Nucleon structure from 2+1 flavor domain wall QCD at a nearly physical pion mass

Shigemi Ohta [*†‡] for RBC and UKQCD Collaborations

Talk at Tropical QCD 2010, Cairns, QLD, September 30, 2010

RBC and UKQCD collaborations are generating new dynamical DWF ensembles:

- Iwasaki + dislocation suppressing determinant ratio (DSDR) gauge action, $\beta = 1.75$, and
- Domain-Wall Fermions (DWF) quarks, $L_s = 32$ and $M_5 = 1.8$,
- $a^{-1} \sim 1.368(7)$ GeV, $m_{\rm res} \sim 0.002$: $m_{\rm strange}a = 0.045$, $m_{\rm ud}a = 0.0042$ and 0.001.

Much closer to physical pion mass than the previous set of Iwasaki+DWF ensembles:

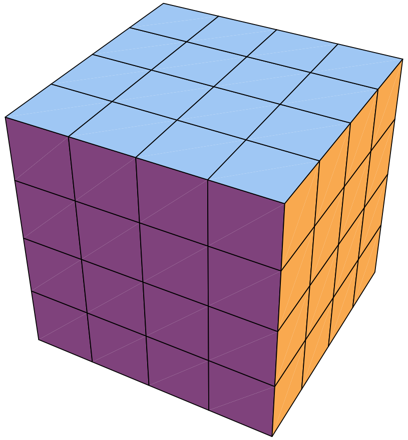
- $m_\pi \sim 180$ and 250 MeV, with large volume, $(\sim 4.6\text{fm})^3$ ($32^3 \times 64$).

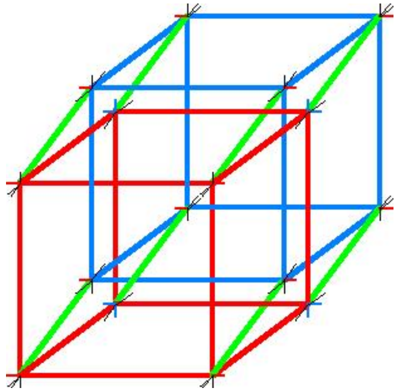
Here we report the current status of our nucleon calculations, by

- Meifeng Lin, Yasumichi Aoki, Tom Blum, Chris Dawson, Taku Izubuchi, Chulwoo Jung, SO, Shoichi Sasaki, Takeshi Yamazaki, James Zanotti ...

[*] Inst. Particle and Nuclear Studies, KEK, Tsukuba, Ibaraki 305-0801, Japan
[†] Department of Particle and Nuclear Physics, Sokendai Graduate University of Advanced Studies, Hayama, Kanagawa 240-0193, Japan
[‡] RIKEN BNL Research Center, Upton, NY 11973, USA

Lattice: 4D simple hyper-cubic lattice, $L_0 L_1 L_2 L_3$, Euclidean





site: $s = (n_0 n_1 n_2 n_3)$, $0 \le n_i \le L_i - 1$ $(i = 0, 1, 2, 3)$.

link: $l = (s, \mu)$, $\mu \in \{0, 1, 2, 3\}$, connects s and $s + \hat{\mu}$.

constant separation (lattice constant) a between neighboring sites.

Taking $a \to 0$ through asymptotic scaling gives exact continuum physics.

Dynamical variables:

quark: $q(s)$, defined on site and forms basis of fundamental (3) representation of SU(3),

gluon: $U(s, \mu) = \exp(ig \int_s^{s+\hat{\mu}} A_\mu(y) \mathrm{d}y_\mu) \in \mathrm{SU}(3)$, now a group element defined on link.

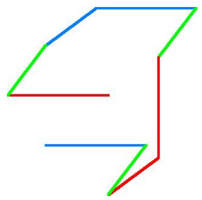
There are many other ways to define lattice (eg. random lattice) with different advantages, but the way q, U and G are defined is basically the same.

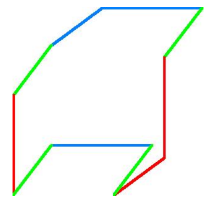
Gauge transformation: $G(s) \in \mathrm{SU}(3)$, defined on site, maps quarks and gluons

$$q(s) \mapsto G(s)q(s) \text{ and } U(s,\mu) \mapsto G(s)U(s,\mu)G(s+\hat{\mu})^{-1}.$$

Gauge invariant objects (QCD action, observables):

- Quark: $\bar{\psi}(x)U(x,\mu)U(x+\hat{\mu},\nu)...U(y-\hat{\rho},\rho)\psi(y), \mapsto \bar{\psi}(x)\underline{G^{-1}(x)G(x)}U(x,\mu)\underline{G^{-1}(x+\hat{\mu})G(x+\hat{\mu})}U(x+\hat{\mu},\nu)...U(y-\hat{\rho},\rho)\underline{G^{-1}(y)G(y)}\psi(y)$.





- Gluon, $\mathrm{Tr}[U(x,\mu)U(x+\hat{\mu},\nu)...U(x-\hat{\rho},\rho)] \mapsto \mathrm{Tr}[\underline{G(x)}U(x,\mu)\underline{G^{-1}(x+\hat{\mu})G(x+\hat{\mu})}U(x+\hat{\mu},\nu)...U(x-\hat{\rho},\rho)\underline{G^{-1}(x)}]$.

Action: $S_{\mathrm{QCD}}[U,q,\bar{q}] = S_{\mathrm{gluon}}[U] + S_{\mathrm{quark}}[U,q,\bar{q}]$, must respect gauge invariance:

gluon part: such as $S_{\mathrm{gluon}}[U] = \frac{6}{g^2}\sum_s \sum_{\mu<\nu} \Box(s,\mu,\nu)$, gives $-\frac{1}{2}\mathrm{Tr}G^{\mu\nu}G_{\mu\nu}$ as $a \to 0$ and $g \to 0$,

- where the plaquette, $\Box(s,\mu,\nu) = 1 - \frac{1}{3}\mathrm{ReTr}U(s,\mu)U(s+\hat{\mu},\nu)U(s+\hat{\nu},\mu)^{-1}U(s,\nu)^{-1}$.

quark part: $S_{\mathrm{quark}}[U,q,\bar{q}] = \sum_{s,s'} \bar{q}(s)M[U](s,s')q(s')$, which should give $\bar{q}(i\gamma^\mu D_\mu - m)q$ as $a \to 0$ and $g \to 0$,

- with $M[U](s,s')$ describing quark propagation between sites s and s'.

Expectation values of any gauge-invariant observable: $\langle O\rangle = N^{-1}\int[\mathrm{d}U][\mathrm{d}q][\mathrm{d}\bar{q}]O[U,q,\bar{q}]\exp(-S_{\mathrm{QCD}}[U,q,\bar{q}])$,

or by integrating over the quark Grassmann variables: $N'^{-1}\int[\mathrm{d}U](\det M[U])\exp(-S_{\mathrm{gluon}}[U])$.
It is often convenient to use effective action: $\tilde{S}[U] = S_{\mathrm{gluon}}[U] - \mathrm{Tr}\log M[U]$.

Finite lattice and compact SU(3) assures finite $\langle O\rangle$.

Continuum limit is well defined because of the asymptotic freedom: consider an observable O with mass dimension,

- the expectation value is described as $\langle O\rangle = a^{-1}f(g)$ with some dimensionless function $f(g)$ of dimensionless coupling g.
- Renormalizability of the theory means the cutoff dependence should vanish $\frac{d\langle O\rangle}{da} \to 0$ as $a \to 0$, or

$$f(g) - f'(g)\left(a\frac{dg}{da}\right) = \beta(g)f'(g) + f(g) \to 0.$$

- This ($df/f = -dg/\beta$) is easily solved to give: $\langle O\rangle a \propto \exp\left(-\int^g \frac{dh}{\beta(h)}\right)$, or

$$\langle O\rangle a \propto (g^2 b_0)^{-\frac{b_1}{2b_0^2}} e^{-\frac{1}{2b_0 g^2}}[1+O(g^2)],$$

where $\beta(g) \equiv -a\frac{dg}{da} = -b_0 g^3 - b_1 g^5 + O(g^7)$ is perturbatively well known.

Chiral symmetry:

- Invariance under global $U(N_f)$ transformations, $q \mapsto \exp(i\theta)q$, $\exp(i\theta'\gamma_5)q$, $\exp(i\alpha^a\frac{\lambda^a}{2})q$ and $\exp(i\beta^a\frac{\lambda^a}{2}\gamma_5)q$.

- Should be preserved in the absence of $m\bar{q}q$, like $U(N_f)_L \times U(N_f)_R = SU(N_f)_V \times SU(N_f)_A \times U(1)_V \times U(1)_A$.

- In fact spontaneously broken for light normal quarks, $m_u \sim m_d \sim 0$, $\langle \bar{u}u + \bar{d}d \rangle \neq 0$.
- Important for Nambu-Goldstone pion, PCAC, etc, $m_\pi^2 f_\pi^2 = m_q \langle \bar{q}q \rangle$.

However, difficult to maintain on regular lattices.

Naive lattice fermion action, with $M_{xy} = \frac{1}{2}a^{D-1}\sum_\mu \gamma_\mu[\delta_{x+\hat{\mu},y} - \delta_{x-\hat{\mu},y}]$, leads to a propagator $\Delta(p) = a[\gamma_\mu \sin(p_\mu a)]^{-1}$, which has 2^D poles at $p_\mu = 0$ or π/a: for $D = 4$, there are $2^4 = 16$ flavors/tastes instead of one.

Shifting of one component of p_μ, such as $\tilde{p}_\mu = p_\mu - \pi/a$, acts like

$$\gamma_\mu \sin(p_\mu a) = -\gamma_\mu \sin(\tilde{p}_\mu a)$$

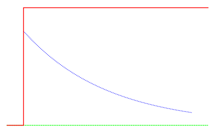
so the chirality $\pm$ states are paired.

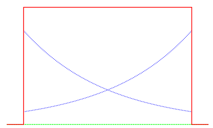
Nielsen and Ninomiya theorem: doubling inevitable (chirality $\pm$ states are paired) for a regular lattice and local, hermitian, and translationally invariant action.

Domain-wall fermions[1]: introduce a 5-th dimension, s, and define a 5D Dirac operator: $D = \gamma_\mu \partial_\mu + \gamma_5 \partial_s + m(s)$,

- With a monotonic $m(s)$ with $m(s=0)=0$, a 4D chiral modes emerge: $\psi_\pm(x,s) = u_p(x)\phi_\pm(s)\chi_\pm$.
- 4D Dirac plane wave u_p and γ_5 eigenstate, $\gamma_5\chi_\pm = \pm\chi_\pm$, indicate the s-dependence,

$$[\pm\partial_s + m(s)]\phi(s) = 0, \quad \text{or} \quad \phi(s) \propto \exp[\mp \int_0^s ds' m(s')],$$

  pinned at the $s=0$ wall, and exponentially decay to $\pm s$ direction. 

- On a finite lattice, two walls, with a pair of $\pm$ chiralities mix. 

- No problem for a vector theory like QCD[2]: mixing exponentially suppressed, described by $m_{\rm res}$.

RIKEN-BNL-Columbia (RBC) Collaboration proved DWF works very well for QCD:

- light hadron mass spectrum,
- electroweak transitions among light hadrons (such as f_π, f_K, B_K and ϵ'/ϵ),

unlike conventional Wilson and staggered fermions.

[1] D.B. Kaplan, Phys. Lett. B288, 342 (1992), hep=lat/9206013.

[2] Y. Shamir, Nucl. Phys. B406, 90 (1993), hep-lat/9303005; V. Furman and Y. Shamir, Nucl. Phys. B439, 54 (1995), hep-lat/9405004; and references cited therein.

QCDSP and QCDOC computers: dedicated for lattice QCD calculations.

QCDSP: completed in 1998, 600 (RBRC) and 400 (Columbia) GFlops configurations

- based on commercial DSP
- assisted by custom designed 4D hypercubic nearest-neighbor communication
- 10$ per MFlops

Demonstrated the use of DWF in (quenched) lattice QCD

- Chiral and flavor symmetries and associated ease in non-perturbative renormalizations,
- hadron spectroscopy: masses and decay constants,
- hadron matrix elements: B_K, ϵ'/ϵ, nucleon form factors and structure functions.

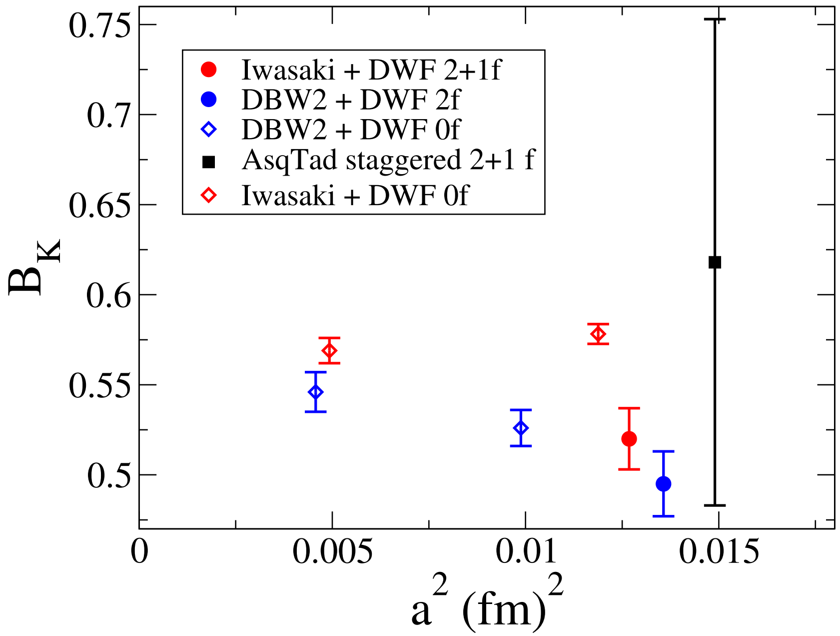
QCDOC: complete in 2005, 10 TFLops configurations in RBRC, BNL and Edinburgh.

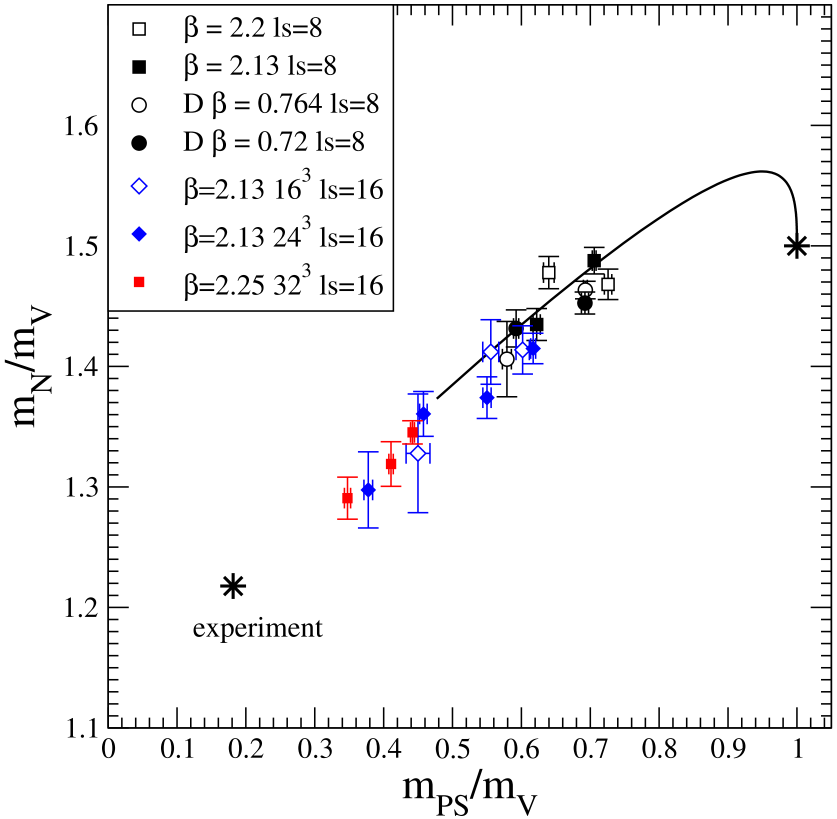
- based on system on a chip technology,
- a QCDSP card was shrunk to be a QCDOC chip, with custom-designed 6D hypercubic communicaitons,
- 1$ per MFlops.

Used for realistic (2+1)-flavor dynamical DWF lattice QCD.

Evolved into BG/L, P and Q.

RBC/UKQCD $N_f = 2 + 1$ dynamical DWF ensembles: a^{-1} = 1.73(2) and 2.28(3) GeV with volumes larger than 2.7 fm across,





Chiral and continuum limit with good flavor and chiral symmetries:

- $f_\pi = 122(2)(5)$ MeV, $f_K/f_\pi = 1.21(3)$; $m_{\rm s}^{\overline{\rm MS}(2{\rm GeV})} = 97(3)$ MeV, $m_{\rm ud}^{\overline{\rm MS}(2{\rm GeV})} = 3.6(2)$ MeV,
- Very accurate constraints on CKM matrix: $B_K^{\overline{\rm MS}}(2{\rm GeV}) = 0.524(10)(28)$, $K_{l3}\ f_+(0) = 0.964(5)$, ...
- Chiral perturbation extrapolation useless from our mass range, $m_\pi \sim 300$ MeV: e.g. NLO $\sim 0.5\times$LO.

Nucleon form factors, measured in elastic scatterings or β decay or muon capture:

$$\langle p|V_\mu^+(x)|n\rangle = \bar{u}_p\left[\gamma_\mu F_V(q^2) + \frac{\sigma_{\mu\lambda}q_\lambda}{2m_N}F_T(q^2)\right]u_n e^{iq\cdot x},$$

$$\langle p|A_\mu^+(x)|n\rangle = \bar{u}_p\left[\gamma_\mu\gamma_5 F_A(q^2) + iq_\mu\gamma_5 F_P(q^2)\right]u_n e^{iq\cdot x}.$$

$$F_V = F_1, F_T = F_2; G_E(q^2) = F_1 - \frac{q^2}{4m_N^2}F_2, G_M = F_1 + F_2.$$

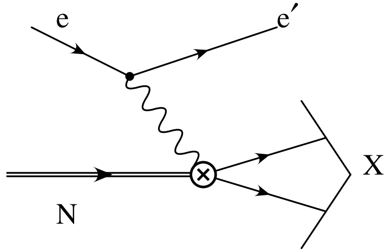
Related to mean-squared charge radius, magnetic moment, $g_V = F_V(0) = G_{\rm Fermi}\cos\theta_{\rm Cabibbo}$, $g_A = F_A(0) = 1.2694(28)g_V$, Goldberger-Treiman relation, $m_N g_A \propto f_\pi g_{\pi NN}$, ... determine much of nuclear physics.

On the lattice, with appropriate nucleon operator, for example, $N = \epsilon_{abc}(u_a^T C\gamma_5 d_b)u_c$, ratio of two- and three-point correlators such as $\dfrac{C_{\rm 3pt}^{\Gamma,O}(t_{\rm sink},t)}{C_{\rm 2pt}(t_{\rm sink})}$ with

$$C_{\rm 2pt}(t_{\rm sink}) = \sum_{\alpha,\beta}\left(\frac{1+\gamma_t}{2}\right)_{\alpha\beta}\langle N_\beta(t_{\rm sink})\bar{N}_\alpha(0)\rangle,$$

$$C_{\rm 3pt}^{\Gamma,O}(t_{\rm sink},t) = \sum_{\alpha,\beta}\Gamma_{\alpha\beta}\langle N_\beta(t_{\rm sink})O(t)\bar{N}_\alpha(0)\rangle,$$

give a plateau in t for a lattice bare value $\langle O\rangle$ for the relevant observable, with appropriate spin ($\Gamma = (1+\gamma_t)/2$ or $(1+\gamma_t)i\gamma_5\gamma_k/2$) or momentum-transfer (if any) projections.

Deep inelastic scatterings 

$: \left|\frac{\mathcal{A}}{4\pi}\right|^2 = \frac{\alpha^2}{Q^4} l^{\mu\nu} W_{\mu\nu},\ W^{\mu\nu} = W^{[\mu\nu]} + W^{\{\mu\nu\}}$

- unpolarized: $W^{\{\mu\nu\}}(x,Q^2) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1(x,Q^2) + \left(P^\mu - \frac{\nu}{q^2} q^\mu\right)\left(P^\nu - \frac{\nu}{q^2} q^\nu\right) \frac{F_2(x,Q^2)}{\nu}$,
- polarized: $W^{[\mu\nu]}(x,Q^2) = i\epsilon^{\mu\nu\rho\sigma} q_\rho \left(\frac{S_\sigma}{\nu}(g_1(x,Q^2) + g_2(x,Q^2)) - \frac{q\cdot S P_\sigma}{\nu^2} g_2(x,Q^2)\right)$,

with $\nu = q\cdot P$, $S^2 = -M^2$, $x = Q^2/2\nu$.

Moments of the structure functions are accessible on the lattice:

$$
\begin{aligned}
2\int_0^1 dx x^{n-1} F_1(x,Q^2) &= \sum_{q=u,d} c_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_q(\mu) + \mathcal{O}(1/Q^2),\\
\int_0^1 dx x^{n-2} F_2(x,Q^2) &= \sum_{f=u,d} c_{2,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_q(\mu) + \mathcal{O}(1/Q^2),\\
2\int_0^1 dx x^{n} g_1(x,Q^2) &= \sum_{q=u,d} e_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_{\Delta q}(\mu) + \mathcal{O}(1/Q^2),\\
2\int_0^1 dx x^{n} g_2(x,Q^2) &= \frac{1}{2}\frac{n}{n+1}\sum_{q=u,d} [e_{2,n}^{q}(\mu^2/Q^2, g(\mu))\, d_n^q(\mu) - 2e_{1,n}^{q}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_{\Delta q}(\mu)] + \mathcal{O}(1/Q^2)
\end{aligned}
$$

- c_1, c_2, e_1, and e_2 are the Wilson coefficients (perturbative),
- $\langle x^n\rangle_q(\mu)$, $\langle x^n\rangle_{\Delta q}(\mu)$ and $d_n(\mu)$ are forward nucleon matrix elements of certain local operators,
- so is $\langle 1\rangle_{\delta q}(\mu) = \langle P,S|\bar\psi i\gamma_5\sigma_{\mu\nu}\psi|P,S\rangle$ which may be measured by polarized Drell-Yan and RHIC Spin.

Unpolarized (F_1/F_2): on the lattice we can measure: $\langle x\rangle_q$, $\langle x^2\rangle_q$ and $\langle x^3\rangle_q$.

$$\frac{1}{2}\sum_s\langle P,S|\mathcal{O}^q_{\{\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = 2\langle x^{n-1}\rangle_q(\mu)[P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n}+\cdots-(\text{trace})]$$

$$\mathcal{O}^q_{\mu_1\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n-1}\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{trace})\right]q$$

Polarized (g_1/g_2): on the lattice we can measure: $\langle 1\rangle_{\Delta q}$ (g_A), $\langle x\rangle_{\Delta q}$, $\langle x^2\rangle_{\Delta q}$, d_1, d_2, $\langle 1\rangle_{\delta q}$ and $\langle x\rangle_{\delta q}$.

$$-\langle P,S|\mathcal{O}^{5q}_{\{\sigma\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{2}{n+1}\langle x^n\rangle_{\Delta q}(\mu)[S_\sigma P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n}+\cdots-(\text{traces})]$$

$$\mathcal{O}^{5q}_{\sigma\mu_1\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n}\gamma_5\gamma_\sigma\overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{traces})\right]q$$

$$\langle P,S|\mathcal{O}^{[5]q}_{[\sigma\{\mu_1]\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{1}{n+1}d^q_n(\mu)[(S_\sigma P_{\mu_1}-S_{\mu_1}P_\sigma)P_{\mu_2}\cdots P_{\mu_n}+\cdots-(\text{traces})]$$

$$\mathcal{O}^{[5]q}_{[\sigma\mu_1]\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n}\gamma_5\gamma_{[\sigma}\overleftrightarrow{D}_{\mu_1]}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{traces})\right]q$$

and transversity (h_1):

$$\langle P,S|\mathcal{O}^{\sigma q}_{\rho\nu\{\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{2}{m_N}\langle x^n\rangle_{\delta q}[(S_\rho P_\nu - S_\nu P_\rho)P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n}+\cdots-(\text{traces})]$$

$$\mathcal{O}^{\sigma q}_{\rho\nu\mu_1\mu_2\cdots\mu_n} = \bar{q}[\left(\frac{i}{2}\right)^{n}\gamma_5\sigma_{\rho\nu}\overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{traces})]q$$

Higher moment operators mix with lower dimensional ones: Only $\langle x\rangle_q$, $\langle 1\rangle_{\Delta q}$, $\langle x\rangle_{\Delta q}$, d_1, and $\langle 1\rangle_{\delta q}$ can be measured with $\vec{P}=0$.

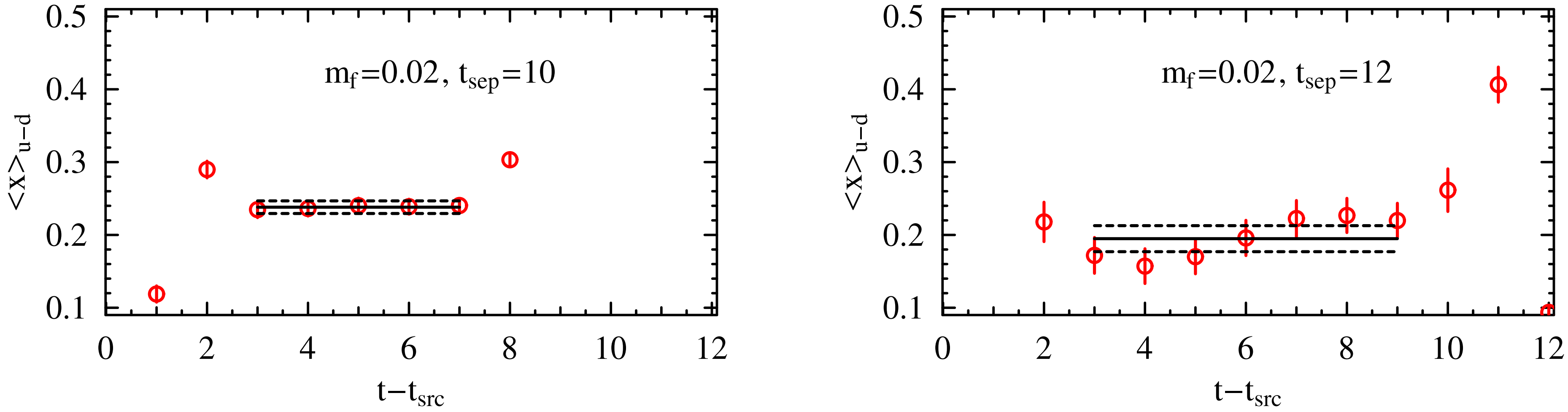
Previous RBC and RBC+UKQCD calculations addressed two important sources of systematics:

- Time separation between nucleon source and sink,
- Spatial volume.

And though not explicitly addressed yet, a better understanding of quark mass dependence is necessary.

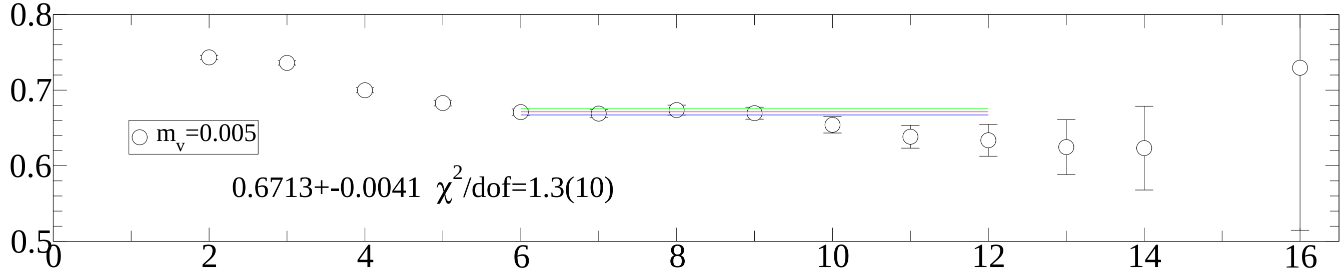
Source/sink time separation:

- If too short, too much contamination from excited states, but if too long, the signal is lost.

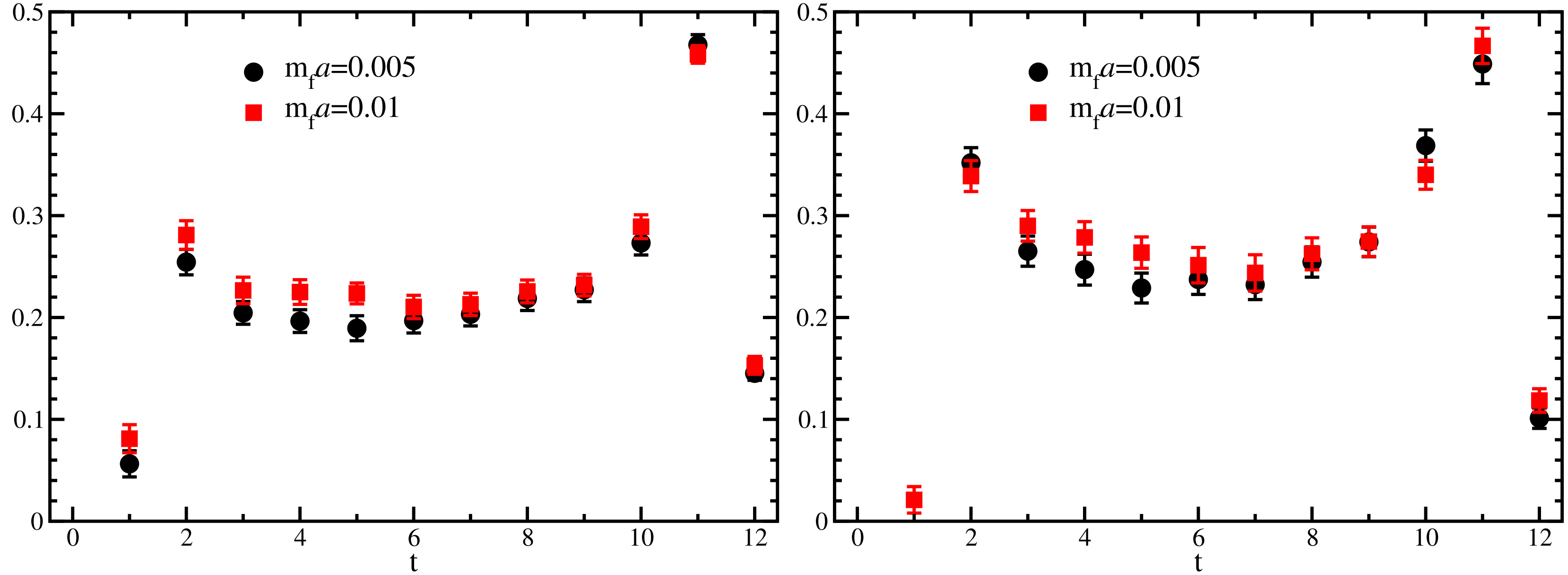


- In an earlier RBC 2-flavor DWF study at $a^{-1} \sim 1.7$ GeV, separation of 10 or 1.1 fm appeared too short.

In the RBC+UKQCD (2+1)-flavor study we choose separation 12 or 13 : ∼1.4 fm:
Mass signal: $m_f = 0.005$

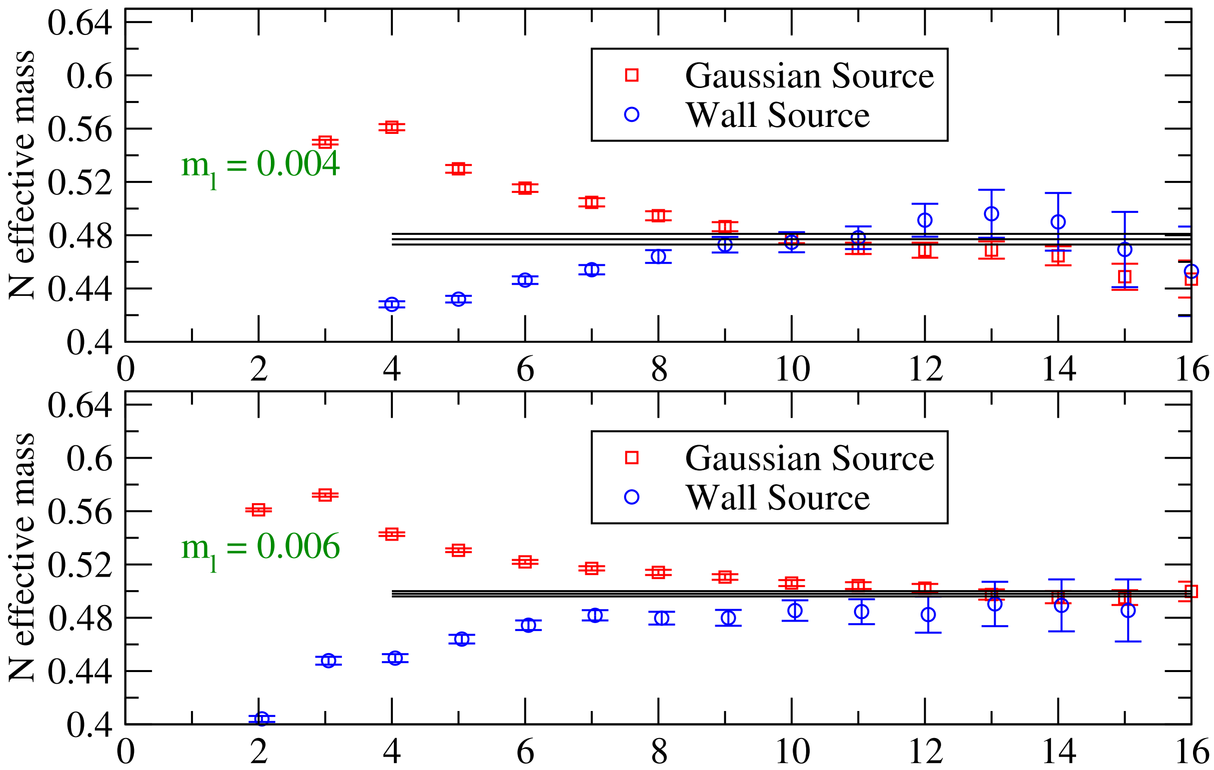


Bare three-point functions: $\langle x \rangle_{u-d}$ (left) and $\langle x \rangle_{\Delta u-\Delta d}$ (right), for $m_f = 0.005$ (red +) and 0.01 (blue ×):



In this study we like to do at least as good, hopefully better: separation of 10 lattice units or longer.

On the other hand, with RBC+UKQCD 2.2-GeV (2+1)-flavor dynamical DWF ensemble:



2-state fits suggest excited-state survives $t_{\text{sink}} \geq 9$.

LHP analysis of vector form factors with $t_{\text{sep}} = 12$ or 1 fm agree with RBC+UKQCD 1.7-GeV results.

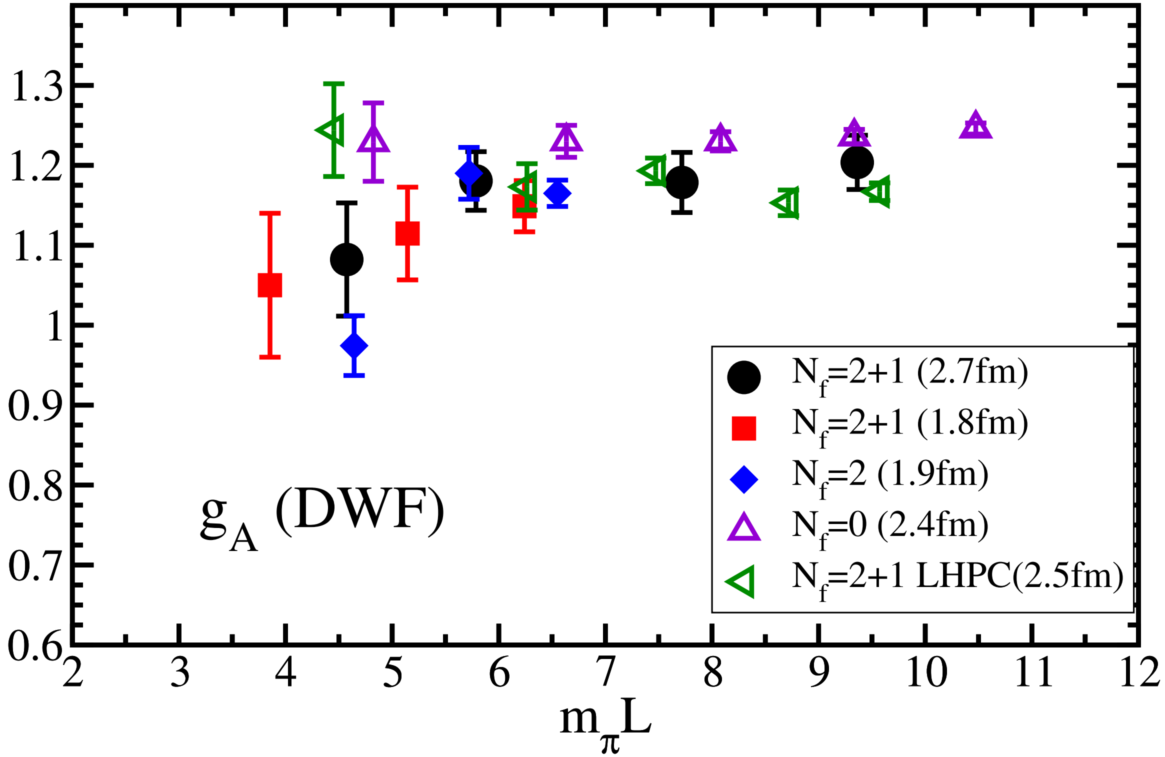
Vector current is less sensitive: conserved charge cannot tell excited-state contamination, for example.

Can we go shorter, $\sim$1 fm, separation, in spite of our lighter masses?

- Perhaps with better tuned source and sink smearing?
- Would be good as we have to fight growing error, $\sim \exp(-3m_\pi t)$.

Spatial volume. In Lattice 2007 Takeshi Yamazaki reported unexpectedly large finite-size effect:

- in axial charge, $g_A/g_V = 1.2694(28)$, measured in neutron β decay, decides neutron life.



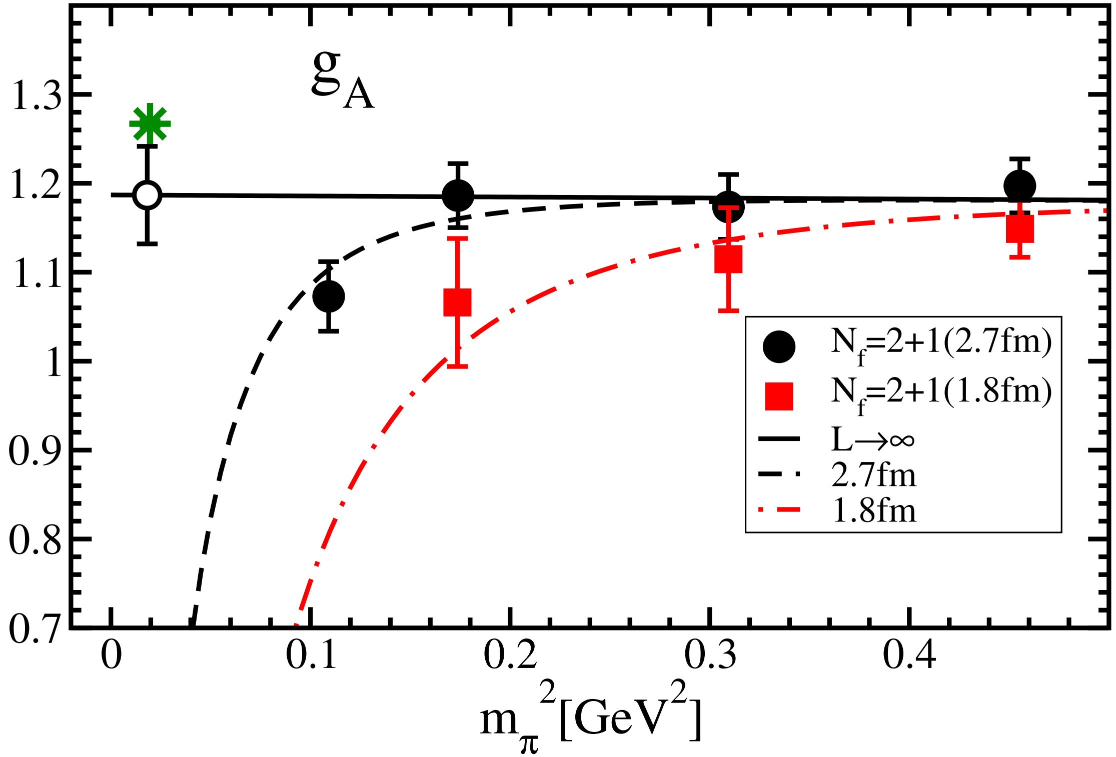
Our DWF on quenched and LHPC DWF on MILC calculations are presented for comparison.

- Heavier quarks: consistent with experiment, no discernible quark-mass dependence.
- Lighter quarks: finite-size sets in as early as $m_\pi L \sim 5$, appear to scale in $m_\pi L$:
  – elastic form factors demand big volumes.

Last year we reported the structure function moments do not seem to suffer so badly, but we need large volume at least for form factors: present $(\sim 4.6\text{fm})^3$ volume is a good start.

RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.73(2)$ GeV, $m_{\rm res} = 0.00315(2)$, $m_{\rm strange} = 0.04$,

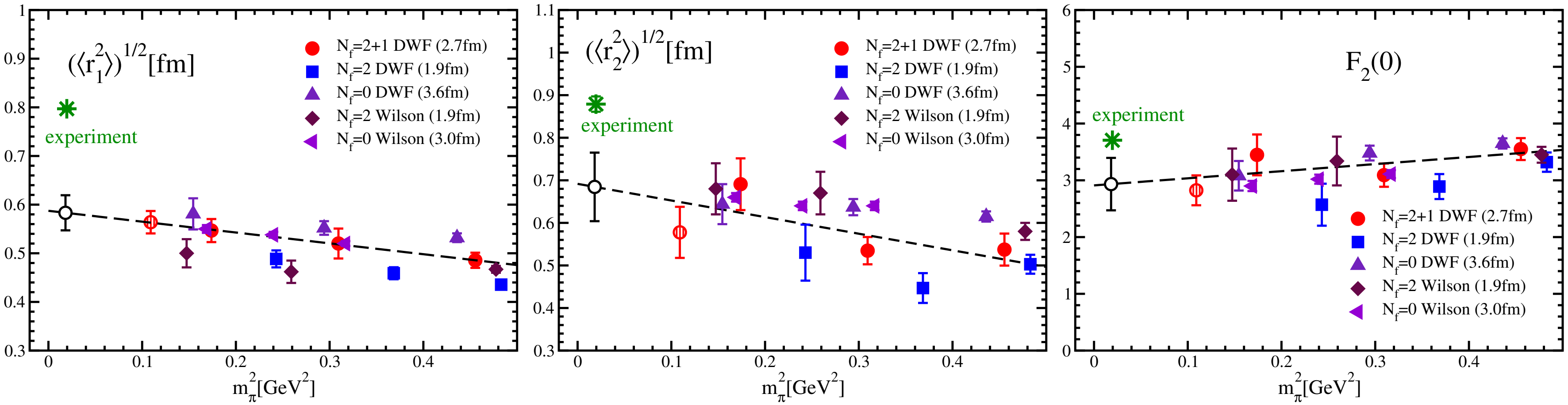
- $m_\pi = 0.67$, 0.56, 0.42 and 0.33 GeV; $m_N = 1.55$, 1.39, 1.22 and 1.15 GeV,
- chiral extrapolation of g_A/g_V and comparison with experiment β decay, $g_A/g_V = 1.2695(29)$.



- A value $1.20(6)_{\rm stat}(4)_{\rm syst}$ is obtained, the systematics from forms such as x^{-3} and $e^{-x}/\sqrt{x}$, $x = m_\pi L$.
- $m_\pi L$ of 6-8 seems necessary to drive the systematics below 1 %.
- 5 fm for the current lightest mass, of 10 for 140 MeV.

RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.73(2)$ GeV, $m_{\rm res} = 0.00315(2)$, $m_{\rm strange} = 0.04$,

- $m_\pi = 0.67$, 0.56, 0.42 and 0.33 GeV; $m_N = 1.55$, 1.39, 1.22 and 1.15 GeV,
- Vector-current form factors: accommodate dipole fit, allow extraction of mean-squared radii.

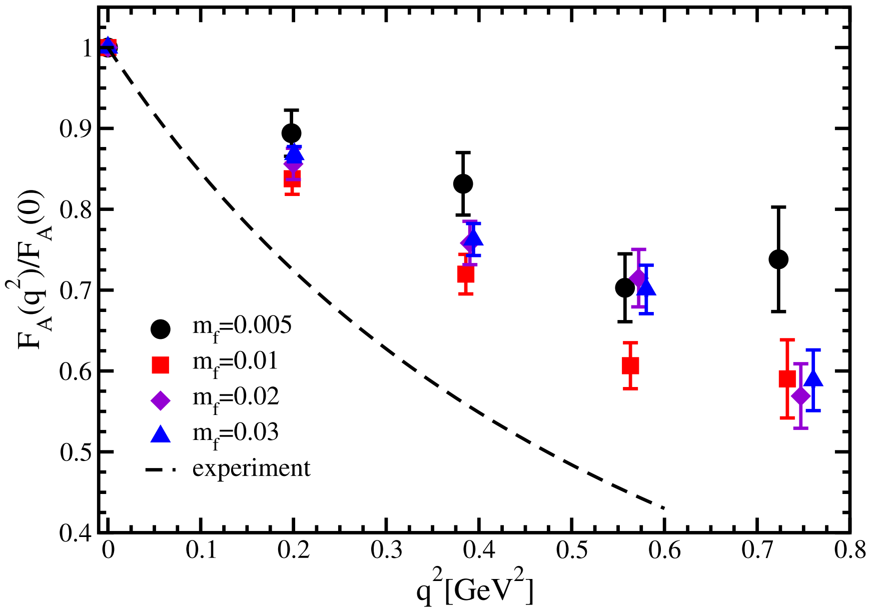


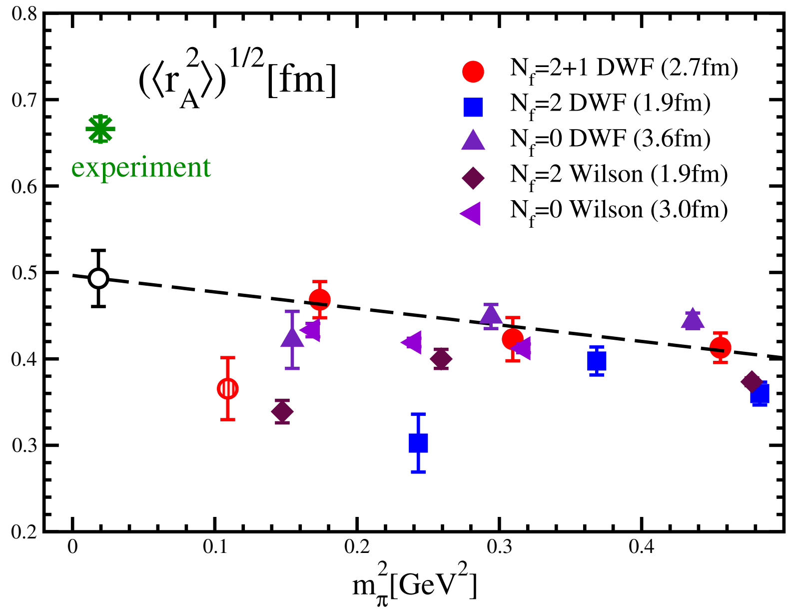
- No singular behavior in m_π^2 seen yet: still too heavy quark mass.
- Anomalous magnetic moment almost consistent with experiment.
- No clear finite-size effect is seen: probably because conserved vector current is too well-behaved.

LHP vector-current calculations on 2.2-GeV RBC/UKQCD ensembles confirm these results.

RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.73(2)$ GeV, $m_{\rm res} = 0.00315(2)$, $m_{\rm strange} = 0.04$,

- $m_\pi = 0.67$, 0.56, 0.42 and 0.33 GeV; $m_N = 1.55$, 1.39, 1.22 and 1.15 GeV,
- Axial-current form factor: note $F_A(0)$ normalization has strong finite-size effect.

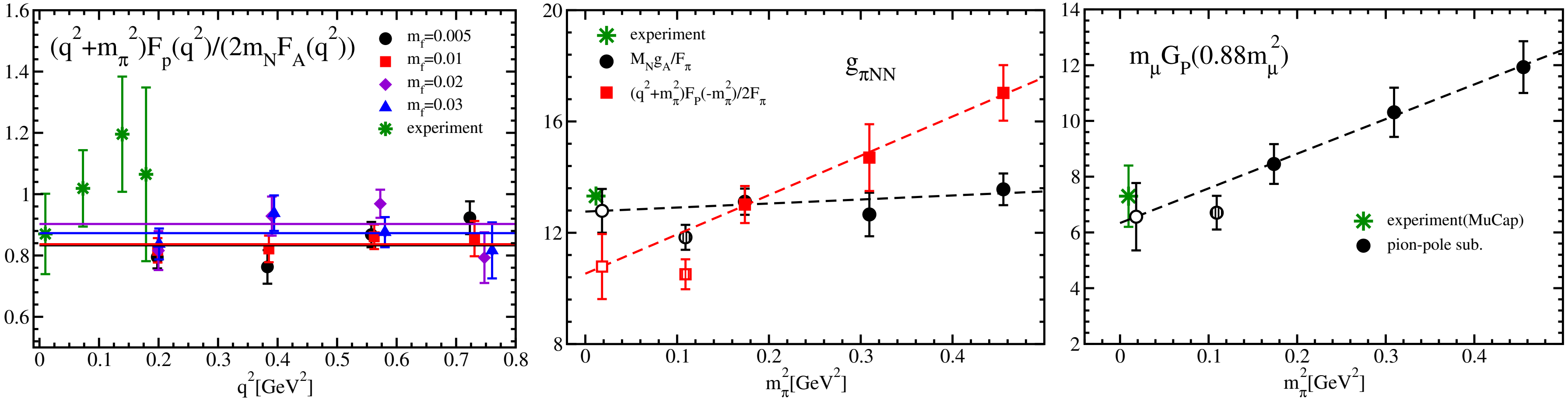




- Needs larger volume.

RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.73(2)$ GeV, $m_{\rm res} = 0.00315(2)$, $m_{\rm strange} = 0.04$,

- $m_\pi = 0.67$, 0.56, 0.42 and 0.33 GeV; $m_N = 1.55$, 1.39, 1.22 and 1.15 GeV,
- Induced pseudoscalar form factor:

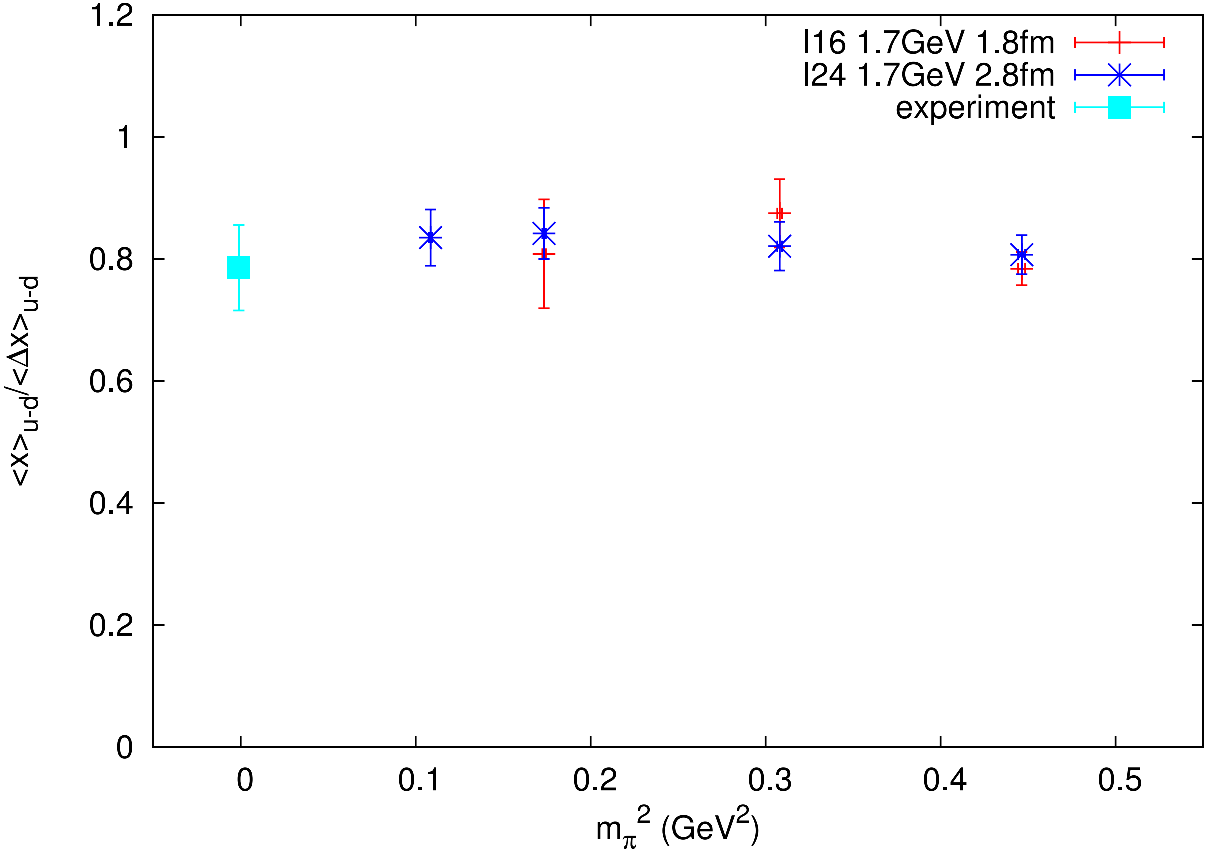


- "Goldberger-Treiman" relation at high momentum transfer?
- $g_{\pi NN}$ from GT and pion pole.
- Muon capture G_P with pion-pole assumption.
- All show finite-size effects at the lightest pion mass.

RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.73(2)$ GeV, $m_{\rm res} = 0.00315(2)$, $m_{\rm strange} = 0.04$,

- $m_\pi = 0.67$, 0.56, 0.42 and 0.33 GeV; $m_N = 1.55$, 1.39, 1.22 and 1.15 GeV,

Ratio, $\langle x \rangle_{u-d}/\langle x \rangle_{\Delta u - \Delta d}$, of momentum and helicity fractions (naturally renormalized on the lattice),

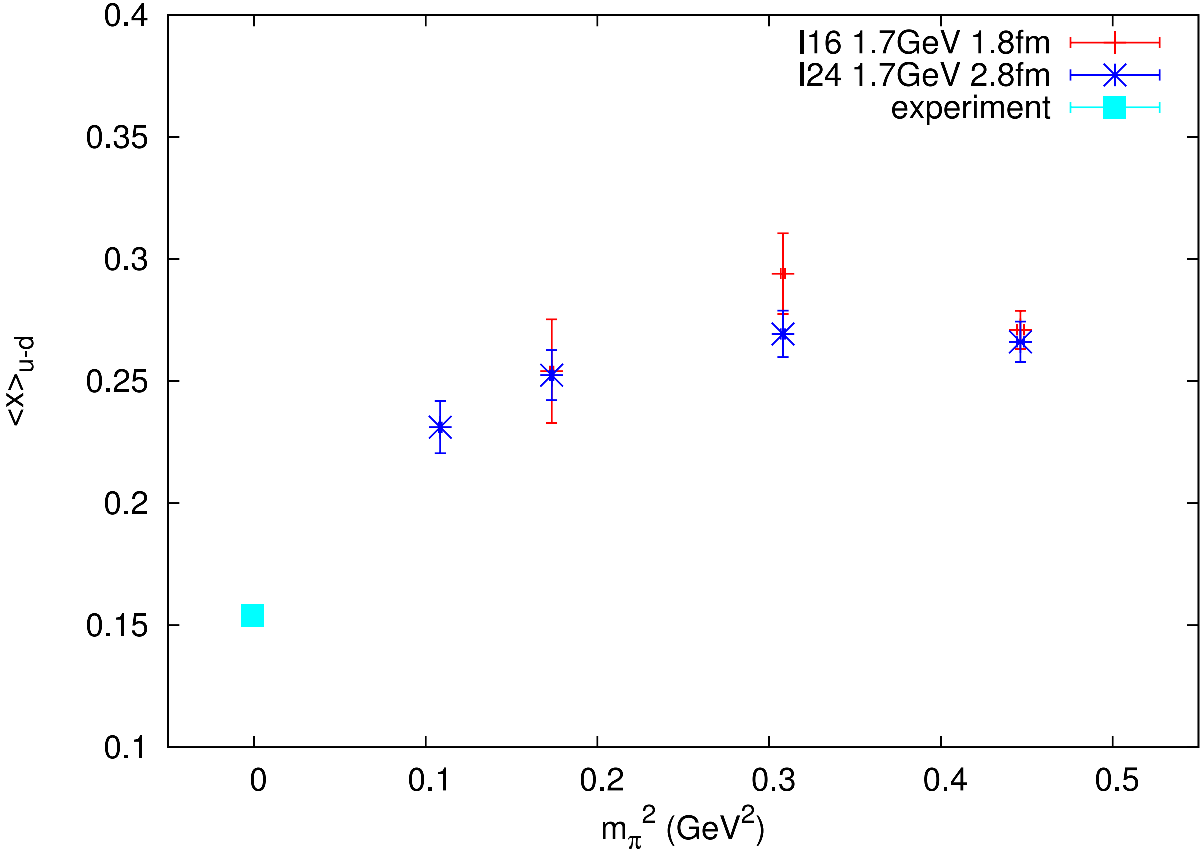


consistent with experiment, no discernible quark-mass dependence.
No finite-size effect seen, in contrast to g_A/g_V which is also naturally renormalized on the lattice.

RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.73(2)$ GeV, $m_{\rm res} = 0.00315(2)$, $m_{\rm strange} = 0.04$,

- $m_\pi = 0.67$, 0.56, 0.42 and 0.33 GeV; $m_N = 1.55$, 1.39, 1.22 and 1.15 GeV,

Momentum fraction, $\langle x \rangle_{u-d}$, with NPR, $Z^{\overline{\rm MS}}(2{\rm GeV}) = 1.15(4)$, plotted against m_π^2,



Absolute values have improved, trending to the experimental values, with NPR, $Z^{\overline{\rm MS}}(2{\rm GeV}) = 1.15(4)$.

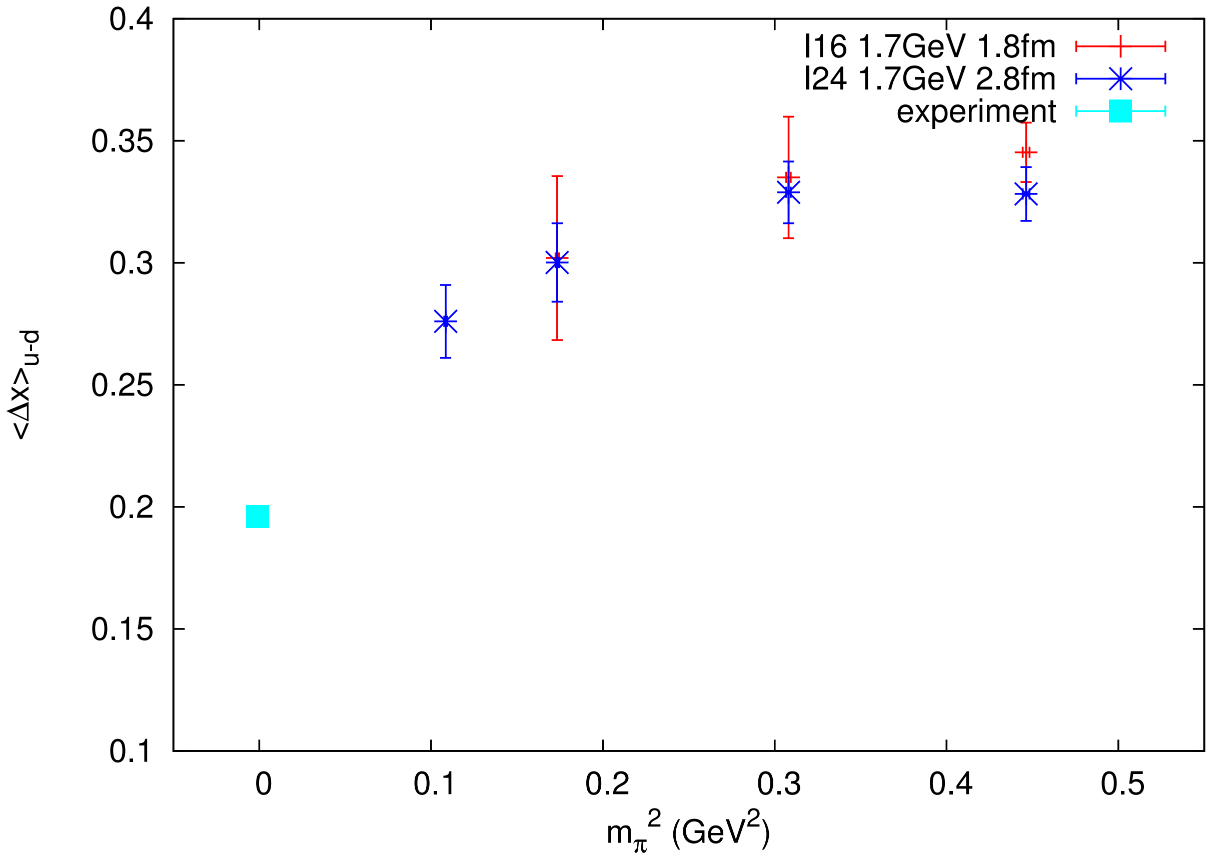
No finite size effect seen (16^3 (+) and 24^3 (×) results agree): Likely physical light-quark effect.

A better understanding of quark mass dependence is necessary.

RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.73(2)$ GeV, $m_{\rm res} = 0.00315(2)$, $m_{\rm strange} = 0.04$,

- $m_\pi = 0.67$, 0.56, 0.42 and 0.33 GeV; $m_N = 1.55$, 1.39, 1.22 and 1.15 GeV,

Helicity fraction, $\langle x \rangle_{\Delta u - \Delta d}$, with NPR, $Z^{\overline{\rm MS}(2{\rm GeV})} = 1.15(3)$, plotted against m_π^2,



Absolute values have improved, trending to the experimental values, with NPR, $Z^{\overline{\rm MS}}(2{\rm GeV}) = 1.15(3)$.

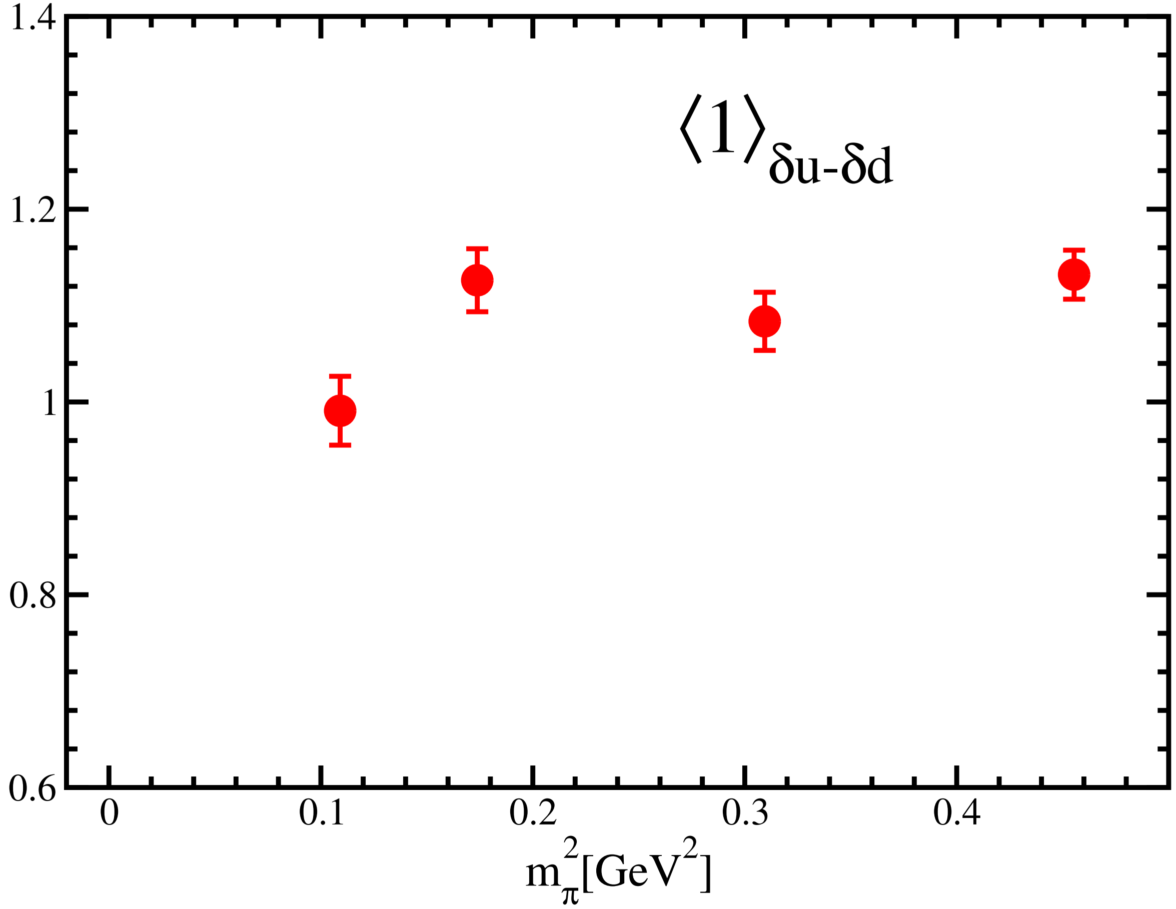
No finite size effect seen (16^3 (+) and 24^3 ($\times$) results agree): Likely physical light-quark effect.

A better understanding of quark mass dependence is necessary.

RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.73(2)$ GeV, $m_{\rm res} = 0.00315(2)$, $m_{\rm strange} = 0.04$,

- $m_\pi = 0.67$, 0.56, 0.42 and 0.33 GeV; $m_N = 1.55$, 1.39, 1.22 and 1.15 GeV,

Transversity fraction, $\langle 1 \rangle_{\delta u - \delta d}$, with preliminary NPR, $Z^{\overline{\rm MS}(2{\rm GeV})} = 0.783(3)$,



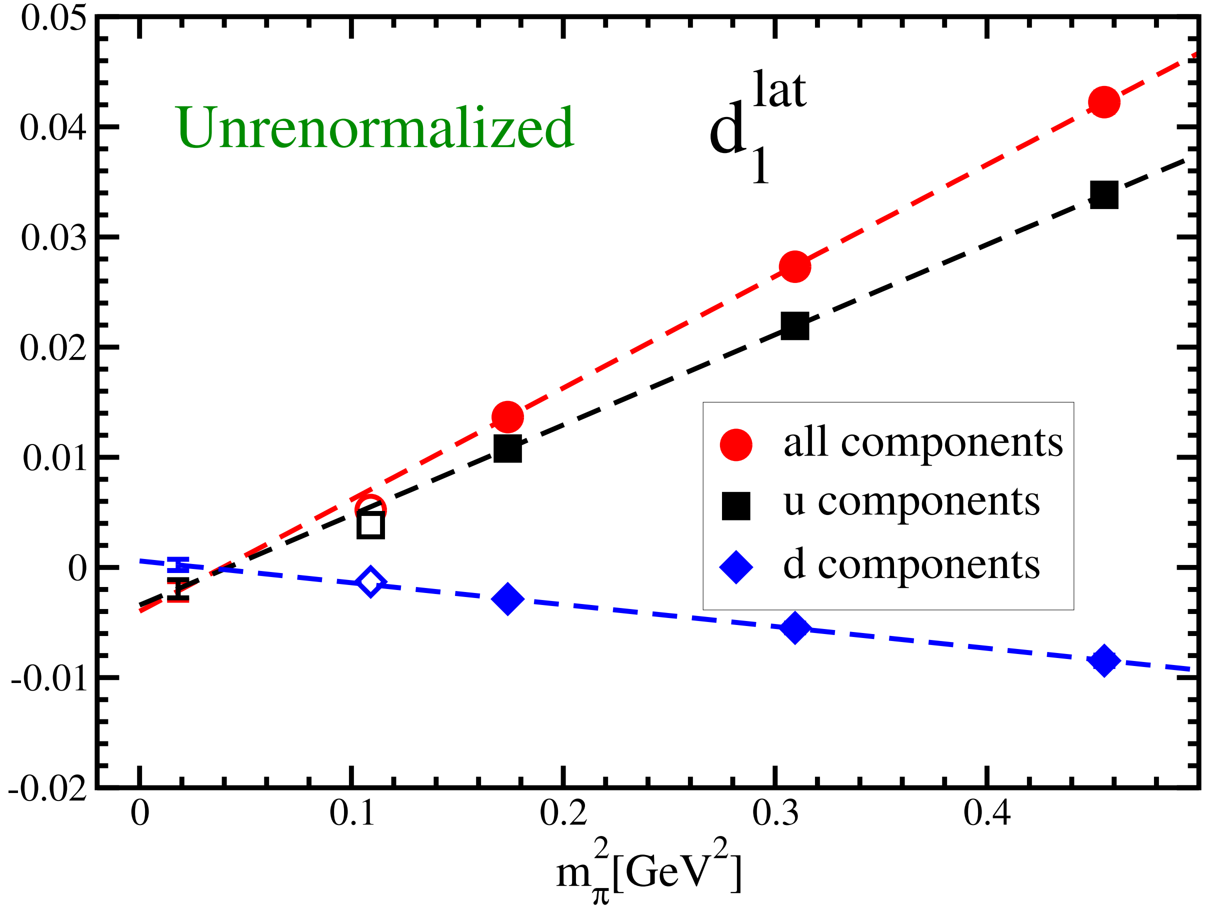
Likely physical light-quark effect.

A better understanding of quark mass dependence is necessary.

RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.73(2)$ GeV, $m_{\rm res} = 0.00315(2)$, $m_{\rm strange} = 0.04$,

- $m_\pi = 0.67$, 0.56, 0.42 and 0.33 GeV; $m_N = 1.55$, 1.39, 1.22 and 1.15 GeV,

Twist-3, d_1, yet to be renormalized



Chirally well-behaved, small, and consistent with Wandzura-Wilczek relation.

Given the severe finite-size effect in axial-current form factors, such as g_A, we abandoned nucleon-structure calculations on the finer, 2.2-GeV, Iwasaki+DWF ensembles.

- LHP reports their analyses on our ensembles: encouraging confirmaiton of our vector-current form factors.

RBC and UKQCD collaborations are jointly generating new (2+1)-flavor DWF ensembles

- with Iwasaki and dislocation-suppressing-determinant-ratio (DSDR) gauge action, $\beta = 1.75$,
- and DWF fermion action, $L_s = 32$ and $M_5 = 1.8$, with $m_{\mathrm{strange}} = 0.045$, $m_{\mathrm{ud}} = 0.0042$ and 0.001,

aiming at lighter mass in a sufficiently large volume: We have reasonable topology distribution while maintaining small residual mass, $m_{\mathrm{res}} \sim 0.002$:

- lattice scale from Ω^-: $a^{-1} = 1.368(7)$ GeV,
- $m_\pi = 0.1816(8)$ and $0.1267(8)$, or ~ 250 and 180 MeV.

$32^3 \times 64$ volume is about 4.6 fm across in space, 9.2 fm in time.

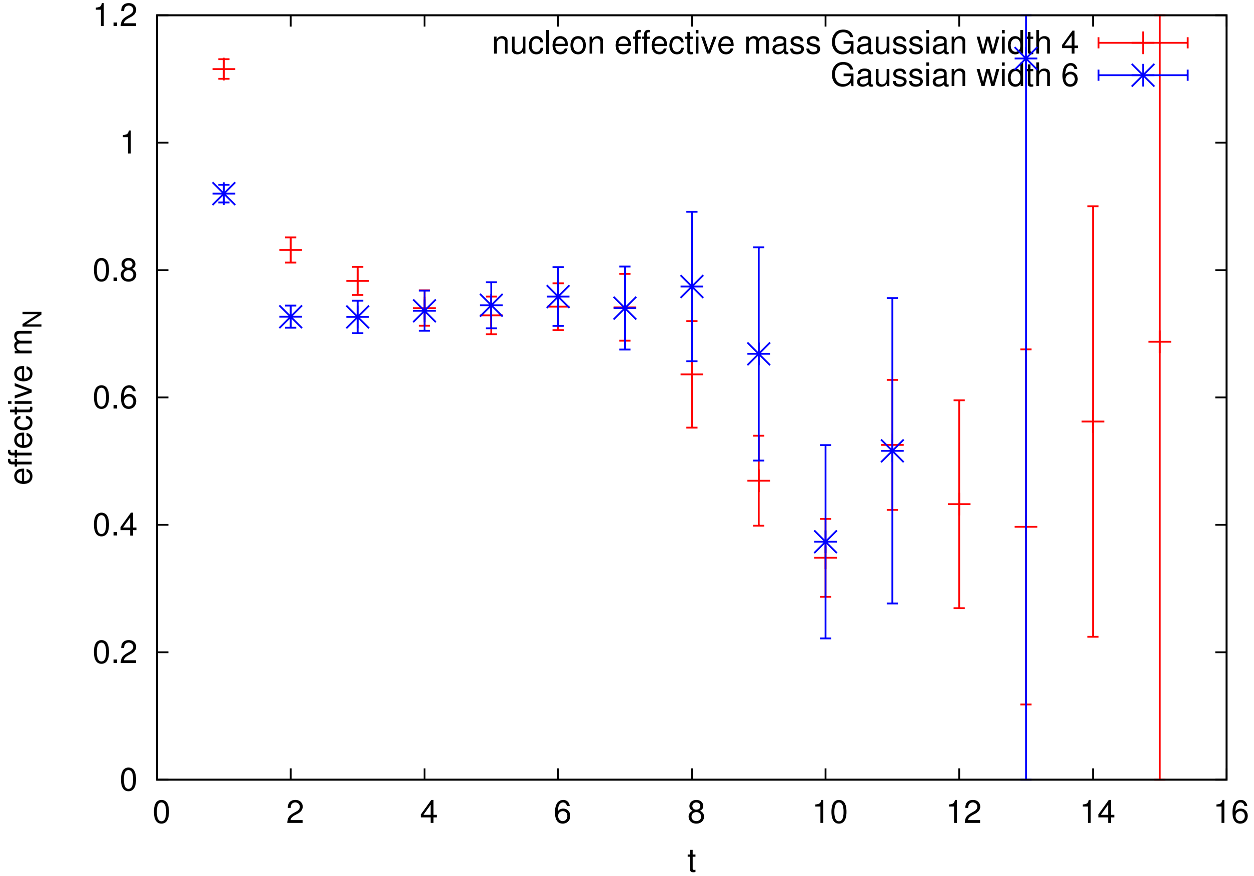
We started nucleon structure calculations using the RICC supercomputing facility at RIKEN, Wako, Japan.

- tuning Gaussian smearing with width 4 and 6,

at this 100-TFlops peak-speed facility.

RBC/UKQCD (2+1)-flavor, ID+DWF dynamical, $a^{-1} = 1.368(7)$ GeV, $m_{\rm strange} = 0.045$,

Nucleon mass signal from the light ($m_{\rm ud} = 0.001$ or $m_\pi = 180$ MeV) ensemble, with ∼30 configurations,



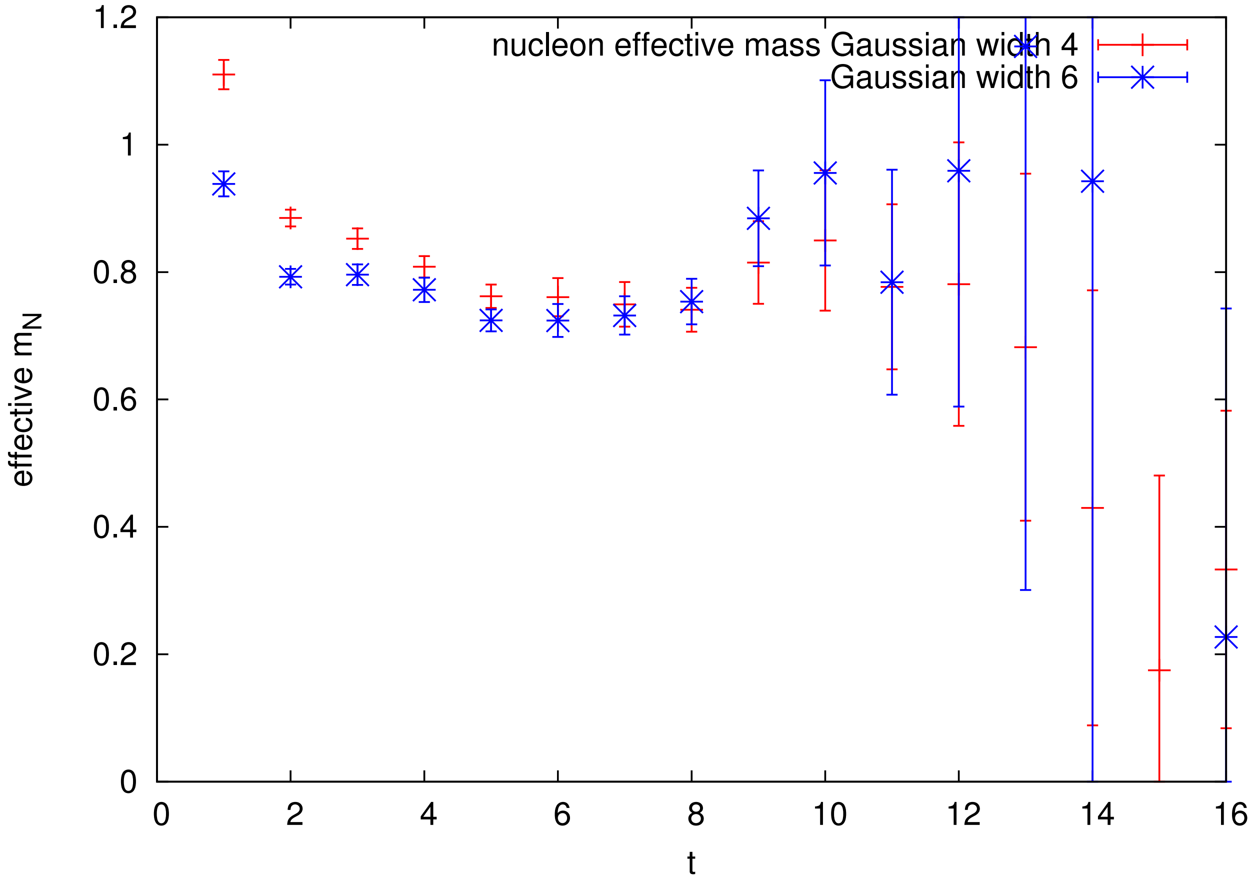
$m_N = 0.721(13)$ or ~ 0.98 GeV,

but probably needs a longer plateau for structure calculation to be free of excited-state contamination,

presently increasing the statistics.

RBC/UKQCD (2+1)-flavor, ID+DWF dynamical, $a^{-1} = 1.368(7)$ GeV, $m_{\rm strange} = 0.045$,

Nucleon mass signal from the heavy ($m_{\rm ud} = 0.0042$ or $m_\pi = 250$ MeV) ensemble, with ∼40–50 configurations

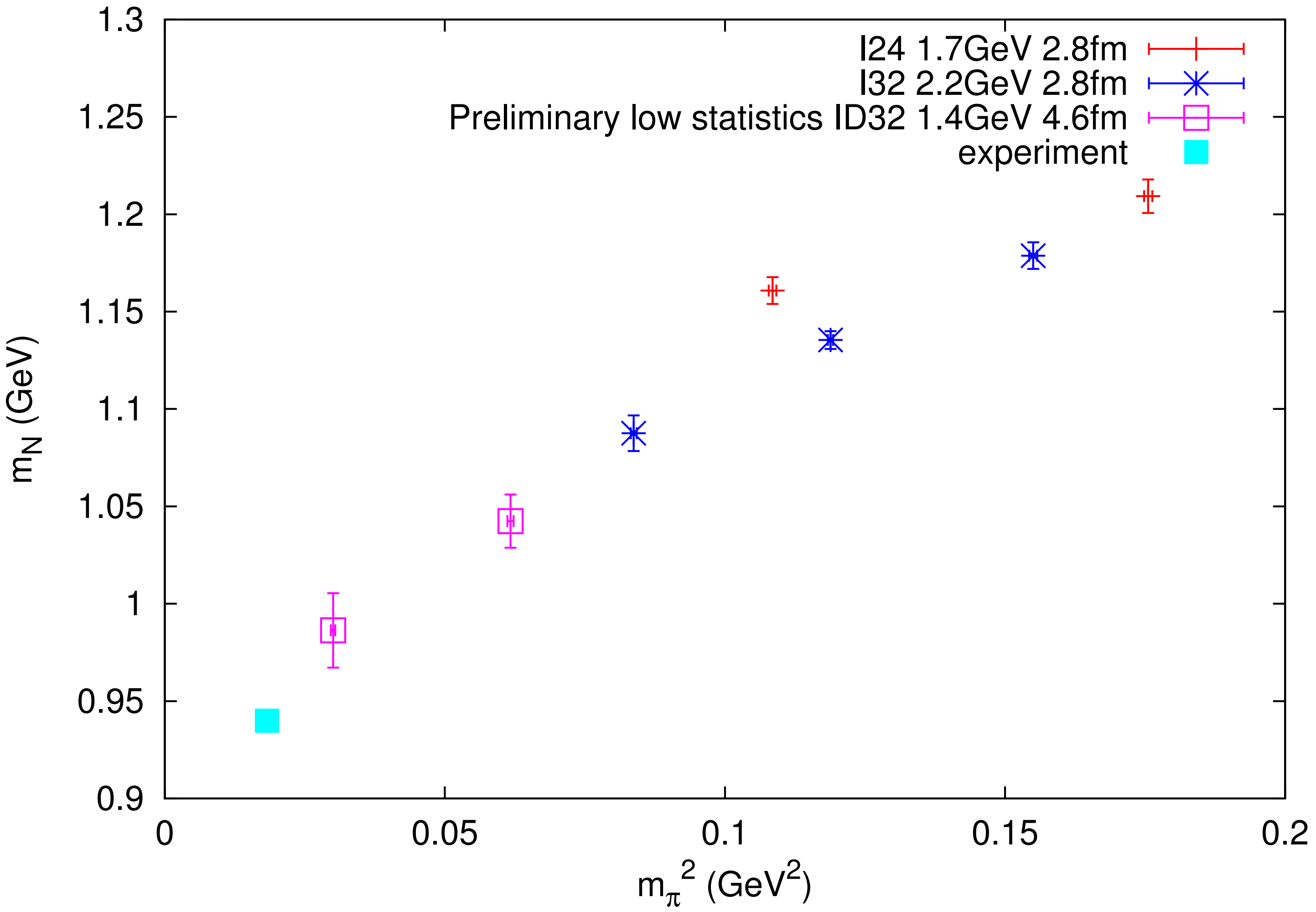


$m_N = 0.763(10)$ or ~ 1.05 GeV,

but needs a better plateau for structure calculation to be free of excited-state contamination,

presently increasing the statistics.

Conclusions: RBC/UKQCD (2+1)-flavor, ID+DWF ensembles are being analyzed for nucleon physics.



with $a^{-1} = 1.368(7)$ GeV, $(\sim 4.6\text{fm})^3$ spatial volume.

Closer to physical mass, $m_\pi = 180$ and 250 MeV, $m_N < 1.0$ GeV,

Isovector form factors and structure function moments will be reported in the near future.