Correlated-basis approach to nuclear four- and five-body problems

Asia-Pacific Few-Body Problems in Physics
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1. Correlated-basis approach
2. Electroweak response functions of ^{4}He
   Resonance, Tensor force
3. ^{16}O as a ^{12}C+n+n+p+p model
   Clustering

Variational calculation for many-body quantum system

- Many-body wave function Ψ has all information of the system
- Solve many-body Schoedinger equation
  ⇔ Eigenvalue problem with Hamiltonian matrix
  $$H\Psi = E\Psi$$
- Variational principle $\langle\Psi|H|\Psi\rangle = E \geqq E_0$ ("Exact" energy)
  (Equal holds if Ψ is the "exact" solution)

- Expand the wave function in the explicitly correlated Gaussian functions
  $$\Psi = \Sigma_k c_k \exp\{- \Sigma_{i,j} \beta^k_{ij}(r_i - r_j)^2\}$$

  – Optimum parameters β^k_{ij} are selected stochastically

  Stochastic Variational Method K. Varga and Y. Suzuki, PRC52, 2885 (1995).

  1. Randomly generate candidates
  2. Calculate energy for each candidate
  3. Select the basis which gives the lowest energy among them
  4. Increase the basis size
  5. Return to 1. and repeat the procedure until energy is converged

  → accurate solution can be obtained with a small basis size

Explicitly correlated basis approach

Correlated Gaussian with two global vectors

Y. Suzuki, W.H., M. Orabi, K. Arai, FBS42, 33-72 (2008)

$$\phi^{\pi}_{(L_1L_2)LM_L}(A,u_1,u_2)=\exp(-\tilde{\boldsymbol{x}}A\boldsymbol{x})[\mathcal{Y}_{L_1}(\tilde{u}_1\boldsymbol{x})\mathcal{Y}_{L_2}(\tilde{u}_2\boldsymbol{x})]_{LM_L}$$

x: any relative coordinates (cf. Jacobi)

$$\mathcal{Y}_\ell(\boldsymbol{r})=r^\ell Y_\ell(\hat{\boldsymbol{r}})$$

$$\tilde{\boldsymbol{x}}A\boldsymbol{x}=\sum_{i,j=1}^{N-1}A_{ij}\boldsymbol{x}_i\cdot\boldsymbol{x}_j$$

$$\tilde{u}_i\boldsymbol{x}=\sum_{k=1}^{N-1}(u_i)_k\boldsymbol{x}_k$$

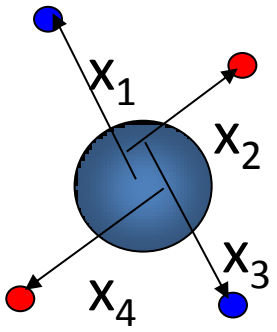
Formulation for N-particle system
Analytical expression for matrix elements

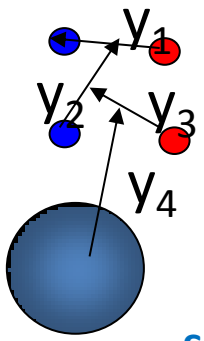
Functional form does not change under any coordinate transformation

$$\boldsymbol{y}=T\boldsymbol{x}\implies\tilde{\boldsymbol{y}}B\boldsymbol{y}=\tilde{\boldsymbol{x}}\tilde{T}BT\boldsymbol{x}$$

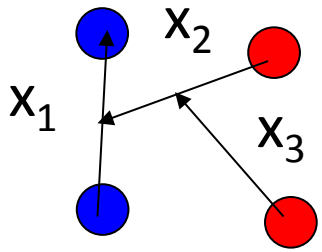
$$\tilde{v}\boldsymbol{y}=\tilde{\tilde{T}}v\boldsymbol{x}$$

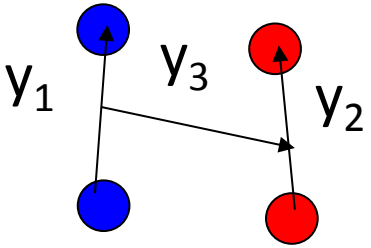
Shell and cluster structure





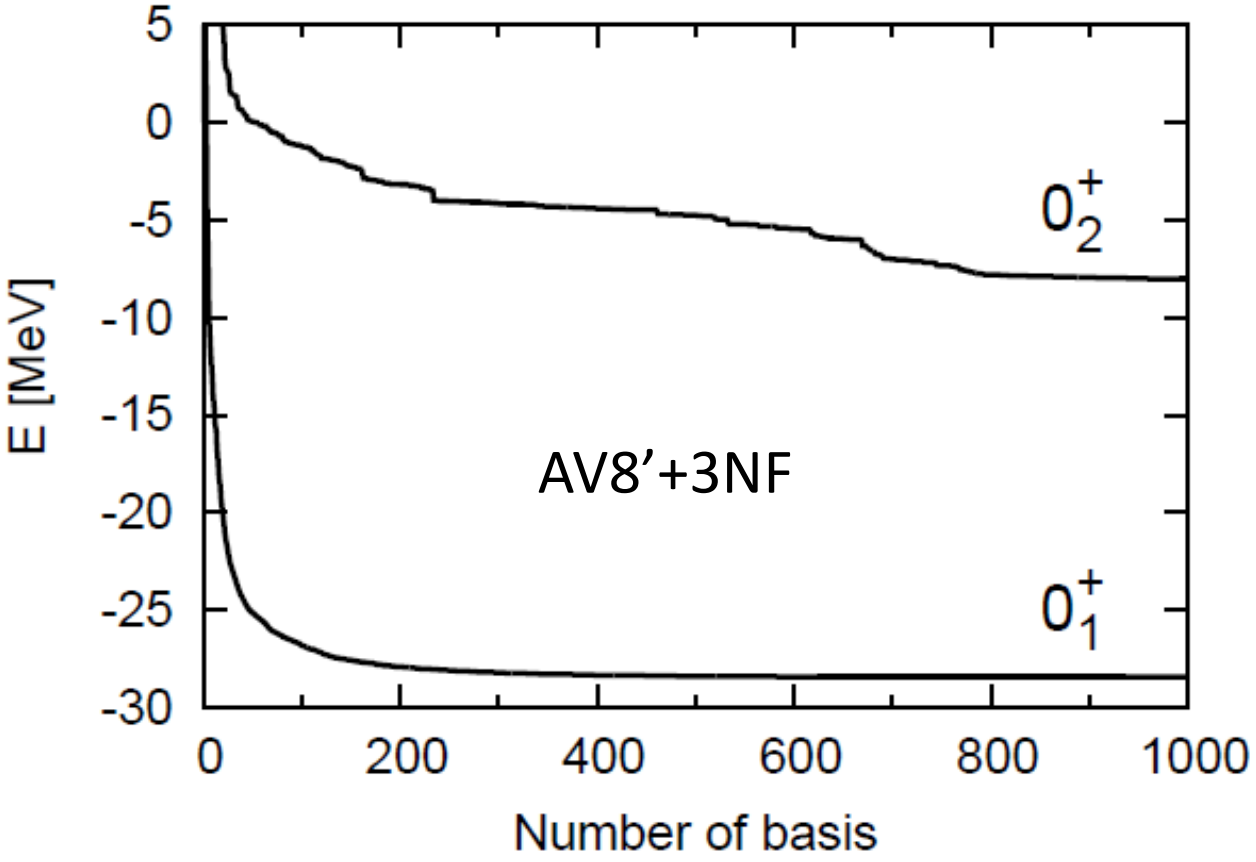
Four particle rearrangement





See Recent Review: J. Mitroy et al., Rev. Mod. Phys. 85, 693 (2013)

Energy convergence of ^{4}He



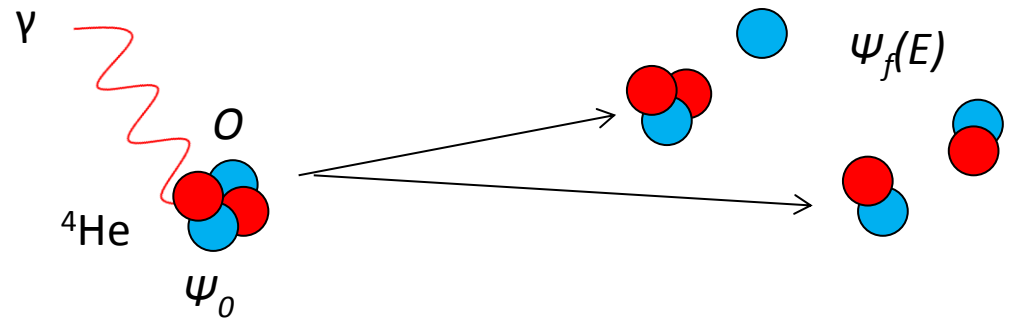
The method can be "*Ab initio*" ; Ground state energy agrees with the other accurate methods (FY, GFMC, NCSM,…)

H. Kamada et al., PRC64, 044001 (2001)

Electroweak response functions

- Response (strength) function
  - Resonant and continuum structure
  - The ground state properties and interactions

$$S(E) = \mathcal{S}_{f\mu} |\langle \Psi_f | \mathcal{O}_{\lambda\mu} | \Psi_0 \rangle|^2 \delta(E_f - E_0 - E)$$



  - Evaluate *S(E)* with *ab initio* theoretical model
    - Nucleon (proton and neutron) degrees of freedom
    - Realistic nuclear force (NN scattering, ^{2}H properties)
    - No specific model assumption

Ab initio calculation for ^{4}He

- ^{4}He nucleus:

  Ideal example of investigating the nuclear interaction

  - The lightest doubly closed nucleus
  - Mostly $(0s)^4$ but having approximately 14% D-state (L=2, S=2) due to tensor component of the nuclear force
  - **Converged *Ab initio* calculation can be done for the ground state**

- Electroweak responses of ^{4}He
  - Electromagnetic (EM) interaction: EM decay, electron scattering,…

    Photoabsorption reaction → Electric dipole (E1) transition
  - Weak interaction: beta decay, neutrino-nucleus reactions,…

    Spin-dipole (SD) transition: similar to the E1 but flips the spin

    Advantageous to probe the tensor correlation?

Variational calculation for many-body system

Hamiltonian

$$H = \sum_{i=1}^{A} T_i - T_{\rm cm} + \sum_{i<j}^{A} v_{ij} + \sum_{i<j<k}^{A} v_{ijk}$$

$$v_{12} = V_c(r) + V_{\rm Coul.}(r) P_{1\pi} P_{2\pi} + V_t(r) S_{12} + V_b(r) \boldsymbol{L} \cdot \boldsymbol{S}$$

- Argonne v8 type interactions (AV8' , G3RS); "bare" interaction
  central, tensor, spin-orbit
- Three-nucleon force (3NF) E. Hiyama et al. PRC70, 031001(R) (2002)
  → reproduce inelastic form factor of the first excited state of ^{4}He.

Basis function

$$\Psi_{(LS)JM_JTM_T} = \mathcal{A}\left\{ \left[\psi_L^{(\rm space)} \psi_S^{(\rm spin)} \right]_{JM_J} \psi_{TM_T}^{(\rm isospin)} \right\}$$

$$\psi_{SM_S}^{(\rm spin)} = \left| [\cdots [[[\tfrac{1}{2}\tfrac{1}{2}]_{S_{12}}\tfrac{1}{2}]_{S_{123}}]\cdots]_{SM_S} \right\rangle$$

$\psi_{LM}^{(\rm space)}$: correlated Gaussian combined with two global vectors

Y. Suzuki, W.H., M. Orabi, K. Arai, FBS42, 33-72 (2008)

$$F_{(L_1L_2)LM}(u_1, u_2, A, \boldsymbol{x}) = \exp\left(-\frac{1}{2} \widetilde{\boldsymbol{x}} A \boldsymbol{x} \right) [\mathcal{Y}_{L_1}(\widetilde{u_1}\boldsymbol{x}) \mathcal{Y}_{L_2}(\widetilde{u_2}\boldsymbol{x})]_{LM}$$

Description of four-body continuum

- Excited states of ^{4}He are all in continuum
  - Gaussian is a square integrable function
    - Boundary condition is not satisfied
  - Bound state approximation is not suitable for quantitative description
    - Decay widths: 0.5-15 MeV
  - Many decay channels
    - Two- (3N+N) and three- (d+p+n) , etc. thresholds open

Complex scaling method (CSM)

Rotated in complex plane
→ outgoing-wave B.C.

$$\boldsymbol{r}_j \to \boldsymbol{r}_j e^{i\theta}, \quad \boldsymbol{p}_j \to \boldsymbol{p}_j e^{-i\theta}$$

Unifying bound and continuum states

Total photoabsorption cross section

Photoabsorption cross section

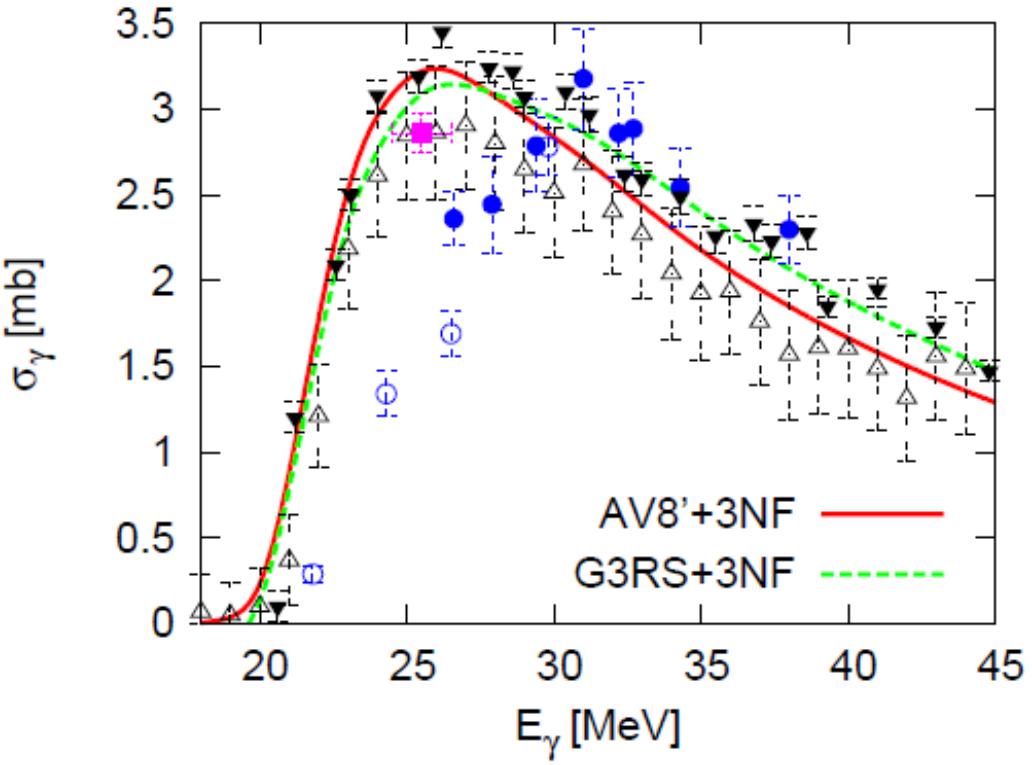
$$\sigma_\gamma(E_\gamma) = \frac{4\pi^2}{\hbar c} E_\gamma \frac{1}{3} S(E_\gamma)$$

Interaction: AV8'+3NF, G3RS+3NF
3NF: E. Hiyama et al., PRC70, 031001(2004).

Final state interactions of two and three-body decay channels are explicitly taken into account.
Non energy-weighted sum rule
99.6 %

Comparison with the measurements
→ good agreement above 30 MeV
Disagree at the low energy with the data by Shima et al.

WH and Y. Suzuki, Phys. Rev. C 87, 034001 (2013)

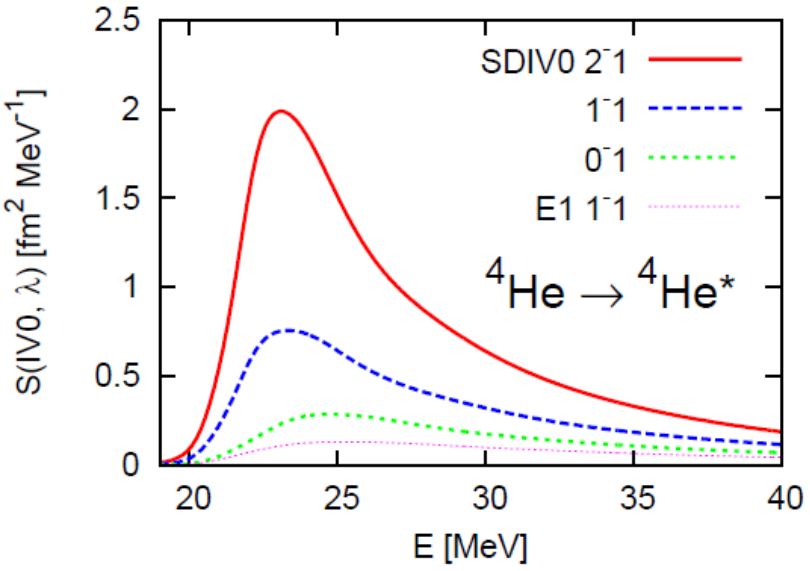


△ S. Nakayama et al., (2007)
■ D.P. Wells et al. (1992)
▼ Y. M. Arkatov et al.,(1974).
○ T. Shima et al., (2005).
● T. Shima et al., new measurement

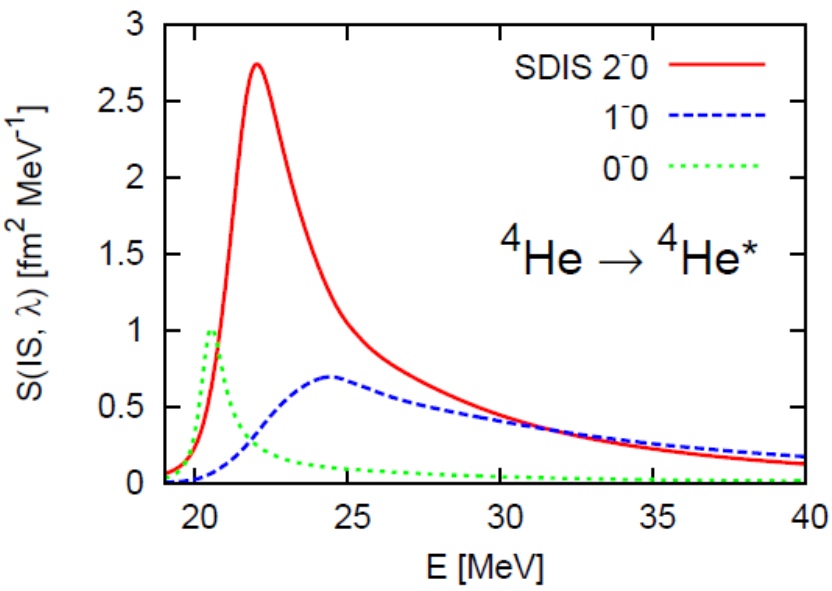
Spin-dipole strength function

Spin-dipole operator $\mathcal{O}^p_{\lambda\mu} = \sum_{i=1}^{4} [(\boldsymbol{r}_i - \boldsymbol{x}_4) \times \boldsymbol{\sigma}_i]_{\lambda\mu} \tau^p_{3i}$

Isovector (IV0)



Isoscalar (IS)



- Relatively small decay widths of 0^-0, 2^-0 (0.84, 2.01 MeV)
- Resonant structure ⇔ Strength function

  Peaks and decay widths are consistent with the observations (7 levels for ^{4}He, 4 levels for ^{4}H and ^{4}Li)

Non energy weighted spin-dipole sum rule

$$m_0(p,\lambda) = \int_0^\infty S(p,\lambda,E)dE = \sum_{\kappa=0}^{2} U_{\lambda\kappa}\langle \mathcal{Q}^p_{(\kappa)0}\rangle$$

$$\mathcal{Q}^p_{(\kappa)0} = \sum_{i,j=1}^{A} \left([\boldsymbol{\rho}_i \times \boldsymbol{\rho}_j]_\kappa \cdot [\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j]_\kappa\right) T_i^{p\dagger} T_j^p, \qquad \boldsymbol{\rho}_i = \boldsymbol{r}_i - \boldsymbol{x}_N$$

$$(U_{\lambda\kappa}) = \begin{pmatrix} \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ \frac{5}{3} & \frac{5}{6} & \frac{1}{6} \end{pmatrix}$$

κ = 0 1 2

λ=0 =1 =2

IV0	
$m_0(p,\lambda)$	SR
4.59	4.59
9.35	9.36
18.36	18.38

Sum rule ～100%

$(0s)^4$: Ratio of SD SR 1 : 3 : 5

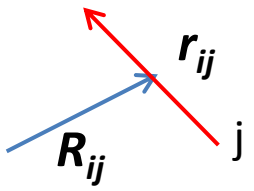
Cal (IV0):1.0 : 2.0 : 4.0 (κ=2 contribution)

cf. $$m_0(p,2) = \frac{5}{3}m_0(p,0) + \frac{1}{2}(5\langle \mathcal{Q}^p_{(1)0}\rangle - 3\langle \mathcal{Q}^p_{(2)0}\rangle) \approx \frac{5}{3}m_0(p,0) - \frac{3}{2}\langle \mathcal{Q}^p_{(2)0}\rangle.$$

Strong evidence of the tensor correlation

Two nucleon relative coordinates

$$\boldsymbol{r}_{ij} = \boldsymbol{r}_i - \boldsymbol{r}_j, \quad \boldsymbol{R}_{ij} = \frac{\boldsymbol{r}_i + \boldsymbol{r}_j}{2} - \boldsymbol{x}_N$$



$$[\boldsymbol{\rho}_i \times \boldsymbol{\rho}_j]_{\kappa\mu} = -\frac{1}{4}[\boldsymbol{r}_{ij} \times \boldsymbol{r}_{ij}]_{\kappa\mu} + [\boldsymbol{R}_{ij} \times \boldsymbol{R}_{ij}]_{\kappa\mu} + \frac{1}{2}(1-(-1)^\kappa)[\boldsymbol{r}_{ij} \times \boldsymbol{R}_{ij}]_{\kappa\mu},$$

κ=2: tensor operator $\left([\boldsymbol{r}_{ij} \times \boldsymbol{r}_{ij}]_2 \cdot [\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j]_2\right)$

From measurement

$$\langle \mathcal{Q}^p_{(\kappa)0}\rangle = \sum_{\lambda=0}^{2} U^{-1}_{\kappa\lambda} m_0(p,\lambda)$$

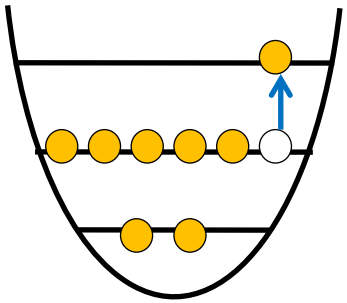
Summary (1)

- Four-body calculations for electroweak strength functions of ^{4}He
  - Explicitly correlated basis with complex scaling method
    - No specific model assumption
    - Bare realistic nuclear force
    - Unified description of bound and unbound states

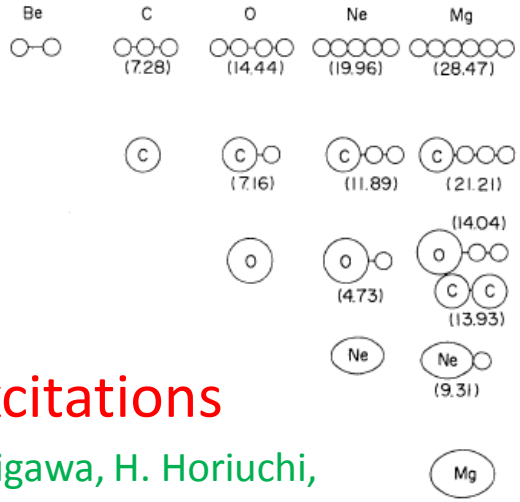
- Dipole type (E1, SD) electroweak strength functions
  - Non-energy-weighted sum rules are fully satisfied
  - Consistent agreement with experimental values
    - Photoabsorption reaction WH, Y. Suzuki, K. Arai, Phys. Rev. C 85, 054002(2012)
    - SD strength function and spectrum WH, Y. Suzuki, Phys. Rev. C 87, 034001 (2013)

    → **Tensor correlation in the ground state**

Motivation: Coexistence of two aspects



Single particle excitations
(Shell model like)

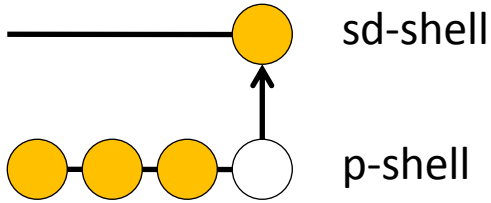


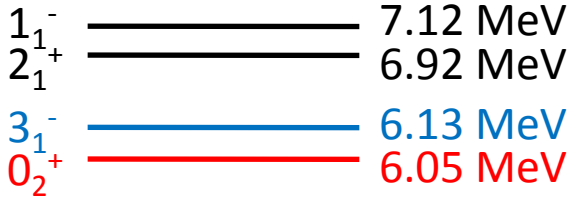
Cluster excitations

K. Ikeda, N. Takigawa, H. Horiuchi,
PTPS52 (1972)

Mysterious 0^+ state in ^{16}O

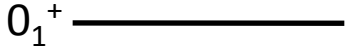
- Conventional 1p-1h negative parity?
- First excited state -> 0^+





Low energy spectrum of ^{16}O

Difficult to reproduce them even in modern large scale calculations

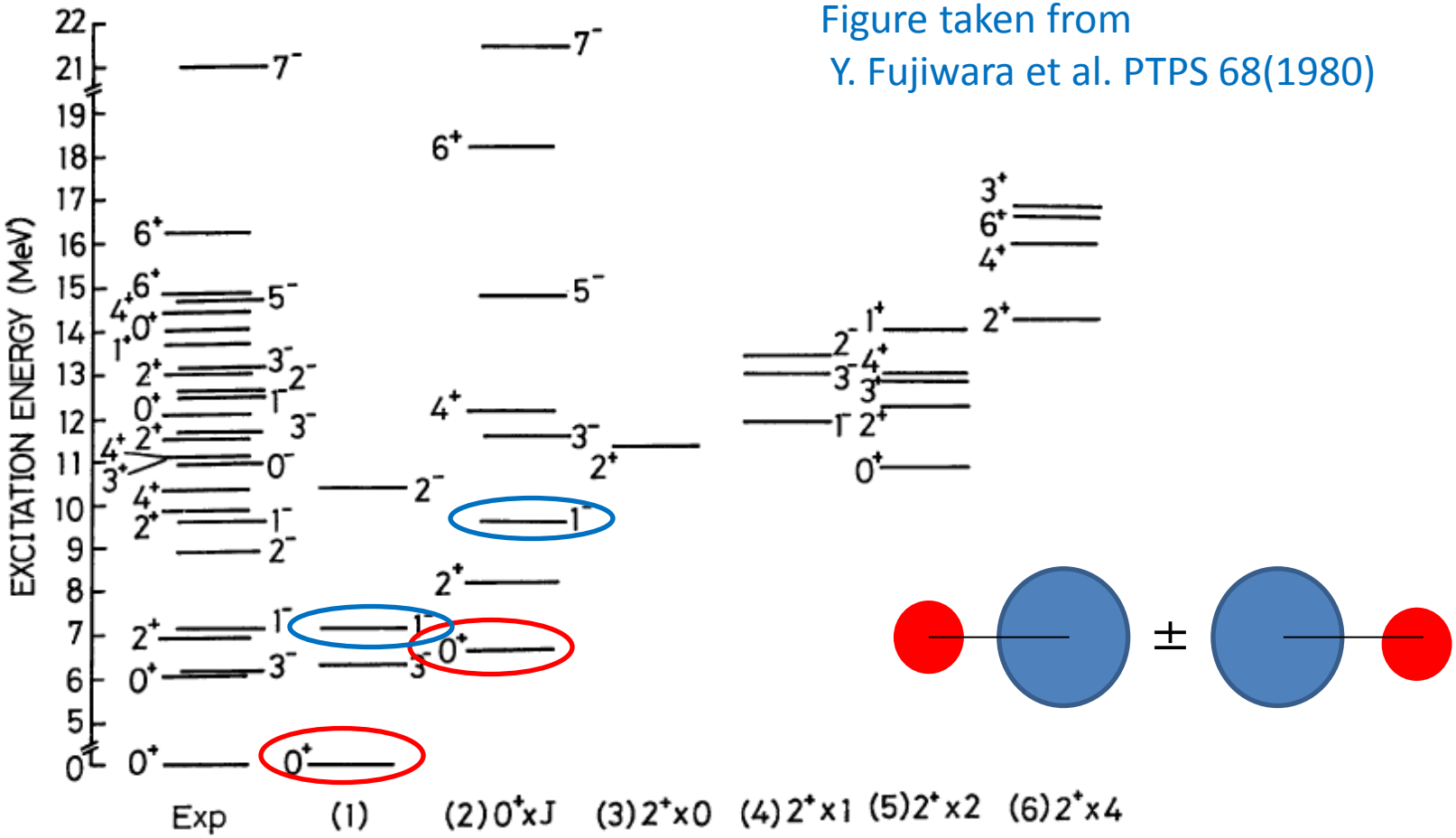


Low-lying spectra of ^{16}O:$^{12}C+\alpha$ picture

$^{12}C+\alpha$ model: coupled channel OCM Y. Suzuki, PTP55 (1976) 1751.

→ Levels are reproduced very well including 0_2^+

Figure taken from
Y. Fujiwara et al. PTPS 68(1980)



Towards resolving the ^{16}O problem

^{12}C+p+p+n+n five body model

Hamiltonian $$H = \sum_{i=1}^{N} T_i - T_{\rm cm} + \sum_{i=1}^{N-1} U_i + \sum_{i<j} v_{ij} + \sum_{i=1}^{N-1} \Gamma_i$$

N-N potential $v_{ij} = v_c + v_{\rm Coul.} P_{i\pi} P_{j\pi}$ Minnesota force

N-^{12}C potential $U_i = U_c + U_{\rm Coul.} P_{i\pi} + U_b \boldsymbol{\ell}_i \cdot \boldsymbol{s}_i$

Woods-Saxon form, reproduce the levels 1/2-, 1/2+, 5/2+ of ^{13}C

Pauli constraint $\Gamma_i = \lambda \sum_{jm} |f_{jm}(i)\rangle \langle f_{jm}(i)|, \quad \lambda \to \infty$

HO 0s1/2 and 0p3/2

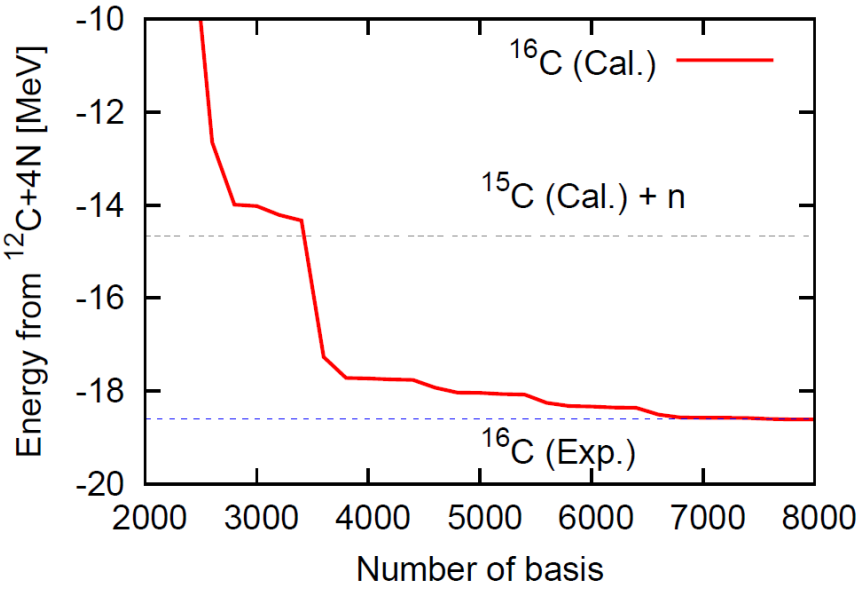
Kukulin and Pomerantsev (1978)

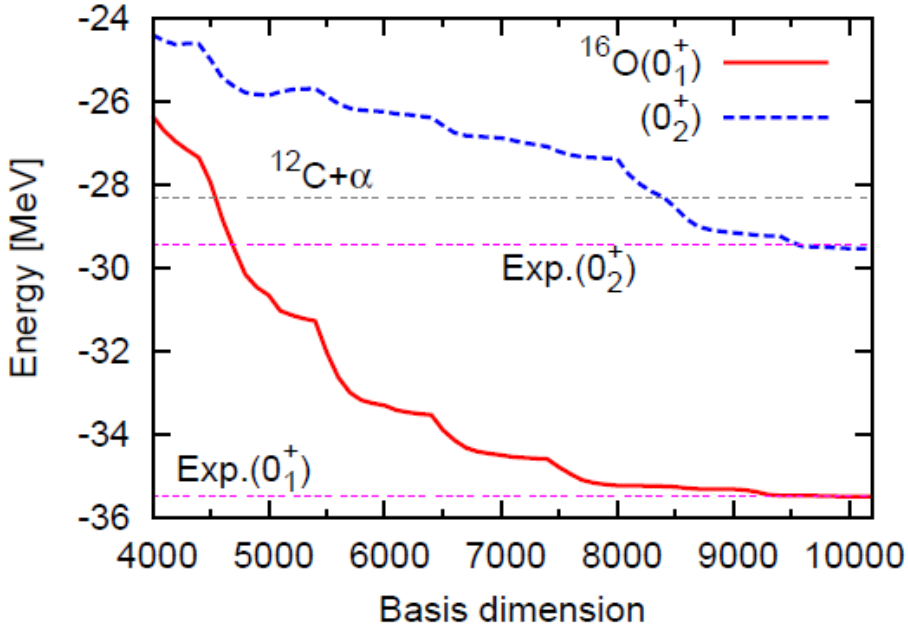
- Four particles can occupy any states except for the occupied orbits in the core
- No assumption of alpha cluster nor shell model conf.

Basis function $$\Phi = \mathcal{A}\left\{ \left[\left[\psi_L^{(\rm space)} \psi_S^{(\rm spin)} \right]_{\mathcal{J}} \phi_I(^{12}{\rm C}) \right]_{JM} \psi_{TM_T}^{(\rm isospin)} \right\}$$

No core excitation (I=0) $$\psi_{SM_S}^{(\rm spin)} = \left| [\cdots [[[\tfrac{1}{2}\tfrac{1}{2}]_{S_{12}} \tfrac{1}{2}]_{S_{123}}] \cdots]_{SM_S} \right\rangle$$

Energy curves of ^{16}C and ^{16}O





- ^{16}C: converged (7000 basis states)
  - Good agreement with experiment
- ^{16}O: converged at around 10000 basis states

Discretizing on grids → Variational parameters $D \sim N^{10+4+4} \times 3 \times 2$

$N=4$, $D \sim 4 \times 10^{11}$

- Energies of the ground and first excited states are reproduced very well

Expectation values of potential terms

	^{16}C (0_1^+)	^{16}O (0_1^+)	^{16}O (0_2^+)	α
E	-18.47	-35.47	-29.52	-28.30
$E_{\rm exp.}$	-18.59	-35.46	-29.41	-28.30
$\langle T_{cv}\rangle$	17.81	11.55	7.16	–
$\langle V_{cv}\rangle$	-82.49	-79.55	-29.22	–
$\langle T_v\rangle$	53.53	72.93	67.46	56.92
$\langle V_v\rangle$	-7.32	-40.41	-74.92	-85.22

N-N potential

$$v_{ij} = v_c + v_{\rm Coul.} P_{i\pi} P_{j\pi}$$

Core-N potential

$$U_i = U_c + U_{\rm Coul.} P_{i\pi} + U_b \ell_i \cdot s_i$$

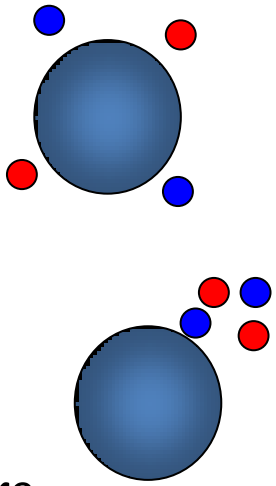
- ^{16}C: shell model like state
- ^{16}O (0_1^+): coexistence of shell and cluster states
  - α cluster is strongly distorted → $\langle V_v\rangle$ ~1/2 of free α
- ^{16}O (0_2^+): well developed α+^{12}C cluster

  $\langle V_{cv}\rangle$ ^{16}O (0_2^+) ~ $\langle V_v\rangle$ ^{4}He

  ~ $\langle V_{cv}\rangle$ ^{16}O(0_1^+)
  - Delicate balance of Core-N and N-N interactions

Density distributions of ^{12}C-4N and 4N

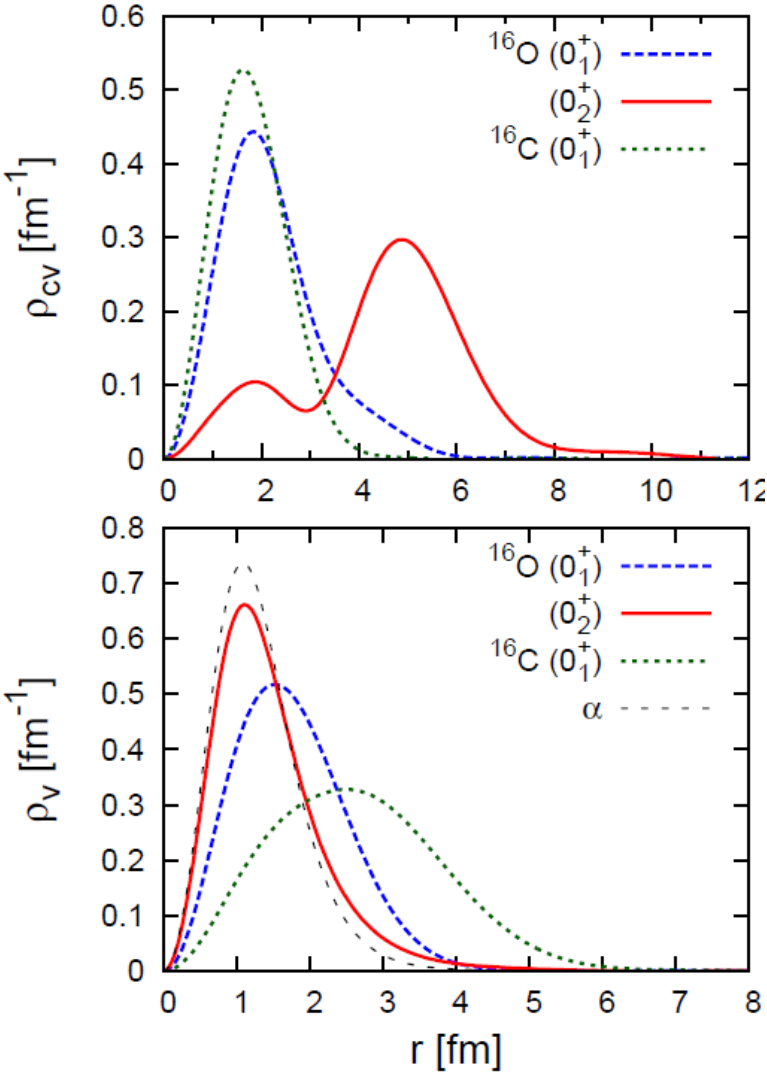
- ^{16}C : weak NN interaction
  - Shell model like states
    - Delocalized 4N density
- ^{16}O: strong NN interaction
  - 0_1^+: Shell and 4N correlation
  - 0_2^+: Strong 4N correlation
    - Second peak far from the core
      Peak at ^{12}C+α touching distance ～4.9 fm
    - Similar 4N density distribution to that of ^{4}He



Rms distance between ^{12}C and 4N and radius of 4N system

	^{16}C (0_1^+)	^{16}O (0_1^+)	^{16}O (0_2^+)	α
$\sqrt{\langle r_{cv}^2 \rangle}$	1.94	2.54	4.86	–
$\sqrt{\langle r_v^2 \rangle}$	2.88	1.90	1.62	1.43

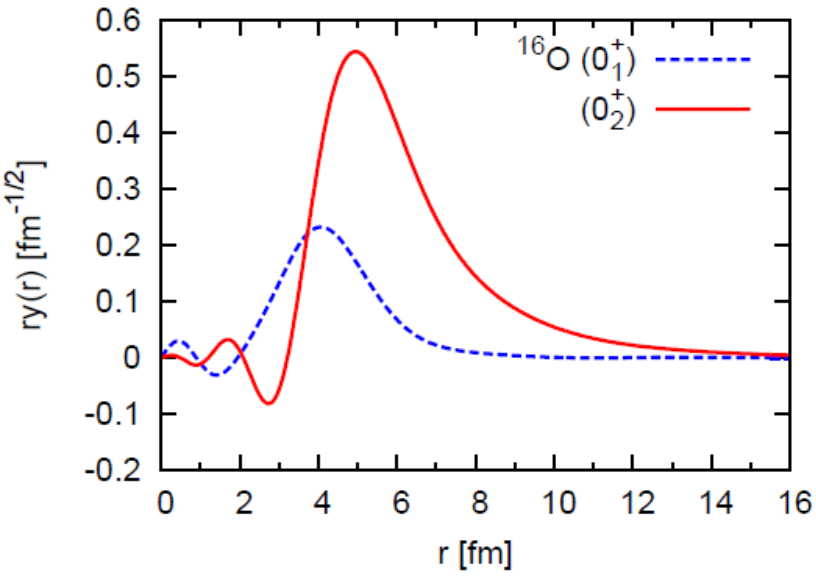
Density distributions between ^{12}C and 4N system



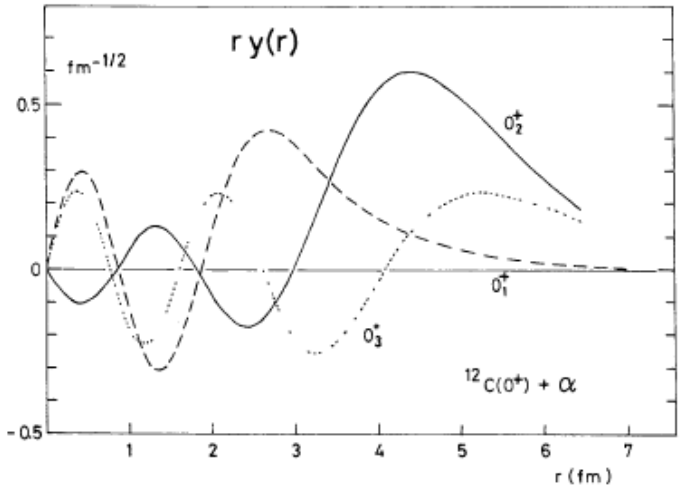
Density distributions of 4N system from their CM

^{12}C+α spectroscopic amplitude

$y(r)=\langle^{12}C+\alpha|^{16}O\rangle$



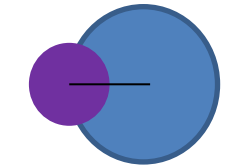
^{12}C+α OCM: Y. Suzuki, PTP56 (1976) 111

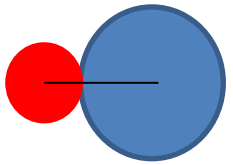


- 0_1^+: Alpha cluster is distorted by the core nucleus
- 0_2^+: Large amplitude beyond the touching distance of ^{12}C+α (∼4.9 fm)
  - Very long tail implying the cluster structure
- Spectroscopic factor

  0_1^+: 0.105 (OCM: 0.300)

  0_2^+: 0.680 (OCM: 0.679)





Distribution of harmonic oscillator quanta

Components of the harmonic oscillator quanta Q in the A=16 wave functions

Oscillator frequency is set to be the same as the occupied (forbidden) states in ^{12}C

^{16}C: $Q \geqq 6$

Shell model state: four neutrons in *p* and *sd* shells

Average: M_Q=7.0

Standard deviation: σ_Q=2.1

^{16}O: $Q \geqq 4$

0_1^+: Shell model or intermediate state between shell and cluster.

M_Q=5.5

σ_Q=2.9

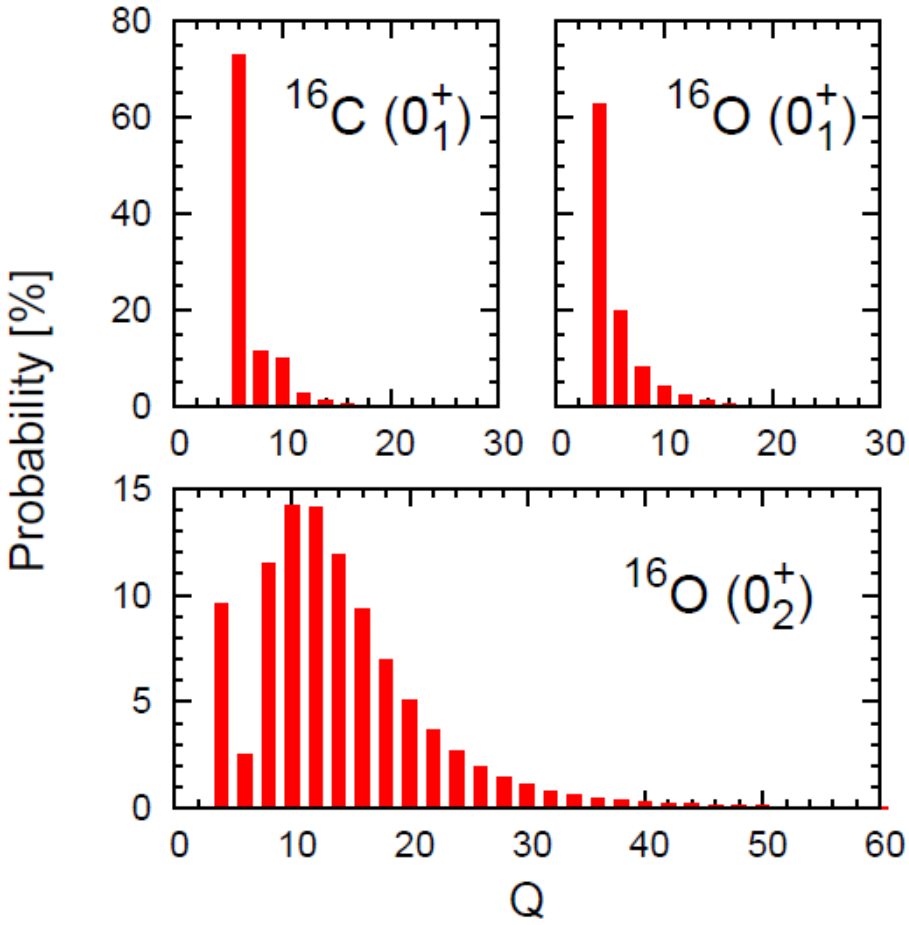
0_2^+: **Cluster state**

Peak : 10-12 hw

2-4 hw more than 4p-4h

M_Q=14.3

σ_Q=8.3



Difficult to describe it with a standard shell model truncation

Monte Carlo, symmetry adaption, importance truncation, etc.

Summary (2)

- Five-body models for A=16 systems: ^{16}O and ^{16}C
  - Correlated Gaussian with global vectors: Both the shell model like and cluster states can be described in a single scheme

- ^{16}O: a ^{12}C+n+n+p+p five body model
  - No assumption of alpha cluster
  - Energies of 0_1^+, 0_2^+ states in ^{16}O are reproduced well
  - Shell model (delocalized) and cluster structure in A=16 systems
    - 0_1^+, 0_2^+: delicate balance of N-^{12}C and N-N potentials
    - Two different aspects coexist in the spectrum of ^{16}O
    - Well developed ^{12}C+α cluster in the first excited states of ^{16}O
      - Density distributions
      - Spectroscopic amplitude (long tail, large α-width)
    - Large major shell mixing in the cluster state

WH and Y. Suzuki, PRC89, 011304(R) (2014)