# Recent developments in covariant baryon chiral perturbation theory

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## ✤ Lattice QCD and Chiral Perturbation Theory (ChPT)

- ✓ A very brief introduction to lattice QCD
- ✓ ChPT in the one-baryon sector — the power counting breaking problem and its recovery

## ✤ One recent application of EOMS BChPT

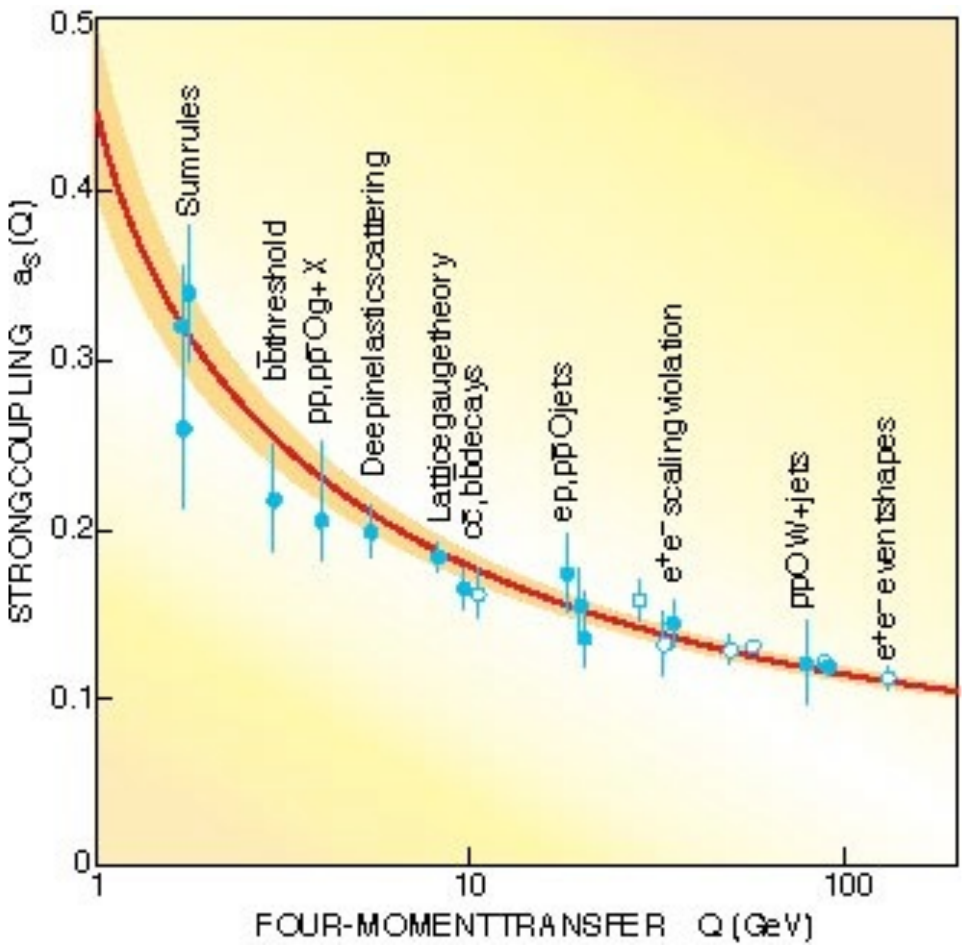
- ✓ Octet baryon masses
  - ➡ Chiral extrapolations
  - ➡ Finite volume corrections
  - ➡ Continuum extrapolations
- ✓ Octet baryon sigma terms

## ✤ Summary and Outlook

# A few words on LQCD and BChPT

# QCD—non-perturbative at low energies

✤ Quantum ChromoDynamics—the theory of the strong interaction



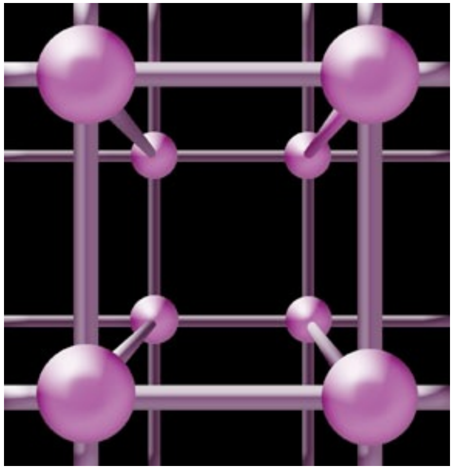

Asymptotic freedom—
Nobel prize in physics 2004

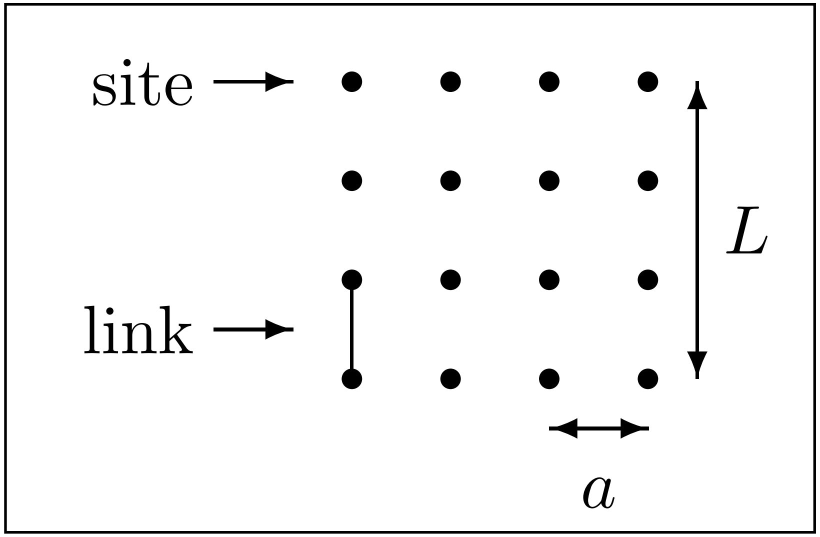
High energy: perturbative QCD successful

Low energy: non-perturbative problematic

- ✓ Models
- ✓ Effective field theories
- ✓ LQCD

# Brute Force: Lattice QCD






**Basic idea： discretize space-time and solve non-perturbative strong interaction physics in a finite hypercube, utilizing monte carlo sampling techniques**

# Calculating path-integral in Euclidean space-time

- **Vacuum**

$$Z = \int [DU] e^{-S_g(U) + \mathrm{Tr} \ln M[U]}$$

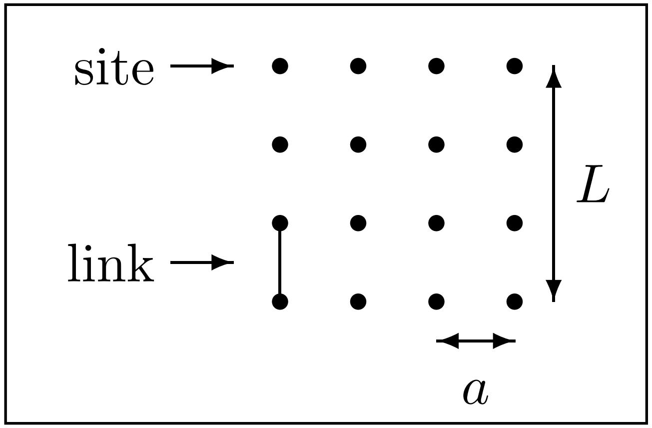
- **Observable**

$$\langle O \rangle = \int [DU] O(U) e^{-S_g(U) + \mathrm{Tr} \ln M[U]}$$

# Parameters and simulation costs

- light quark masses: $m_u/m_d$
- lattice spacing: $a$
- lattice volume: $V=L^4$

$$\text{cost} \propto \left(\frac{L}{a}\right)^4 \frac{1}{a}\frac{1}{m_\pi^2 a}$$




- To reduce cost: employ larger than physical light quark masses, finite lattice spacing and volume.
- To obtain physical quantities, multiple extrapolations are needed

# Multiple extrapolations

- **Chiral extrapolations:** light quark masses to their physical values

$$m_q \to m_q(\text{Phys.})$$

- **Finite volume corrections:** infinite space-time

$$L \to \infty$$

- **Continuum extrapolations:** zero lattice spacing

$$a \to 0$$

# Chiral Perturbation Theory

- **The low-energy effective field theory of QCD**
  - **provides a bridge to link LQCD simulations to the physical world**
  - **helps/guides to perform the aforementioned extrapolations**

# Interplay between ChPT and LQCD Simulations

- ☑ As the low-energy EFT of QCD, ChPT provides a model-independent description of low-energy strong interaction phenomena by itself
- ☑ At higher orders, which are needed to achieve accuracy at the few percent level, there might be too many unknown low-energy constants (LECs), which can not easily be determined by experimental data alone
- ☑ LQCD simulations provide a solution to overcome the above difficulty

# Chiral Perturbation Theory (ChPT) in essence

- **Maps quark (u, d, s) dof's to those of the asymptotic states, hadrons**

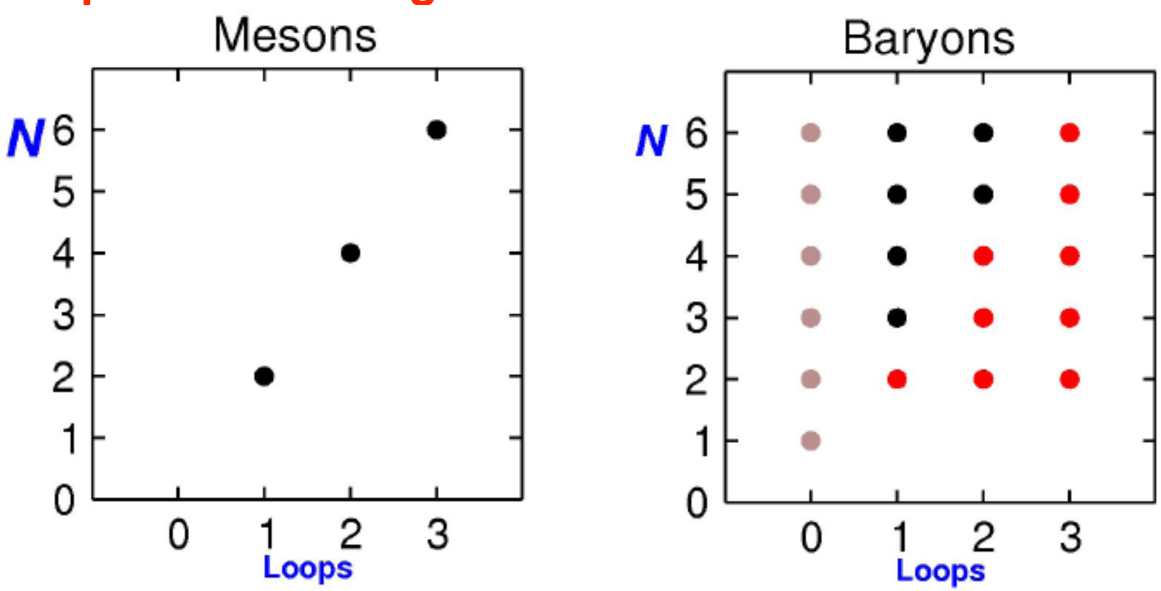
$$\mathcal{L}_{\rm QCD}[q,\bar{q};G] \to \mathcal{L}_{\rm ChPT}[U,\partial U,\ldots,\mathcal{M},N]$$

- $U$ parameterizes the Nambu-Goldstone bosons
- $\partial U$ vanishes at $E = \vec{p} = 0$ (Nambu-Goldstone theorem)
- $M$ parameterizes the explicit symmetry breaking
- $N$ denotes interactions with matter fields
- Exact mapping via chiral Ward identities

- **ChPT exploits the symmetry of the QCD Lagrangian and its ground state; in practice, one solves in a perturbative manner the constraints imposed by chiral symmetry and unitarity by expanding the Green functions in powers of the external momenta and of the quark masses. (J. Gasser, 2003)**

# Power-counting-breaking (PCB) in the one-baryon sector

- **ChPT very successful in the study of Nanbu-Goldstone boson self-interactions. (at least in SU(2))**
- **In the one-baryon sector, things become problematic because of the nonzero (large) baryon mass in the chiral limit, which leads to the fact that high-order loops contribute to lower-order results, i.e., a systematic power counting is lost!**




***red dots* denote possible PCB terms (pion-nucleon scattering)**

*J. Gasser et al., NPB 307, 779(1988)*

Chiral order = $4L - 2N_M - N_B + \sum_k kV_k$

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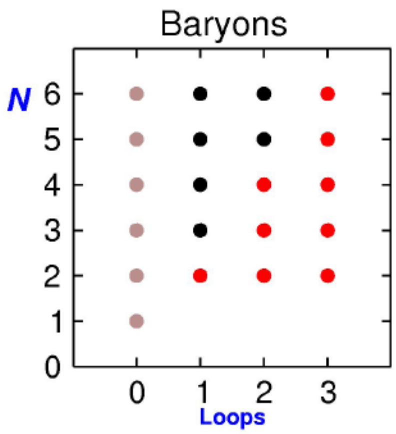
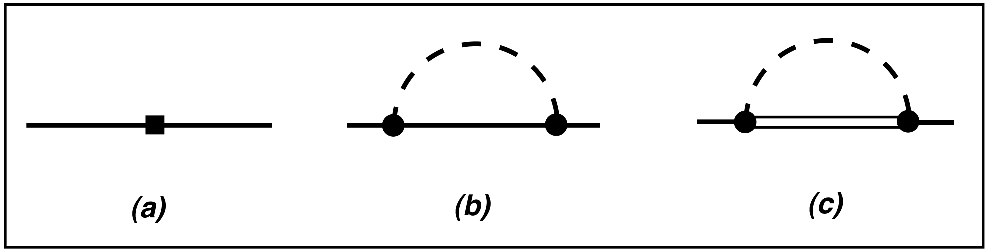
# Nucleon mass up to O(p³)

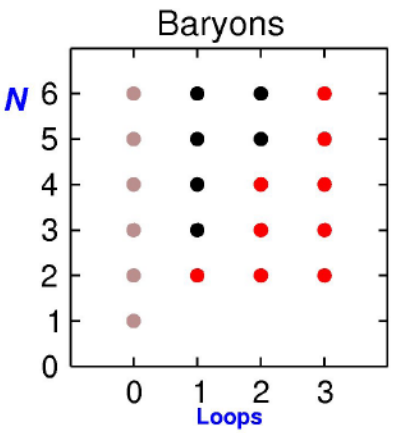


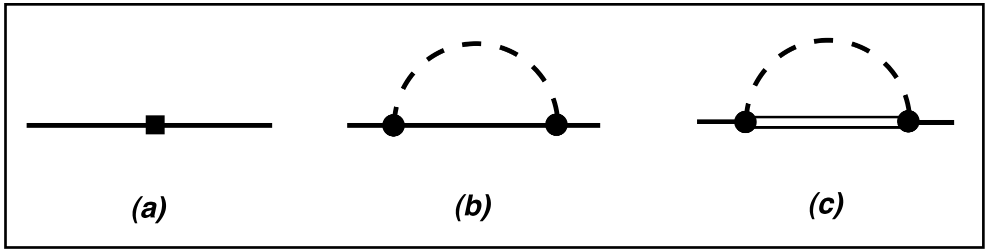




$\text{order of the loop} = 1 + 1 + 4 - 1 - 2 = 3$

Chiral order = $4L - 2N_M - N_B + \sum_k kV_k$.

# Nucleon mass up to $O(p^3)$




order of the loop $= 1 + 1 + 4 - 1 - 2 = 3$

**Naively (no PCB)**

$$M_N = M_0 + bm_\pi^2 + \text{loop}$$

$$\text{loop}(= cm_\pi^3 + \cdots)$$

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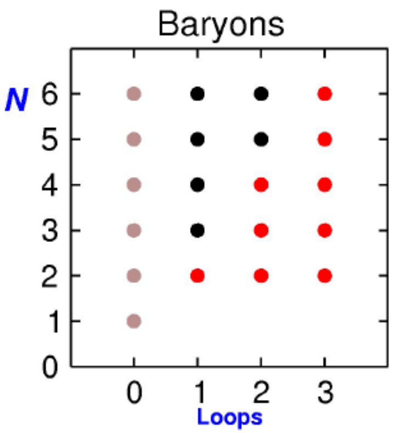


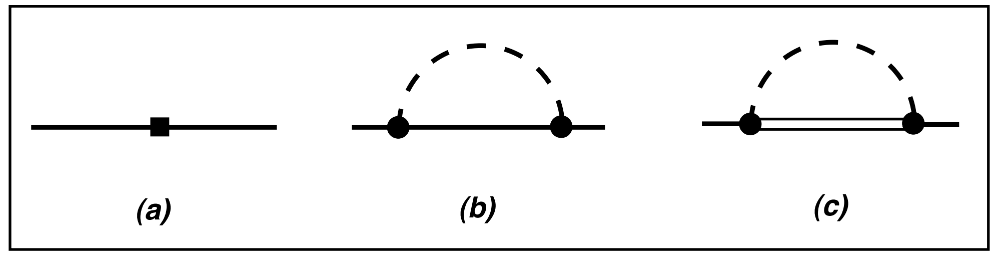


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**However** $$\text{loop} = aM_0^3 + b' M_0 m_\pi^2 + cm_\pi^3 + \cdots$$

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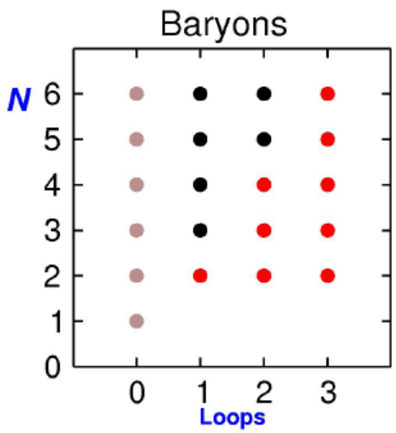
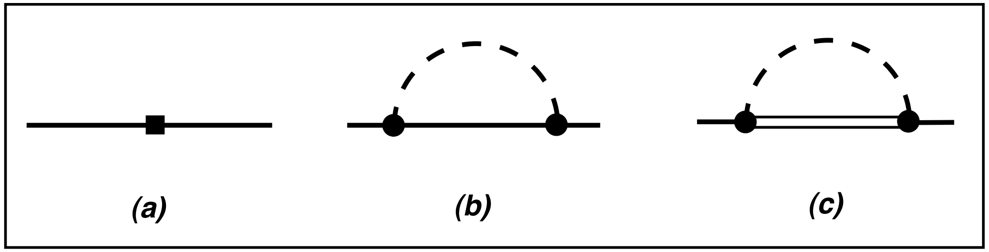



# Nucleon mass up to $O(p^3)$

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**However** $\text{loop} = aM_0^3 + b' M_0 m_\pi^2 + cm_\pi^3 + \cdots$

**No need to calculate, simply recall that *$M_0 \sim O(p^0)$***

# Power-counting-restoration methods

- **Heavy Baryon ChPT**: baryons are treated "semi-relativistically" by a simultaneous expansion in terms of external momenta and $1/M_N$ (*Jenkins et al., 1993*). It converges slowly for certain observables!

- **Relativistic baryon ChPT**: removing power counting breaking terms but retaining higher-order relativistic corrections, thus, keeping relativity.
  - Infrared baryon ChPT (*T. Becher and H. Leutwyler, 1999*)
  - Fully relativistic baryon ChPT–Extended On-Mass-Shell (EOMS) scheme (*J. Gegelia et al., 1999; T. Fuchs et al.,2003*)

- IR scheme separates the full integral into the Infrared and Regular parts:

$$H = \frac{1}{ab} = \int_0^1 dz \frac{1}{[(1-z)a + zb]^2} \equiv I + R = \int_0^\infty \ldots dz - \int_1^\infty \ldots dz$$

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$$H = \text{Infrared}$$

# Extended-on-Mass-Shell (EOMS)

- "Drop" the PCB terms

$$\text{tree} = M_0 + b m_\pi^2 \quad + \quad \text{loop} = a M_0^3 + b' M_0 m_\pi^2 + c m_\pi^3 + \cdots$$

$$\Downarrow \quad a = 0; b' = 0$$

$$M_N = M_0 + b\ m_\pi^2 + c m_\pi^3 + \cdots \quad (\mathcal{O}(p^3))$$

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$$\text{tree} = M_0 + b m_\pi^2 \quad + \quad \text{loop} = a M_0^3 + b' M_0 m_\pi^2 + c m_\pi^3 + \cdots$$

$$\Downarrow \quad M_0^r = M_0(1 + a M_0^2); b^r = b^0 + b' M_0$$

$$M_N = M_0^r + b^r m_\pi^2 + c m_\pi^3 + \cdots \quad (\mathcal{O}(p^3))$$

# Extended-on-Mass-Shell (EOMS)

- **"Drop" the PCB terms**

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**ChPT contains all possible terms allowed by symmetries, therefore whatever analytical terms come out from a loop amplitude, they must have a corresponding LEC**

# HB vs. Infrared vs. EOMS

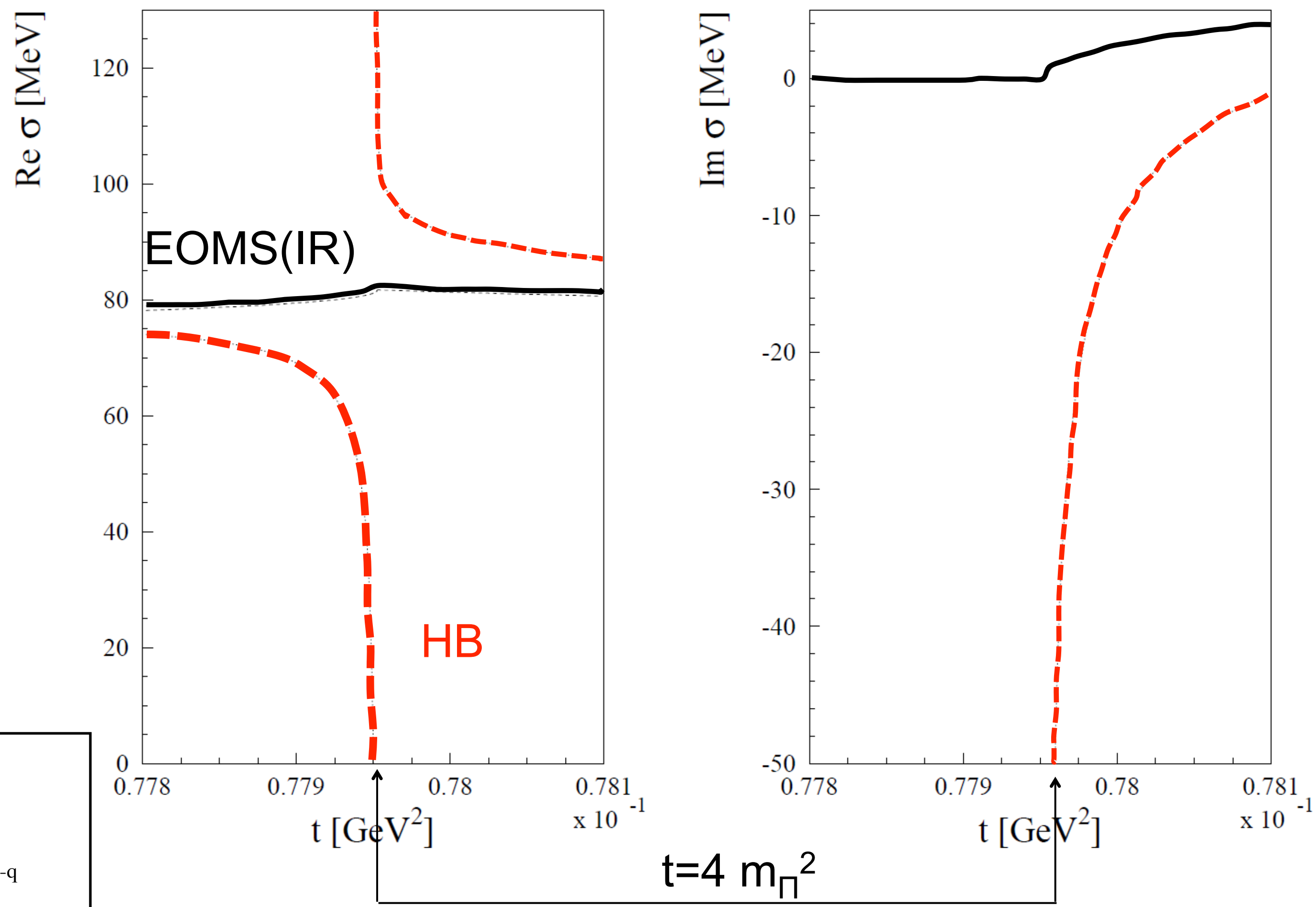
- **Heavy baryon (HB) ChPT**
  - non-relativistic
  - breaks analyticity of loop amplitudes
  - converges slowly (particularly in three-flavor sector)
  - strict PC and simple nonanalytical results
- **Infrared BChPT**
  - breaks analyticity of loop amplitudes
  - converges slowly (particularly in three-flavor sector)
  - analytical terms the same as HBChPT
- **Extended-on-mass-shell (EOMS) BChPT**
  - satisfies all symmetry and analyticity constraints
  - converges relatively faster--an appealing feature

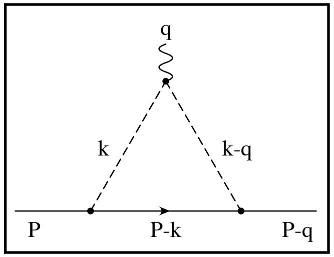
# The nucleon scalar form factor at $q^3$

t

$$\langle p(p',s')|\mathcal{H}_{\rm sb}(0)|p(p,s)\rangle = \bar{u}(p',s')u(p,s)\sigma(t), \quad t=(p'-p)^2$$

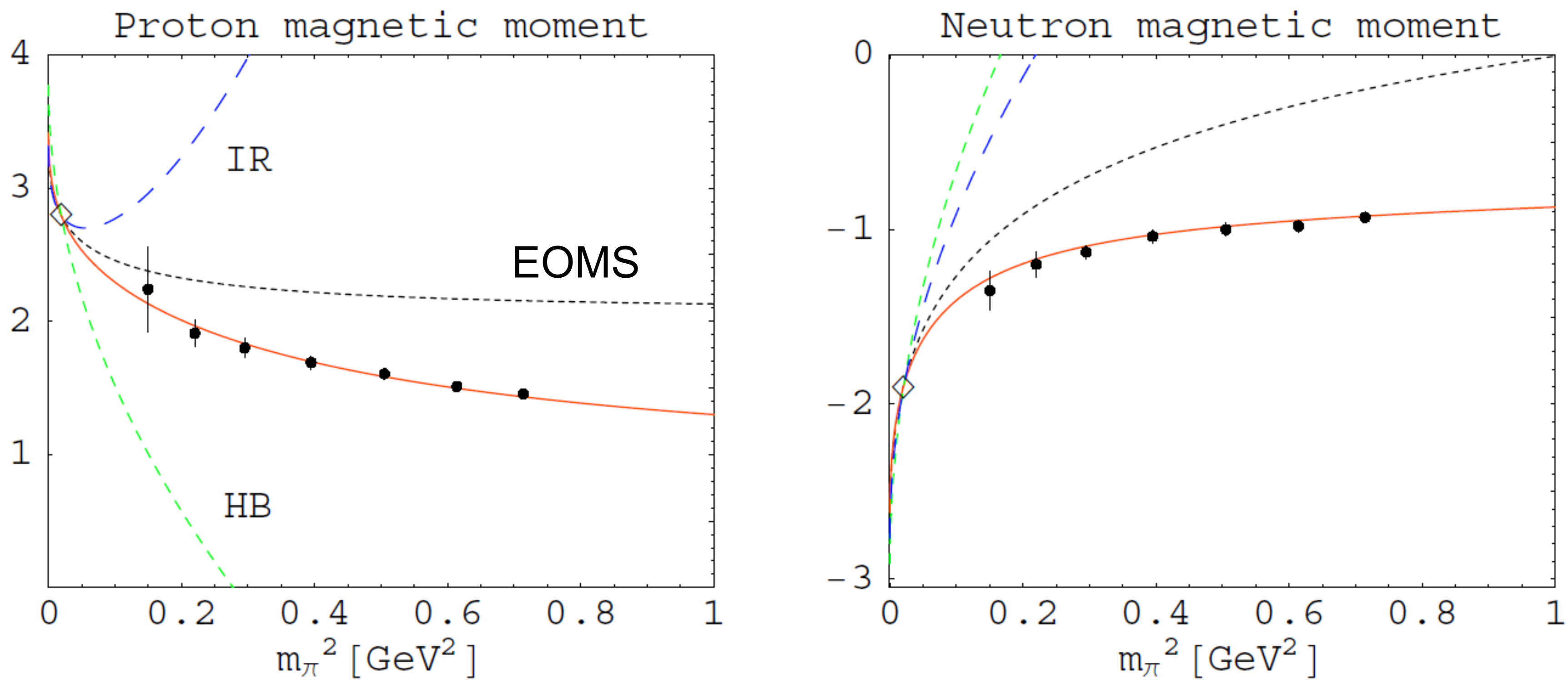
$$\mathcal{H}_{\rm sb} = \hat{m}(\bar{u}u + \bar{d}d)$$




S. Scherer, Prog.Part.Nucl.Phys.64:1-60,2010

# Proton and neutron magnetic moments: chiral extrapolation




V. Pascalutsa et al., Phys.Lett.B600:239-247,2004.

# Octet baryon magnetic moments at NLO BChPT

$$\chi^2 = \sum(\mu_{th} - \mu_{exp})^2$$

| | $p$ | $n$ | $\Lambda$ | $\Sigma^-$ | $\Sigma^0$ | $\Sigma^+$ | $\Xi^-$ | $\Xi^0$ | $\Lambda\Sigma^0$ | $\chi^2$ |
|---|---|---|---|---|---|---|---|---|---|---|
| C-G | 2.56 | -1.60 | -0.80 | -0.97 | 0.80 | 2.56 | -0.97 | -1.60 | 1.38 | **0.46** |
| HB | 3.01 | -2.62 | -0.42 | -1.35 | 0.42 | 2.18 | -0.52 | -0.70 | 1.68 | **1.01** |
| IR | 2.08 | -2.74 | -0.64 | -1.13 | 0.64 | 2.41 | -1.17 | -1.45 | 1.89 | **1.83** |
| EOMS | 2.58 | -2.10 | -0.66 | -1.10 | 0.66 | 2.43 | -0.95 | -1.27 | 1.58 | **0.18** |
| Exp. | 2.79 | -1.91 | -0.61 | -1.16 | — | 2.46 | -0.65 | -1.25 | 1.61 | — |

- Contribution of the chiral series [LO(1+NLO/LO)]:

$$\mu_p = 3.47(1-0.257), \quad \mu_n = -2.55(1-0.175), \quad \mu_\Lambda = -1.27(1-0.482),$$

$$\mu_{\Sigma^-} = -0.93(1+0.187), \quad \mu_{\Sigma^+} = 3.47(1-0.300), \quad \mu_{\Sigma^0} = 1.27(1-0.482),$$

$$\mu_{\Xi^-} = -0.93(1+0.025), \quad \mu_{\Xi^0} = -2.55(1-0.501), \quad \mu_{\Lambda\Sigma^0} = 2.21(1-0.284).$$

LSG, J. Martin Camalich , L. Alvarez-Ruso, M.J. Vicente Vacas, Phys.Rev.Lett. 101:222002,2008

# Some successful applications of covariant BChPT (in the three-flavor sector)

- **Octet (decuplet) baryon magnetic moments:**

  Phys.Rev.Lett.101:222002,2008; Phys.Lett.B676:63-68,2009; Phys.Rev.D80:034027,2009

- **Octet and Decuplet baryon masses**

  Phys.Rev.D82:074504,2010; Phys.Rev.D84:074024,2011; JHEP12(2012)073; Phys.Rev.D D87:074001 (2013); Phys.Rev. D89:054034,2014 ; Eur.Phys.J. C74:2754,2014

- **Hyperon vector coupling $f_1(0)$**

  Phys.Rev.D79:094022,2009;arXiv:1402.7133

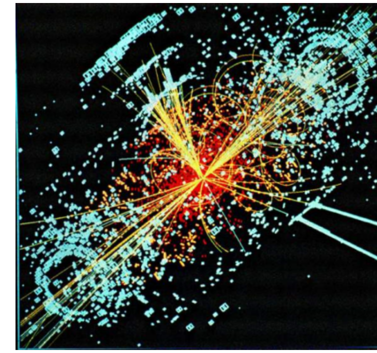
- **Nucleon-Delta axial coupling**

  Phys.Rev.D78:014011,2008

# Lattice QCD light-hadron spectrum

*Phys.Rev.D82:074504,2010;*
*Phys.Rev.D84:074024,2011;*
*JHEP12(2012)073;*
*Phys.Rev.D D87:074001 (2013)*
*Eur.Phys.J. C74:2754,2014*

# Origin of nucleon(baryon) masses



1) **Mass of its constituents—quarks**

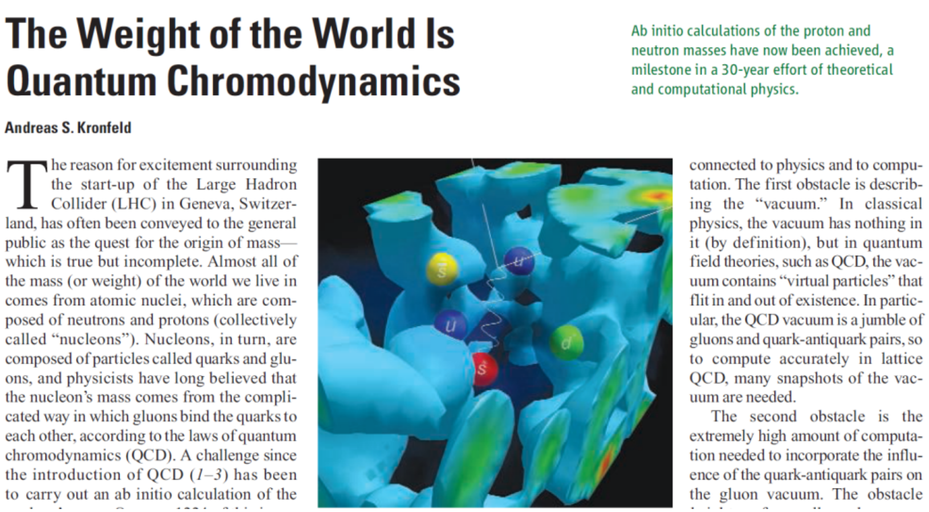
**In SM, due to the Higgs mechanism → LHC@CERN**

Nobel prize 2013

2)**Strong interaction—lattice QCD**

LHC@CERN

**mass of proton (940 MeV) ≠ sum of current quark masses (~10 MeV).**



# The Weight of the World Is Quantum Chromodynamics

Ab initio calculations of the proton and neutron masses have now been achieved, a milestone in a 30-year effort of theoretical and computational physics.

Andreas S. Kronfeld

The reason for excitement surrounding the start-up of the Large Hadron Collider (LHC) in Geneva, Switzerland, has often been conveyed to the general public as the quest for the origin of mass—which is true but incomplete. Almost all of the mass (or weight) of the world we live in comes from atomic nuclei, which are composed of neutrons and protons (collectively called "nucleons"). Nucleons, in turn, are composed of particles called quarks and gluons, and physicists have long believed that the nucleon's mass comes from the complicated way in which gluons bind the quarks to each other, according to the laws of quantum chromodynamics (QCD). A challenge since the introduction of QCD (1–3) has been to carry out an ab initio calculation of the

connected to physics and to computation. The first obstacle is describing the "vacuum." In classical physics, the vacuum has nothing in it (by definition), but in quantum field theories, such as QCD, the vacuum contains "virtual particles" that flit in and out of existence. In particular, the QCD vacuum is a jumble of gluons and quark-antiquark pairs, so to compute accurately in lattice QCD, many snapshots of the vacuum are needed.

The second obstacle is the extremely high amount of computation needed to incorporate the influence of the quark-antiquark pairs on the gluon vacuum. The obstacle

*Budapest-Marseille-Wuppertal Collaboration*

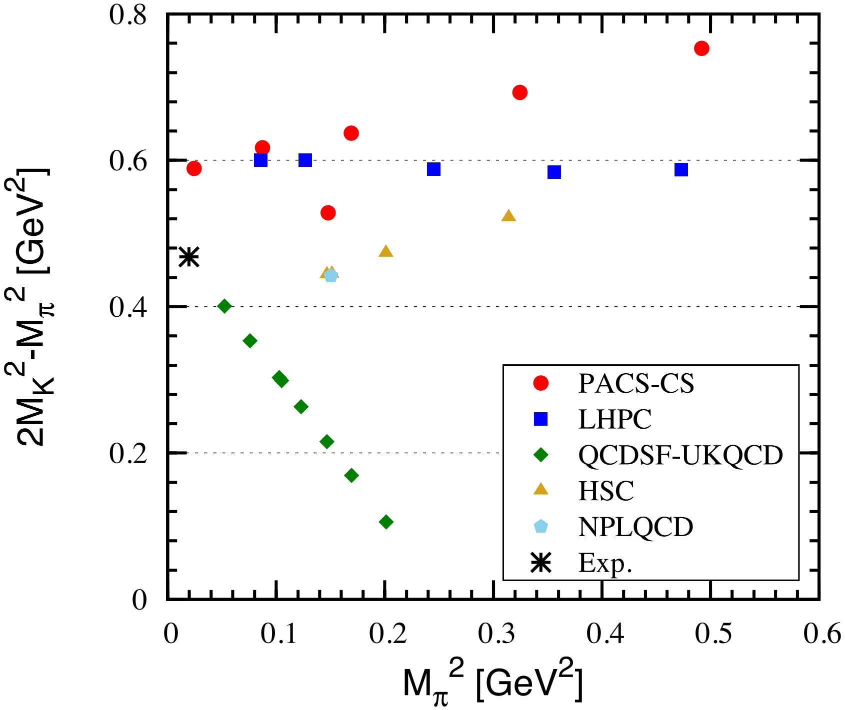
**BMW**

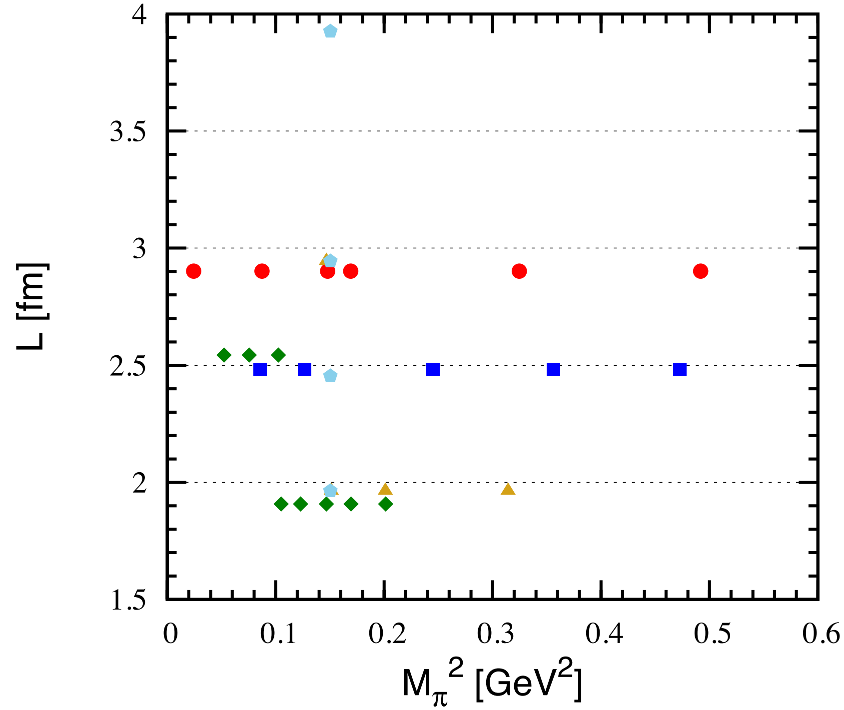

ongoing projects on Blue Gene/P,
total sustained performance for QCD:
Jülich Supercomputing Centre: 82.5 Teraflops,
IDRIS/CNRS: 51,5 Teraflops

CPU and GPU clusters,
Bergische Universität Wuppertal
and at CNRS Marseille
31 Teraflops (sustained for QCD)

S. Durr et al., Science 322, 1224(2008)

# landscape of latest 2+1 f LQCD simulations of g.s. octet baryon masses




**To obtain g.s. baryon masses in the physical world**

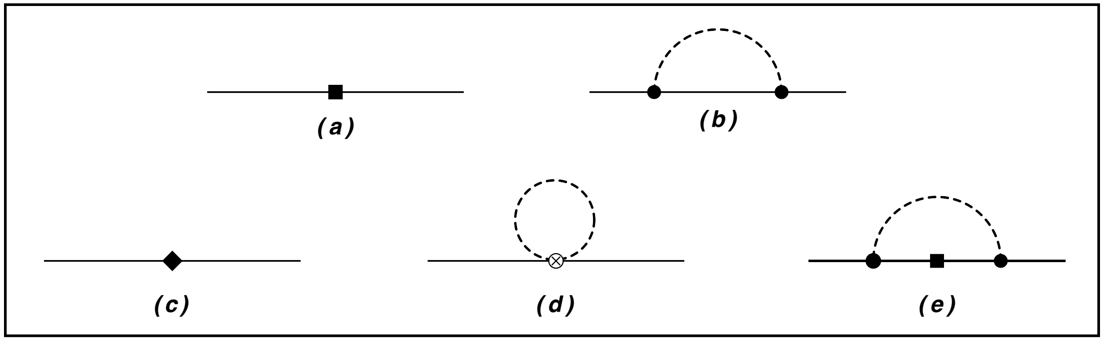
- Extrapolate to the continuum: $a \to 0$
- Extrapolate to physical light quark masses: $m_q \to m_q(\text{Phys.})$
- Extrapolate to infinite space-time: $L \to \infty$

# Many Studies in BChPT: HB, IR, EOMS

- NNLO HBChPT - failed to describe the lattice data
  - LHPC (A. Walker-Loud et al.), PRD79:054502, 2009
  - PACS-CS (K.-I. Ishikawa), PRD80:054502, 2009.
- NNLO EOMS BChPT - improved description of the LHP and PACS-CS data, particularly, in comparison with HBChPT
  - J. Martin-Gamalich, LSG, et al., PRD80:054502, 2009.
- ✓ $N^3LO$ EOMS BChPT - the first global study of all the publicly available LQCD data X.-L. Ren, LSG, et al., JHEP12(2012)073, PRD87, 074001 (2013)
- Studies based on other alternative formulations of BChPT:
  - NNLO finite-range-regularized HB ChPT — nice description of the PACS-CS and LHPC data—R.D. Young and A. W. Thomas, PRD 81:014503 (2010)
  - $N^3LO$ partial summation BChPT — nice description of the BMW, PACS-CS, and UKQCD data—A. Semeke and M.F.M Lutz, PRD 85:034001(2012)
  - $N^3LO$ infrared BChPT — Peter C. Bruns, Ludwig Greil, and Andreas Schäfer, PRD 87: 052005(2012)

# Diagrams and Lagrangians

- **Diagrams (up to $N^3LO$):**




- **Lagrangians at NNLO (3 LECs)—tree**

$$\mathcal{L}_{\phi B}^{(2,\mathrm{sb})} = b_0\langle\chi_+\rangle\langle B\bar{B}\rangle + b_{D/F}\langle\bar{B}[\chi_+, B]_\pm\rangle,$$

# Diagrams and Lagrangians

- **Lagrangians at NNLO (8 LECs)—tadpole**

$$
\begin{aligned}
\mathcal{L}_{\phi B}^{(2)\prime} = {} & b_1\langle \bar{B}[u_\mu,[u^\mu,B]]\rangle + b_2\langle \bar{B}\{u_\mu,\{u^\mu,B\}\}\rangle \\
& + b_3\langle \bar{B}\{u_\mu,[u^\mu,B]\}\rangle + b_4\langle \bar{B}B\rangle\langle u^\mu u_\mu\rangle \\
& + ib_5\left(\langle \bar{B}[u^\mu,[u^\nu,\gamma_\mu D_\nu B]]\rangle - \langle \bar{B}\overleftarrow{D}_\nu[u^\nu,[u^\mu,\gamma_\mu B]]\right) \\
& + ib_6\left(\langle \bar{B}[u^\mu,\{u^\nu,\gamma_\mu D_\nu B\}]\rangle - \langle \bar{B}\overleftarrow{D}_\nu\{u^\nu,[u^\mu,\gamma_\mu B]\}\right) \\
& + ib_7\left(\langle \bar{B}\{u^\mu,\{u^\nu,\gamma_\mu D_\nu B\}\}\rangle - \langle \bar{B}\overleftarrow{D}_\nu\{u^\nu,\{u^\mu,\gamma_\mu B\}\}\rangle\right) \\
& + ib_8\left(\langle \bar{B}\gamma_\mu D_\nu B\rangle - \langle \bar{B}\overleftarrow{D}_\nu\gamma_\mu B\rangle\right)\langle u^\mu u^\nu\rangle + \cdots
\end{aligned}
$$

- **Lagrangians at N$^3$LO (7 LECs)—tree**

$$
\begin{aligned}
\mathcal{L}_{\phi B}^{(4)} = {} & d_1\langle \bar{B}[\chi_+,[\chi_+,B]]\rangle + d_2\langle \bar{B}[\chi_+,\{\chi_+,B\}]\rangle \\
& + d_3\langle \bar{B}\{\chi_+,\{\chi_+,B\}\}\rangle + d_4\langle \bar{B}\chi_+\rangle\langle \chi_+ B\rangle \\
& + d_5\langle \bar{B}[\chi_+,B]\rangle\langle \chi_+\rangle + d_7\langle \bar{B}B\rangle\langle \chi_+\rangle^2 \\
& + d_8\langle \bar{B}B\rangle\langle \chi_+^2\rangle.
\end{aligned}
$$

# Analytical results and PCB

$$m_B = m_0 + m_B^{(2)} + m_B^{(3)} + m_B^{(4)}.$$

$$m_B^{(2)} = \sum_{\phi=\pi,\ K} \xi_{B,\phi}^{(a)} M_\phi^2.$$

$$m_B^{(3)} = \frac{1}{(4\pi F_0)^2} \sum_{\phi=\pi,\ K,\ \eta} \xi_{B,\phi}^{(b)} H_B^{(b)}(M_\phi)$$

$$\begin{aligned} m_B^{(4)} = {} & \xi_{B,\pi}^{(c)} M_\pi^4 + \xi_{B,K}^{(c)} M_K^4 + \xi_{B,\pi K}^{(c)} M_\pi^2 M_K^2 \\ & + \frac{1}{(4\pi^2 F_\phi)^2} \sum_{\phi=\pi,K,\eta} \left[ \xi_{B,\phi}^{(d,1)} H_B^{(d,1)}(M_\phi) + \xi_{B,\phi}^{(d,2)} H_B^{(d,2)}(M_\phi) + \xi_{B,\phi}^{(d,3)} H_B^{(d,3)}(M_\phi) \right] \\ & + \frac{1}{(4\pi^2 F_\phi)^2} \sum_{\substack{\phi=\pi,K,\eta \\ B'=N,\Lambda,\Sigma,\Xi}} \xi_{BB',\phi}^{(e)} \cdot H_{B,B'}^{(e)}(M_\phi). \end{aligned}$$

# Analytical results and PCB

$$m_B = m_0 + m_B^{(2)} + m_B^{(3)} + m_B^{(4)}.$$

$$m_B^{(2)} = \sum_{\phi=\pi,\ K} \xi_{B,\phi}^{(a)} M_\phi^2.$$

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$$\begin{aligned} m_B^{(4)} = & \ \xi_{B,\pi}^{(c)} M_\pi^4 + \xi_{B,K}^{(c)} M_K^4 + \xi_{B,\pi K}^{(c)} M_\pi^2 M_K^2 \\ & + \frac{1}{(4\pi^2 F_\phi)^2} \sum_{\phi=\pi,K,\eta} \left[ \xi_{B,\phi}^{(d,1)} H_B^{(d,1)}(M_\phi) + \xi_{B,\phi}^{(d,2)} H_B^{(d,2)}(M_\phi) + \xi_{B,\phi}^{(d,3)} H_B^{(d,3)}(M_\phi) \right] \\ & + \frac{1}{(4\pi^2 F_\phi)^2} \sum_{\substack{\phi=\pi,K,\eta \\ B'=N,\Lambda,\Sigma,\Xi}} \xi_{BB',\phi}^{(e)} \cdot H_{B,B'}^{(e)}(M_\phi). \end{aligned}$$

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$$\begin{aligned} m_B^{(4)} = &\ \xi_{B,\pi}^{(c)} M_\pi^4 + \xi_{B,K}^{(c)} M_K^4 + \xi_{B,\pi K}^{(c)} M_\pi^2 M_K^2 \\ &+ \frac{1}{(4\pi^2 F_\phi)^2} \sum_{\phi=\pi,K,\eta} \left[ \xi_{B,\phi}^{(d,1)} H_B^{(d,1)}(M_\phi) + \xi_{B,\phi}^{(d,2)} H_B^{(d,2)}(M_\phi) + \xi_{B,\phi}^{(d,3)} H_B^{(d,3)}(M_\phi) \right] \\ &+ \frac{1}{(4\pi^2 F_\phi)^2} \sum_{\substack{\phi=\pi,K,\eta \\ B'=N,\Lambda,\Sigma,\Xi}} \xi_{BB',\phi}^{(e)} \cdot H_{B,B'}^{(e)}(M_\phi). \end{aligned}$$

# Analytical results and PCB

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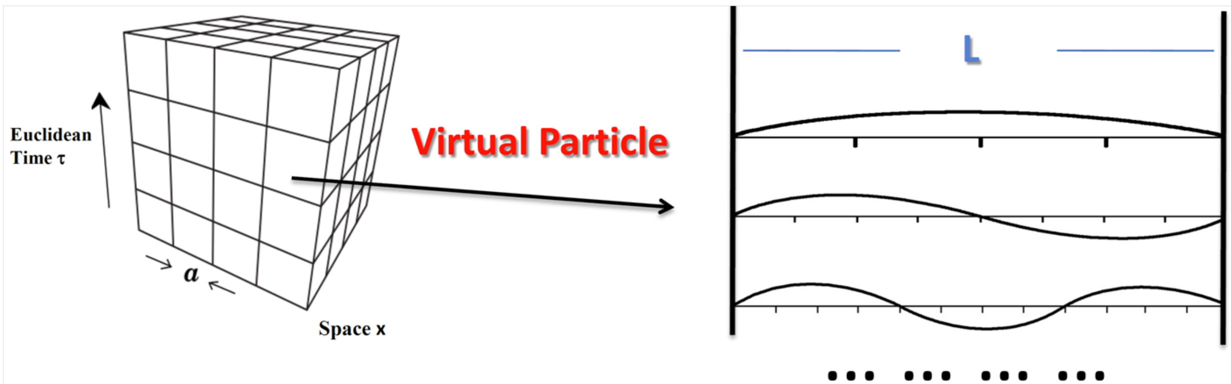
$$\begin{aligned} m_B^{(4)} = & \ \xi_{B,\pi}^{(c)} M_\pi^4 + \xi_{B,K}^{(c)} M_K^4 + \xi_{B,\pi K}^{(c)} M_\pi^2 M_K^2 \\ & + \frac{1}{(4\pi^2 F_\phi)^2} \sum_{\phi=\pi,K,\eta} \left[ \xi_{B,\phi}^{(d,1)} H_B^{(d,1)}(M_\phi) + \xi_{B,\phi}^{(d,2)} H_B^{(d,2)}(M_\phi) + \xi_{B,\phi}^{(d,3)} H_B^{(d,3)}(M_\phi) \right] \\ & + \frac{1}{(4\pi^2 F_\phi)^2} \sum_{\substack{\phi=\pi,K,\eta \\ B'=N,\Lambda,\Sigma,\Xi}} \xi_{BB',\phi}^{(e)} \cdot H_{B,B'}^{(e)}(M_\phi). \end{aligned}$$

Power-counting-breaking (PCB) terms removed by the extended-on-mass-shell (EOMS) scheme

# Results in a finite box -finite volume corrections

- **Physical origin: existence of boundary conditions**




- **Momenta of virtual particles are discretized**

$$k_i = 2\pi \frac{n_i}{L},\ (i = 0, 1, 2, 3)$$

$$\int_{-\infty}^{\infty} dk \Rightarrow \sum_{n=-\infty}^{\infty} \left(\frac{2\pi}{L}\right) \cdot n.$$

LSG, Xiu-Lei Ren, Jorge Martin-Camalich, Wolfram Weise, Phys. Rev. D 84:074024,2011

# Low energy constants (LECs)

- Unknown—to be fitted (19)
  - $m_0$,
  - $b_0$, $b_D$, $b_F$, $b_1$, $b_2$, $b_3$, $b_4$, $b_5$,$b_6$,$b_7$,$b_8$
  - $d_1$,$d_2$,$d_3$,$d_4$,$d_5$,$d_7$,$d_8$
- Reasonably well-known
  - $f_0$=0.0871 GeV
  - D=0.46
  - F=0.8
  - μ=1 GeV

Only 4 data points at the physical point! LQCD simulations needed!

# Selection of the lattice data

- **$N^3$LO BChPT cannot describe all the lattice data with arbitrarily large light quark mass / small volumes**

- **Two criteria: light quark masses/NGB masses ($m_M$) and $m_M L$**

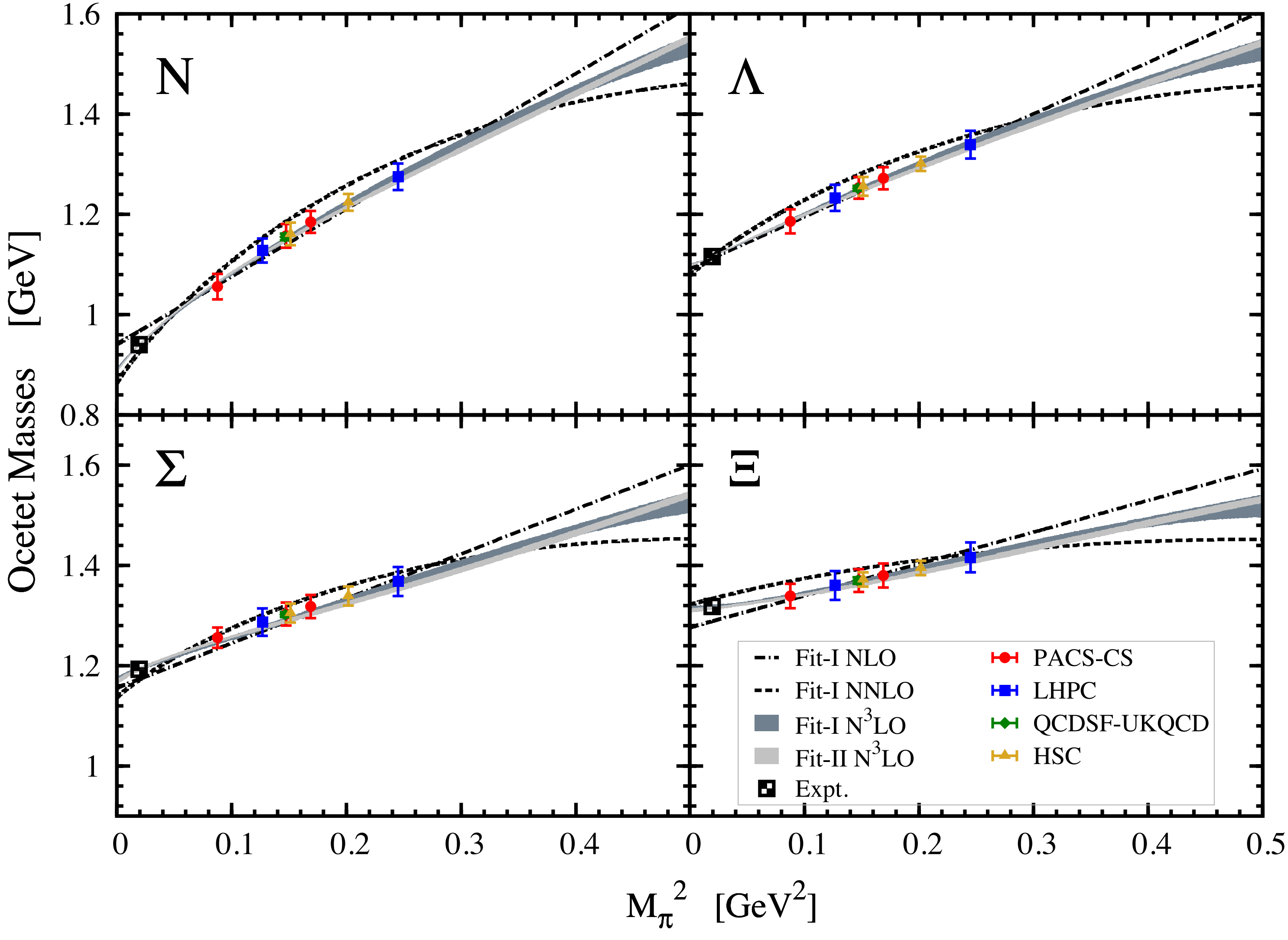
$$m_\pi^2 < 0.25\mathrm{GeV}^2$$

$$m_\phi L > 4 (\phi = \pi, \eta, K)$$

**11 sets of data (44 points) from five collaborations:**

**LHPC, PACS-CS, QCDSF-UKQCD, HSC, and NPLQCD**

# Results: physical data included in the fits

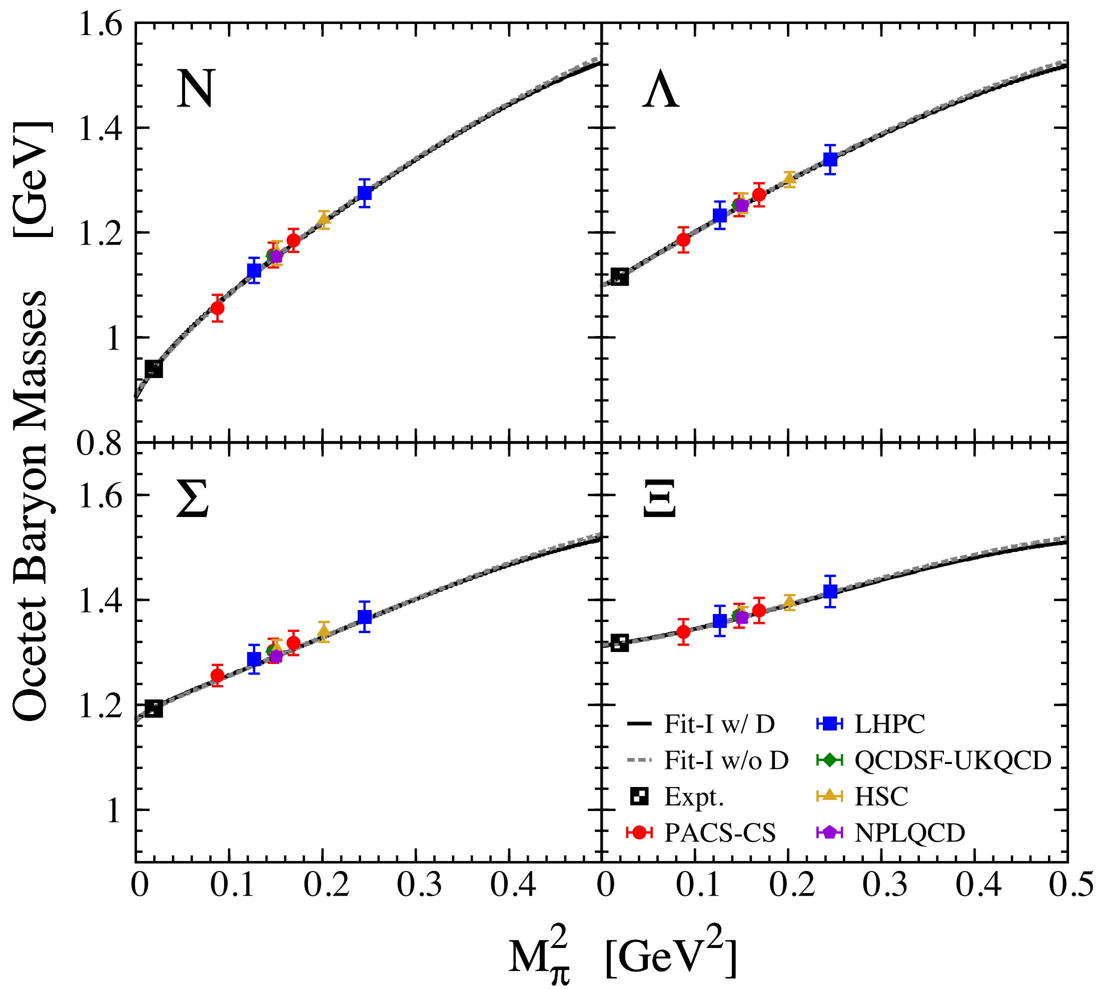


# Results: physical data included in the fits

| | Fit - $\mathcal{O}(p^2)$ | Fit - $\mathcal{O}(p^3)$ | Fit I - $\mathcal{O}(p^4)$ |
|---|---|---|---|
| $m_0$ [MeV] | 900(6) | 767(6) | 880(22) |
| $b_0$ [GeV$^{-1}$] | $-0.273(6)$ | $-0.886(5)$ | $-0.609(19)$ |
| $b_D$ [GeV$^{-1}$] | 0.0506(17) | 0.0482(17) | 0.225(34) |
| $b_F$ [GeV$^{-1}$] | $-0.179(1)$ | $-0.514(1)$ | $-0.404(27)$ |
| $b_1$ [GeV$^{-1}$] | – | – | 0.550(44) |
| $b_2$ [GeV$^{-1}$] | – | – | $-0.706(99)$ |
| $b_3$ [GeV$^{-1}$] | – | – | $-0.674(115)$ |
| $b_4$ [GeV$^{-1}$] | – | – | $-0.843(81)$ |
| $b_5$ [GeV$^{-2}$] | – | – | $-0.555(144)$ |
| $b_6$ [GeV$^{-2}$] | – | – | 0.160(95) |
| $b_7$ [GeV$^{-2}$] | – | – | 1.98(18) |
| $b_8$ [GeV$^{-2}$] | – | – | 0.473(65) |
| $d_1$ [GeV$^{-3}$] | – | – | 0.0340(143) |
| $d_2$ [GeV$^{-3}$] | – | – | 0.296(53) |
| $d_3$ [GeV$^{-3}$] | – | – | 0.0431(304) |
| $d_4$ [GeV$^{-3}$] | – | – | 0.234(67) |
| $d_5$ [GeV$^{-3}$] | – | – | $-0.328(60)$ |
| $d_7$ [GeV$^{-3}$] | – | – | $-0.0358(269)$ |
| $d_8$ [GeV$^{-3}$] | – | – | $-0.107(32)$ |
| $\chi^2$/d.o.f. | 11.8 | 8.6 | 1.0 |

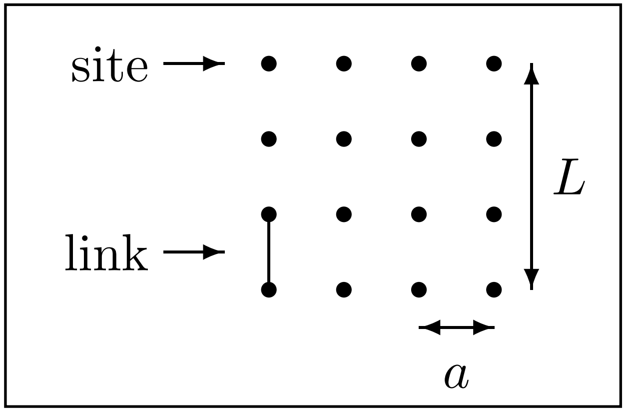
- **$O(p^4)$ is much better than $O(p^3)$ and $O(p^2)$ fit**
- **All LECs look natural and consistent with each other**
- **Neglecting Finite-Volume-Corrections would lead to** $\chi^2_{d.o.f} = 1.9$

# Effects of virtual decuplet baryons small




*Xiu-Lei Ren, LSG, Jie Meng, Hiroshi Toki, Phys.Rev.D D87:074001 (2013)*

# Continuum extrapolation/ discretization effects




- In principle, all the aforementioned studies of the LQCD simulations should be performed after they have been extrapolated to the continuum, since ChPT refers to continuum QCD
- At the lattice spacing of the order of 0.1 fm discretization effects are usually assumed to be small
- Nevertheless, explicit studies are still missing

Xiu-Lei, LSG, Jie Meng, Eur.Phys.J. C74 (2014) 2754

# ChPT with Wilson fermions

- **Close to the continuum limit, LQCD can be described by the Symanzik action**

$$
\begin{aligned}
S_{\rm eff} &= S_0 + aS_1 + a^2 S_2 + \cdots \\
&= \int d^4x (\mathcal{L}^{(4)} + a\mathcal{L}^{(5)} + a^2\mathcal{L}^{(6)} + \cdots),
\end{aligned}
$$

- **To take into account discretization effects, one can then construct ChPT in accordance with the Symanzik effective field theory, instead of the continuum QCD**

# Hierarchy of Energy Scales and Power Counting

- **In LQCD simulations, the following hierarchy of energy scales is satisfied**

$$m_q \ll \Lambda_{\mathrm{QCD}} \ll \frac{1}{a}.$$

- **If one assumes that the size of chiral symmetry breaking due to finite light quark masses and the discretization effects are of comparable size**

$$p^2 \sim \frac{m_q}{\Lambda_{\mathrm{QCD}}} \sim a\Lambda_{\mathrm{QCD}},$$

# Lattice spacing dependent chiral Lagrangians

- **Chiral Lagrangians up to O(a²) and O(am$_q$)**

$$\mathcal{L}_a^{\rm eff} = \mathcal{L}_a^{(1)} + \mathcal{L}_a^{(2)},$$

$$\mathcal{L}_a^{(1)} = \mathcal{L}^{\mathcal{O}(a)} + \mathcal{L}^{\mathcal{O}(am_q)},$$

$$\mathcal{L}_a^{(2)} = \mathcal{L}_1^{\mathcal{O}(a^2)} + \mathcal{L}_2^{\mathcal{O}(a^2)} + \mathcal{L}_3^{\mathcal{O}(a^2)} + \mathcal{L}_4^{\mathcal{O}(a^2)} + \mathcal{L}_5^{\mathcal{O}(a^2)},$$

- **Re-calculate the following Feynman diagrams**

# LQCD simulations performed with Wilson fermions

- PACS-CS: a=0.0907 fm, $c_{SW}$=1.715
- QCDSF-UKQCD: a=0.0795fm, $c_{SW}$=2.65
- HSC and NPLQCD: $a^s$=0.1227, $a^t$=0.03506; $c^s_{SW}$=2.6, $c^t_{SW}$=1.8
- All the simulations are O(a) improved, meaning that discretization effects start at $O(am_q)$ or $O(a^2)$

$$m_B = m_0 + m_B^{(2)} + m_B^{(3)} + m_B^{(4)} + m_B^{(a)}$$

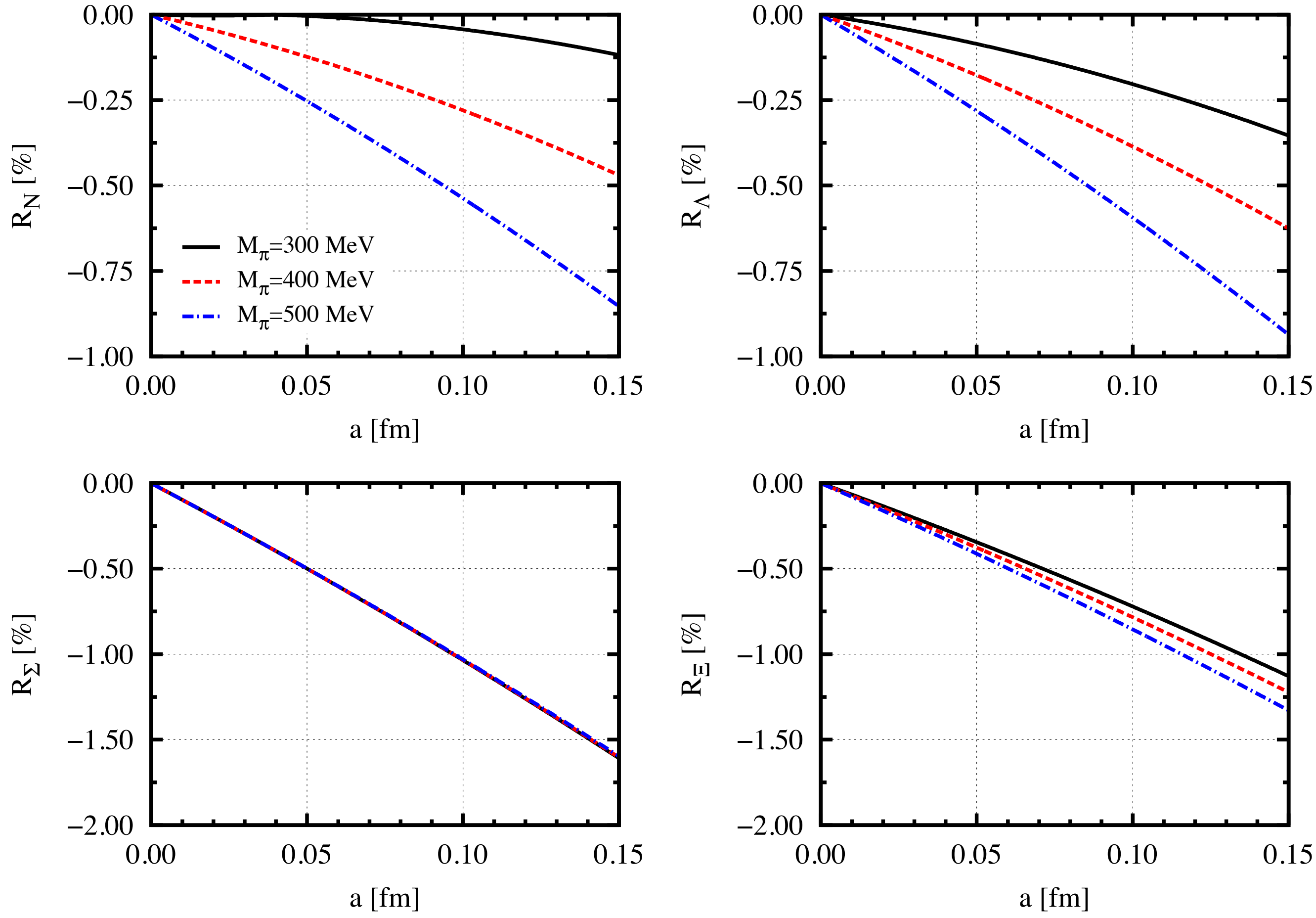
$$\begin{aligned} m_B^{(a)} &= m_B^{\mathcal{O}(am_q)} + m_B^{\mathcal{O}(a^2)} \\ &= -8ac_{\rm SW}W_0\left(\xi_l M_\pi^2 + \xi_s(2M_K^2 - M_\pi^2)\right) - 16a^2 c_{\rm SW}^2 W_0^2 \bar{X}, \end{aligned}$$

# Fits with and without discretization effects taken into account

| | BChPT | WBChPT | | BChPT | WBChPT |
|---|---|---|---|---|---|
| $m_0$ [MeV] | 910(20) | 915(20) | $d_1$ [GeV$^{-3}$] | 0.0295(124) | $-0.0196(121)$ |
| $b_0$ [GeV$^{-1}$] | $-0.579(56)$ | $-0.557(50)$ | $d_2$ [GeV$^{-3}$] | 0.342(65) | 0.230(58) |
| $b_D$ [GeV$^{-1}$] | 0.211(56) | 0.201(48) | $d_3$ [GeV$^{-3}$] | $-0.0314(63)$ | $-0.0557(56)$ |
| $b_F$ [GeV$^{-1}$] | $-0.434(43)$ | $-0.359(41)$ | $d_4$ [GeV$^{-3}$] | 0.372(114) | 0.304(1008) |
| $b_1$ [GeV$^{-1}$] | 0.730(10) | 0.810(8) | $d_5$ [GeV$^{-3}$] | $-0.401(110)$ | $-0.237(88)$ |
| $b_2$ [GeV$^{-1}$] | $-1.21(18)$ | $-0.819(26)$ | $d_7$ [GeV$^{-3}$] | $-0.0913(58)$ | $-0.104(48)$ |
| $b_3$ [GeV$^{-1}$] | $-0.340(153)$ | $-0.357(12)$ | $d_8$ [GeV$^{-3}$] | $-0.132(79)$ | $-0.0417(67)$ |
| $b_4$ [GeV$^{-1}$] | $-0.776(16)$ | $-0.780(15)$ | $\bar{B}_1$ [GeV$^{-3}$]$\times 10^{-2}$ | – | $-0.121(103)$ |
| $b_5$ [GeV$^{-2}$] | $-1.15(287)$ | $-1.34(23)$ | $\bar{B}_2$ [GeV$^{-3}$]$\times 10^{-2}$ | – | $-0.467(109)$ |
| $b_6$ [GeV$^{-2}$] | 0.778(390) | 0.889(199) | $\bar{B}_3$ [GeV$^{-3}$]$\times 10^{-2}$ | – | 0.344(267) |
| $b_7$ [GeV$^{-2}$] | 0.899(26) | 0.787(14) | $\bar{X}$ [GeV$^{-3}$]$\times 10^{-4}$ | – | 0.606(5723) |
| $b_8$ [GeV$^{-2}$] | 0.627(37) | 0.817(28) | | | |
| $\chi^2$ | 30.0 | 28.0 | $\chi^2$/d.o.f. | 0.91 | 0.97 |

- Slight reduction of $\chi^2$ but not $\chi^2$/d.o.f.
- Discretization effects are not important for the description of the present LQCD simulations
- Different from finite volume corrections, which are essential to obtain a $\chi^2$/d.o.f. around 1

# Lattice spacing evolutions




$$R_B = m_B^{(a)}/m_B$$

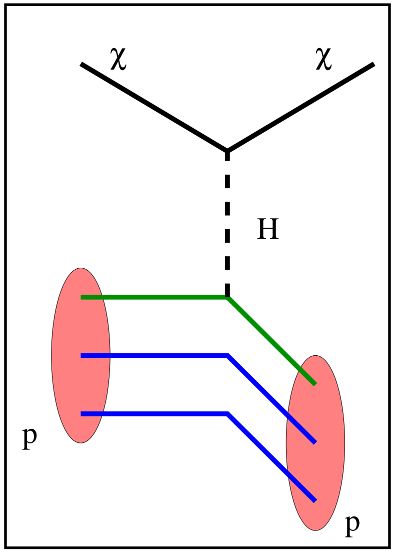
- **For LQCD simulations with $m_\pi$<500 MeV and a<0.15 fm, discretization effects are about 1 to 2 percent**

# Predictions: octet baryon sigma terms

**Important for**

➡**understanding the composition of baryons**

➡**direct dark matter searches**




$$\mathcal{L}_{int} = \lambda_N \bar{n} n \bar{\chi} \chi \to \mathcal{L}_{int} = \lambda_q \bar{q} q \bar{\chi} \chi$$

$$\lambda_N \longrightarrow \sum_{q=1}^{6} f_q^N \lambda_q$$

Spin indep. WIMP-N X-section

$$\sigma_{SI} = \frac{4M^2}{\pi} \left[ Z f_P + (A - Z) f_N \right]$$

with

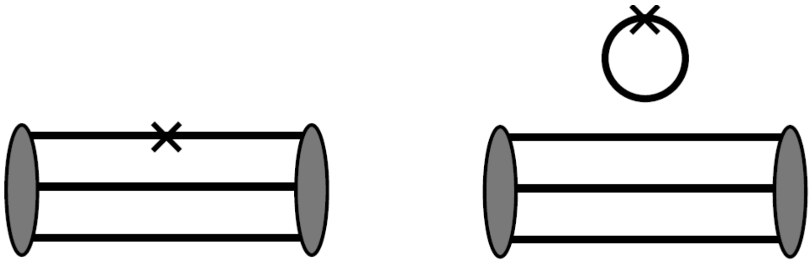
$$\frac{f_N}{M_N} = \sum_q f_q^N \frac{\lambda_q}{m_q}$$

$$f_{ud}^N M_N = \sigma_{\pi N} = \langle N(p) | (\bar{u} u + \bar{d} d) | N(p$$

$$f_s^N M_N = \sigma_{\bar{s}sN}/2 = \langle N(p) | \bar{s} s | N(p) \rangle$$

# LQCD determination of sigma terms

- **Direct method**—calculates the 3-point connected and disconnect diagrams



– JLQCD coll., PRD83,114506 (2011)
– R. Babich *et al.*, PRD85,054510 (2012)
– QCDSF coll., PRD85, 054502 (2012)
– ETM coll., JHEP 1208,037(2012)
– M. Engelhardt *et al.*, PRD86, 114510 (2012)
– JLQCD coll., PRD87, 034509 (2013)

- **Spectrum method**-calculates the baryon masses, and relates the sigma terms to their quark mass dependence via the Feynman Hellman theorem

$$\sigma_{\pi B} = m_l \langle B(p)|\bar{u}u + \bar{d}d|B(p)\rangle = m_l \frac{\partial M_B}{\partial m_l}$$

$$\sigma_{sB} = m_s \langle B(p)|\bar{s}s|B(p)\rangle = m_s \frac{\partial M_B}{\partial m_s}.$$

– JLQCD coll., PRD83,114506 (2011)
– R. Babich *et al.*, PRD85,054510 (2012)
– QCDSF coll., PRD85, 054502 (2012)
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# Selection of LQCD data

- All $n_f$=2+1 LQCD simulations
  - PACS-CS, LHPC, QCDSF-UKQCD, HSC, NPLQCD, BWM
  - BWM—not publicly available
  - HSC and NPLQCD—Low statistics

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PACS-CS, LHPC, QCDSF-UKQCD

# An accurate determination of baryon sigma terms

- **Scale setting:** mass independent (given by the LQCD simulations or self-consistently determined) vs. mass dependent ($r_0$, $r_1$, $X_\pi$)
- **Isospin breaking effects:** better constrain the LQCD LECs
- **Theoretical uncertainties caused by truncating chiral expansions:** NNLO vs. N3LO; EOMS vs. FRR

# Scale-setting effects on the determination of baryon sigma terms

arXiv:1301.3231
P.E. Shanahan*, A.W. Thomas and R.D. Young

- **Lattice-scale setting**
  - **PACS-CS data with mass independent scale-setting:**

$$\sigma_{sN} = 59 \pm 7 \ (\text{MeV})$$

  - **PACS data with mass dependent ($r_0$) scale-setting:**

$$\sigma_{sN} = 21 \pm 6 \ (\text{MeV})$$

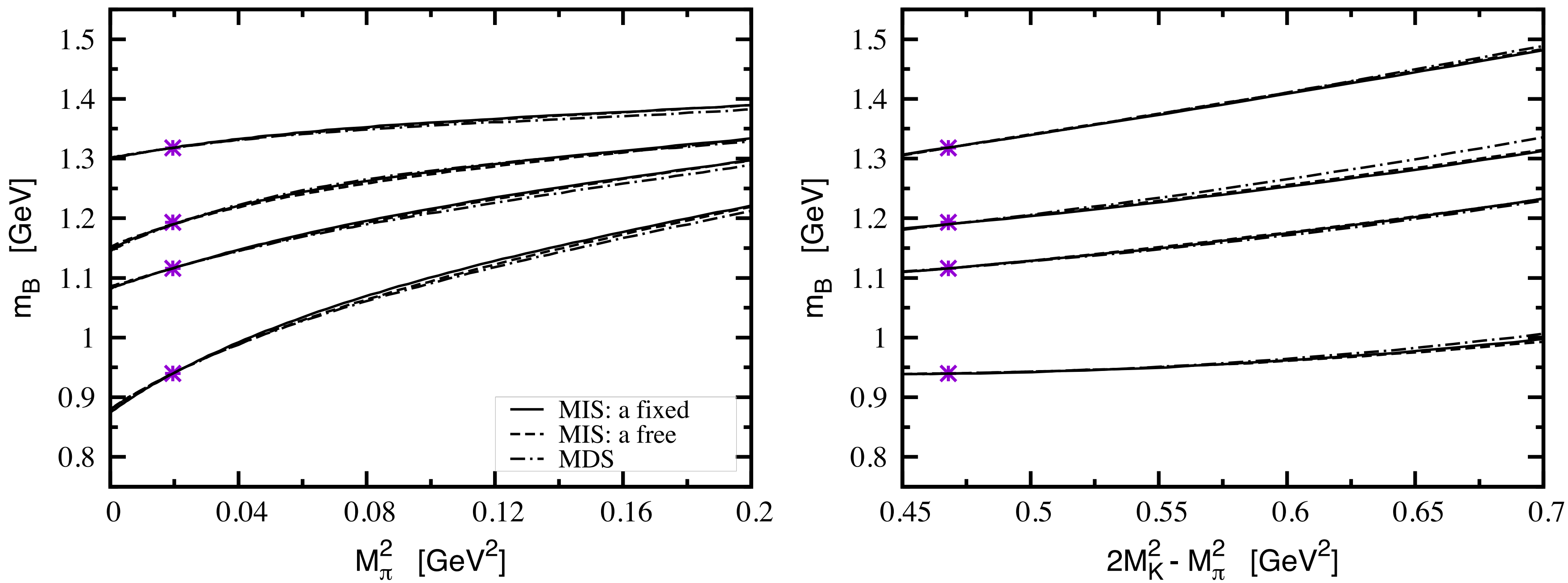
- **Whether other LQCD data will show the same trend?**

# Three different fits at $N^3LO$

| | MIS | | MDS |
|---|---|---|---|
| | $a$ fixed | $a$ free | |
| $m_0$ [MeV] | 884(11) | 877(10) | 887(10) |
| $b_0$ [GeV$^{-1}$] | −0.998(2) | −0.967(6) | −0.911(10) |
| $b_D$ [GeV$^{-1}$] | 0.179(5) | 0.188(7) | 0.039(15) |
| $b_F$ [GeV$^{-1}$] | −0.390(17) | −0.367(21) | −0.343(37) |
| $b_1$ [GeV$^{-1}$] | 0.351(9) | 0.348(4) | −0.070(23) |
| $b_2$ [GeV$^{-1}$] | 0.582(55) | 0.486(11) | 0.567(75) |
| $b_3$ [GeV$^{-1}$] | −0.827(107) | −0.699(169) | −0.553(214) |
| $b_4$ [GeV$^{-1}$] | −0.732(27) | −0.966(8) | −1.30(4) |
| $b_5$ [GeV$^{-2}$] | −0.476(30) | −0.347(17) | −0.513(89) |
| $b_6$ [GeV$^{-2}$] | 0.165(158) | 0.166(173) | −0.0397(1574) |
| $b_7$ [GeV$^{-2}$] | −1.10(11) | −0.915(26) | −1.27(8) |
| $b_8$ [GeV$^{-2}$] | −1.84(4) | −1.13(7) | 0.192(30) |
| $d_1$ [GeV$^{-3}$] | 0.0327(79) | 0.0314(72) | 0.0623(116) |
| $d_2$ [GeV$^{-3}$] | 0.313(26) | 0.269(42) | 0.325(54) |
| $d_3$ [GeV$^{-3}$] | −0.0346(87) | −0.0199(81) | −0.0879(136) |
| $d_4$ [GeV$^{-3}$] | 0.271(30) | 0.230(24) | 0.365(23) |
| $d_5$ [GeV$^{-3}$] | −0.350(28) | −0.302(50) | −0.326(66) |
| $d_7$ [GeV$^{-3}$] | −0.435(10) | −0.352(8) | −0.322(7) |
| $d_8$ [GeV$^{-3}$] | −0.566(24) | −0.456(30) | −0.459(33) |
| $\chi^2$/d.o.f. | 0.87 | 0.88 | 0.53 |

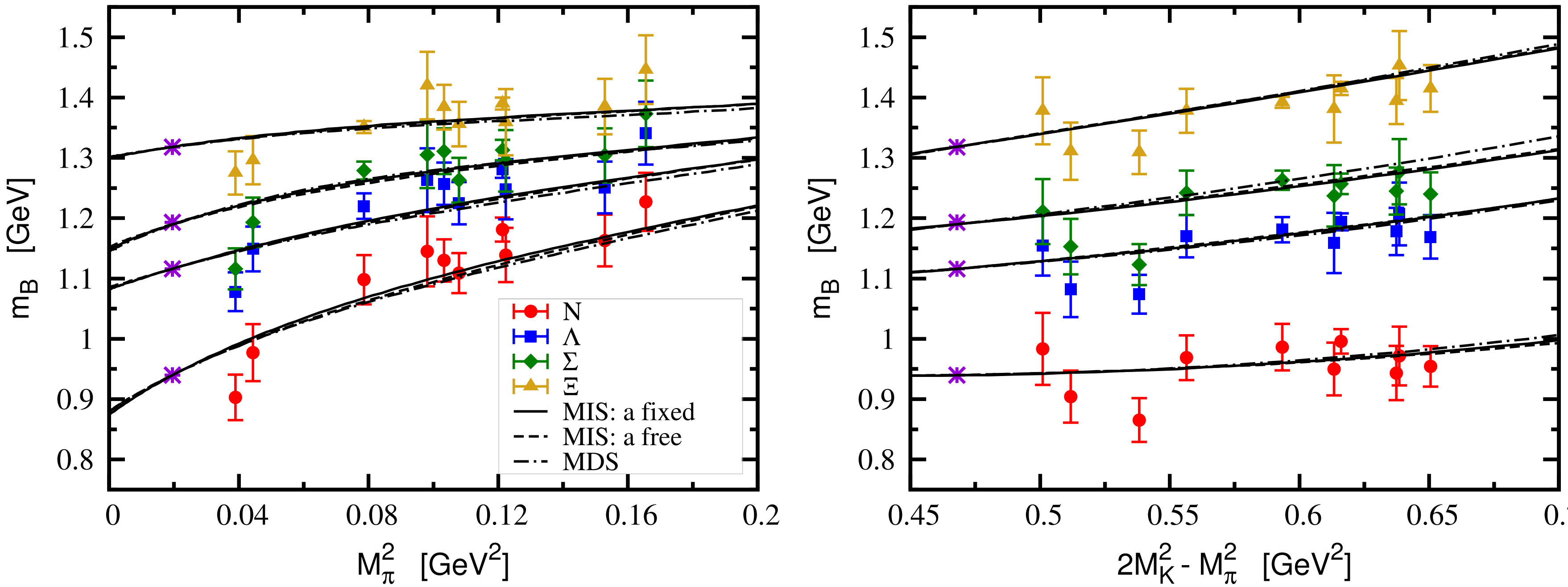
- **Mass independent**
  - Lattice spacing a fixed to the published value
  - Lattice spacing a determined self-consistently
- **Mass dependent**
  - $r_0$ for PACS-CS
  - $r_1$ for LHPC
  - $X_\pi$ for QCDSF-UKQCD

# Evolution of baryon masses with u/d and s quark masses




Only central values are shown!

# Evolution of baryon masses with u/d and s quark masses in comparison with the BMW data




S. Durr et al., BMW collaboration, Phys.Rev. D85 (2012) 014509

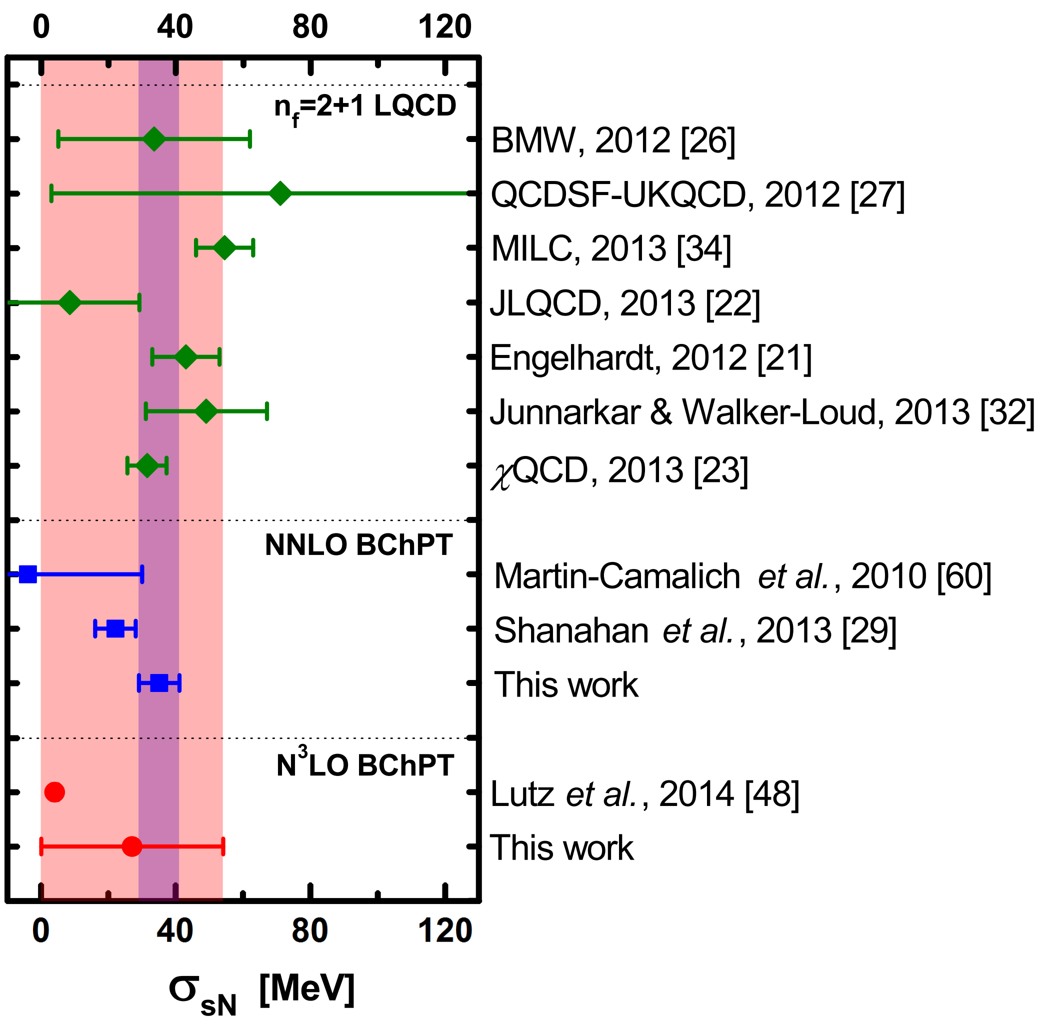
# Baryon sigma terms from N$^3$LO BChPT

| | Ref. [48] | MIS | | MDS |
|---|---|---|---|---|
| | | $a$ fixed | $a$ free | |
| $\sigma_{\pi N}$ [MeV] | 40(0) | 55(1)(4) | 54(1) | 51(2) |
| $\sigma_{\pi \Lambda}$ [MeV] | 23(0) | 32(1)(2) | 32(1) | 30(2) |
| $\sigma_{\pi \Sigma}$ [MeV] | 18(0) | 34(1)(3) | 33(1) | 37(2) |
| $\sigma_{\pi \Xi}$ [MeV] | 6(1) | 16(1)(2) | 18(2) | 15(3) |
| $\sigma_{s N}$ [MeV] | 4(1) | 27(27)(4) | 23(19) | 26(21) |
| $\sigma_{s \Lambda}$ [MeV] | 83(3) | 185(24)(17) | 192(15) | 168(14) |
| $\sigma_{s \Sigma}$ [MeV] | 228(3) | 210(26)(42) | 216(16) | 252(15) |
| $\sigma_{s \Xi}$ [MeV] | 355(5) | 333(25)(13) | 346(15) | 340(13) |

- All three scale-setting methods yield similar baryon sigma terms

[48] M. F. M. Lutz, R. Bavontaweepanya, C. Kobdaj and K. Schwarz, arXiv:1401.7805 [hep-lat].

# Comparison with earlier studies




- Consistent with most recent LQCD studies and those of NNLO ChPT, e.g., that of Young and Shanahan
- Uncertainties at $N^3LO$ substantially larger, because of the extra LECs

# Summary and Outlook

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✤ We have performed a systematic study of the LQCD simulations of octet baryon masses, in terms of chiral extrapolations, finite volume corrections, and continuum extrapolations, and predicted their sigma terms

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- We have performed a systematic study of the LQCD simulations of octet baryon masses, in terms of chiral extrapolations, finite volume corrections, and continuum extrapolations, and predicted their sigma terms
- Our studies showed
  - The extended-on-mass-shell (EOMS) BChPT provides a reliable framework to study the properties of the ground-state octet baryons
  - LQCD simulations can help determine the many unknown low-energy constants which otherwise cannot be fixed

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- ✤ Our studies showed
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  - LQCD simulations can help determine the many unknown low-energy constants which otherwise cannot be fixed
- ✤ Many interesting observables remain unexplored within the EOMS framework
  - Axial, Vector, and Electromagnetic form factors of the g.s. octet baryons
  - TMDs and GMDs of the octet baryons
  - Hyperon-nucleon (hyperon) forces
  - ...

# NNLO fits

TABLE I. Values of the LECs obtained from the best fits to the LQCD simulations and the experimental octet baryon masses and the corresponding $\chi^2$/d.o.f.. The underlined numbers denote the values at which they are fixed.

| | EOMS | | FRR | |
|---|---|---|---|---|
| | Fit-I | Fit-II | Fit-III | Fit-IV |
| $m_0$ [MeV] | $757(7)$ | $808(1)$ | $829(7)$ | $805(9)$ |
| $b_0$ [GeV$^{-1}$] | $-0.907(6)$ | $-0.710(2)$ | $-0.820(7)$ | $-0.922(20)$ |
| $b_D$ [GeV$^{-1}$] | $0.0582(22)$ | $0.0570(22)$ | $0.101(2)$ | $0.116(3)$ |
| $b_F$ [GeV$^{-1}$] | $-0.508(2)$ | $-0.411(11)$ | $-0.464(2)$ | $-0.510(8)$ |
| $f_0$ [GeV] | $\underline{0.0871}$ | $0.105(3)$ | $\underline{0.0871}$ | $\underline{0.0871}$ |
| $\Lambda$ or $\mu$ [GeV] | $\underline{1.0}$ | $\underline{1.0}$ | $\underline{1.0}$ | $1.24(5)$ |
| $\chi^2$/d.o.f. | $3.0$ | $1.6$ | $2.4$ | $1.8$ |

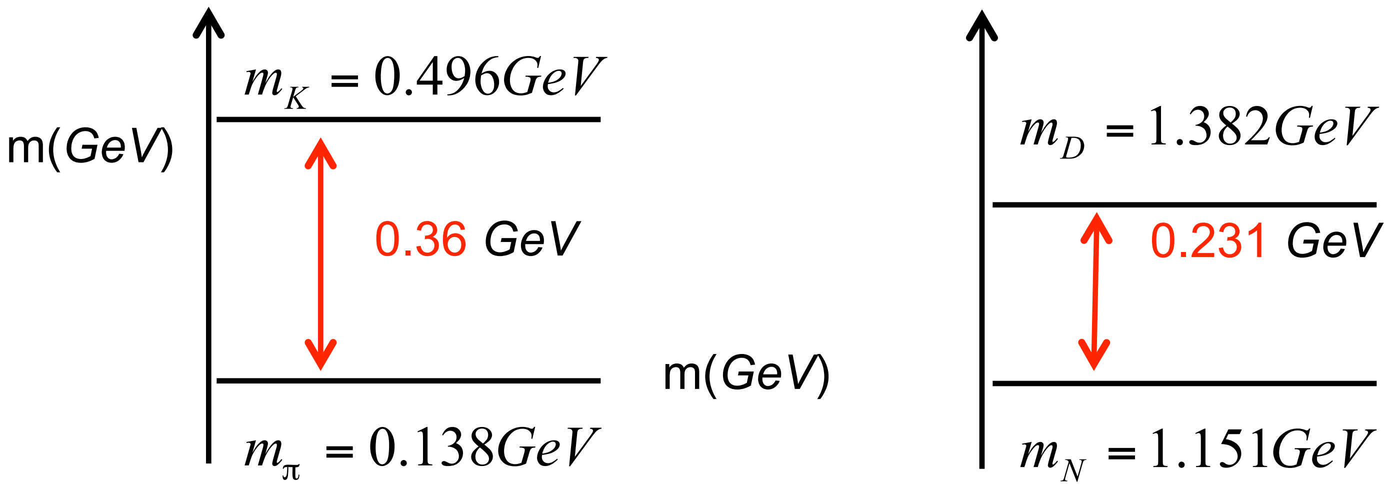
# NNLO sigma terms

TABLE II. Sigma terms of the octet baryons at the physical point, predicted by the NNLO BChPT with the LECs of Table I.

| | EOMS | | FRR | |
|---|---|---|---|---|
| | Fit-I | Fit-II | Fit-III | Fit-IV |
| $\sigma_{\pi N}$ [MeV] | 56(0) | 47(1) | 47(0) | 53(1) |
| $\sigma_{\pi \Lambda}$ [MeV] | 35(1) | 30(1) | 31(1) | 34(1) |
| $\sigma_{\pi \Sigma}$ [MeV] | 32(0) | 27(1) | 25(0) | 27(1) |
| $\sigma_{\pi \Xi}$ [MeV] | 13(1) | 12(1) | 13(1) | 13(1) |
| $\sigma_{s N}$ [MeV] | 35(6) | 27(7) | 21(6) | 20(7) |
| $\sigma_{s \Lambda}$ [MeV] | 147(7) | 152(7) | 162(7) | 153(7) |
| $\sigma_{s \Sigma}$ [MeV] | 218(7) | 222(7) | 226(7) | 214(7) |
| $\sigma_{s \Xi}$ [MeV] | 295(7) | 313(8) | 332(7) | 312(8) |

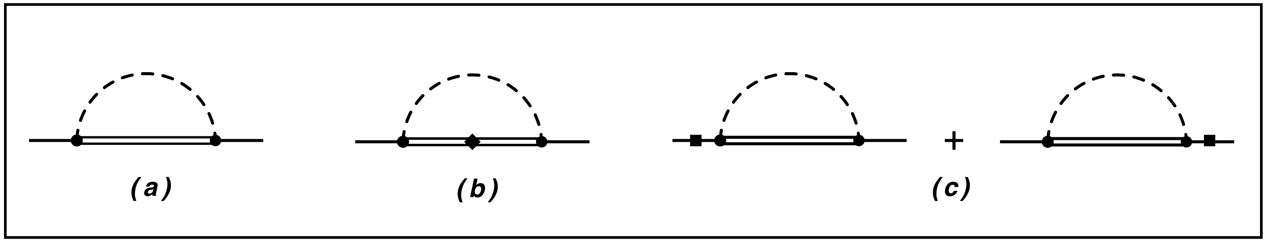
# Effects of dynamical decuplet baryons

- **ChPT relies on the assumption that all high-energy degrees of freedom can be integrated out--not necessarily true for SU(3) BChPT**



# Feynman diagrams/Lagrangians-no new unknown LECs

- **Feynman diagrams**




- **Lagrangians**
  - **Octet-Decuplet-Pseudoscalr coupling**

$$\mathcal{L}^{(1)}_{\phi BT} = \frac{i\mathcal{C}}{m_D F_\phi} \varepsilon^{abc} (\partial_\alpha \bar{T}^{ade}_\mu) \gamma^{\alpha\mu\nu} B^e_c \partial_\nu \phi^d_b + \text{H.c.},$$

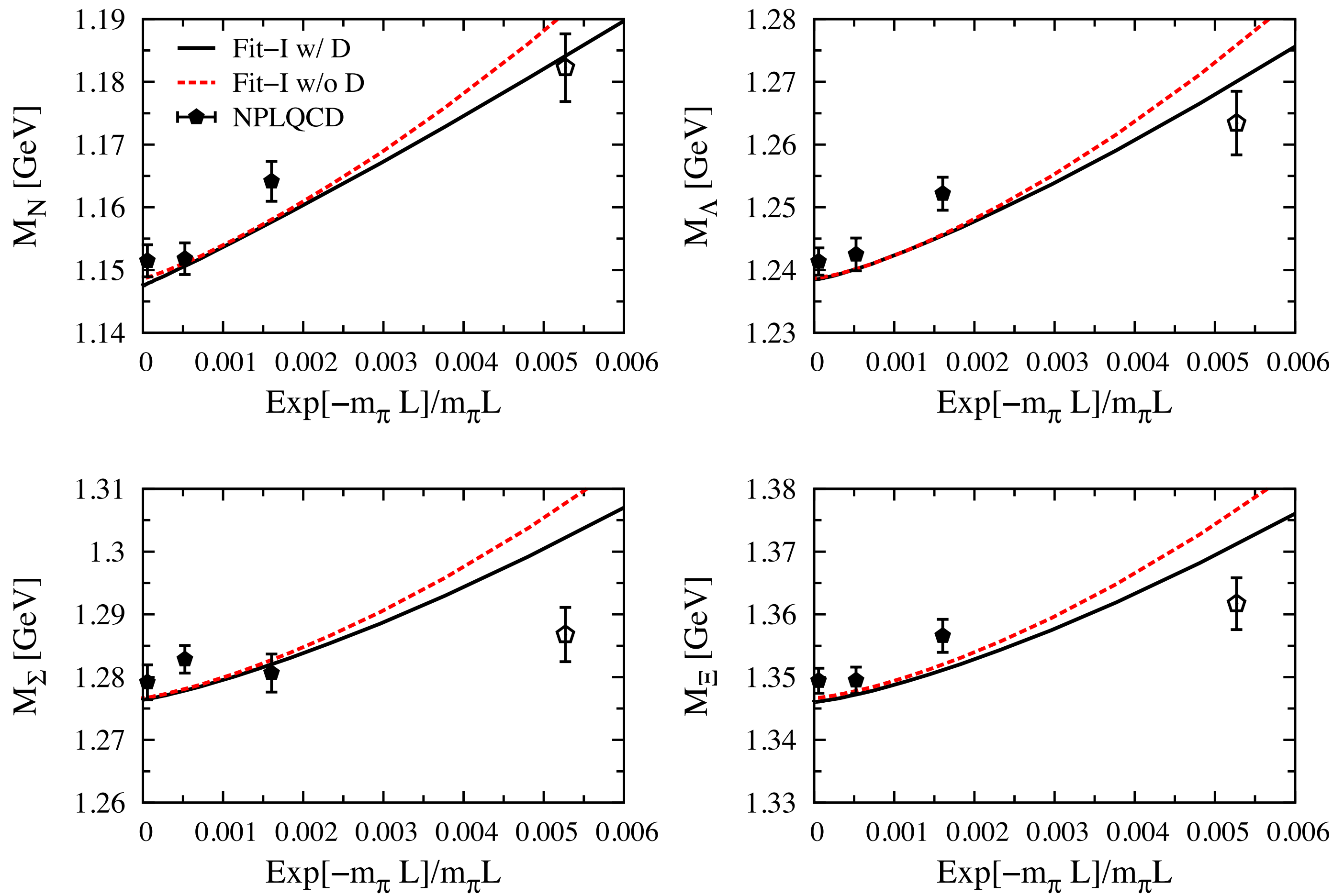
*fixed from decay of a decuplet into an octet baryon and a pseudoscalar*

  - **mass corrections**

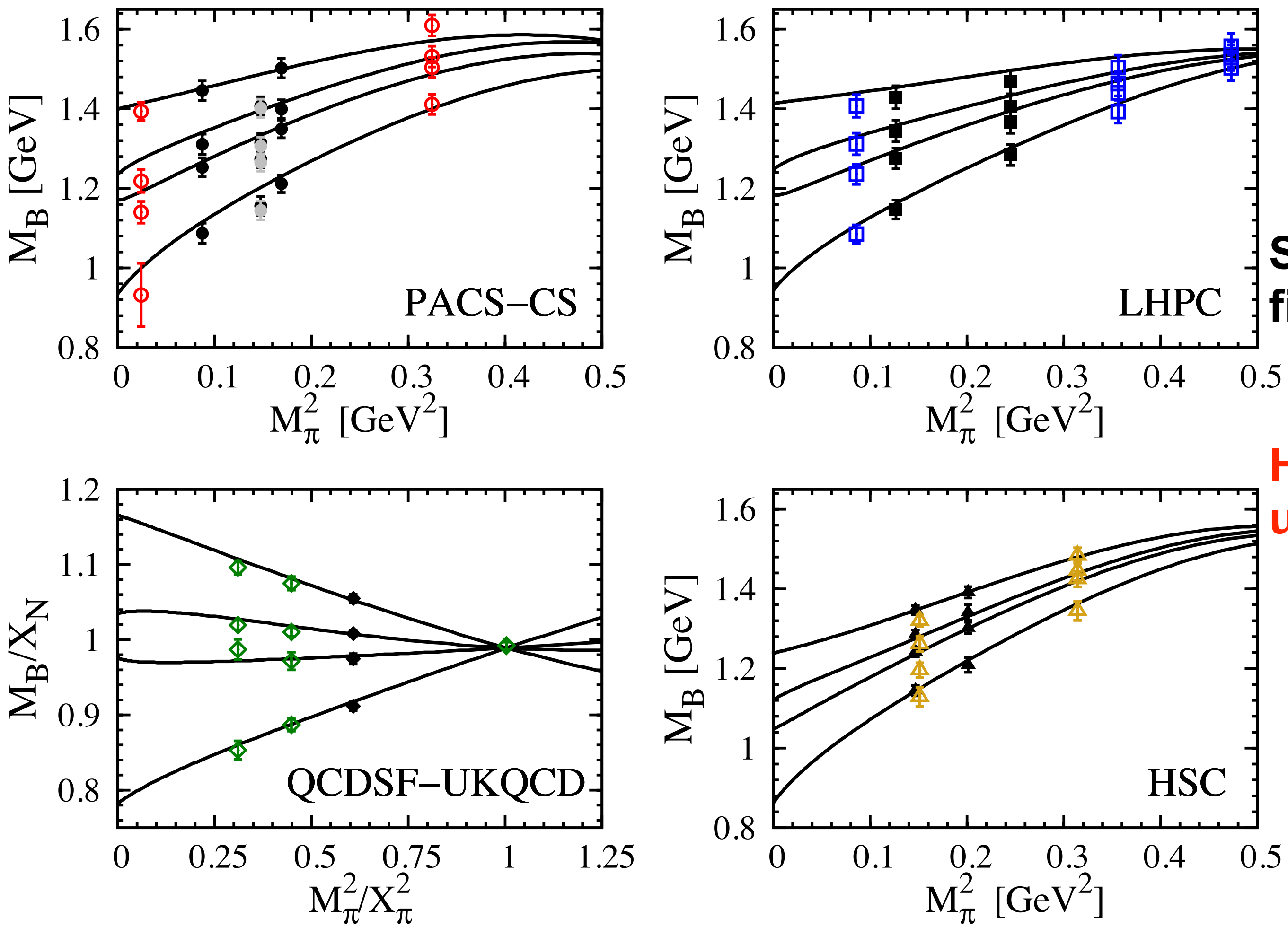
$$\mathcal{L}^{(2)}_T = \frac{t_0}{2} \bar{T}^{abc}_\mu g^{\mu\nu} T^{abc}_\nu \langle \chi_+ \rangle + \frac{t_D}{2} \bar{T}^{abc}_\mu g^{\mu\nu} (\chi_+, T_\nu)^{abc},$$

*fixed from the experimental decuplet masses*

# Slightly better description of the volume dependence of the NPLQCD data



# Unfitted data can also reasonably well described




**Solid black: fitted**

**Hollow colored: unfitted**

# Baryon Pion and Strangeness Sigma terms

- **Feynman-Hellmann theorem states**

$$\sigma_{\pi B} = m_l \langle B(p) | \bar{u}u + \bar{d}d | B(p) \rangle = m_l \frac{\partial M_B}{\partial m_l}$$

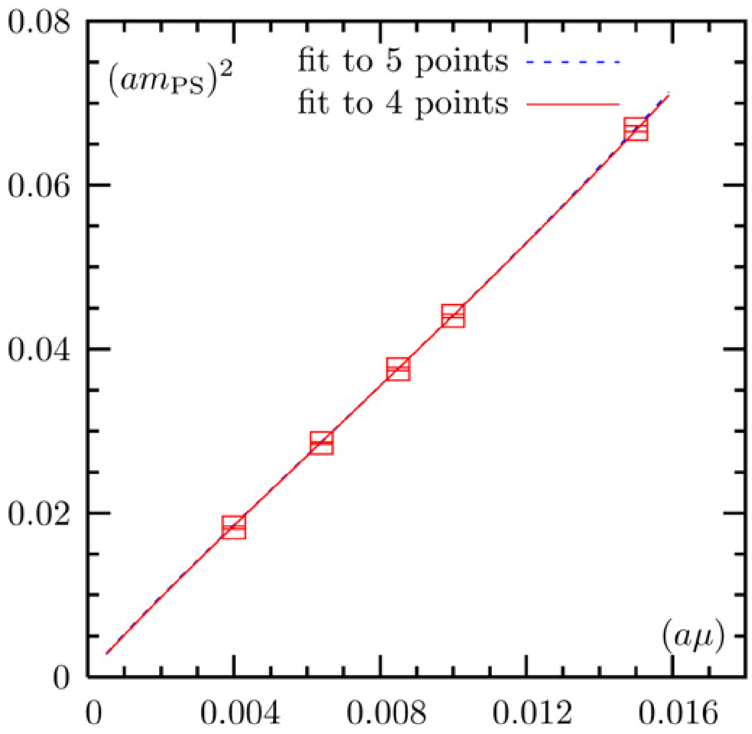
$$\sigma_{sB} = m_s \langle B(p) | \bar{s}s | B(p) \rangle = m_s \frac{\partial M_B}{\partial m_s}.$$

- **Using leading-order ChPT meson masses**

$$\sigma \pi B = \frac{m_\pi^2}{2} \left( \frac{1}{m_\pi} \frac{\partial}{\partial m_\pi} + \frac{1}{2m_K} \frac{\partial}{\partial m_K} + \frac{1}{3m_\eta} \frac{\partial}{\partial m_\eta} \right) m_B$$

$$\sigma_s = \left( m_K^2 - \frac{m_\pi^2}{2} \right) \left( \frac{1}{2m_K} \frac{\partial}{\partial m_K} + \frac{2}{3m_\eta} \frac{\partial}{\partial m_\eta} \right) m_B$$

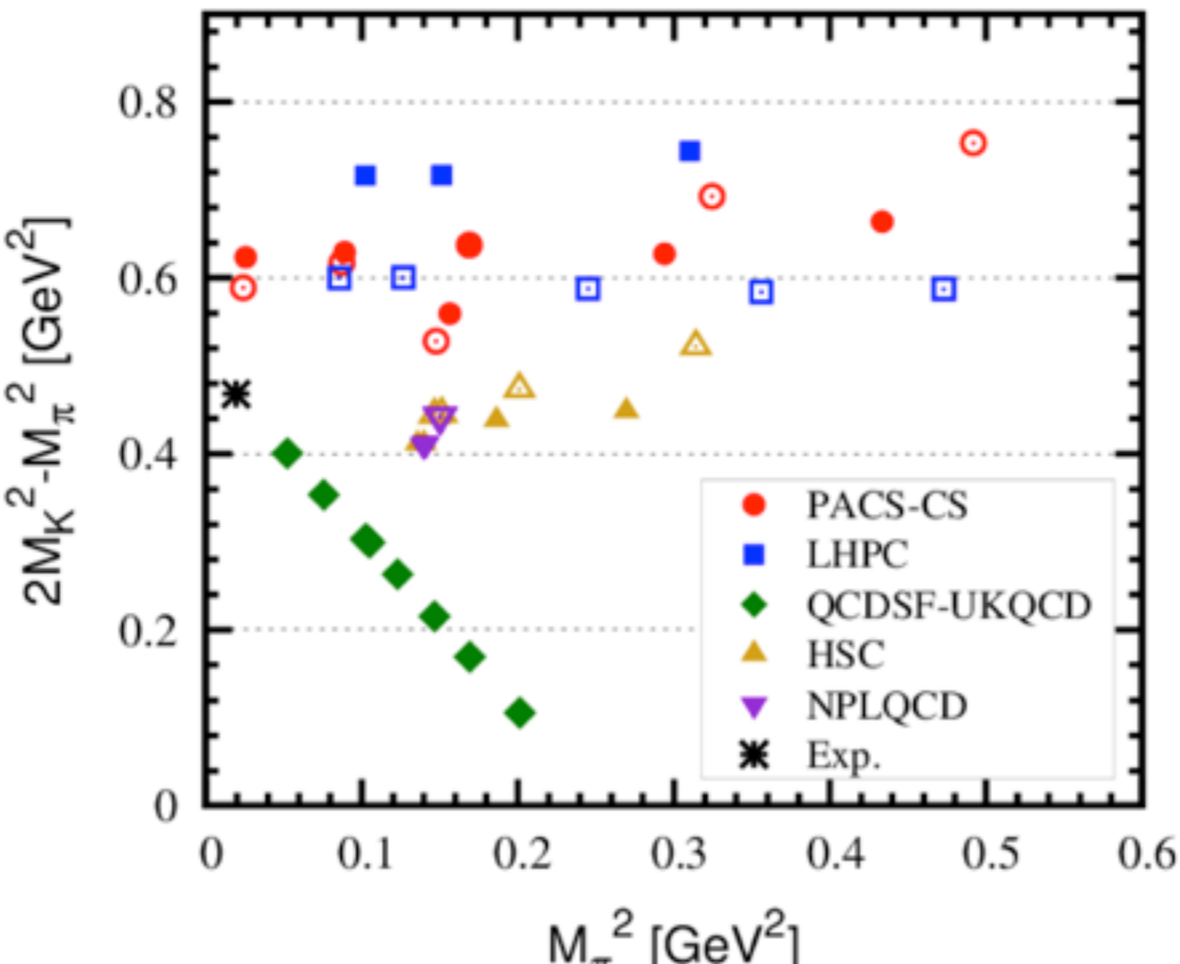
# Pion mass vs. light quark mass




ETM collaboration, hep-lat/0701012

$$m_\pi^2 \propto m_q$$

# Scale-setting effects on the octet baryon masses




- **Full symbols:** scale dependent
- **Hollow symbols:** scale independent