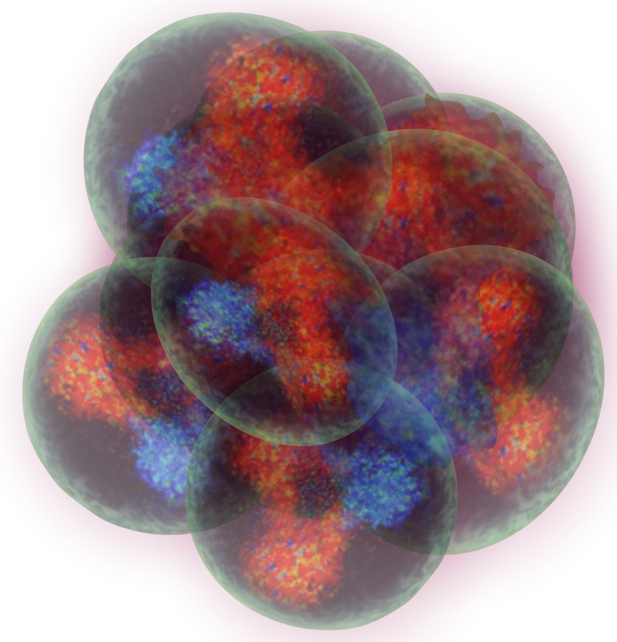
(Properties of) light nuclei in lattice QCD

William Detmold
MIT

From quarks to nuclei

- Few-body nuclear physics emerges from the underlying Standard Model
  - How exactly does this happen?
    What does it take to make a quantitative connection?
- Recent progress: focus on BB interactions and light nuclei
- New direction: nuclear matrix elements



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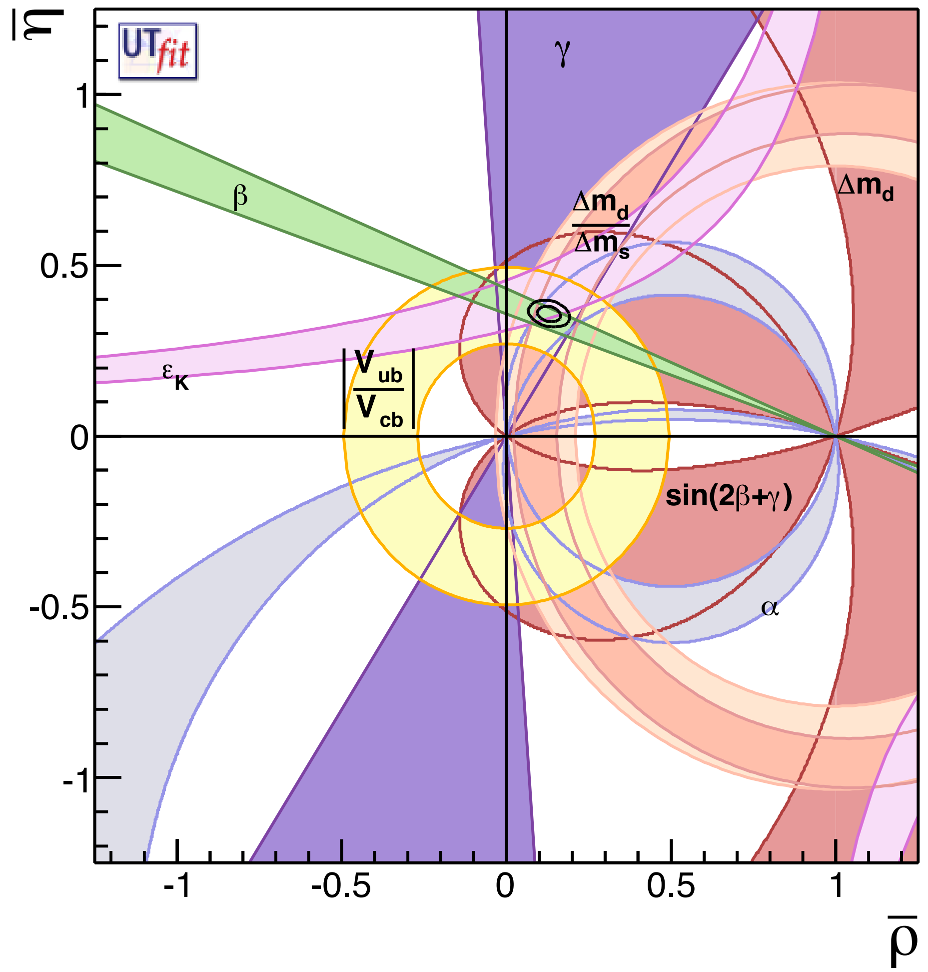

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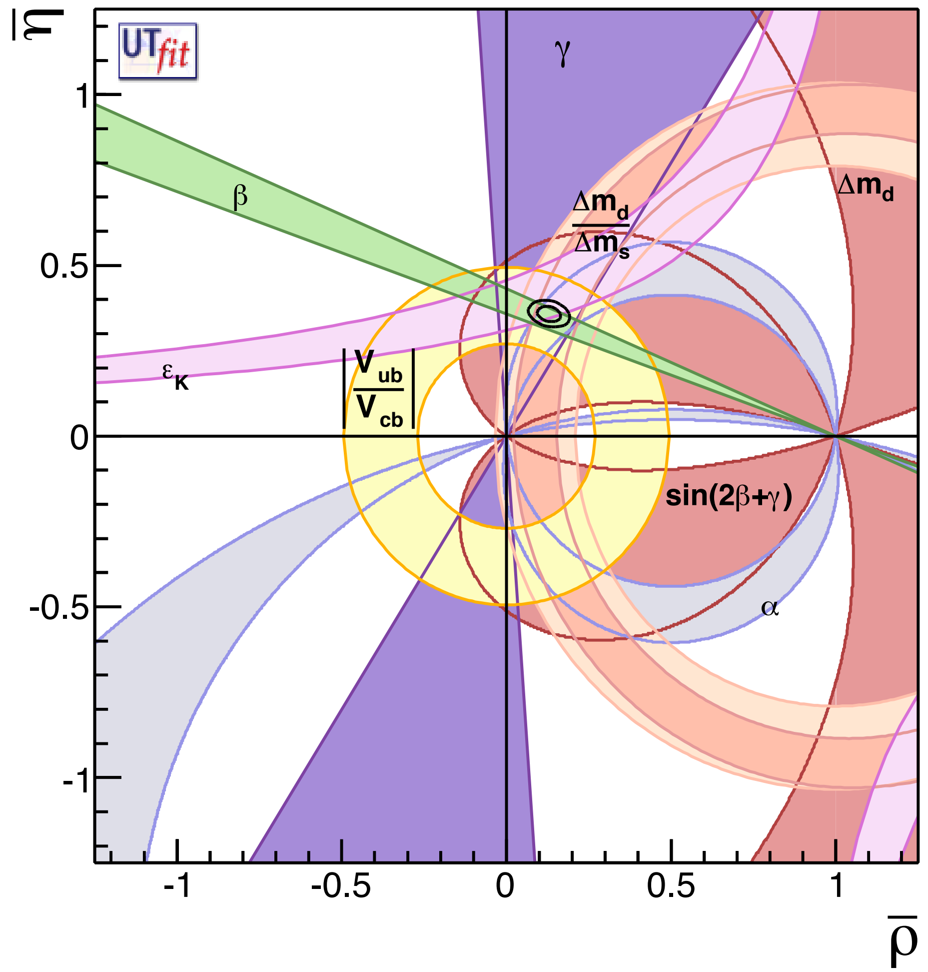

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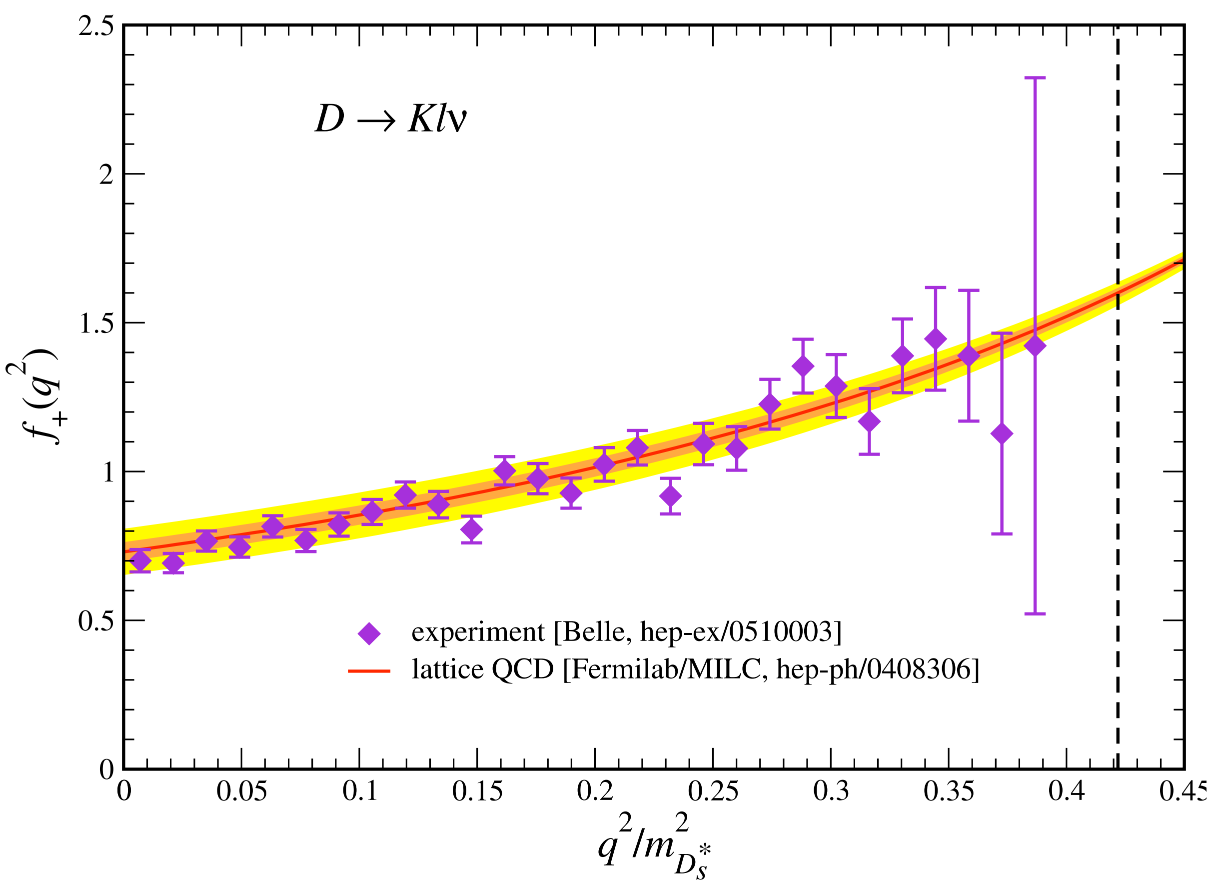
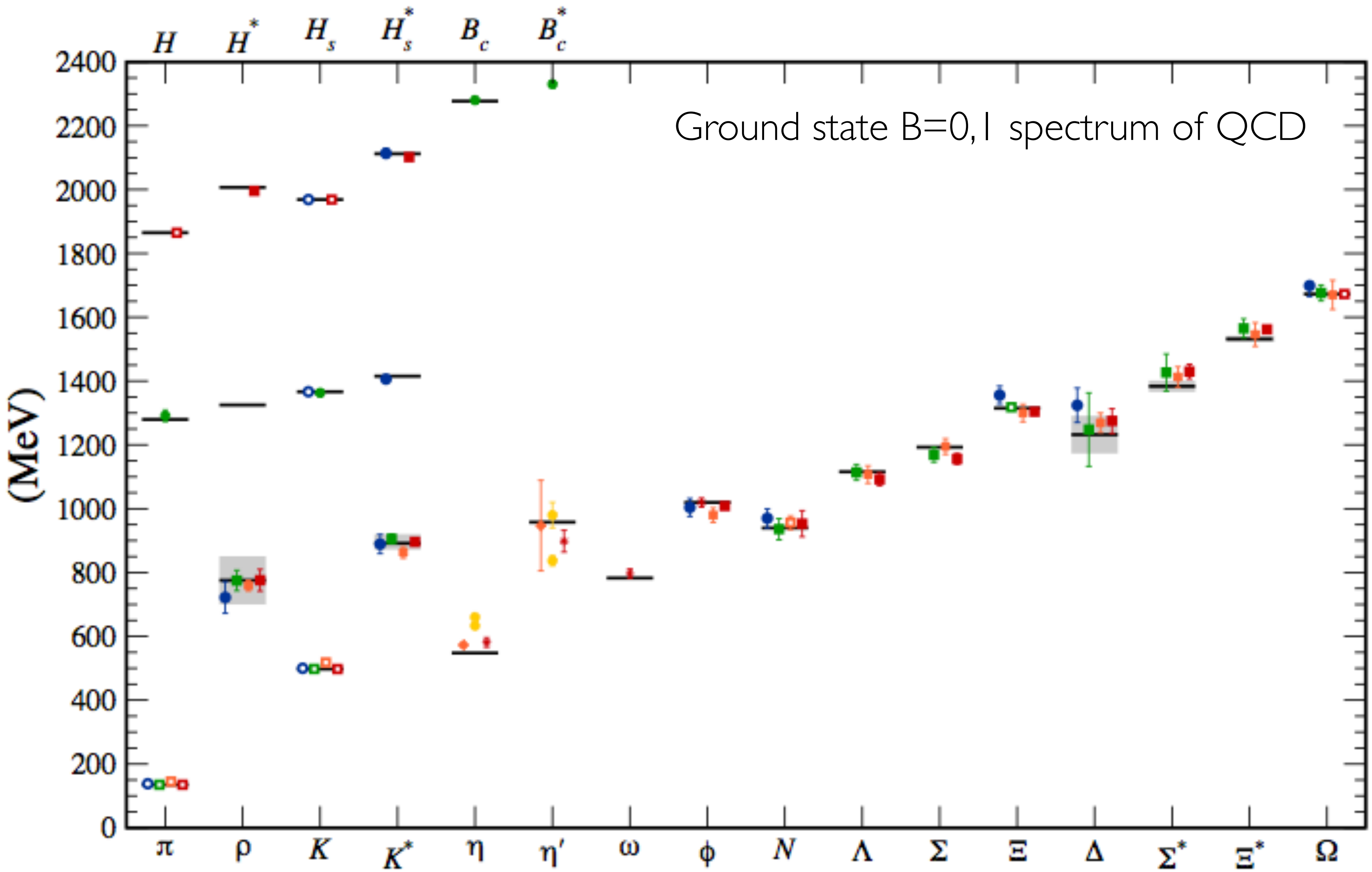
  - SM predictions with reliable uncertainty quantification



QCD: meson/baryon spectrum



[A Kronfeld, 1209.3468]

points correspond to different sets of calculations

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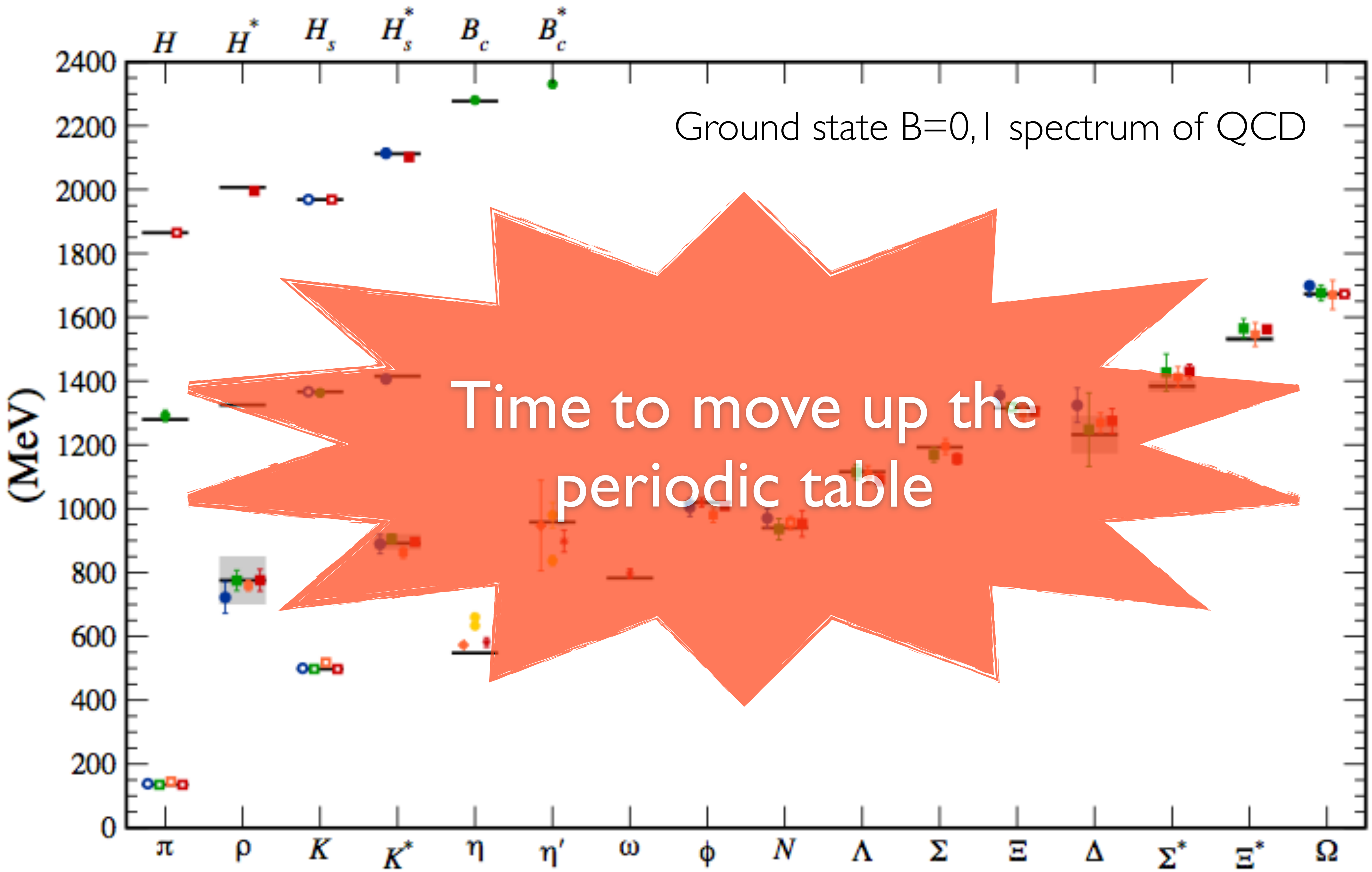

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    - Nuclear axial form factors
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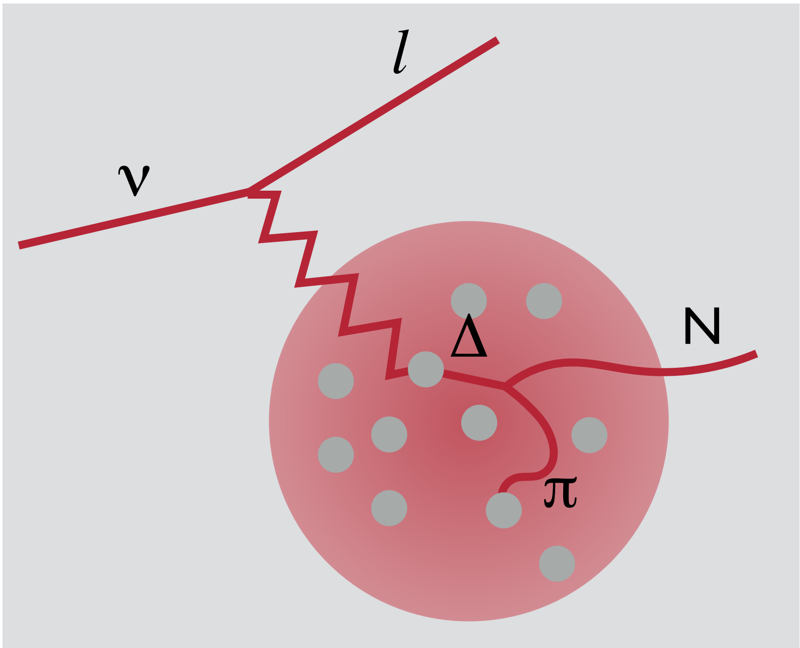
    - Nuclear structure in neutrino DIS



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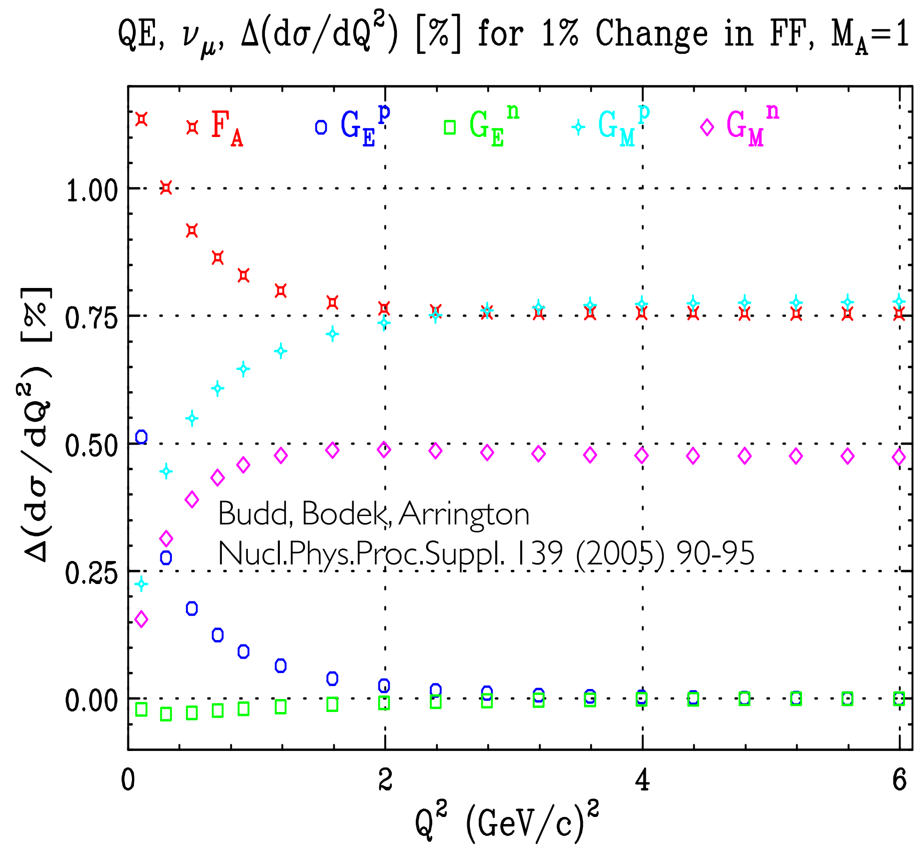

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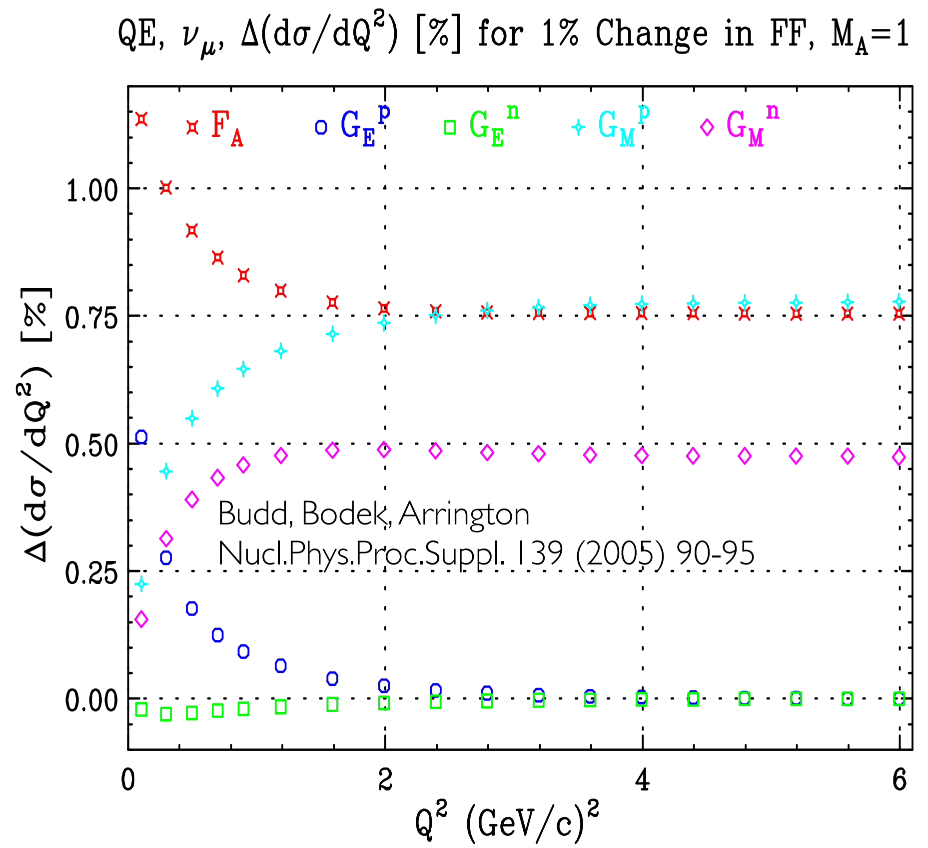
  - $0\nu\beta\beta$ experiments need double weak decay matrix elements

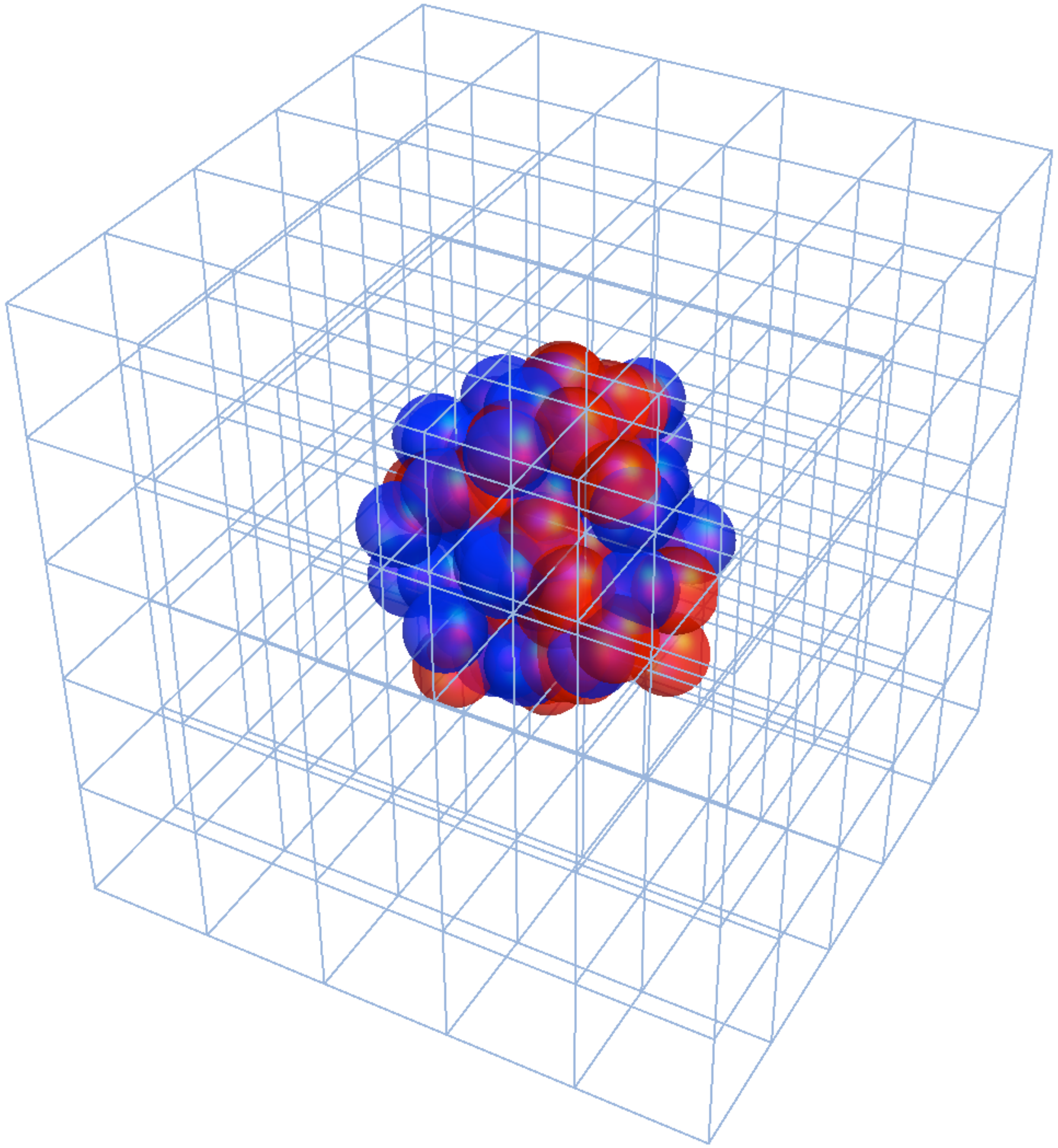


The intensity frontier

- Searches for new physics
  - Dark matter detection: nuclear recoils as signal
    Nuclear matrix elements of exchange current
  - Proposed mu2e conversion expt: similar requirements
- If(when) we detect new physics we will need precision nuclear matrix elements to learn what it is
- Nuclear physics will be the new flavour physics
  - Need to develop the tools for precision predictions

LQCD to the rescue?

- Nuclear physics is Standard Model physics
  - ... so calculate ab initio???



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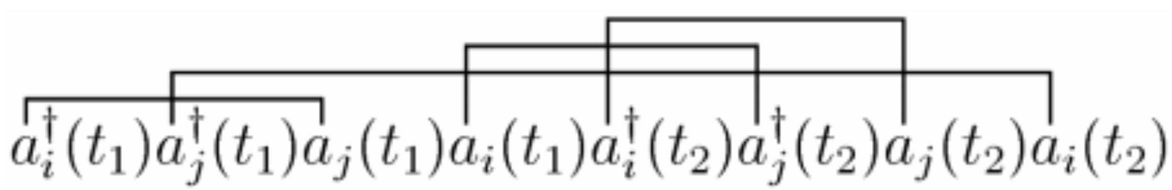

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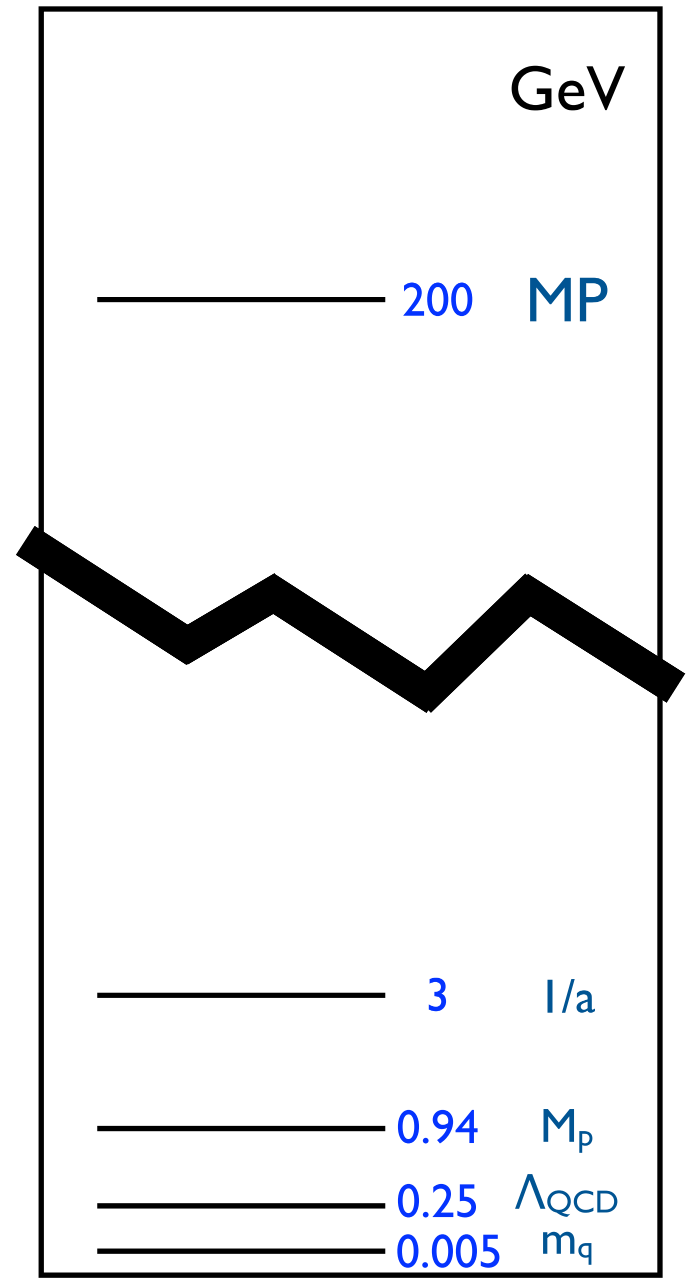

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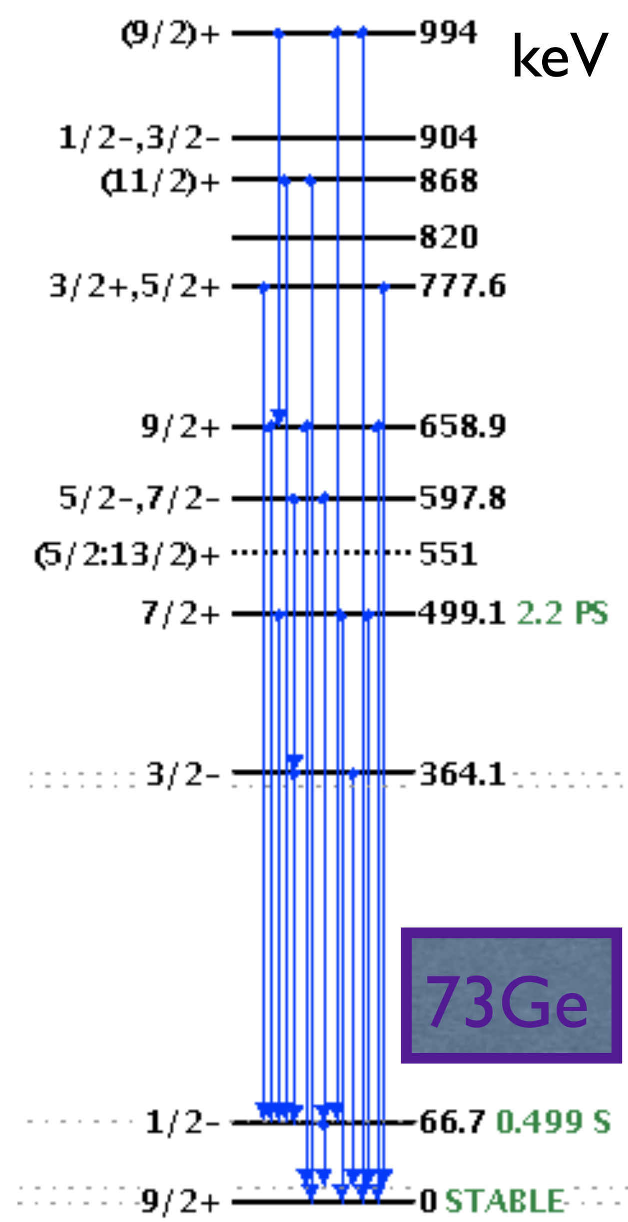

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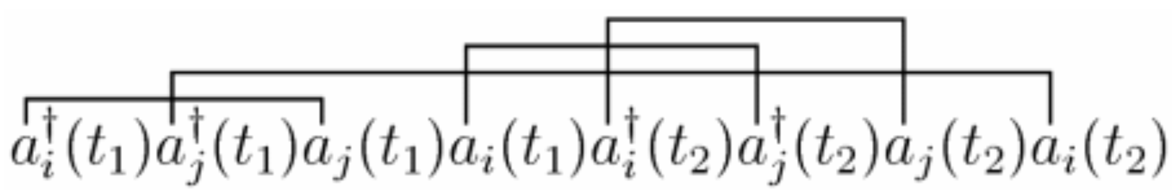

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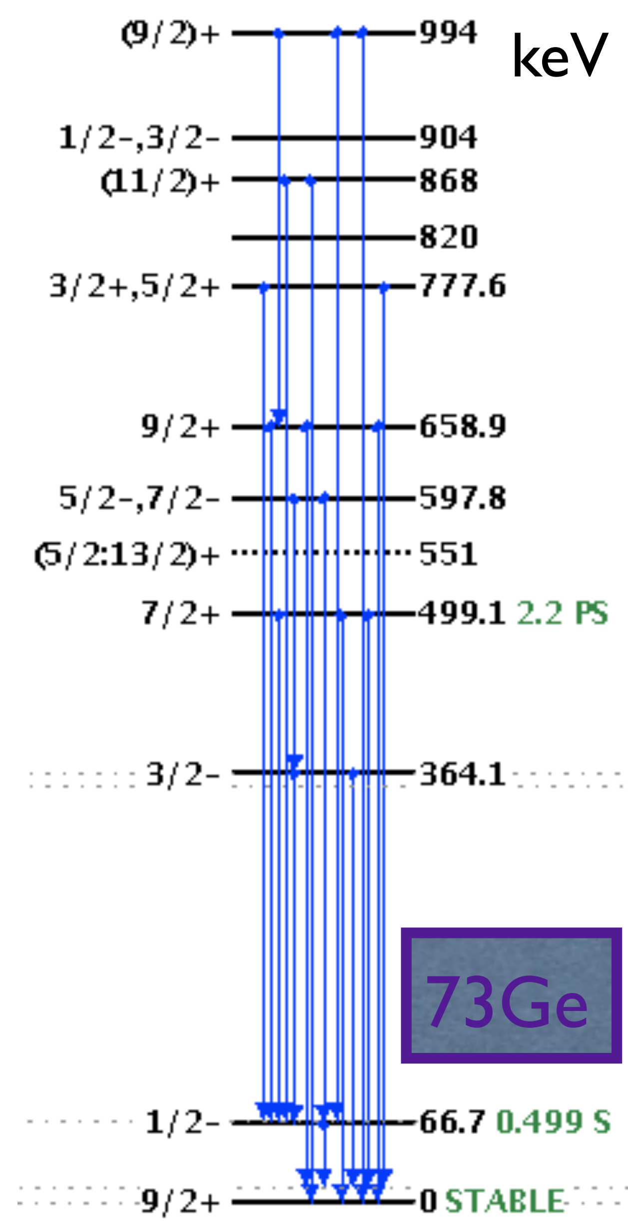
- Importance sampling Monte Carlo: statistical noise exponentially increases with A



The trouble with baryons

[Lepage '89]

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  ➤ correlators determined stochastically

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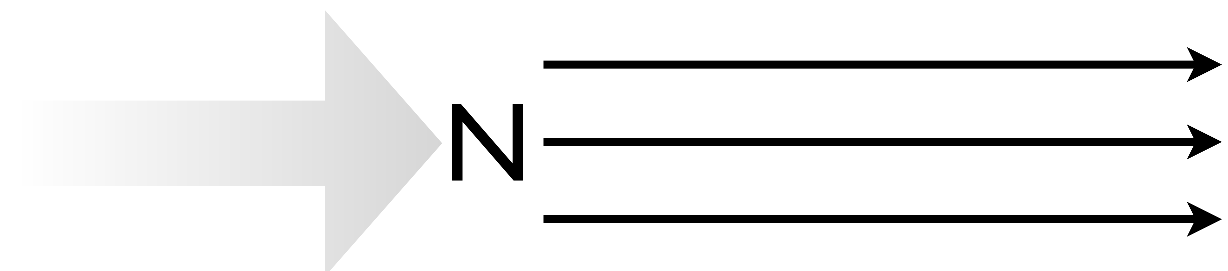

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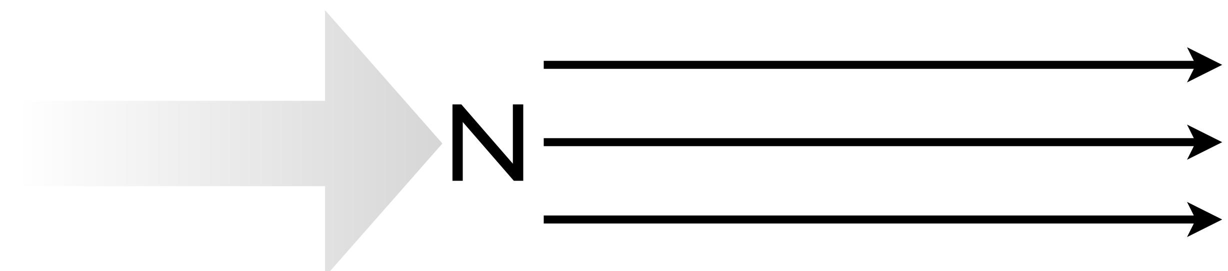

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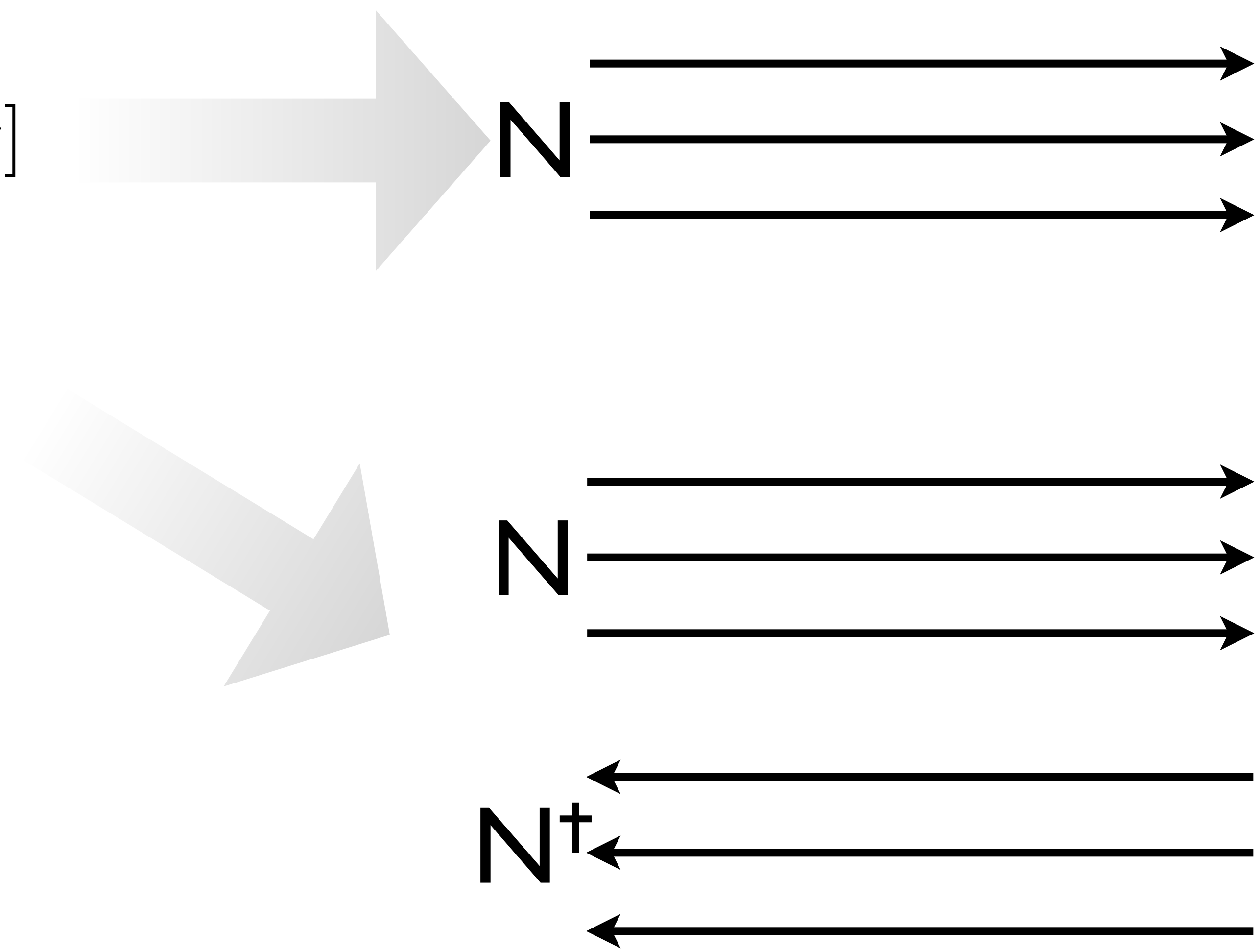

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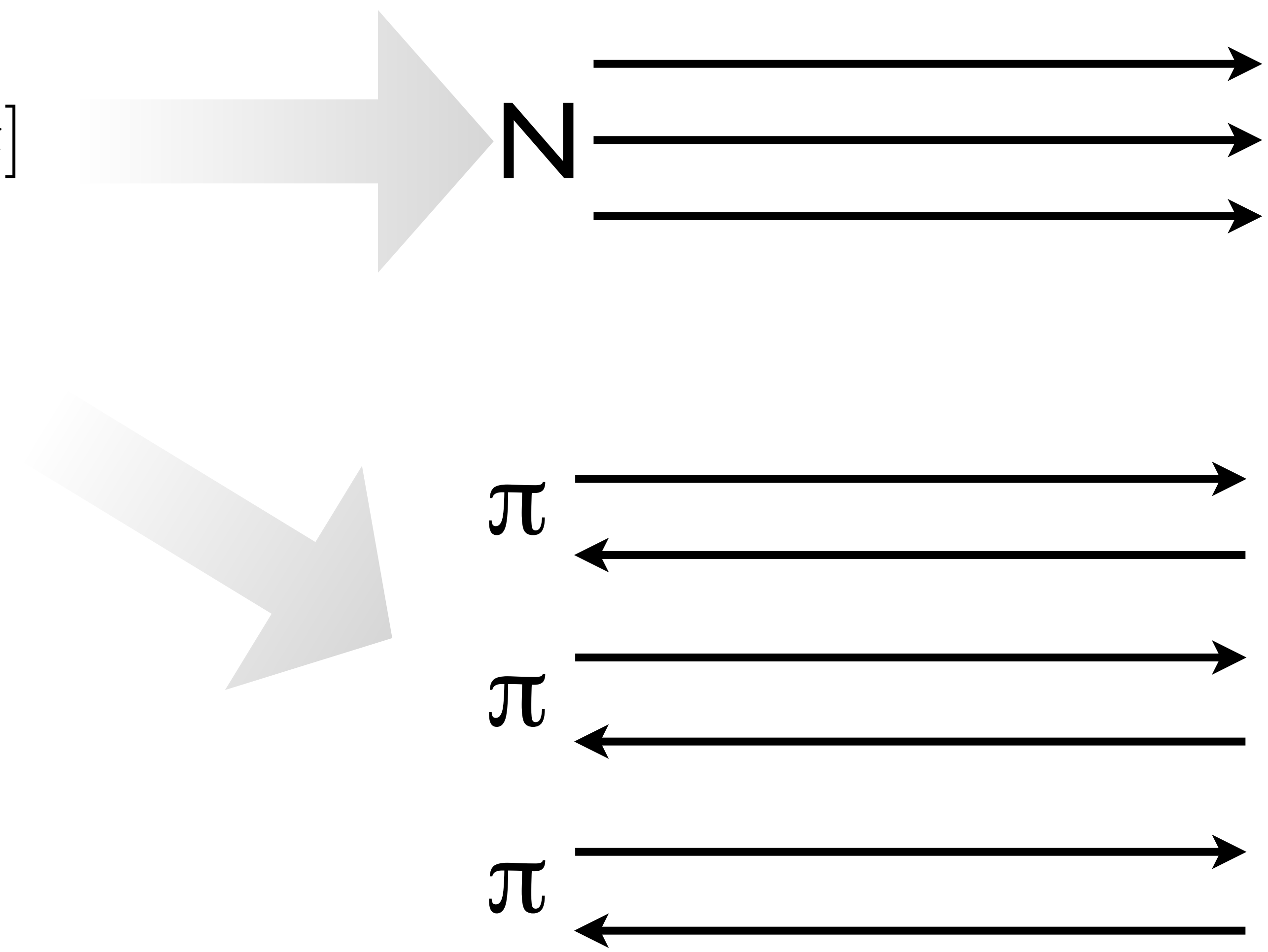

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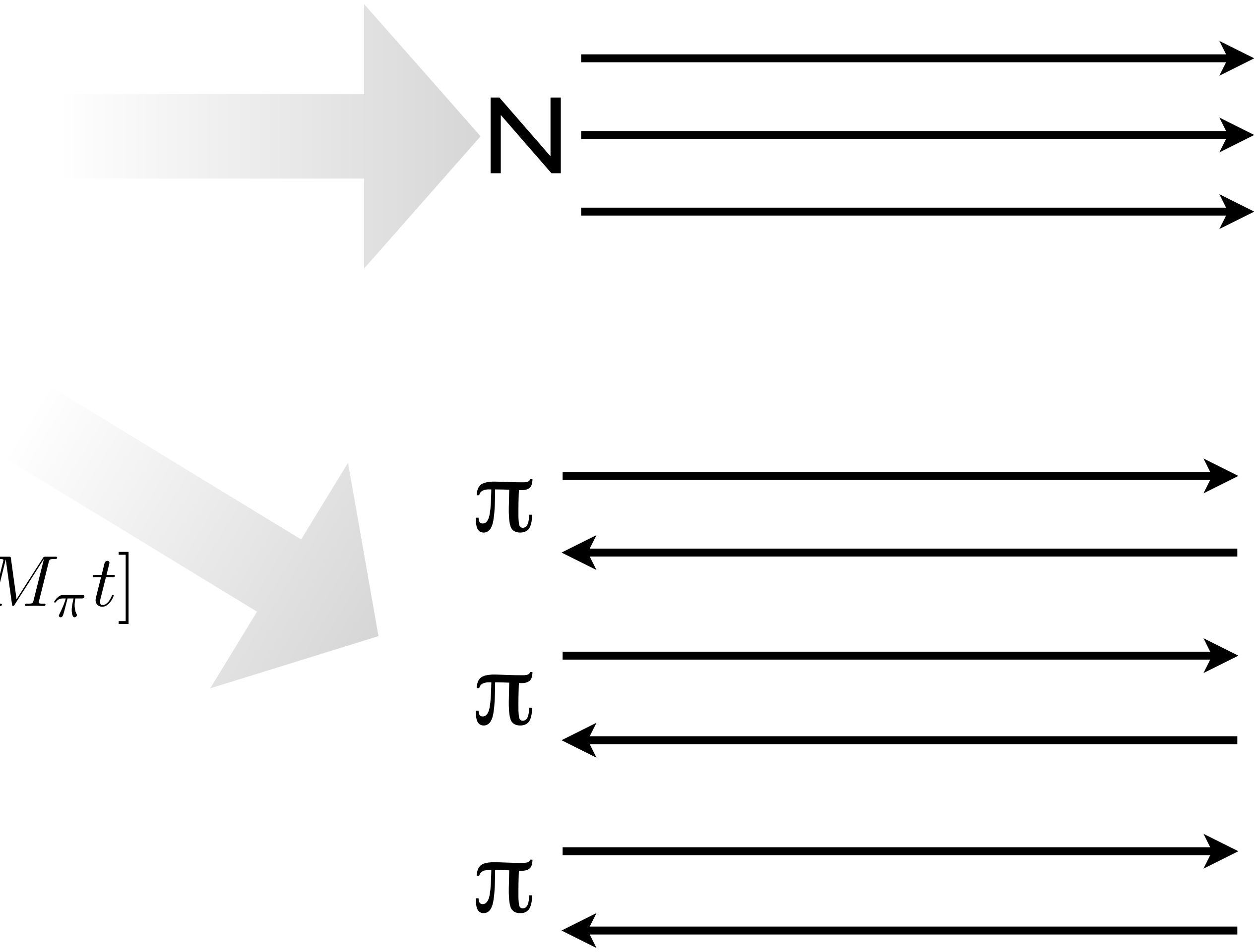

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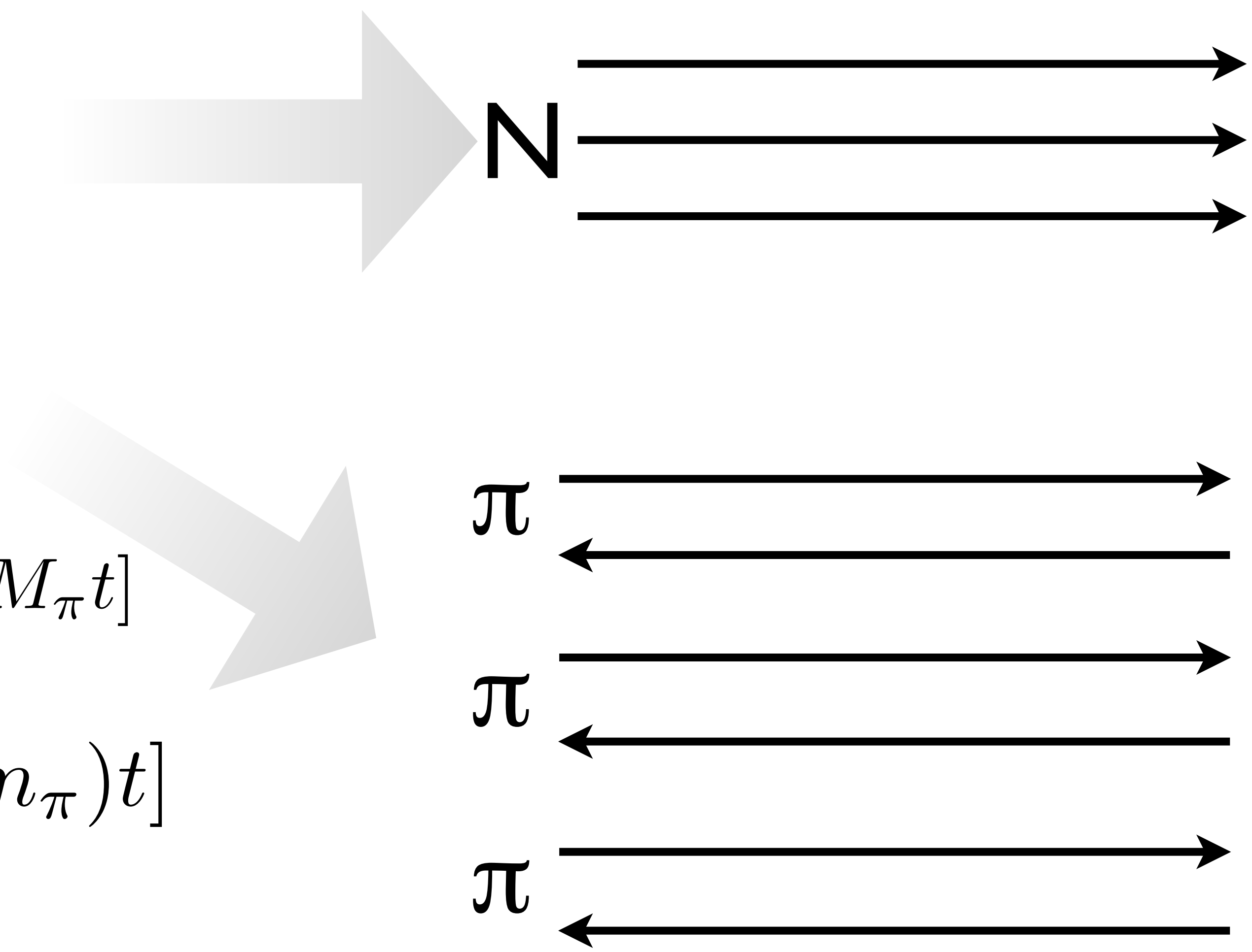

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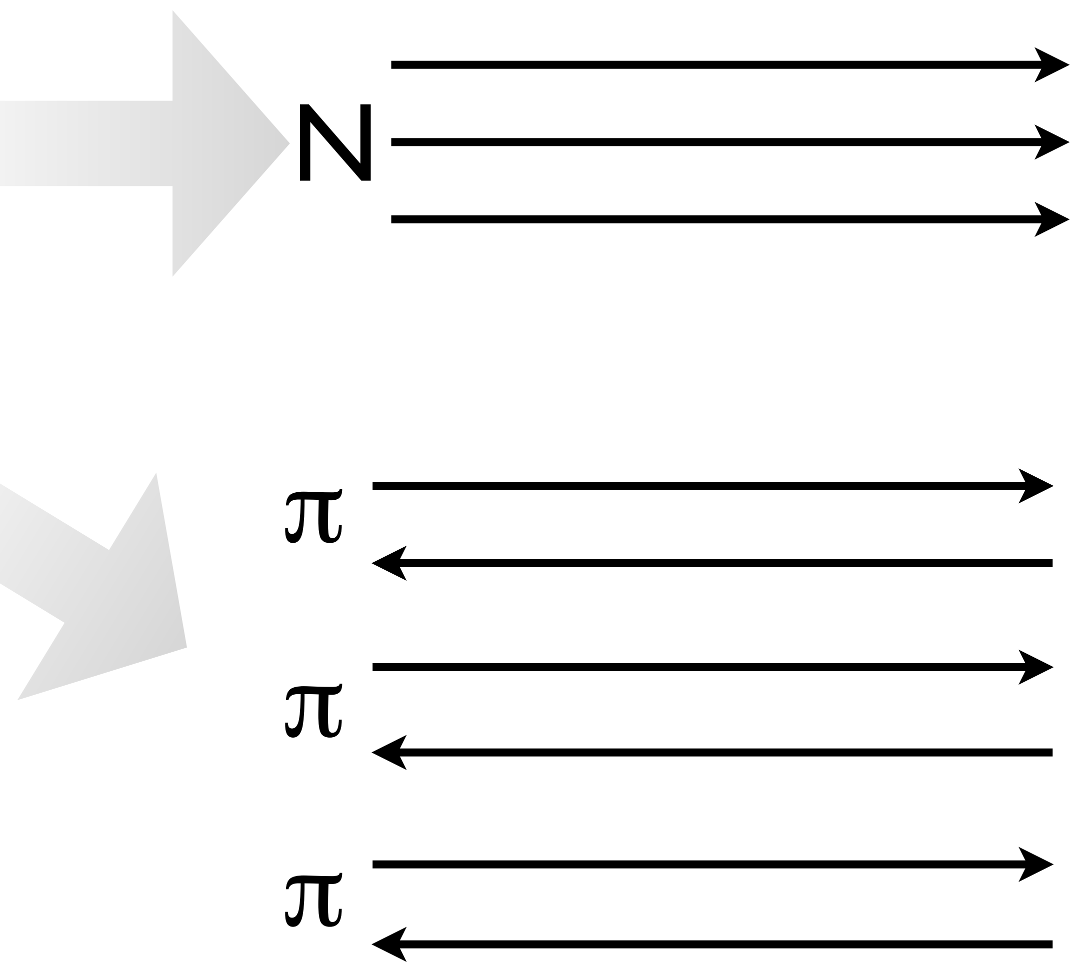
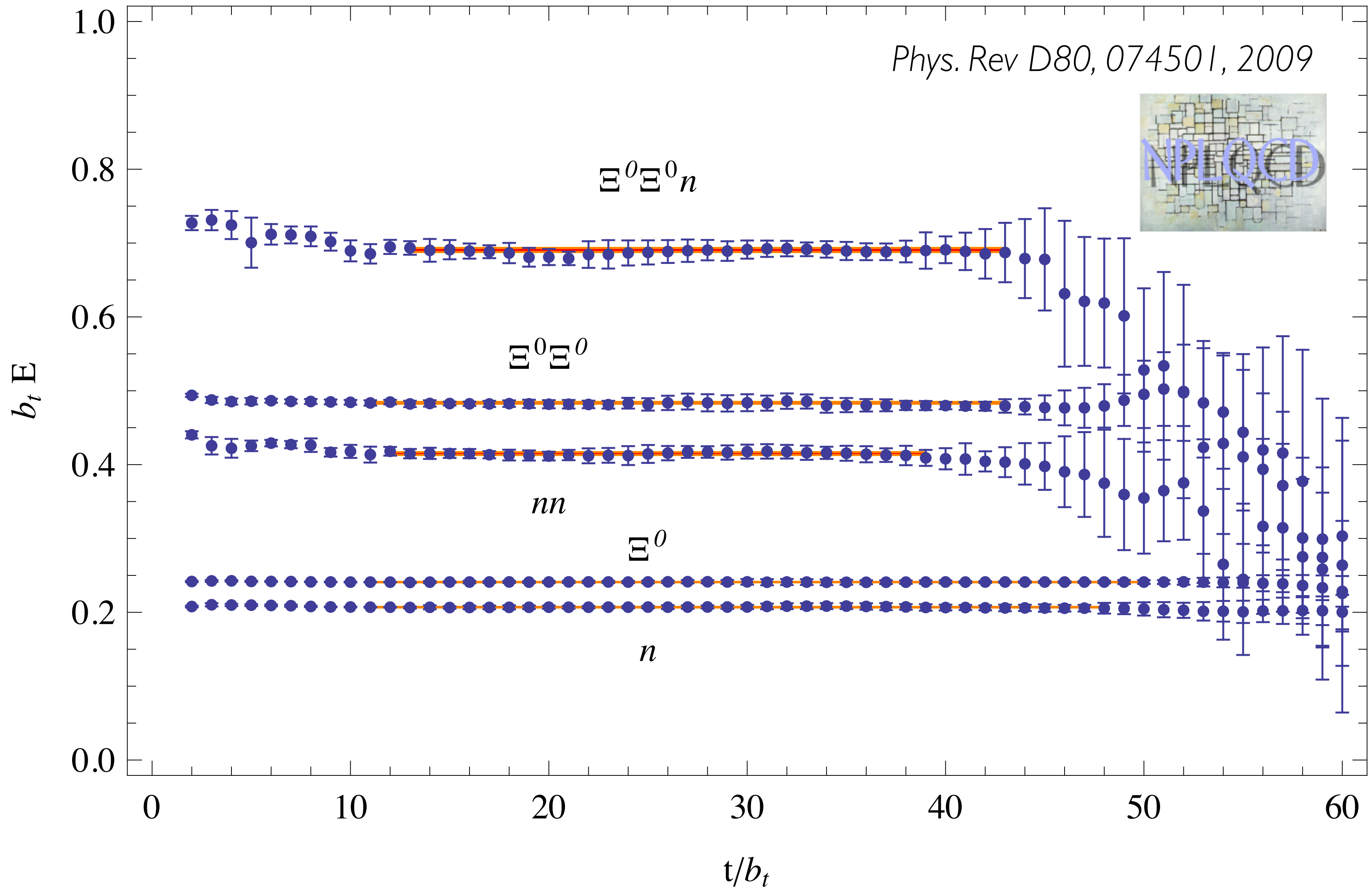


- For nucleus A:

$$\frac{\text{signal}}{\text{noise}} \sim \exp\left[-A(M_N - 3/2 m_\pi) t\right]$$

[Lepage '89]

High statistics study using anisotropic lattices (fine temporal resolution)



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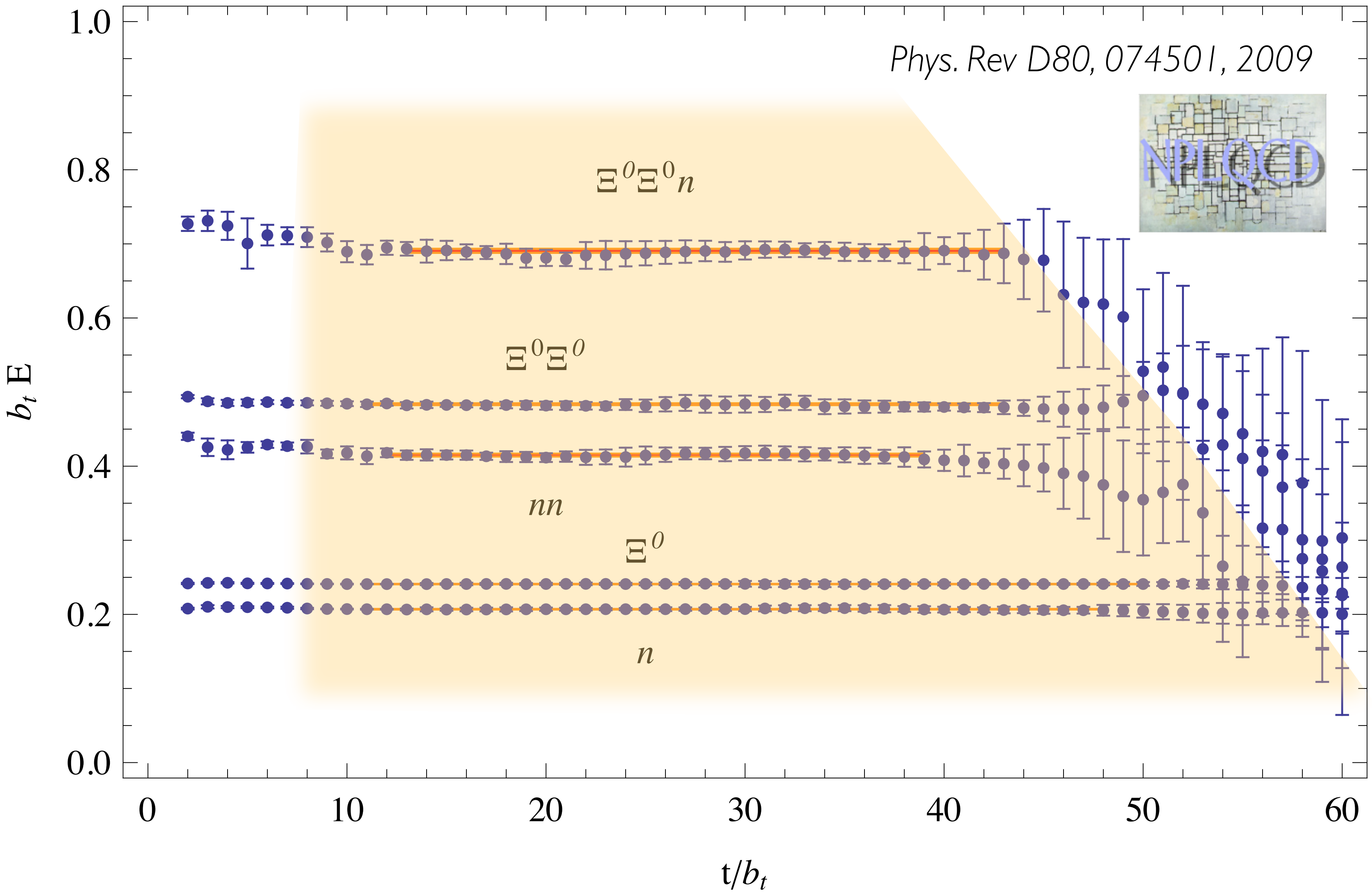

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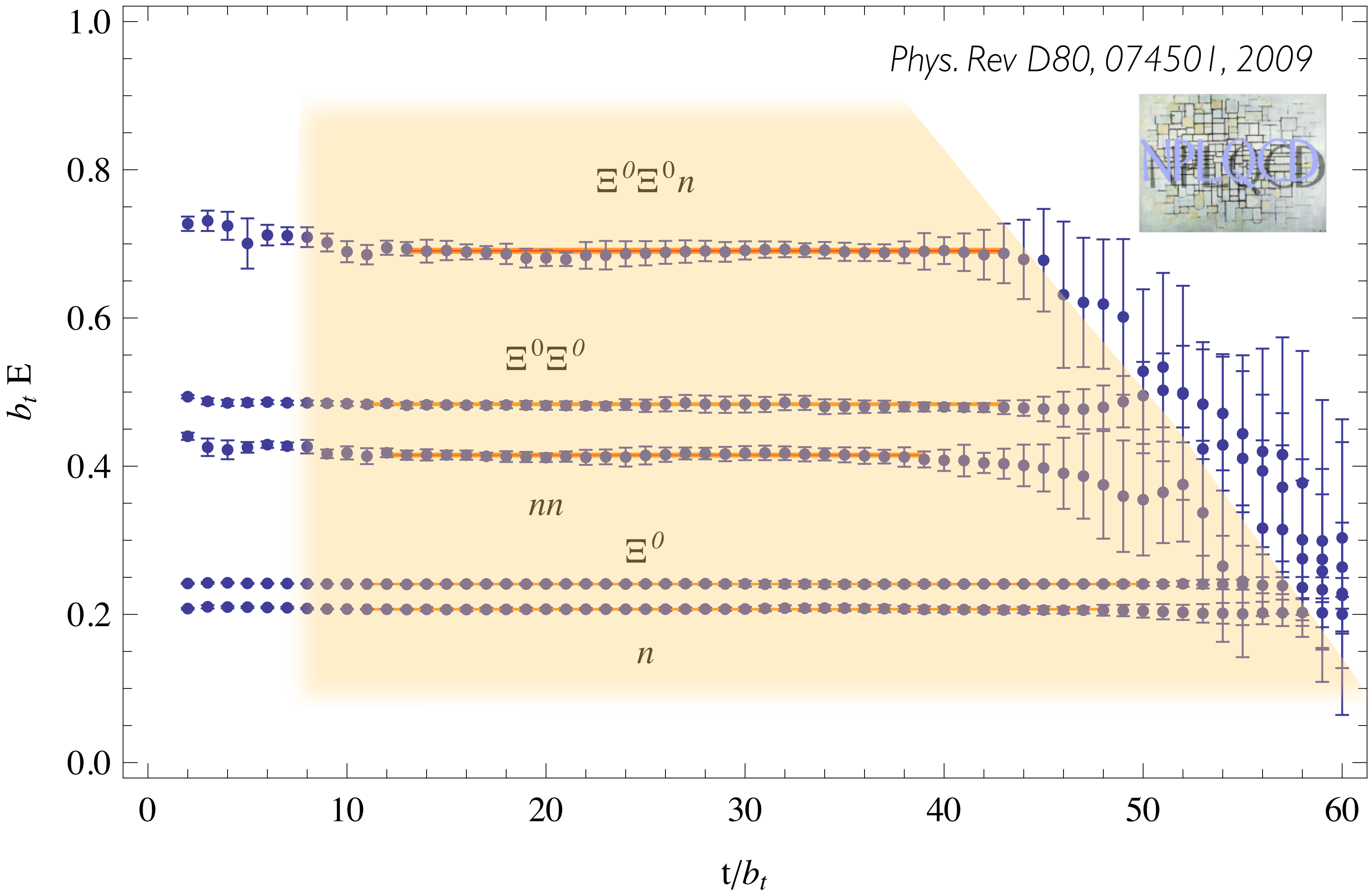

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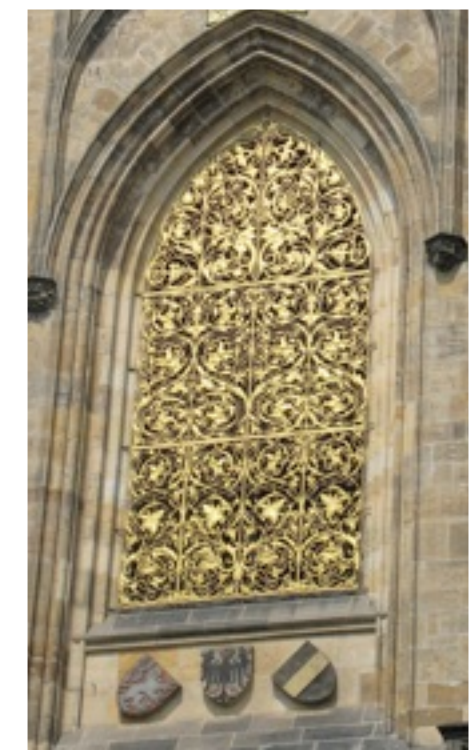

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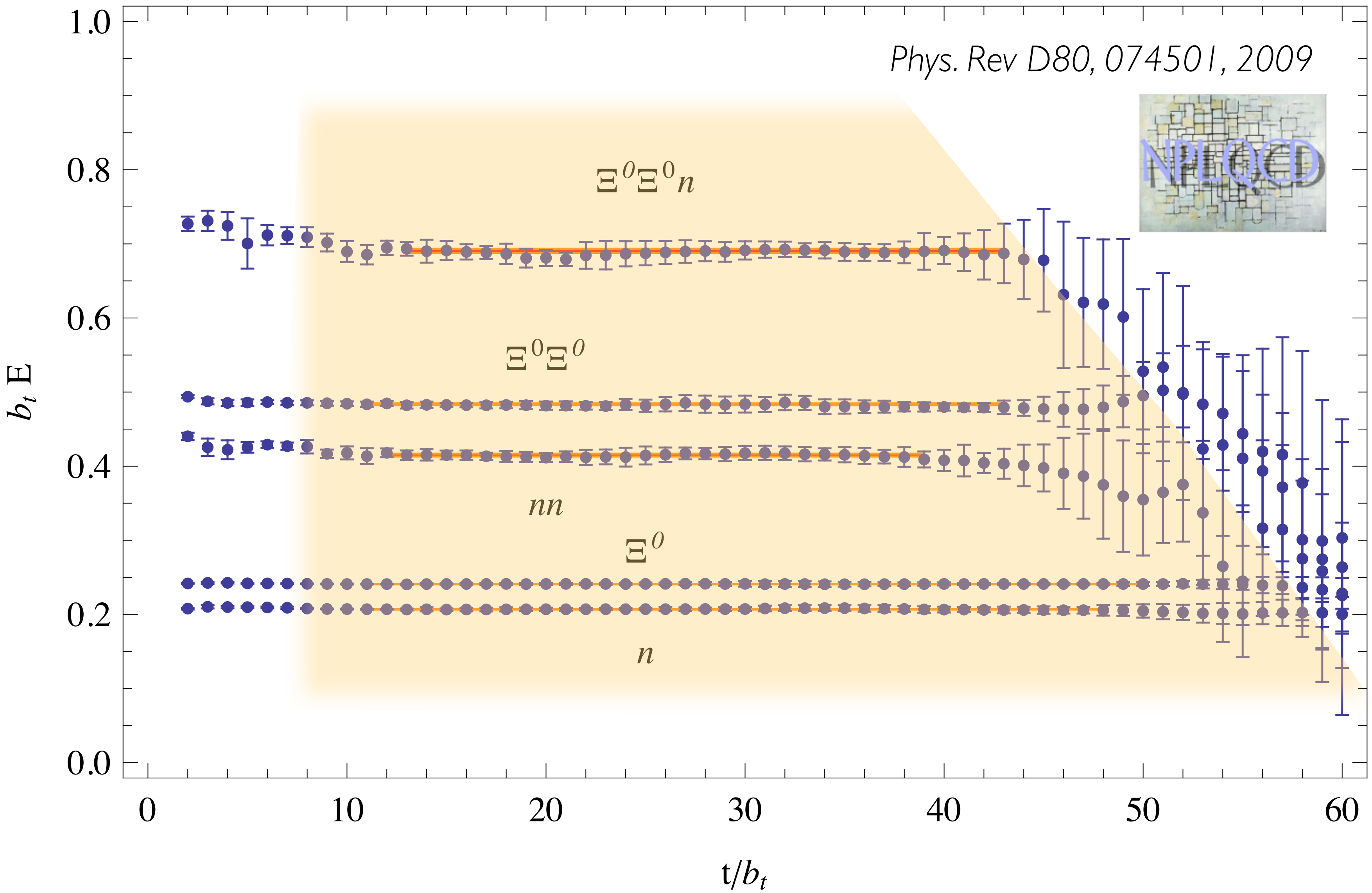

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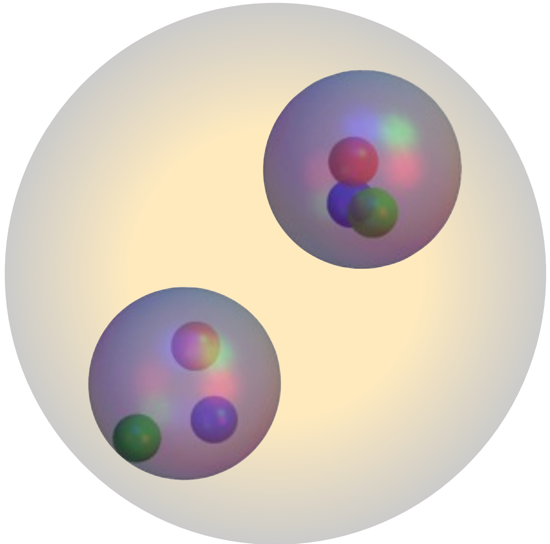
Interpolator choice can be used to suppress noise

Bound states at finite volume



- Focus on bound states
- Two particle scattering amplitude in infinite volume

$$\mathcal{A}(p) = \frac{8\pi}{M}\frac{1}{p\cot\delta(p) - ip}$$

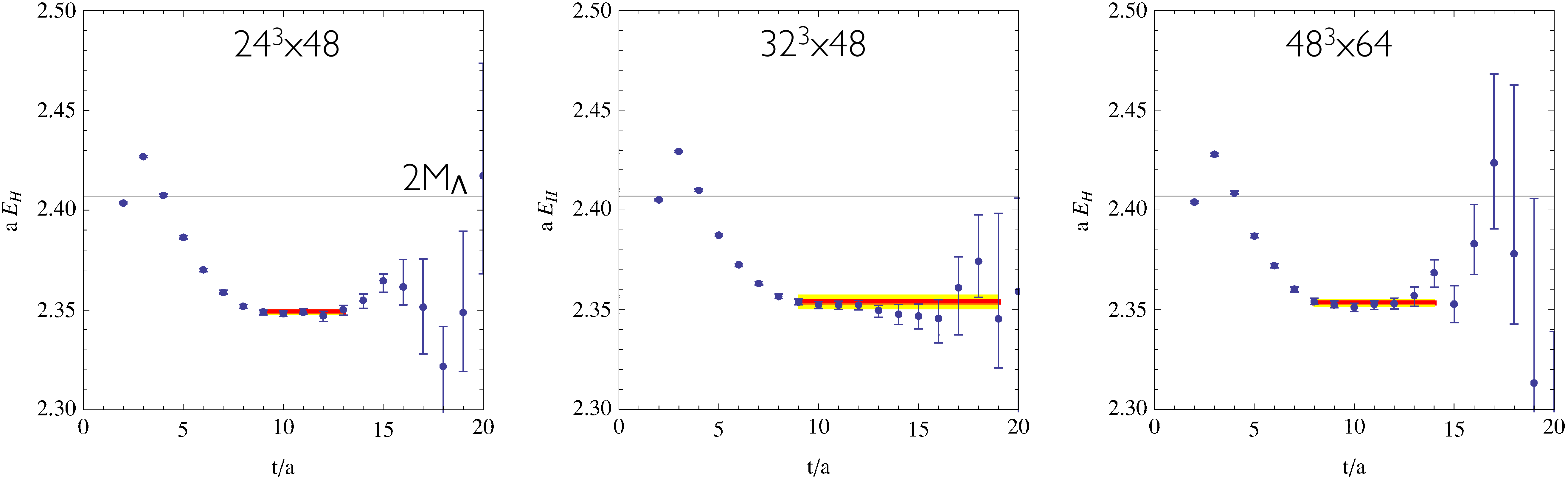
bound state at $p^2 = -\gamma^2$ when $\cot\delta(i\gamma) = i$

- Scattering amplitude in finite volume (Lüscher method) $\kappa \xrightarrow{L\to\infty} \gamma$

$$\cot\delta(i\kappa) = i - i\sum_{\vec{m}\neq 0}\frac{e^{-|\vec{m}|\kappa L}}{|\vec{m}|\kappa L}$$

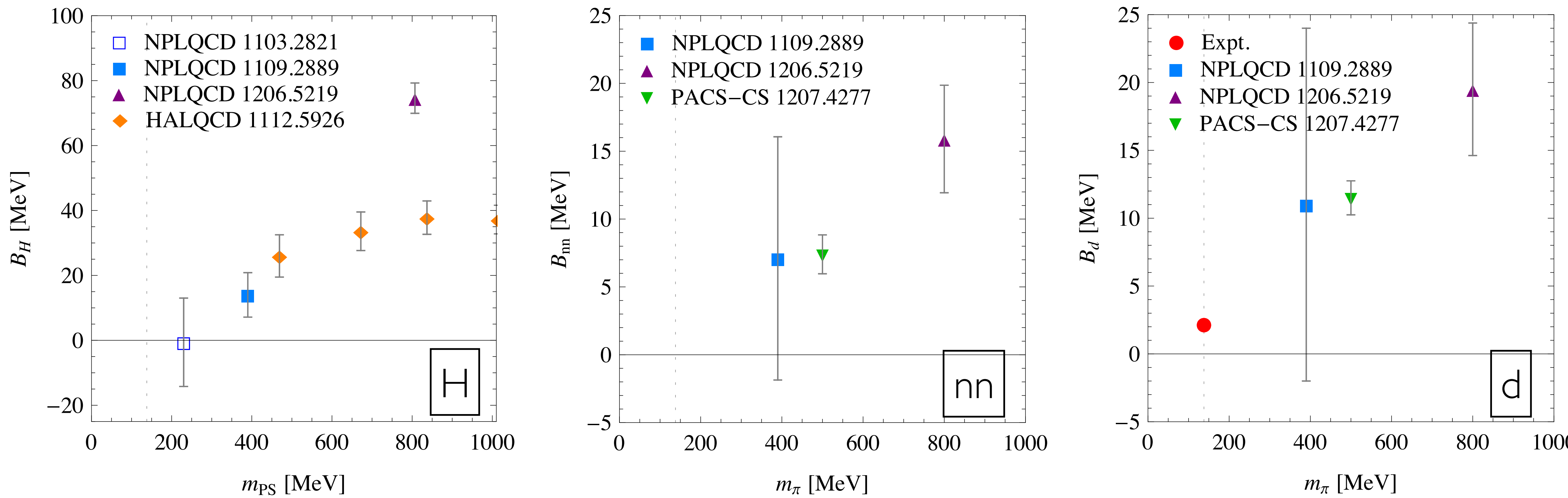
  - Need multiple volumes
- More complicated for n>2 body bound states

Ex: H dibaryon



- Effective mass plots of energy shifts
- First dibaryon bound state calculated in QCD [NPLQCD 2010]
- Multiple volumes needed to disentangle bound state from attractive scattering state

Dibaryons



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- More exotic channels also considered (**ΞΞ**, n**Ω** and **ΩΩ**)
- Clearly more work needed at lighter masses

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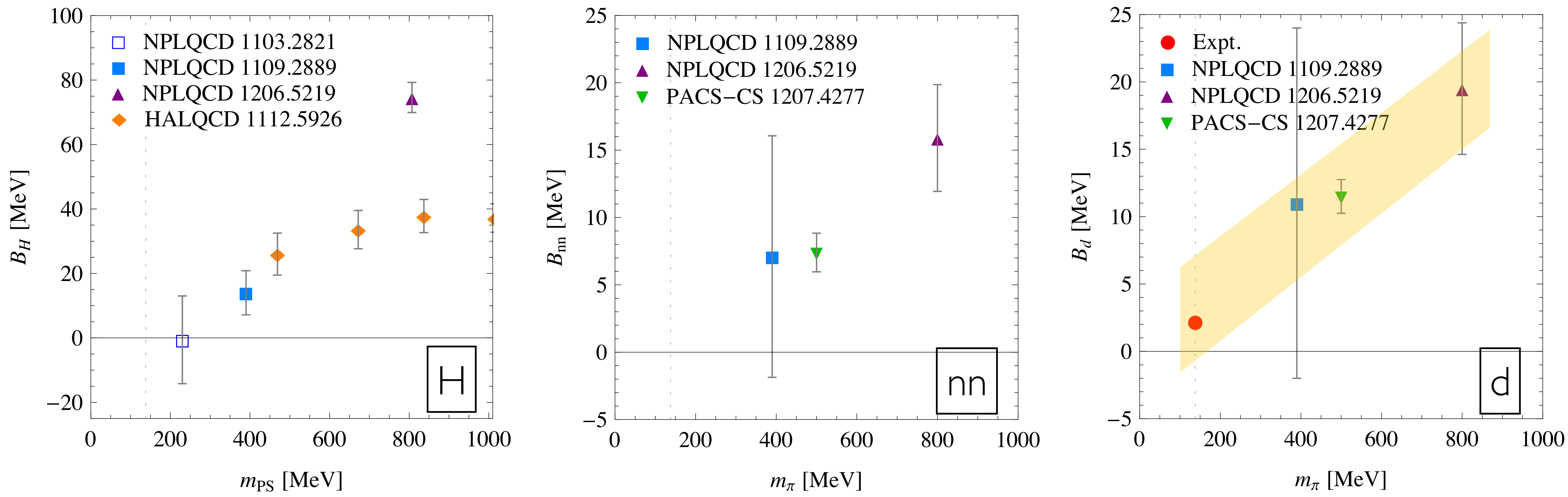

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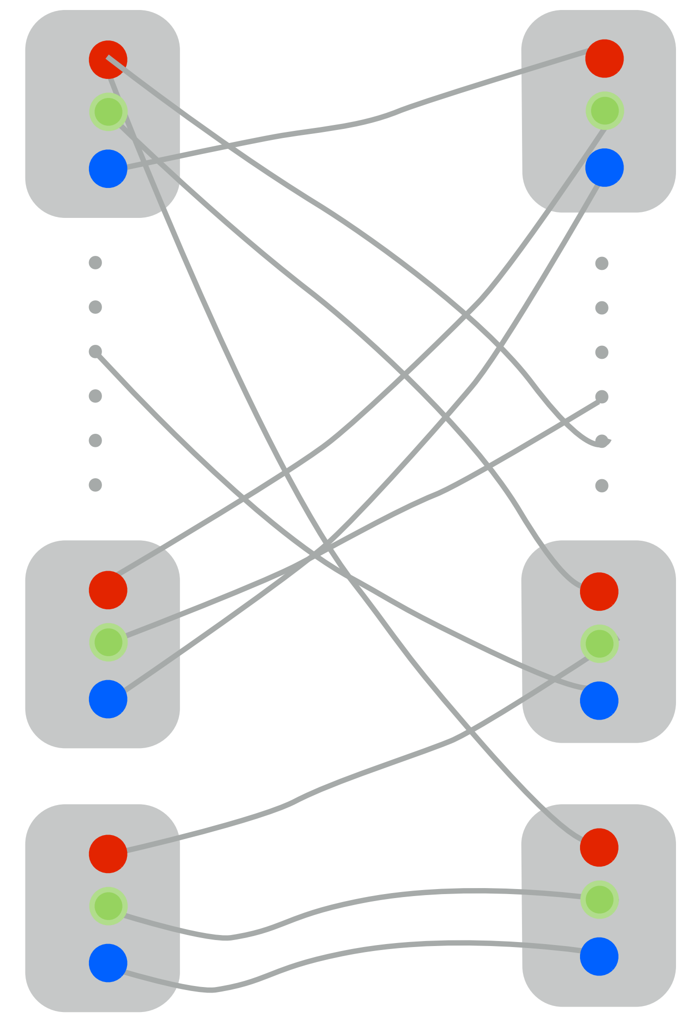
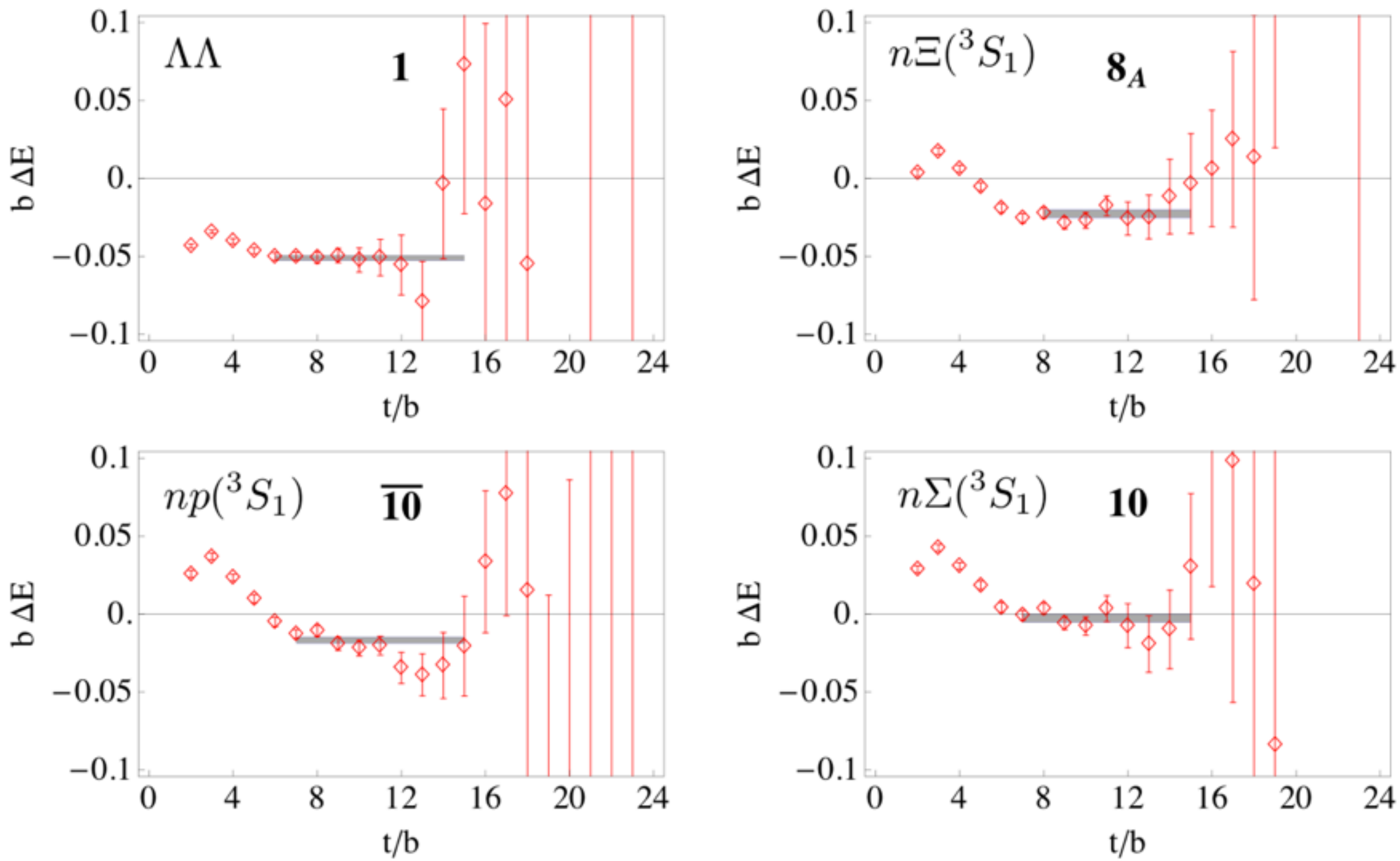
Many baryon systems

- Many baryon correlator construction is messy and expensive
  - Techniques learnt in many-pion studies
    [WD & M. Savage; WD,, K Orginos, Z. Shi]
  - New tricks
    [T. Doi & M. Endres.; WD, K Orginos; Gunther et al]
- Enables study of few (and many) baryon systems
- NPLQCD collaboration study
  - Unphysical SU(3) symmetric world @ m_s^{phys}
  - Multiple big volumes, single lattice spacing



Nuclei (A=2)



NPLQCD Phys.Rev. D87 (2013), 034506

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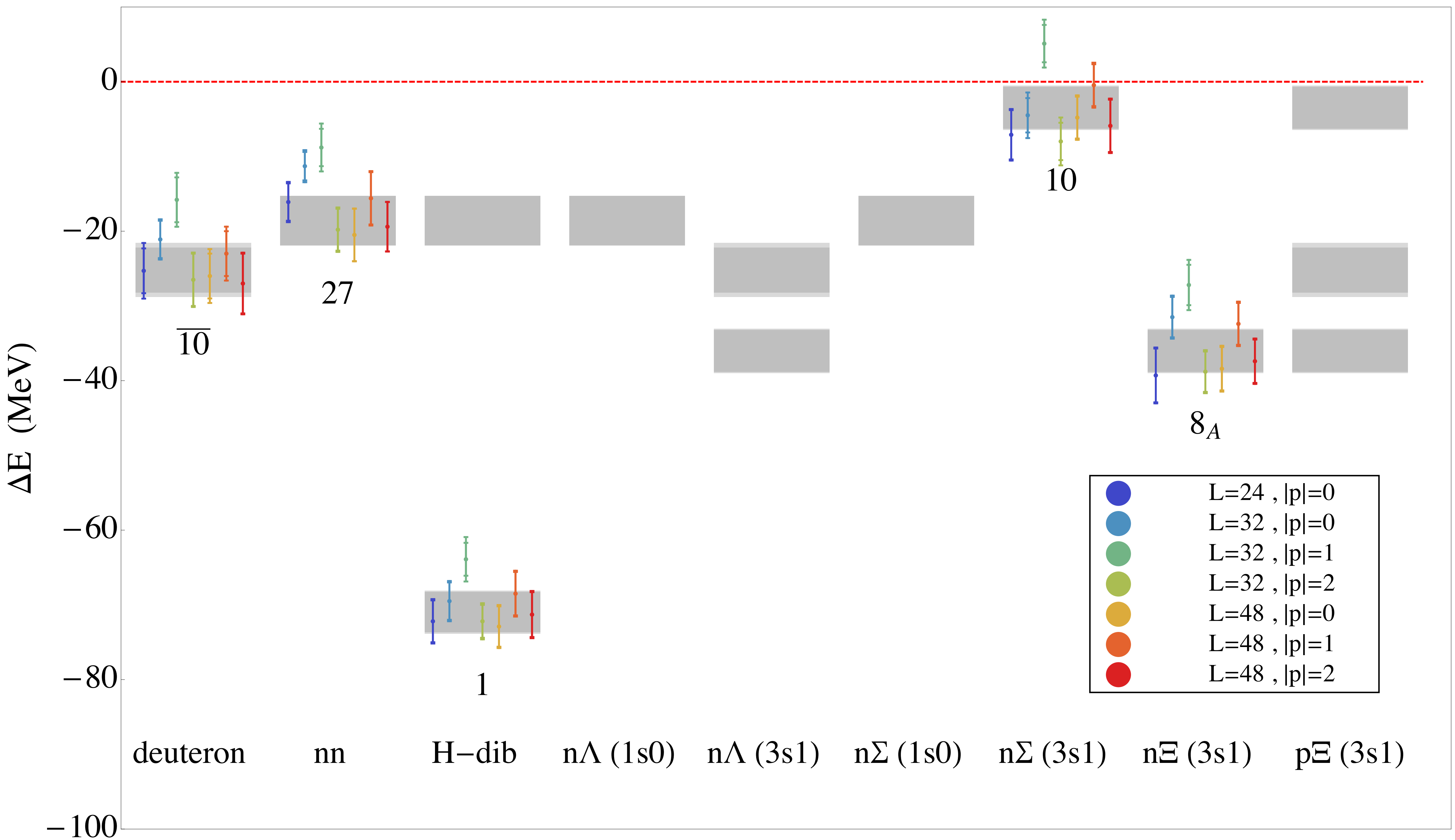

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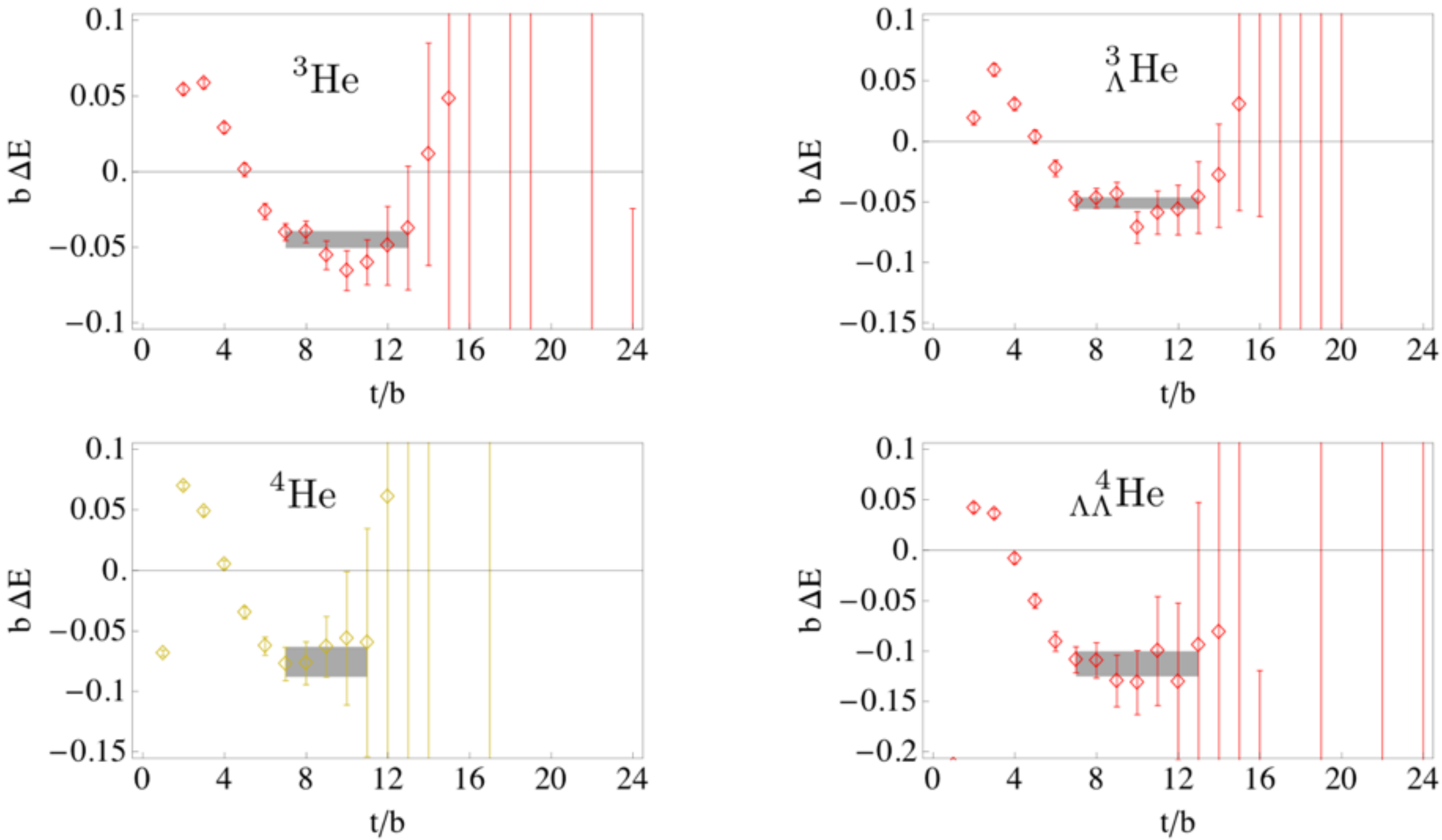
Nuclei (A=3,4)



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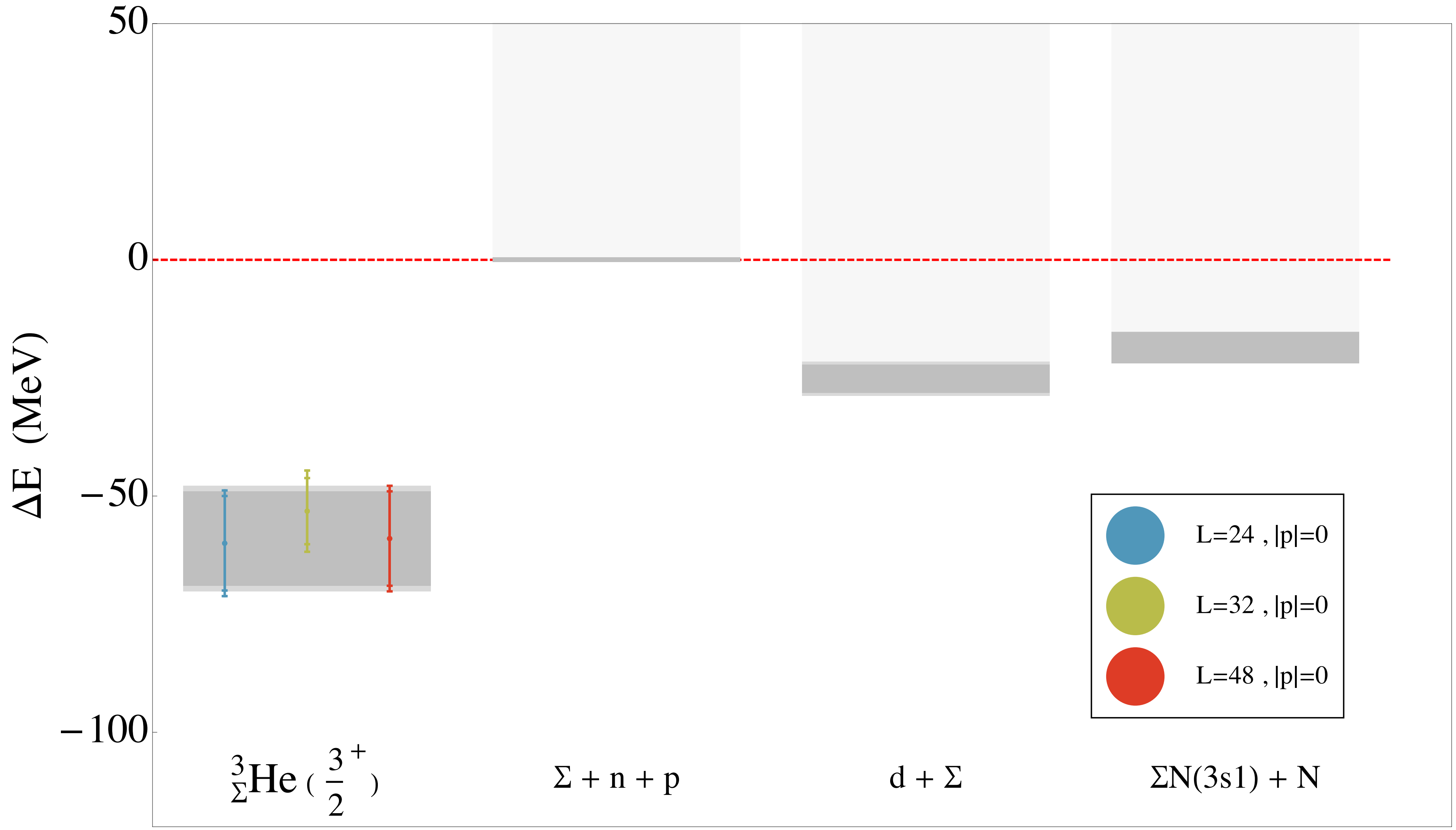
Nuclei (A=3,4)

- Empirically investigate volume dependence
- Need to ask if this is a 2+1 or 3+1 or 2+2 etc scattering state



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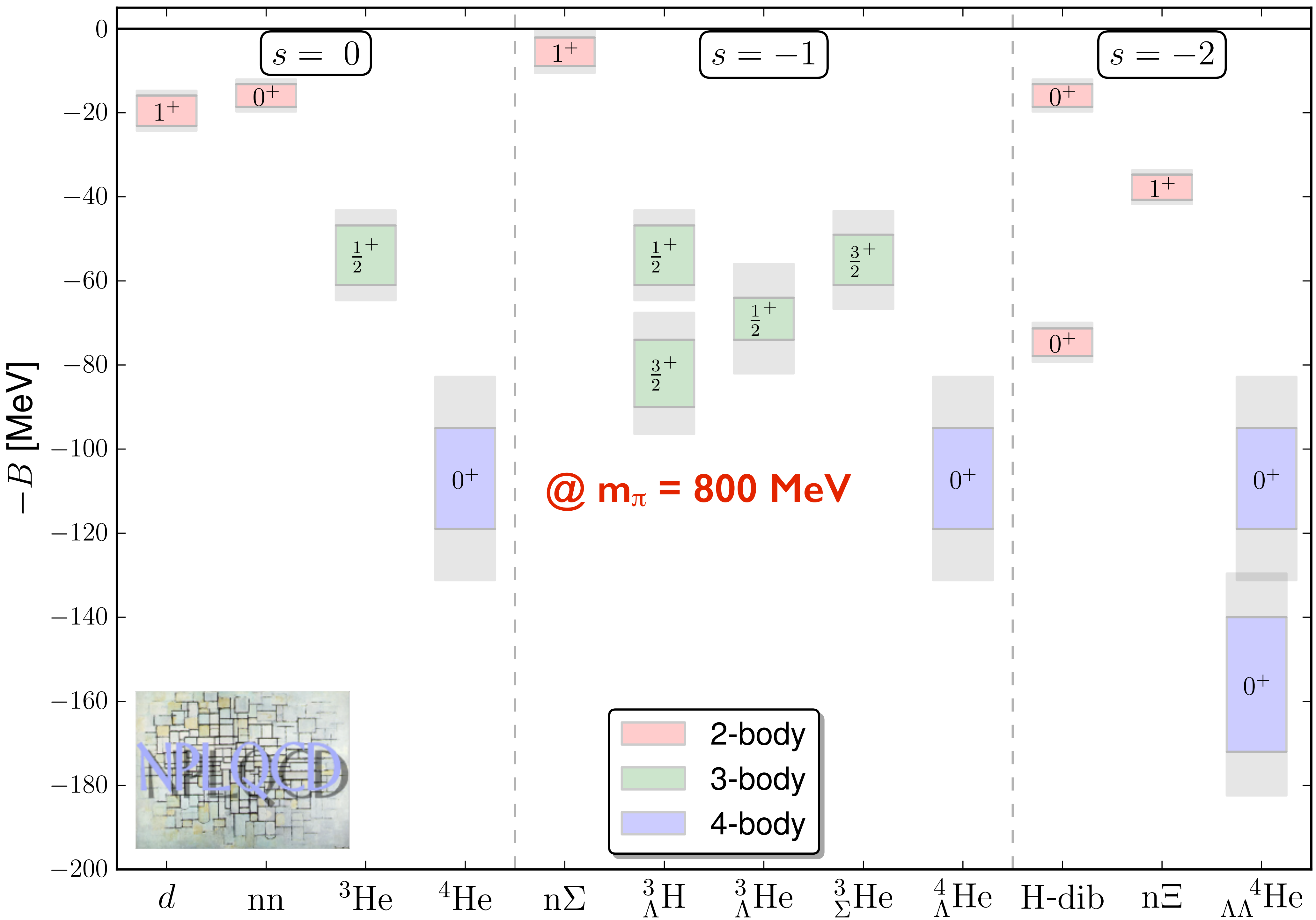

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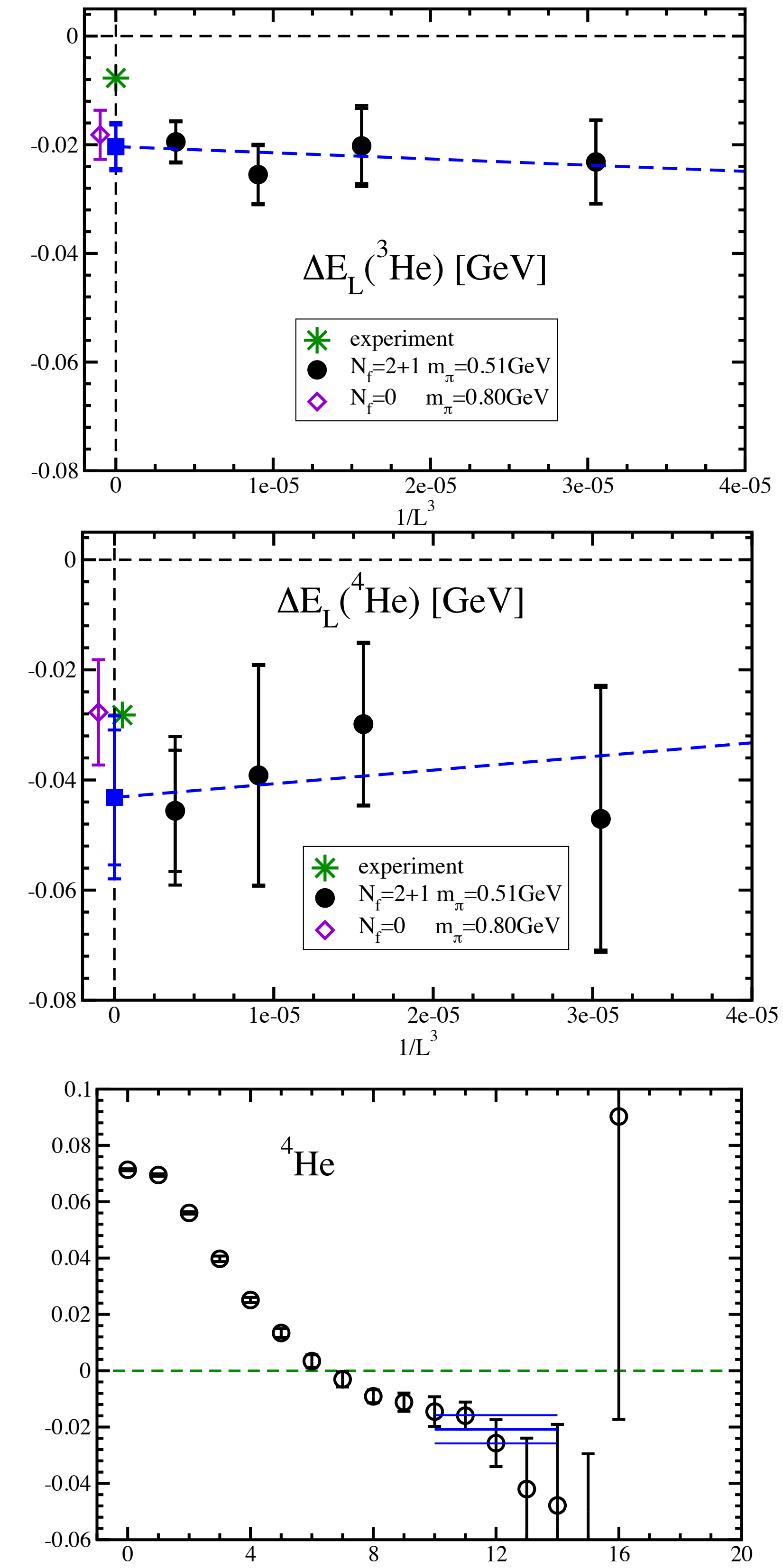
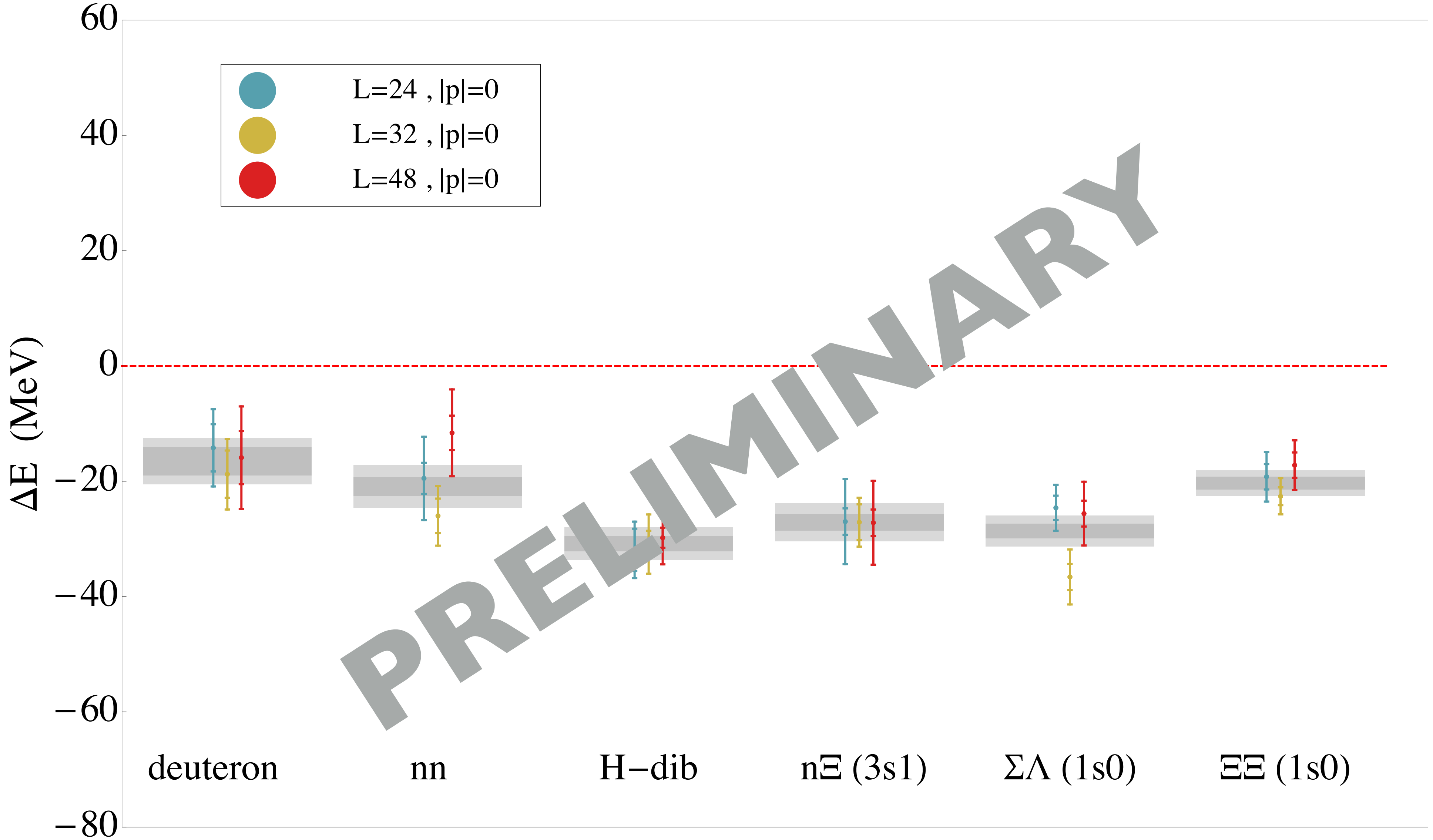
d, nn, 3He, 4He

- PACS-CS: bound d,nn, ^{3}He, ^{4}He
  - Previous quenched work
  - Unquenched study at m_π=500 MeV
  - Working on m_π=300 MeV [Lattice 2013]
- HALQCD
  - Extract an NN potential
  - Strong enough to bind H, ^{4}He at m_{PS}=490 MeV SU(3) pt
  - d, nn not bound

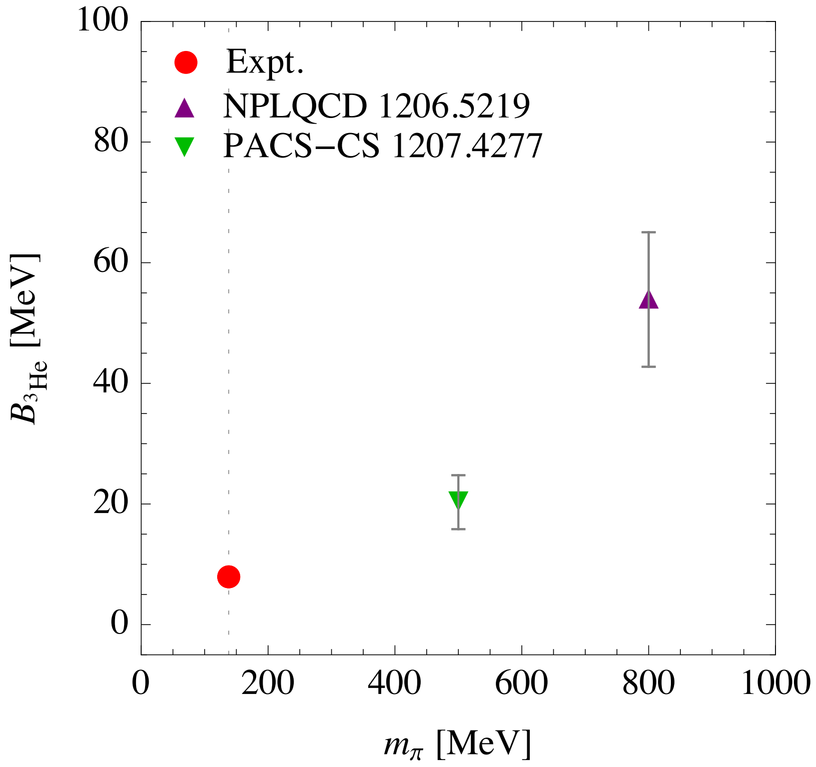


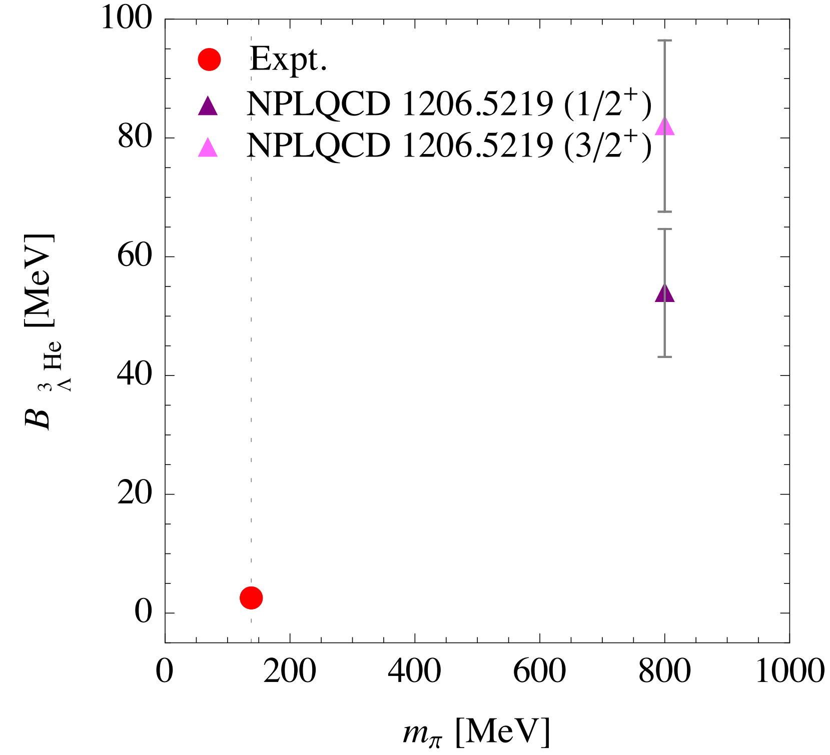
[Yamazaki et al. 1207.4277]

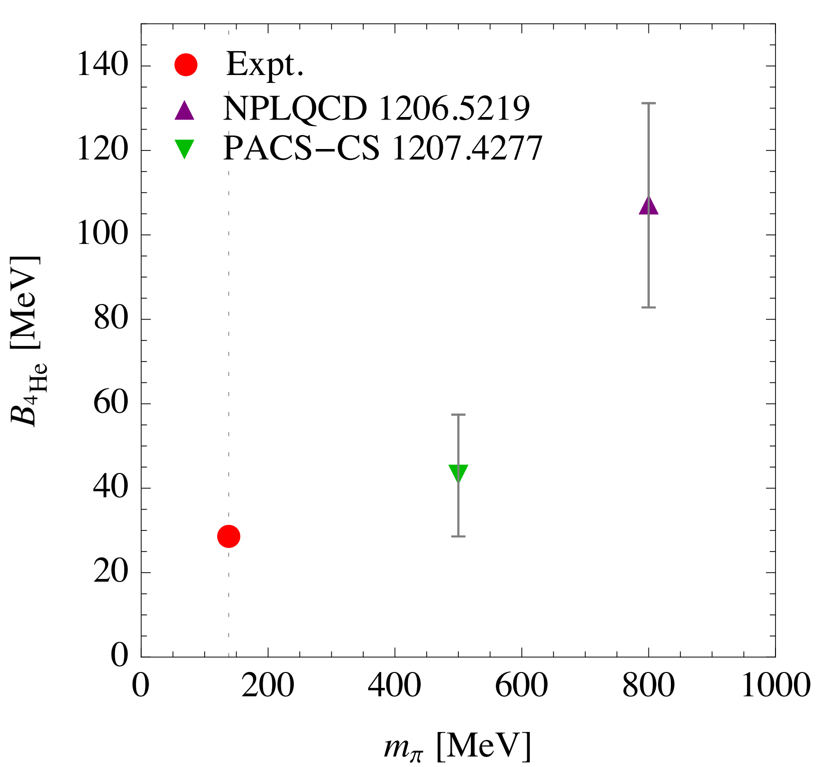
- Pion mass of ~400 MeV

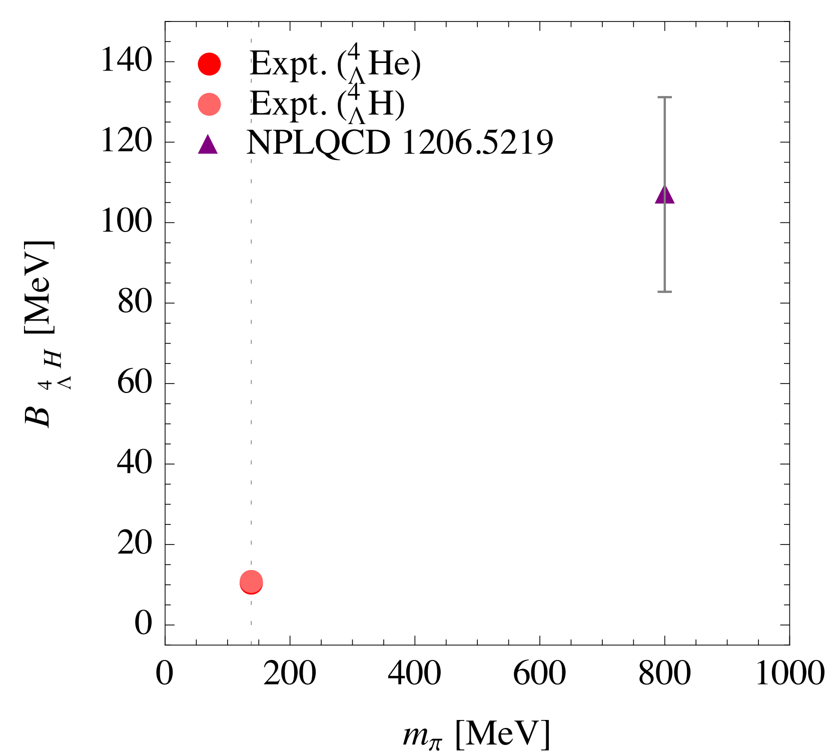


QCD Nuclei (s=0,-1)



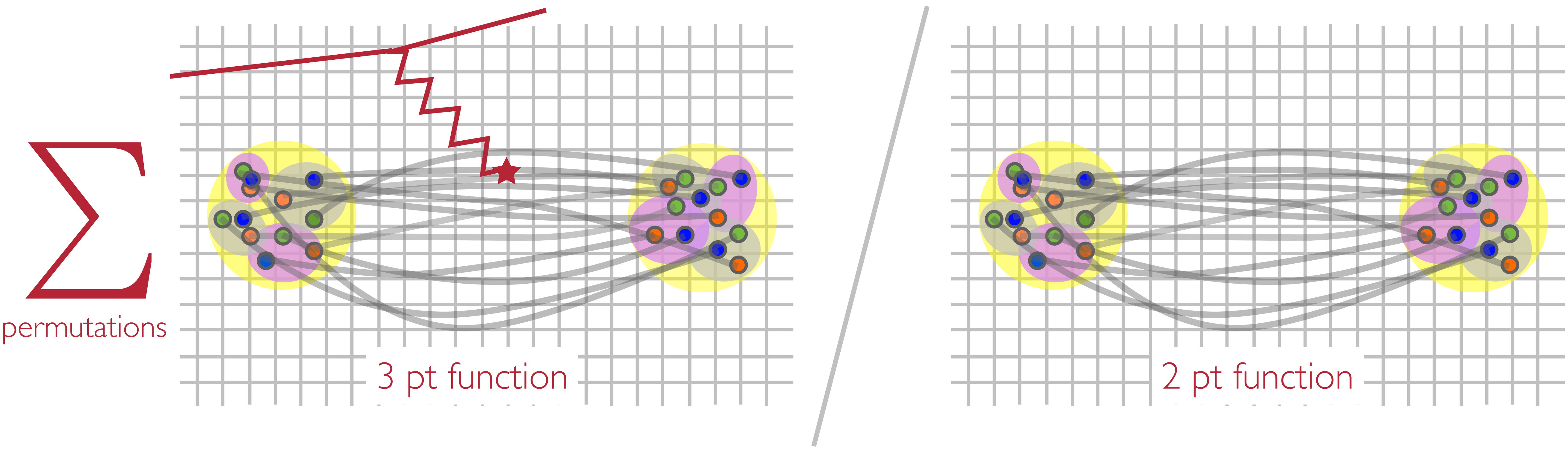






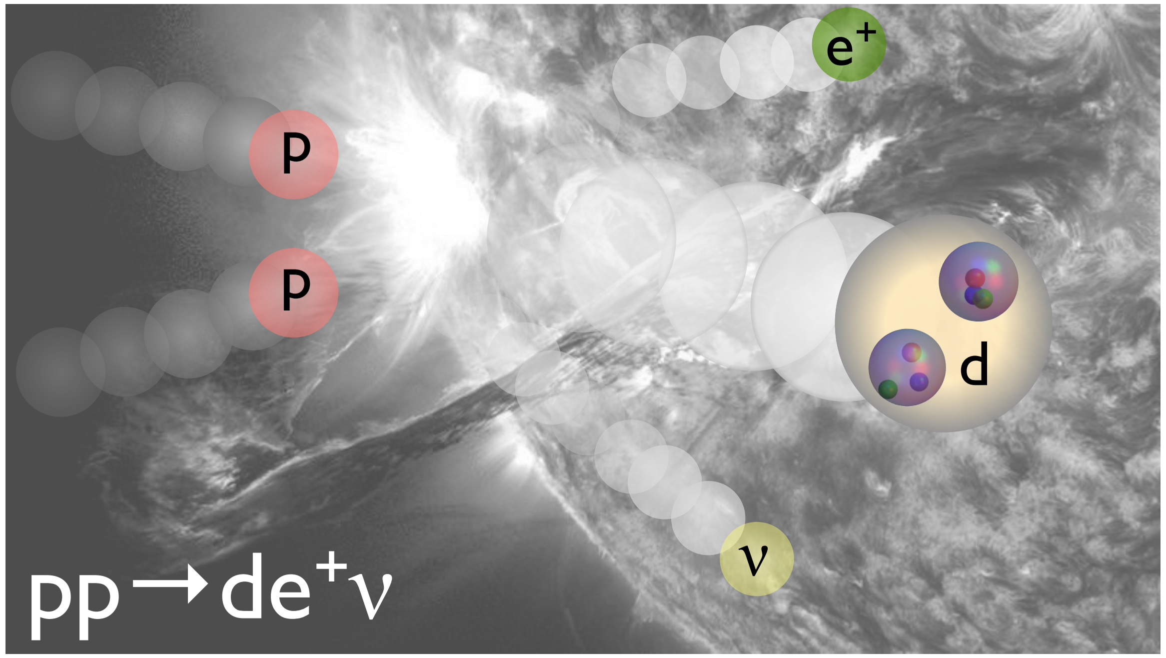
Nuclear matrix elements

- Calculations of matrix elements of currents in light nuclei just beginning for A<5
- For deeply bound nuclei, use the same techniques as for single hadron matrix elements



  - At large time separations gives matrix element of current
- For near threshold states, need to be careful with volume effects

Nuclear matrix elements

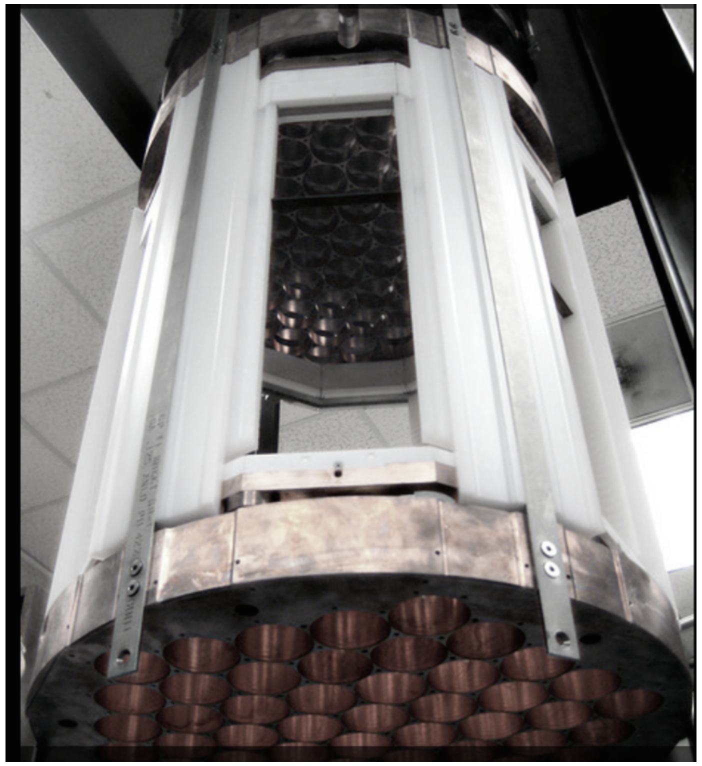


- Axial coupling to NN system
  - pp fusion: "Calibrate the sun"
  - Muon capture: MuSun @ PSI
  - $d\boldsymbol{\nu} \rightarrow n\,n\,e^{+}$: SNO
- Twist-2 operators: eg EMC effect

$$\langle N, Z|\bar{q}\gamma_{\{\mu_1}D_{\mu_2}\ldots D_{\mu_n\}}q|N, Z\rangle$$

  - Proof of principle (moments of pion PDF in pion gas) [WD, HW Lin 1112.5682]

Nuclear sigma terms



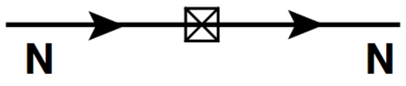
- Dark matter direct detection experiments look for DM interactions with nuclei (Si, Xe, ...)
- One possible interaction is through scalar exchange

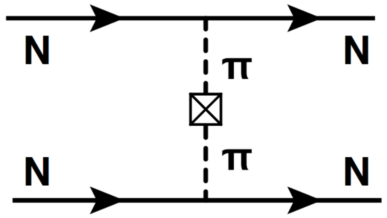
$$\mathcal{L} = \frac{G_F}{2} \sum_q a_S^{(q)} (\overline{\chi}\,\chi)(\overline{q}\,q)$$

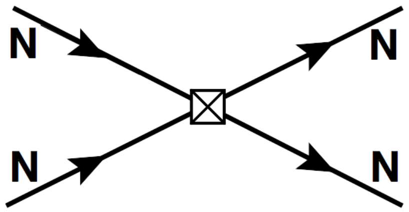
- Accessible via Feynman-Hellman theorem
- At hadronic/nuclear level

$$\mathcal{L} \to G_F\,\overline{\chi}\chi\,\Big(\frac{1}{4}\langle 0|\overline{q}q|0\rangle\ \mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right]\ +\ \frac{1}{4}\langle N|\overline{q}q|N\rangle N^\dagger N \mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right]$$
$$-\ \frac{1}{4}\langle N|\overline{q}\tau^3 q|N\rangle\left(N^\dagger N\mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right]\ -\ 4N^\dagger a_{S,\xi}N\right)\ +\ \ldots\Big)$$

- Contributions:







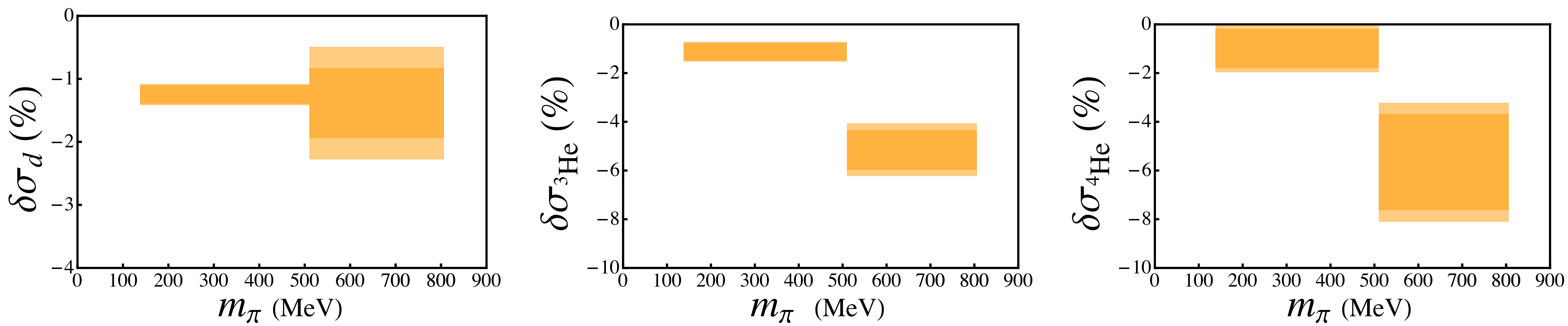
Nuclear sigma terms

- Previous work suggested scalar dark matter couplings to nuclei have O(50%) uncertainty arising from MECs [Prezeau et al 2003]

- Quark mass dependence of nuclear binding energies bounds such contributions

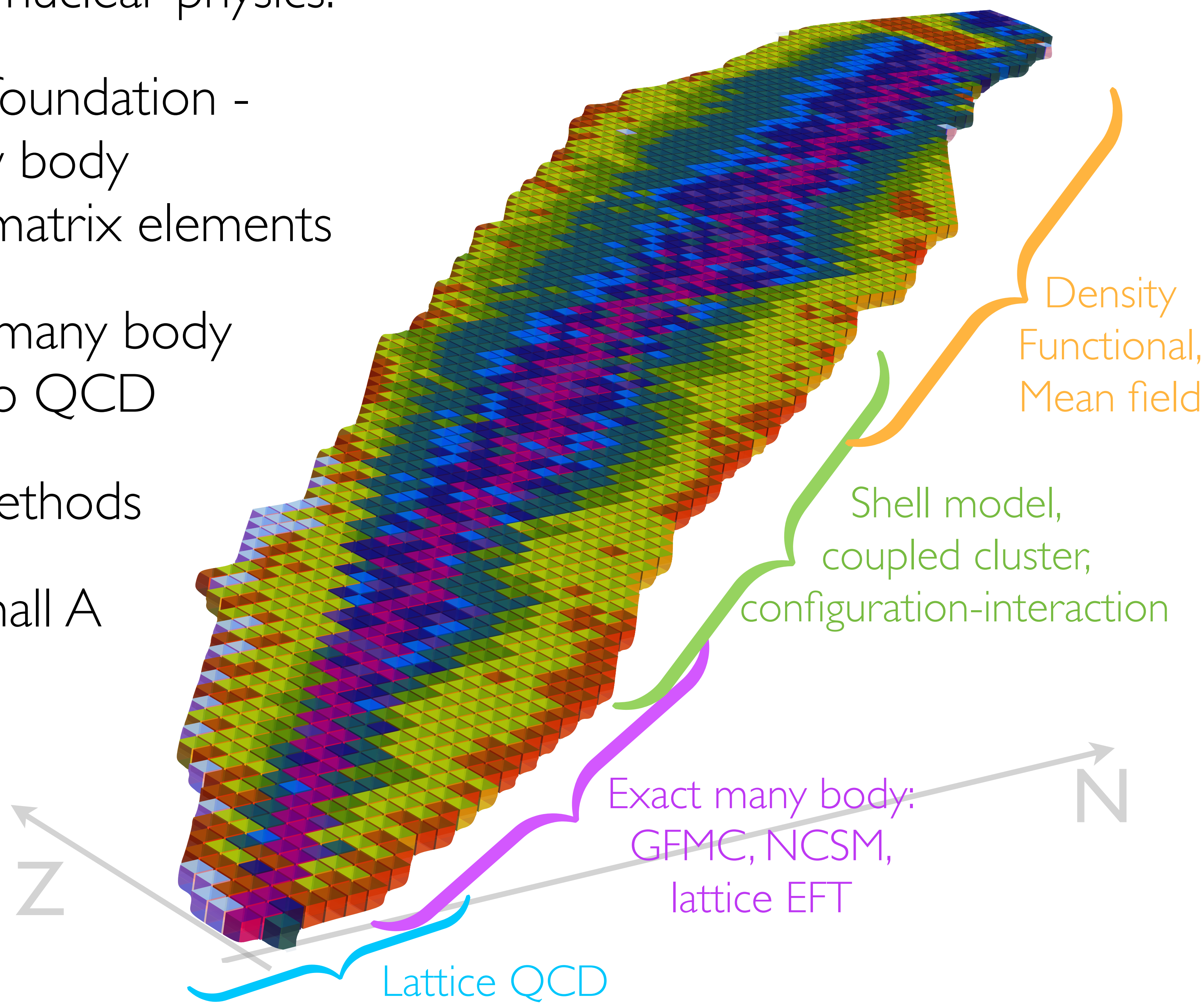
$$\delta\sigma_{Z,N} = \frac{\langle Z,N(\mathrm{gs})|\ \overline{u}u+\overline{d}d|Z,N(\mathrm{gs})\rangle}{A\ \langle N|\ \overline{u}u+\overline{d}d|N\rangle} - 1 = -\frac{1}{A\sigma_N}\frac{m_\pi}{2}\frac{d}{dm_\pi}B_{Z,N}$$

  - Lattice calculations + physical point suggest such contributions are O(10%) or less for light nuclei (A<4)



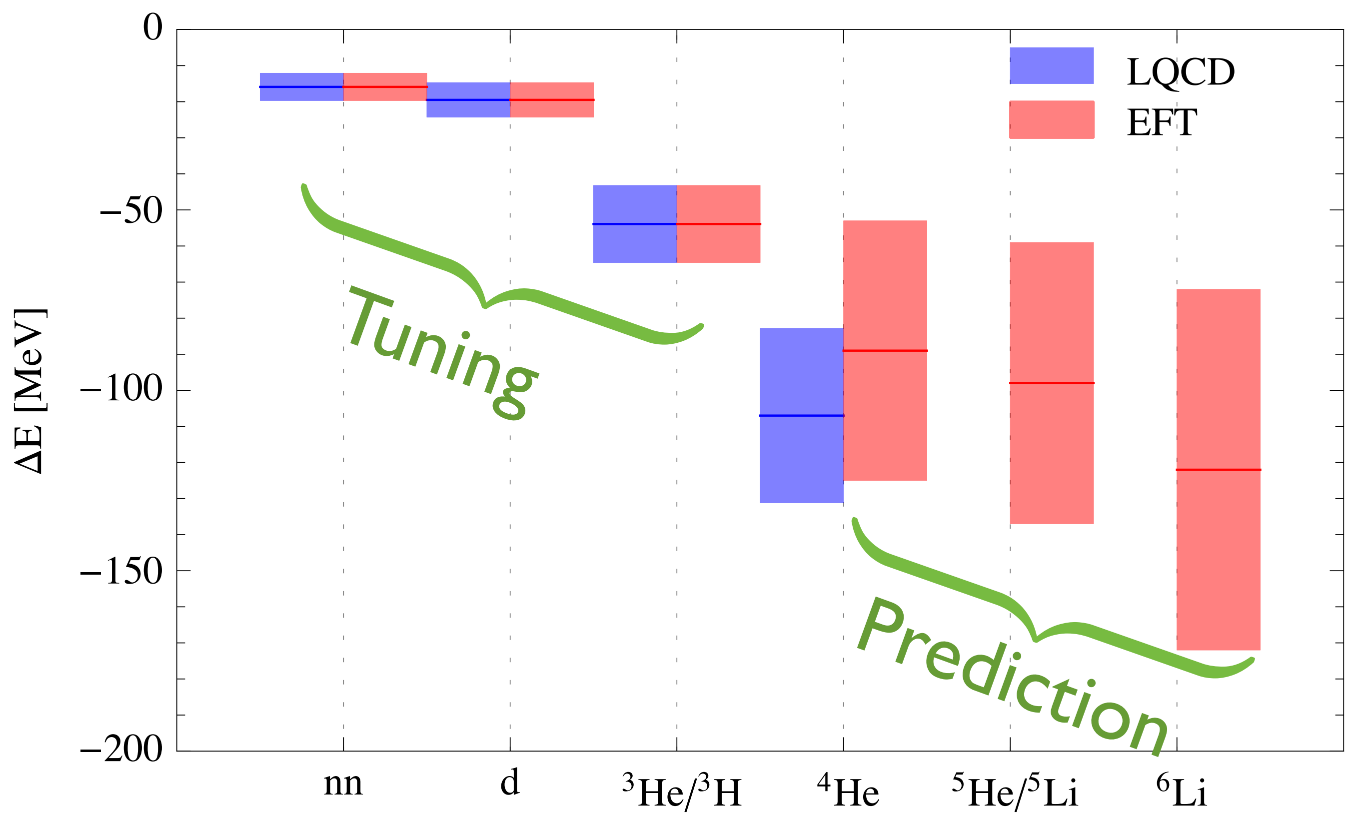
[NPLQCD PRD to appear]

- A path to *ab initio* nuclear physics:
  - QCD forms a foundation - determines few body interactions & matrix elements
  - Match existing many body techniques onto QCD
  - Hierarchy of methods
- QCD: focus on small A
  - ... for now ...



[Barnea, et al. 1311.4966]

- Already seeing LQCD and nuclear EFT coming together
- For heavy quarks, even spectroscopy requires QCD matching



In a world @ m_π = 800 MeV

- Equally important for matrix elements at the physical quark mass

Matrix elements

- Power counting of nuclear effective field theory:
  - 1-body currents are dominant
  - 2-body currents are sub-leading *but non-negligible*
    Higher-body currents are even less important
- Determine one body contributions from single nucleon
- Determine few-body contributions from A=2,3,4...
- Match EFT and many body methods to LQCD to extend to larger nuclei

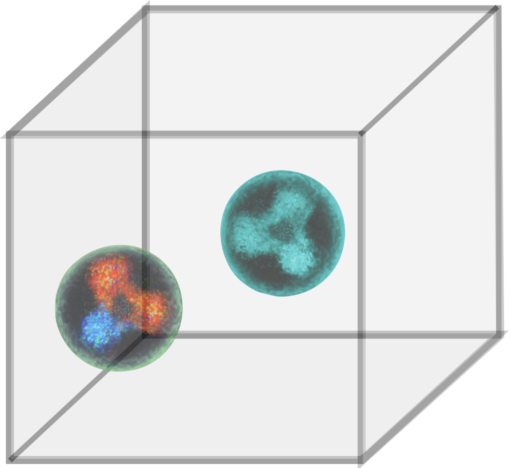
The road ahead...

- What does the future hold?
  - Physical quark masses, isospin breaking, E&M
  - Spectroscopy
    - Precision YN, YY phase shifts
    - p-shell and larger nuclei
    - Three body information: nnn, YNN, ...
    - Nuclear reactions(?): eg d+d in 4He channel
  - Properties of light nuclei (moments/structure) and electroweak interactions

fin

Acknowledgements

Silas Beane, Emmanuel Chang, Saul Cohen, Parry Junnarkar, Huey-wen Lin, Tom Luu, Kostas Orginos, Assumpta Parreño, Martin Savage, Andre Walker-Loud



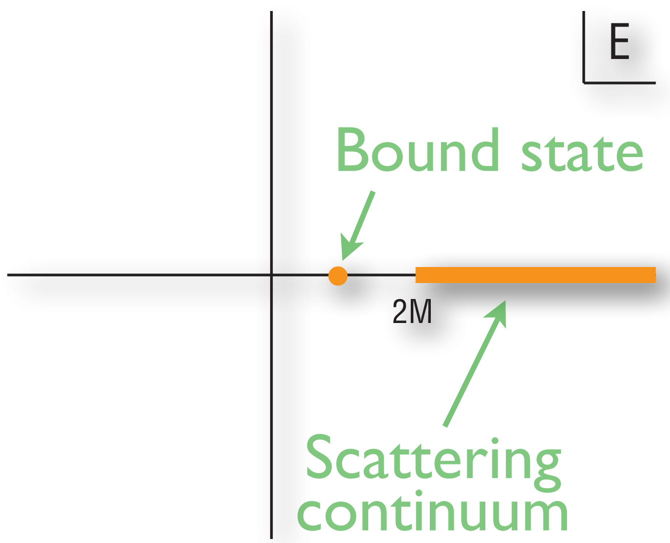
- Maiani-Testa: extracting multi-hadron S-matrix elements from Euclidean lattice calculations of Green functions in infinite volume is impossible

- Lüscher: volume dependence of two-particle energy levels
  ⇒ scattering phase-shift, **δ**(p), up to inelastic threshold

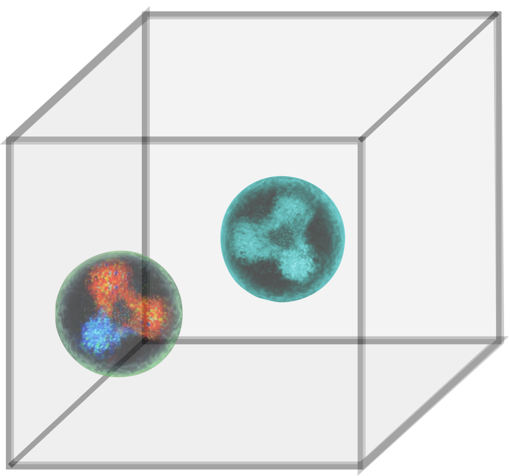
$$\Delta E_{(n)} = \sqrt{|\mathbf{q}_{(n)}|^2 + m_A^2} + \sqrt{|\mathbf{q}_{(n)}|^2 + m_B^2} - m_A - m_B$$

$$q_{(n)} \cot \delta(q_{(n)}) = \frac{1}{\pi\, L} S\left(\frac{q_{(n)} L}{2\pi}\right)$$

$$S(\eta) = \lim_{\Lambda\to\infty}\left[\sum_{\vec{n}}^{|\vec{n}|<\Lambda} \frac{1}{|\vec{n}|^2 - \eta^2} - 4\pi\Lambda\right]$$



Hadron scattering



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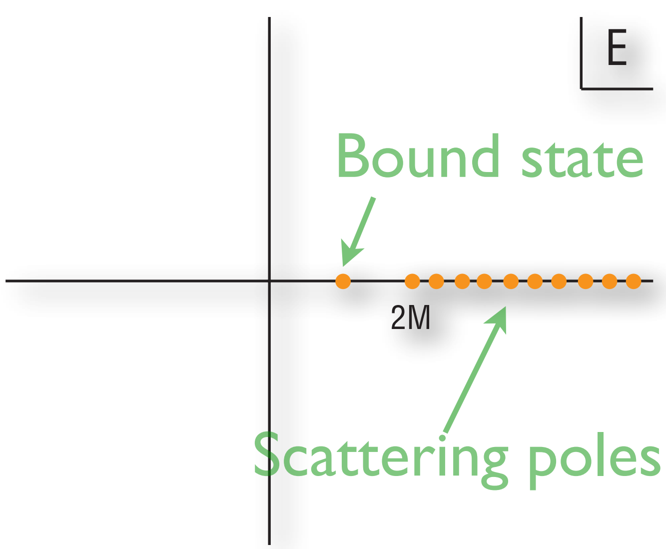

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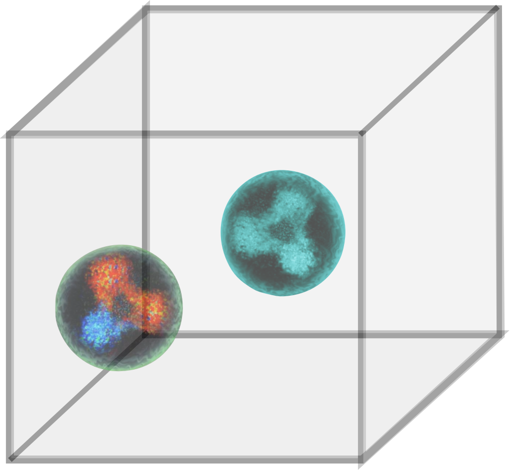

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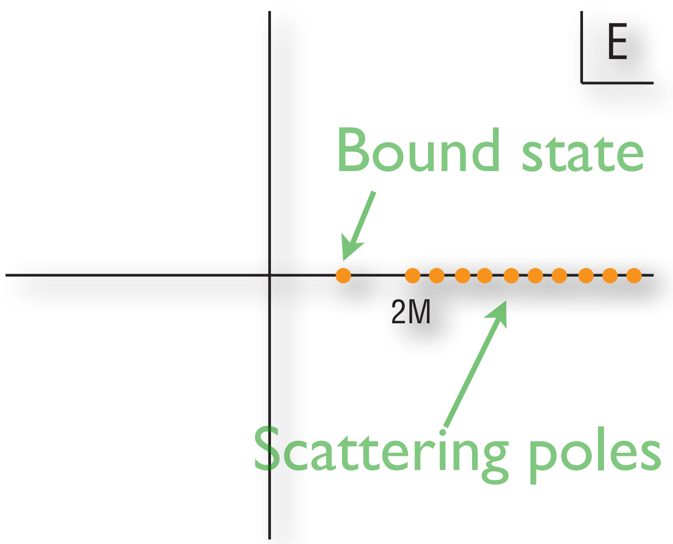
- Lüscher: volume dependence of two-particle energy levels
  ⇒ scattering phase-shift, δ(p), up to inelastic threshold
- Exact relation provided r«L
- Used for ππ, KK, ...
  - A precision science for stretched states
- Known for many years in QM, NP



NN phase shifts

[NPLQCD 1301.5790]

- Recent calculation of NN phase shifts at m_π=800 MeV
- Scattering length and effective range extracted with O(10%) precision

$$a^{(^3S_1)} = 1.82^{+0.14+0.17}_{-0.13-0.12}\ \mathrm{fm} \qquad r^{(^3S_1)} = 0.906^{+0.068+0.068}_{-0.075-0.084}\ \mathrm{fm}$$

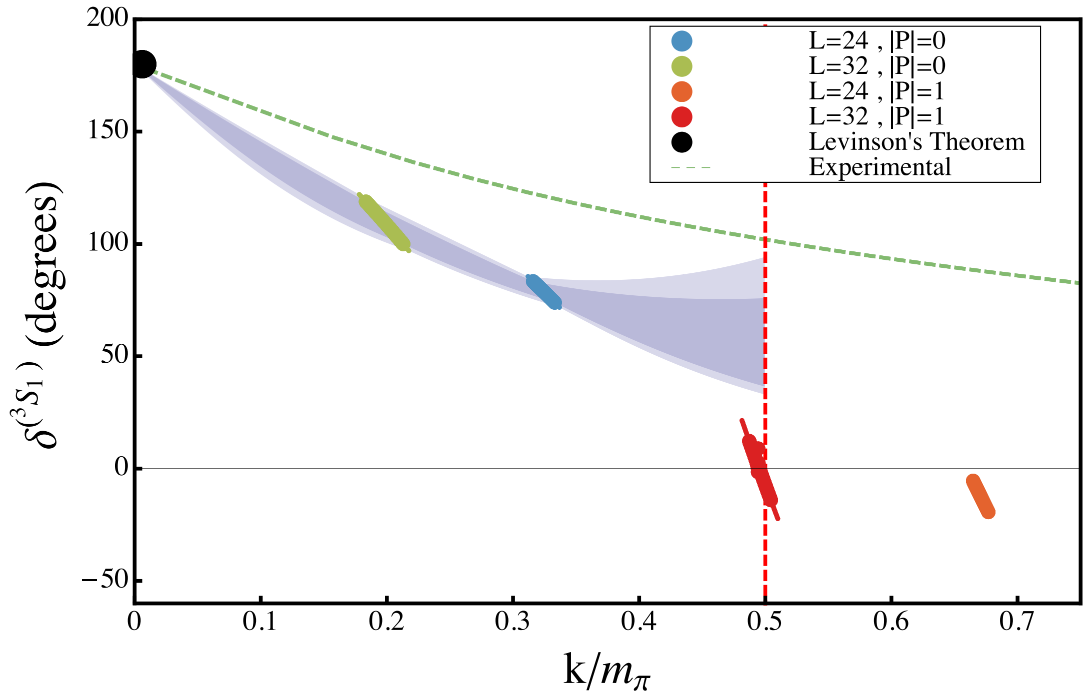
$$a^{(^1S_0)} = 2.33^{+0.19+0.27}_{-0.17-0.20}\ \mathrm{fm} \qquad r^{(^1S_0)} = 1.130^{+0.071+0.059}_{-0.077-0.063}\ \mathrm{fm}$$

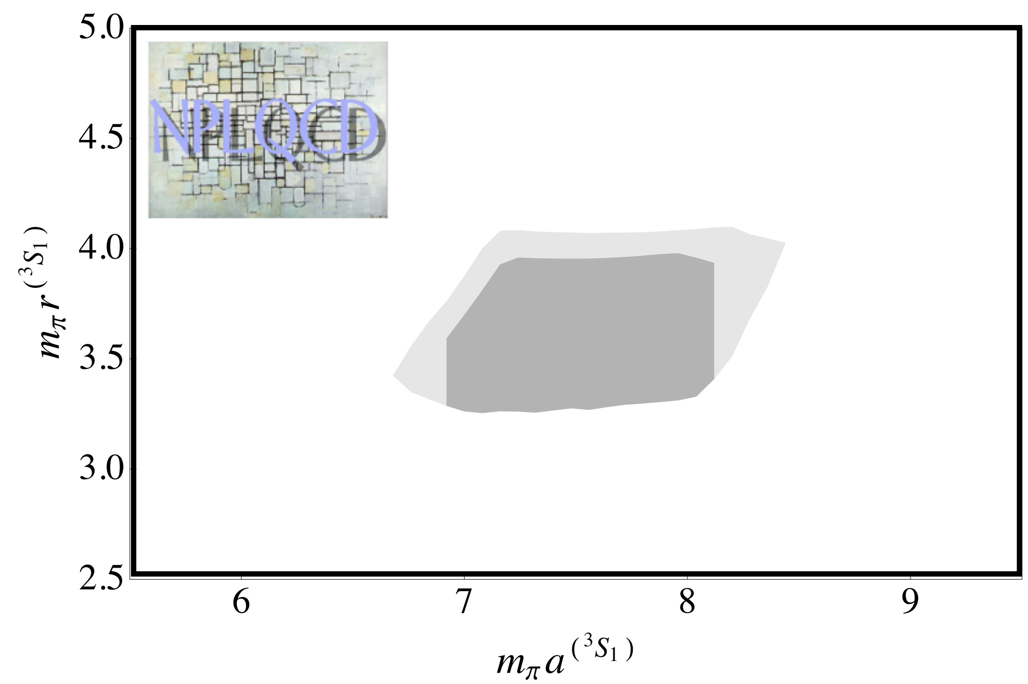
- Fine-tuning of NN at physical mass?

$$a^{(^3S_1)}/r^{(^3S_1)} = 2.06^{+0.22+0.25}_{-0.18-0.19}$$

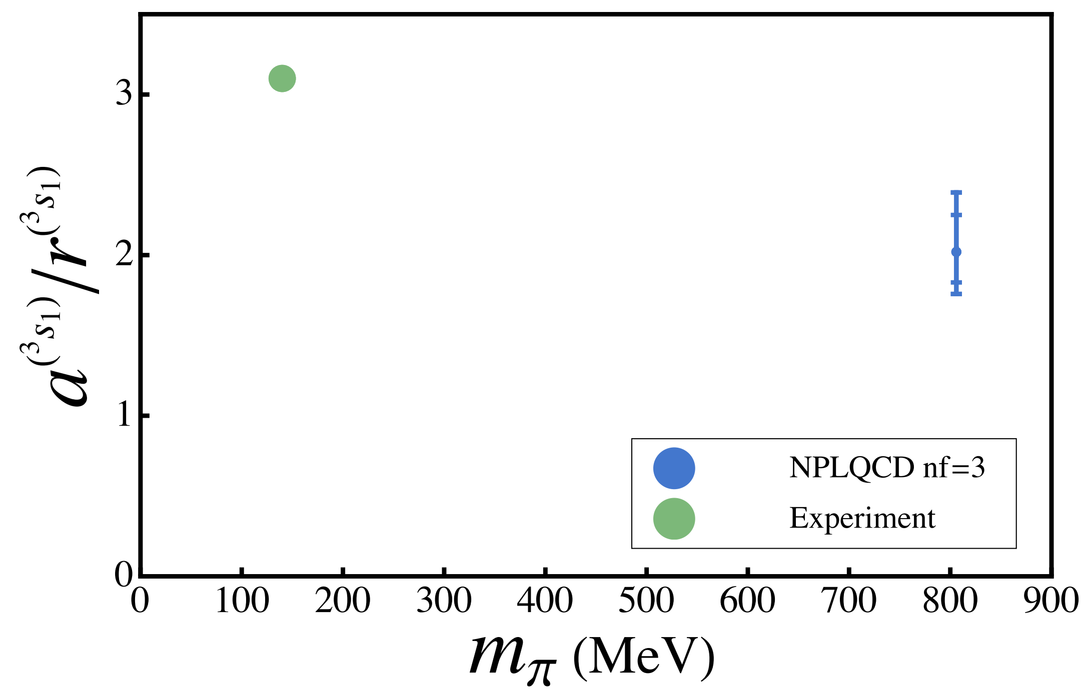
$$a^{(^1S_0)}/r^{(^1S_0)} = 2.02^{+0.23+0.29}_{-0.19-0.18}$$

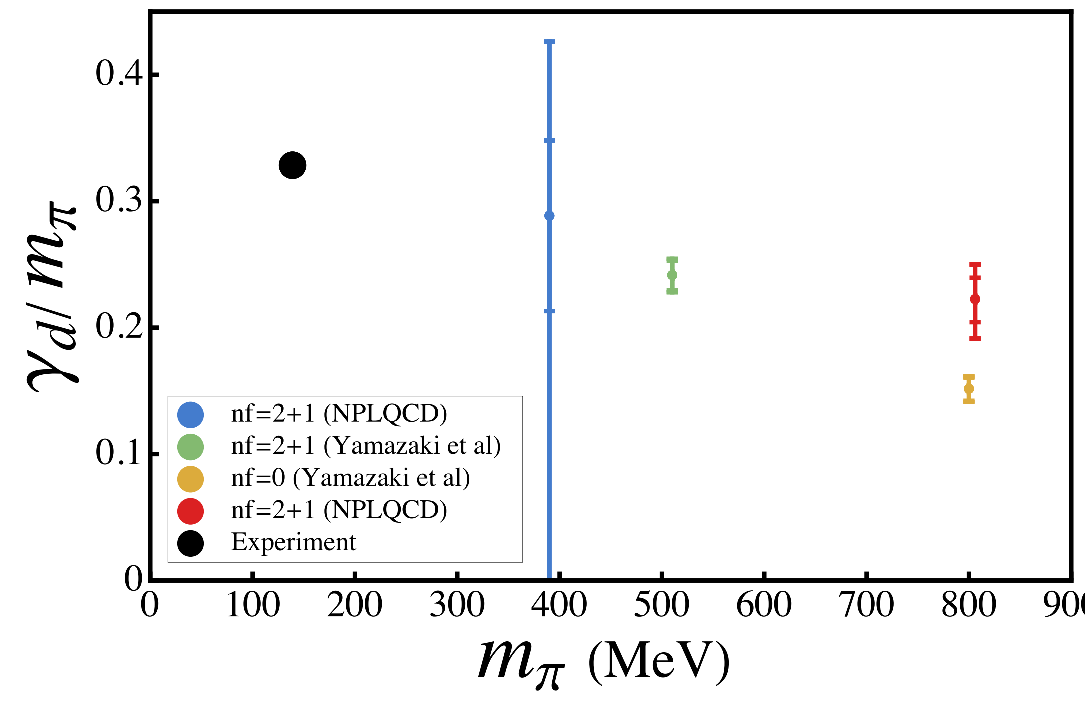
- Wigner SU(4) symmetry

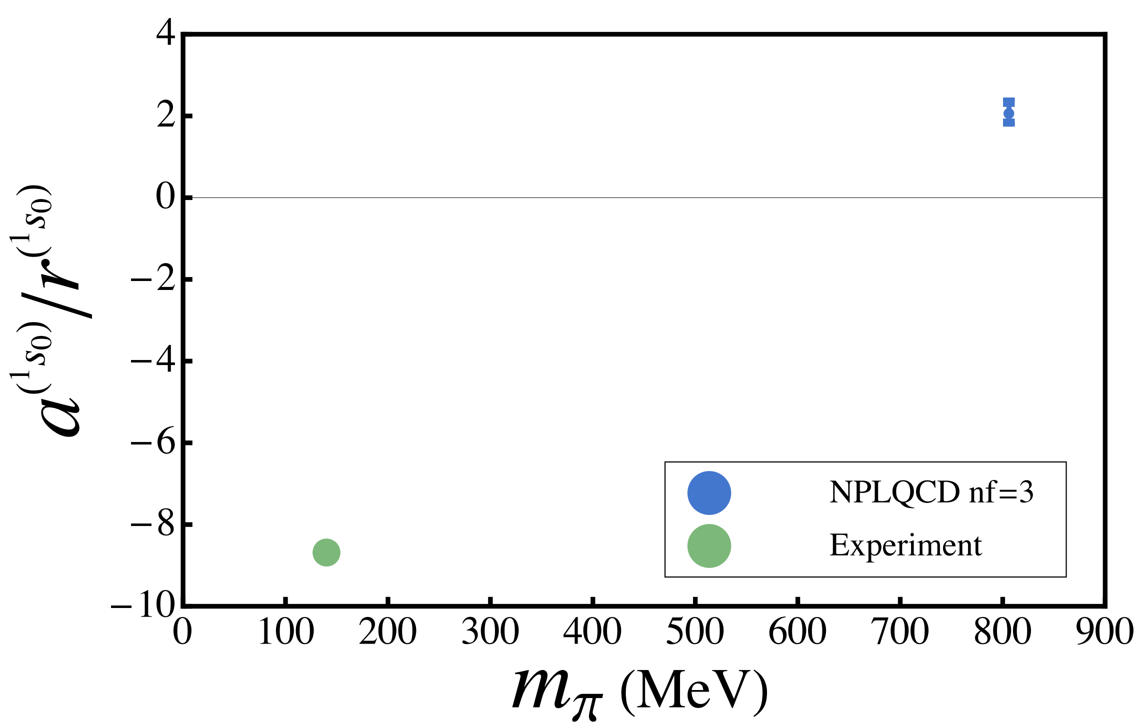


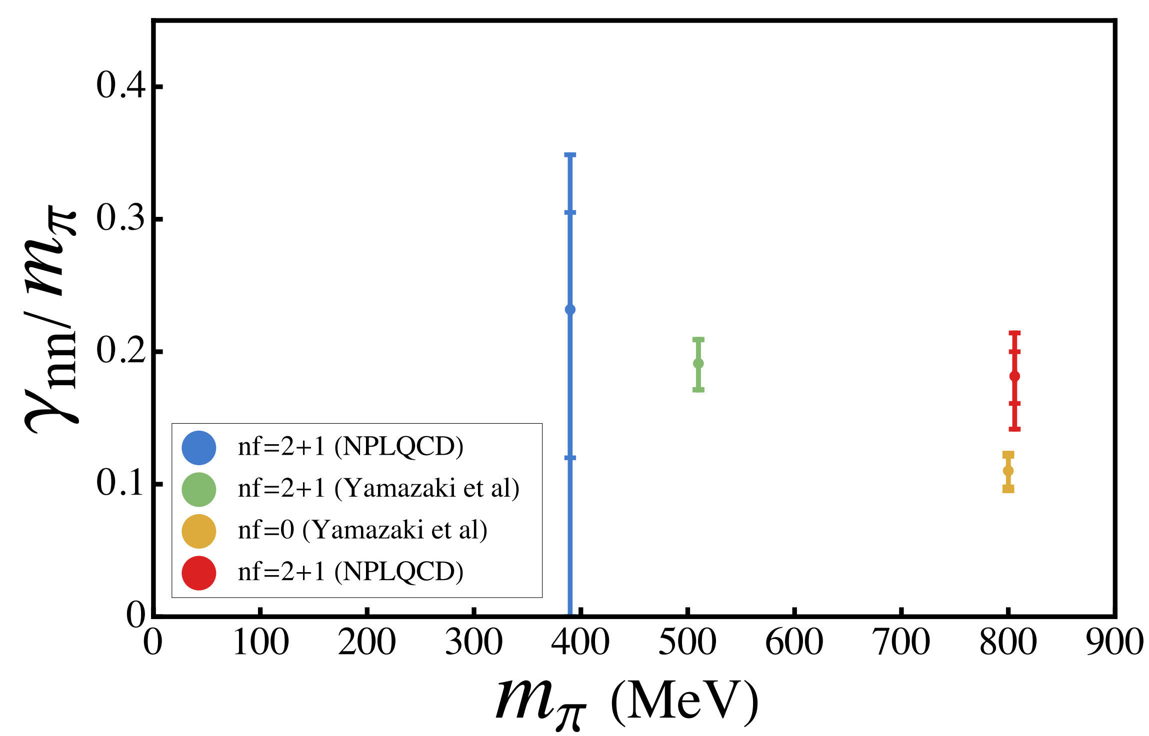


NN fine tuning



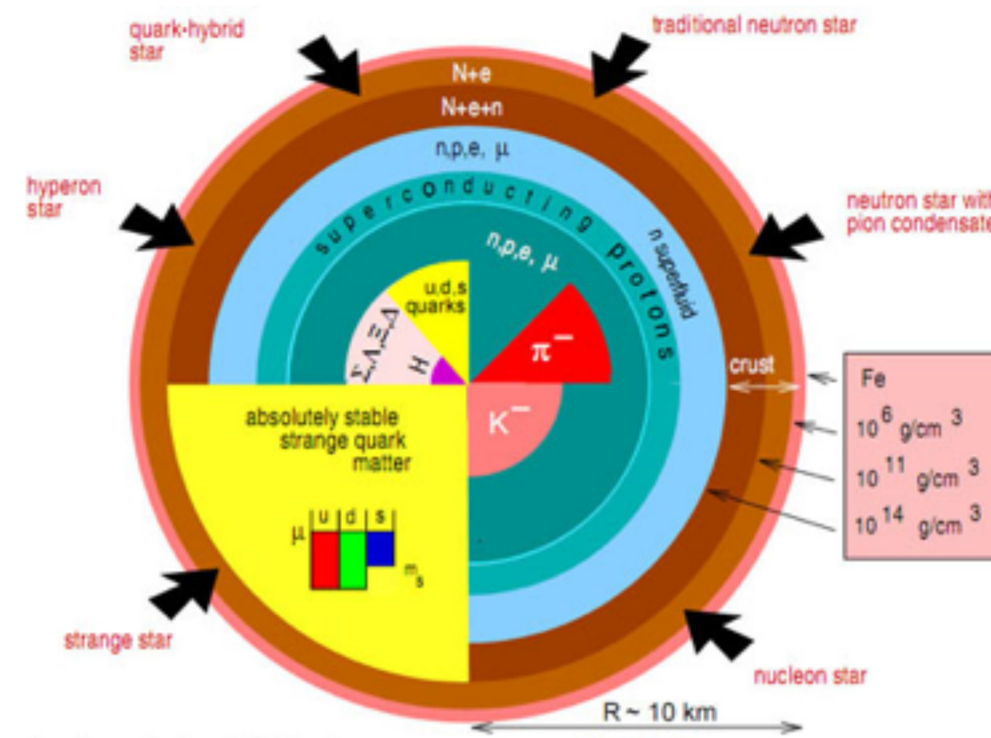


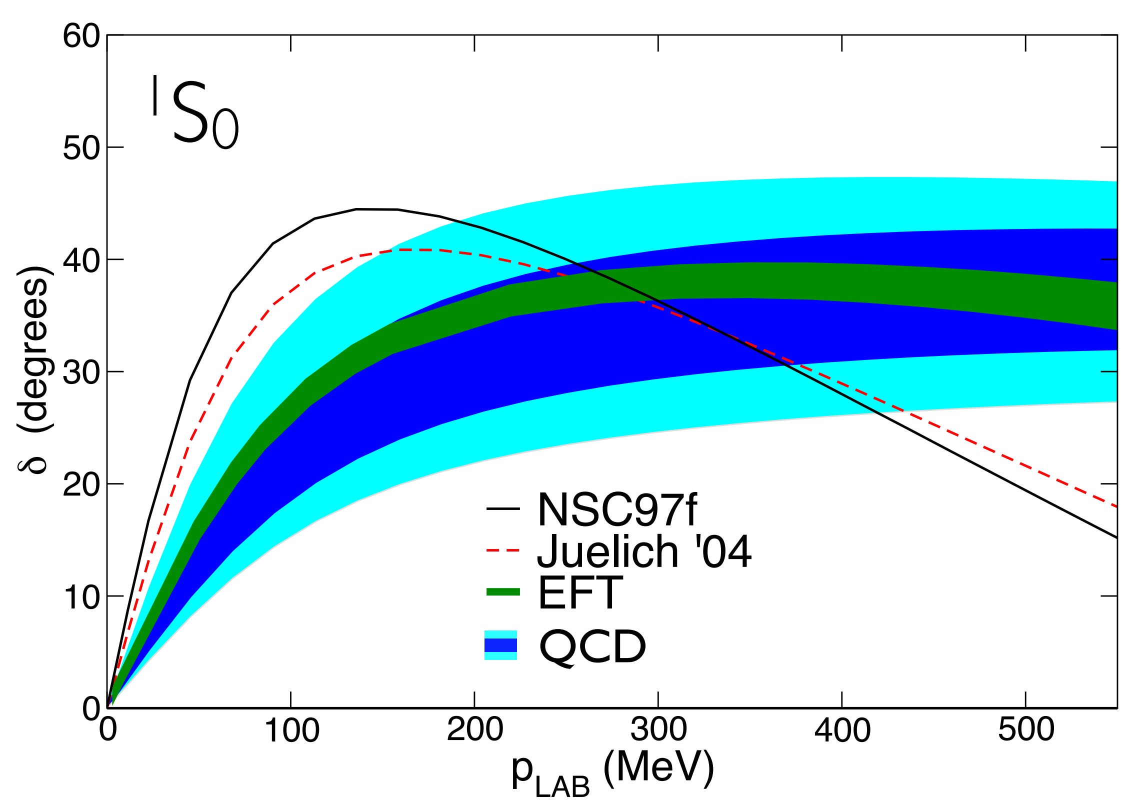


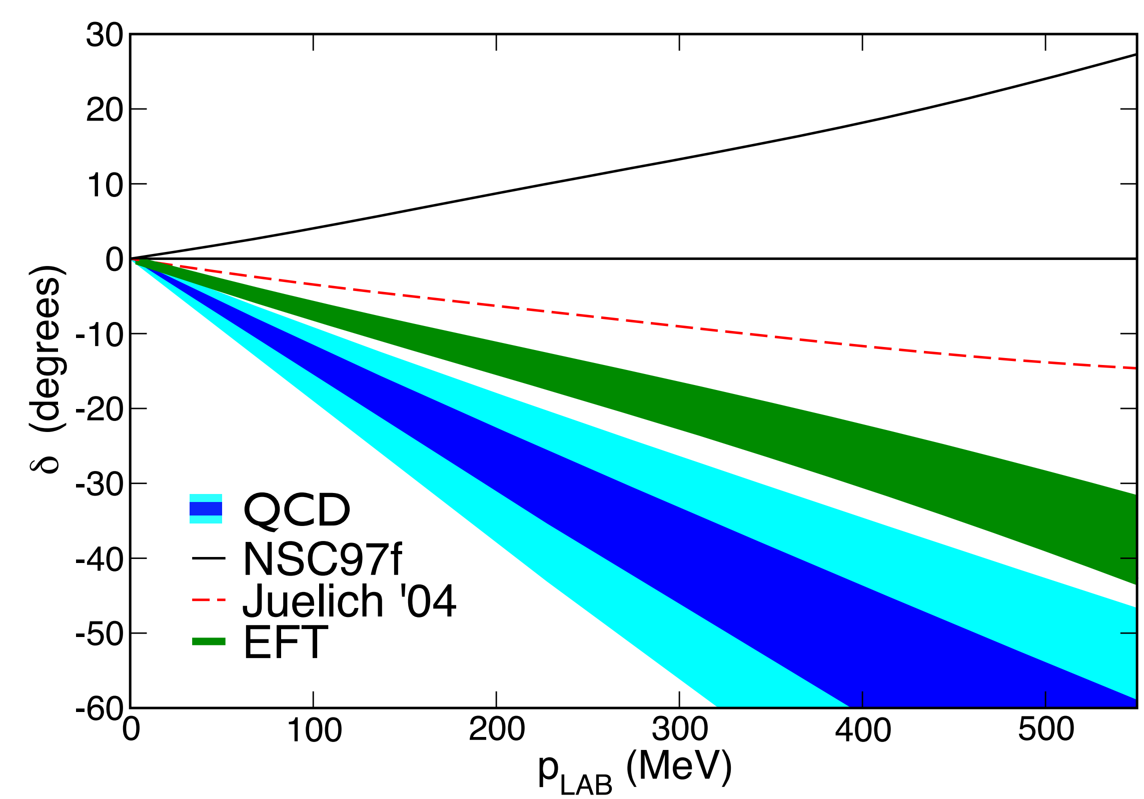


$\Sigma^{-}n$ (I=3/2) phase shifts

- Hyperon-nucleon phase shifts important EoS of neutron stars
- Determine at one quark mass
- Match to effective field theory to extract phase shift at physical mass







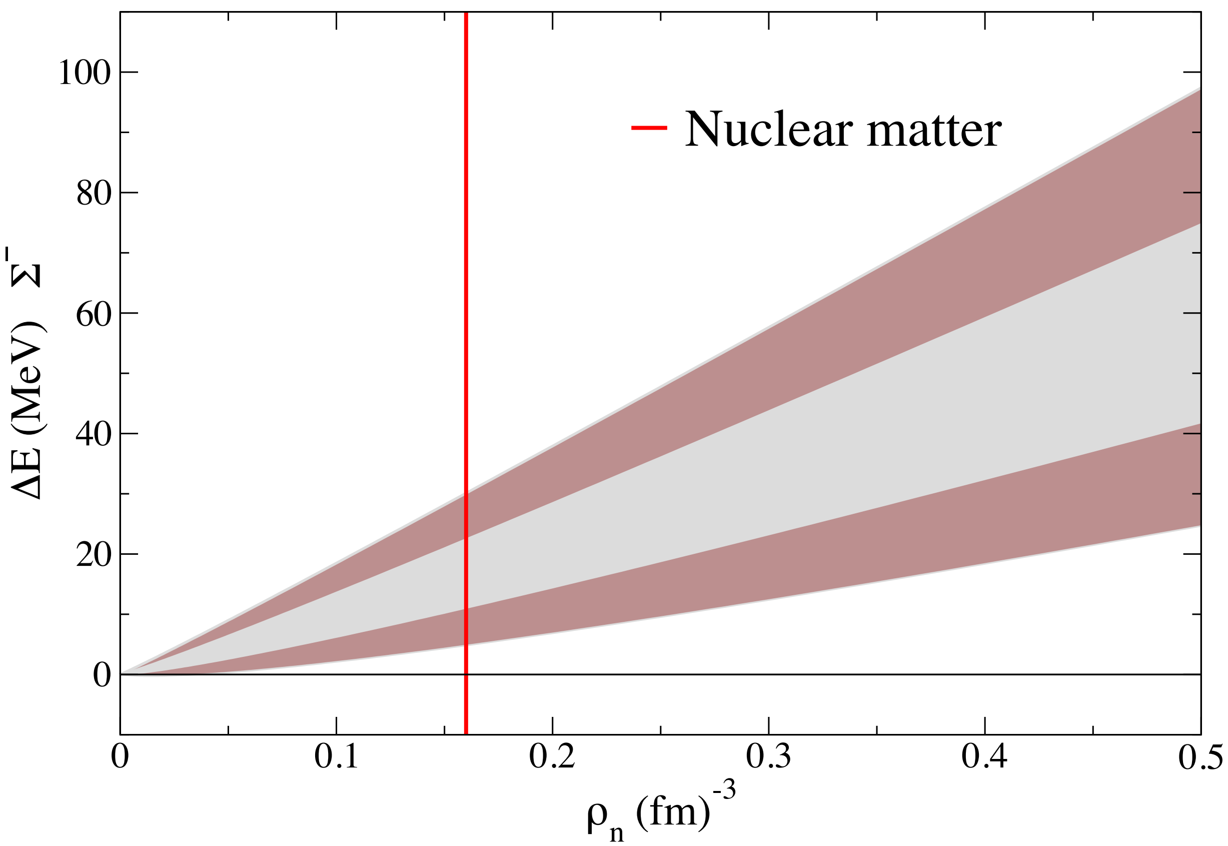
Phys. Rev. Lett. 109 (2012) 172001

Σ^-n (I=3/2) phase shifts

- Influence on EoS is complex
  - Crude approx: Fumi's theorem

$$\Delta E = -\frac{1}{\pi\mu}\int_0^{k_f} dk\, k\left[\frac{3}{2}\delta_{^3S_1}(k) + \frac{1}{2}\delta_{^1S_0}(k)\right]$$

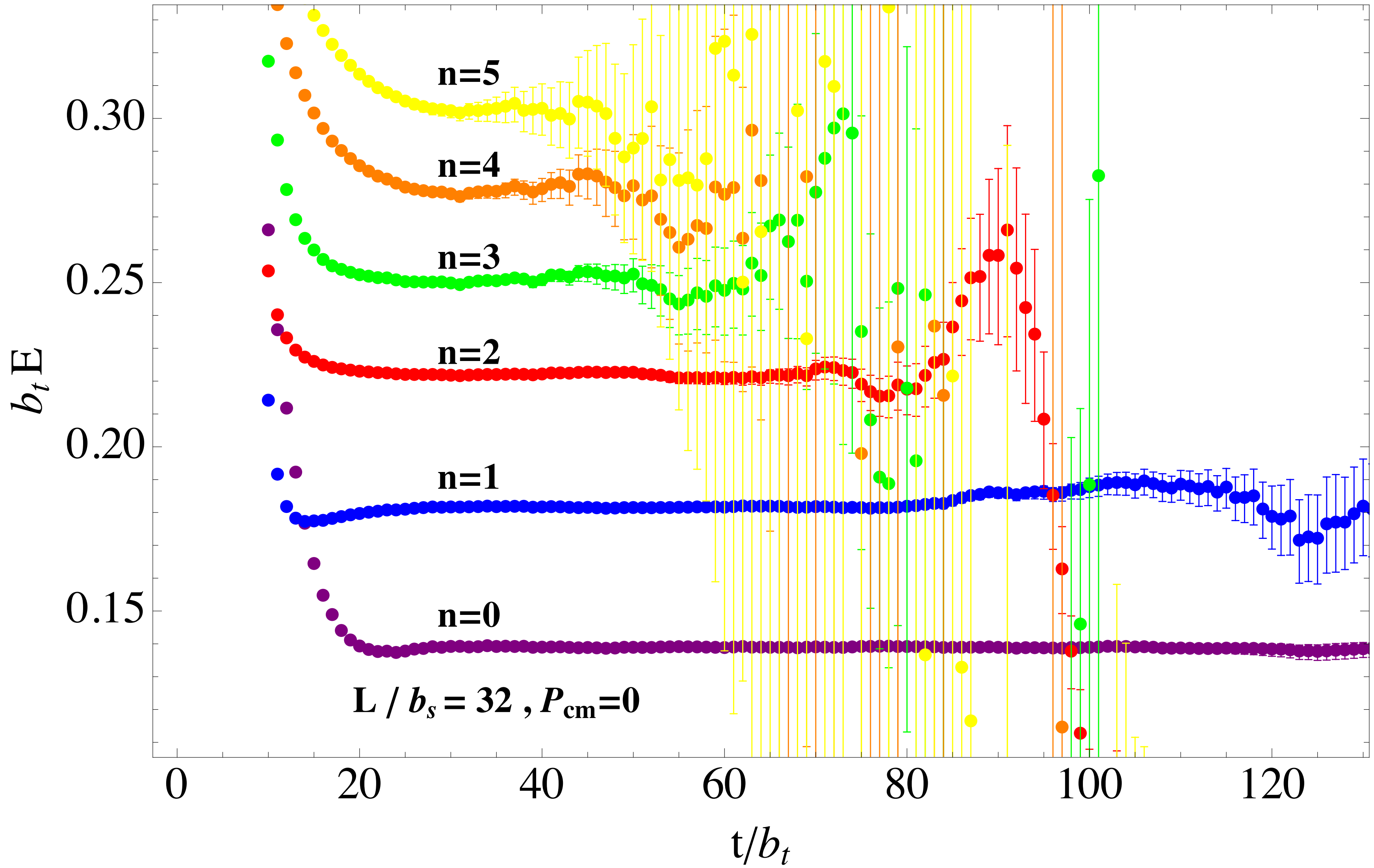
- For $\rho_n \sim 0.4\ \mathrm{fm}^{-3}$,
  $\mu_n + \mu_{e^-} \sim 1290\ \mathrm{MeV}$
- If $\mu_{\Sigma^-} = M_\Sigma + \Delta E \lesssim 1290\ \mathrm{MeV}$ then **Σ**⁻s probably relevant to n-star structure



Phys. Rev. Lett. 109 (2012) 172001

Example: I=2 ππ

- Study multiple energy levels of two pions in a box for multiple volumes and with multiple P_{CM}



@ mπ = 390 MeV

1107.5023 [prd]

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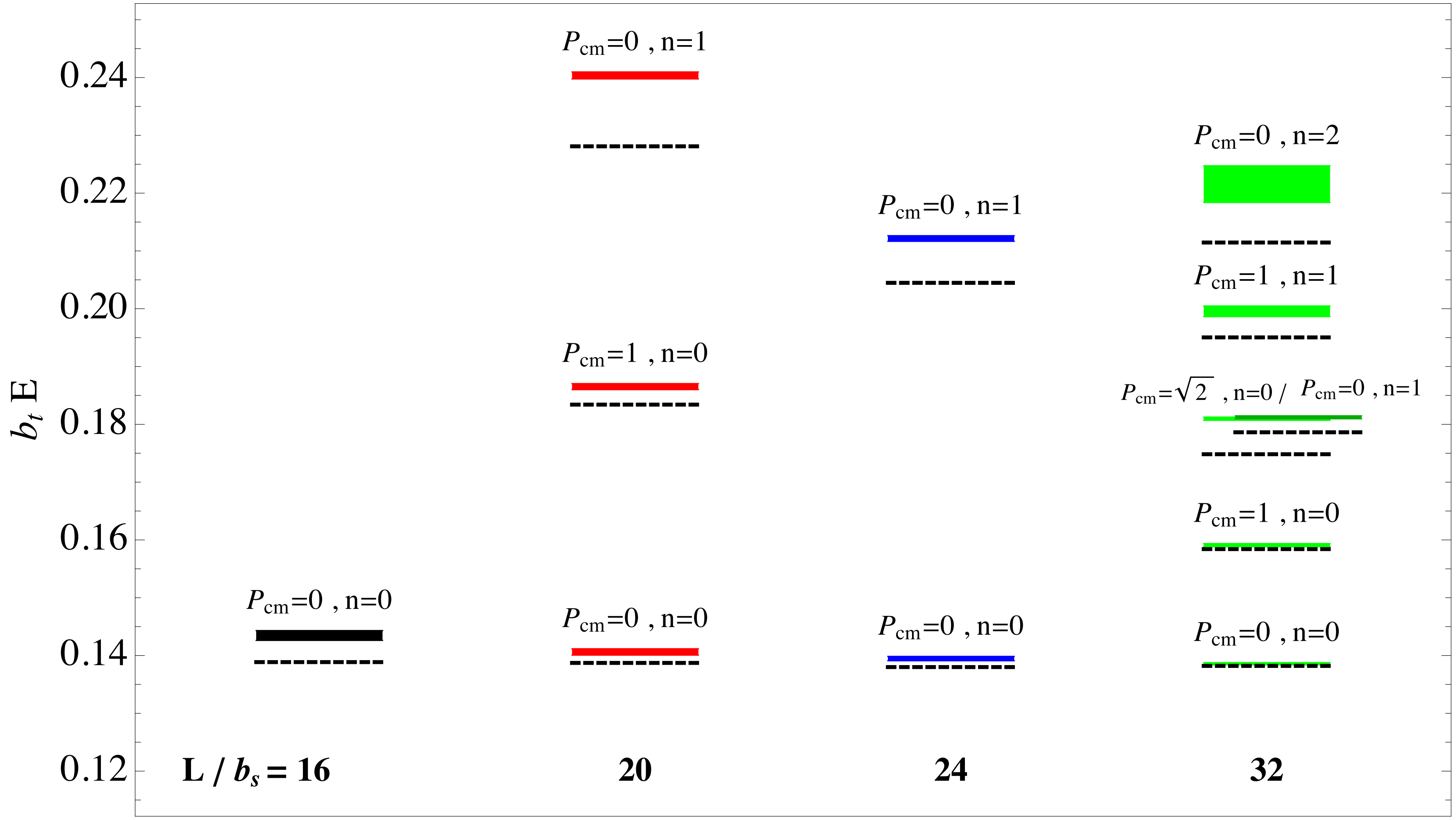


Dashed lines are non-interacting energy levels

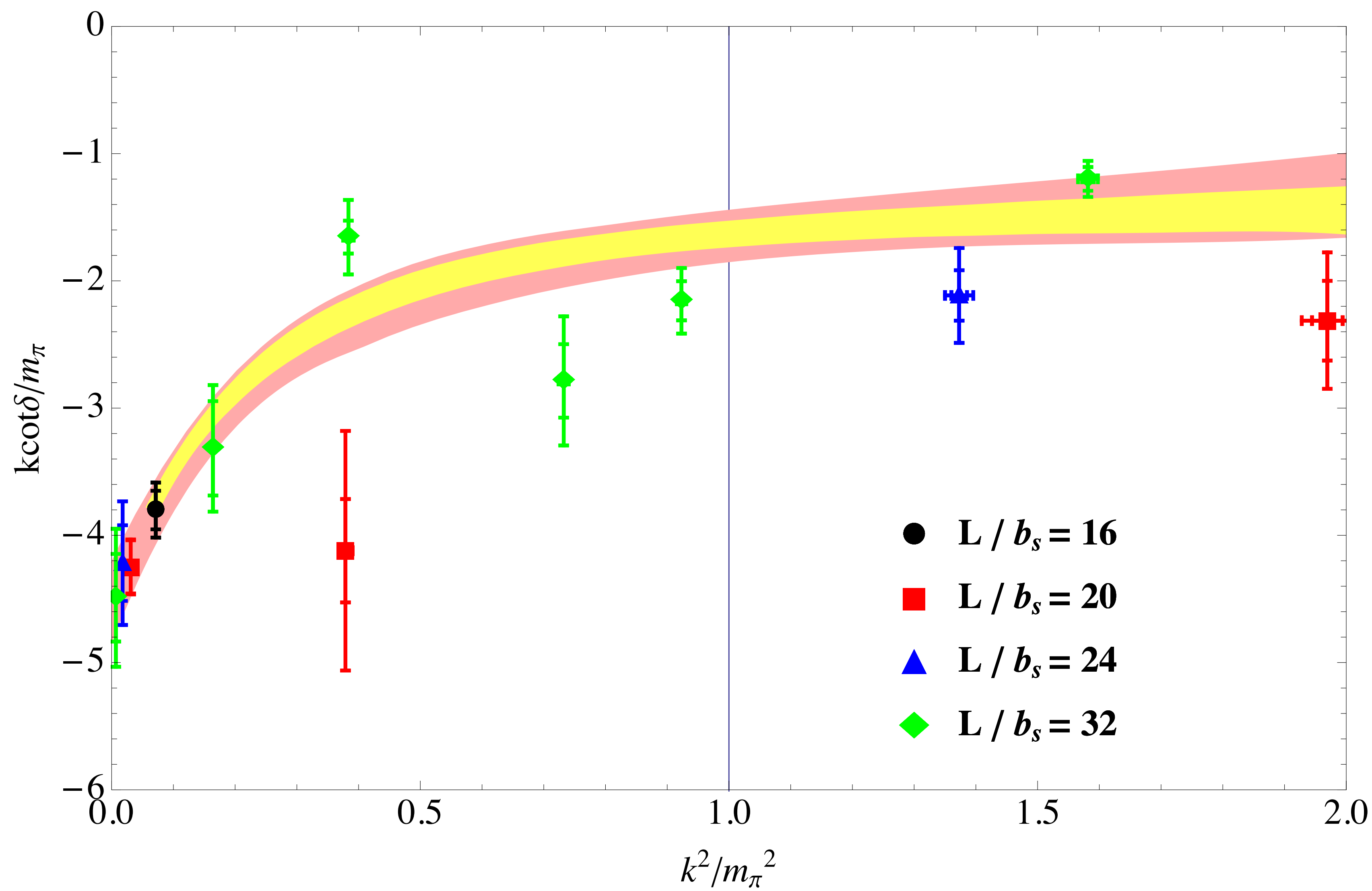
@ mπ = 390 MeV

NPLQCD

1107.5023 [prd]

Example: I=2 ππ

- Allows phase shift to be extracted at multiple energies



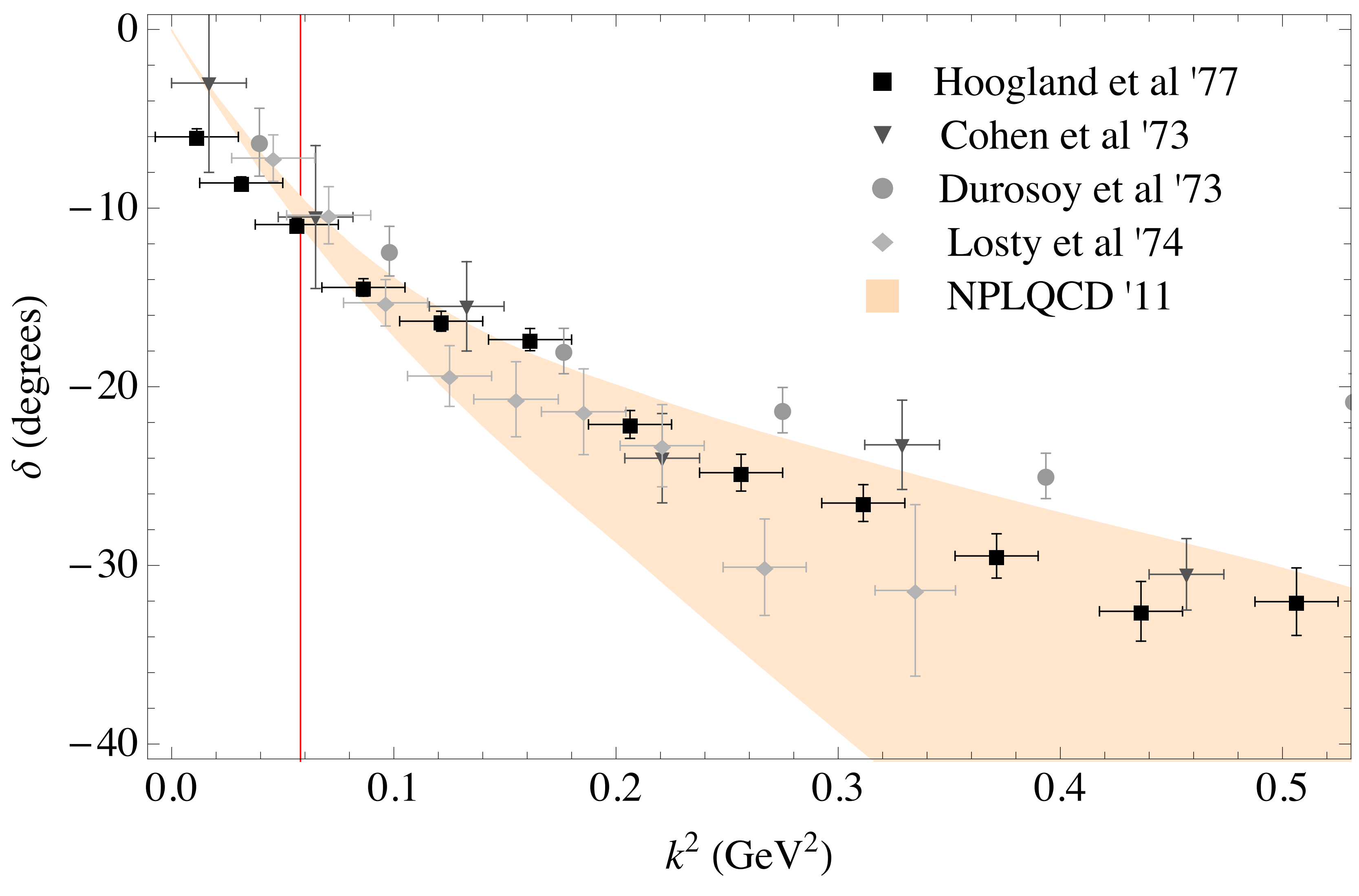
@ mπ = 390 MeV

NPLQCD

1107.5023 [prd]

Example: I=2 ππ

- Combine with chiral perturbation theory to interpolate to physical pion mass
- D wave phase shift also extracted [JLab]



NPLQCD

1107.5023 [prd]

Lattice QCD potentials?

- HALQCD collaboration determine a Bethe-Salpeter (BS) wavefunction from QCD correlation functions

$$G(\mathbf{r}, t-t_0; J^P) = \sum_{\mathbf{x}} \left\langle 0 \left| h^{(1)}(\mathbf{x},t) h^{(2)}(\mathbf{x}+\mathbf{r},t) \overline{J}(t_0;\{Q\}) \right| 0 \right\rangle ,$$
$$= \sum_{n=0}^{\infty} A_n \psi^{(n)}(\mathbf{r};\{Q\})\, e^{-E_n(t-t_0)}$$
$$\psi^{(n)}(\mathbf{r};\{Q\}) \equiv \sum_{\mathbf{x}} \langle 0 | h_a^{(1)}(\mathbf{x},0) h_b^{(2)}(\mathbf{x}+\mathbf{r},0) | n \rangle$$

- Satisfies Schrödinger equation

$$(E_{n=0} - H_0)\, \psi^{(n=0)}(\mathbf{r},\{Q\}) = \int d^3\mathbf{r}'\, U(\mathbf{r},\mathbf{r}')\, \psi^{(n=0)}(\mathbf{r}',\{Q\}) \,.$$

$$U(\mathbf{r},\mathbf{r}') = V(\mathbf{r},-i\nabla)\delta^{(3)}(\mathbf{r}-\mathbf{r}') \qquad V(\mathbf{r},-i\nabla) = V_0(r) + \mathcal{O}(\nabla^2/M^2)$$

$$V_0^{(n=0)}(\mathbf{r}) = \frac{1}{M} \frac{(\nabla^2 + |\mathbf{k}|^2)\psi^{(n=0)}(\mathbf{r},\{Q\}))}{\psi^{(n=0)}(\mathbf{r},\{Q\})}$$

Lattice QCD potentials?

- Potential is energy dependent: only guaranteed to reproduce phase shift at the energy of the NN system in the calculation
- Potential is dependent on choice of sink operators
- Complicated analysis in the presence of statistical uncertainty
- Serious issues with excited states and finite volume effects
- Caveat emptor!