Kaon Elastic Form Factor and Parton Distribution Functions

Parada Hutauruk[1], I. C. Cloet[2] and Anthony Thomas[1]

[1] CSSM and ARC Center of Excellence in Particle Physics at the Terascale

[2] Physics Division, Argonne National Laboratory

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Outline

- Motivation

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- Motivation
- Nambu-Jona-Lasinio (NJL) Model
  - Dynamical quark Mass (Gap equation)
  - Bethe-Salpeter Equation $\rightarrow$ bound states

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- Conclusion

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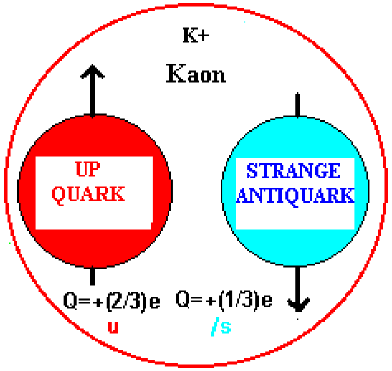
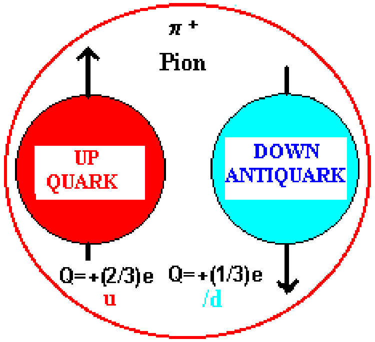
Background

- The K^+ consists of a quark-antiquark pair with spin-0





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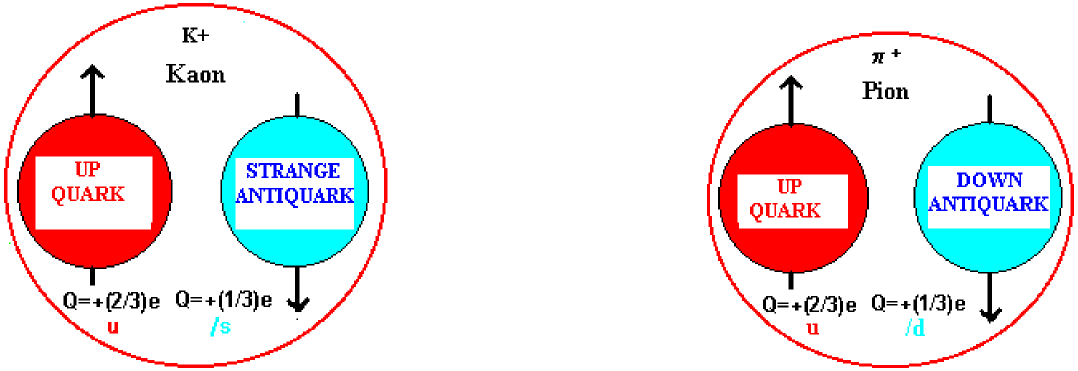

- The EM form factors and PDFs play a key role in revealing the internal structure of hadrons

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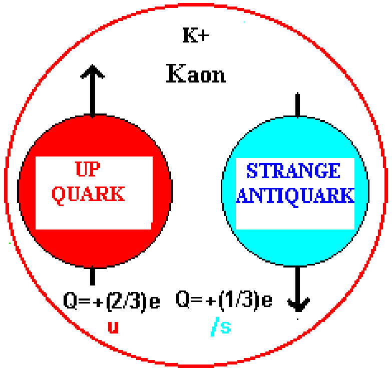

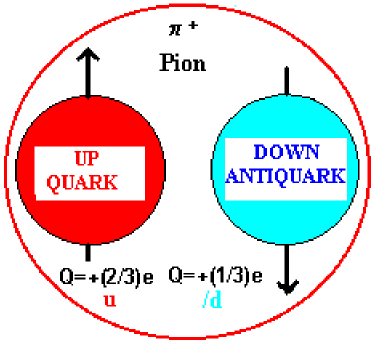

- The EM form factors and PDFs play a key role in revealing the internal structure of hadrons
- New data from COMPASS, JLAB 12 GeV for PDFs and FFs will be coming soon

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NJL Model

- The NJL model is built mimicking QCD symmetries properties [Prog. Part. Nucl. Phys. 27 (1991) 195]

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NJL Model

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- Therefore the NJL Lagrangian must fulfill the QCD symmetries

$$\mathcal{S}_{NJL} = SU(3)_c \otimes SU(N_f)_V \otimes SU(N_f)_A,$$
$$\otimes\ U(1)_V \otimes \mathcal{C} \otimes \mathcal{P} \otimes \mathcal{T}, \quad (2)$$

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NJL Model

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$$\begin{aligned} \mathcal{S}_{NJL} &= SU(3)_c \otimes SU(N_f)_V \otimes SU(N_f)_A, \\ &\otimes \; U(1)_V \otimes \mathcal{C} \otimes \mathcal{P} \otimes \mathcal{T}, \end{aligned} \tag{3}$$

- The NJL model is non-renormalizable, therefore $\longrightarrow$ it removes by using proper time regularization scheme.

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NJL Model

The effective Lagrangian density of SU(2) flavour NJL model is

$$\mathcal{L}_{NJL} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + \sum_\alpha G_\alpha(\bar{\psi}\Gamma_\alpha\psi)^2, \tag{4}$$

where

$$\Gamma_\alpha = [1, \gamma_5, \gamma^\mu, \gamma_5\gamma^\mu, \sigma^{\mu\nu}], \tag{5}$$

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NJL Model

The NJL model is non-renormalizable $\longrightarrow$ must be regularized $\longrightarrow$ Proper Time Regularization (PTR) scheme.

$$\frac{1}{X^n} = \frac{1}{(n-1)!}\int_0^\infty d\tau\tau^{n-1}e^{-\tau X},$$

$$\longrightarrow \frac{1}{(n-1)!}\int_{\frac{1}{\Lambda_{UV}^2}}^{\frac{1}{\Lambda_{IR}^2}} d\tau\tau^{n-1}e^{-\tau X}, \tag{6}$$

- We need Λ_{UV} to render the theory finite

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NJL Model

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- We need Λ_{UV} to render the theory finite
- Λ_{IR} plays an important role, preventing quarks going on their mass shell $\rightarrow$ confinement.

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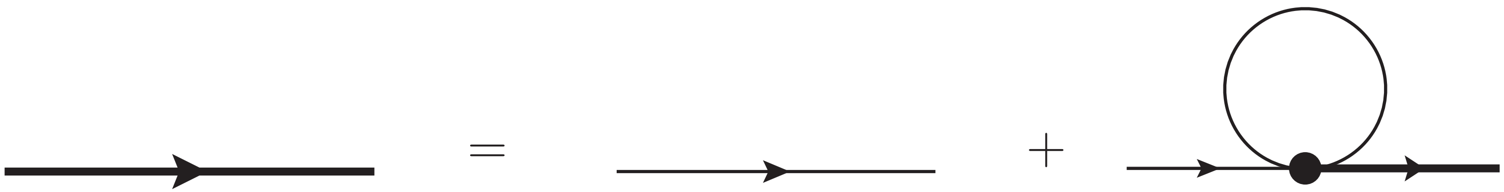
NJL Model

The Feynman diagram for dynamical mass (gap equation) of the NJL model :



- NJL gap equation can be written as

$$
\begin{aligned}
S^{-1}(k) &= S_0^{-1}(k) - \Sigma(k), \\
S_0^{-1}(k) &= [\not{k} - m], \\
\Sigma(k) &= \sum_j \int \frac{d^4 l}{(2\pi)^4} Tr\left[S(l)\bar{\Omega}^j\right]\Omega^j,
\end{aligned} \tag{8}
$$

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NJL Model

- Based on self-energy equation, the dynamical (constituent) quark mass is expressed

$$M = m + 48iG_\pi M \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - M^2 + i\epsilon},$$

$$= m + \frac{3G_\pi M}{\pi^2} \int_{\frac{1}{\Lambda_{UV}^2}}^{\frac{1}{\Lambda_{IR}^2}} \frac{d\tau}{\tau^2} e^{-\tau M^2}, \quad (9)$$

where $G_\pi \to$ pion coupling constant, m $\to$ current quark mass

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Bethe-Salpeter Equation (BSE)

In the NJL model T-Matrix can be expressed

$$\mathcal{T}(q)_{\alpha\beta,\gamma\delta} = \mathcal{K}_{\alpha\beta,\gamma\delta} + \int \frac{d^4k}{(2\pi)^4}\mathcal{K}_{\alpha\beta,\lambda\epsilon}S(q+k)_{\epsilon\epsilon'}, \times S(k)_{\lambda\lambda'}\mathcal{T}(q)_{\epsilon'\lambda',\gamma\delta'}, \tag{10}$$

$$\mathcal{T}(q)^i_{\alpha\beta,\gamma\delta} = (\gamma_5\tau_i)_{\alpha\beta}\frac{2iG_\pi}{1-2G_\pi\Pi_K(q^2)}(\gamma_5\tau_i)_{\gamma\delta}, \tag{11}$$

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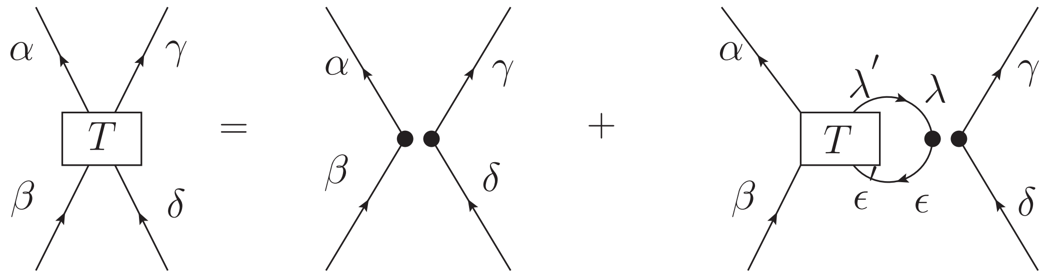

Bethe-Salpeter Equation (BSE)

From T-matrix, the kaon mass is determined (N_C is color numbers, S_1 and S_2 are the propagator for quark 1 and 2 respectively)

$$1 - 2G_\pi \Pi_K(q^2 = m_K^2) = 0. \qquad (12)$$

where bubble graph is defined as

$$\Pi^K_{ps}(k^2) = -2iN_C \int \frac{d^4p}{(2\pi)^4} tr[\gamma_5 S_2(p+\frac{k}{2}), \times \gamma_5 S_1(p-\frac{k}{2})], \qquad (13)$$

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Bethe-Salpeter Equation (BSE)

Shortly the kaon mass is expressed as

$$m_K^2 = -\left[\frac{m_2}{M_2} + \frac{m_1}{M_1}\right] \frac{1}{16iGN_C\mathcal{I}_{21}} + (M_2 - M_1)^2.$$

(14)

$$\mathcal{I}_{21} = 4\int \frac{d^4p}{(2\pi)^4} \int_0^1 dx \times \frac{(p_\mu - k_\mu(x - \frac{1}{2})(M_1 - M_2) - \frac{1}{2}k_\mu(M_1 + M_2)}{[p^2 + k^2(x - x^2) - xM_2^2 + (x-1)M_1^2]^2}$$

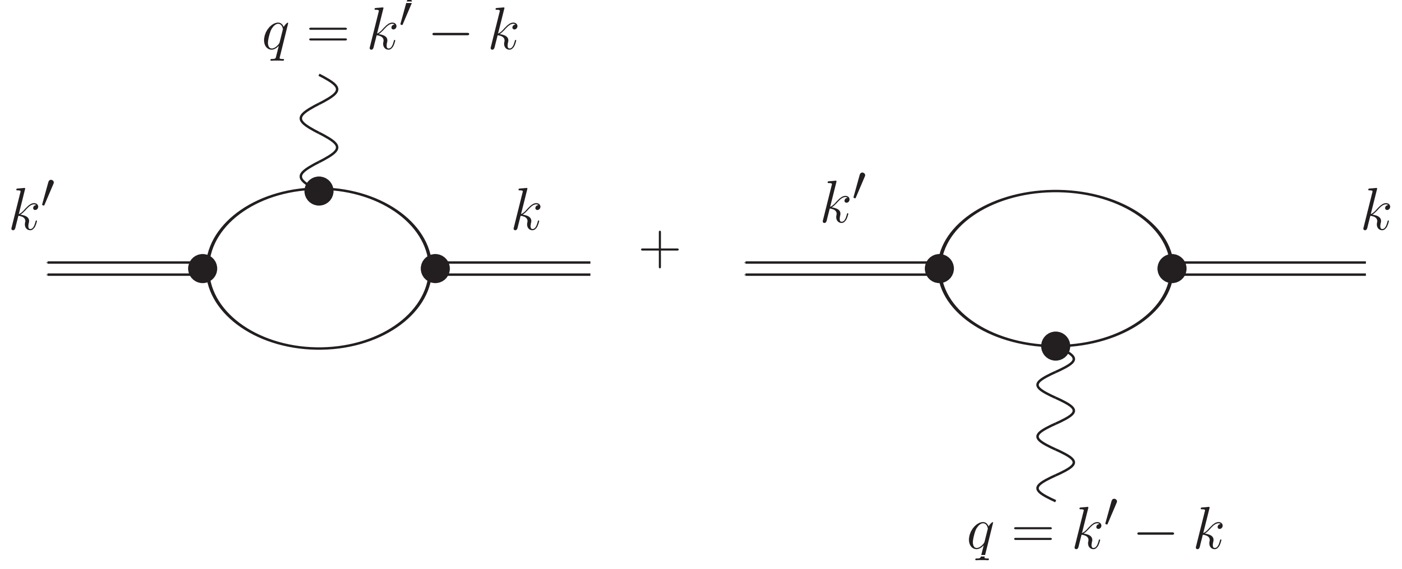
(15)

Kaon Form Factor

The matrix element of the EM current is expressed

$$\mathcal{J}^{\mu,i,j}_{K^+}(k,k') = \langle K^j(k)|\mathcal{J}^\mu|K^i(k')\rangle, \tag{16}$$
$$= (k+k')^\mu \mathcal{F}_{K^+}(Q^2)$$

where $\mathcal{F}_{K^+}(Q^2)$ is the kaon form factor.



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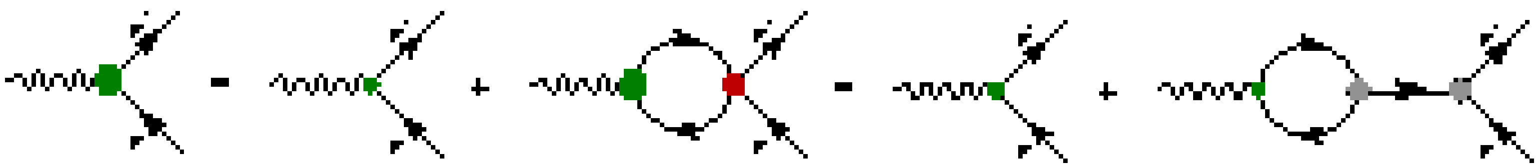

Kaon Form Factor

The total kaon form factor from the Feynman diagram ($C_{K^+} = 2iN_C g_{Kq\bar{q}}^2$ is constant):

$$\begin{aligned}
\mathcal{J}_{K^+}^{\mu,i,j}(k,k') &= e_u \Lambda_1^\mu(k,k') f_\rho + e_s \Lambda_2^\mu(-k,-k') f_\phi, \\
\Lambda_1^\mu(k,k') &= C_{K^+} \int \frac{d^4p}{(2\pi)^4} Tr[\gamma_5 S_1(p+k')\gamma^\mu \\
&\times S_1(p+k)\gamma_5 S_2(p)], \\
\Lambda_2^\mu(-k,-k') &= C_{K^+} \int \frac{d^4p}{(2\pi)^4} Tr[\gamma_5 S_2(p-k)\gamma^\mu, \\
&\times S_2(p-k')\gamma_5 S_1(p)].
\end{aligned} \tag{18}$$

where $f_i = \frac{m_i}{m_i+Q^2}$ where $i = \rho, \phi$.

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PDFs

- The quark distributions of the K^+ is

$$q_K(x) = p^+ \int \frac{d\xi^-}{2\pi} e^{ixp^+\xi^-}, \times \langle p, s|\bar{\psi}_q(0)\gamma^+\psi_q(\xi^-)|p, s\rangle_c, \quad (19)$$

PDFs

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(21)

- The moment of the PDFs can be written as

$$\langle x^{n-1} q_K \rangle = \int_0^1 dx x^{n-1} q_K(x).$$

(22)

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PDFs

- The moments of the PDFs must satisfy the *Baryon Number* and *Momentum Sum Rules*

$$
\begin{aligned}
\langle u_K(x) - \bar{u}_K(x) \rangle &= 1, \\
\langle \bar{s}(x) - s(x) \rangle &= 1, \\
\langle x u_K(x) + x \bar{s}_K \rangle &= 1.
\end{aligned}
$$

(23)

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(24)

- The first and the second lines are baryon number sum rules and the third line is the momentum sum rule.

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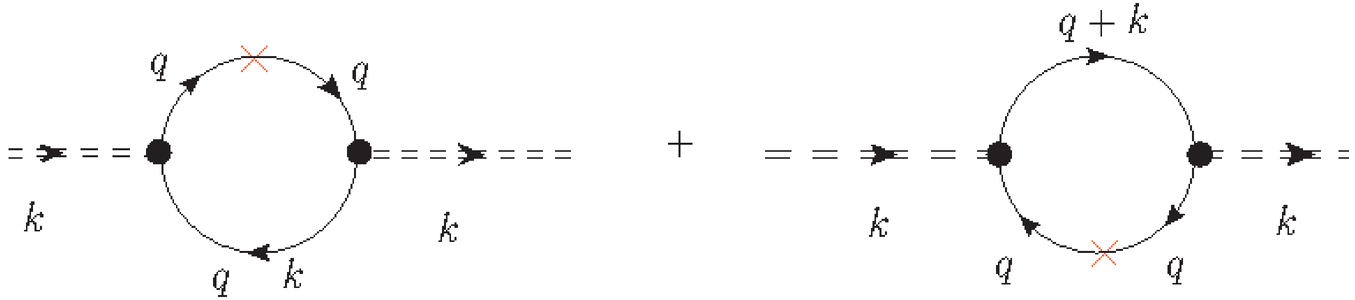
Kaon PDFs in NJL model

The valence quark distributions of K^+ constituents are formulated as

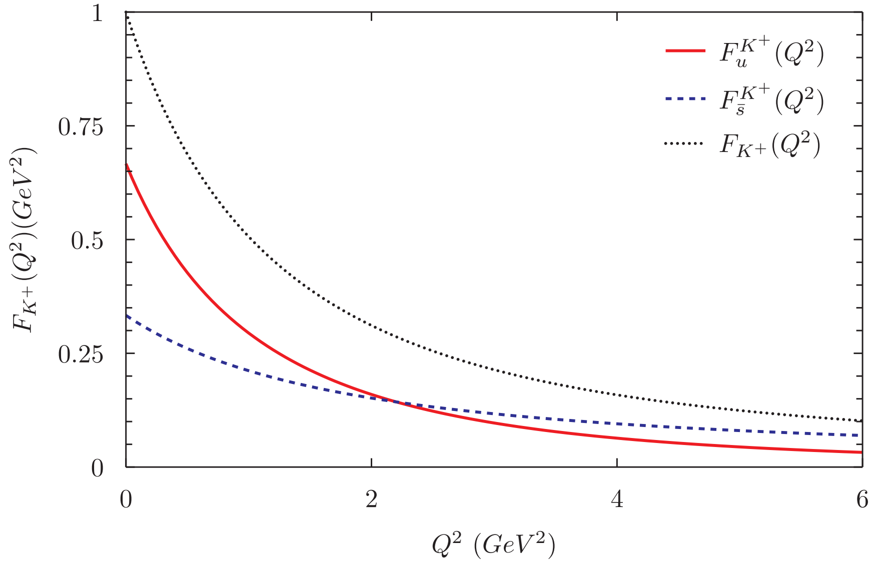
$$\begin{aligned} u_{K^+} &= \Lambda_K(x,k^2), \quad \bar{s}_{K^+} = \Lambda_K(x,k^2), \\ \Lambda_K(x,k^2) &= -2iN_C g_{Kqq}^2 \frac{\partial}{\partial p^2} \int \frac{d^4q}{(2\pi)^4} \delta\left(x - \frac{q_-}{k_-}\right), \\ &\times \; Tr[\gamma_5 S_1(q) \gamma_5 S_2(q-k)]. \end{aligned}$$

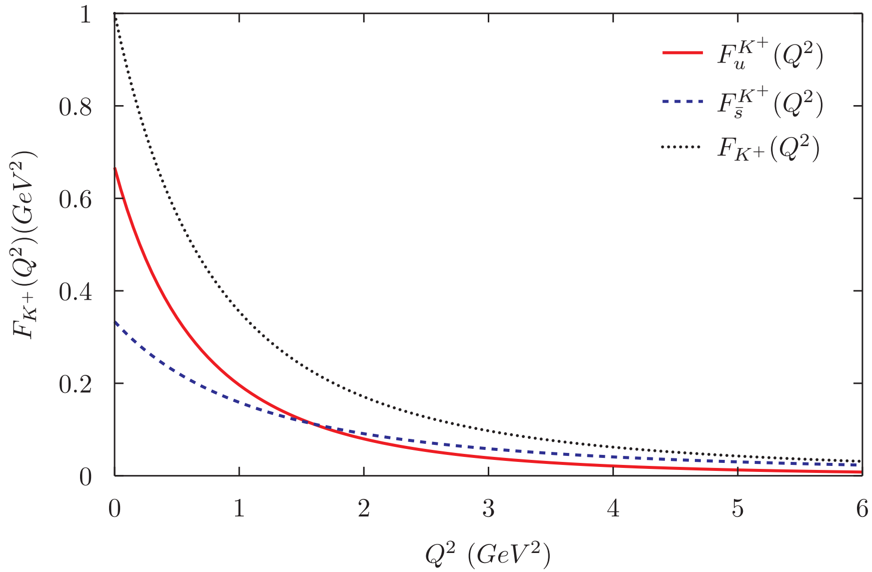
(25)



Results and Discussions

The form factors of the quark constituents in the K^+





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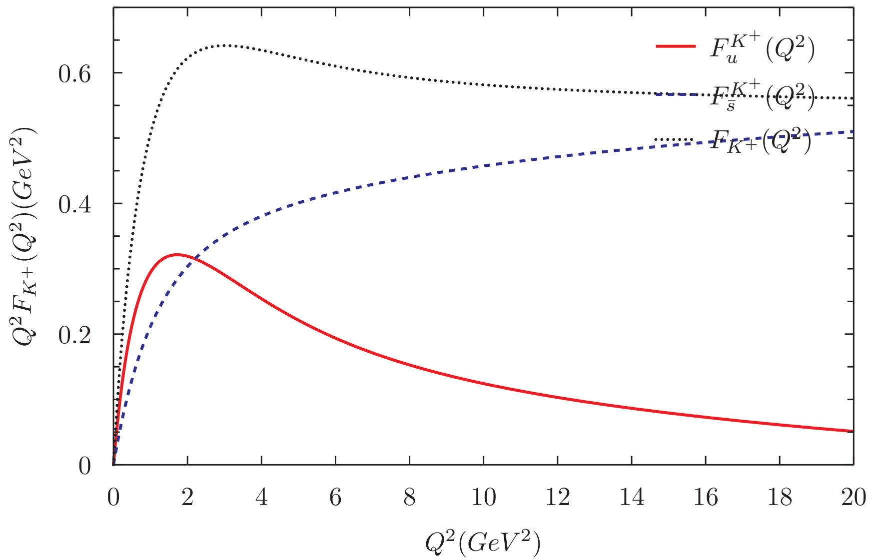
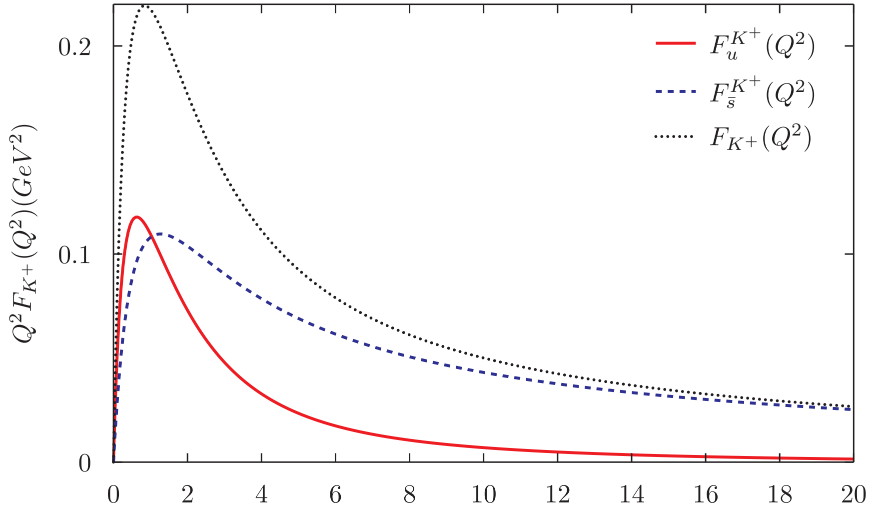
Results and Discussions

The form factors of the quark constituents in the K^+ and show dramatic differences





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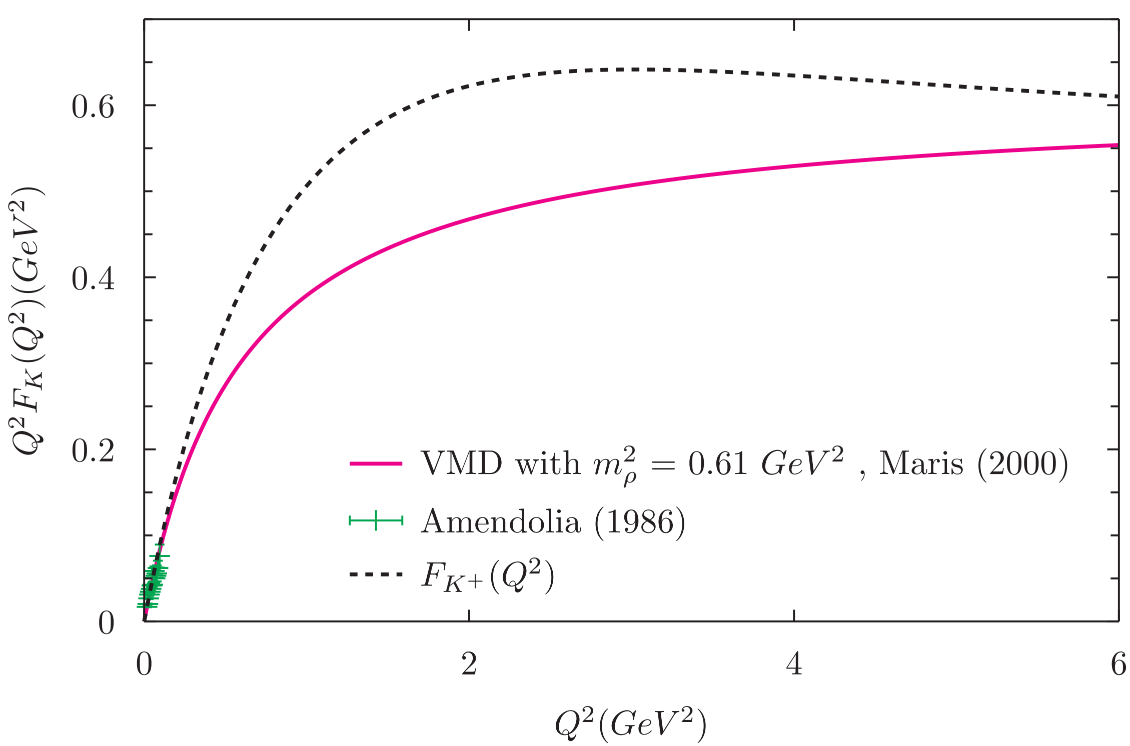

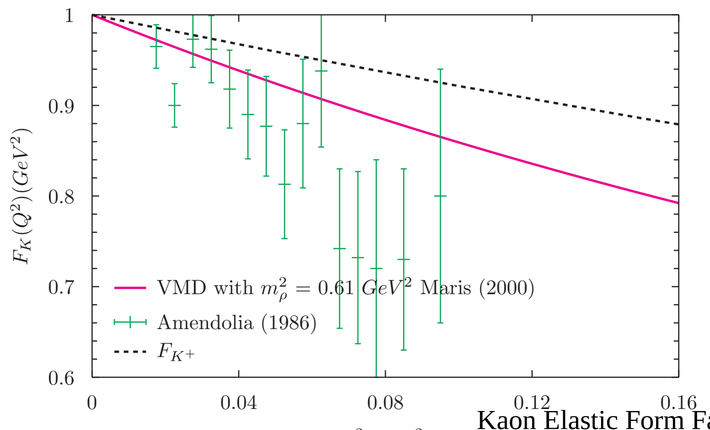

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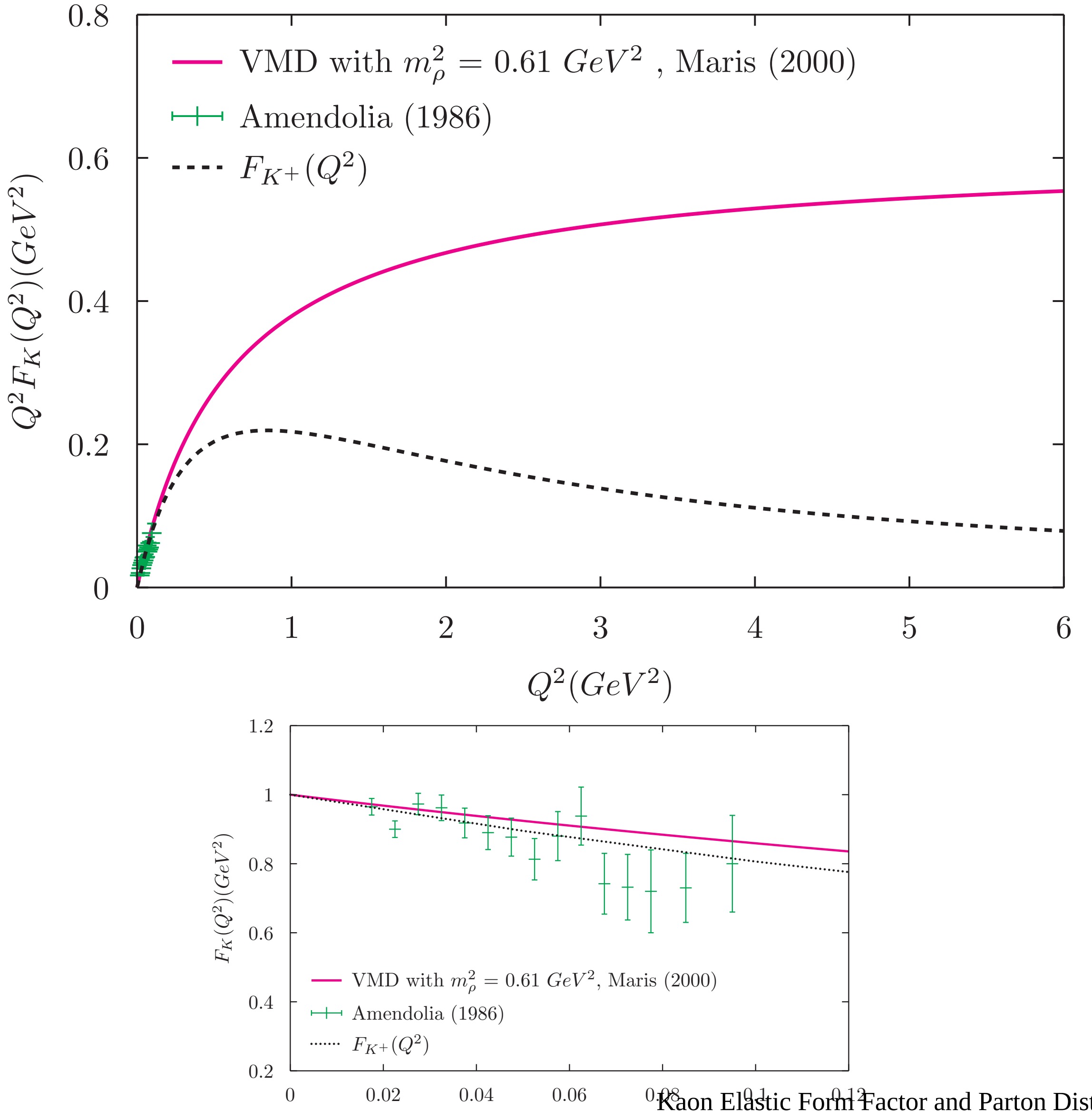
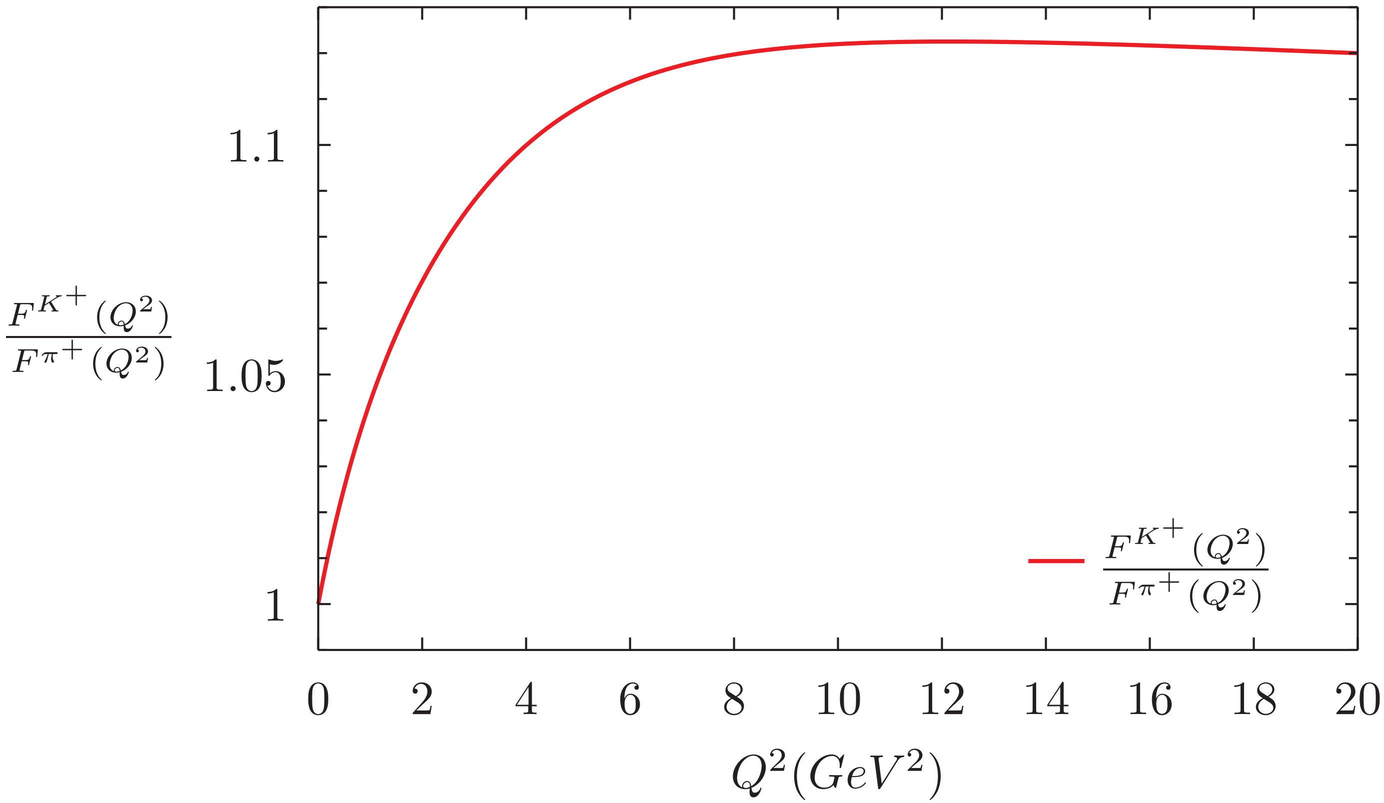


Results and Discussions

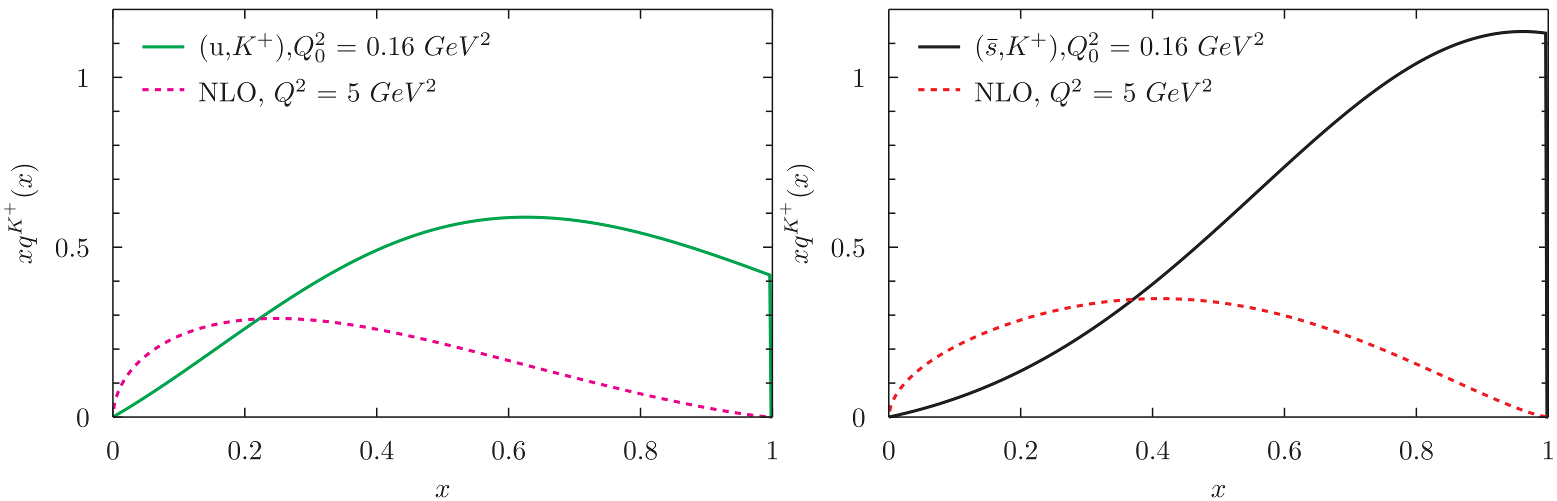
The ratio of the K^+ and π^+ form factors



Results and Discussions

The valence quark distribution of each quark constituent in the K^+

- The valence u and $\bar{s}$ quark distribution of the Kaon for Q_0 =0.16 GeV^2 and their evolution for the NLO, Q^2 = 5 GeV^2



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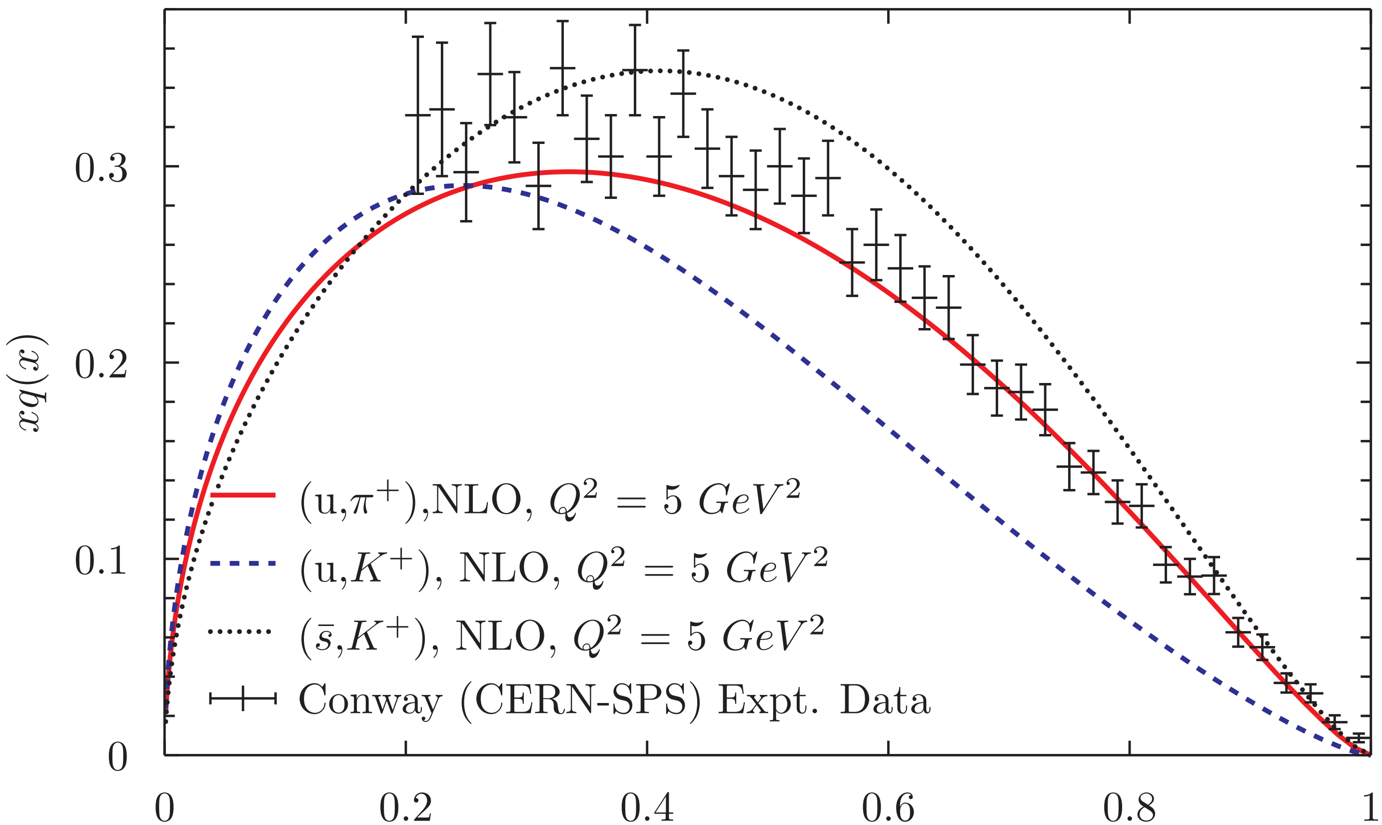

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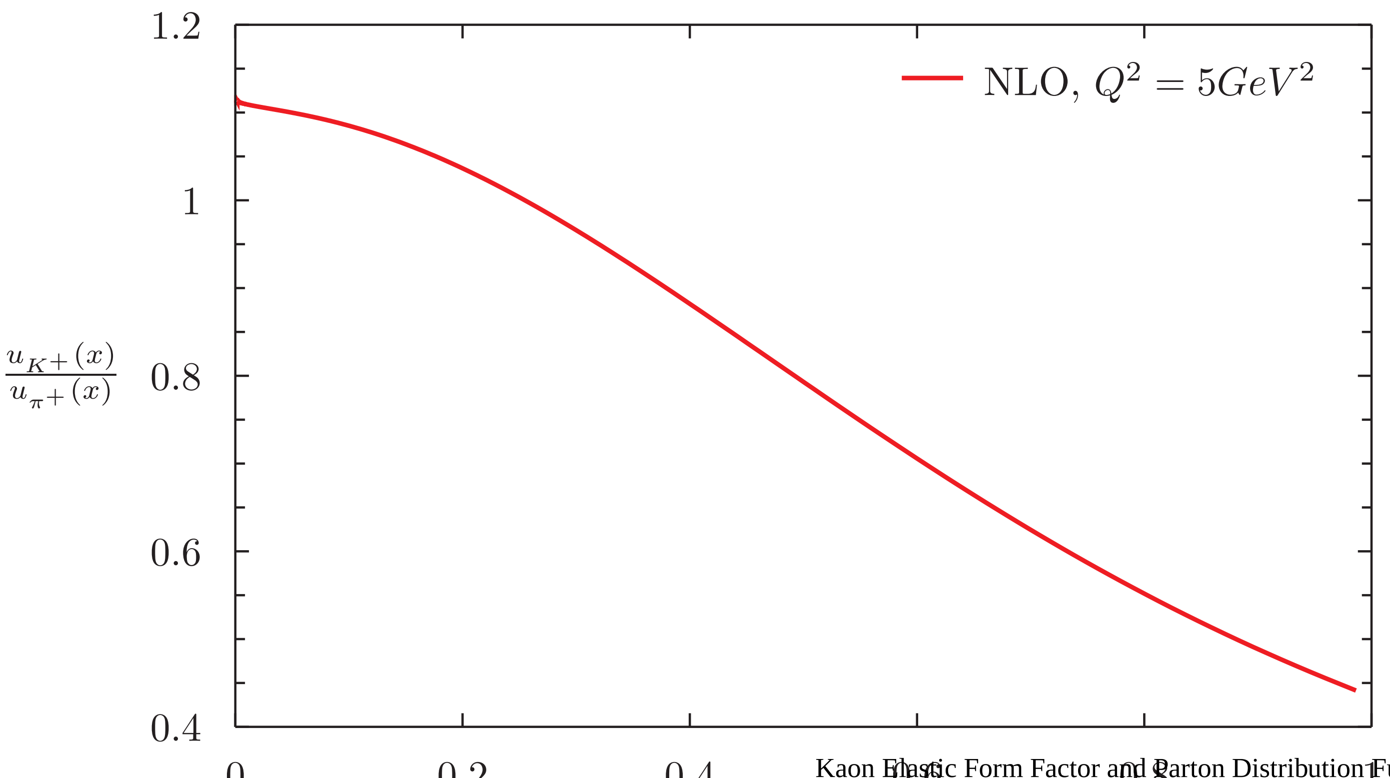

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- The u and $\bar{s}$ quark contributions to the kaon form factor show significant differences
- The valence up quark distributions of the π^+ shows good agreement with experimental data

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Backup Slide

Fitting parameters are obtained from NJL model :

- Λ_{UV} = 644.874 MeV, G_π = 0.0000190 MeV^2, mu or m_d = 16.4311 MeV, m_s = 355.882 MeV
- M_s = 610.539 MeV, $g_{\pi q\bar{q}}$ = 4.22529, $g_{Kq\bar{q}}$ = 4.57046, f_K = 97.3529 MeV
- m_π = 140 MeV, f_π = 93 MeV, m_K = 495 MeV,Λ_{IR} = 240 MeV, M_u or M_d = 400 MeV
- and we choose $m_\phi = 1.020 GeV$ and $m_\rho = 0.770 GeV$.

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