NNpion Faddeev Calculation

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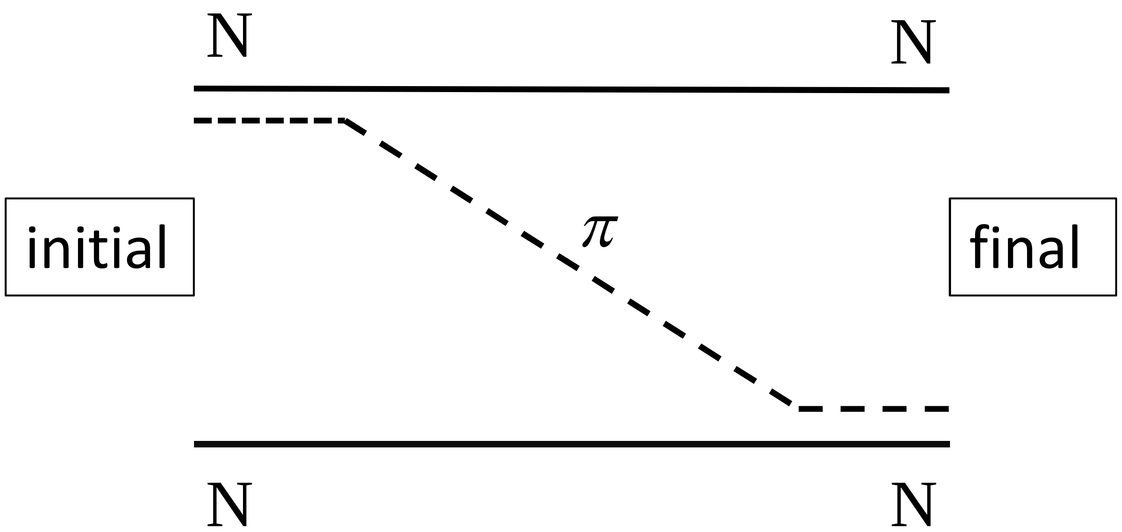
Motivation

- The study of the 3N system is carried out very often
  1) the 3-body equation is concise
     because the 3N can be treated as 3 identical particles,
  2) input data is nuclear force,
  3) fruitful experimental data are existed, etc.
- One of the most fundamental 3-body systems is NNpion system
  because it is the origin of a nuclear force.
  The NNpion system refers to or leads to
  1) pion-N potential, 2) pion-D scattering,
  3) NN scattering, 4) Deuteron state etc.
- Pioneers for the NNpion system are in Adelaide.
  We would like to revisit the system.

introduction

- One of our aims is to investigate the low energy NN interaction by 3-body equation.



outline

type A pion-N potential

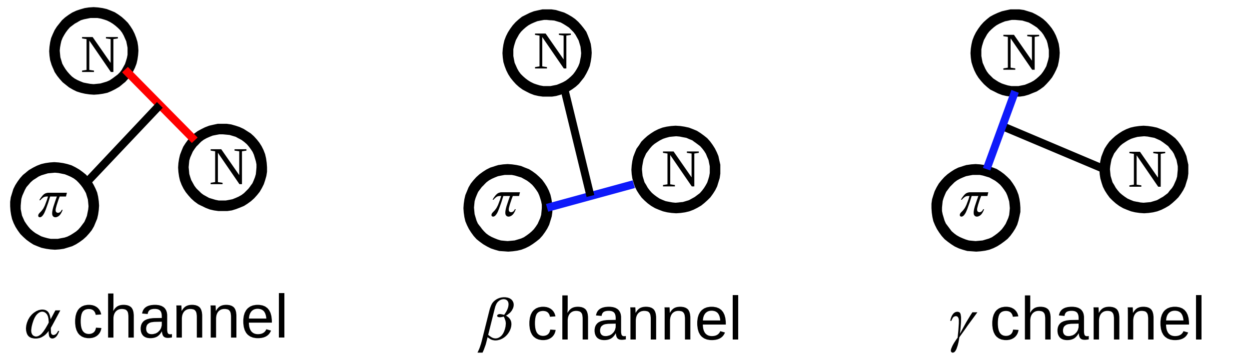
- π^+D elastic scattering with $P_{33}(1232)$ resonance.
- π^+D elastic scattering with P_{11} bound state and $S_{11}(1535)$, $P_{11}(1440)$, $P_{33}(1232)$ resonances.

type B pion-N potential

- πD scattering length.
- Energy dependent 2-body Quasi potential(E2Q).
- neutron-proton scattering length by E2Q.
- πD scattering length by E2Q.
- deuteron by E2Q (preliminary calculation).

π^+D elastic scattering

potential of $\pi^{+}\mathrm{D}$ elastic scattering



①Nuclear potential $\longrightarrow$ Argonne $v18$

R. B. Wiringa et al., PRC 51, 38 (1995)

②pion-N potential $\longrightarrow$ $S_{11}, S_{31}, P_{11}, P_{13}, P_{31}, P_{33} = \ell_{2t2j}$ { type A / type B }

ℓ : angular momentum
t : pair isospin
j : total angular momentum

pion-Nucleon phase shift(type A)
pion Lab kinetic energy (MeV) vs phase shift (deg)

ℓ : angular momentum
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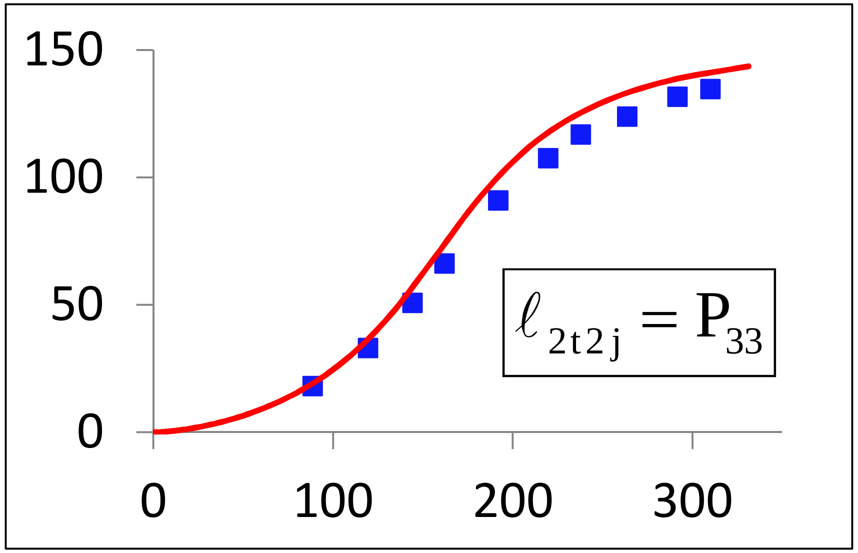
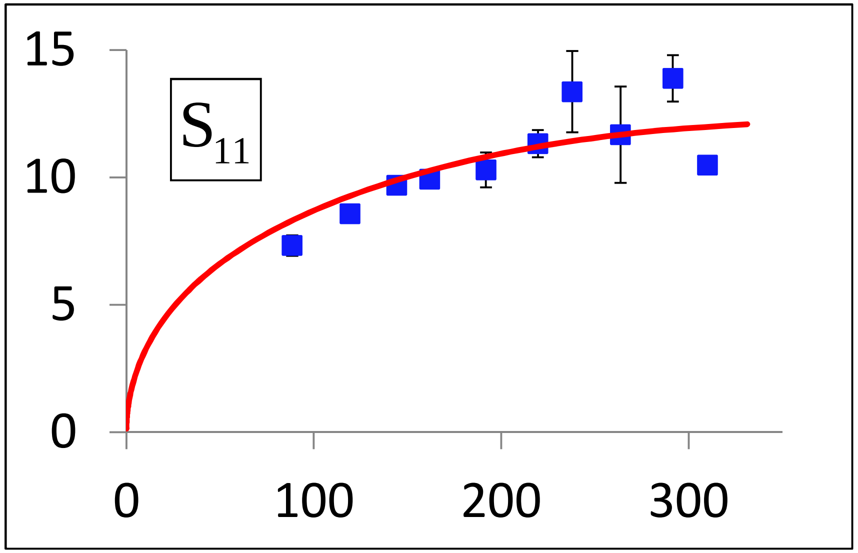
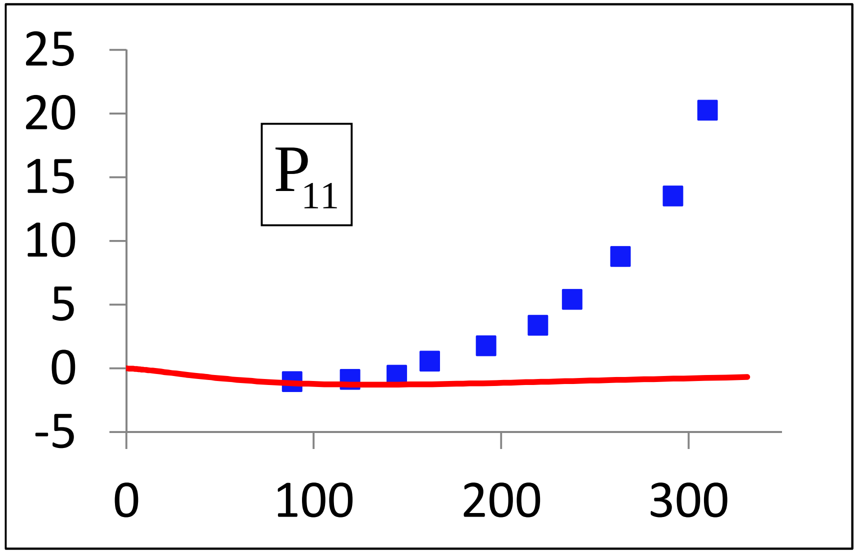
j : total angular momentum



①type A
A. W. Thomas, NPA258, 417 (1976)

②EXP
J. R. Carter et.al., NP B58, 378 (1973)





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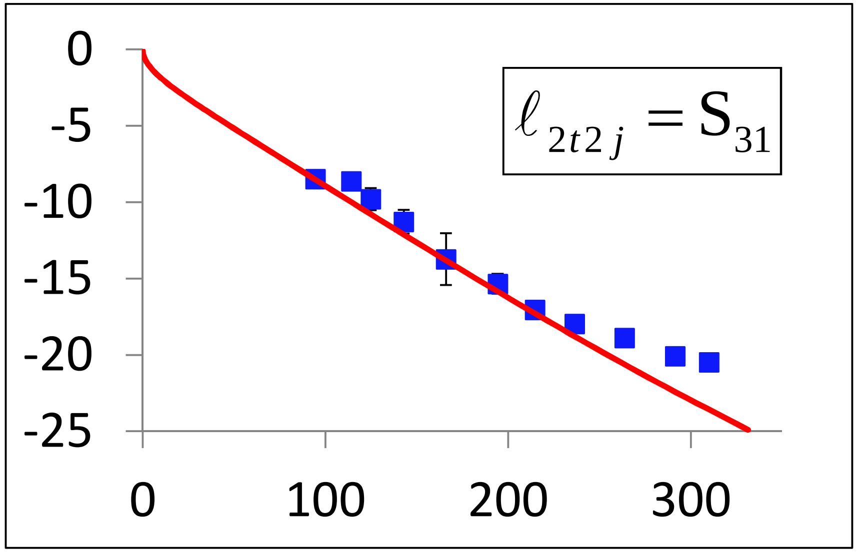

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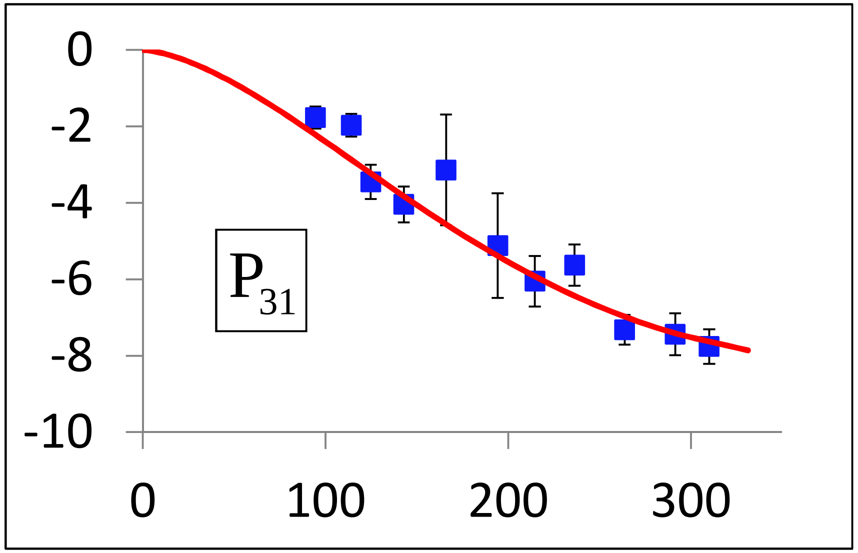
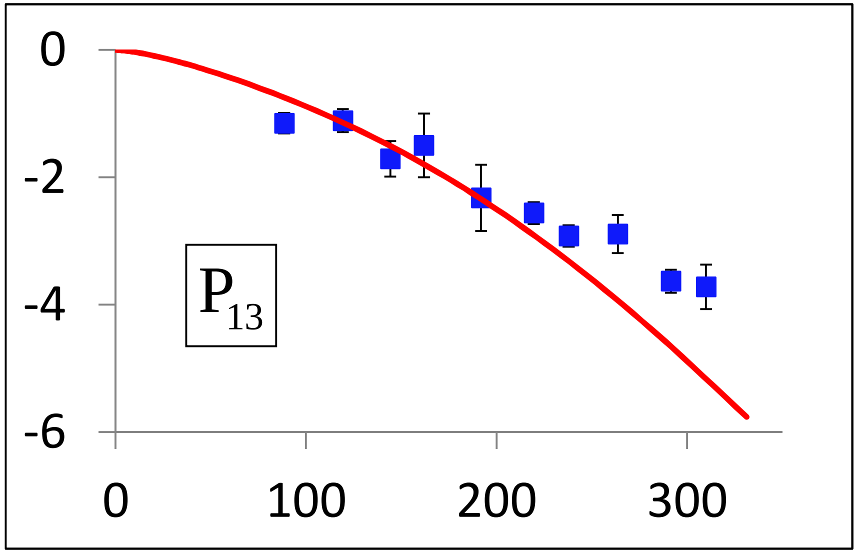
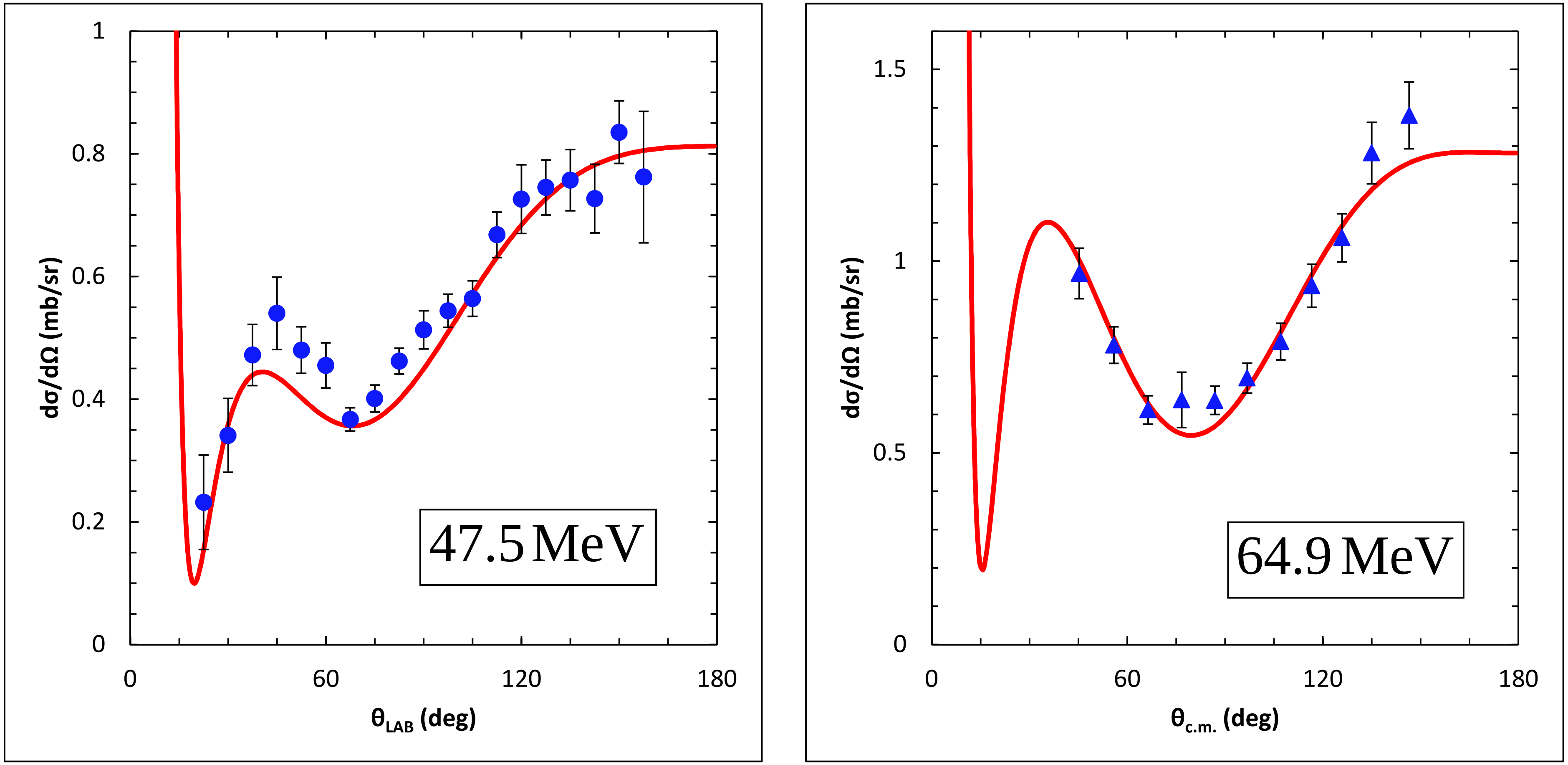


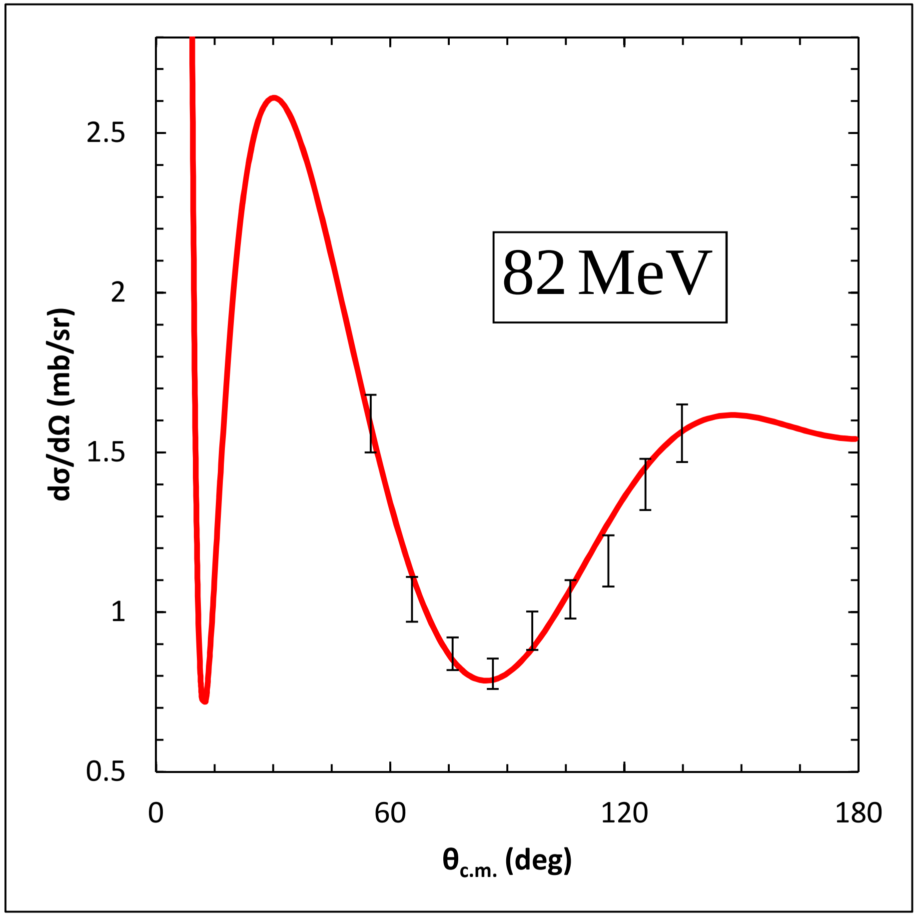


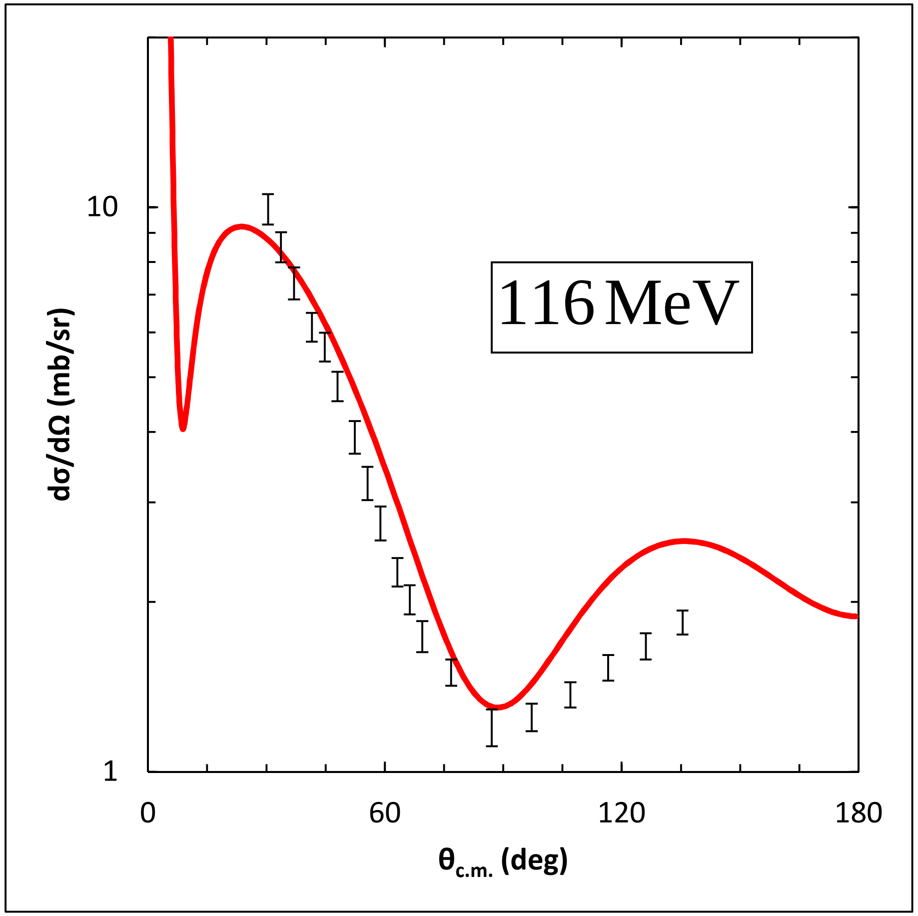
π^+D elastic scattering (type A; P_{33} resonance)



D. Axen et al., Nucl. Phys. A256, 387-413 (1976);
B. Balestri et al., Nucl. Phys. A392, 217-321 (1976).

π^{+}D elastic scattering (type A; P_{33} resonance)





K. Gabathuler et al., Nucl. Phys. A350, 253-264 (1980).

back to introduction

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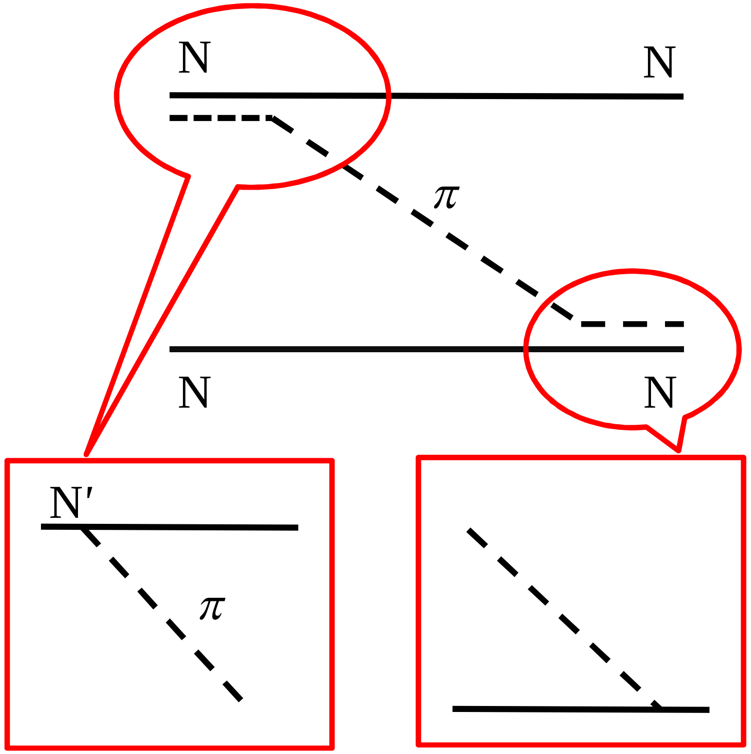


P_{11}

pion creation & annihilation/absorption

type B

M. G. Fuda, PRC52, 2875(1995)

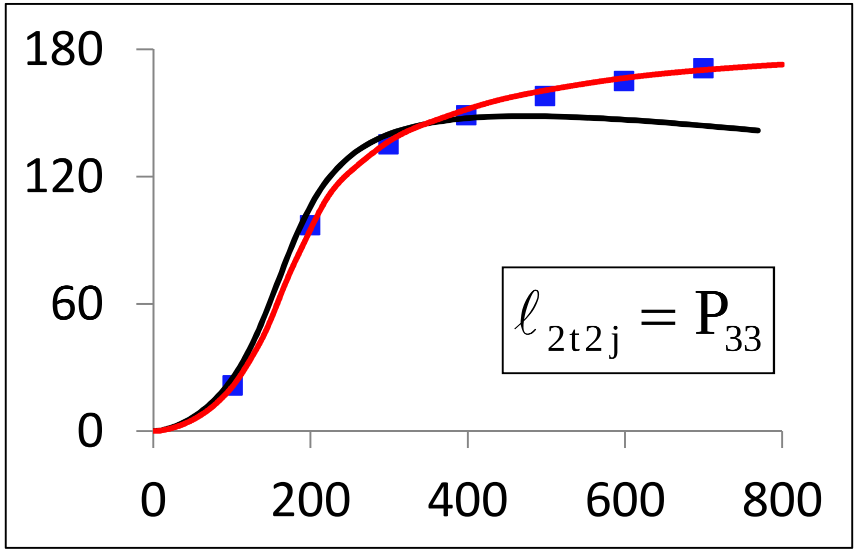
① P_{11} bound state

② $S_{11}(1535)$, $P_{11}(1440)$, $P_{33}(1232)$ resonances

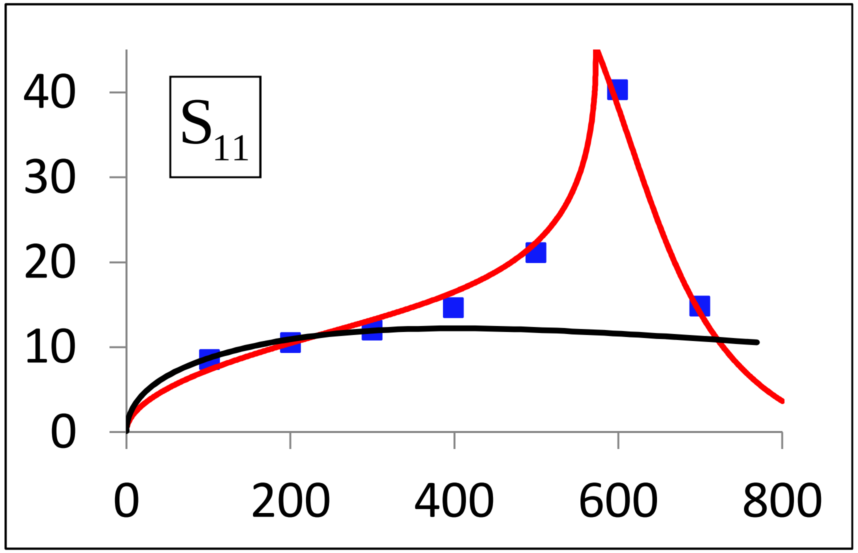
③ high energy experimental data are reproduced

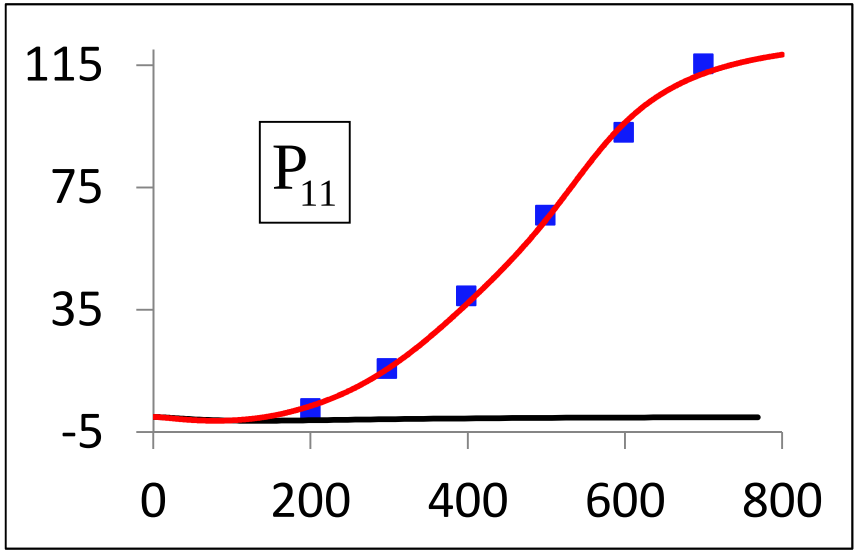
pion-Nucleon phase shift(type B)
pion Lab kinetic energy (MeV) vs phase shift (deg)

ℓ : angular momentum
t : pair isospin
j : total angular momentum



①type A
A. W. Thomas
②type B
Michael G. Fuda, PRC, 52, 2875(1995)
③EXP
R. A. Arndt et al., SAID program (1995)





pion-Nucleon phase shift(type B)
pion Lab kinetic energy (MeV) vs phase shift (deg)

ℓ : angular momentum
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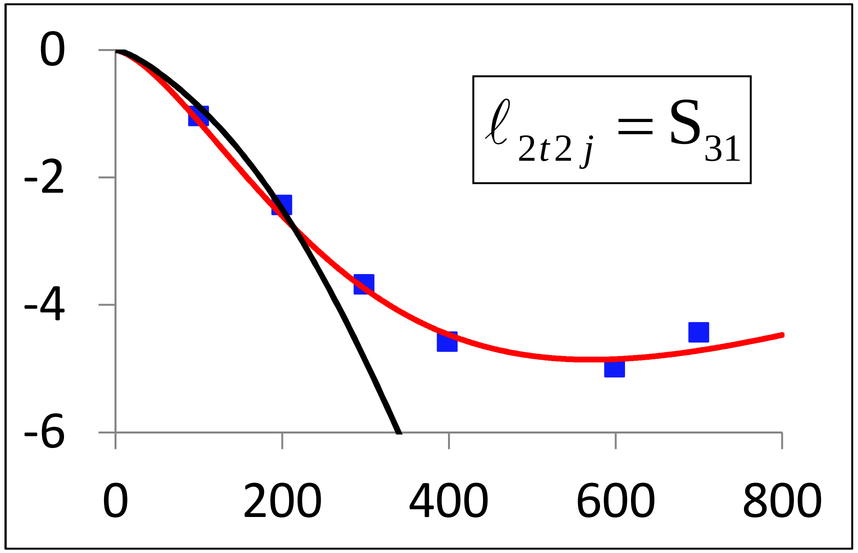

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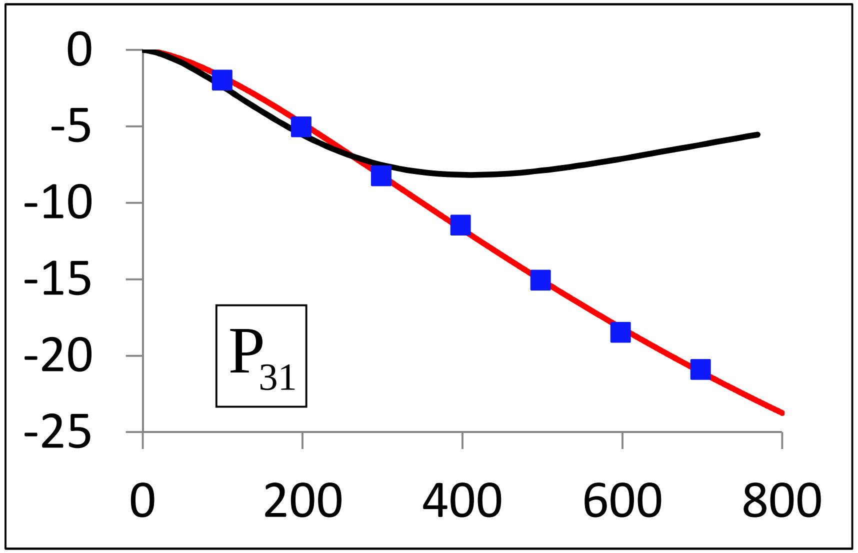
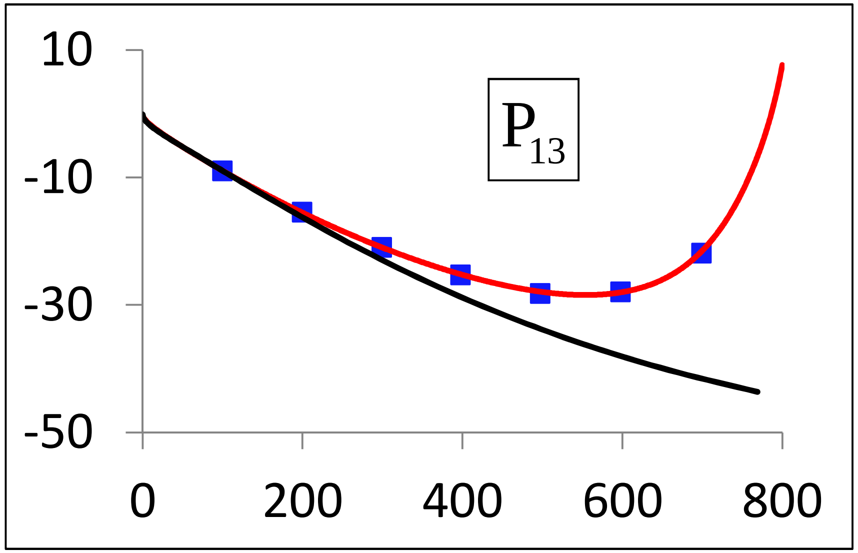




πD scattering length

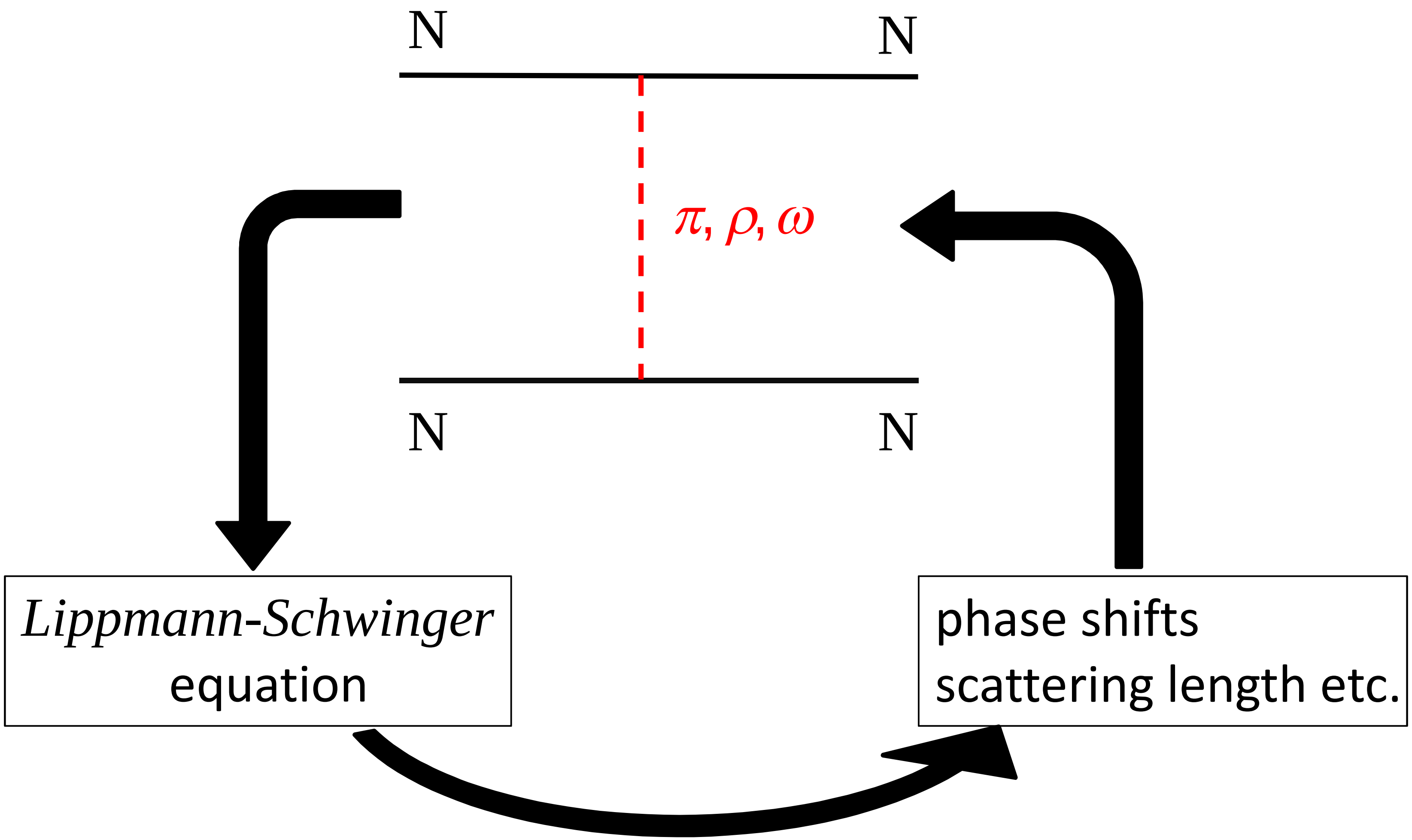
πD scattering length by 3-body

	Scattering length [fm]
type A P_{33} resonance	0.033
type B S_{11}, P_{11}, P_{33} resonance P_{11} bound state	$-0.019 \quad +0.019i$
EXP	$-0.038 \quad +0.009i^{(1)}$ $-0.038 \quad +0.008i^{(2)}$

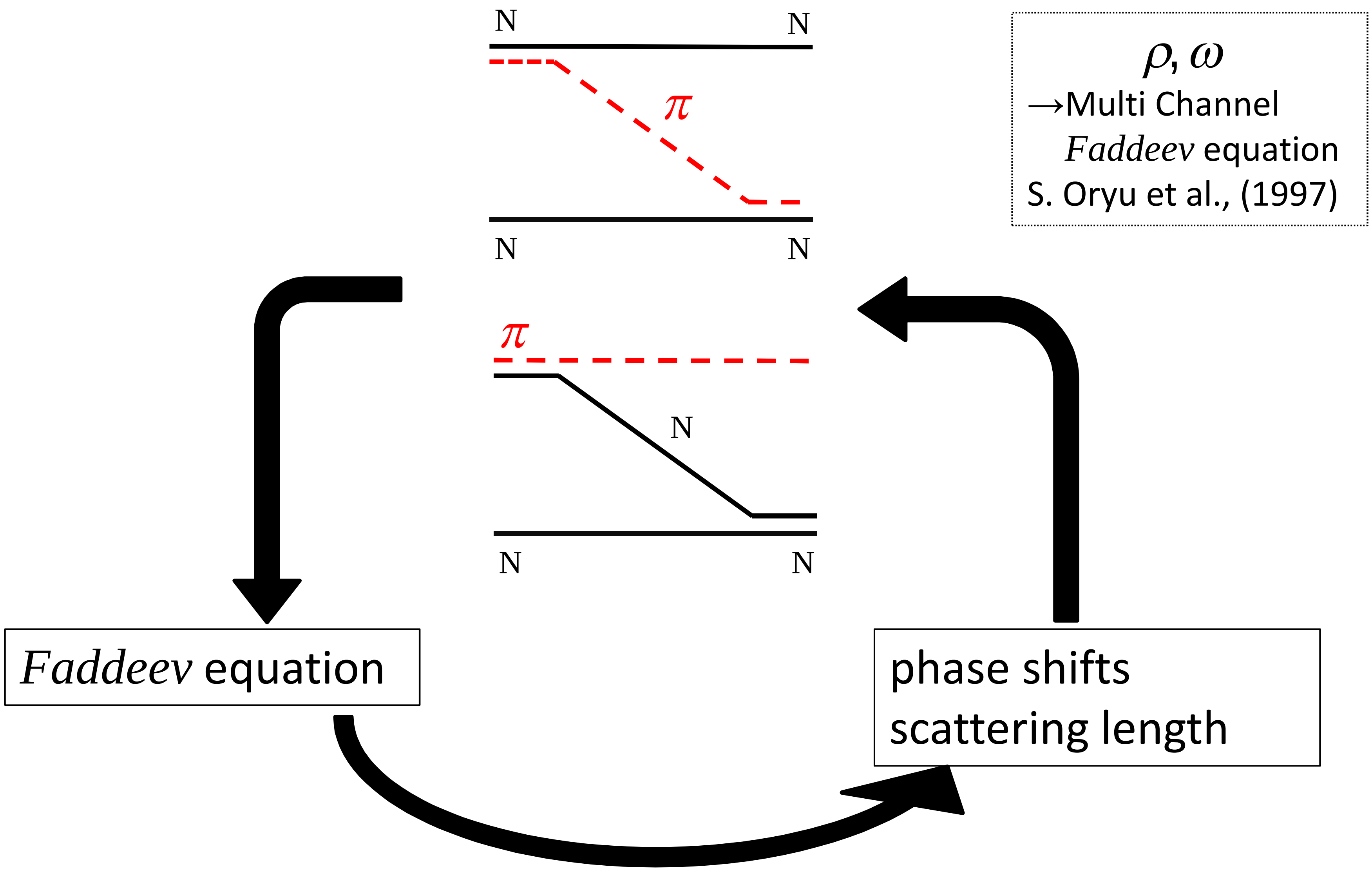
(1) P. Hauser et al., Phys. Rev. C58, R1869 (1998);
(2) D. Chatellard et al., Nucl. Phys. A625, 855 (1997).

neutron-proton scattering length
&
deuteron

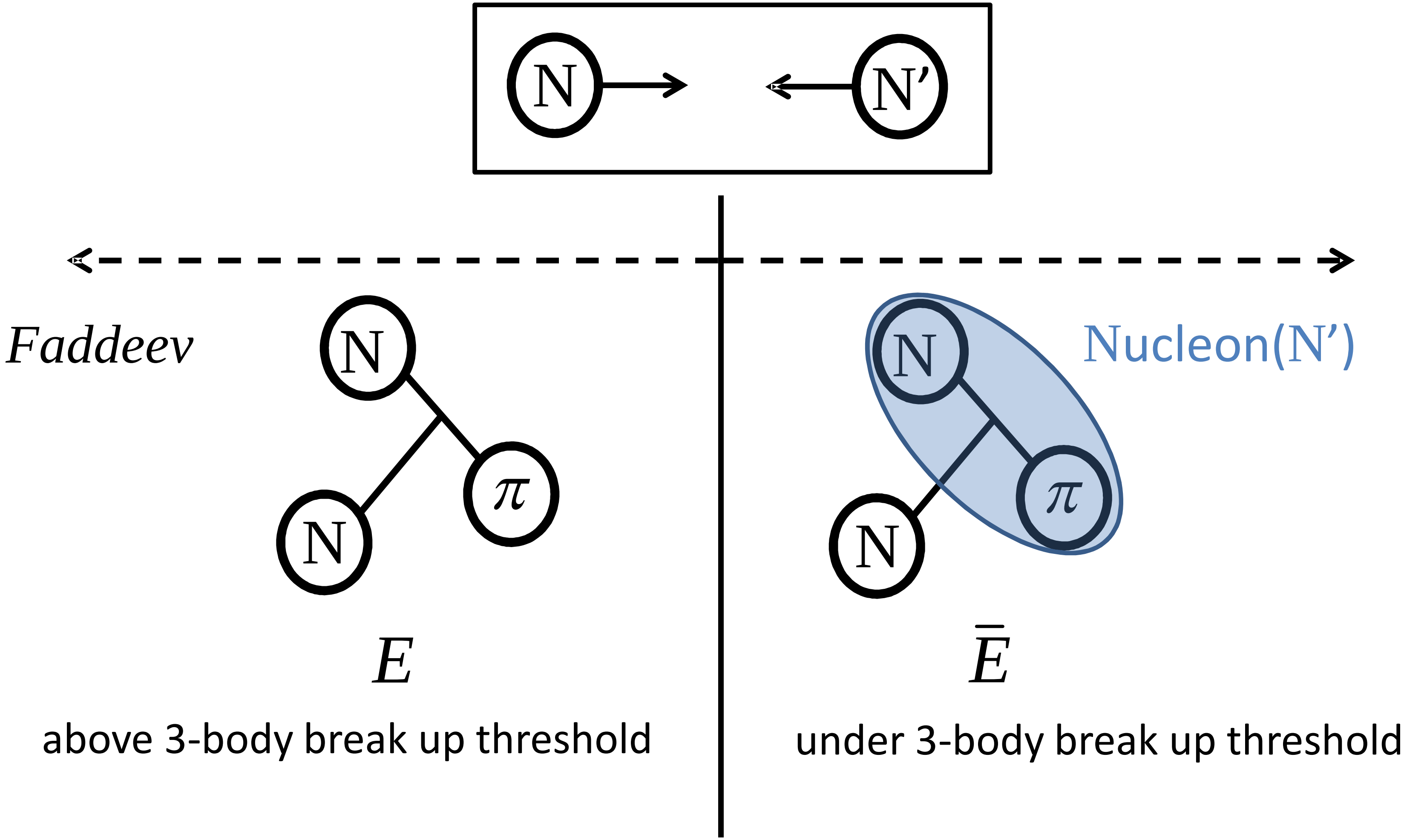
NN interaction in 2-body effective potential



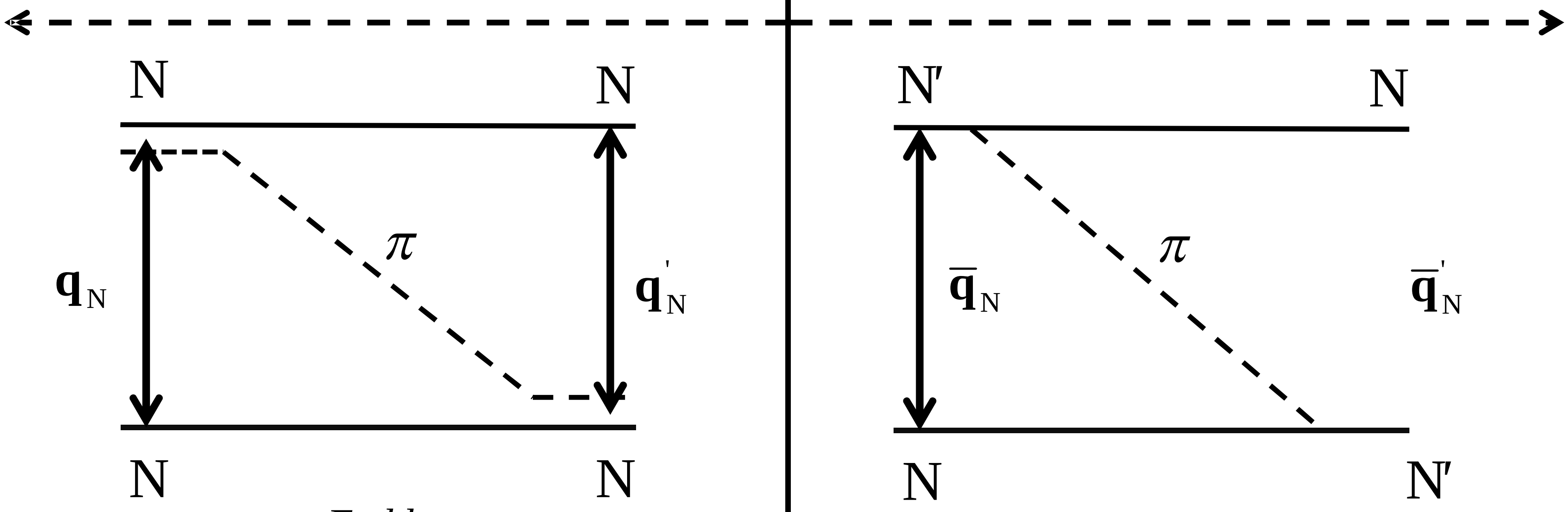
NN interaction in 3-body



3-body energy of NNπ in NN' scattering



$$E = \bar{E} - m_\pi$$



Faddeev

above 3-body breakup threshold

$$\frac{g\left(\mathbf{p}_{\pi N}\right) g\left(\mathbf{p}_{\pi N}\right)}{E - \frac{\mathbf{q}_N^2}{2m_N} - \frac{\mathbf{q}_N'^2}{2m_N} - \frac{\left(\mathbf{q}_N + \mathbf{q}_N'\right)^2}{2m_\pi}}$$

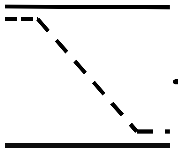
under 3-body breakup threshold

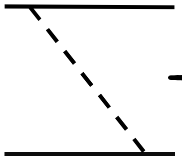
$$\frac{g\left(\bar{\mathbf{p}}_{\pi N}\right) g\left(\bar{\mathbf{p}}_{\pi N}\right)}{\bar{E} - \frac{\bar{\mathbf{q}}_N^2}{2m_N} - \frac{\bar{\mathbf{q}}_N'^2}{2m_N} - \frac{\left(\bar{\mathbf{q}}_N + \bar{\mathbf{q}}_N'\right)^2}{2m_\pi}}$$

Energy dependent
2-body Quasi potential (E2Q)

S. Oryu, PRC86, 044001(2012)

neutron-proton triplet scattering length by 3-body

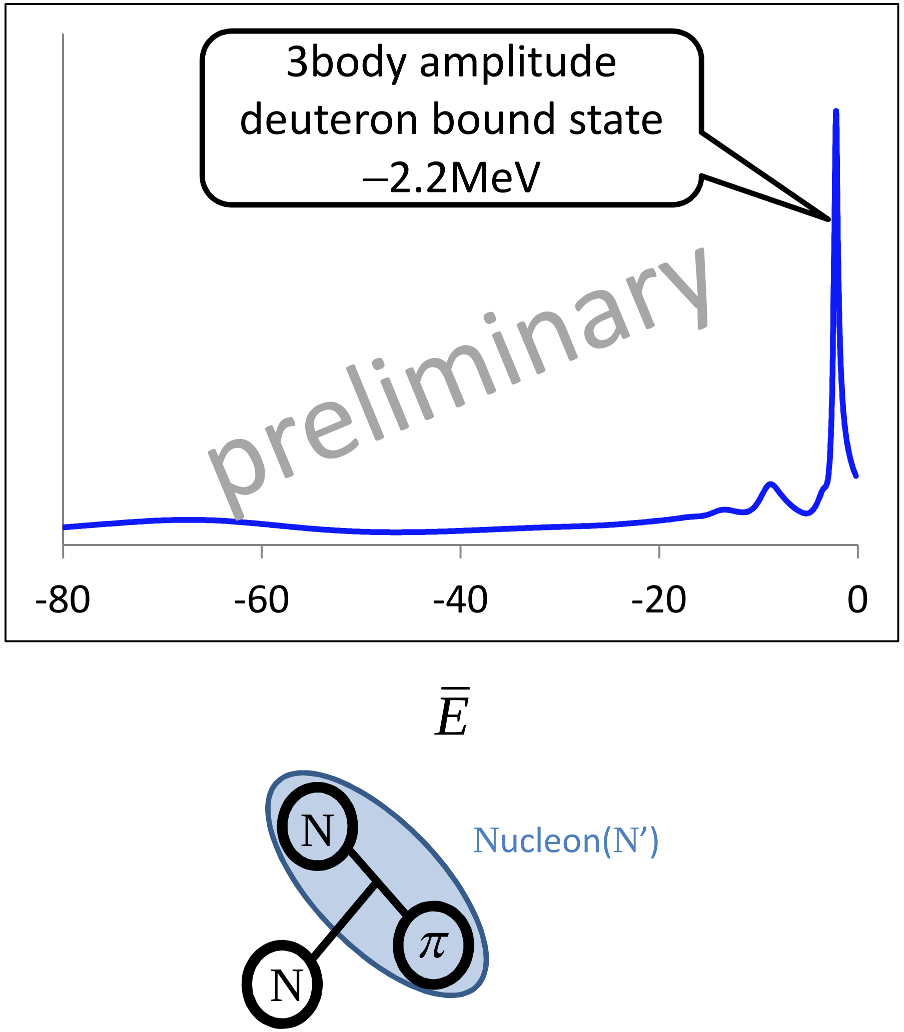


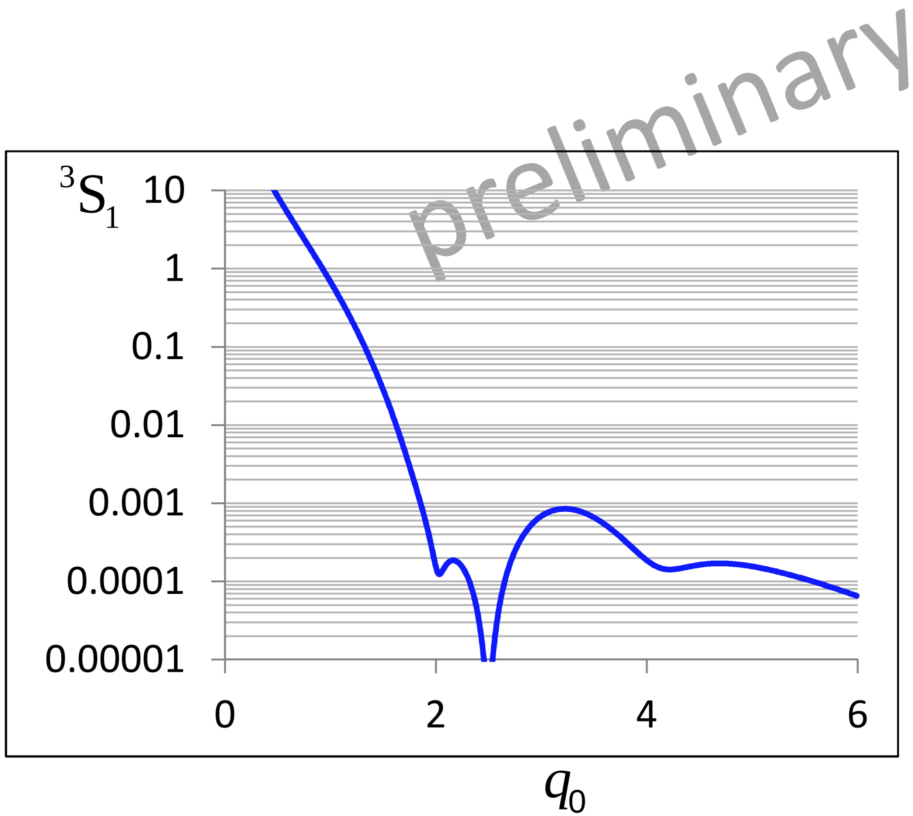


	Scattering length [fm]
Faddeev (type A)	0.280
Faddeev (type B; S_{11}, P_{11} resonance P_{11} bound state)	2.85
E2Q (type B; S_{11}, P_{11} resonance P_{11} bound state)	4.66
EXP	5.419±0.007

T. L. Houk, PRC3, 1886 (1971); W. Dilg, PRC11,103 (1975);
S. Klarsfeld et al., JPG10, 165 (1984)

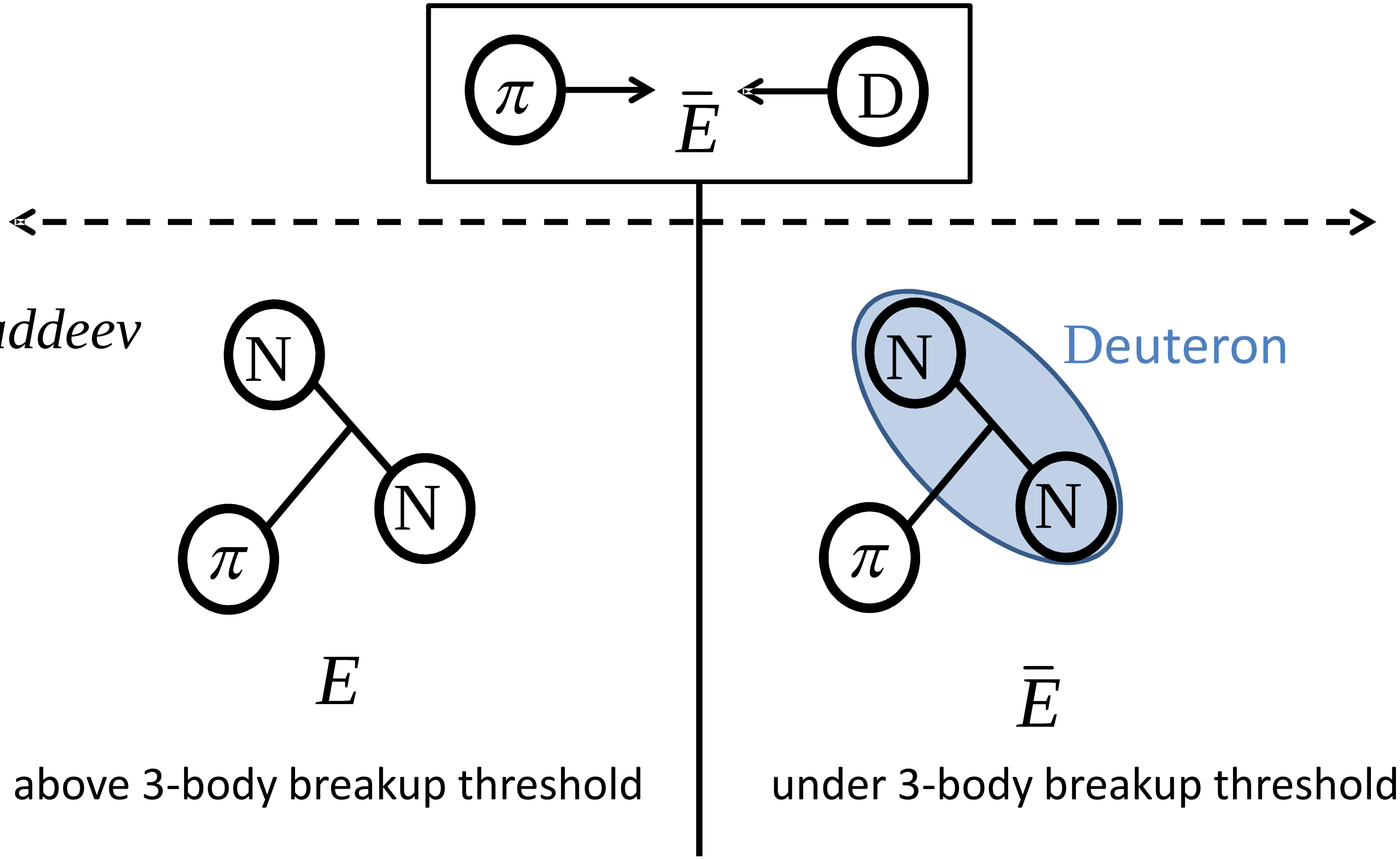
deuteron
by E2Q & type B pion-Nucleon potential





Back to πD scattering

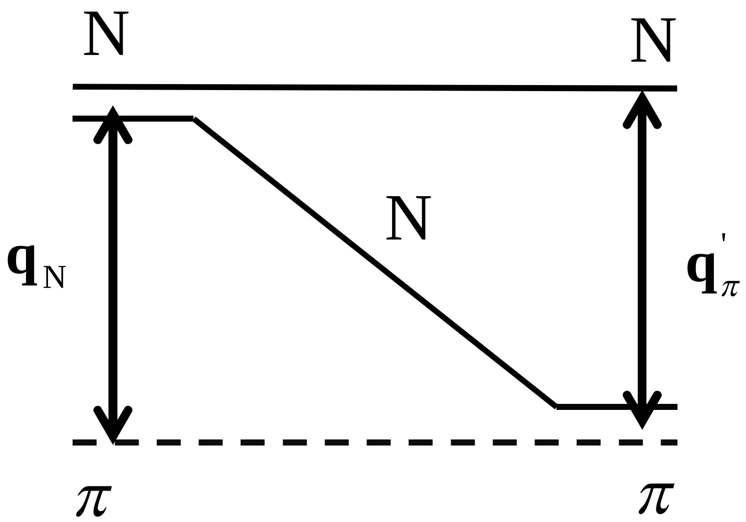
3-body energy of NNπ in πD scattering



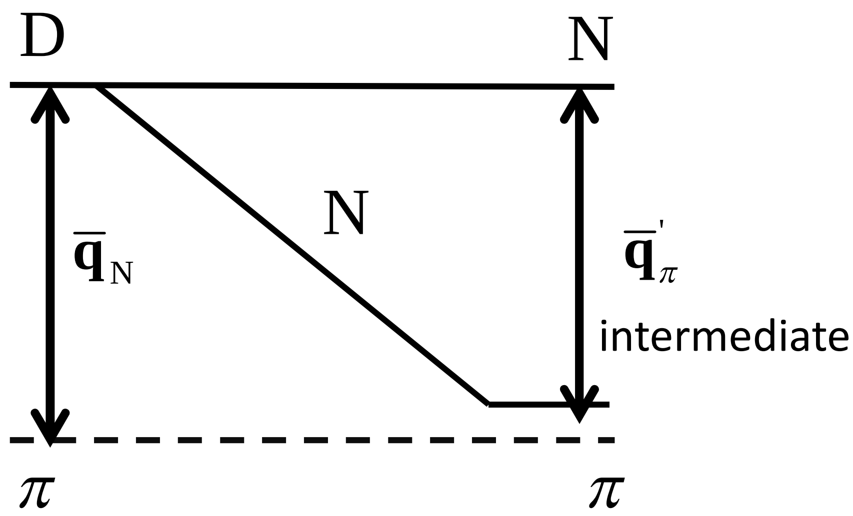
$$E = \bar{E} - \varepsilon_{\mathrm{D}}$$

Deuteron binding energy

πD elastic scattering



Faddeev
above 3-body breakup threshold



E2Q
under 3-body breakup threshold

$$\frac{g\left(\mathbf{p}_{\mathrm{NN}}\right)g\left(\mathbf{p}_{\mathrm{N}\pi}\right)}{E-\frac{\mathbf{q}_{\mathrm{N}}^{2}}{2m_{\mathrm{N}}}-\frac{\mathbf{q}_{\pi}^{'2}}{2m_{\pi}}-\frac{\left(\mathbf{q}_{\mathrm{N}}+\mathbf{q}_{\pi}^{'}\right)^{2}}{2m_{\mathrm{N}}}}$$

$$\frac{g\left(\bar{\mathbf{p}}_{\mathrm{NN}}\right)g\left(\bar{\mathbf{p}}_{\mathrm{N}\pi}\right)}{\bar{E}-\frac{\bar{\mathbf{q}}_{\mathrm{N}}^{2}}{2m_{\mathrm{N}}}-\frac{\bar{\mathbf{q}}_{\pi}^{'2}}{2m_{\pi}}-\frac{\left(\bar{\mathbf{q}}_{\mathrm{N}}+\bar{\mathbf{q}}_{\pi}^{'}\right)^{2}}{2m_{\mathrm{N}}}}$$

πD scattering length by 3-body

	Scattering length [fm]
Faddeev (type A; P_{33} resonance)	0.033
Faddeev (type B; S_{11}, P_{11}, P_{33} resonance P_{11} bound state)	-0.019 $+0.019i$
E2Q (type B; S_{11}, P_{11}, P_{33} resonance P_{11} bound state)	-0.023 $+0.019i$
EXP	-0.038 $+0.009i$ -0.038 $+0.008i$

P. Hauser et al., Phys. Rev. C58, R1869 (1998);
D. Chatellard et al., Nucl. Phys. A625, 855 (1997).

summary

- In 1976, A. W. Thomas accomplished 47.5MeV $\pi^+\mathrm{D}$ elastic scattering.
- In 1995, M. G. Fuda proposed a new type $\pi\mathrm{N}$ potential which is including P_{11} bound state etc.
- For NN and $\pi\mathrm{D}$ scattering length, the Fuda's potential brings about good results. The Fuda's potential is represented by only rank 1. Our results may be improved by more ranks potential or off-shell effects.
- In 2012, S. Oryu insisted that a 3-body diagram should be changed under a 3-body breakup threshold. We call it E2Q. For the scattering lengths, E2Q leads to good results.
- Although our deuteron calculation by E2Q is preliminary, we may find the binding energy and wave function. On the other hand, it is difficult to find the deuteron state by a non-relativistic Faddeev.