Gauge-invariance constraints for photoproduction processes

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Collaborators:
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Overview

- Motivation and Goal
- Basics of gauge invariance
- Single-pion photoproduction
- Bremsstrahlung
- Two-pion photoproduction
- Three-meson photoproduction
- Conclusions

NB: Pions and nucleons here are placeholders for arbitrary mesons and baryons.
Note also, the formulation is valid for real *and* virtual photons.

Overview

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- Basics of gauge invariance
- Single-pion photoproduction
- Bremsstrahlung
- Two-pion photoproduction
- Three-meson photoproduction
- Conclusions

To describe the production of three or more pions is straightforward following the procedures given here

NB: Pions and nucleons here are placeholders for arbitrary mesons and baryons.

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Motivation

- Multi-meson production processes are being measured now. Examples: $\gamma N \to \pi\pi N$, $\gamma N \to \pi\pi\pi N$, etc.
- Understanding of the generic dynamics of such processes is essential for strangeness production in processes like $\gamma N \to KK\Xi$, $\gamma N \to KKK\Omega$
- No comprehensive formulation of multi-meson photoproduction processes exists that is at the same level of rigor as single-pion production

Goal

- Provide guidance for microscopically consistent formulations of meson photoproduction processes off the nucleon valid for real *and* virtual photons within the framework of a coupled-channel formalism

- **Full implementation of gauge invariance at the microscopic level**

- In practice, approximations will be necessary: Provide prescriptions to retain microscopic consistency and gauge invariance even then

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Procedure

- Use field theory based on hadronic Lagrangians
- Employ LSZ-type mechanisms to couple electromagnetic field to fully dressed hadronic propagators and vertices
- Obey full *local* gauge invariance

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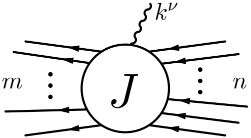
- Obey full *local* gauge invariance **Important**

Gauge Invariance

$(N+1)$-point current:

$J^\mu \ =$ 

one incoming photon
n incoming hadrons
m outgoing hadrons
$N = m + n$

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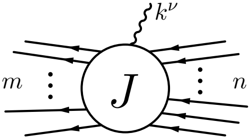

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Vanishing four-divergence:

$$\boxed{\partial_\mu J^\mu = 0}$$

alle external hadrons on-shell

Equivalent continuity equation:

$$\frac{\partial \rho}{\partial t} - \boldsymbol{\nabla} \cdot \mathbf{J} = 0$$

$J^\mu = (\rho, \mathbf{J})$

Implies charge conservation:

$$Q = \int d^3 r \; \rho = \text{const}$$

Gauge Invariance

Where does it come from?

Gauge Invariance $\equiv$ U(1) Invariance

Global gauge invariance: Physics invariant under global phase transformation of field

$$\phi \to \phi\, \mathrm{e}^{-i\lambda} \qquad \lambda\text{: real, constant}$$

implies a conserved charge

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This is not very helpful from a dynamical point of view since charge conservation is built into the models explicitly anyway, either via the isospin formalism or by employing an explicit particle basis.

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In addition, it implies the very existence of the electromagnetic field A^μ, i.e., it provides the Maxwell equations:

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Violation of local gauge invariance tampers with consistent implementation of the electromagnetic field!

Gauge Invariance

Use the requirement of *local* gauge invariance as a tool for finding all reaction mechanisms that must be considered simultaneously

Current Conservation vs. Local Gauge Invariance

Current conservation: $\boxed{k_\mu J^\mu = 0}$

Decompose current into transverse and longitudinal pieces: $J^\mu = J^\mu_{\mathrm{T}} + J^\mu_{\mathrm{L}}$

Current conservation: $k_\mu J^\mu_{\mathrm{T}} = 0$ (trivial) and $\boxed{k_\mu J^\mu_{\mathrm{L}} = 0}$ (non-trivial)

Amplitude for real photons ($k^2 = 0$ and $k_\mu \varepsilon^\mu = 0$): $\boxed{\varepsilon_\mu J^\mu = \varepsilon_\mu J^\mu_{\mathrm{T}}}$

Real photons never see the non-trivial part of the current.

Current Conservation vs. Local Gauge Invariance

Current conservation: $\boxed{k_\mu J^\mu = 0}$ on-shell condition

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- Dynamically consistent framework:

  Local gauge invariance appears as **off-shell conditions**
  in the form of **generalized Ward-Takahashi identities** that
  link longitudinal and transverse current contributions dynamically

First, a simple problem: $\gamma N \to \pi N$

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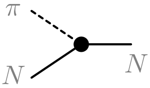
Attach photon to πNN vertex:



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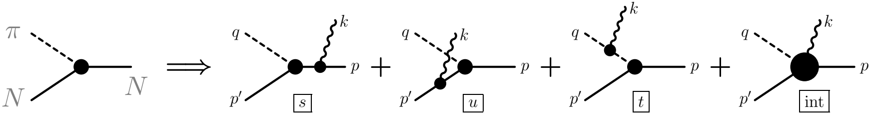

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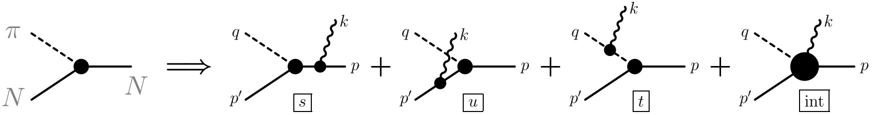

at tree level: $-g_\pi \frac{\gamma_5 \gamma^\mu}{2m} e_\pi$

Kroll-Ruderman term

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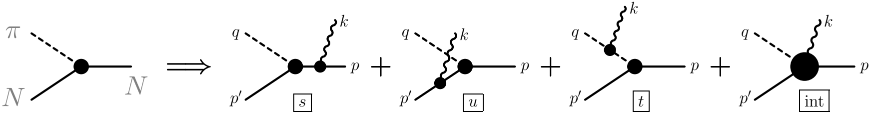
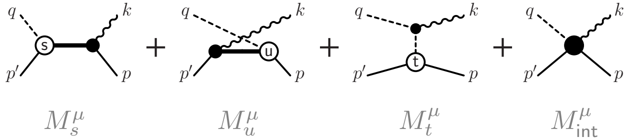


Redraw equivalently:

$M^\mu =$



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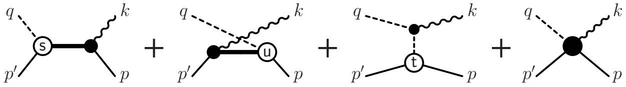
Photoproduction current:

$M^\mu =$ 

M_s^μ $\quad$ M_u^μ $\quad$ M_t^μ $\quad$ M_{int}^μ

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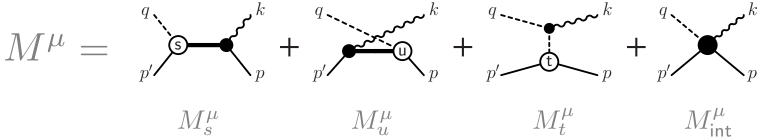

NB: This is the full topological structure of the current — even in the most sophisticated implementation of the problem

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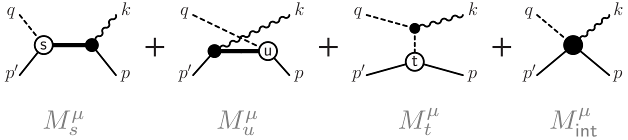

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Input needed:

- πNN vertices F_s, F_u, and F_t
- Propagators
- Electromagnetic nucleon and pion currents
- Interaction current M_{int}^μ ($=$ contact-type current)

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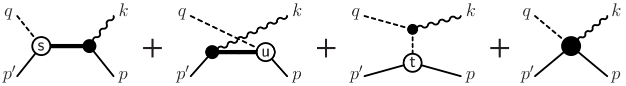

$$M^\mu = M_s^\mu + M_u^\mu + M_t^\mu + M_{\text{int}}^\mu$$

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Input needed:

- πNN vertices F_s, F_u, and F_t
- Propagators
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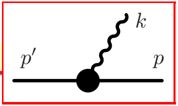
Full implementation requires incorporating all possible dressing effects

⇒ more details to come

Electromagnetic Current Γ^μ of the Nucleon



$$\Gamma^\mu(p',p) = e\delta_N\gamma^\mu F_1(k^2) + e\kappa_N \frac{\sigma^{\mu\nu}k_\nu}{2m} F_2(k^2)$$

F_1: Dirac form factor
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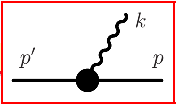

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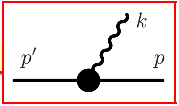

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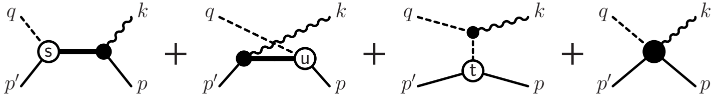
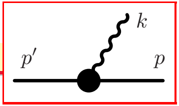
- In general, that's not the case.

Example: Pion photoproduction



Electromagnetic Current Γ^μ of the Nucleon



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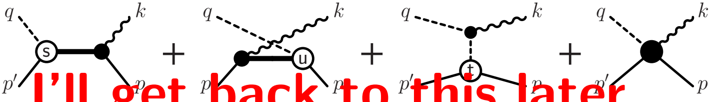
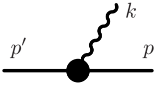


I'll get back to this later. . .

Electromagnetic Current Γ^μ of the Nucleon



For a Bethe-Salpeter-type approach:

- General Lorentz-covariant structure of Γ^μ requires **12 form factors**.
  Bincer, PR118,855(1960)
- Applying gauge invariance, this reduces to **8 form factors**.
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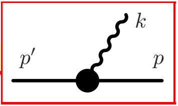

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$F_1,\ F_2,\ f_1,\ f_2,\ g_1,\ g_2$

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$$\Gamma^\mu(p',p) = e\left[\delta_N\gamma^\mu + \delta_N\gamma_{\mathrm{T}}^\mu(F_1-1) + \frac{i\sigma^{\mu\nu}k_\nu}{2m}\kappa_N F_2 \right.$$
$$+ \frac{S^{-1}(p')}{2m}\left(\gamma_{\mathrm{T}}^\mu f_1 + \frac{i\sigma^{\mu\nu}k_\nu}{2m}\kappa_N f_2\right) + \left(\gamma_{\mathrm{T}}^\mu f_1 + \frac{i\sigma^{\mu\nu}k_\nu}{2m}\kappa_N f_2\right)\frac{S^{-1}(p)}{2m}$$
$$\left. + \frac{S^{-1}(p')}{2m}\left(\gamma_{\mathrm{T}}^\mu g_1 + \frac{i\sigma^{\mu\nu}k_\nu}{2m}\kappa_N g_2\right)\frac{S^{-1}(p)}{2m}\right]$$

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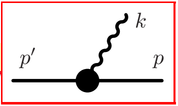

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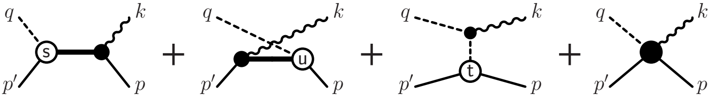
Constraints: no kinematic singularity: $f_1(k^2) \xrightarrow{k^2=0} 0$ and $g_1(k^2) \xrightarrow{k^2=0} 0$

chiral-symmetry limit: $f_1 \to \dfrac{g_A - G_A(k^2)}{g_A}$ and $f_2 \to 1$

Implications of off-shell structure: Pion photoproduction



s-channel:

$$F_s S(p+k)\Gamma_i^\mu(p+k,p) = F_s S(p+k)\left(e\delta_i\gamma^\mu + \frac{i\sigma^{\mu\nu}k_\nu}{2m}e\kappa_i\right) + \underbrace{F_s \frac{i\sigma^{\mu\nu}k_\nu}{2m}\frac{e\kappa_i}{2m}f_{2i}}_{\text{contact terms}}$$

u-channel:

$$\Gamma_f^\mu(p',p'-k)S(p'-k)F_u = \left(e\delta_f\gamma^\mu + \frac{i\sigma^{\mu\nu}k_\nu}{2m}e\kappa_f\right)S(p'-k)F_u + \overbrace{\frac{i\sigma^{\mu\nu}k_\nu}{2m}\frac{e\kappa_f}{2m}f_{2f}F_u}$$

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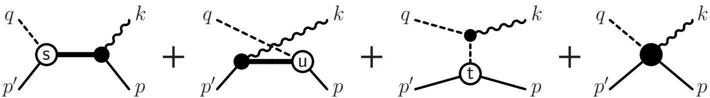

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No connection to low-energy χPT results possible without such contact terms

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A Word about “Off-shell Effects”

It is often stated that “off-shell effects are not measurable” and that, therefore, any such effects should be summarily banished from any theory.

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Panel discussion at the "17th European Conference on Few-Body Problems in Physics," Evora, Portugal, September 11–16, 2000, NPA689 (2001)

Franz Gross on off-shell effects

"It is commonly stated that "off-shell effects" are unobservable. This is of course true, but so are wave functions, potentials, and most of the theoretical tools we use to describe physics. A better point is that off-shell effects are *meaningless without a theory or model to define them*. Almost all models provide such a definition, and off-shell effects should be discussed only in the context of a particular model that defines these effects *uniquely*."

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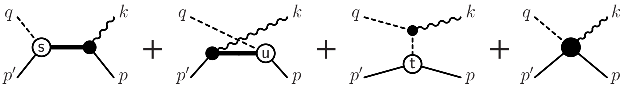
$\Longrightarrow$ Within the Bethe-Salpeter-type equations that originate from effective Lagrangian formulations, the off-shell structure of the nucleon current arises naturally as an integral part of the description of the reaction dynamics.

Gauge Invariance: Generalized Ward-Takahashi identities

$M^\mu =$



M^μ_s $\quad M^\mu_u$ $\quad M^\mu_t$ $\quad M^\mu_{\text{int}}$

- **Generalized WTI for the full current M^μ:**

$k_\mu M^\mu = 0$ on-shell

$$k_\mu M^\mu = -\underbrace{F_s S(p+k) Q_i S^{-1}(p)}_{s-\text{channel}} + \underbrace{S^{-1}(p') Q_f S(p'-k) F_u}_{u-\text{channel}} + \underbrace{\Delta_\pi^{-1}(q) Q_\pi \Delta_\pi(q-k) F_t}_{t-\text{channel}}$$

- **WTI for the nucleon current Γ^μ:**

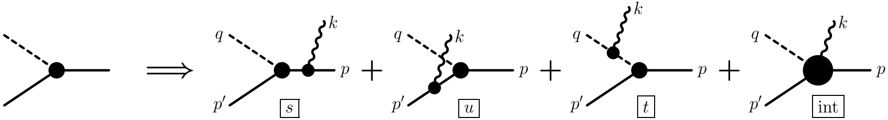
$$k_\mu \Gamma^\mu = S^{-1}(p+k) Q_N + Q_N S^{-1}(p)$$

- **Generalized WTI for the interaction current M^μ_{int}:**

$$k_\mu M^\mu_{\text{int}} = -F_s Q_i + Q_f F_u + Q_\pi F_t$$

Construct Single-pion Production Current

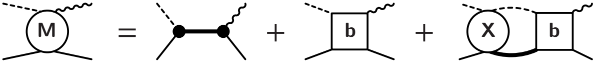
Attach photon to πNN vertex:



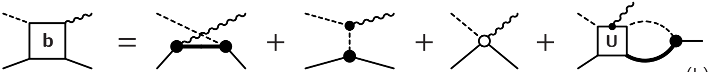
contains full FSI

- Simple at tree level
- *Very* complicated for *dressed* vertex

- **Pion-production current M^μ:**

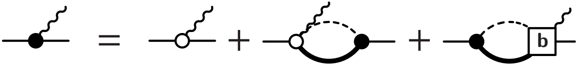


(a)



(b)

- **Nucleon current Γ^μ:**

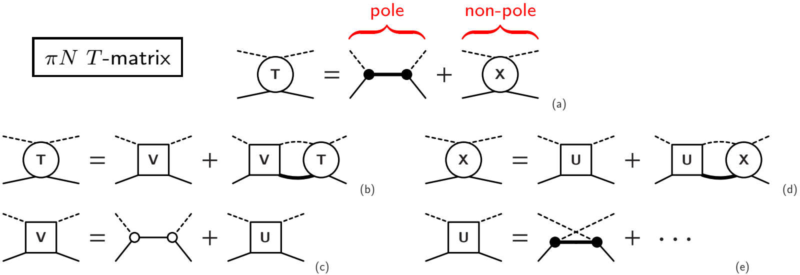


⇒ The internal structures of the dressed nucleon current can be understood by the dynamics of the pion production current.

- Tower of *nonlinear* Dyson-Schwinger-type equations

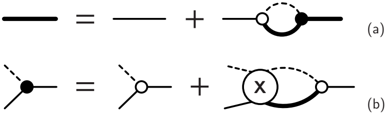
Pions, Nucleons, and Photons

πN T-matrix



dressed nucleon propagator

dressed πNN vertex



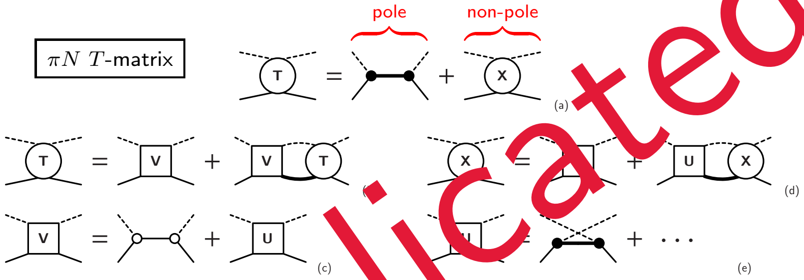
propagator determines current

- Tower of *nonlinear* Dyson-Schwinger-type equations

Pions, Nucleons, and Photons

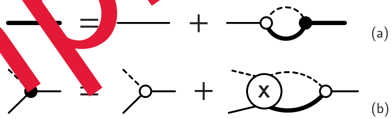
HH, PRC 5… 2041 (1997)

πN T-matrix



dressed nucleon propagator

dressed πNN verte…



propagator determines current

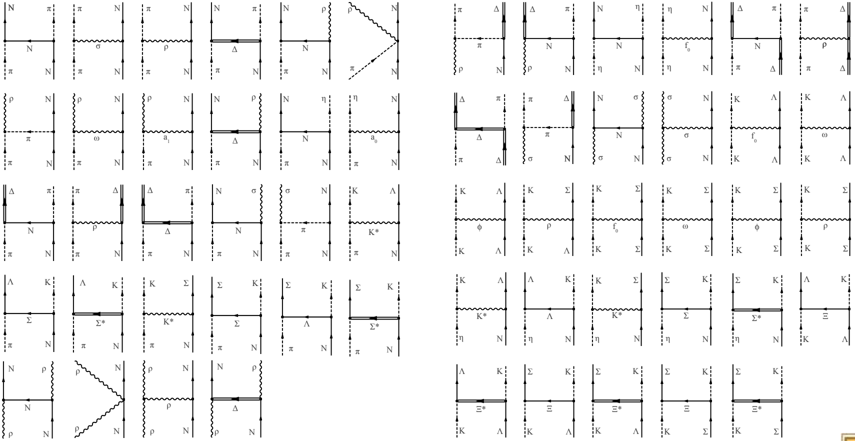
Complicated

- Tower of *nonlinear* Dyson-Schwinger-type equations

Hadronic Input: The Jülich Model (the latest incarnation)

Coupled system of equations with Lippmann-Schwinger structure:

$$T = V + VG_0T$$



D. Rönchen, M. Döring, F. Huang, H. Haberzettl, J. Haidenbauer, C. Hanhart, S. Krewald, U.-G. Meißner, K. Nakayama, EPJA**49**:44(2013)

. . . from the Jülich–Athens–Washington Collaboration

Jülich:	M. Döring	Forschungszentrum Jülich
	J. Haidenbauer	
	Ch. Hanhart	
	S. Krewald	
	U.-G. Meißner	
	D. Rönchen	
Athens:	F. Huang	The University of Georgia
	K. Nakayama	
Washington:	H. Haberzettl	The George Washington University


The
JAW
Collaboration
Jülich–Athens/GA–Washington/DC

. . . from the Jülich–Athens–Washington Collaboration

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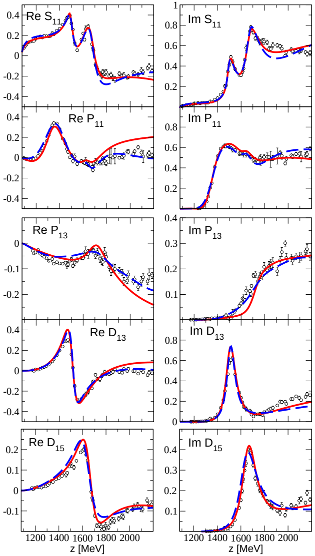
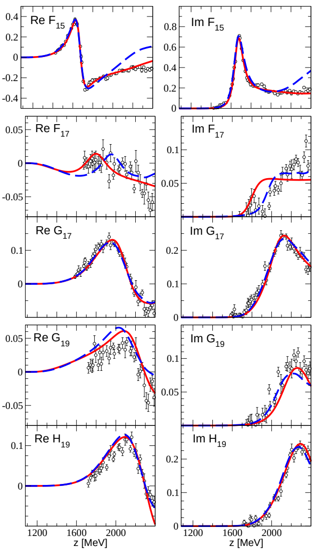
	U.-G. Meißner	
	D. Rönchen	
Athens:	K. Nakayama	The University of Georgia
Washington:	M. Doring	The George Washington University
	H. Haberzettl	
	F. Huang	UCAS, Beijing

Hadronic Input: The Jülich Model (the latest incarnation)



Reaction $\pi N \to \pi N$, S- to D-wave.

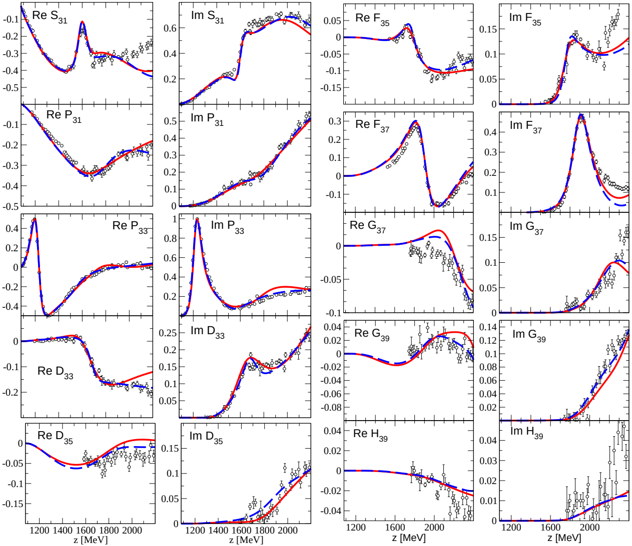


Reaction $\pi N \to \pi N, F$- to H-wave.

$\pi N \to \pi N$

Isospin 1/2

Hadronic Input: The Jülich Model (the latest incarnation)



Reaction $\pi N \rightarrow \pi N$, S- to D-wave.

Reaction $\pi N \rightarrow \pi N$, F- to H-wave.

$\pi N \rightarrow \pi N$

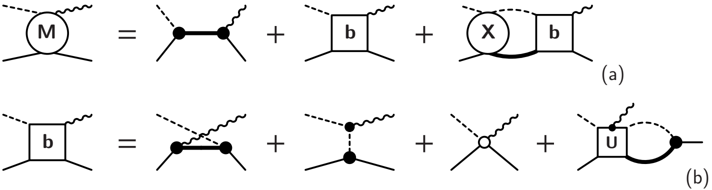
Isospin 3/2

Back to

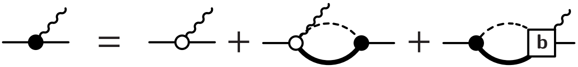
Pion Photoproduction

HH, PRC **56**, 2041 (1997)

- **Pion-production current M^μ:**



- **Nucleon current Γ^μ:**



⇒ The internal structures of the dressed nucleon current can be understood by the dynamics of the pion production current.

- Tower of *nonlinear* Dyson-Schwinger-type equations

Problems?

- Everything is exact!
- Everything is nonlinear!
- Everything is hideously complicated!

Approximations?

- In general, indiscriminate approximations will destroy gauge invariance and thus do damage to the consistent description of the electromagnetic interaction

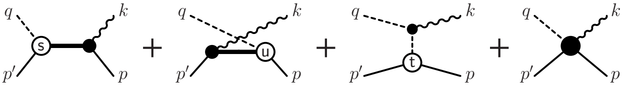
Approximations?

- In general, indiscriminate approximations will destroy gauge invariance and thus do damage to the consistent description of the electromagnetic interaction

☑ Approximations must be carefully constructed to preserve the generalized Ward-Takahashi identities necessary to maintain full local gauge invariance

Gauge Invariance: Generalized Ward-Takahashi identities

$M^\mu =$



M_s^μ $\quad$ M_u^μ $\quad$ M_t^μ $\quad$ M_{int}^μ

- **Generalized WTI for the full current M^μ:** $\quad$ $k_\mu M^\mu = 0$ on-shell

$$k_\mu M^\mu = -\underbrace{F_s S(p+k) Q_i S^{-1}(p)}_{s-\text{channel}} + \underbrace{S^{-1}(p') Q_f S(p'-k) F_u}_{u-\text{channel}} + \underbrace{\Delta_\pi^{-1}(q) Q_\pi \Delta_\pi(q-k) F_t}_{t-\text{channel}}$$

- **WTI for the nucleon current Γ^μ:**

$$k_\mu \Gamma^\mu = S^{-1}(p+k) Q_N + Q_N S^{-1}(p)$$

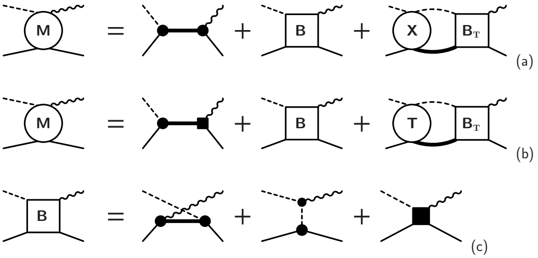
- **Generalized WTI for the interaction current M_{int}^μ:**

$$\boldsymbol{k_\mu M_{\text{int}}^\mu = -F_s Q_i + Q_f F_u + Q_\pi F_t}$$

Key relation to make approximations work

Rewrite the Production Current

- **Pion-production current M^μ:**

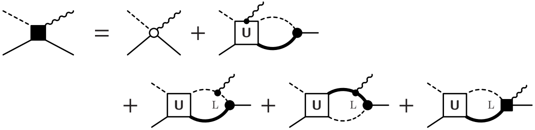


X

equivalent

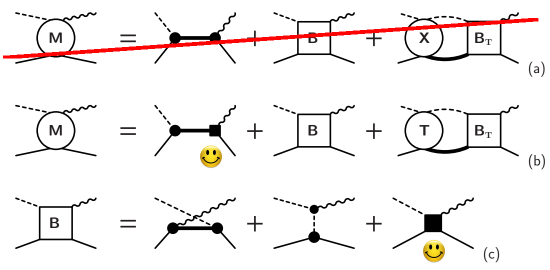
T

- **Contact-type current M_c^μ:**



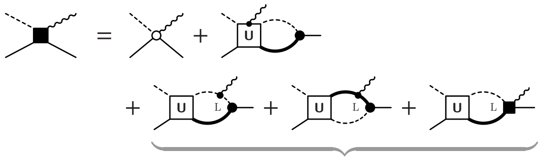
Where to make approximations?

HH, F. Huang, K. Nakayama, PRC**83**,065502(2011)

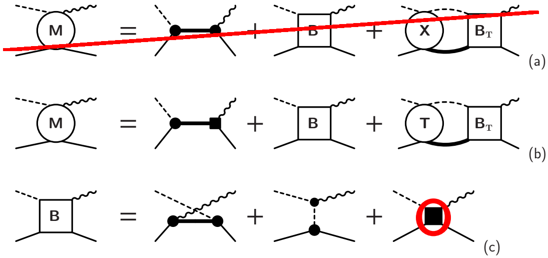


Do not use X.
Work with full T.

$$M_c^\mu$$



Approximating M_c^μ



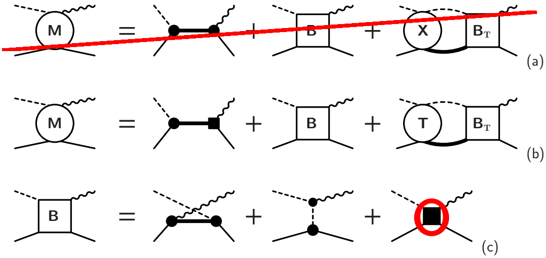
M_c^μ

- **Lowest-order approximation in terms of phenomenological form factors:**

$$M_c^\mu = ge\gamma_5 \frac{i\sigma^{\mu\nu}k_\nu}{4m^2}\tilde{\kappa}_N - (1-\lambda)g\frac{\gamma_5\gamma^\mu}{2m}\tilde{F}_t e_\pi - G_\lambda \left[e_i(2p+k)^\mu \frac{\tilde{F}_s - \hat{F}}{s-p^2} \right.$$
$$\left. + e_f(2p'-k)^\mu \frac{\tilde{F}_u - \hat{F}}{u-p'^2} + e_\pi(2q-k)^\mu \frac{\tilde{F}_t - \hat{F}}{t-q^2} e \right]$$

Phenomenological contact current
— no poles

Approximating M_c^μ



Approximation preserves full off-shell gauge invariance.

- **Lowest-order approximation in terms of phenomenological form factors:**

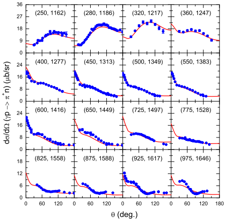
Don't try to read the details. What is important is that this is a simple expression, easy to evaluate, and that it helps preserve gauge invariance of the entire production current.

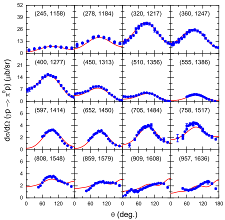
Approximation can be made more sophisticated if necessary. . .

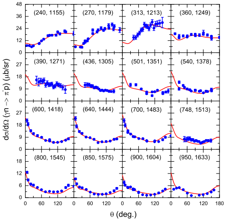
Does it work? — Yes!

- **Results for $\gamma N \to \pi N$**

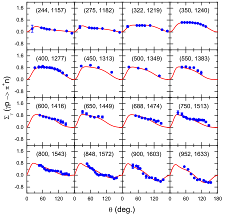
$\frac{d\sigma}{d\Omega}$

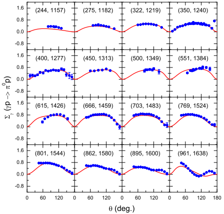


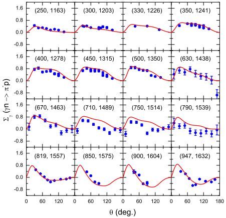




Σ







$\gamma p \to \pi^+ n$ $\qquad$ $\gamma p \to \pi^0 p$ $\qquad$ $\gamma n \to \pi^- p$

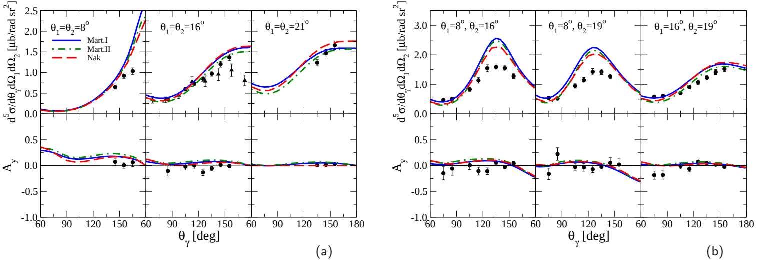
F. Huang, M. Döring, H. Haberzettl, J. Haidenbauer, C. Hanhart, S. Krewald, U.-G. Meißner, and K. Nakayama, PRC**85**, 054003 (2012)

Bremsstrahlung $NN \to NN\gamma$

KVI data — a longstanding problem (for theorists)

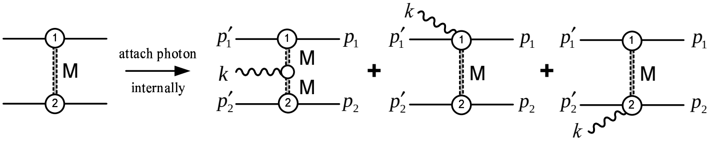
$\frac{d^5\sigma}{d\theta_\gamma d\Omega_1 d\Omega_2}$

A_y



Theory was unable to describe KVI data

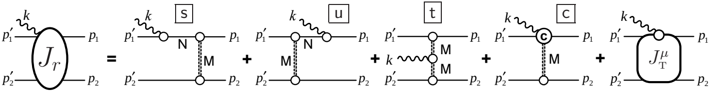
Interaction current for meson-exchange potential:



Bremsstrahlung $NN \to NN\gamma$

- **Bremsstrahlung Current:**

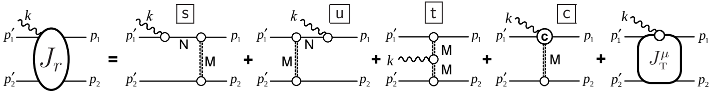
$$J^\mu_{\mathsf{B}} = (TG_0 + 1)J^\mu_r(1 + G_0T) \qquad T\text{: } NN\ T\text{-matrix}$$



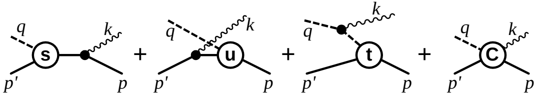
Similar currents along the lower nucleon line

Bremsstrahlung Current:

$$J_{\mathrm{B}}^{\mu} = (TG_0 + 1){\color{red}J_r^{\mu}}(1 + G_0T) \qquad T:\ \ NN\ T\text{-matrix}$$

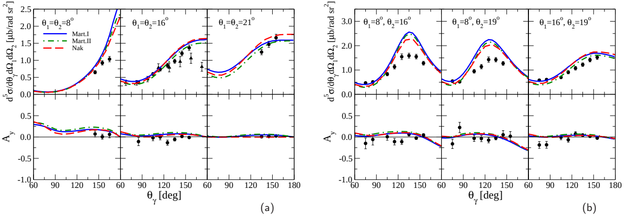


Compare the photon processes along the top nucleon line above to the meson production diagrams below



⇒ Essential parts of the process can be described as a meson capture process — i.e., as an inverse photoproduction process — in the presence of a spectator nucleon.

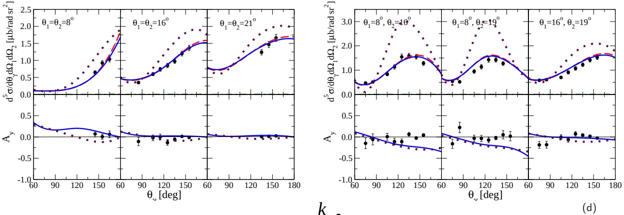
Application to KVI data — Or: Resolving a longstanding problem



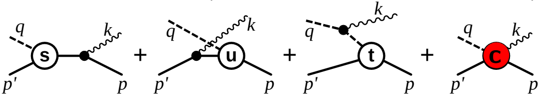
Old

$$\frac{d^5\sigma}{d\theta_\gamma d\Omega_1 d\Omega_2}$$

A_y



New

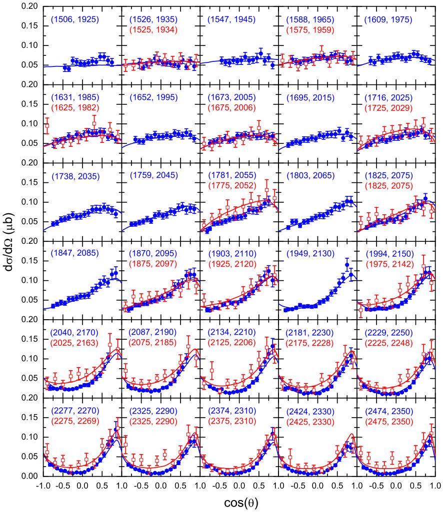


Contact current missing from old calculations

Also available. . .

- η photoproduction
- η' photoproduction

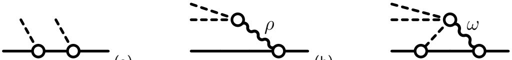
Combined Analysis of η' Production



Combined analysis of η' production reactions: $\gamma N \to \eta' N$, $NN \to NN\eta'$, and $\pi N \to \eta' N$,
F. Huang, HH, K. Nakayama,
Phys. Rev. C **87**, 054004 (2013)

Two-pion Production

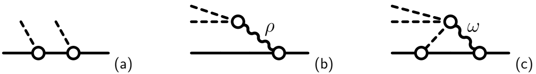
Basic Hadronic Two-pion Production Processes



(a) (b) (c) etc.

(a) sequential production off nucleon
(b) production off intermediate vector meson
(c) production off intermediate three- or more-pion vertex

Basic Hadronic Two-pion Production Processes



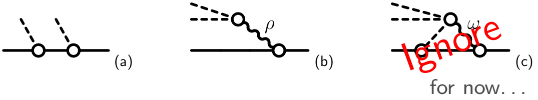
etc.

(a) sequential production off nucleon
(b) production off intermediate vector meson
(c) production off intermediate three- or more-pion vertex

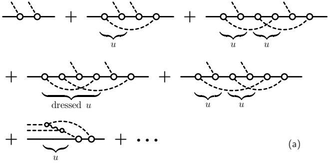
Procedure

(1) Iterate bare hadronic processes and sum up to obtain dressed mechanisms
(2) Attach photon — employ (gauge-invariant!!) single-pion amplitudes

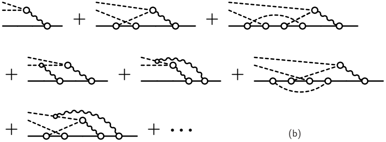
Iterated Hadronic Two-pion Production Processes



etc.

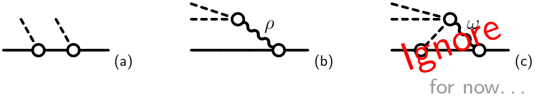


(a)

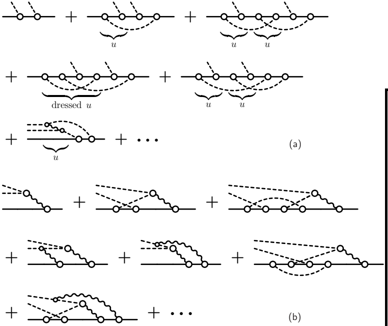


(b)

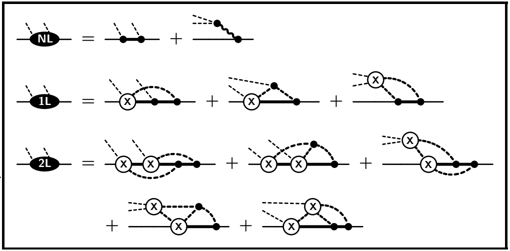
Iterated Hadronic Two-pion Production Processes



etc.

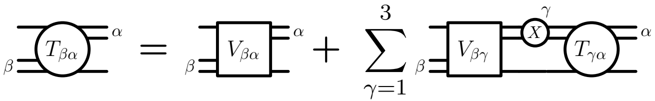


Lowest orders of
3-body multiple scattering series

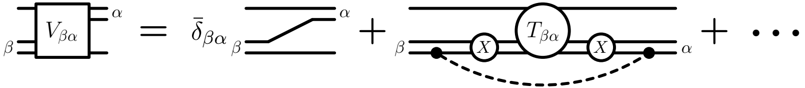


Faddeev-type Alt-Grassberger-Sandhas Equations

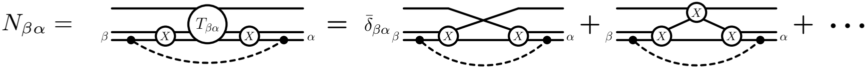
NP **B2**, 167 (1967)



$\bar{\delta}_{\beta\alpha} = 1 - \delta_{\beta\alpha}$



Non-linear contributions:



Cluster indices:

$$
\begin{aligned}
\alpha, \beta, \gamma : \quad & \text{“1”} = (\pi_1 N, \pi_2) \\
& \text{“2”} = (\pi_2 N, \pi_1) \\
& \text{“3”} = (\pi_1 \pi_2, N)
\end{aligned}
$$

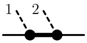
$$
T_{\beta\alpha} = V_{\beta\alpha} + \sum_{\gamma=1}^{3} V_{\beta\gamma} G_0 X_\gamma G_0 T_{\gamma\alpha}
$$

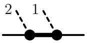
Matrix LS structure: $\mathrm{T} = \mathrm{V} + \mathrm{V} G_0 \mathrm{T}$

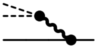
Closed-form Expression for $N \to \pi\pi N$ 'Vertex'

$$M_\beta = \sum_\alpha \left(\delta_{\beta\alpha} + \sum_\gamma T_{\beta\gamma} G_0 X_\gamma \bar{\delta}_{\gamma\alpha} \right) f_\alpha + \sum_\gamma \left(\delta_{\beta\gamma} + T_{\beta\gamma} G_0 X_\gamma G_0 \right) \sum_\alpha N_{\gamma\alpha} G_0 f_\alpha$$

AGS amplitudes $T_{\beta\alpha}$ subsume all explicit three-body effects, with contributions of infinitely many mesons provided by their non-linear driving terms $N_{\beta\alpha}$

where $f_1 =$ 

$f_2 =$ 

$f_3 =$ 

Closed-form Expression for $N \to \pi\pi N$ 'Vertex'

$$M_\beta = \sum_\alpha \left(\delta_{\beta\alpha} + \sum_\gamma T_{\beta\gamma} G_0 X_\gamma \bar{\delta}_{\gamma\alpha} \right) f_\alpha + \sum_\gamma \left(\delta_{\beta\gamma} + T_{\beta\gamma} G_0 X_\gamma G_0 \right) \sum_\alpha N_{\gamma\alpha} G_0 f_\alpha$$

Loop Expansion...

$$= f_\beta$$

$\Leftarrow$ no loop

$$+ \sum_{\gamma,\alpha} \bar{\delta}_{\beta\gamma} \bar{\delta}_{\gamma\alpha} X_\gamma G_0 f_\alpha$$

$\Leftarrow$ one loop

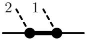
$$+ \sum_{\gamma,\kappa,\alpha} \bar{\delta}_{\beta\gamma} \bar{\delta}_{\gamma\kappa} \bar{\delta}_{\kappa\alpha} X_\gamma G_0 X_\kappa G_0 f_\alpha$$

$\Leftarrow$ two loops

$$+ \sum_\alpha N_{\beta\alpha} G_0 f_\alpha \cdots$$

appear only at 2-loop level — *Relief!*

where $f_1 =$ 

$f_2 =$ 

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$$= f_\beta$$ ⇐ no loop

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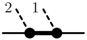
$$+ \sum_{\gamma,\kappa,\alpha} \bar{\delta}_{\beta\gamma} \bar{\delta}_{\gamma\kappa} \bar{\delta}_{\kappa\alpha} X_\gamma G_0 X_\kappa G_0 f_\alpha$$ ⇐ two loops

$$+ \sum_\alpha N_{\beta\alpha} G_0 f_\alpha \cdots$$

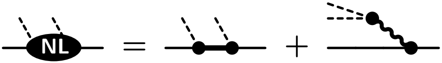
attach photon order by order

where $f_1 =$ 

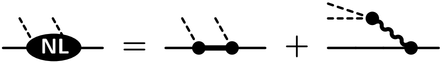
$f_2 =$ 

$f_3 =$ 

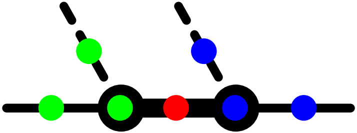
Attach Photon — No-loop Graphs



Attach Photon — No-loop Graphs



Example for attaching photon:



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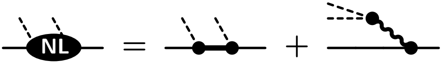

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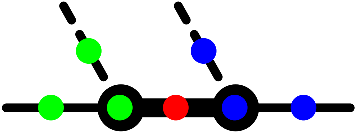
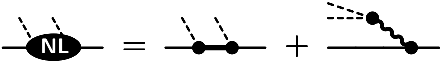
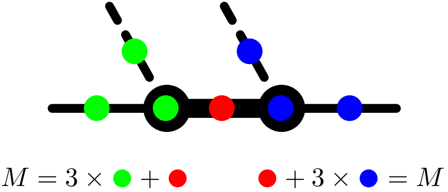


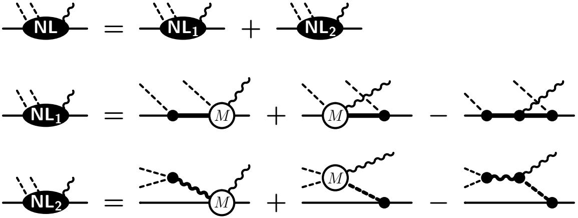
$M = 3 \times$ ● $+$ ●   ● $+ 3 \times$ ● $= M$

Attach Photon — No-loop Graphs



⇓ attach photon





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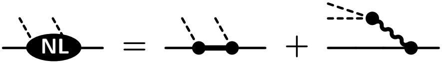
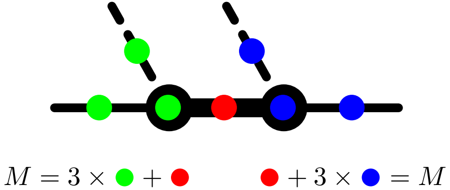
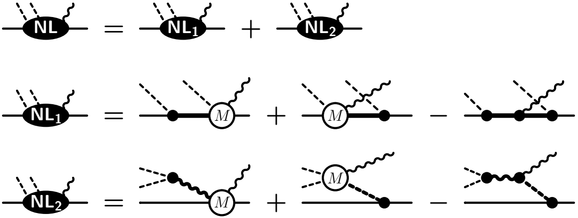


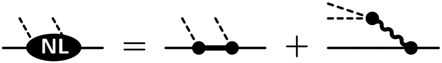
$\Downarrow$ attach photon





Subtraction necessary to avoid double-counting of

Attach Photon — No-loop Graphs



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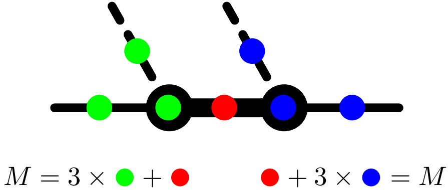
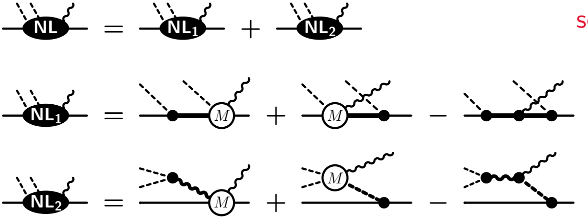
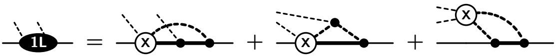




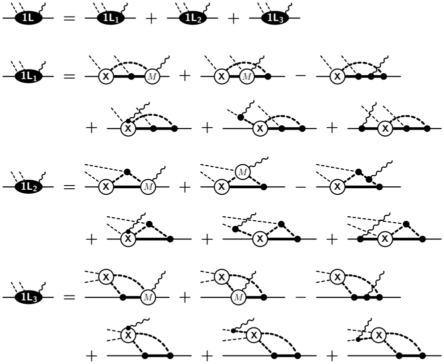
separately gauge invariant!

Local gauge invariance follows as a matter of course since all ingredients satisfy their respective off-shell WTIs ☺

Attach Photon — One-loop Graphs



⇓ attach photon



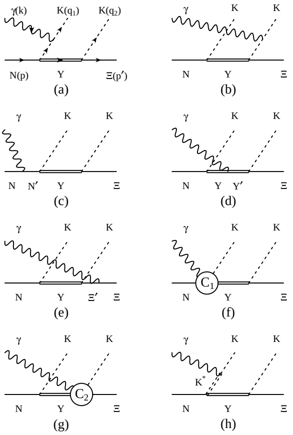
separately gauge invariant!

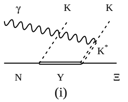
10 graphs for each group

In general, for n loops, there are $7 + 3n$ graphs in each group

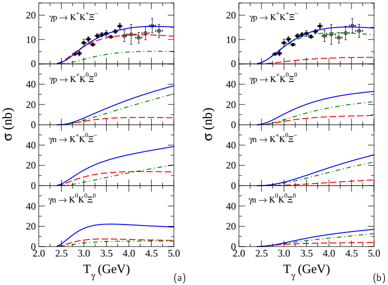
Application: $\gamma N \to KK\Xi$

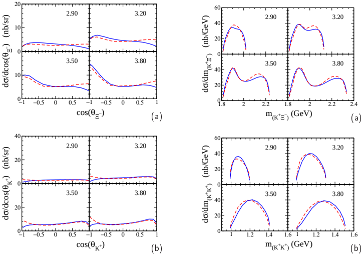
Y. Oh, K. Nakayama, HH, PRC**74**, 035205(2006)



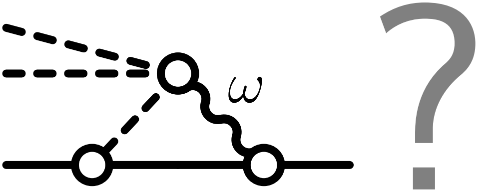


No-loop
approach

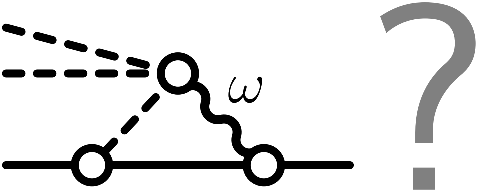




What about. . .



What about...



Can be done. Complicated. — I'll skip this...

Examples of Three-meson Production Current

Example: Ω baryon photoproduction process

$$\gamma p \to K^+ K^+ K^0 \Omega^-$$

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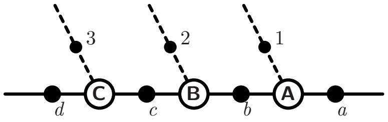
Generically, to model this process, start from underlying hadronic 3-meson'vertex':



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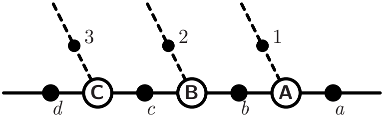

Next, attach photon in all ten topologically distinct places:

- three external meson lines 1, 2, 3
- two external and two internal baryon lines a, b, c, d
- three vertices A, B, C

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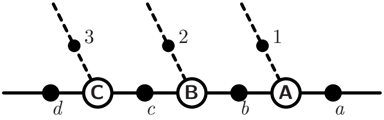

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The three interaction currents M^μ_A, M^μ_B, M^μ_C must be modeled to preserve overall gauge invariance

Examples of Three-meson Production Current

Example: Ω baryon photoproduction process

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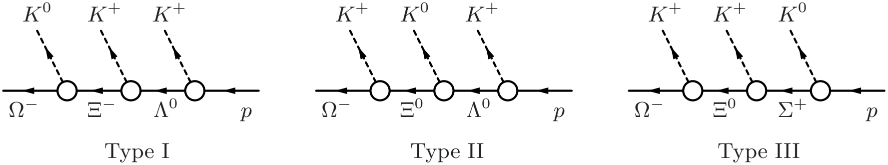
In detail, one needs to consider:



Calculation by Huiyoung Ryu shows basic cross section is in the picobarn range. Inclusion of resonance contributions can push this into the nanobarn range (preliminary).

Summary

- Formalism presented provides a complete description of multi-meson production processes. In principle, the formalism could be implemented to an arbitrary degree of sophistication for any given set of effective interaction Lagrangians.

- Full implementation of *local* gauge invariance order by order in terms of *Generalized Ward–Takahashi Identities* at all levels of the reaction dynamics.

  $\Longrightarrow$ Essential for the microscopic consistency of all reaction mechanisms.

- Valid for hadronic two- and three-point functions dressed by arbitrary internal mechanisms — even nonlinear ones.

- Extension to the production of any number of mesons straightforward.

- **Key steps for preserving local gauge invariance:**
  (1) All currents related to the same underlying hadronic mechanism **must** be treated together
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Thank you!