ρ^+ Form Factors in the NJL Model

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THE UNIVERSITY of ADELAIDE

SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

Outline

ρ^+ features

NJL Model

Constituent Quark Form Factors

ρ^+ Form Factors

Results

Conclusions

Quark content, spin, mass and width

- Up and down quark
  as constituents

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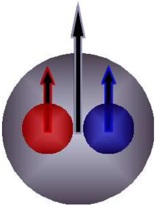

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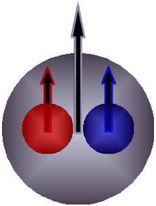

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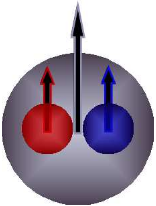

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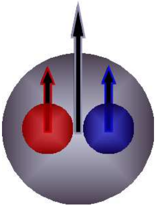

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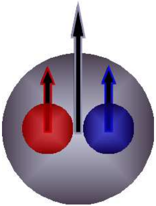

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- Resonances
  1. $\Gamma = 149.1 \pm 0.8$ MeV. $\sim$ $\tau = 10^{-24}$ s.
  2. $M = 775.11 \pm 0.34$ MeV.

Outline

NJL[1]

- The Nambu-Jona-Lasinio (NJL) model simpler than full QCD.
- Keeps QCD symmetries $\rightarrow$ χ sym. breaking $\rightarrow m_q$.

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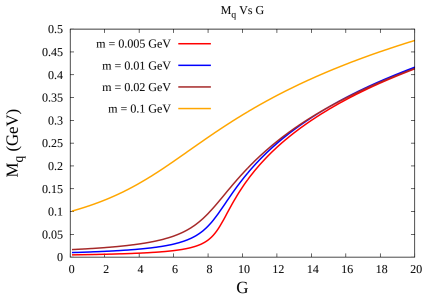
- Main feature $\rightarrow$ hadronic masses $\rightarrow (M_q) \rightarrow$ Gap Eq.



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The NJL Lagrangian SU(2)[2]

- Original SU(2) NJL Lagrangian

$$\mathcal{L}_{NJL} = G\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\vec{\tau}\psi\right)^2\right] \quad (1)$$

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- In our model just SU(2) (Mesons) $\rightarrow$ Fierz Transformations

$$\mathcal{L}_{NJL} = \frac{1}{2}G_\pi\left[\left(\bar{\psi}\psi\right)^2 - \left(\bar{\psi}\gamma_5\vec{\tau}\psi\right)^2\right] - \frac{1}{2}G_\omega\left(\bar{\psi}\gamma^\mu\psi\right)^2 - \frac{1}{2}G_\rho\left[\left(\bar{\psi}\gamma^\mu\vec{\tau}\psi\right)^2 + \left(\bar{\psi}\gamma^\mu\gamma_5\vec{\tau}\psi\right)^2\right] \quad (2)$$

- Feynman Rules.

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Regularization in proper time

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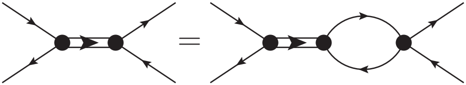
- $\Lambda_{IR} \rightarrow$ QCD scale Λ_{QCD}.
- $\Lambda_{UV} \rightarrow$ pion decay constant f_π.

The Bethe Salpeter Equations for mesons in NJL

$$\mathcal{T}(p) = \int \frac{d^4k}{(2\pi)^4} \mathcal{K} S(p+k)\, S(k) \mathcal{T}(p)$$



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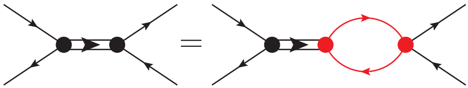

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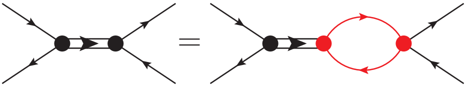
- Vertices depend on the nature of the meson $\rightarrow$ NJL Lagrangian (Feynman Rules)



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- Solution reduced t-matrix pion.

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with

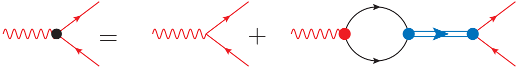
$$g_\pi = 2 \left(\left. \frac{\partial \Pi_\pi}{\partial p^2} \right|_{p^2 = M_\pi^2} \right)^{-1} \tag{6}$$

- Same for $\rho \rightarrow$ include Lorentz structure.

Outline

The BSE for quarks

- Vector coupling γ^μ photon to quark → Vector meson



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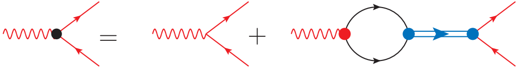

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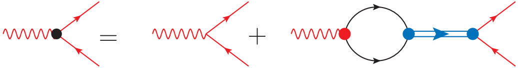

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where $\Lambda^{\mu}_{\rho(\omega)} = \gamma^\mu F_{1\rho(\omega)}\left(q^2\right) + \frac{i\sigma^{\mu\nu}q_\nu}{2M} F_{2\rho(\omega)}\left(q^2\right)$

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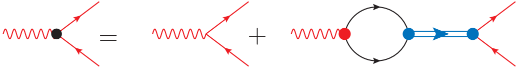

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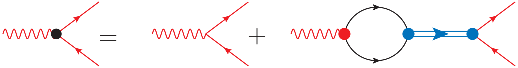

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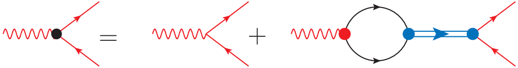

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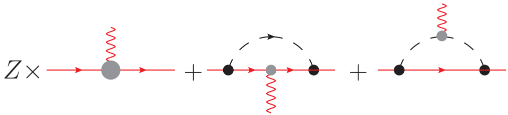
$F_{1\rho(\omega)} = \frac{1}{1+2G_{\rho(\omega)}\Pi_{\rho(\omega)}(q^2)}$ $\qquad F_{2\rho(\omega)} = 0$
- Use pole approximation: $F_{1\rho(\omega)} = \frac{1}{1+Q^2/m^2_{\rho(\omega)}}$.

Pion Cloud

- Solving following diagrams.



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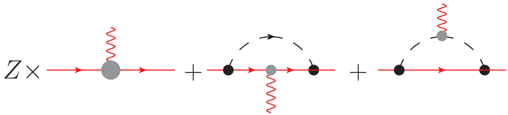

- Includes: Z (quark wave function renormalization), π FF. and reduced pion t-matrix.
- Dressed Quark sector Form Factors given by

$$F_{1U(D)} = Z\left[\frac{1}{6}F_{1\omega} + (-)\frac{1}{2}F_{1\rho}\right] + \left[F_{1\omega} - (+)F_{1\rho}\right] f_1^q + (-)F_{1\rho} f_1^\pi \tag{8}$$

$$F_{2U(D)} = \left[F_{1\omega} - (+)F_{1\rho}\right] f_2^{(q)} + (-)F_{1\rho} f_2^\pi \tag{9}$$

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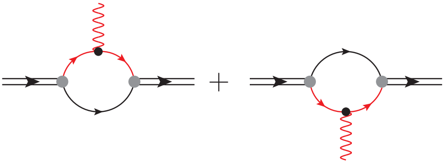

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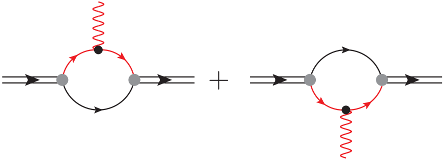

- Three Form Factors:

$$j_\rho^{\mu,\alpha\beta}(p,p') = \left[g^{\alpha\beta} F_{1\rho}\left(Q^2\right) - \frac{q^\alpha q^\beta}{2M_\rho^2} F_{2\rho}\left(Q^2\right) \right] \left(p'+p\right)^\mu - \left(q^\alpha g^{\mu\beta} - q^\beta g^{\mu\alpha}\right) F_{3\rho}\left(Q^2\right) \tag{10}$$

with

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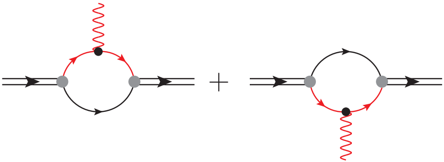

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with

$$F_{i\rho}(Q^2) = \left[F_{1U}(Q^2) - F_{1D}(Q^2)\right] f_i^V(Q^2) + \left[F_{2U}(Q^2) - F_{2D}(Q^2)\right] f_i^T(Q^2). \quad (11)$$

$i = 1, 2, 3.$

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- Sachs-like charge, magnetic and quadrupole Form Factors

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$$G_C\left(Q^2\right) = F_1\left(Q^2\right) + \frac{2}{3}\eta G_Q\left(Q^2\right), \tag{12}$$

$$G_M\left(Q^2\right) = F_3\left(Q^2\right), \tag{13}$$

$$G_Q\left(Q^2\right) = F_1\left(Q^2\right) + (1+\eta)\,F_2\left(Q^2\right) - F_3\left(Q^2\right). \tag{14}$$

$\eta = \frac{Q^2}{4M_\rho^2}$

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$\eta = \frac{Q^2}{4M_\rho^2}$

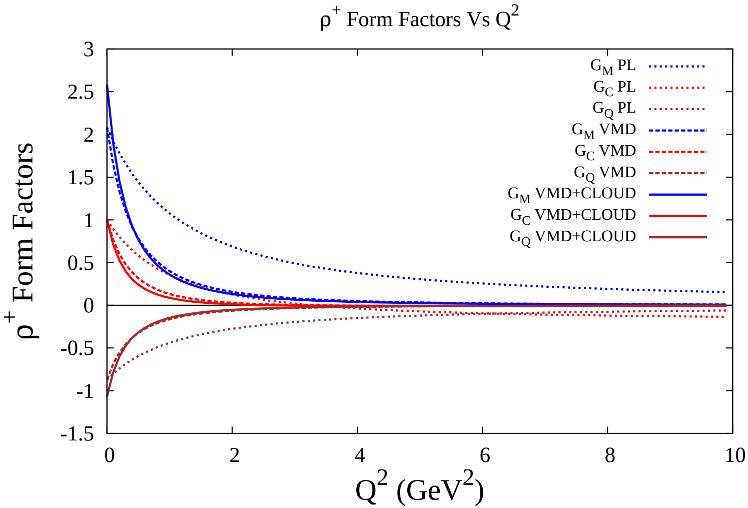
- Charged squared radius $\left\langle r^2\right\rangle$ and magnetic moment μ_ρ

$$\left\langle r^2\right\rangle = -6\left.\frac{\partial G_C\left(Q^2\right)}{\partial Q^2}\right|_{Q^2=0} \tag{15}$$

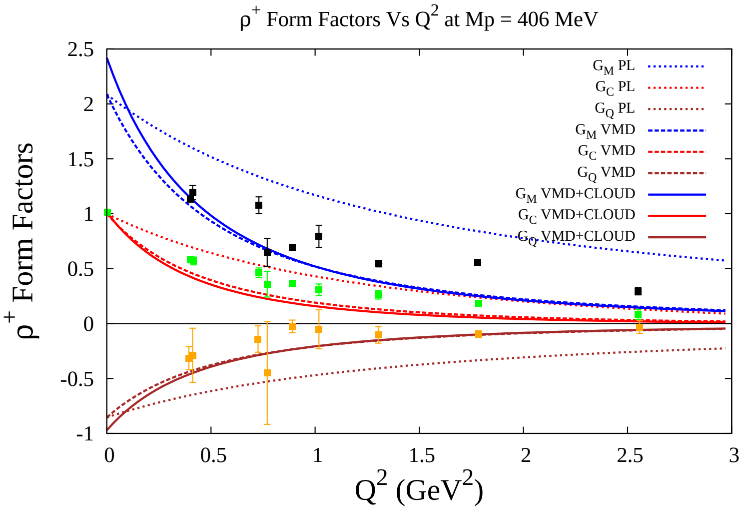
$$\mu_\rho\left(\mu_N\right) = \left[\frac{m_N}{m_\rho\left(m_\pi\right)} G_M\left(0, m_\pi\right)\right]\mu_N \tag{16}$$

Outline

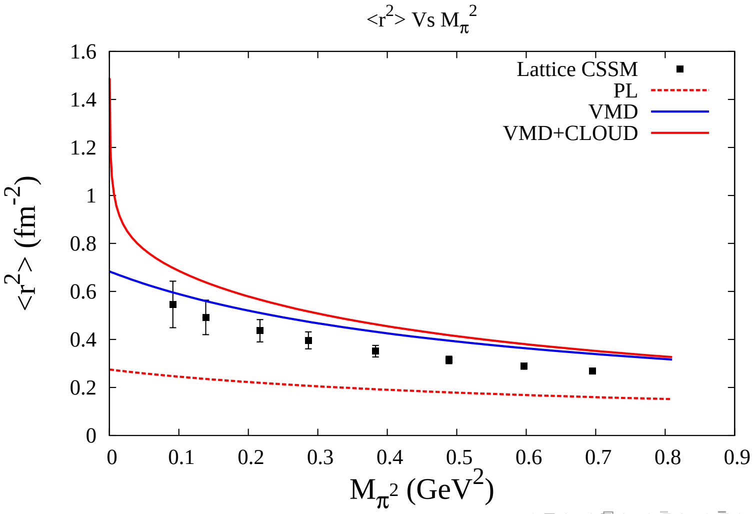
Form Factor Results



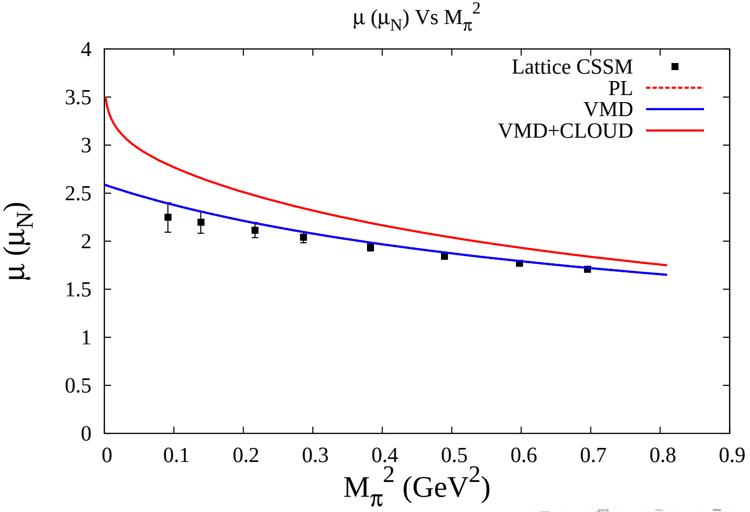
QCDSF PoS LAT2008 (2008) 051 [arXiv:hep-lat/0611029v1]



$\langle r^2 \rangle$ Lattice at $Q^2 = 0.22$ Mev (CSSM): Phys. Rev. **D** 75, 094504, 2007.



μ_ρ (CSSM) Phys. Rev. **D** 75, 094504, 2007.



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Conclusions

- Results for $\langle r^2 \rangle$ and μ_ρ promising.
- Look forward to more detailed Lattice results.
- Ability to calculate FF for many $m_\pi^2 \rightarrow$ Lattice.
- Look forward to experiments?