Structure of the Roper in Lattice QCD

Waseem Kamleh

Collaborators

Dale Roberts, Derek Leinweber, Adrian Kiratidis

Selim Mahbub, Peter Moran and Tony Williams

CSSM, University of Adelaide

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THE UNIVERSITY OF ADELAIDE AUSTRALIA

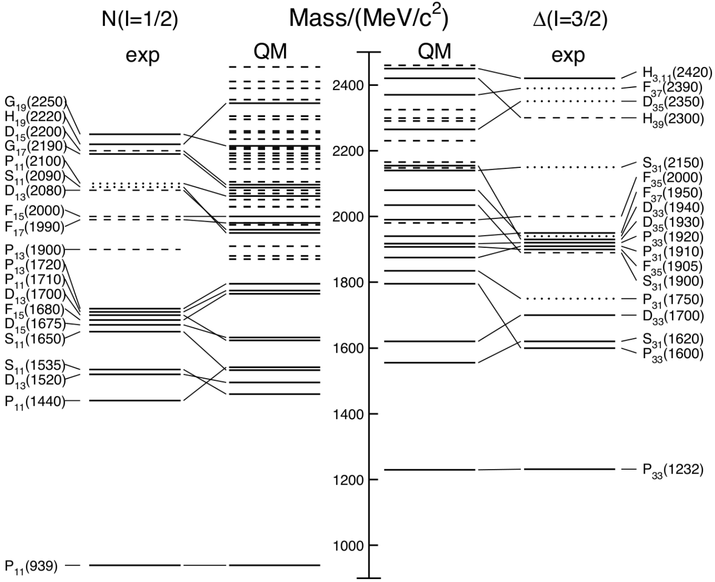
SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

$N(1440)\ 1/2^+$

$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

Breit-Wigner mass = 1420 to 1470 (≈ 1440) MeV
Breit-Wigner full width = 200 to 450 (≈ 300) MeV
$p_{beam} = 0.61$ GeV/c $\quad 4\pi\lambda\!\!\!^{-2} = 31.0$ mb
Re(pole position) = 1350 to 1380 (≈ 1365) MeV
−2Im(pole position) = 160 to 220 (≈ 190) MeV

***N*(1440) DECAY MODES**	Fraction (Γ_i/Γ)	p (MeV/c)
$N\pi$	55–75 %	398
$N\eta$	(0.0±1.0) %	†
$N\pi\pi$	30–40 %	347
$\Delta\pi$	20–30 %	147
$\Delta(1232)\pi$, P-wave	15–30 %	147
$N\rho$	<8 %	†
$N\rho$, S=1/2, P-wave	(0.0±1.0) %	†
$N(\pi\pi)^{I=0}_{S\text{-wave}}$	10–20 %	–
$p\gamma$	0.035–0.048 %	414
$p\gamma$, helicity=1/2	0.035–0.048 %	414
$n\gamma$	0.02–0.04 %	413
$n\gamma$, helicity=1/2	0.02–0.04 %	413

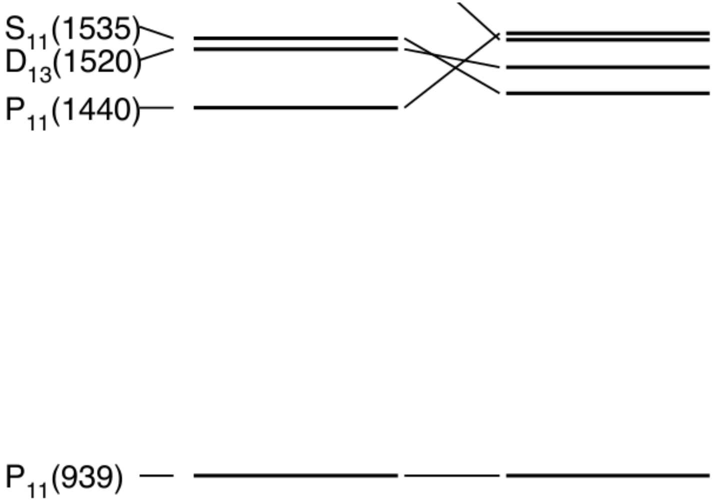


N(I=1/2)
Mass/(MeV/c2)
Δ(I=3/2)
exp
QM
QM
exp
2400
2200
2000
1800
1600
1400
1200
1000
G19(2250)
H19(2220)
D15(2200)
G17(2190)
P11(2100)
S11(2090)
D13(2080)
F15(2000)
F17(1990)
P13(1900)
P13(1720)
P11(1710)
D13(1700)
F15(1680)
D15(1675)
S11(1650)
S11(1535)
D13(1520)
P11(1440)
P11(939)
H3,11(2420)
F37(2390)
D35(2350)
H39(2300)
S31(2150)
F35(2000)
F37(1950)
D33(1940)
D35(1930)
P33(1920)
P31(1910)
F35(1905)
S31(1900)
P31(1750)
D33(1700)
S31(1620)
P33(1600)
P33(1232)

Roper Resonance

- Quark model: $N = 2$ radial excitation of the nucleon.
- Much lower in mass than simple quark model predictions.

Roper Resonance



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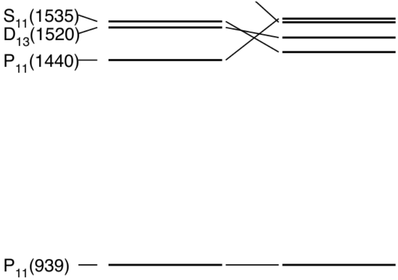

- Experiment: Lighter than $N = 1$ radial excitation of the nucleon, the negative parity $S_{11}(1535)$.
- “Exotic” in nature.

Roper Resonance

- It has proven difficult to isolate this state on the lattice.
- Consider the nucleon interpolators,

$$\chi_1(x) = \epsilon^{abc}(u^{Ta}(x)\, C\gamma_5\, d^b(x))\, u^c(x)\,,$$
$$\chi_2(x) = \epsilon^{abc}(u^{Ta}(x)\, C\, d^b(x))\, \gamma_5\, u^c(x)\,,$$
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  - We will see that this is wrong!
- Key to isolating this elusive state is an appropriate variational basis.
  - Phys.Lett. B707 (2012) 389-393, “Roper Resonance in 2+1 Flavor QCD”

Correlation Functions

- Start with the two point correlation function:

$$G(t,\vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}.\vec{x}} \langle \Omega | T\{\chi(x)\bar{\chi}(0)\} | \Omega \rangle.$$

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- Asymptotically, Euclidean time evolution isolates the ground state

$$G^{\pm}(t, \vec{0}) \overset{t \to \infty}{=} \lambda_0 \bar{\lambda}_0 e^{-M_0 t}.$$

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- Construct an $n \times n$ correlation matrix,

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- Solve a generalised eigenproblem to find the linear combination of interpolating fields,

$$\bar{\phi}^{\alpha} = \sum_{i=1}^{N} u_i^{\alpha}\, \bar{\chi}_i, \qquad \phi^{\alpha} = \sum_{i=1}^{N} v_i^{\alpha}\, \chi_i$$

such that the correlation matrix is diagonalised,

$$v_i^{\alpha} G_{ij}(t) u_j^{\beta} = \delta^{\alpha\beta} z^{\alpha} \bar{z}^{\beta} e^{-m_{\alpha} t}.$$

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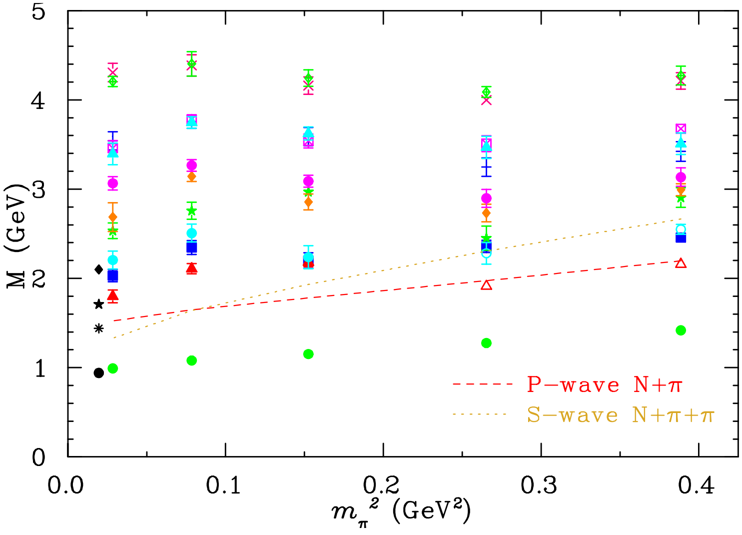
- Solution: Use different levels of gauge-invariant quark smearing to expand the operator basis.
- Analogy: Can expand any radial function using a basis of Gaussians of different widths

$$f(|\vec{r}|) = \sum_i c_i e^{-\varepsilon_i r^2}.$$

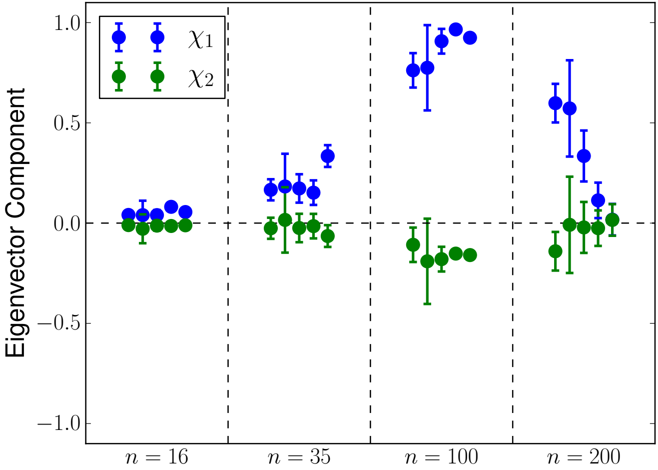
Simulation Details

- PACS-CS Configs (via ILDG)
  - S. Aoki, et al., Phys. Rev. **D79** (2009) 034503.
  - $2 + 1$ flavour dynamical-fermion QCD
  - Lattice volume: $32^3 \times 64$
  - $a = 0.0907$ fm, $\sim (2.9 \text{ fm})^3$
  - $m_\pi = \{$ 156, 293, 413, 572, 702 $\}$ MeV
- Combined 8×8 correlation matrix analysis using χ_1, χ_2 and χ_1, χ_4 with 4 different levels ($n = 16, 35, 100, 200$) of smearing.
- Corresponds to RMS radii of 2.37, 3.50, 5.92 and 8.55 lattice units.

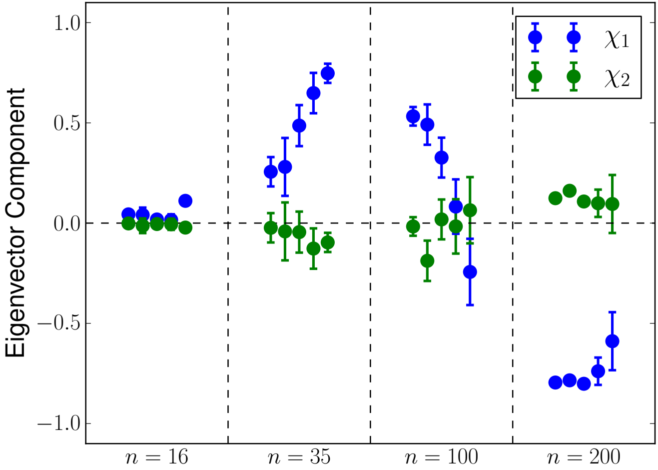
N^+ spectrum



Eigenvector analysis – State 1 (Ground)

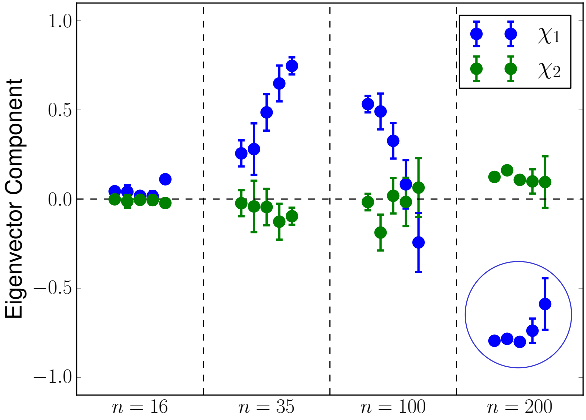


Eigenvector analysis – State 2 (First Excited)



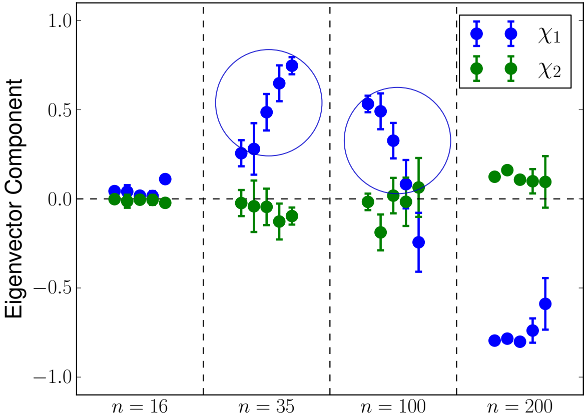
Eigenvector analysis – First Excited State

- Dominant contribution is from $\chi_1, n = 200$.



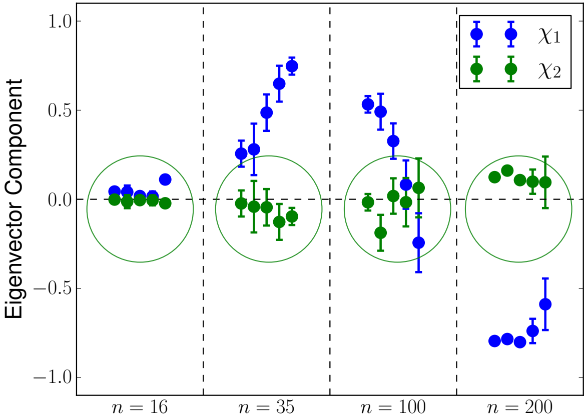
Eigenvector analysis – First Excited State

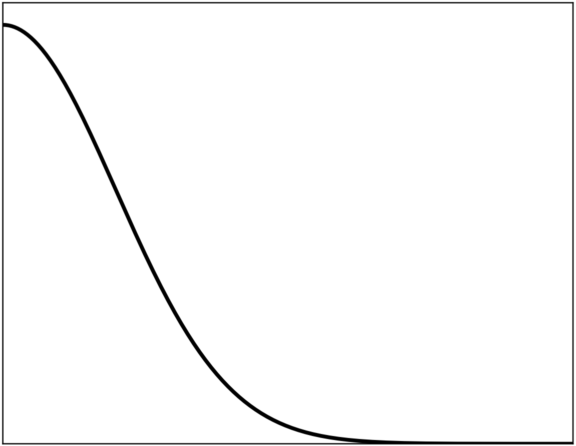
- Opposite contribution from a mix of $\chi_1, n = 100$ and $\chi_1, n = 35$.

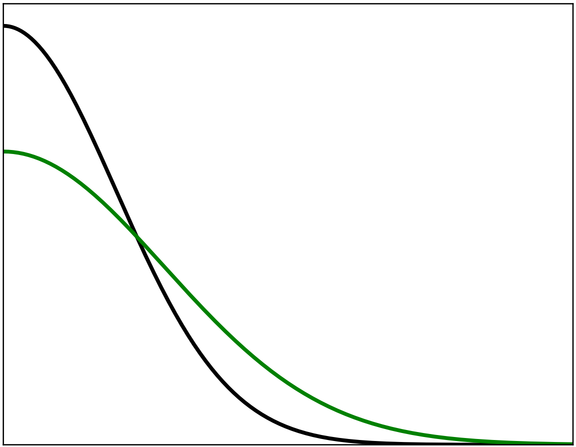


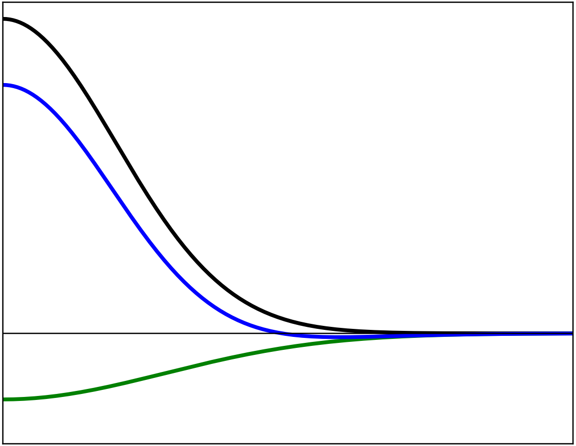
Eigenvector analysis – First Excited State

- Negligible contribution from $\chi_1, n = 16$ and all χ_2 operators.

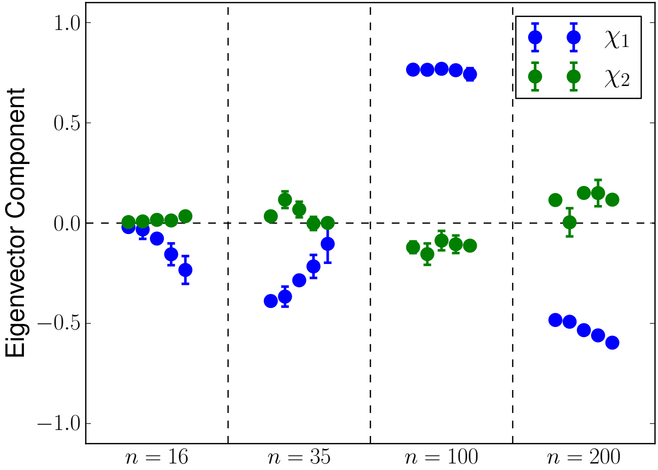








Eigenvector analysis – State 4



Eigenvector analysis

- First positive-parity excited state couples strongly to χ_1.
- Large smearing values are critical.
- χ_2 coupling to the Roper is negligible.
- Transition from scattering state to resonance as quark mass drops.
  - The 3-quark coupling to meson-baryon scattering states is suppressed by the lattice volume $\sim 1/\sqrt{V}$.
- At light quark mass the Roper mass is pushed up due to finite volume effects?
- How can we learn more?
  - Study multiple lattice volumes.

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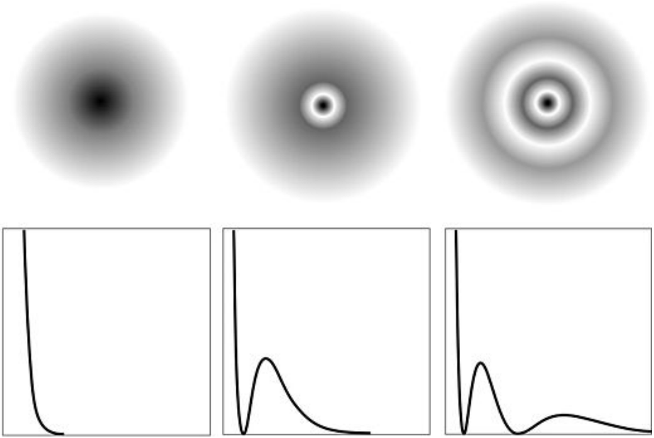
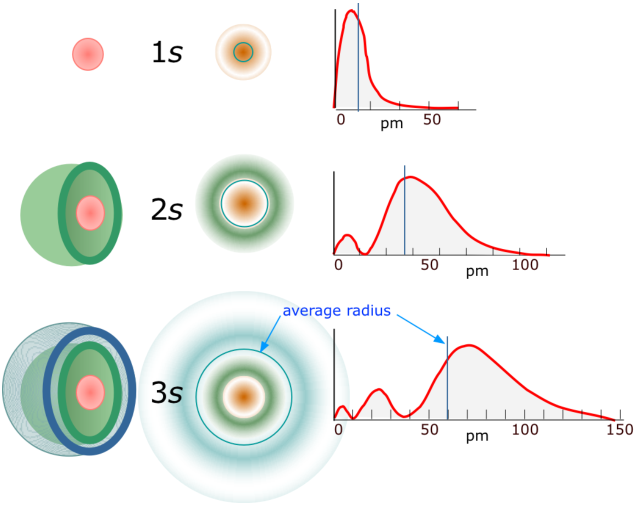
  - Look at the excited state structure via the wave function.



Hydrogen S states



Wave function of the Roper

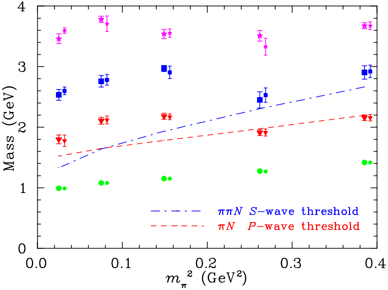
- We explore the structure of the nucleon excitations by examining the Bethe-Salpeter amplitude.
- The baryon wave function is built by giving each quark field in the annihilation operator a spatial dependence,

$$\chi_1(\vec{x},\vec{y},\vec{z},\vec{w}) = \epsilon^{abc}\,(\,u_a^T(\vec{x}+\vec{y})\,C\gamma_5\,d_b(\vec{x}+\vec{z})\,)\,u_c(\vec{x}+\vec{w}).$$

- The creation operator remains local.
- The resulting construction is gauge-dependent.
  - We choose to fix to Landau gauge.

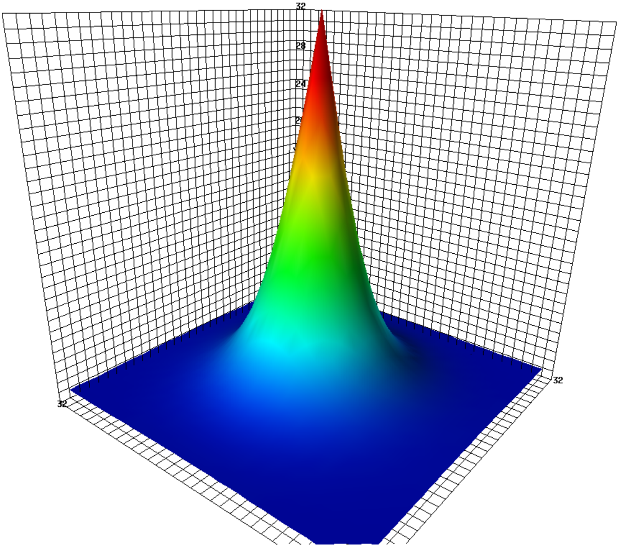
Wave function of the Roper

- Non-local sink operator cannot be smeared.
- Construct states using right eigenvector u^{α} only.
- Eigenvectors from 4×4 CM analysis using χ_1 only.



- The position of the u quarks is fixed and we measure the d quark probability distribution.

Ground state probability distribution



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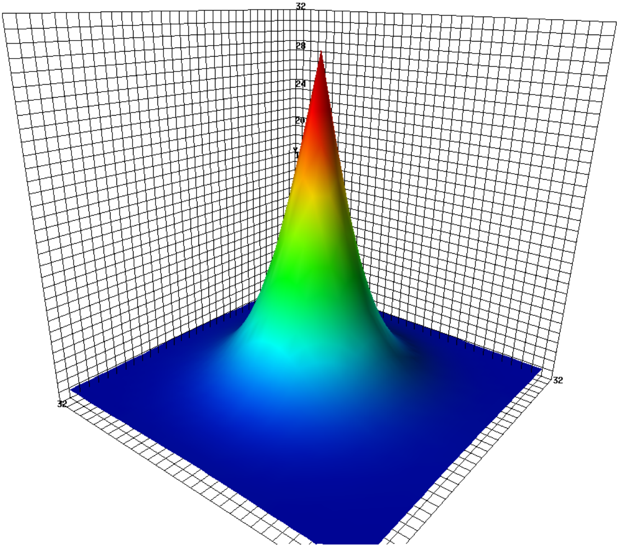

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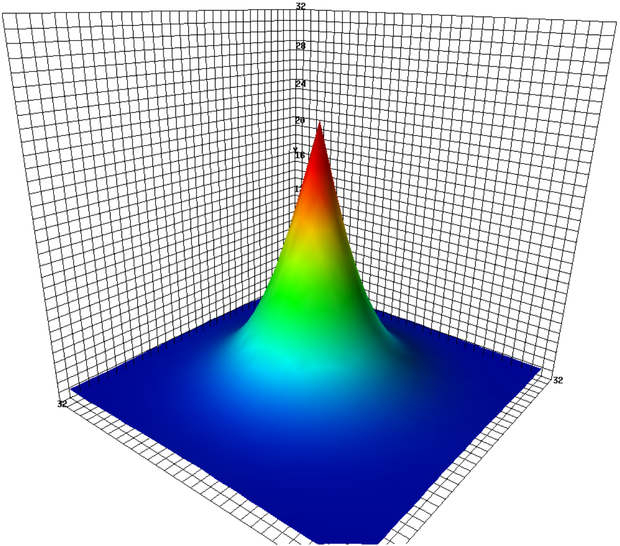

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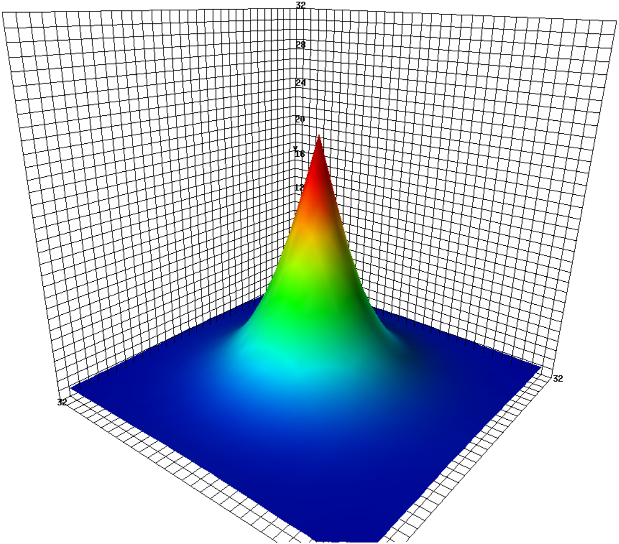

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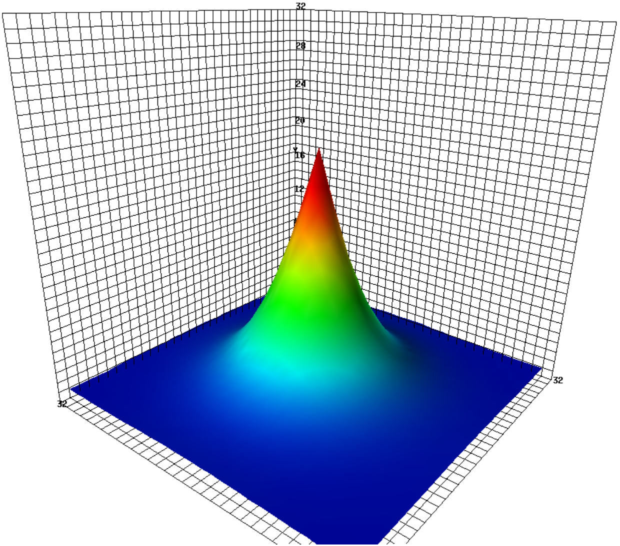
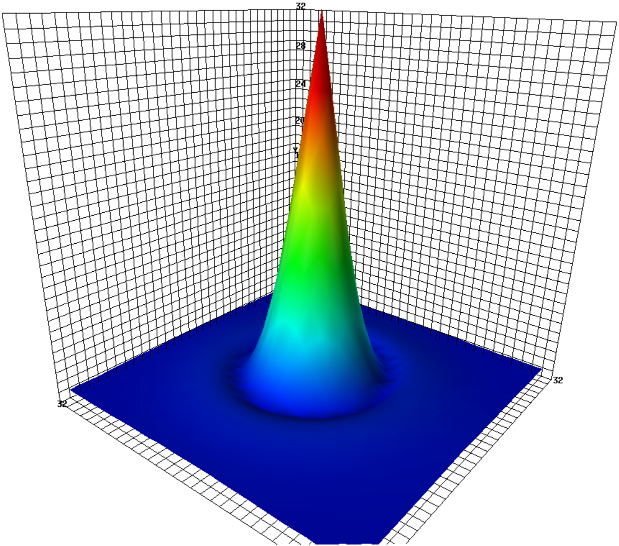


First excited state probability distribution



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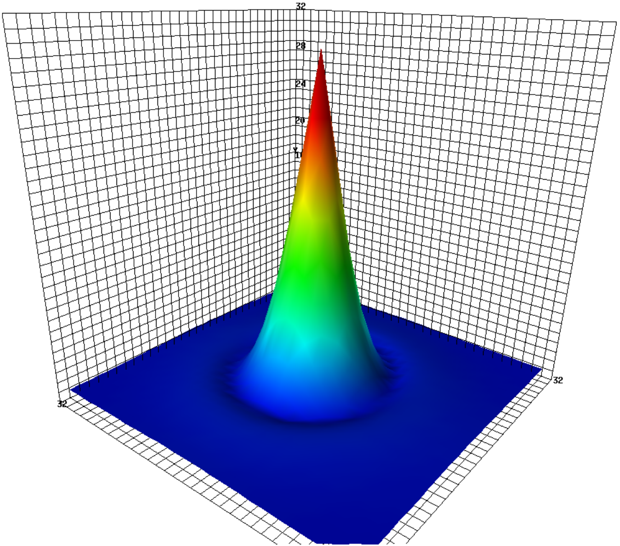

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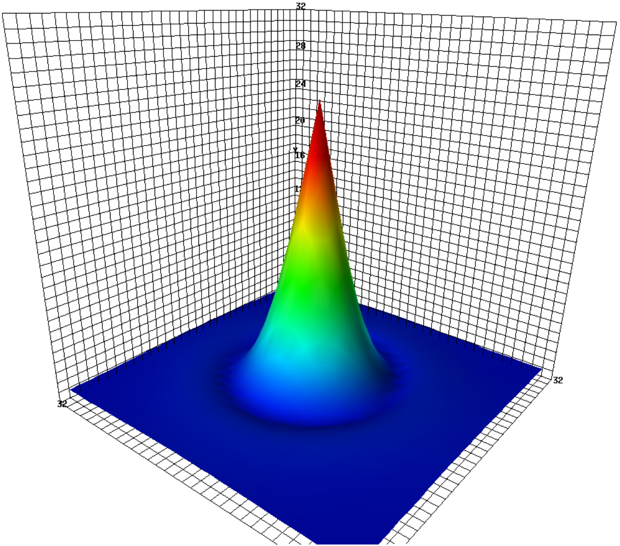

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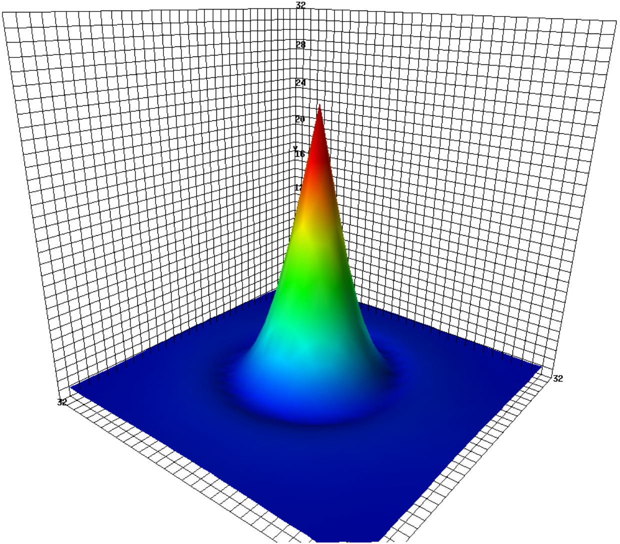

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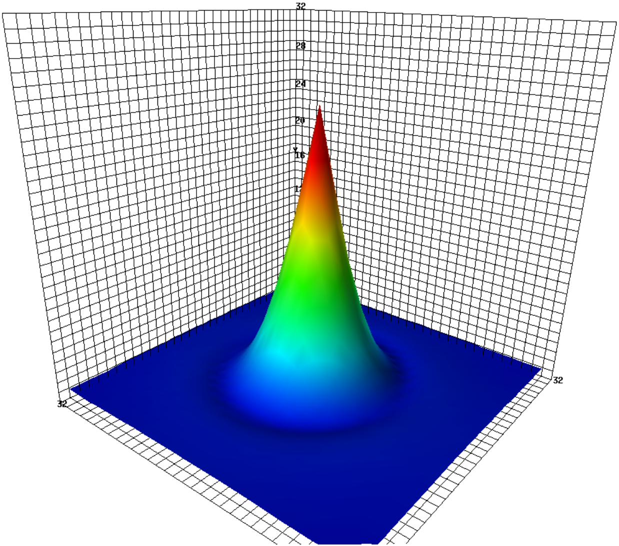
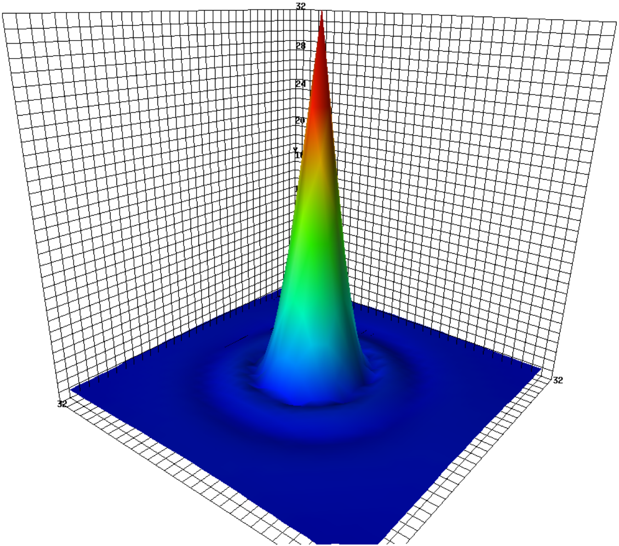


Second excited state probability distribution



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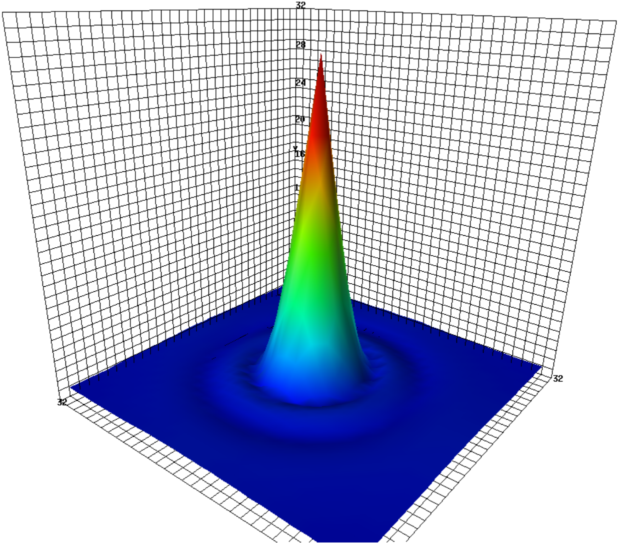

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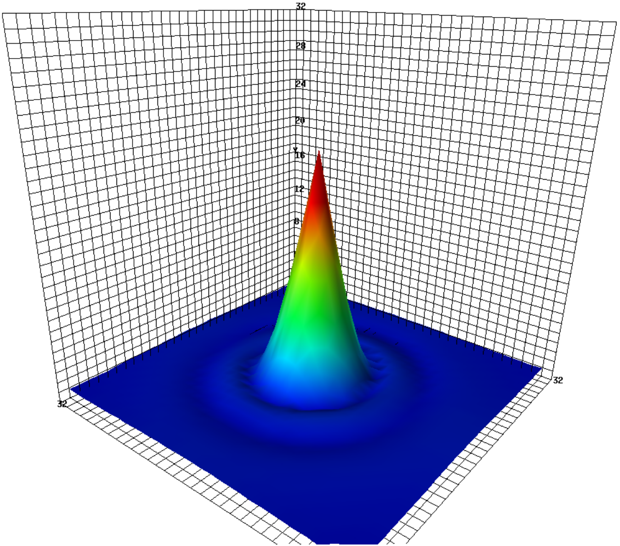

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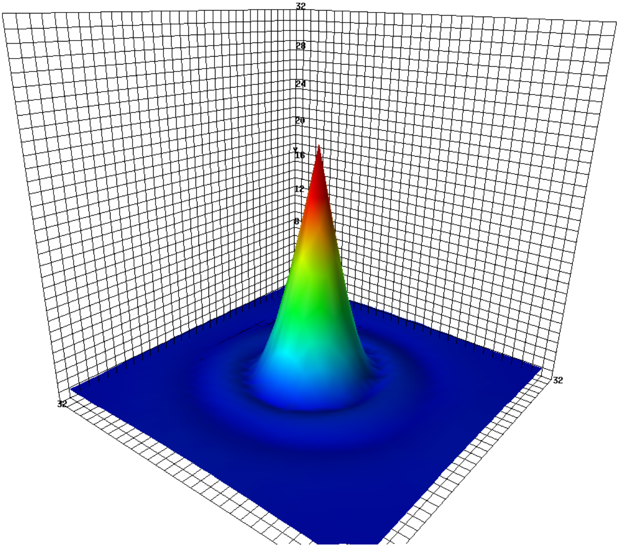

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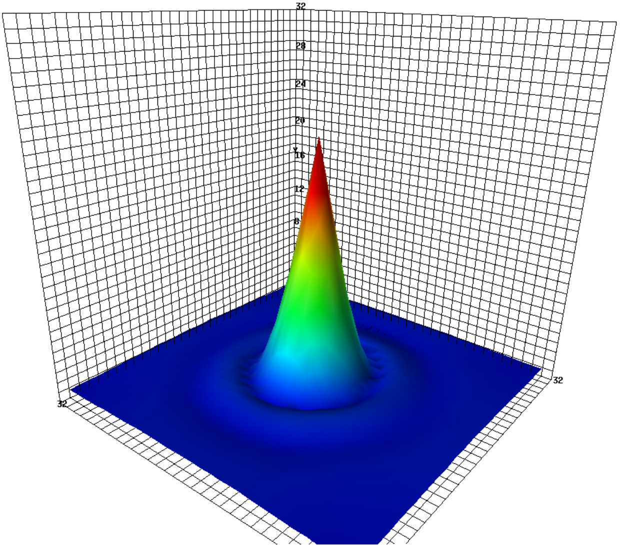

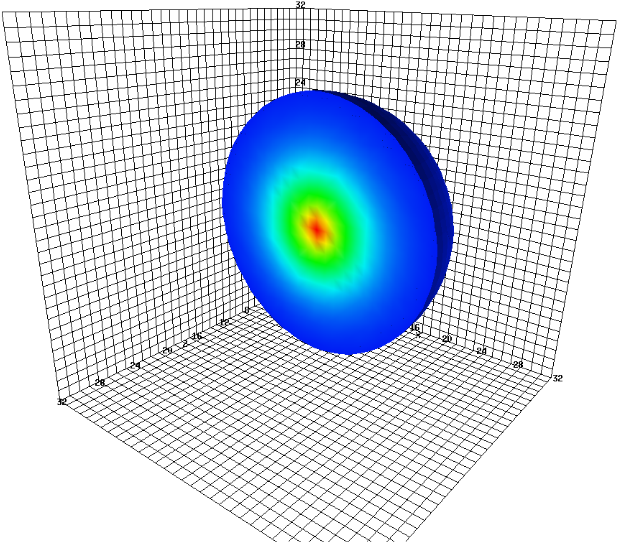

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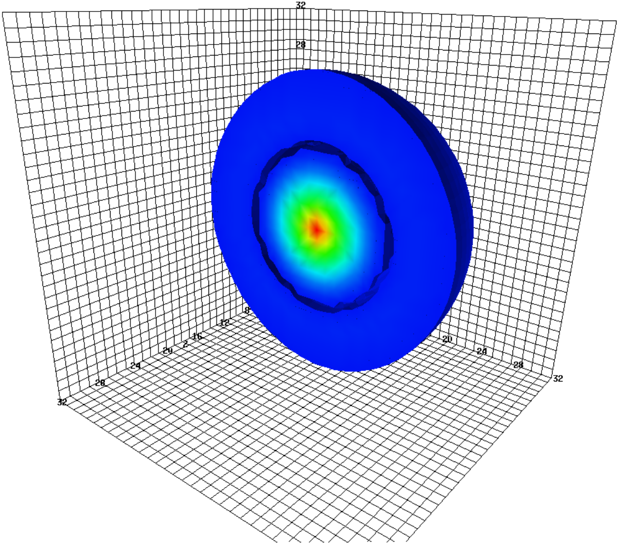
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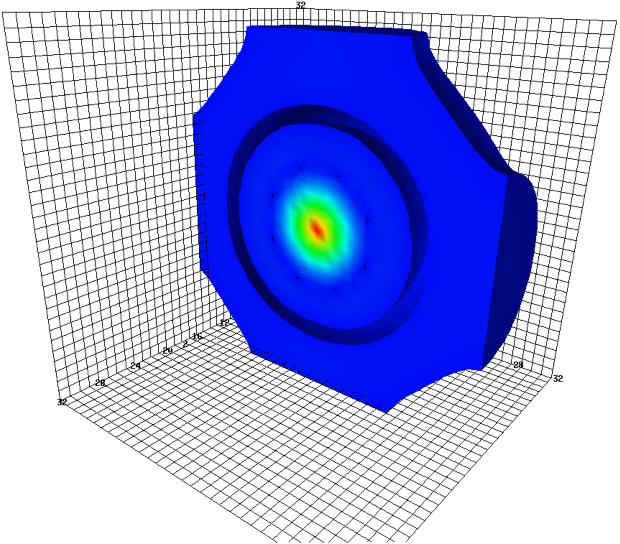

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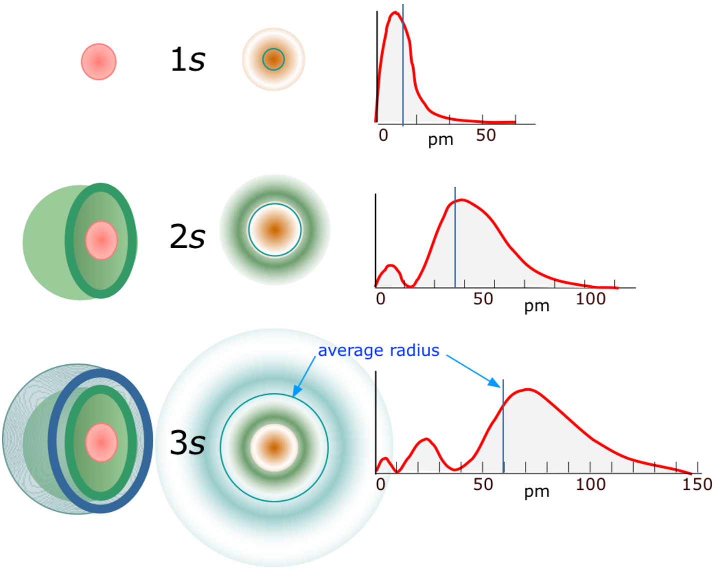
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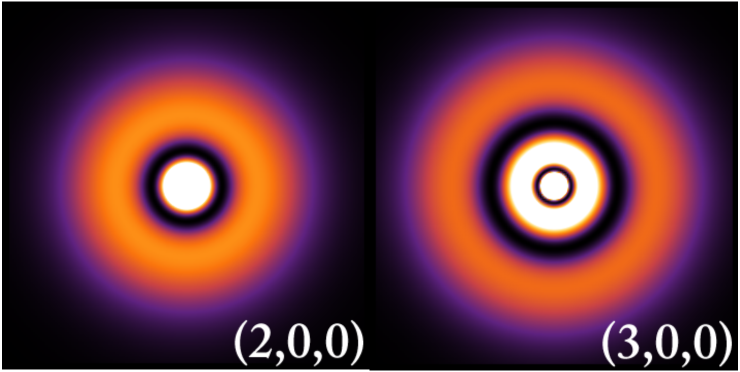
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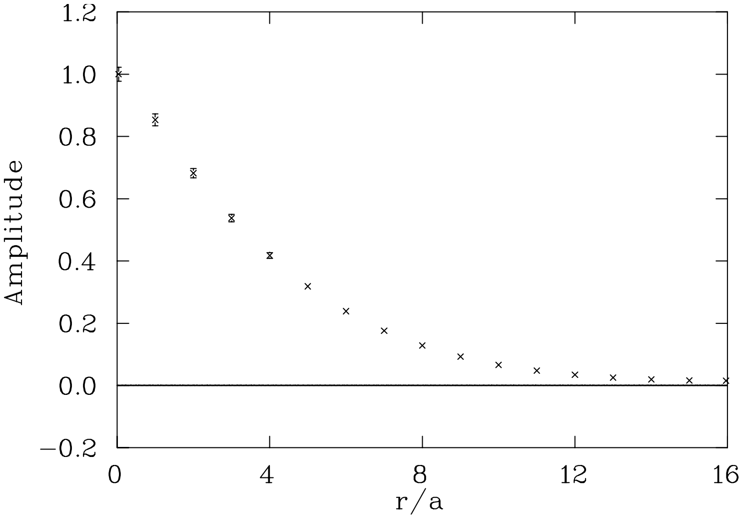
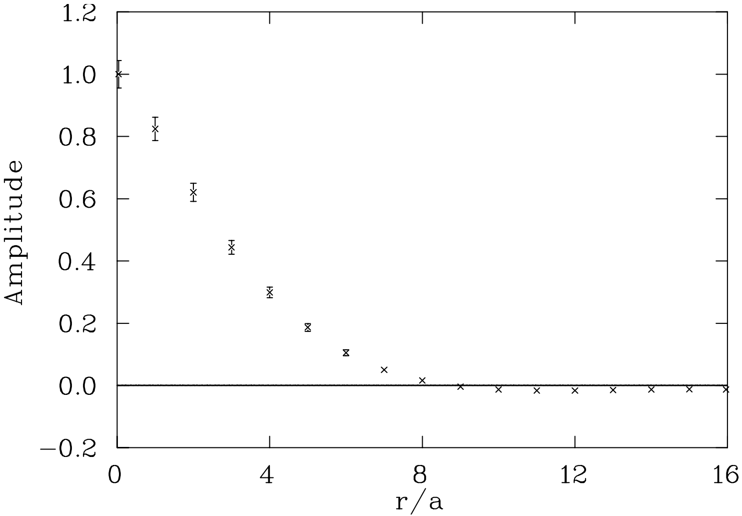
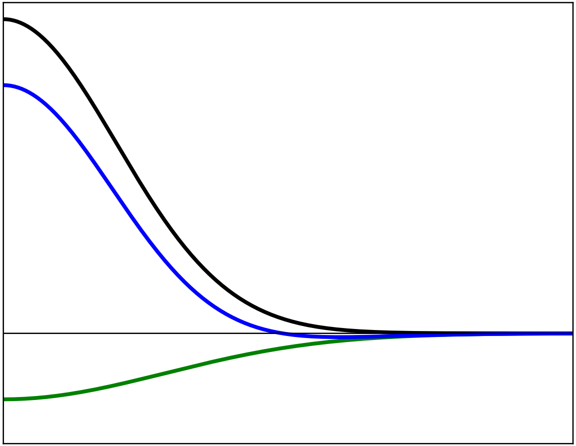
Wave Function



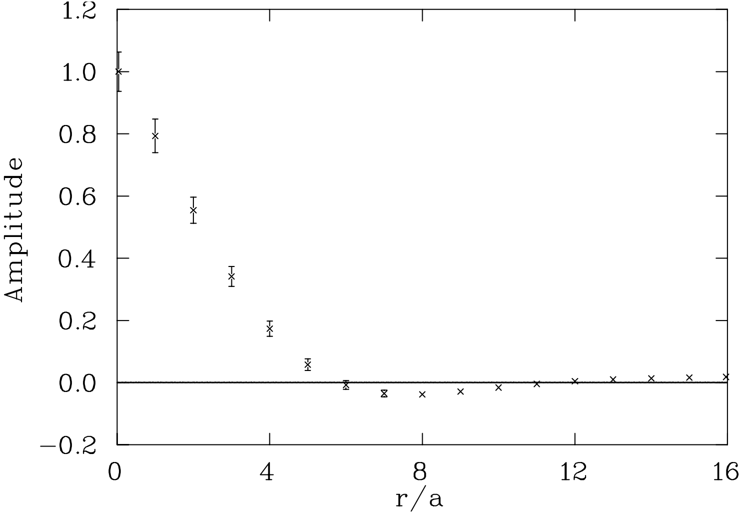
Wave Function



Wave Function



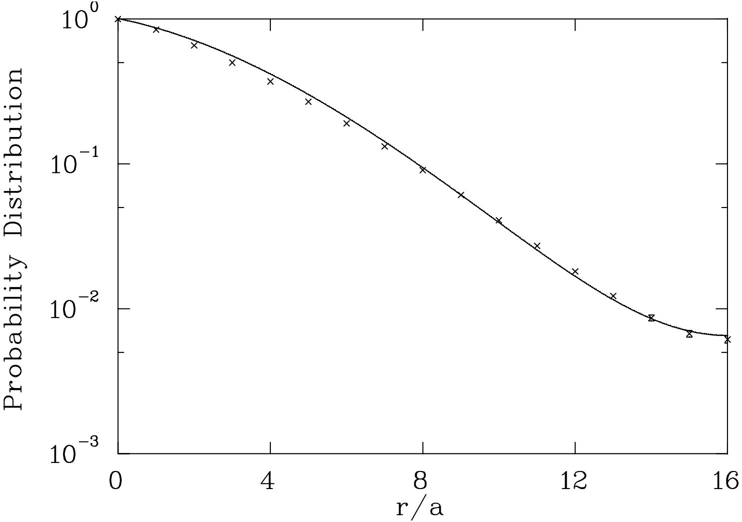
Wave Function



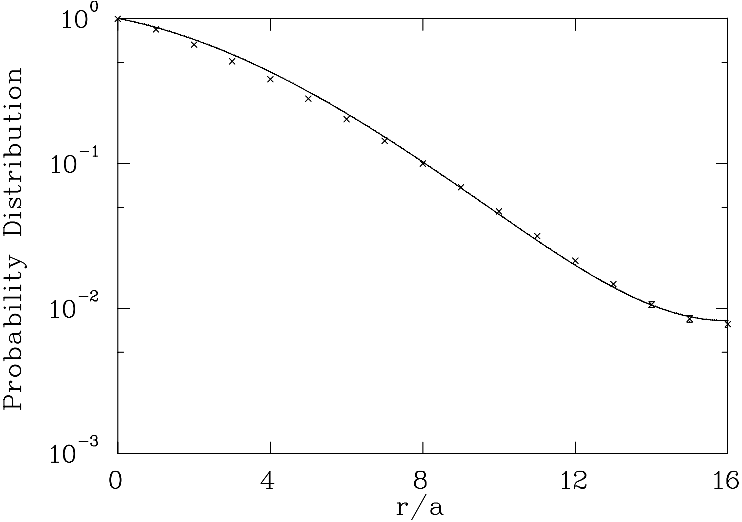
Quark Model comparison

- Compare to a non-relativistic constituent quark model.
  - One-gluon-exchange motivated Coulomb + ramp potential.
  - Spin dependence in R. K. Bhaduri, L. E. Cohler and Y. Nogami, Phys. Rev. Lett. 44 (1980) 1369.
- The radial Schrodinger equation is solved with boundary conditions relevant to the lattice data.
  - The derivative of the wave function is set to vanish at a distance $L_x/2$.
- Two parameter fit to the nucleon radial wave function yields:
  - String tension $\sqrt{\sigma} = 400$ MeV
  - Constituent quark mass $m_q = 360$ MeV
- These parameters are held fixed for the excited states.

Ground state comparison



Ground state comparison



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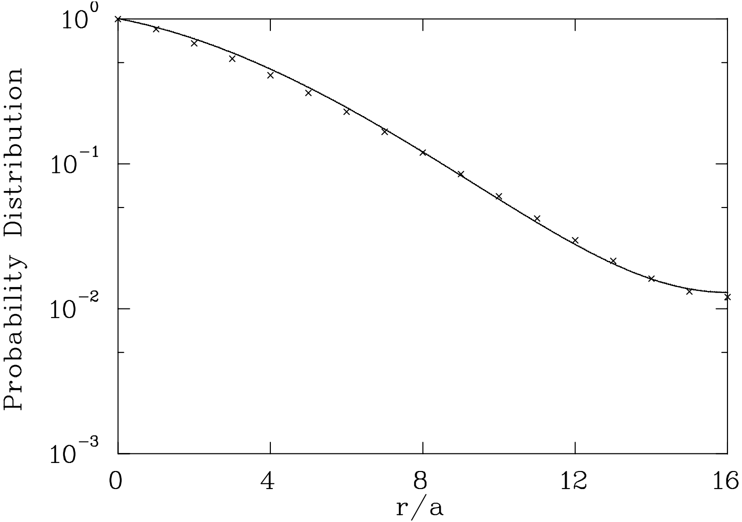
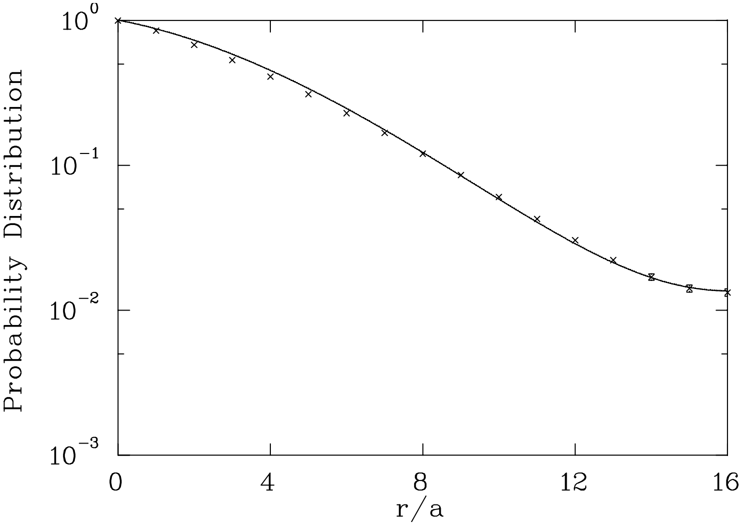
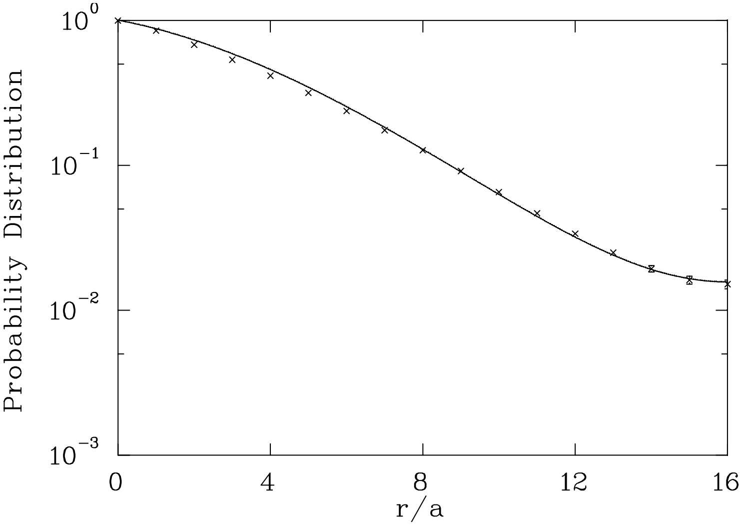


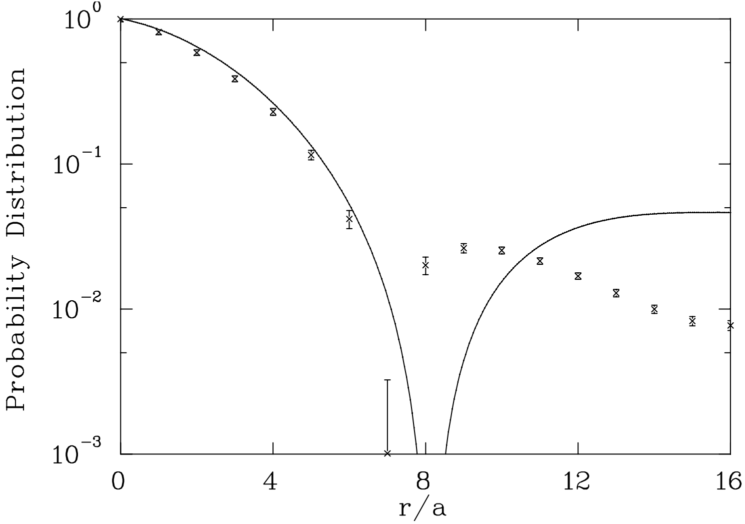
Ground state comparison



Ground state comparison



First excited state comparison



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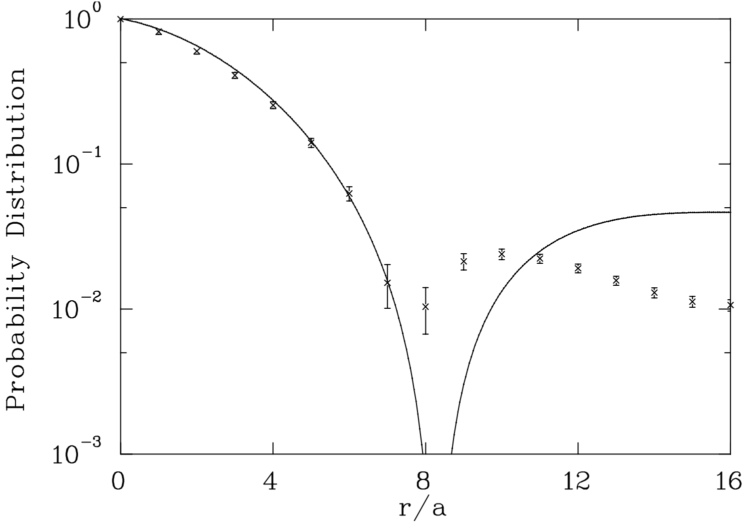

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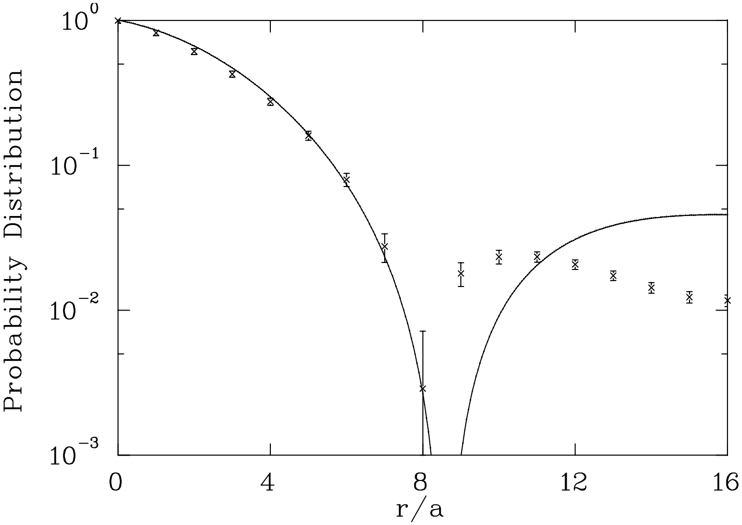

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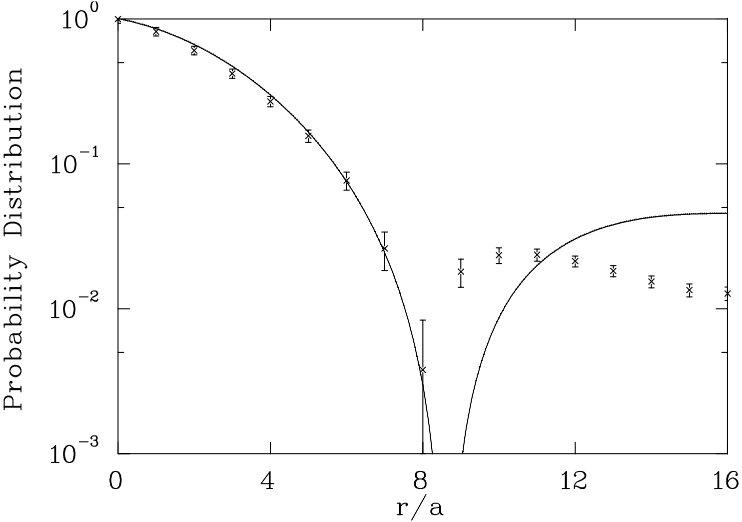

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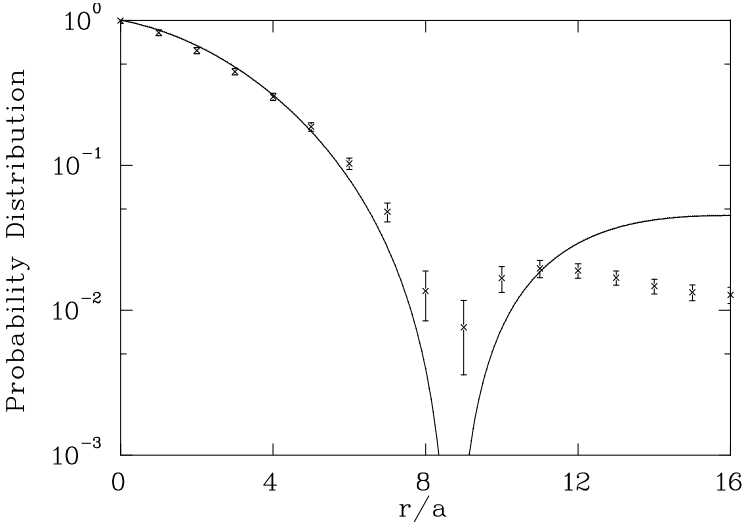
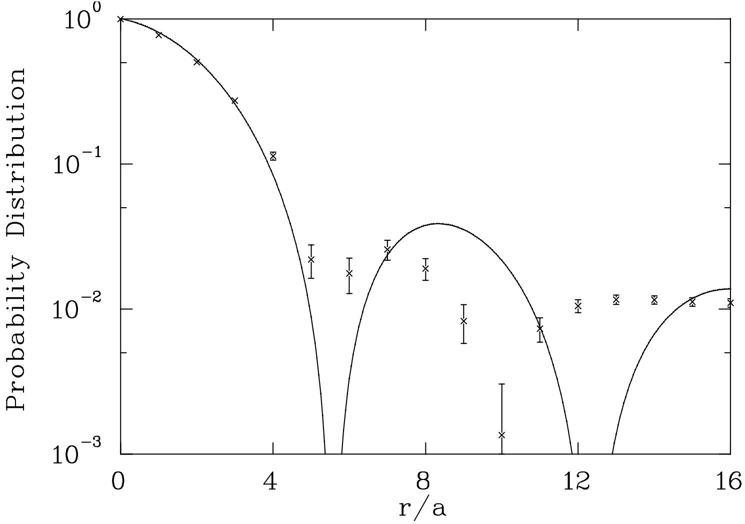


Second excited state comparison



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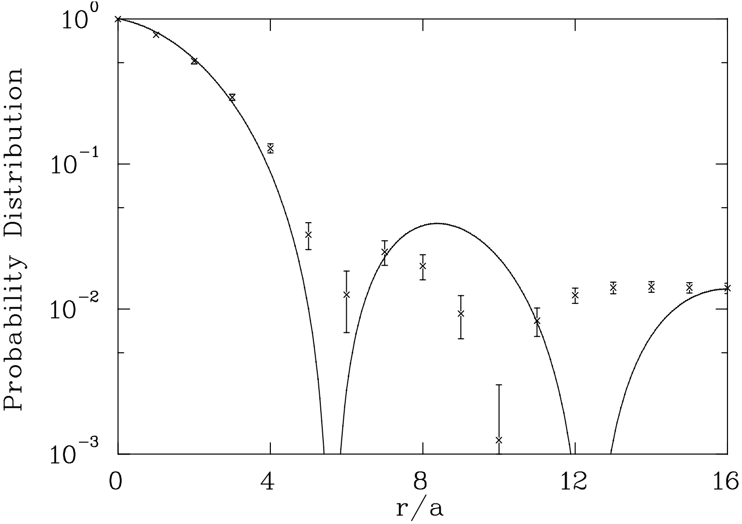

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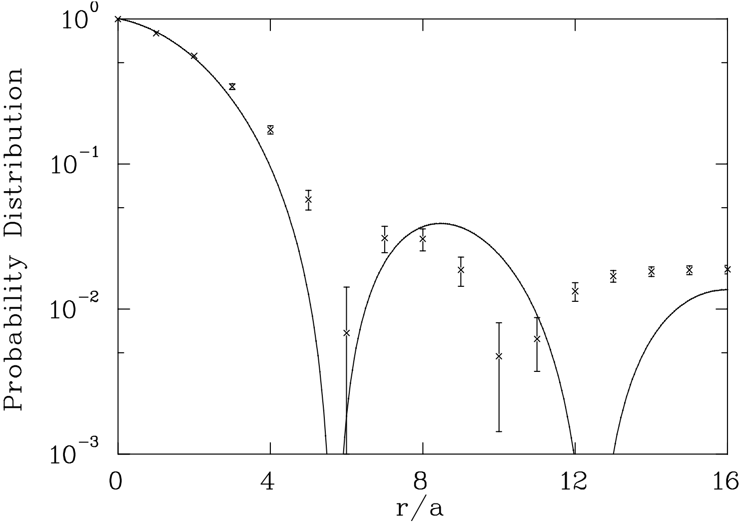

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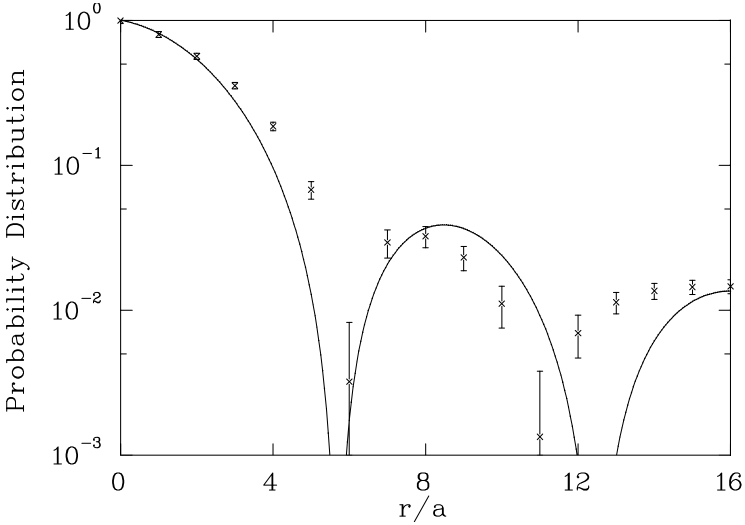

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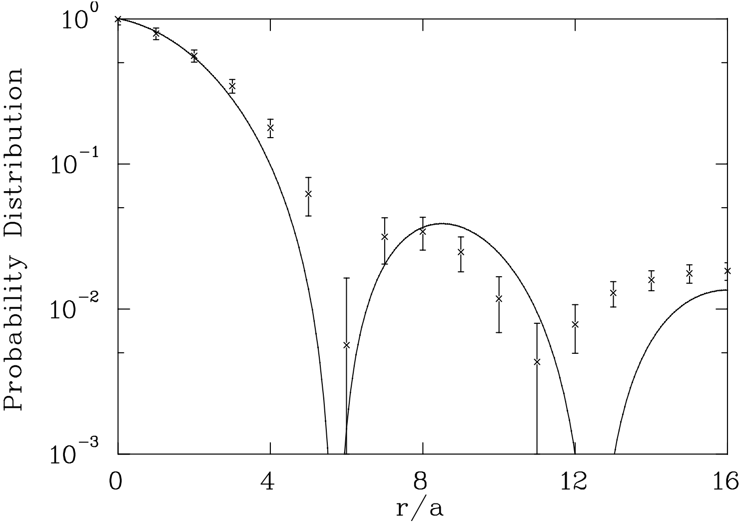
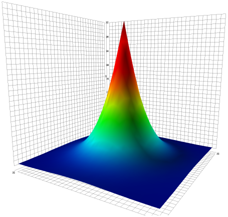
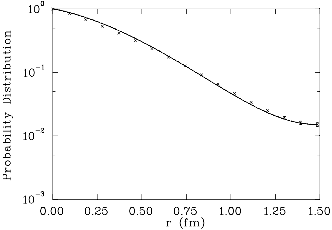


Quark Model comparison

- Ground state QM agrees well (as expected).





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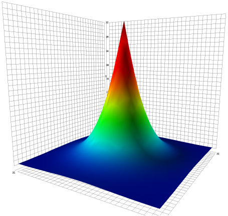
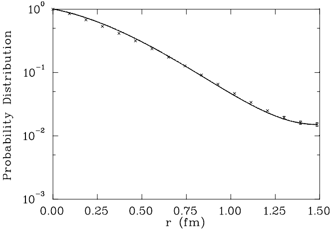
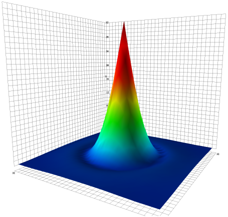
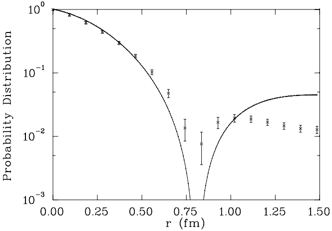




- First excited state shows a node structure.





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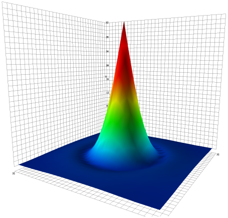

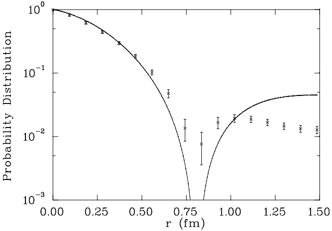

- Consistent with $N = 2$ radial excitation.
- QM predicts node position fairly well.
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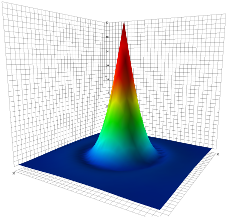

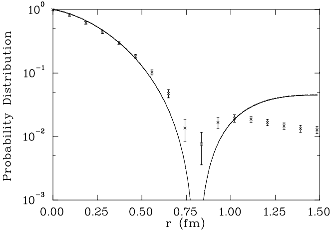

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  - Reveals why an overlap of two broad Gaussians with opposite sign is needed to form the Roper.
- Second excited state shows a double node structure.
  - Consistent with $N = 3$ radial excitation.
  - Similar story for QM comparison.

Quark Model comparison

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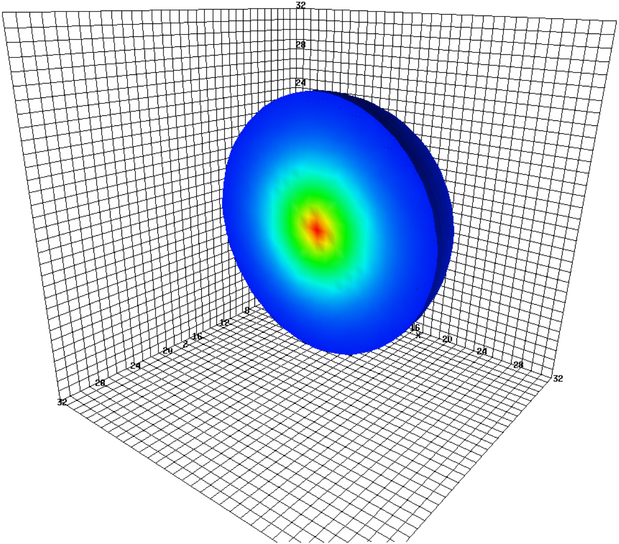
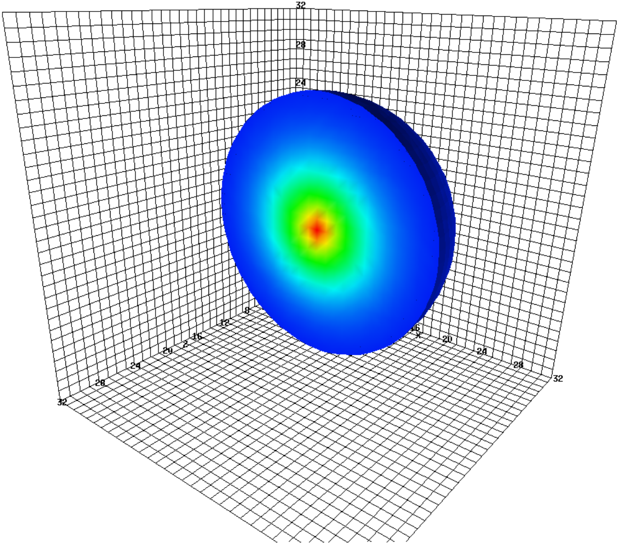
- First excited state shows a node structure.
  - Consistent with $N = 2$ radial excitation.
  - QM predicts node position fairly well.
  - QM disagrees near the boundary.
  - Reveals why an overlap of two broad Gaussians with opposite sign is needed to form the Roper.
- Second excited state shows a double node structure.
  - Consistent with $N = 3$ radial excitation.
  - Similar story for QM comparison.
- Finite volume effects?

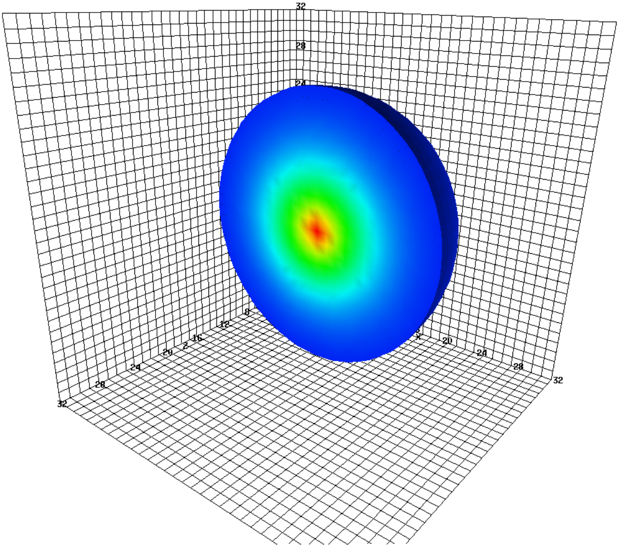
Ground state probability distribution



Ground state probability distribution



Ground state probability distribution



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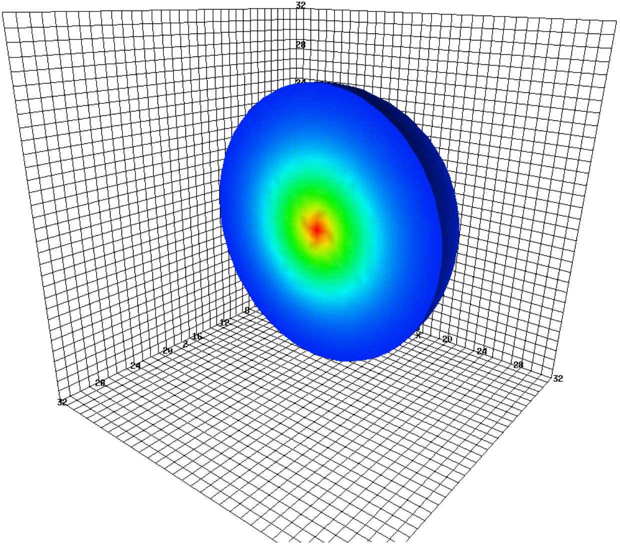
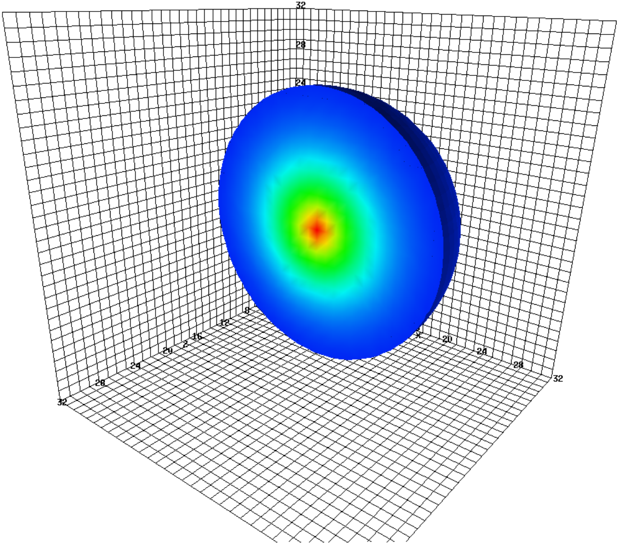
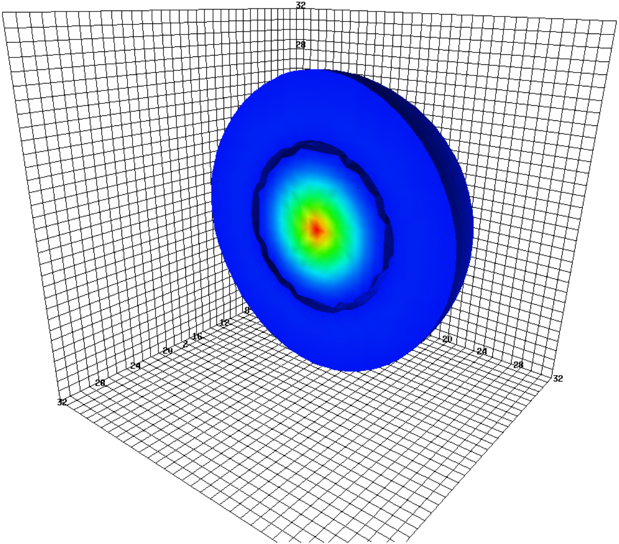


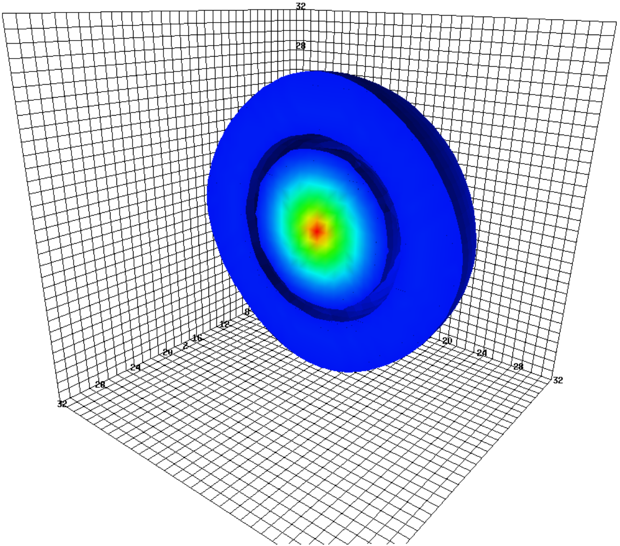
Ground state probability distribution



First excited state probability distribution



First excited state probability distribution



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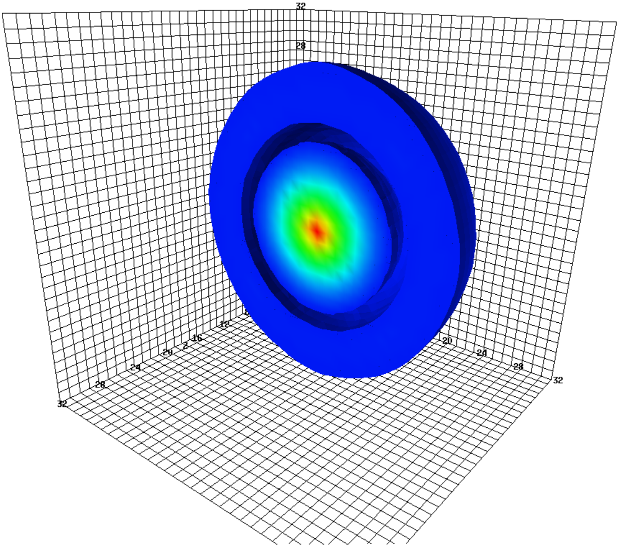

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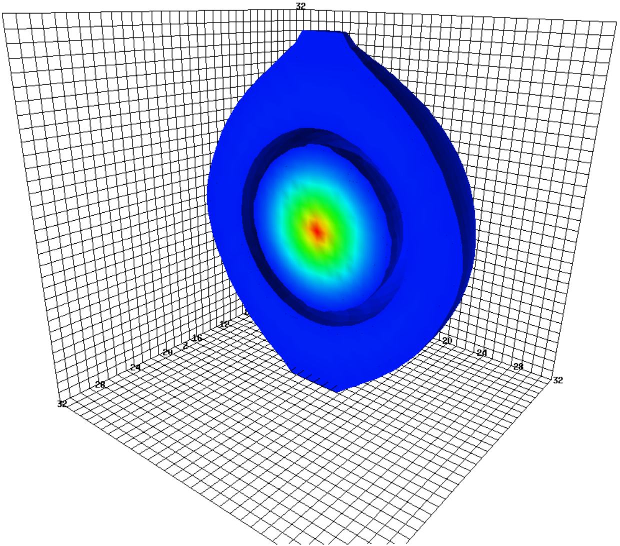
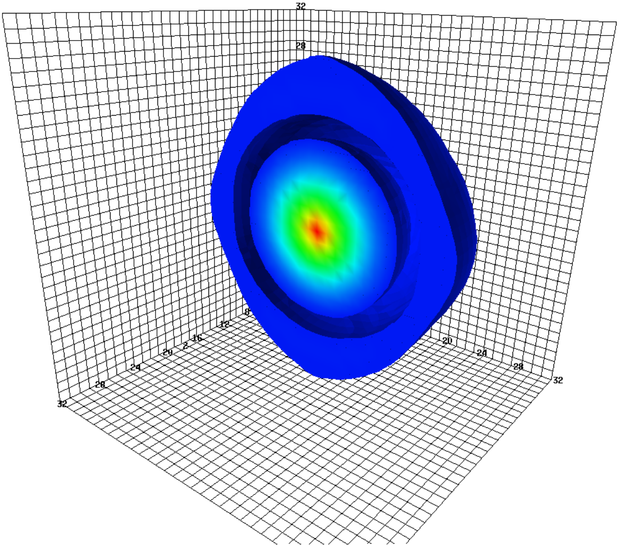
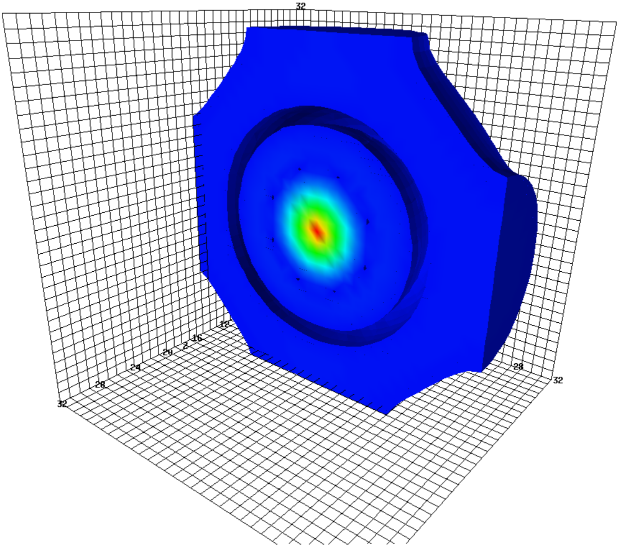


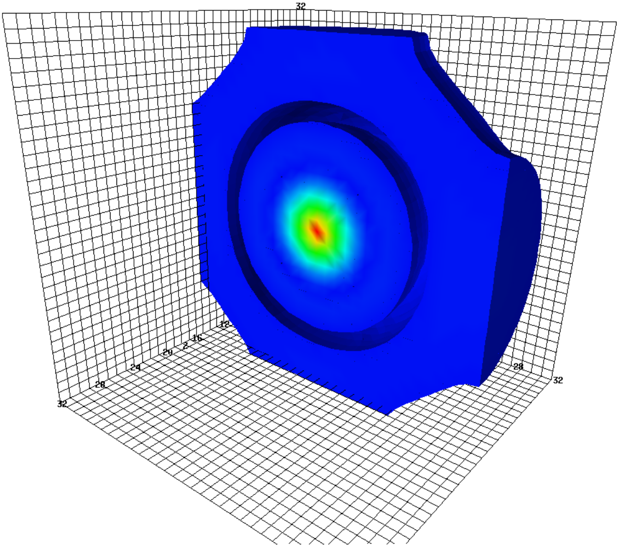
First excited state probability distribution



Second excited state probability distribution



Second excited state probability distribution



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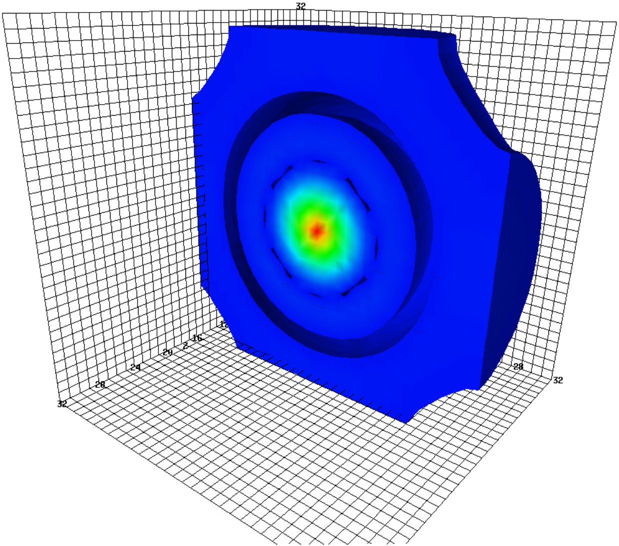
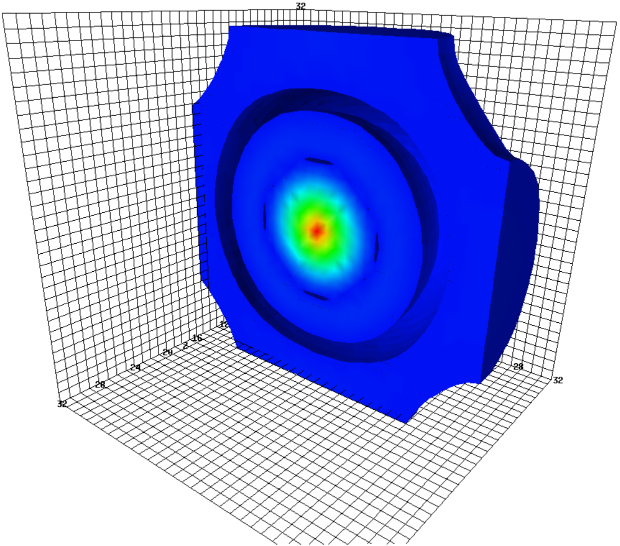
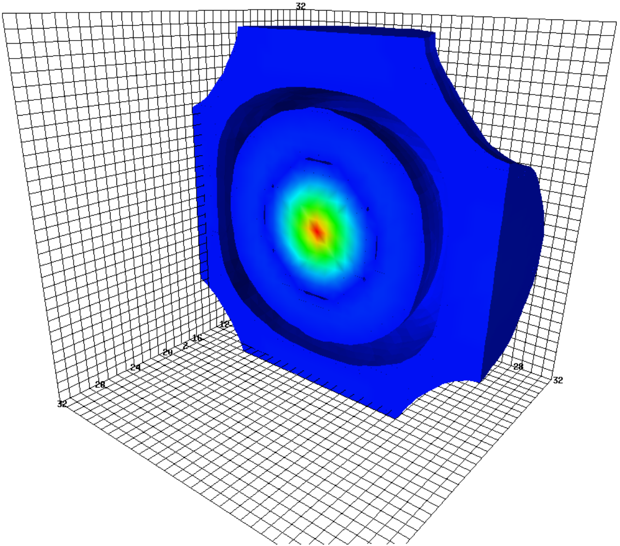


Second excited state probability distribution

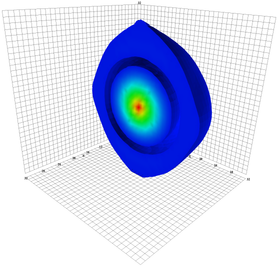


Second excited state probability distribution



Quark Model comparison

- Wave function should be spherically symmetric.



- Outer shell of Roper wave function clearly reveals distortion due to finite volume.
- Effective field theory arguments suggest the small volume will drive up the energy.

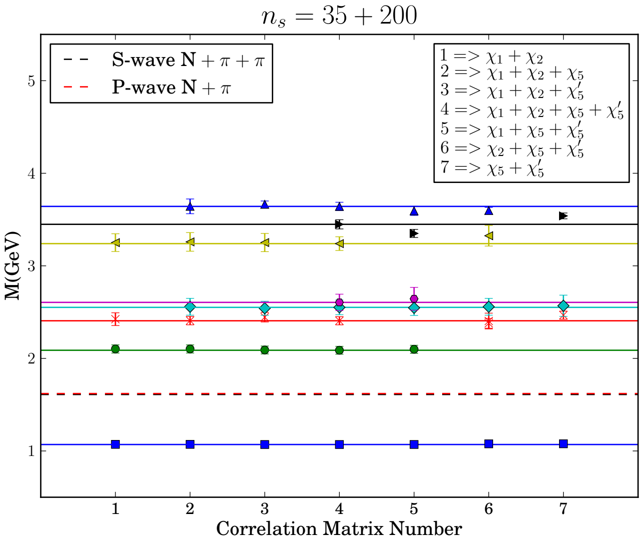
5-quark operators

- What if the Roper has a large 5-quark component?
- Dynamical gauge fields – can create $q\bar{q}$ from glue.
- Maybe we can do a better job by introducing 5-quark operators?
- Take χ_1 and χ_2 operators and couple a π to get $N^+_{\frac{1}{2}}$ quantum numbers:

$$\chi_1 + \pi \to \chi_5$$
$$\chi_2 + \pi \to \chi_5'$$

- Preliminary results at $m_\pi = 293$ MeV with two smearings $n = 35, 200$.

N+ spectrum with 5 quark operators



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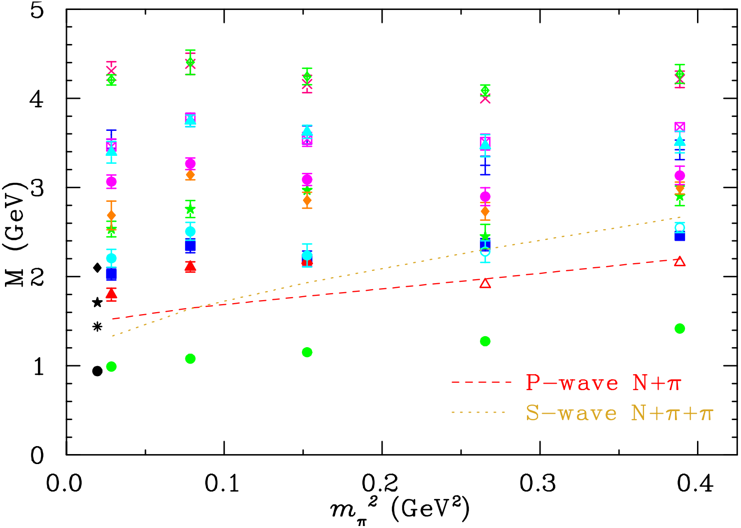
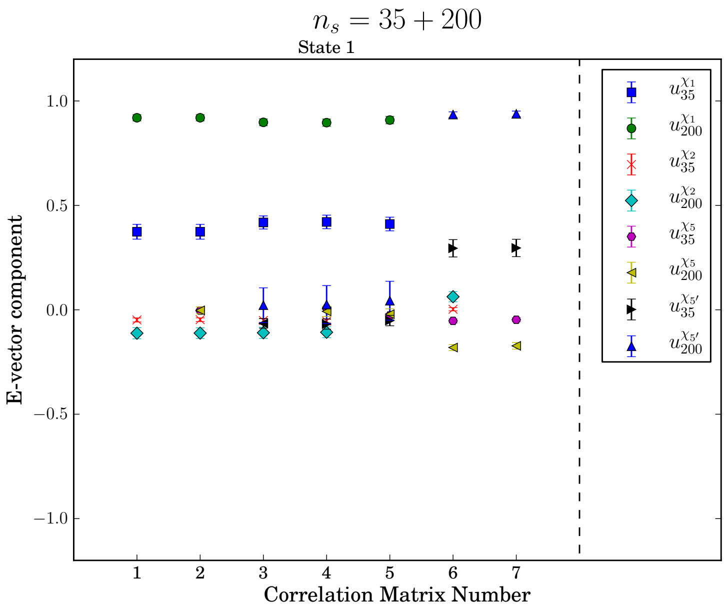
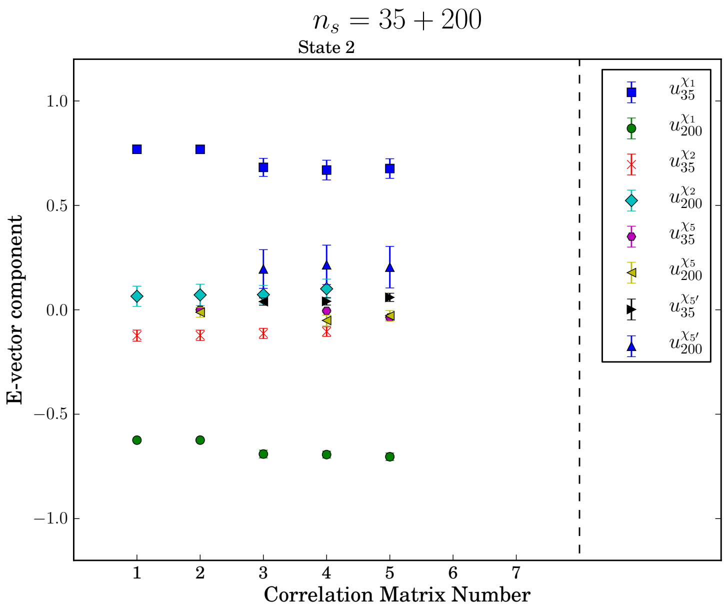




$n_s = 35 + 200$
State 1
1.0
0.5
0.0
−0.5
−1.0
E-vector component
1
2
3
4
5
6
7
Correlation Matrix Number
$u_{35}^{\chi_1}$
$u_{200}^{\chi_1}$
$u_{35}^{\chi_2}$
$u_{200}^{\chi_2}$
$u_{35}^{\chi_5}$
$u_{200}^{\chi_5}$
$u_{35}^{\chi_{5'}}$
$u_{200}^{\chi_{5'}}$



$n_s = 35 + 200$
State 2
E-vector component
Correlation Matrix Number
1.0
0.5
0.0
−0.5
−1.0
1
2
3
4
5
6
7
$u_{35}^{\chi_1}$
$u_{200}^{\chi_1}$
$u_{35}^{\chi_2}$
$u_{200}^{\chi_2}$
$u_{35}^{\chi_5}$
$u_{200}^{\chi_5}$
$u_{35}^{\chi_{5'}}$
$u_{200}^{\chi_{5'}}$

Summary

- A basis of multiple Gaussian smearings is well-suited to isolating radial excitations of the nucleon.
- The variational method allows us to access a state that is consistent with the $N = 2$ Roper excitation with standard three-quark interpolators.
- Probing the Roper wave function reveals a nodal structure.
  - χ_2 has negligible coupling to the Roper.
  - Multiple χ_1 operators at large smearings are critical to form the correct structure.

Summary

- Qualitative agreement with QM predictions for the Roper radial wave function.
- Finite volume effects clearly evident in the Roper probability distribution.
  - Larger lattice volumes needed!
- Preliminary results do not indicate a strong coupling to 5-quark operators.