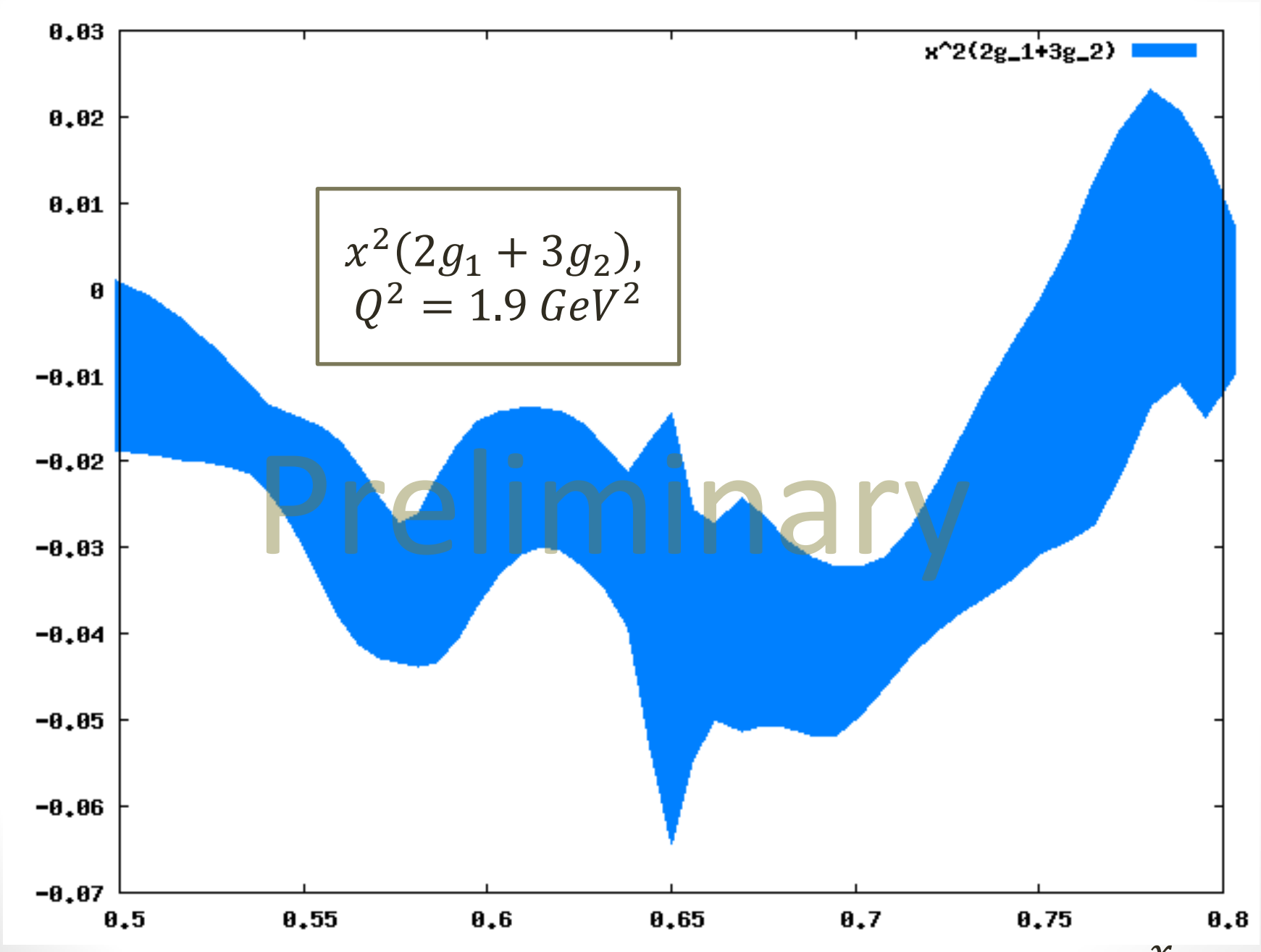
OPE relates the twist-3 matrix element d_2 to the following:

$$d_2 = 3\int_0^1 x^2(g_2 - g_2{}^{WW})dx = \int_0^1 x^2(2g_1 + 3g_2)dx$$

Twist-3 matrix element d_2



Twist-3 matrix element d_2



Twist-3 matrix element d_2

Very preliminary result of the limited x range is

$$\overline{d_2} = -0.0072 \pm 0.0004$$

$$\overline{d_2} = \int_{0.5}^{0.8} x^2(2g_1 + 3g_2)dx$$

A_1, A_2 fit evaluated at $Q^2 = 1.9\ GeV^2$

SANE collaboration

U. Basel, Florida International U., Hampton U., Norfolk S. U., North Carolina A&T S. U., IHEP-Protvino, U. of Regina, Rensselaer Polytechnic I., Rutgers U., Seoul National U., Temple U., TJNAF, U. of Virginia, College of William & Mary, Yerevan Physics I.

Spokespersons: S. Choi (Seoul), M. Jones(Jlab), Z-E. Meziani (Temple), O. A. Rondon (U. of Virginia)

Summary

- SANE-HMS data covers various kinematic regions and it can produce meaningful results, besides BETA(main detector) data.
- Preliminary spin asymmetry A_1 and structure function g_1 shows good agreement with previous experiments.
- But A_2 and g_2 show remarkable Q^2 dependency.
- Preliminary twist-3 matrix element d_2 of the limited x range is -0.0072 ± 0.0004.
- World first proton transverse spin data on this kinematic region is expected to improve global fits.

Backup slides

Experimental goal

The goal is to get the proton spin structure functions g_1 and g_2 over broad range of

the Bjorken scaling variable

$$0.3 \leq x \leq 0.8$$

and 4-momentum transfer

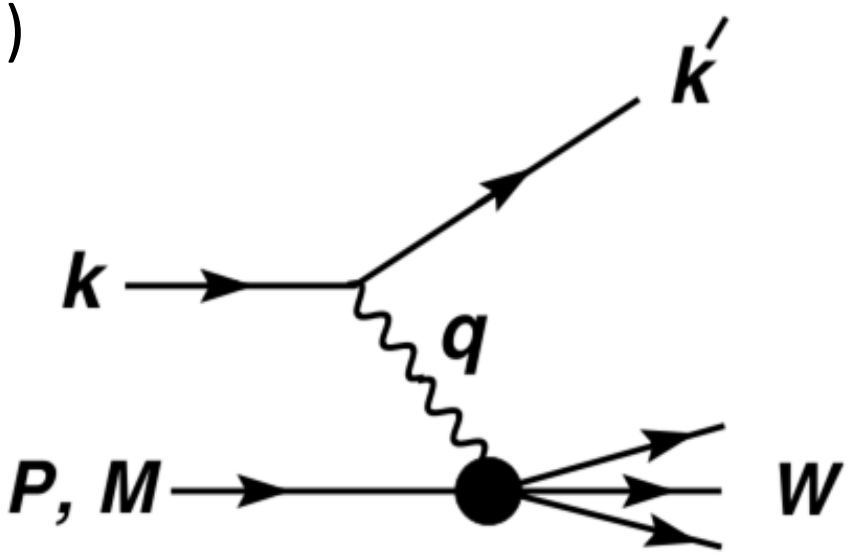
$$2.5 GeV^2 \leq Q^2 \leq 6.5 GeV^2$$

ep deep inelastic scattering

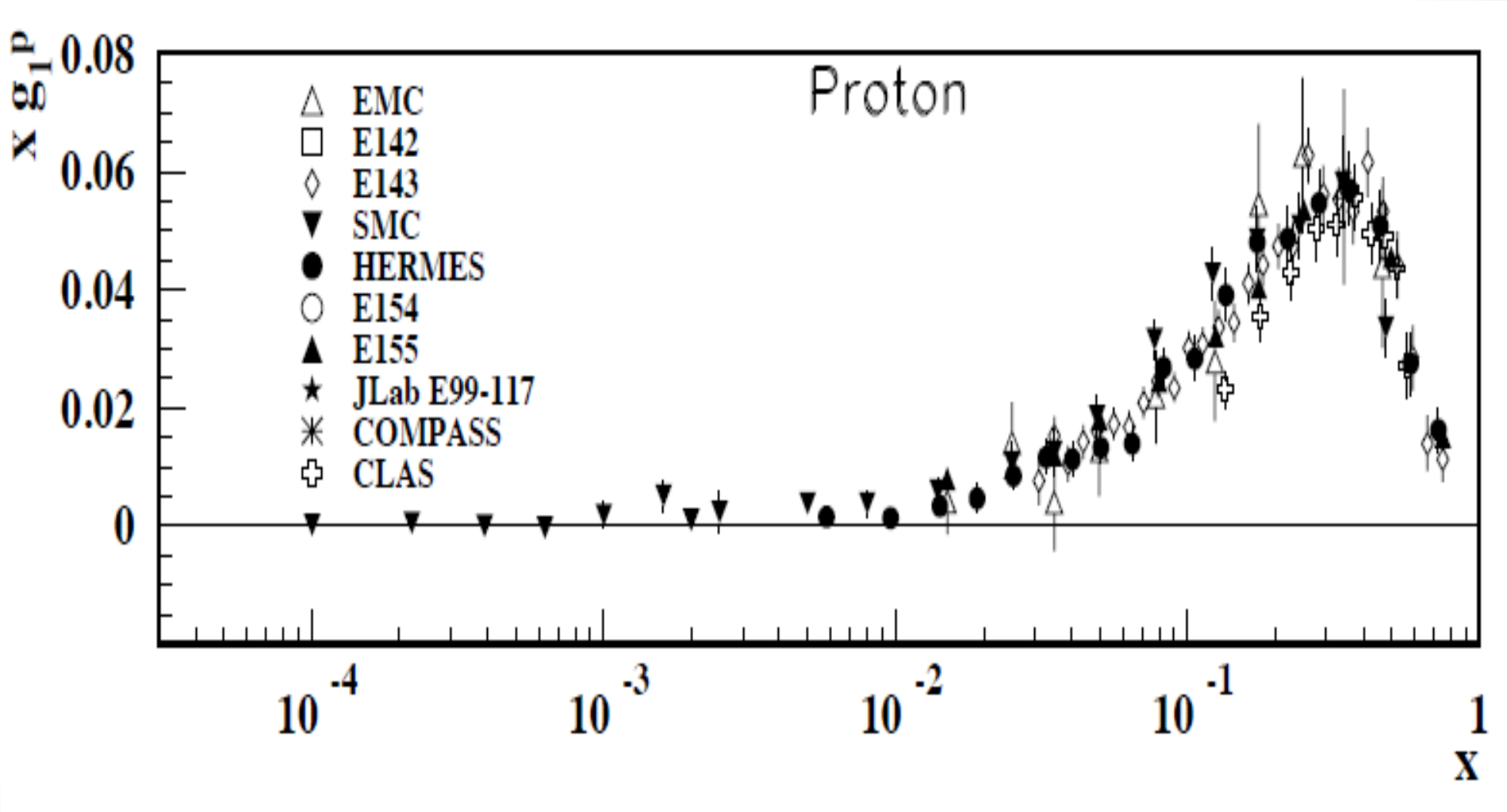
High-energy electron-nucleon scattering(Deep Inelastic Scattering) ep → e'X

k and k' are the four-momenta of the incoming and outgoing electrons, P is the four-moemntum of a proton with mass M, and W is the mass of the recoiling system X.

q is the four-momentum of the virtual photon(the exchanged particle). ($Q^2 = -q^2$)



World Data of g_1



C. Amsler et al., Physics Letters B667, 1 (2008)

Jefferson Lab

Thomas Jefferson National Accerator Facility

Jefferson Lab
Exploring the Nature of Matter

Located in Newport News, Virginia, USA

Funded by the U.S. Department of Energy's Office of Science

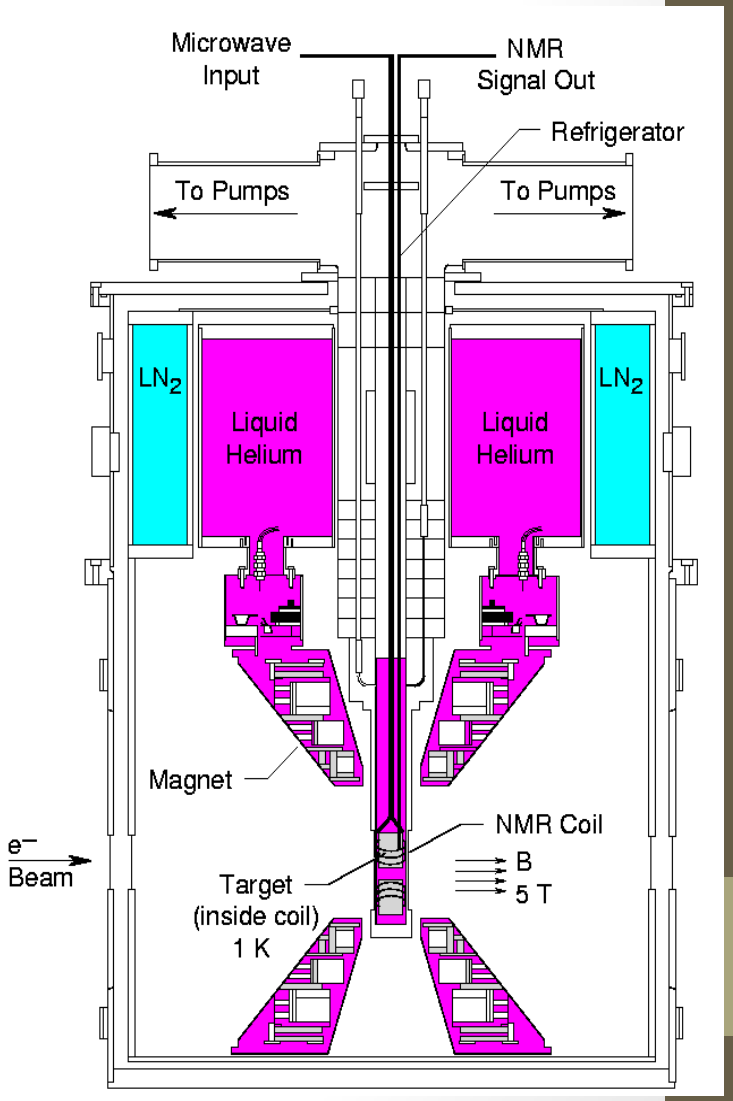
Having CEBAF (Continuous Electron Beam Accelerator Facility) and three experimental halls

UVA Target

Superconducting magnet applies 5-T magnetic field.

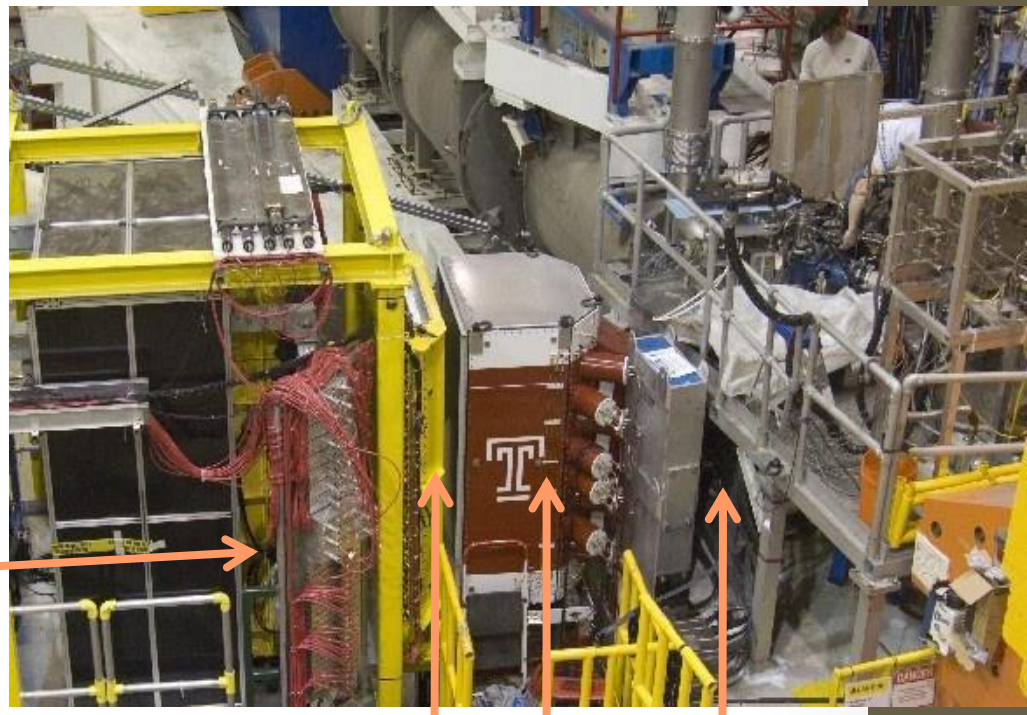
The target and the magnet are cooled down by liquid helium.

~140 Ghz microwave frequency causes dynamic nuclear polarization up to 90%.

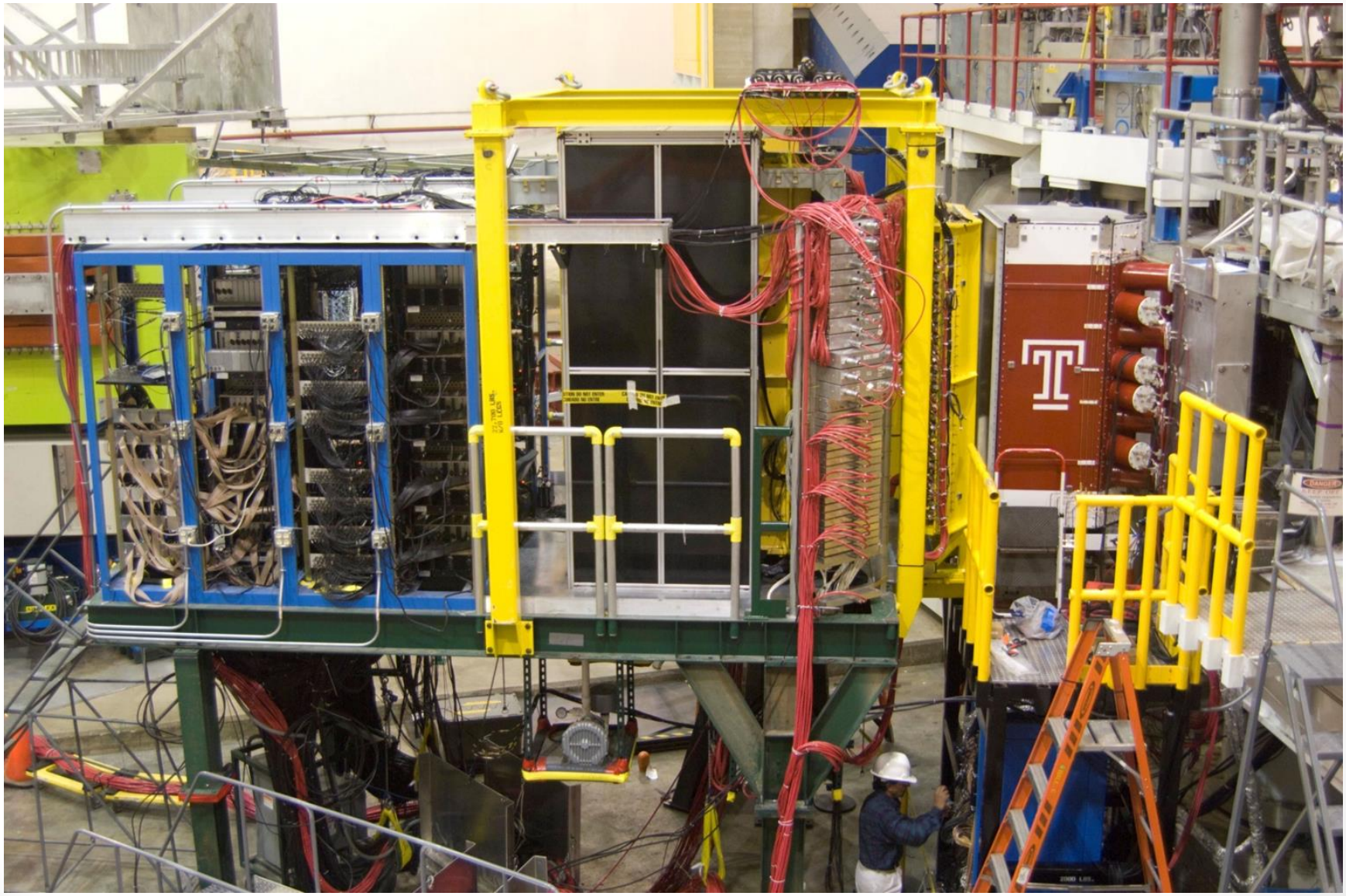


Big Electron Telescope Array - BETA

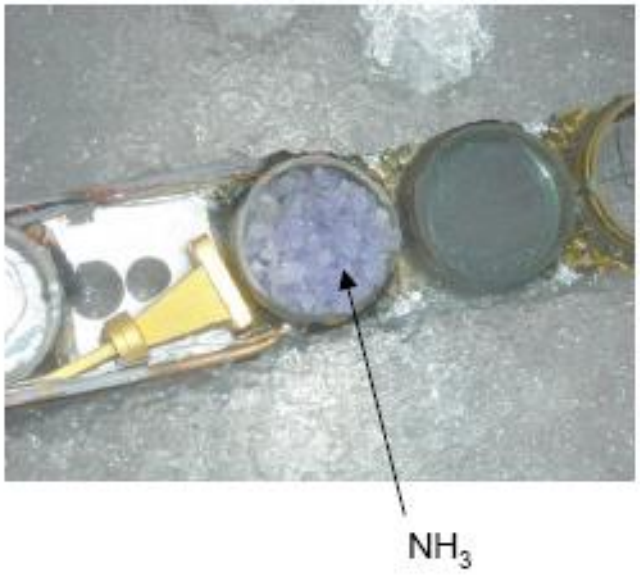
- **BigCal** lead glass calorimeter: main detector, being built for *GEp-III*.
- **Gas Cherenkov**: additional pion rejection
- Tracking **Lucite hodoscope**
- BETA's characteristics
  - Effective solid angle = 0.194 sr
  - Energy resolution $5\%/\sqrt{E(\text{GeV})}$
  - angular resolution < 0.8°
  - 1000:1 pion rejection
- Added: **forward hodoscope**
  - vertex resolution ~ 5 mm
  - angular resolution ~ 1 mr
- Target field sweeps low *E* background

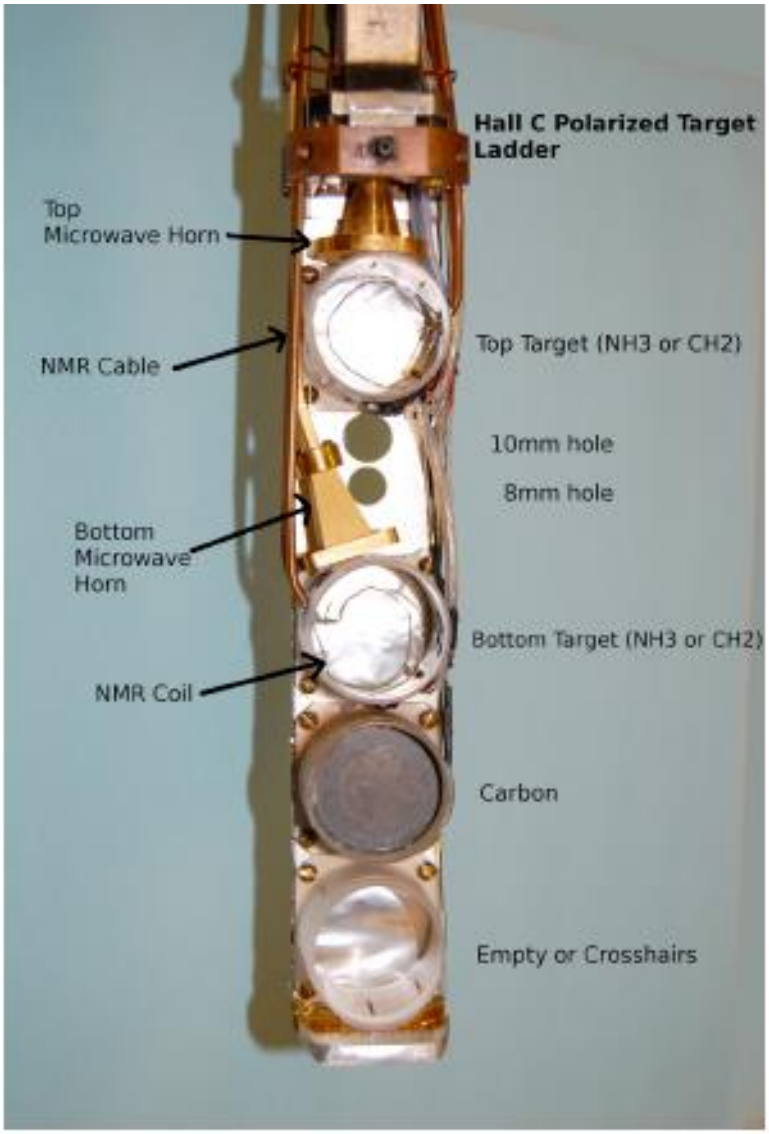


BETA



Target insert



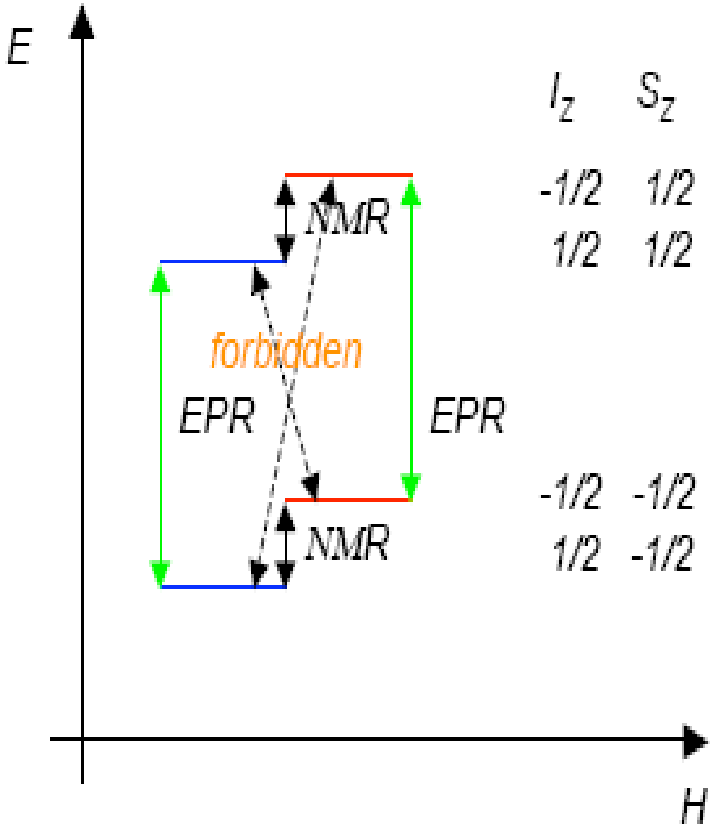


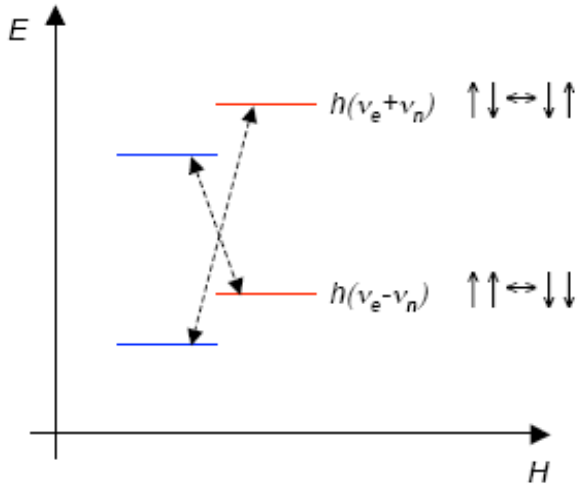
Dynamic Nuclear Polarization

The DNP process for polarizing protons, deuterons, or any nucleus possessing a magnetic moment, requires temperatures of ;1 K or less and large magnetic holding fields. For thermal equilibrium at 1 K and 5 T, the proton polarization is only about 0.5%. However, the polarization of the “free” electrons, associated with the paramagnetic radicals introduced into the target material, is greater than 99%. The electron polarization can be transferred to the proton through a hyperfine transition by irradiating the target with microwaves at appropriate frequencies.

K. Abe et al., Physical Review D 58, 11203 (1998)

Dynamic Nuclear Polarization





The rf field drives the forbidden transitions.

flip-flops: ↑↓↔↑↓

flip-flops: ↑↑↔↓↓

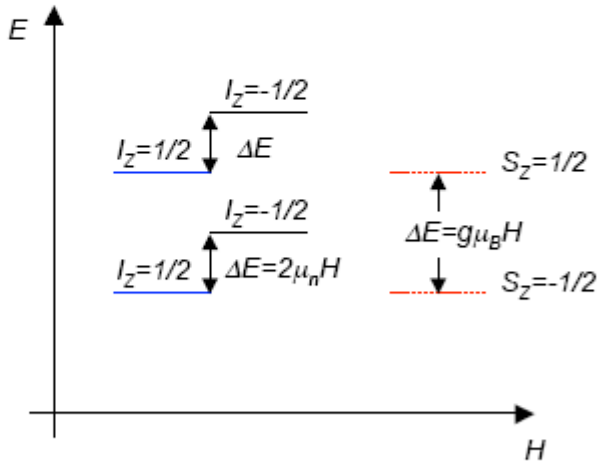
$\Delta(S_Z+I_Z) = 0,2$

Dynamic Nuclear Polarization

Nucleons also possess magnetic moment, although:

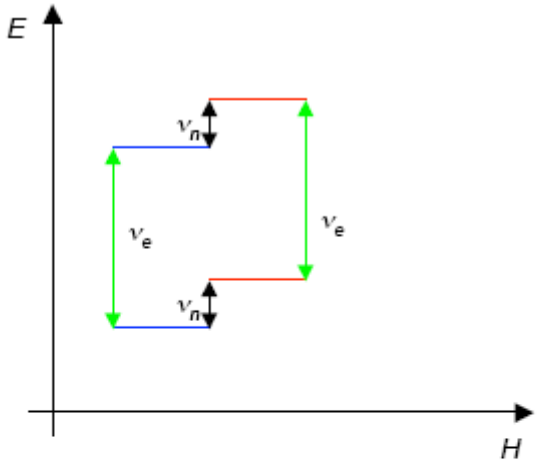
Nuclear moments/Electron Moments ~ 10^{-3}

Including nuclear moments => Hyperfine Splitting



S_z: Electrons

I_z: Nucleons, Nuclei



1) $\nu_{rf} \sim \nu_e$ get transitions -- -> -+
++ -> +-

where $\Delta E = g\mu_B H = h\nu_e$

2) $\nu_{rf} \sim \nu_n$ get transitions +- -> ++
++ -> -+

where $\Delta E = 2\mu_n H = h\nu_n$

Dilution Factor from Packing Fraction

Total yield has linear relation with packing fraction:

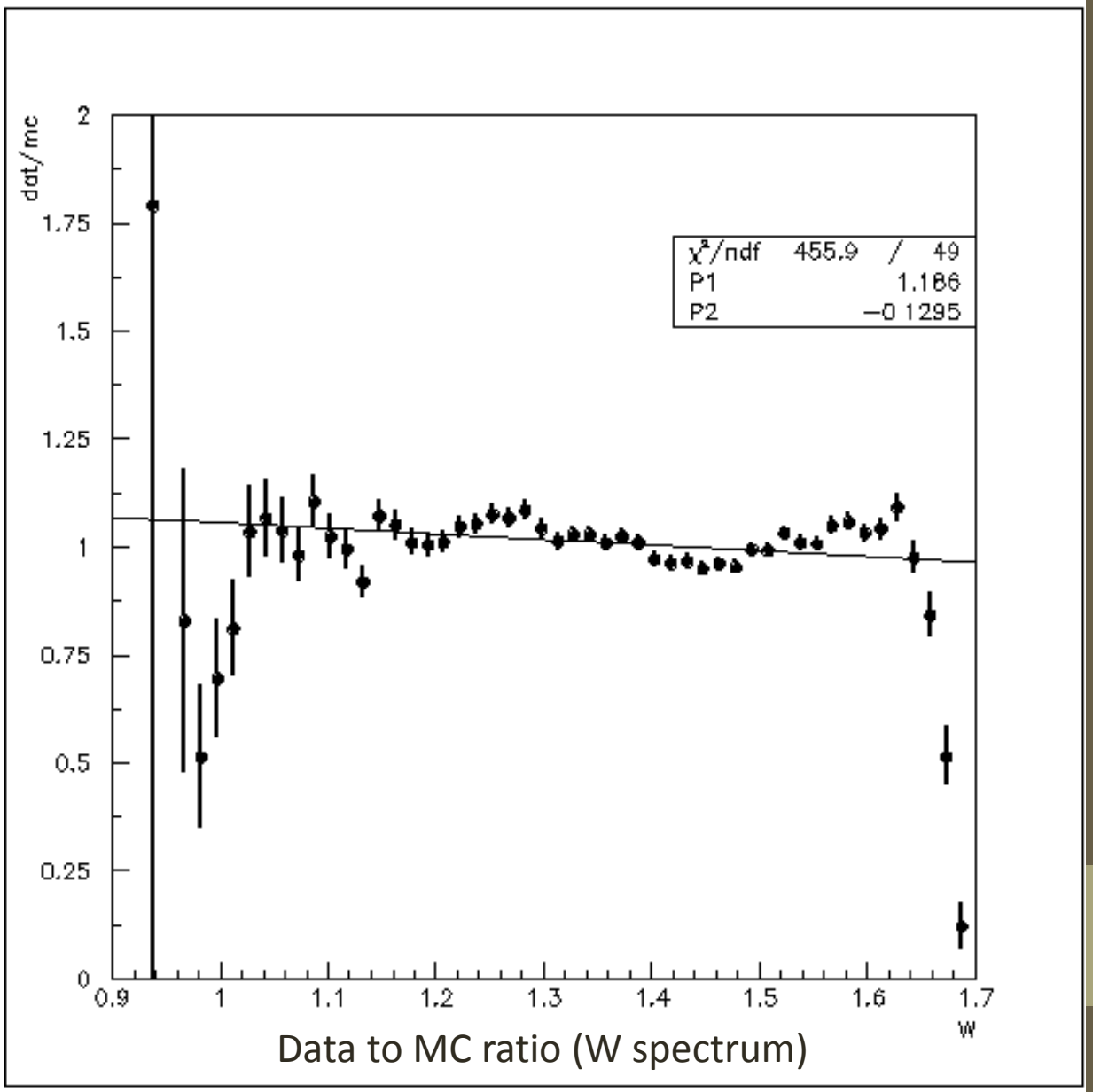
$$Y_T = m\, pf + b$$

Using MC (P. E. Bosted and M. E. Christy, Phys. Rev., C 77, 065206 (2008)) assuming two different *pf*, the slope(m) and intercept(b) can be calculated and then the yield of real data produces *pf* of real target.

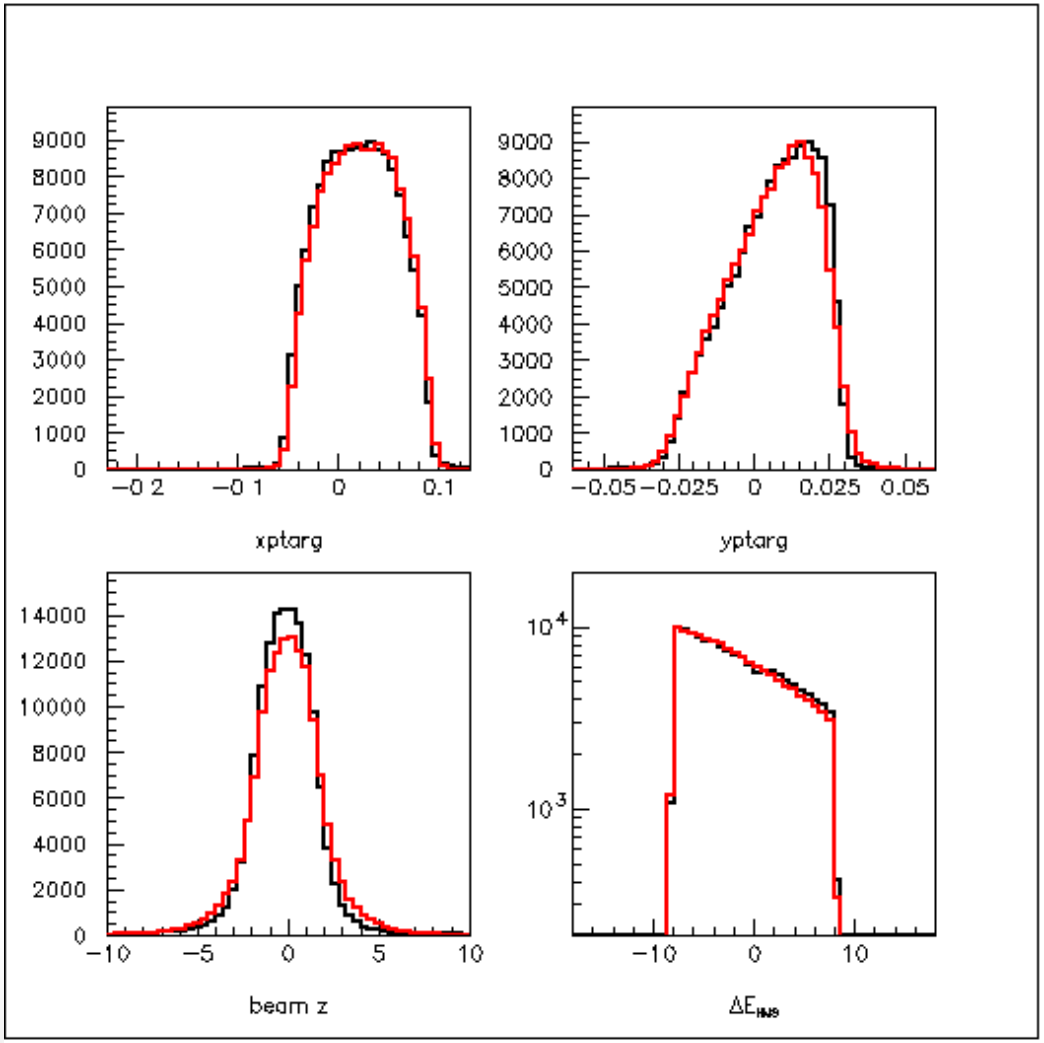
SANE packing fractions are 56% - 62% with ~4.5% error.

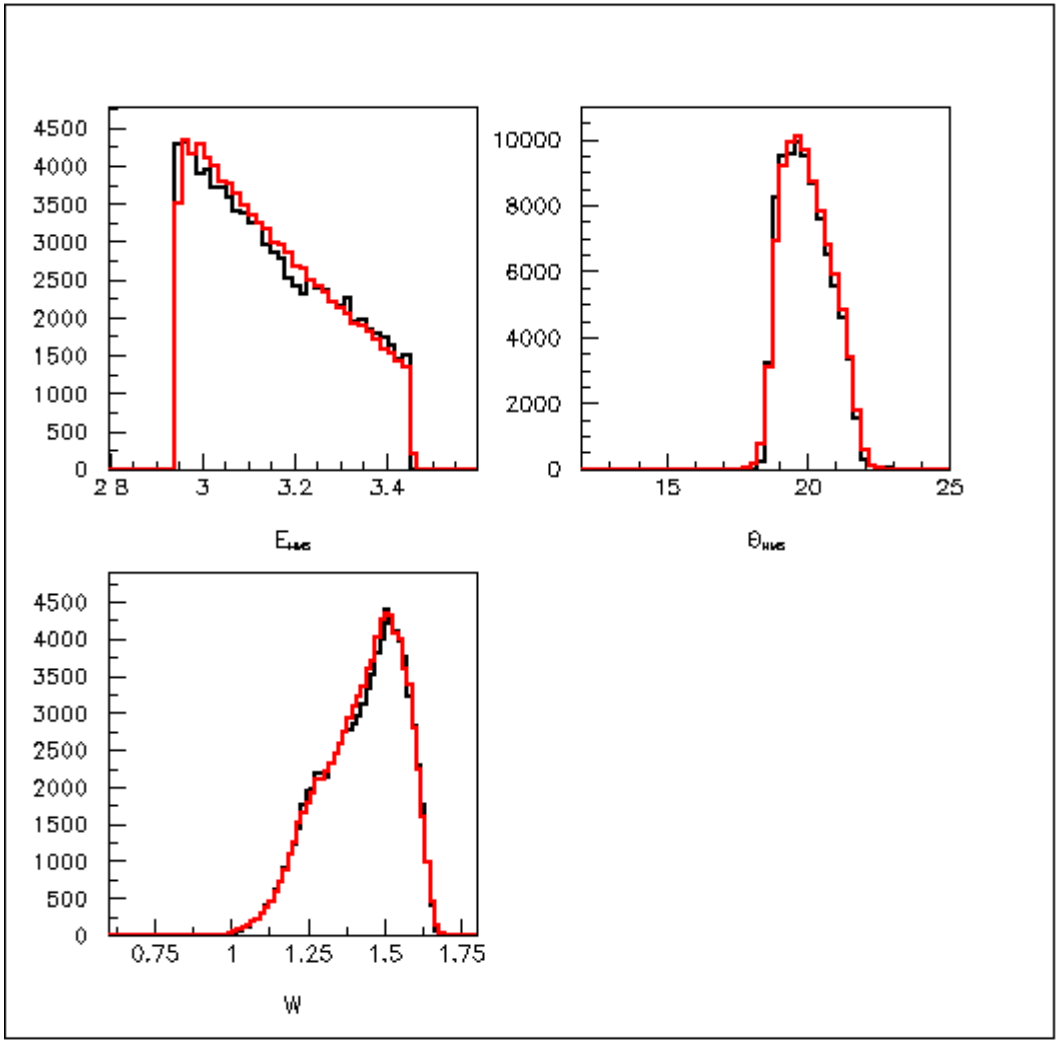
Packing Fraction

Comparing data with Monte Carlo results assuming 50% and 60% packing fraction of target, 60.9% packing fraction is determined for the target material #9 6-28-07 14NH3.



Packing Fraction





Data and MC comparison (Red is MC)

Fitting Function

$$\text{fit} = \underbrace{\sum_{i=1}^{4} BW_i}_{\text{Resonances}} + \underbrace{x^{\alpha} \sum_{n=0}^{3} \beta_n \, x^n}_{\text{DIS}} \times \underbrace{\frac{1}{\sqrt{Q^2}}}_{A_2 \text{ only}}$$

where

$$BW_i = \frac{a_i \; \kappa_i^2 \; w_i^2 \; \Gamma_i \; \Gamma_i^{\gamma}}{\kappa_{cm}^2 \; [(w_i^2 - W^2)^2 + w_i^2 \; \Gamma_i^2]}$$

$$= g_i \left(\frac{q_{cm}}{q_i}\right)^{(2l_i+1)} \left(\frac{q_i^2 + X_i^2}{q_{cm}^2 + X_i^2}\right)^{l_i}$$

$$= g_i \left(\frac{\kappa_{cm}}{\kappa_i}\right)^{(2j_i)} \left(\frac{\kappa_i^2 + X_i^2}{\kappa_{cm}^2 + X_i^2}\right)^{j_i}$$

$$\kappa_i = \sqrt{\frac{(w_i^2 + M^2 + Q^2)^2}{4w_i^2} - M^2}$$

$$q_i = \sqrt{\frac{(w_i^2 + M^2 - m_\pi^2)^2}{4w_i^2} - M^2}$$

$$\kappa_{cm} = \sqrt{\frac{(W^2 + M^2 + Q^2)^2}{4W^2} - M^2}$$

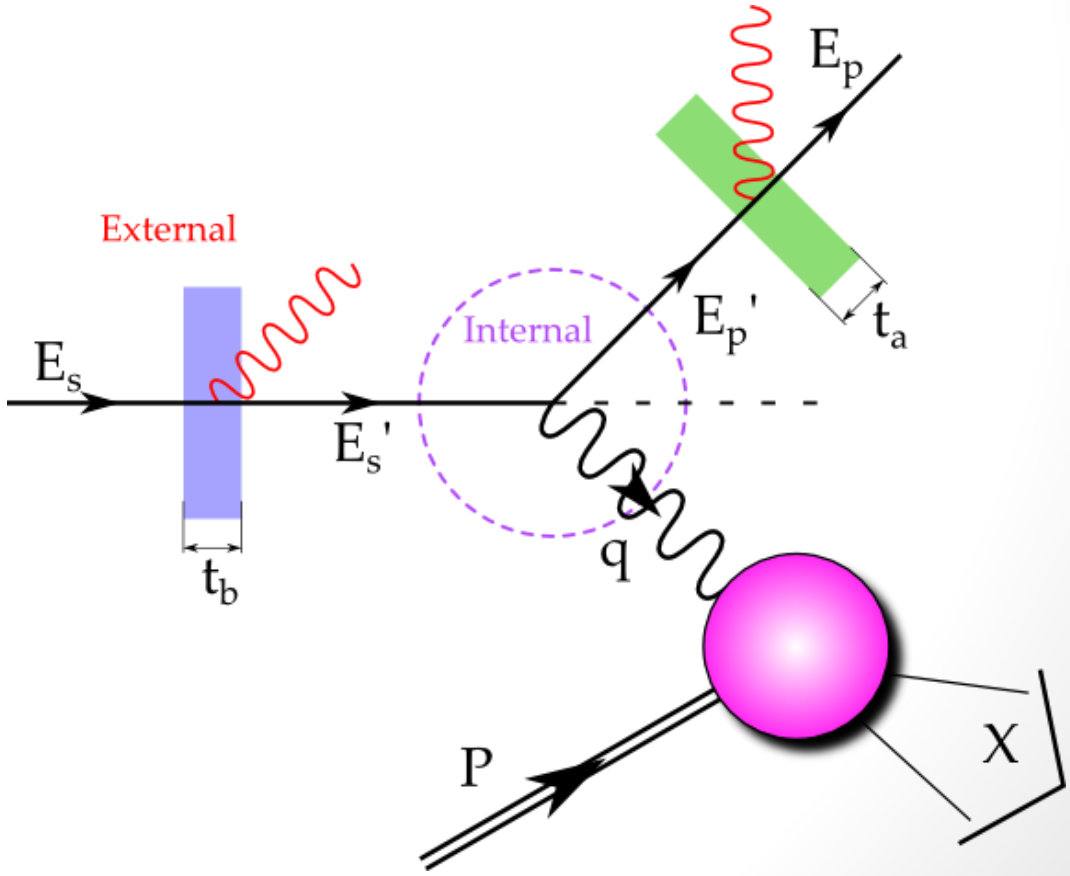
$$q_{cm} = \sqrt{\frac{(W^2 + M^2 - m_\pi^2)^2}{4W^2} - M^2}$$

Radiative Correction

$$A = \frac{1}{f C_N P_b P_t f_{RC}} \frac{d\sigma^{\downarrow\uparrow} - d\sigma^{\uparrow\uparrow}}{d\sigma^{\downarrow\uparrow} + d\sigma^{\uparrow\uparrow}} + A_{RC}$$

Radiative Correction

1. Incoming and outgoing electron lose energy before and after scattering.
2. Elastic tail should be subtracted.
3. QED processes other than Born contributes to data.



ep deep inelastic scattering

Invariant quantities:

$\nu = \dfrac{q \cdot P}{M} = E - E'$ is the lepton's energy loss in the nucleon rest frame (in earlier literature sometimes $\nu = q \cdot P$). Here, E and E' are the initial and final lepton energies in the nucleon rest frame.

$Q^2 = -q^2 = 2(EE' - \vec{k} \cdot \vec{k}') - m_\ell^2 - m_{\ell'}^2$ where $m_\ell (m_{\ell'})$ is the initial (final) lepton mass. If $EE' \sin^2(\theta/2) \gg m_\ell^2, m_{\ell'}^2$, then

$\approx 4EE' \sin^2(\theta/2)$, where θ is the lepton's scattering angle with respect to the lepton beam direction.

$x = \dfrac{Q^2}{2M\nu}$ where, in the parton model, x is the fraction of the nucleon's momentum carried by the struck quark.

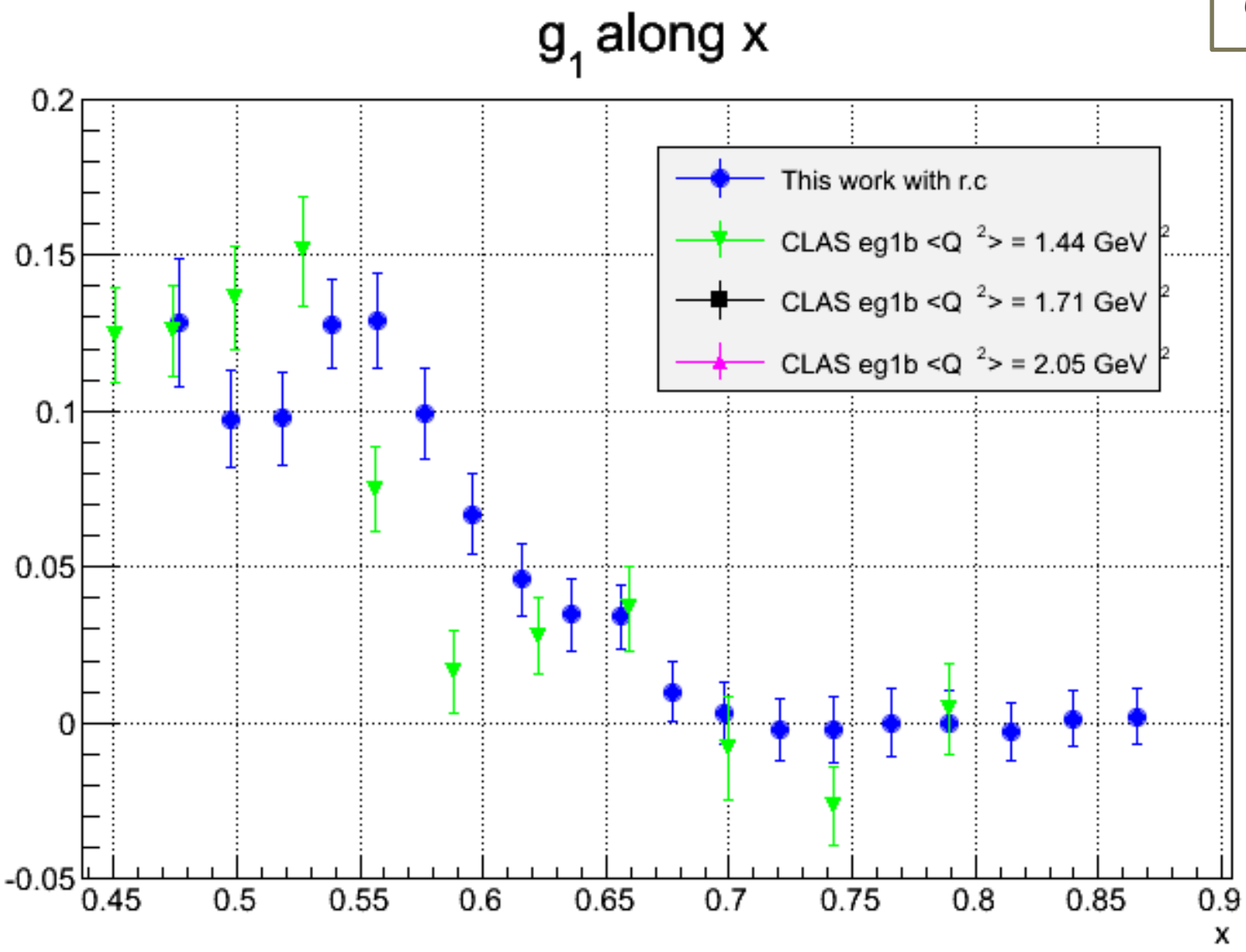
$y = \dfrac{q \cdot P}{k \cdot P} = \dfrac{\nu}{E}$ is the fraction of the lepton's energy lost in the nucleon rest frame.

$W^2 = (P+q)^2 = M^2 + 2M\nu - Q^2$ is the mass squared of the system X recoiling against the scattered lepton.

C. Amsler et al., Physics Letters B667, 1 (2008)

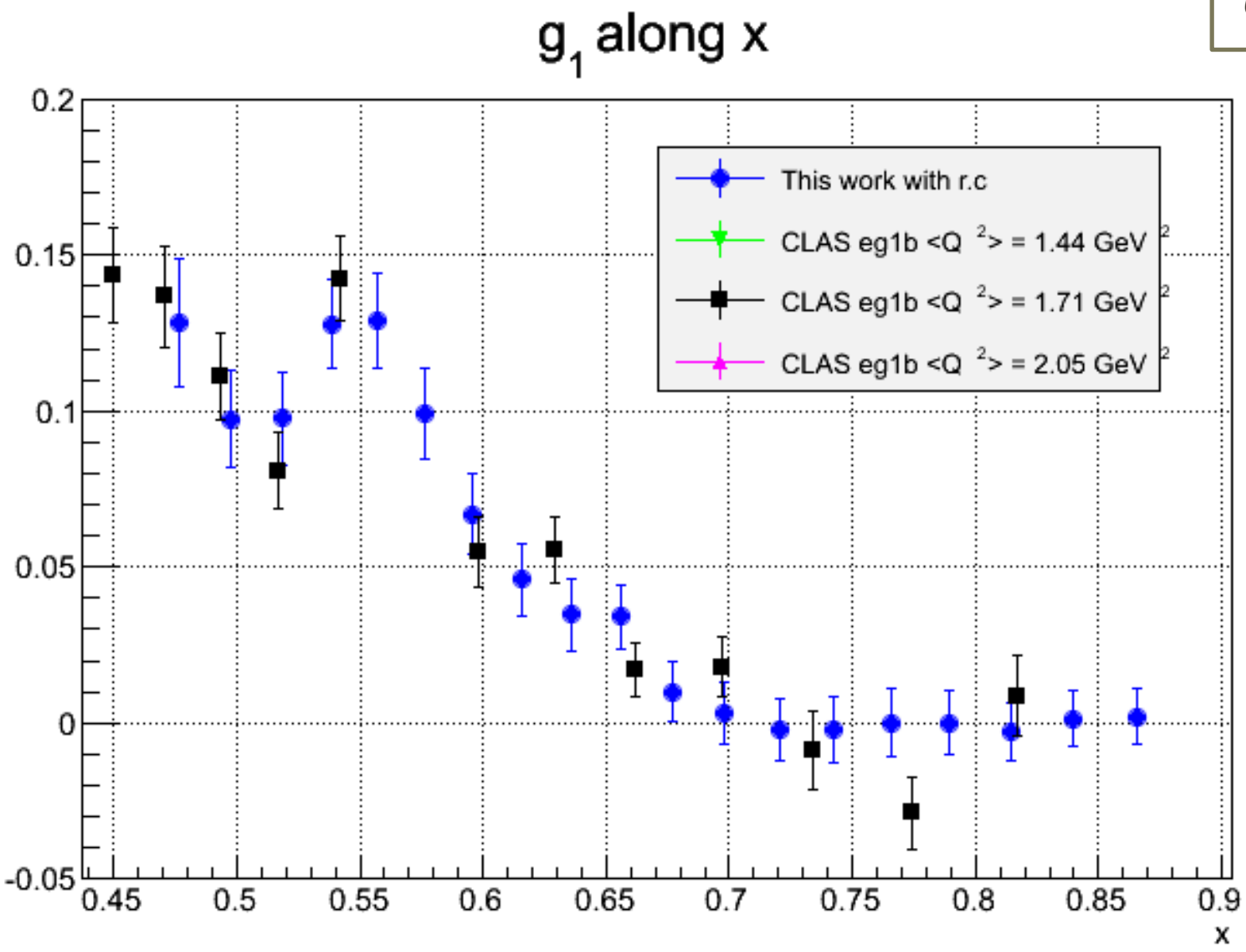
Structure Function g_1

$Q^2 = 1.86\ GeV^2$



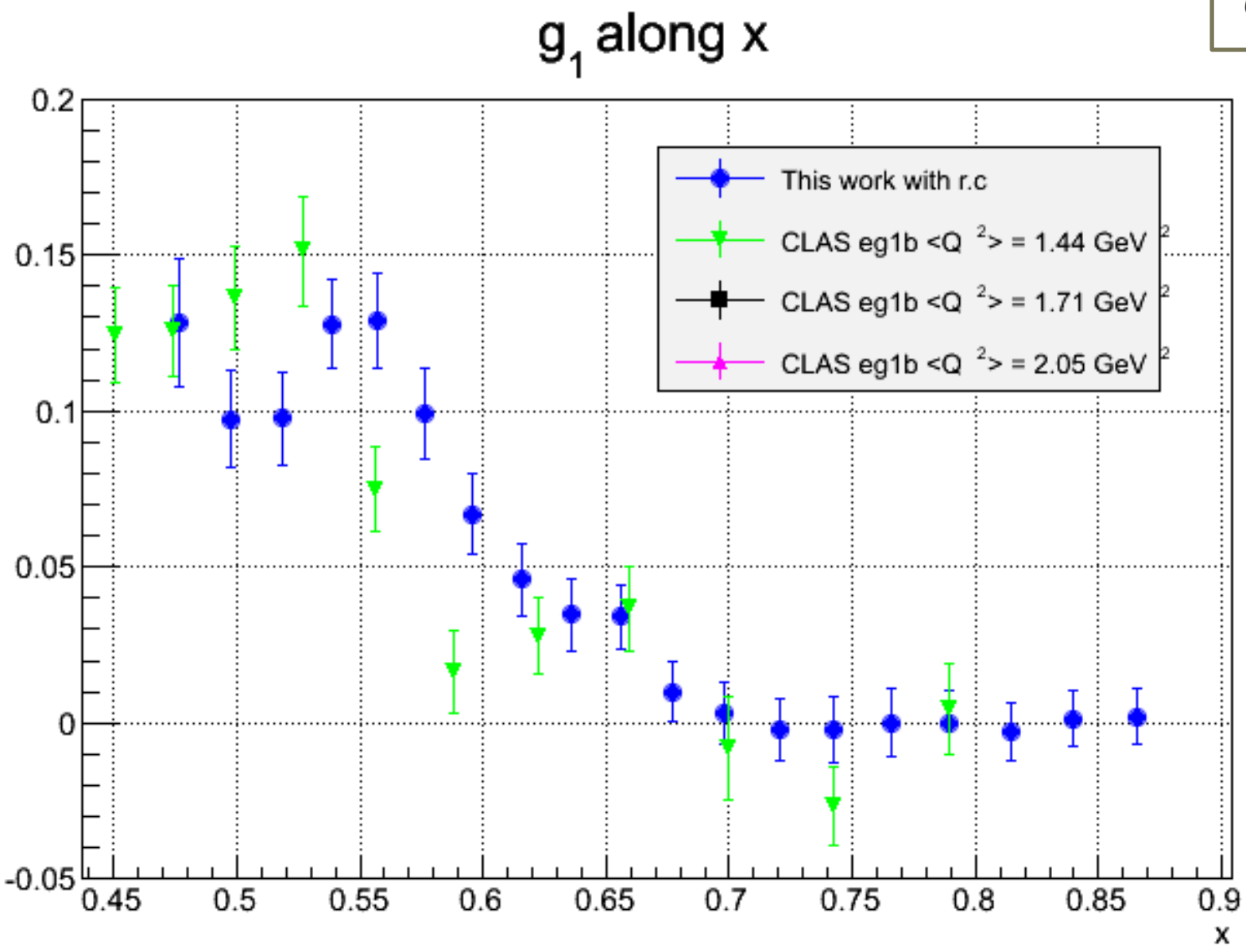
Structure Function g_1

$Q^2 = 1.86\ GeV^2$



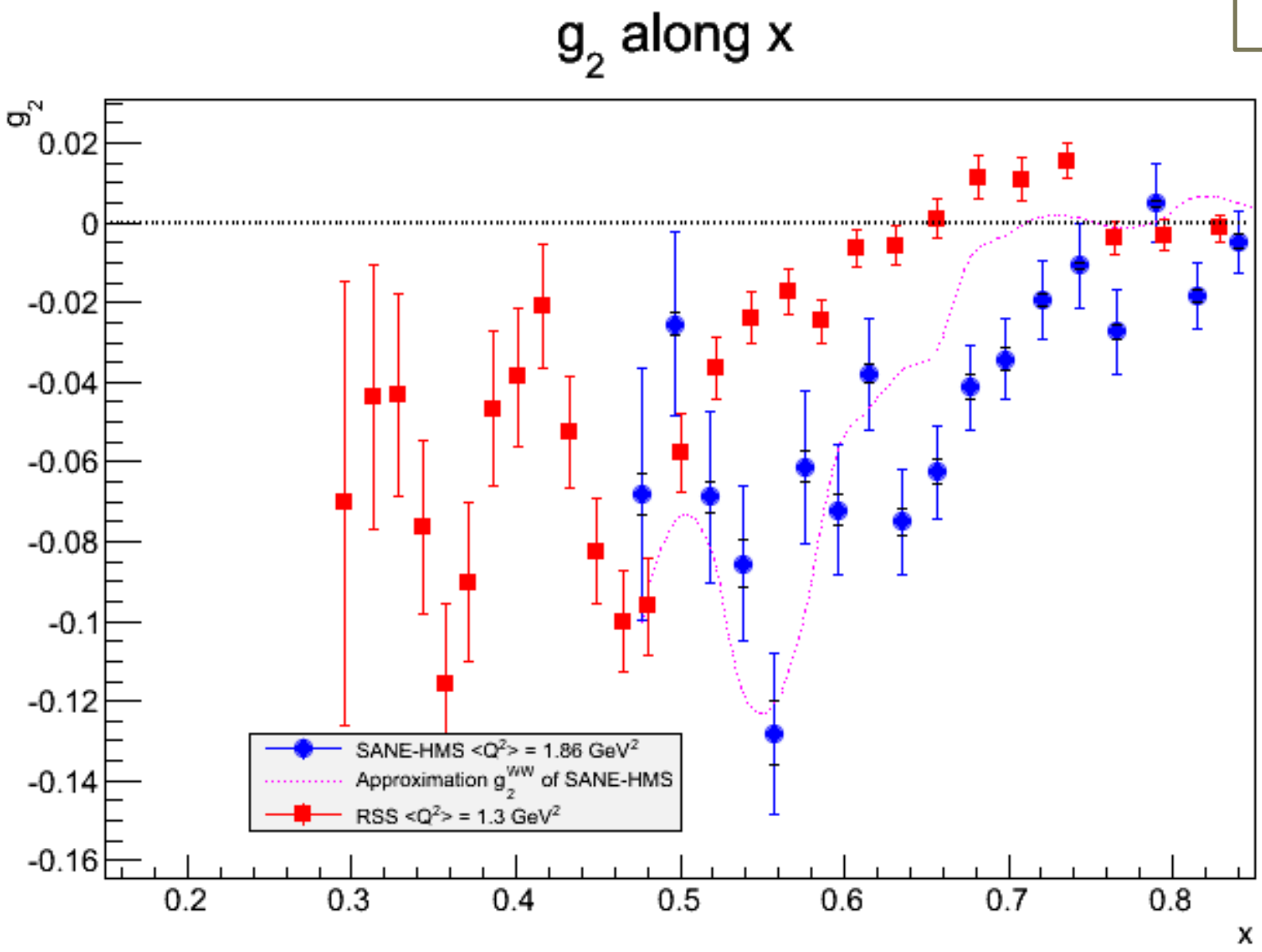
Structure Function g_1

$Q^2 = 1.86\ GeV^2$

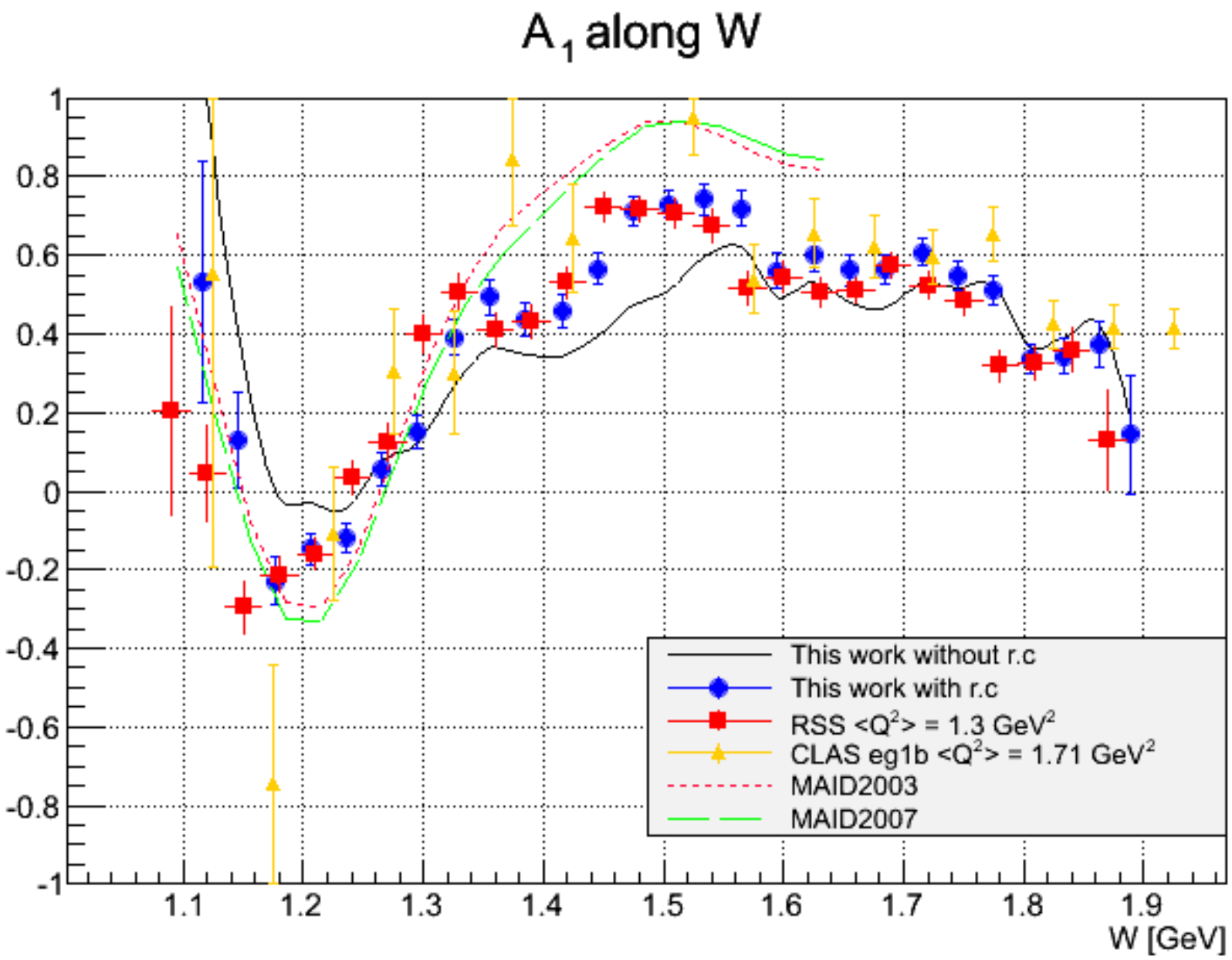


Structure Function g_2

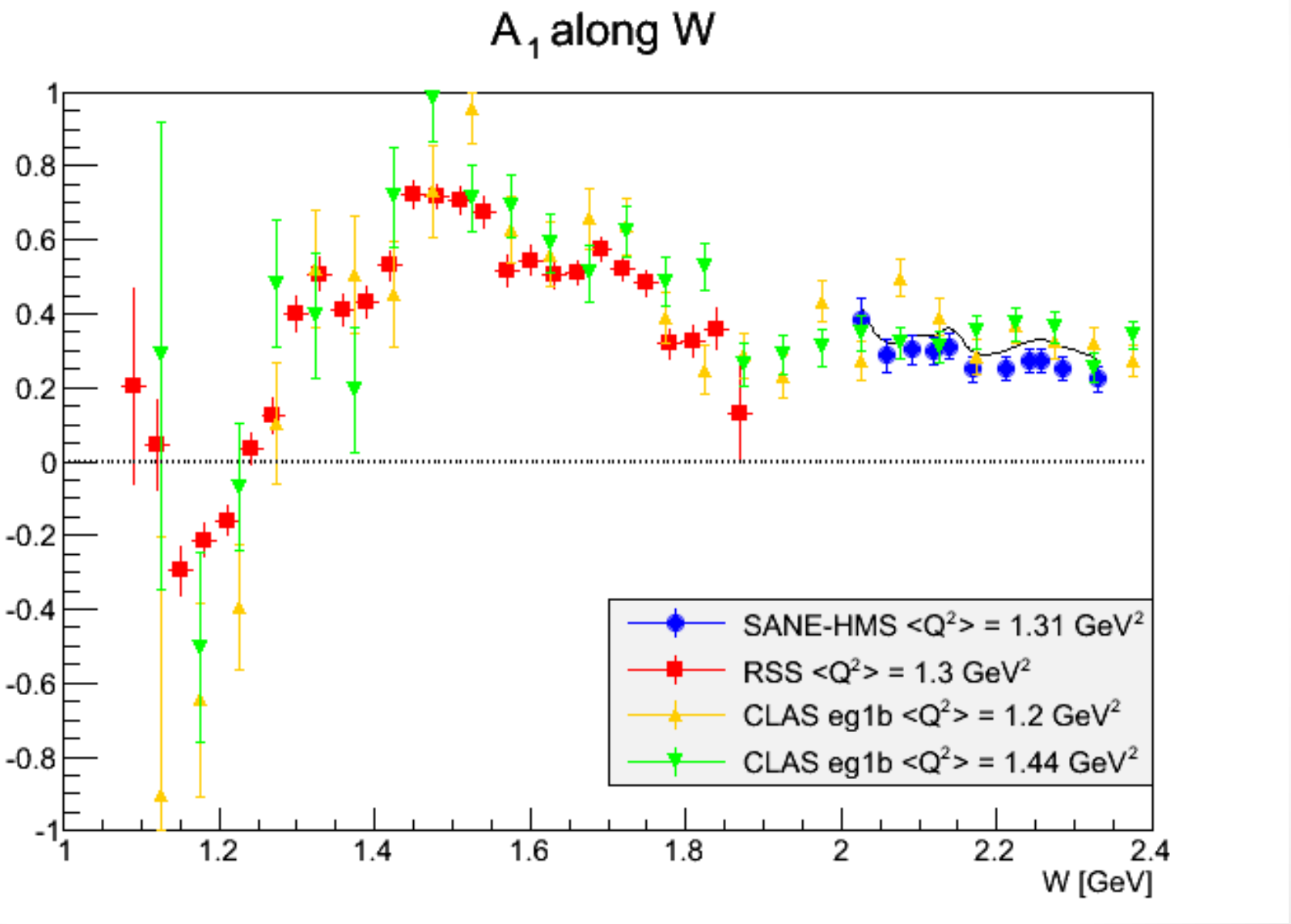
$Q^2 = 1.86\ GeV^2$



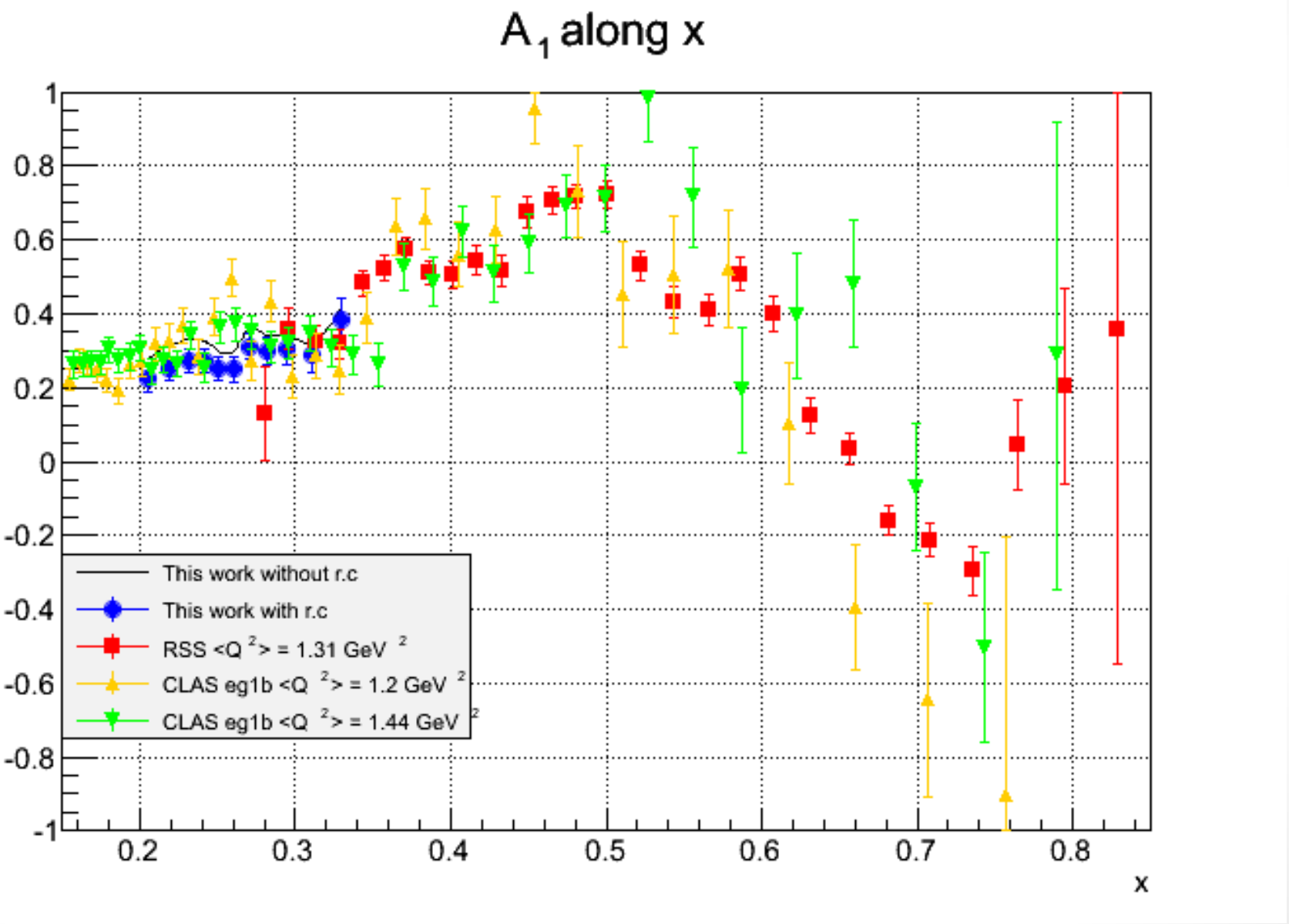
Regenerated RSS result



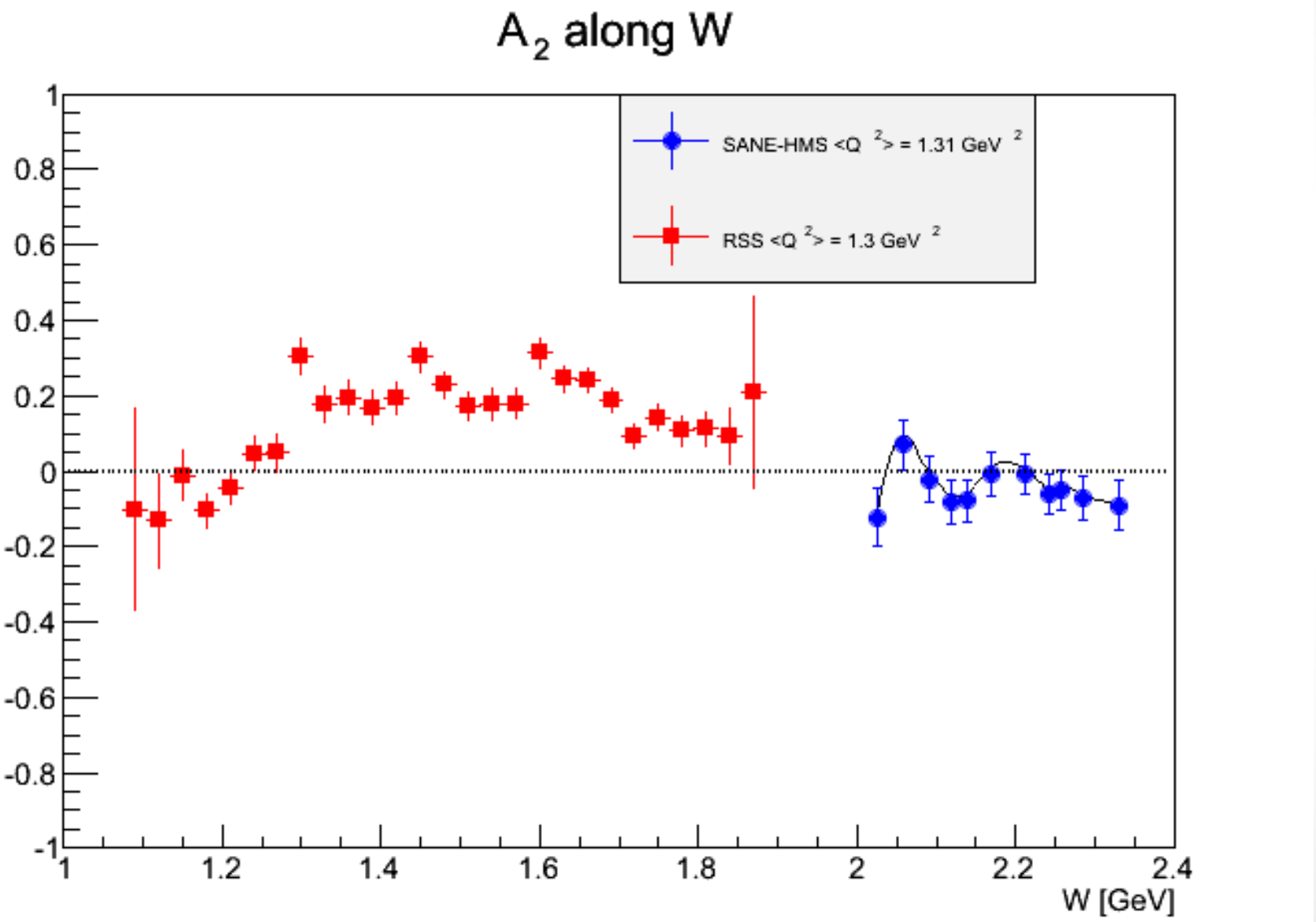
Continuity with RSS and CLAS



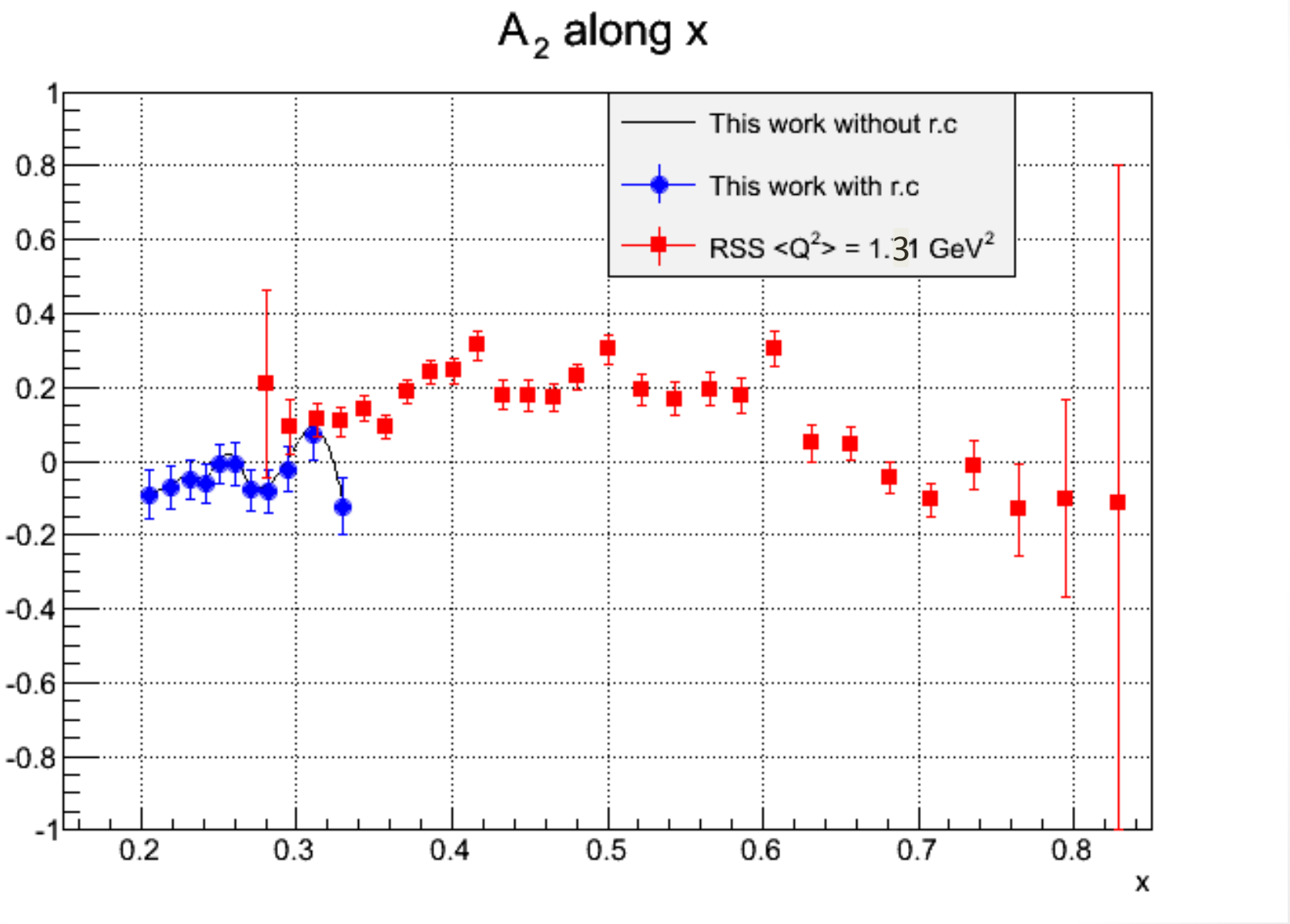
Continuity with RSS and CLAS



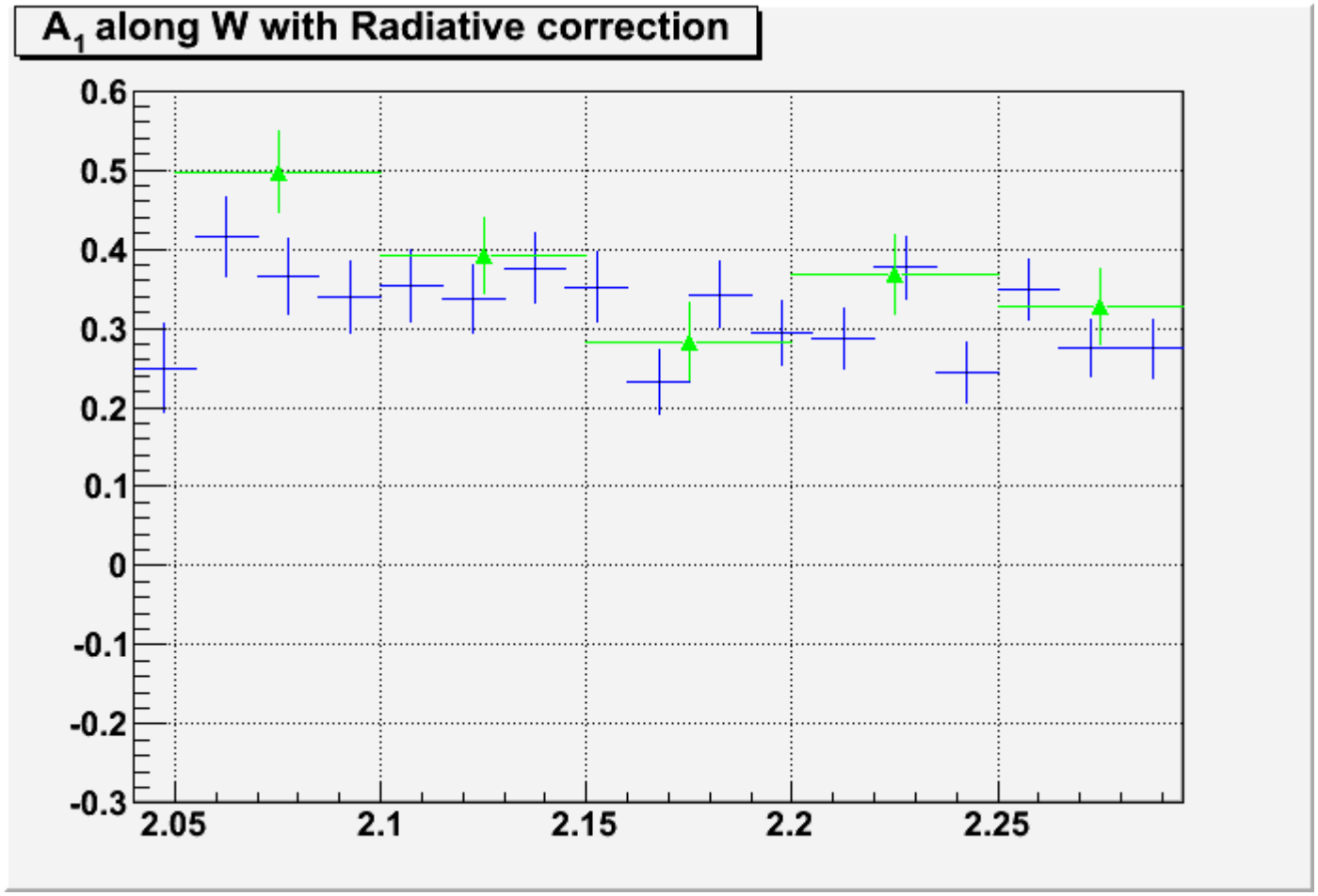
Continuity with RSS



Continuity with RSS

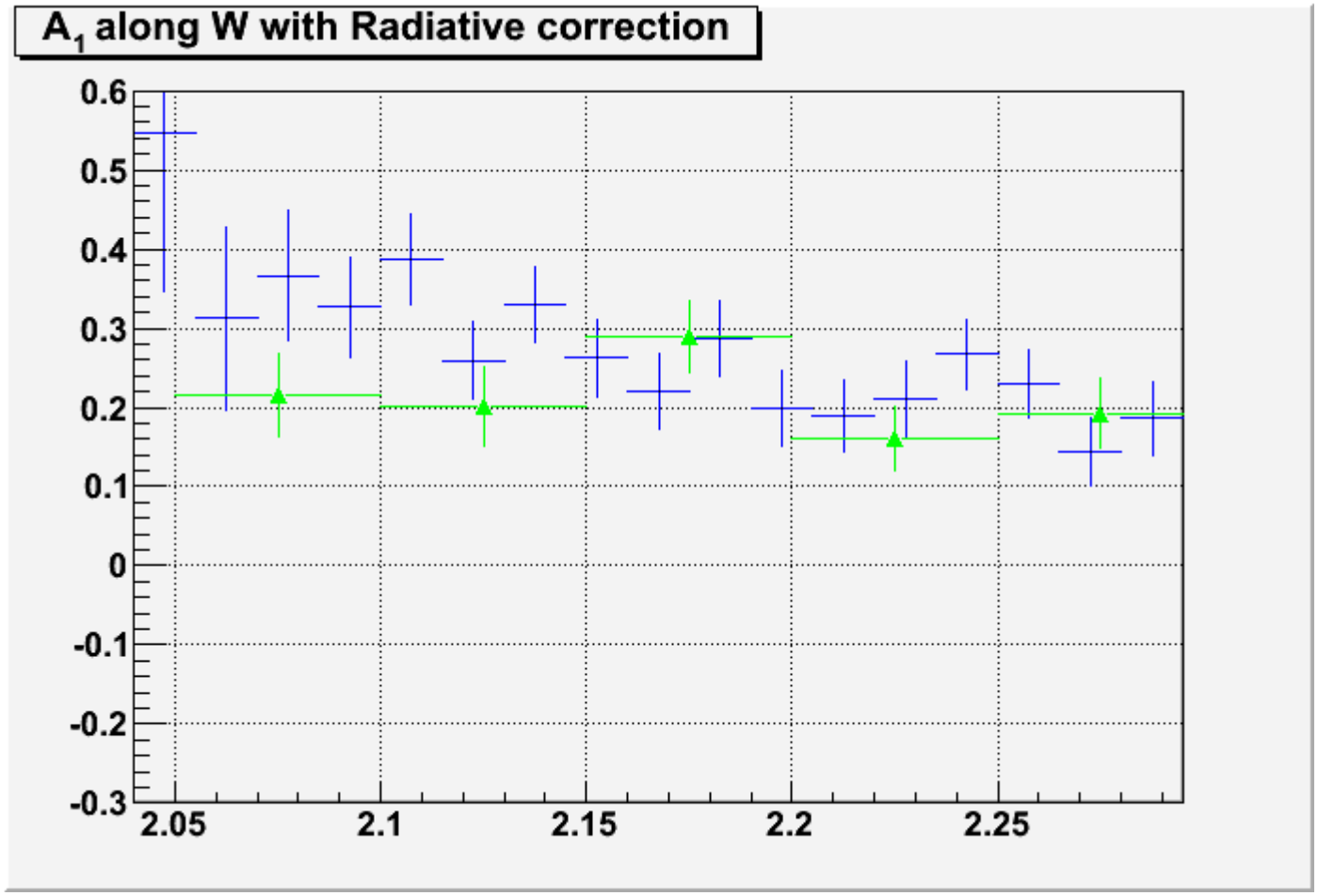


Asymmetry $A_1 = \frac{\sigma_{TT}}{\sigma_T}$



- Green triangles : CLAS EG1b result with Q^2 = 1.2 GeV^2
- Blue line : This work with Q^2 = 1.313 GeV^2

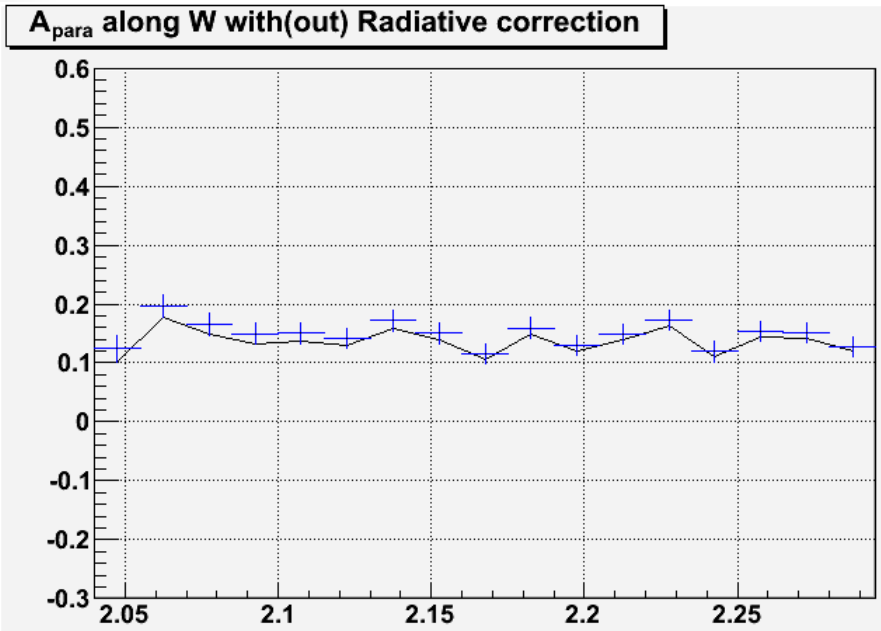
Asymmetry $A_1 = \frac{\sigma_{TT}}{\sigma_T}$

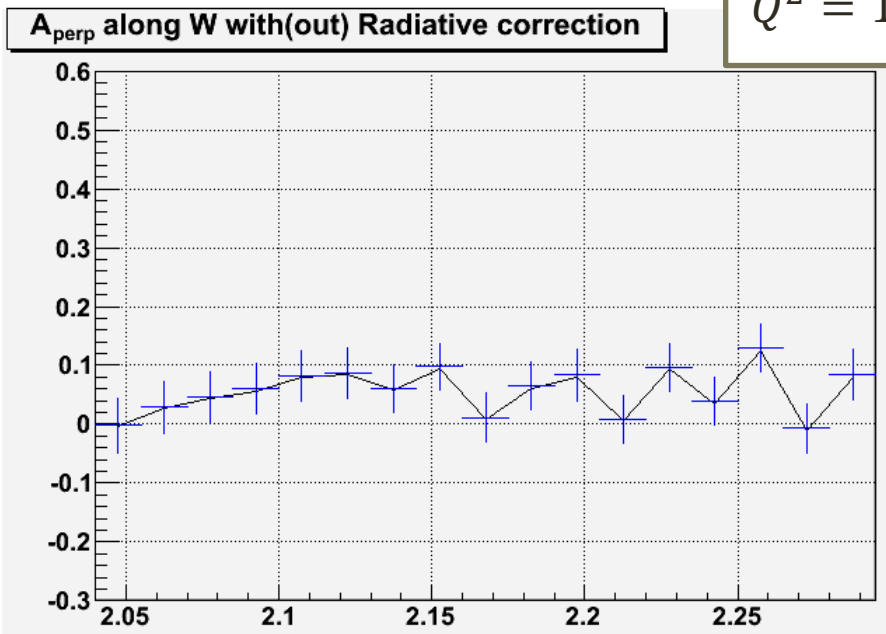


- Green triangles : CLAS EG1b result with $Q^2 = 0.844$ GeV^2
- Blue line : This work with $Q^2 = 0.806$ GeV^2

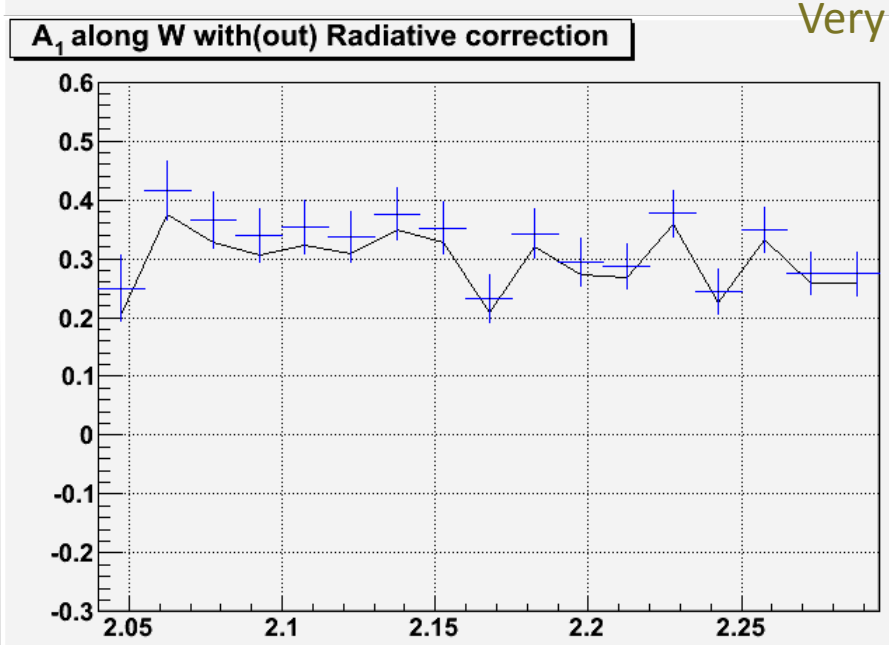
Preliminary Asymmetries (2)

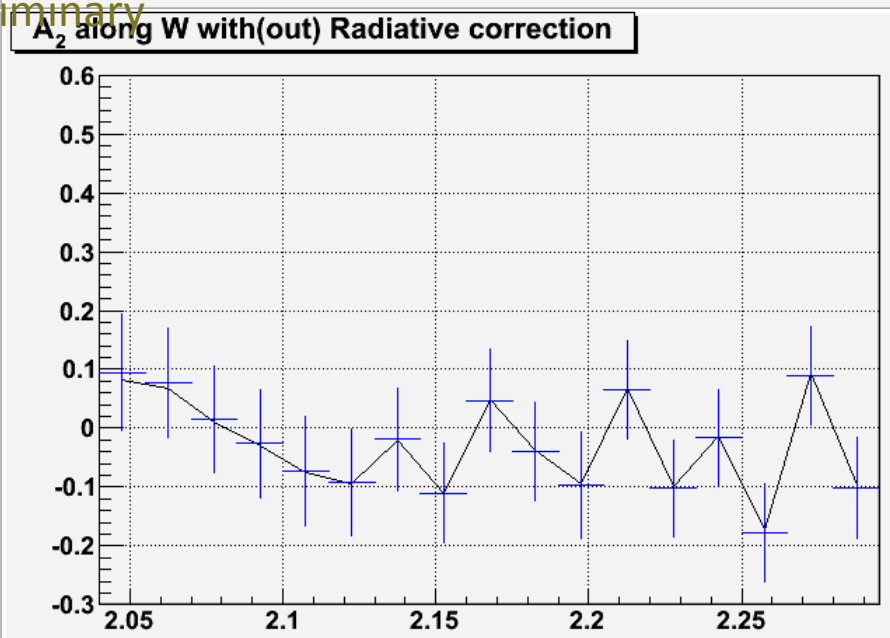
$Q^2 = 1.313\ GeV^2$





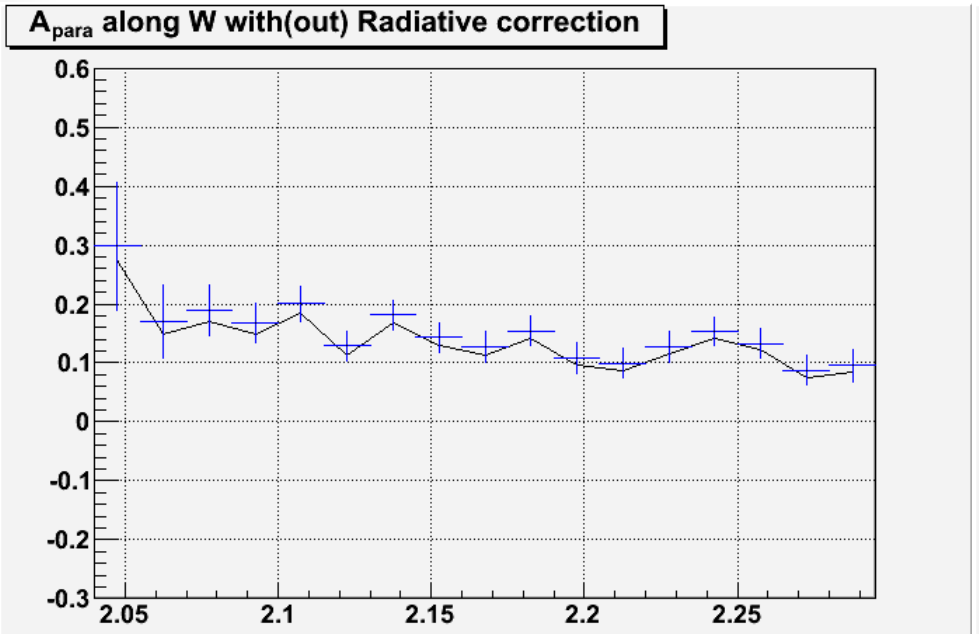
Very Preliminary

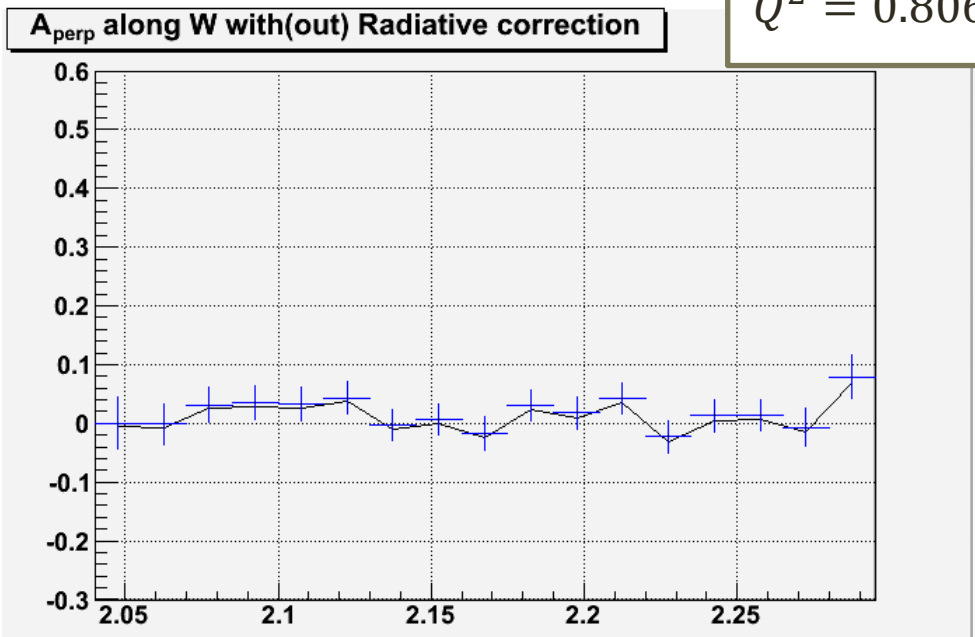




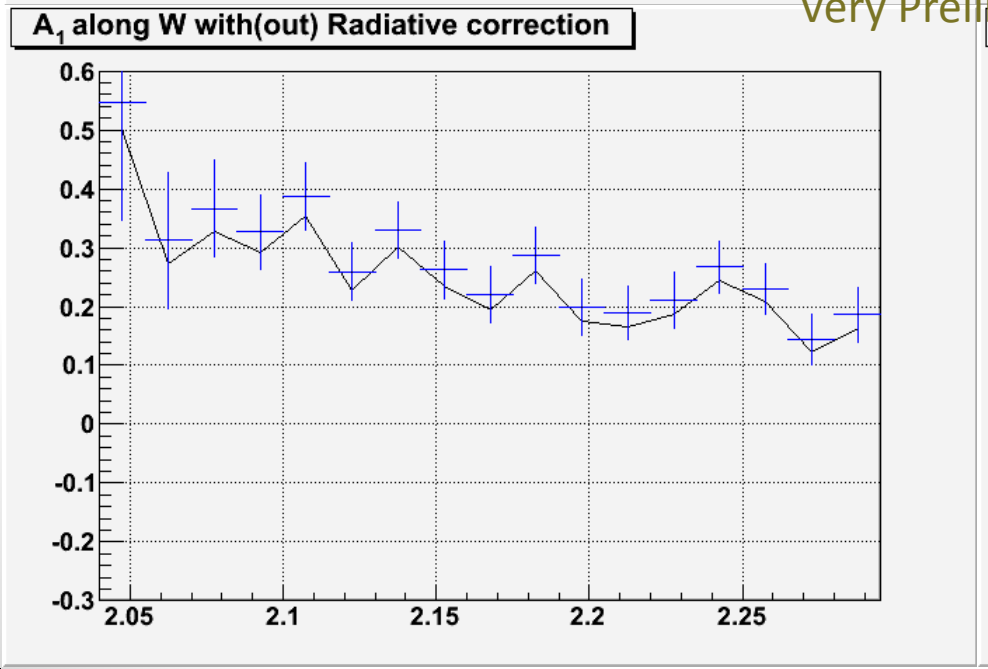
Preliminary Asymmetries (3)

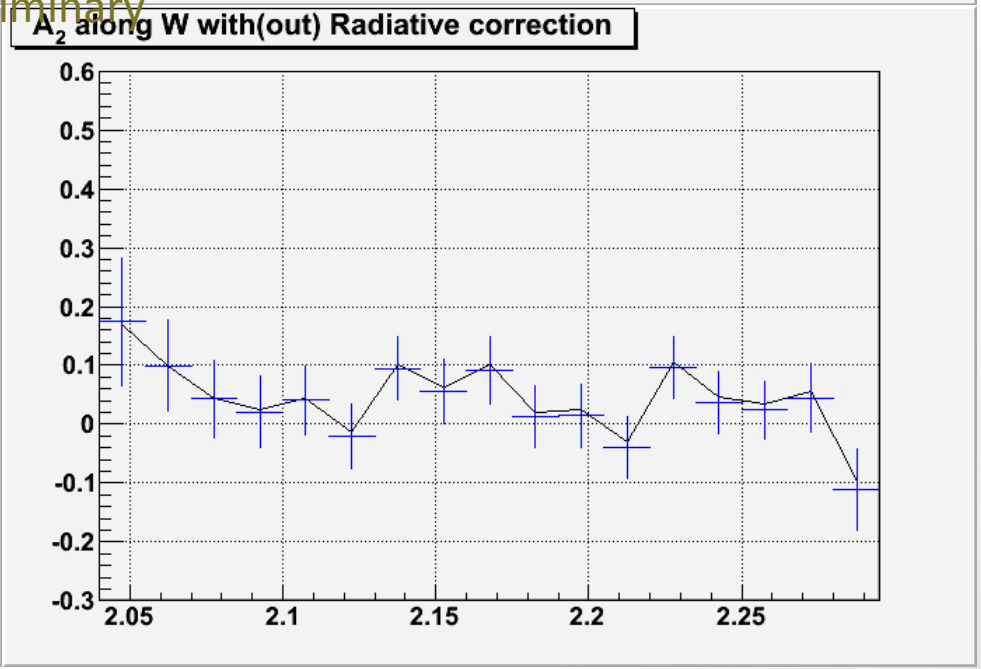
$Q^2 = 0.806\ GeV^2$



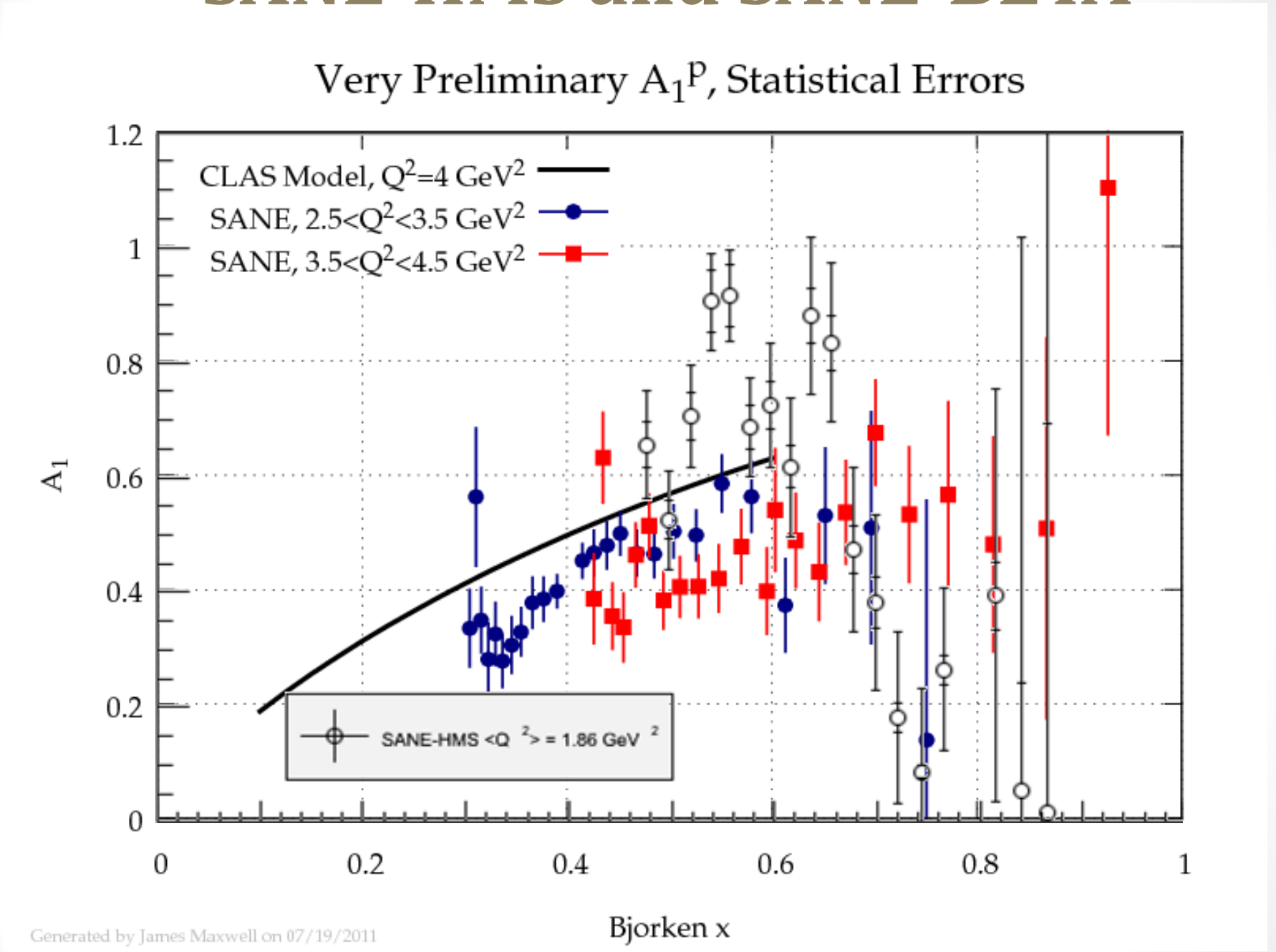


Very Preliminary



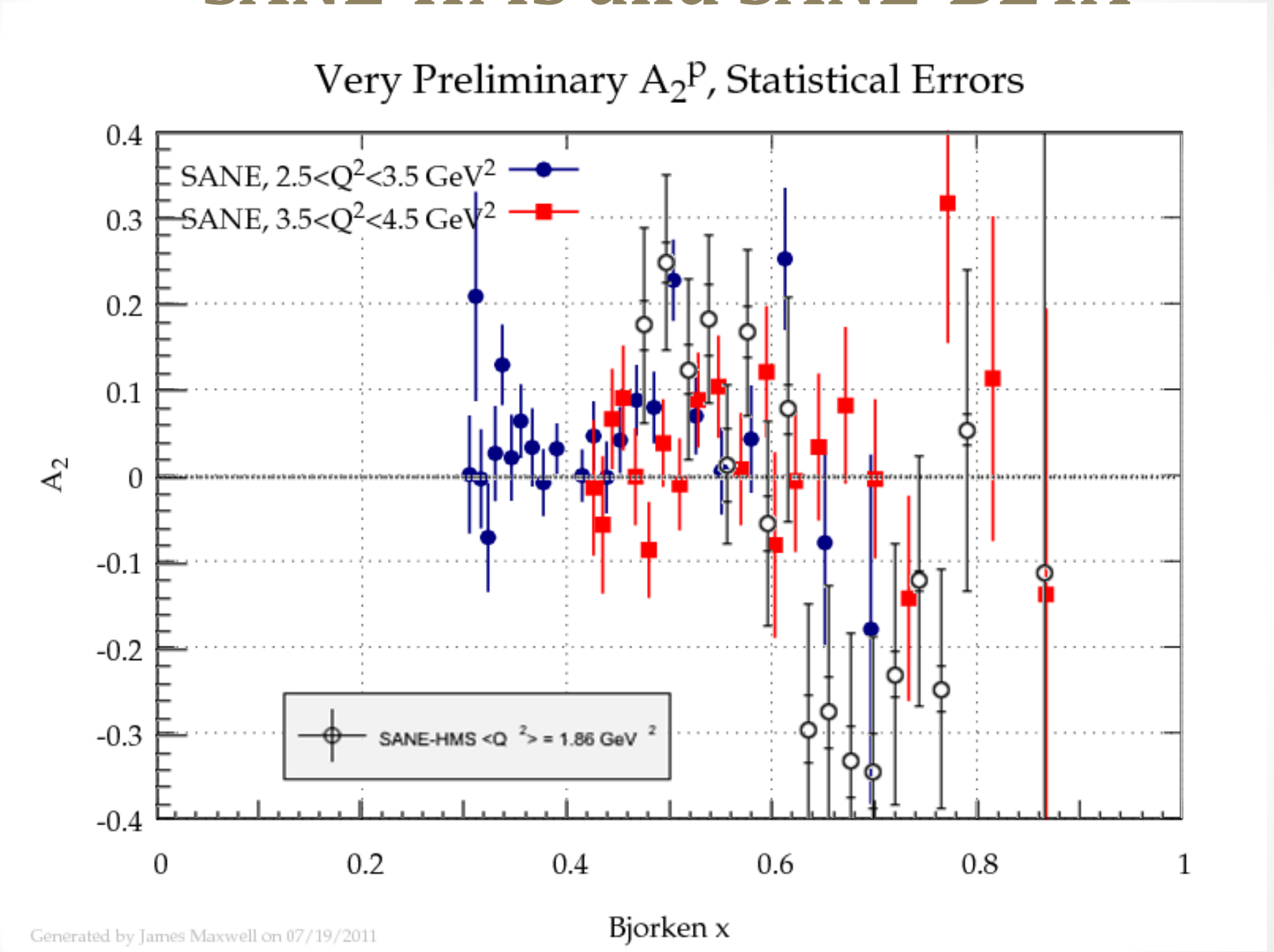


SANE-HMS and SANE-BETA



BETA analyzed by J. Maxwell (UVA)

SANE-HMS and SANE-BETA



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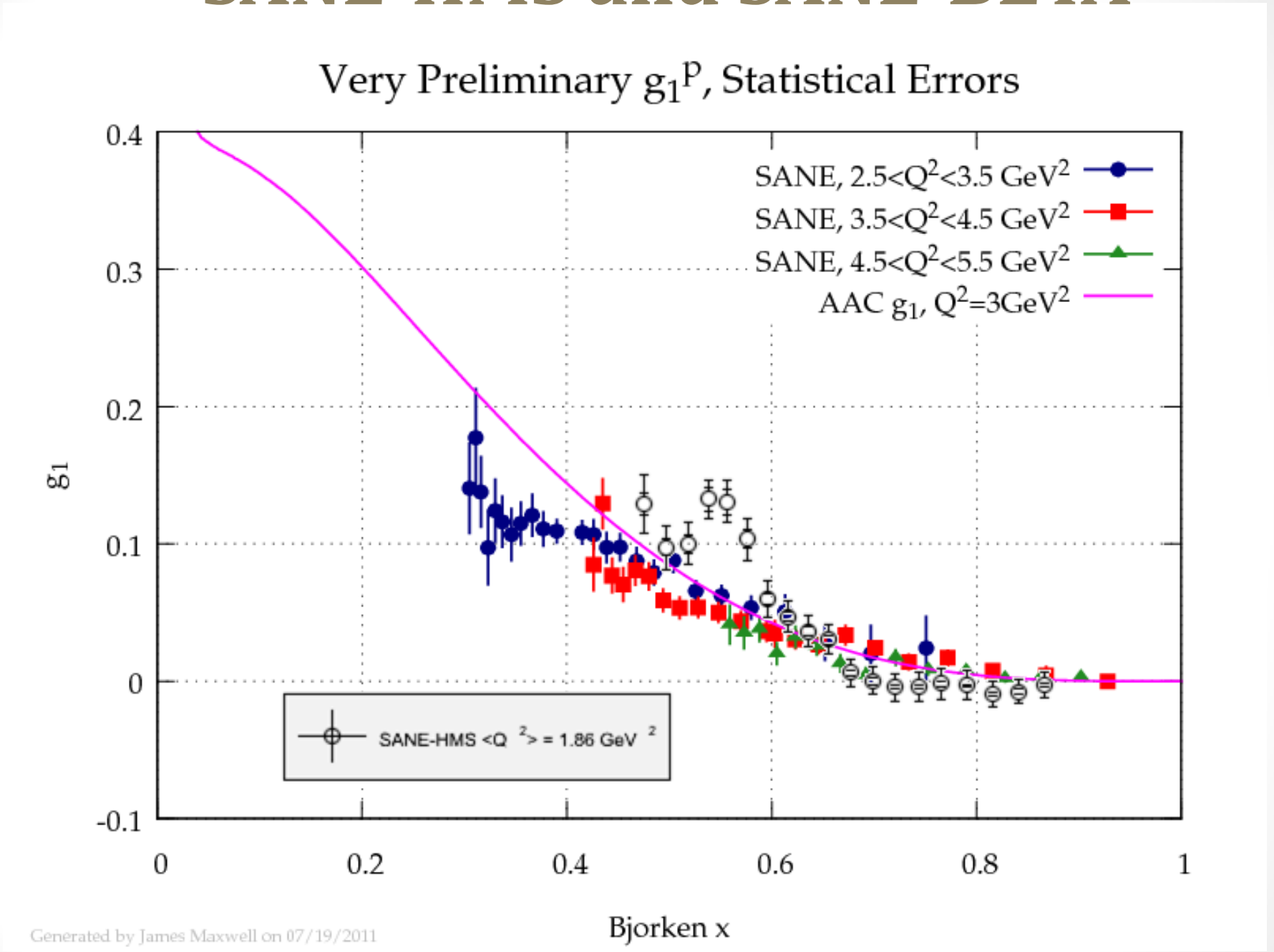
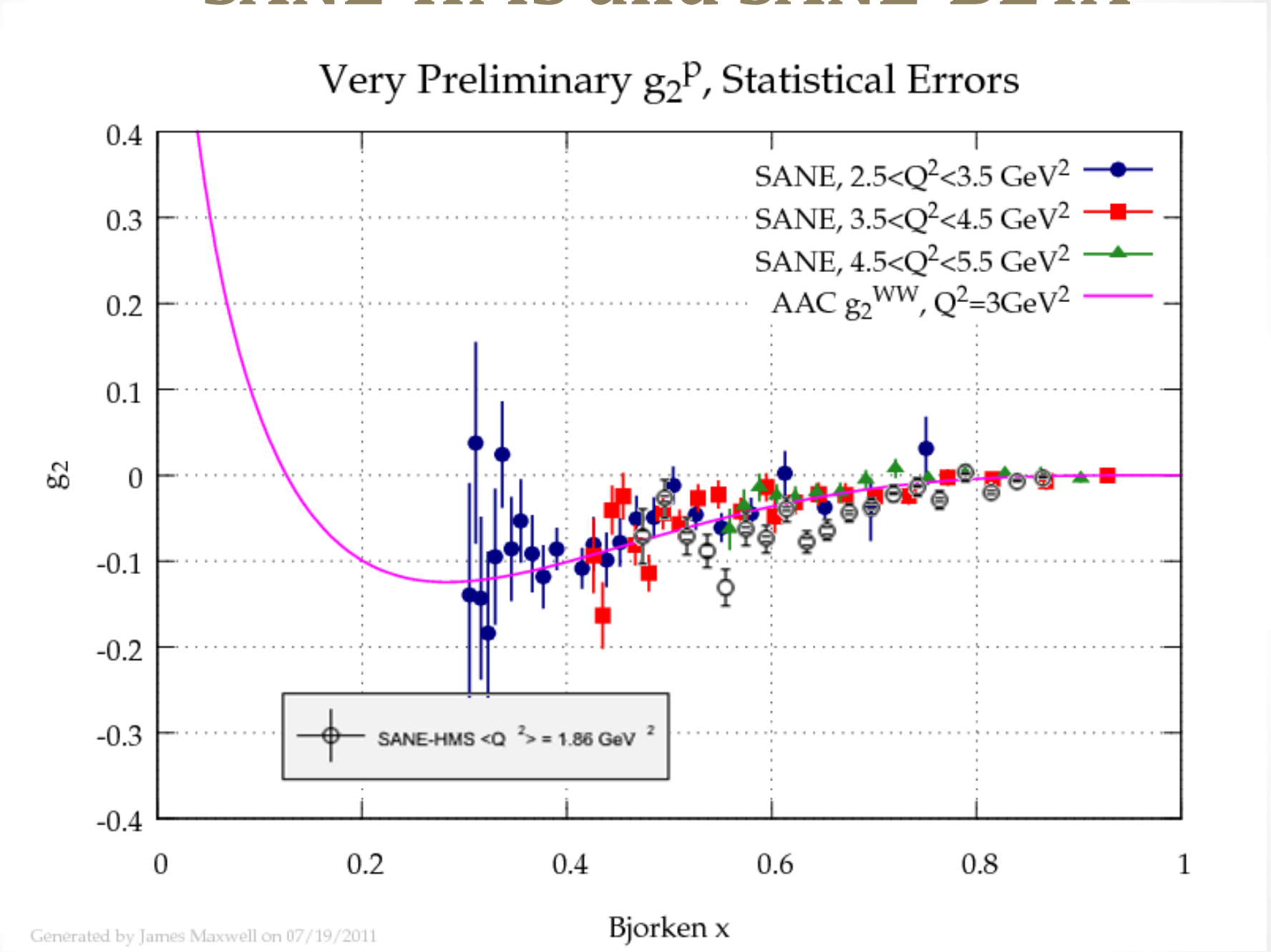
SANE-HMS and SANE-BETA



BETA analyzed by J. Maxwell (UVA)

SANE-HMS and SANE-BETA

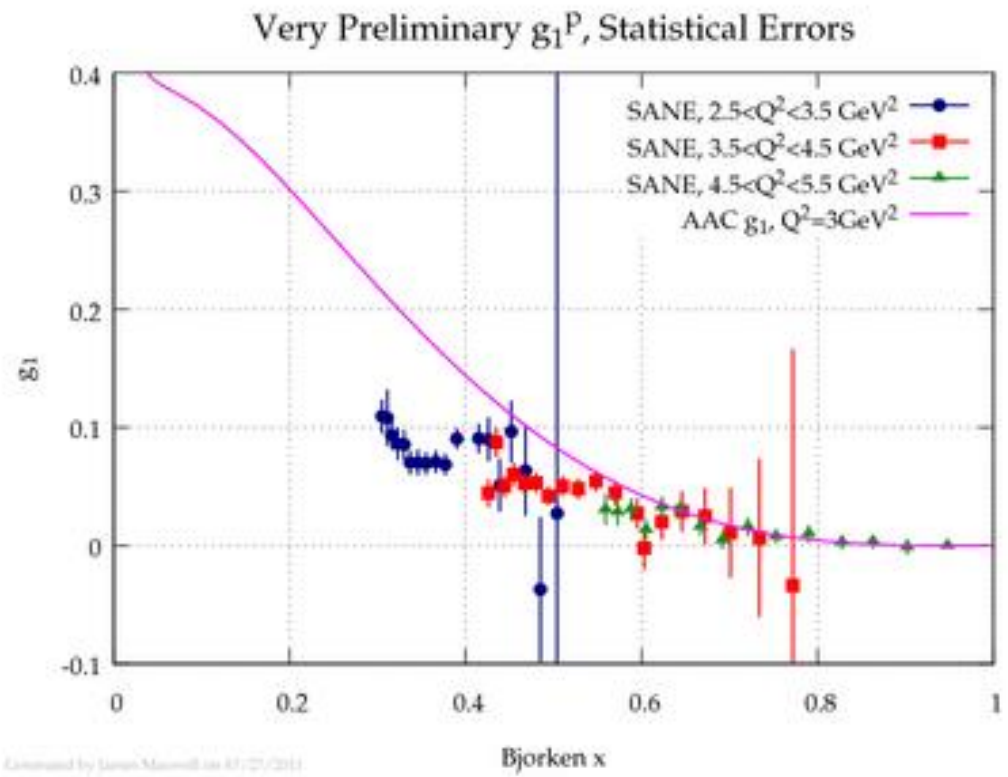


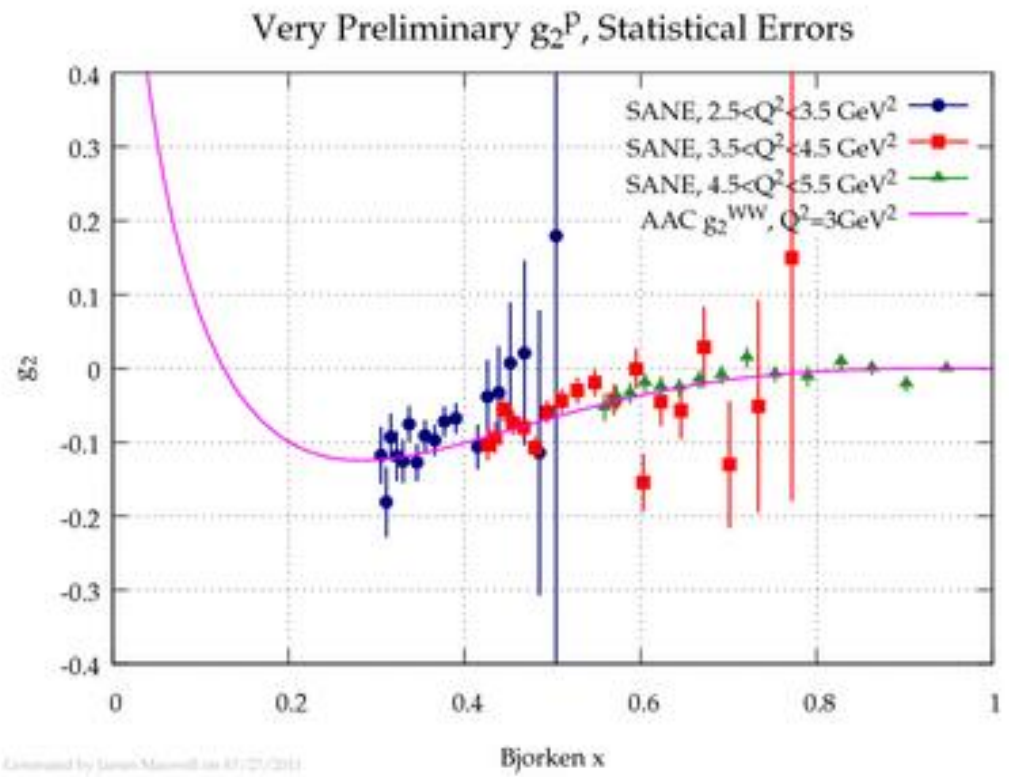
BETA analyzed by J. Maxwell (UVA)

Systematic Errors

Error Source	Average
Target Polarization	4.0%
Beam polarization	1.5%
Dilution Factor	3.0%
Radiative Corrections	10.4%
Kinematic Reconstruction	0.4%

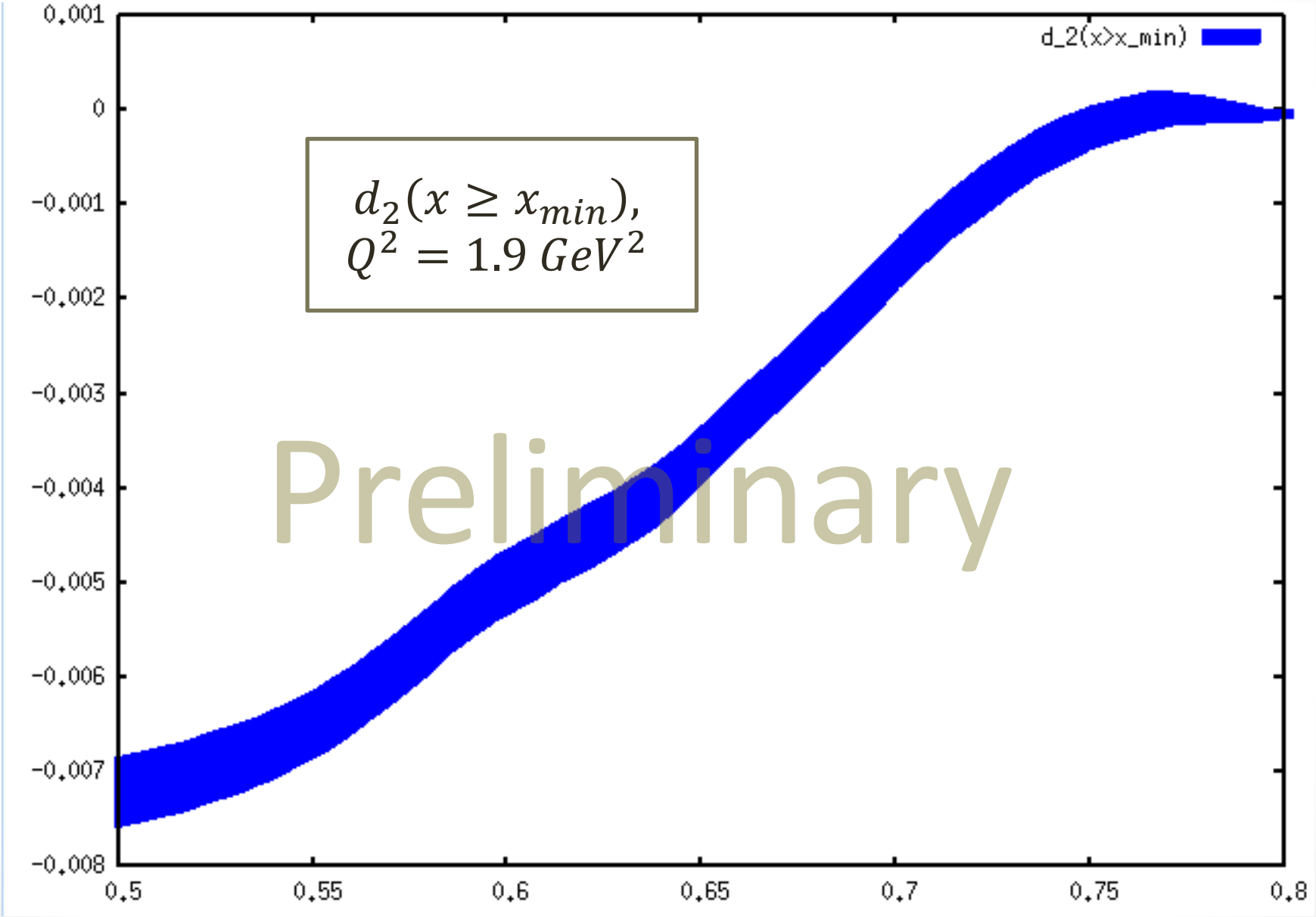
Preliminary g_1 and g_2 from BETA



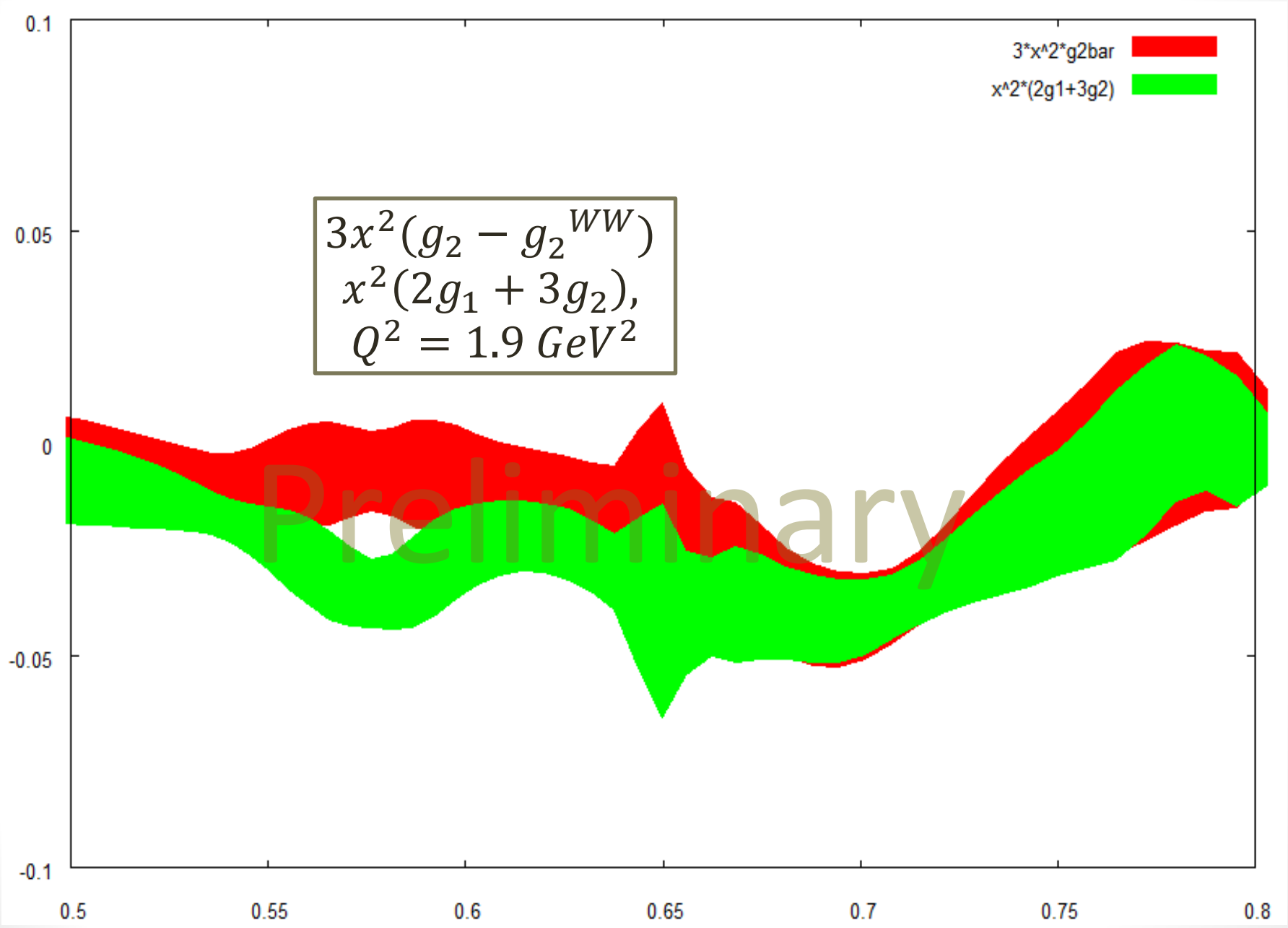


J. Maxwell (UVA)

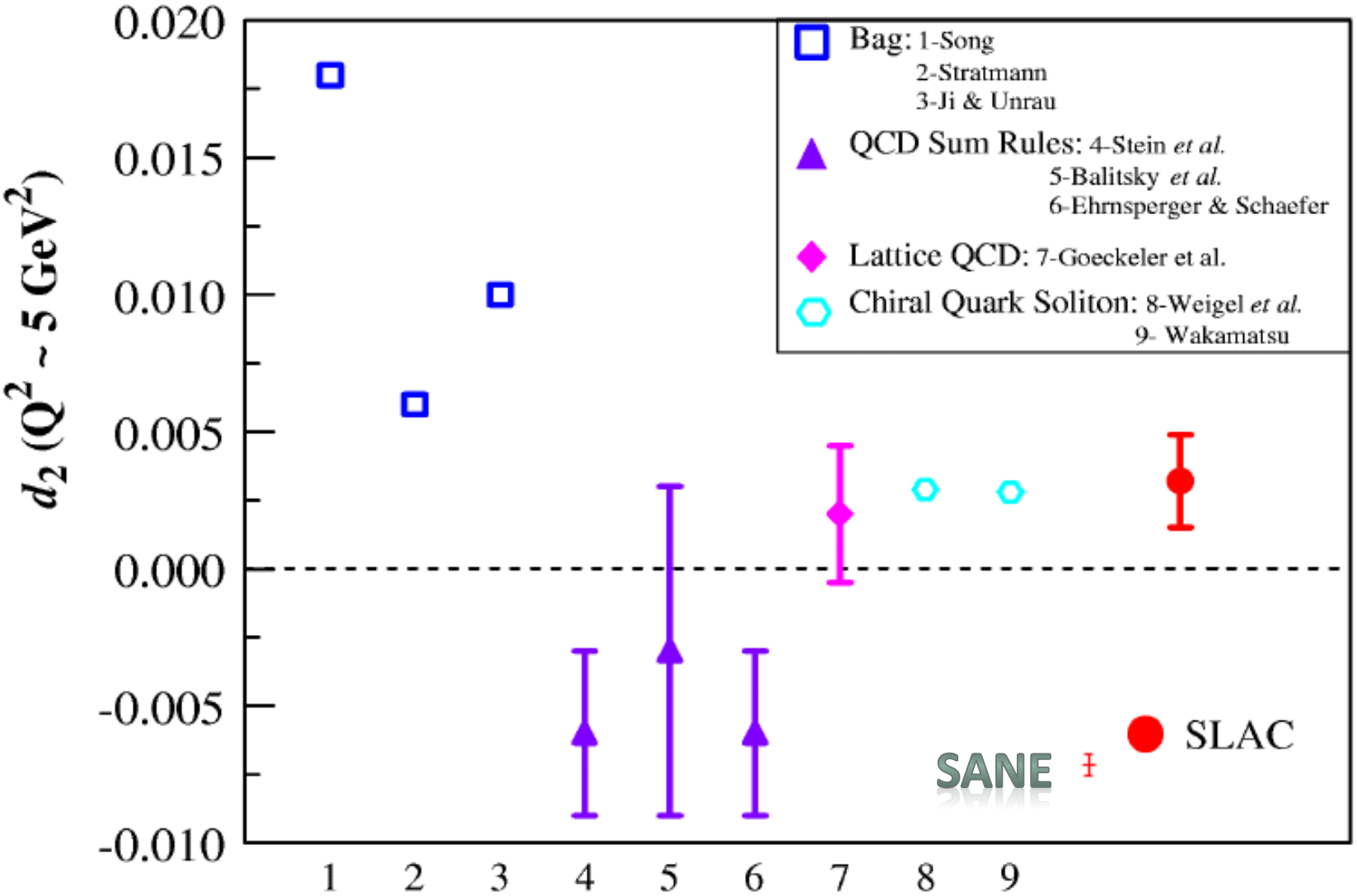
Twist-3 matrix element d_2



Twist-3 matrix element d_2



Twist-3 matrix element d_2



RSS Experiment Results

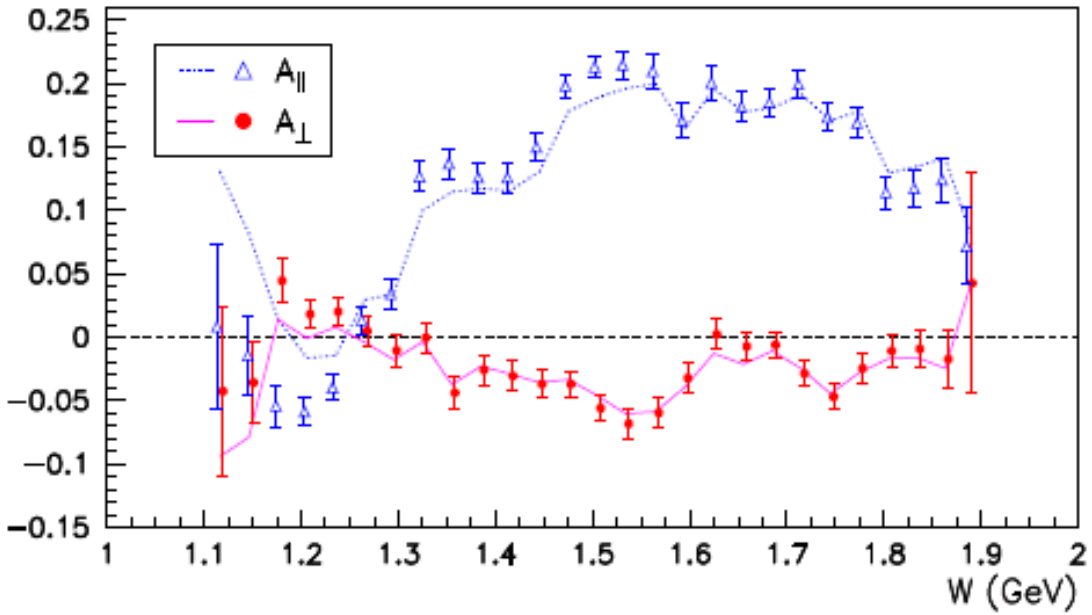


FIG. 1: Our measured asymmetries $A_{\parallel}$ and $A_{\perp}$, fully corrected (points) and without radiative corrections (curves).

TABLE I: Averaged systematic errors in the asymmetries.

Error Source	$A_{\parallel}$	$A_{\perp}$
Target Polarization	} 1.1 %	2.9 %
Beam Polarization		1.3 %
Dilution Factor	4.9 %	4.9 %
Radiative Corrections	2.7 %	12.9 %
Kinematic Reconstruction	0.4 %	0.4 %

Phys. Rev. Lett. 98, 132003 (2007)

RSS Experiment Results

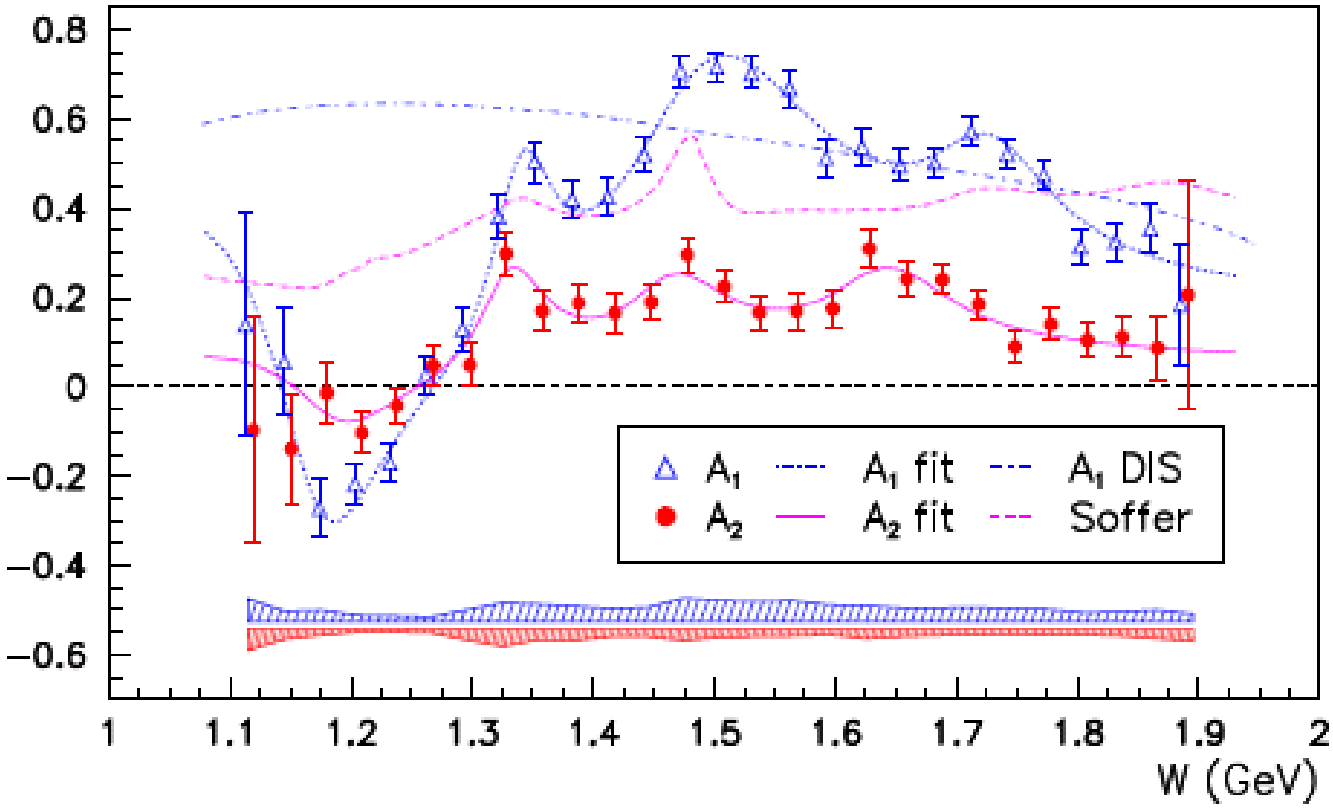


FIG. 2: Virtual photon asymmetries A_1 and A_2 from our data and corresponding fits. Also shown is the E155 fit to DIS data [25, 26], evaluated at our (x, Q^2), and the Soffer limit for A_2 [31], based on our A_1 fit. The upper error band indicates the systematic error in A_1, the lower one A_2.

Phys. Rev. Lett. 98, 132003 (2007)

RSS Experiment Results

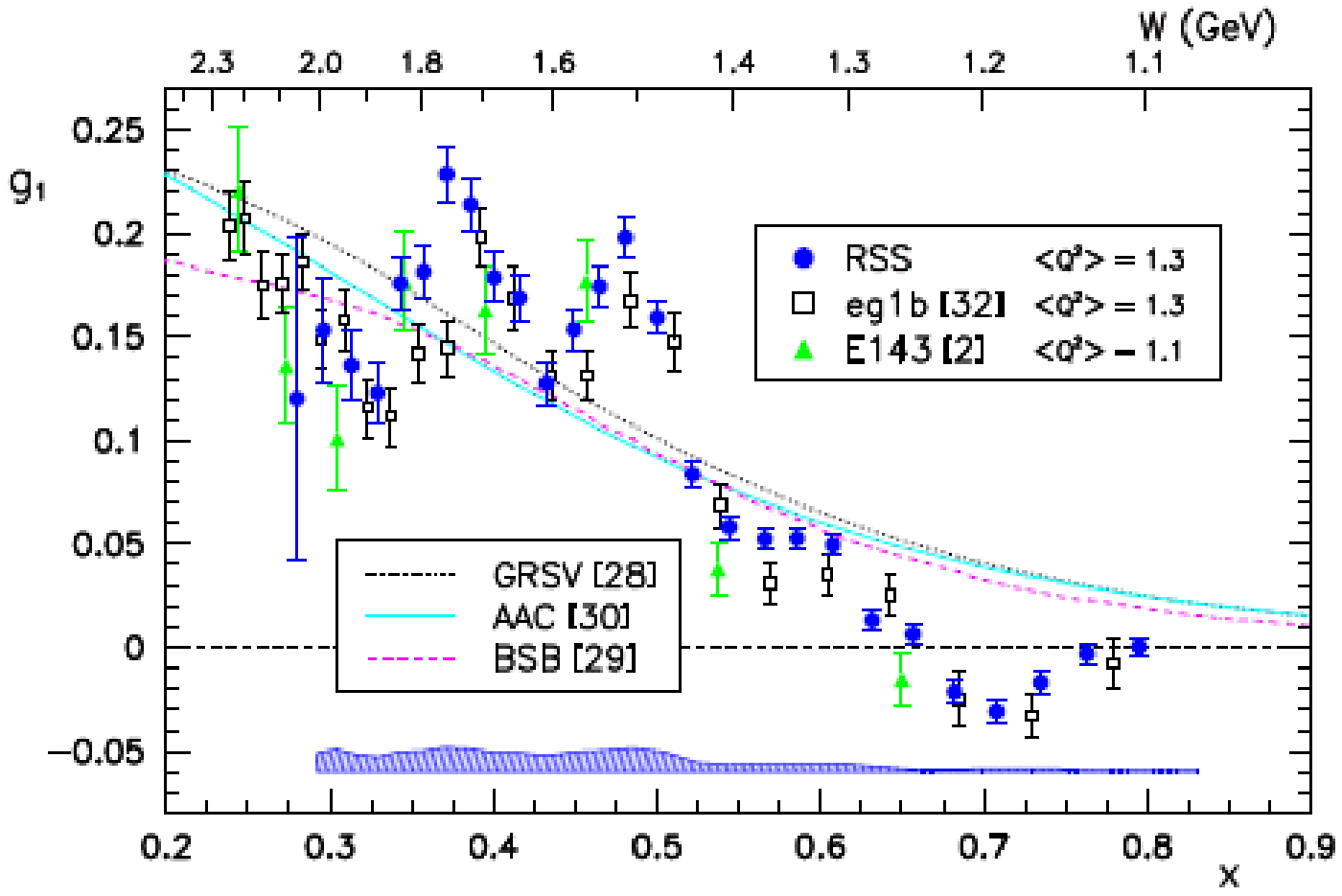


FIG. 3: Results for g_1 from this experiment (RSS) and other relevant data [2, 32], as well as target mass corrected NLO PDFs. The upper scale shows W (at $Q^2 = 1.3$ GeV^2) for reference.

Phys. Rev. Lett. 98, 132003 (2007)

RSS Experiment Results

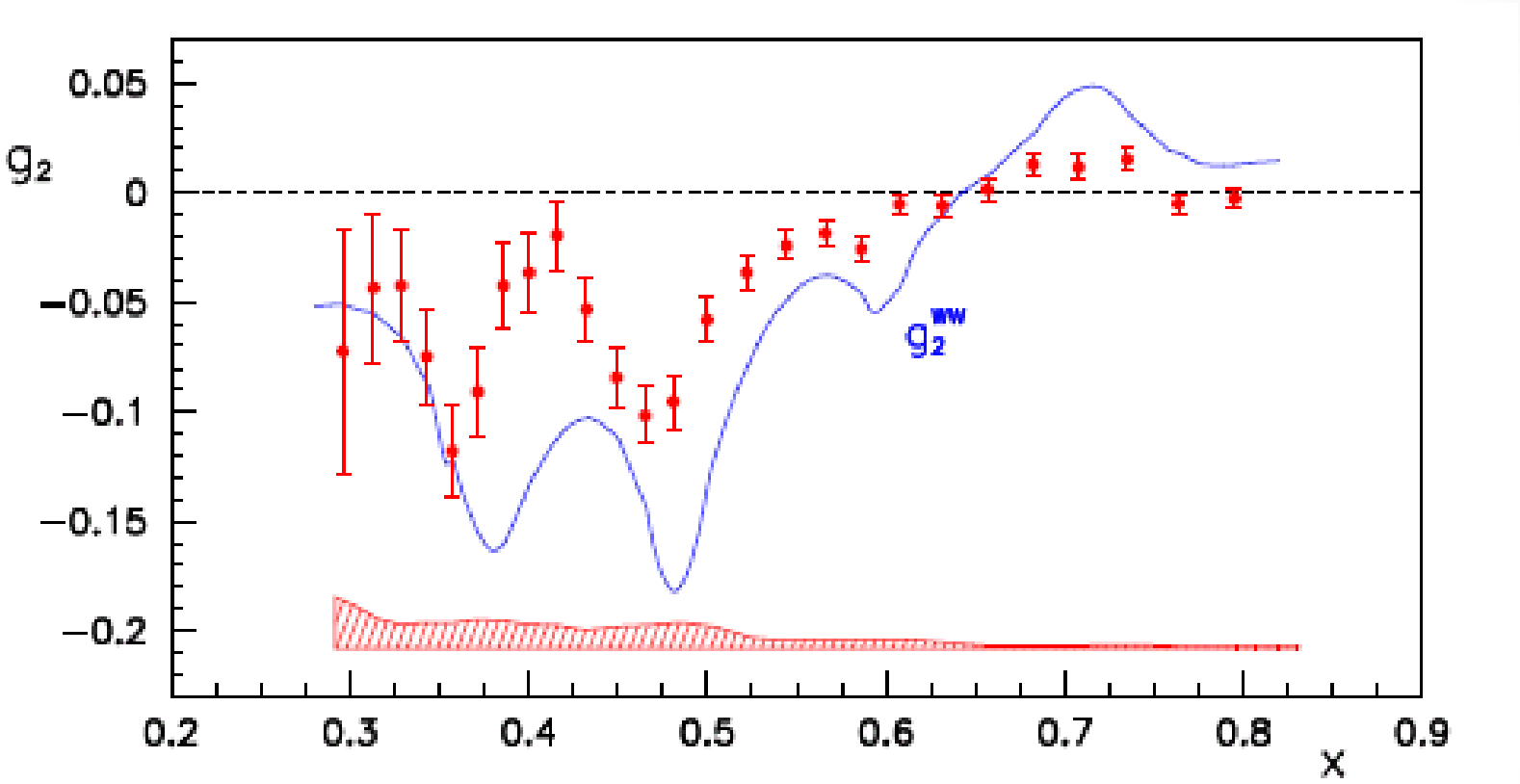
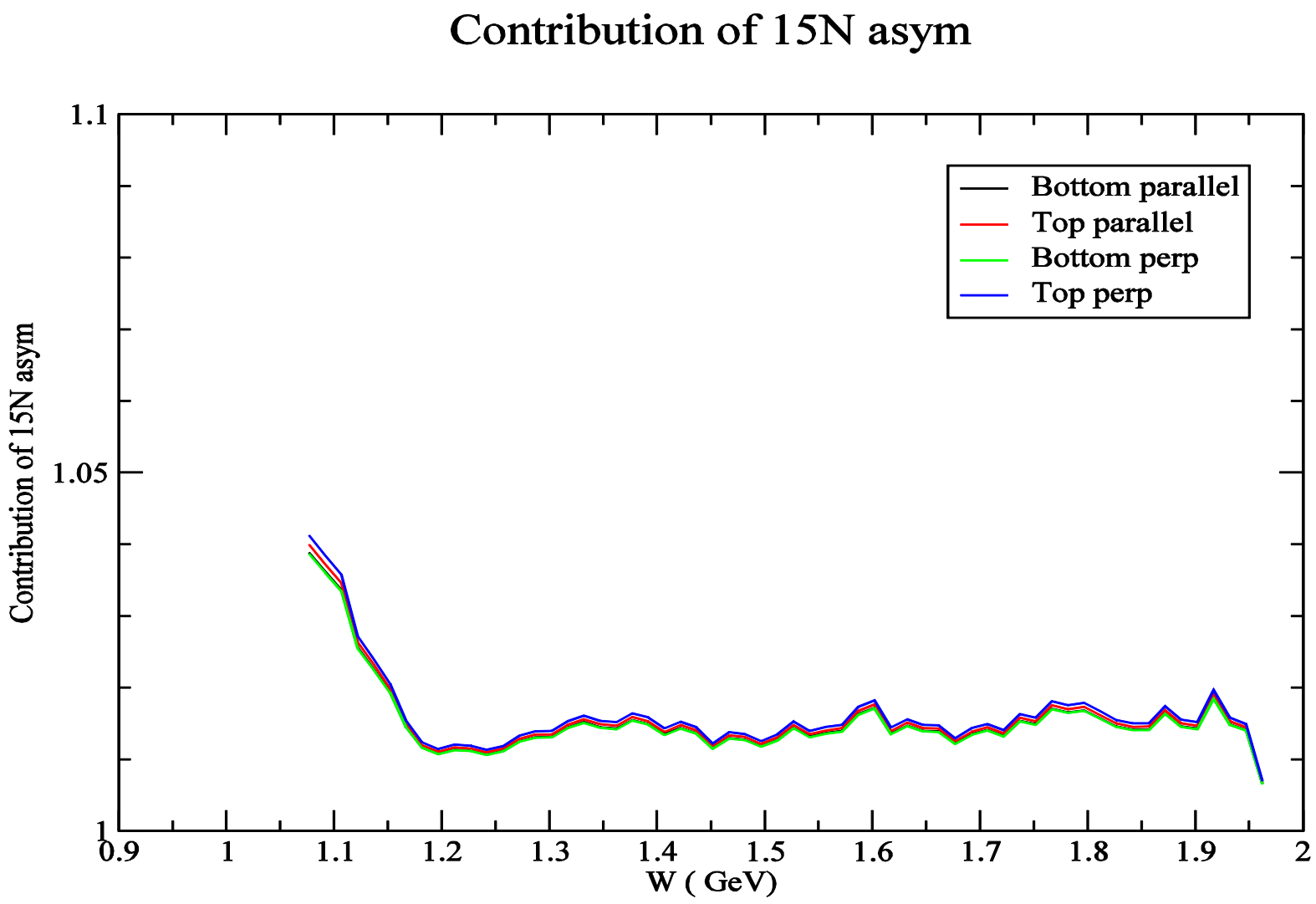


FIG. 4: Our (RSS) values for g_2 and the approximation g_2^{ww} (Eq. 1) as evaluated from our data.

Phys. Rev. Lett. 98, 132003 (2007)

Nitrogen Correction



Borrwed Slides

Transverse Spin Structure Function

- Polarized longitudinal structure function has simple parton model interpretation

$$g_1(x) = \sum e_i^2 \Delta q_i(x), \qquad i = u, \bar{u}, d, \bar{d} \ldots$$

- g_2 is combination of twist-2 and twist-3 components:

$$g_2(x,Q^2) = g_2^{WW}(x,Q^2) + \overline{g_2}(x,Q^2)$$

$$= -g_1(x,Q^2) + \int_x^1 g_1(x',Q^2)\frac{dx'}{x'} - \int_x^1 \frac{\partial}{\partial x'}\left[\frac{m}{M}h_T(x',Q^2) + \xi(x',Q^2)\right]\frac{dx'}{x'}$$

  - *W*andzura-*W*ilczek g_2^{WW} depends on g_1; h_T is twist-2 chiral odd transversity
  - ξ represents quark-gluon correlations (twist-3).

- Transverse spin structure function g_T measures spin distribution normal to virtual γ

$$g_T = g_1 + g_2 = \int_x^1 \left[g_1 - \frac{\partial}{\partial x'}\left(\frac{m}{M}h_T + \xi\right)\right]\frac{dx'}{x'} = \frac{\nu}{\sqrt{Q^2}}F_1(x,Q^2)A_2(x,Q^2)$$

Borrowed Slides

Transverse Spin Structure Sum Rules

- OPE: moments of g_1, g_2 related to twist-2 (a_N), twist-3 (d_N) matrix elements.

$$\int_0^1 x^N g_1(x,Q^2)\,dx = \frac{1}{2} a_N + O(M^2/Q^2), \qquad N = 0, 2, 4, \ldots$$

$$\int_0^1 x^N g_2(x,Q^2)\,dx = \frac{N}{2(N+1)}(d_N - a_N) + O(M^2/Q^2), \quad N = 2, 4, \ldots$$

- d_N measure twist-3 contributions (related to for $m \ll M$ and h_T not too large.)

$$d_N(Q^2) = \frac{2(N+1)}{N} \int_0^1 x^N \overline{g_2}(x,Q^2)\,dx$$

- Burkhardt-Cottingham
  - not from OPE

$$\int_0^1 g_2(x)\,dx = 0$$

- Efremov-Leader-Teryaev
  - valence quarks combining with $g_{2,1}{}^n$ from Hall A

$$\int_0^1 x\left(g_1^V(x) + 2\,g_2^V(x)\right)dx = 0$$