Hadron Resonances and Decays from the Lattice

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With

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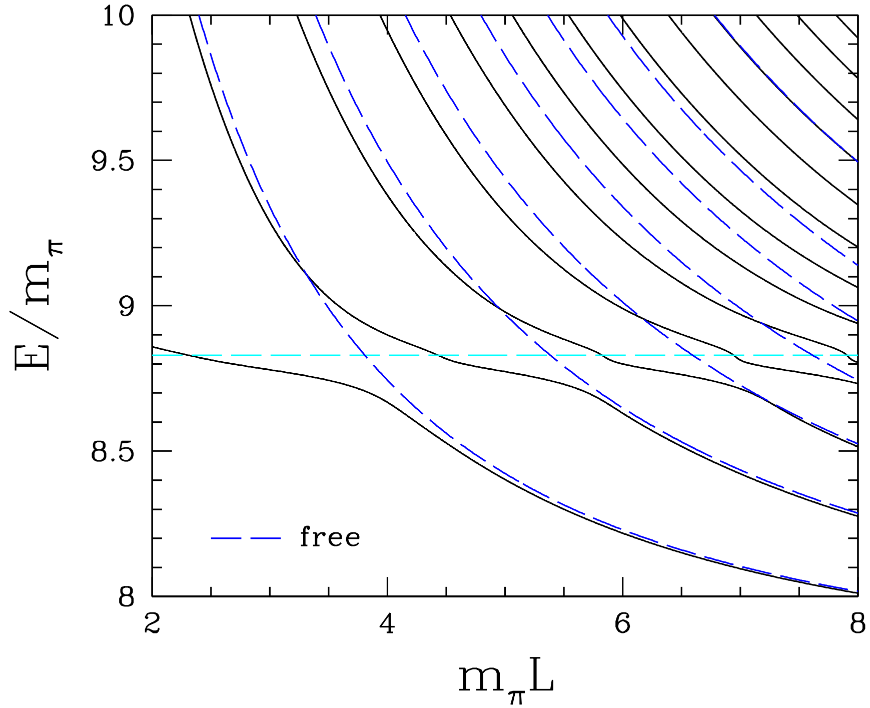
The Problem

- Apart from the nucleon, hadrons of most phenomenological interest are resonances

- Resonance states cannot be identified with a single energy eigenstate of the lattice Hamiltonian

- The method of choice is to compute mass and width from the volume dependence of the energy levels

Lüscher, Wiese



$\Delta \rightarrow N\pi$

This Talk

					$\underline{SU(2)_F}$
$\rho(770)$	$\longrightarrow$	$\pi\pi$	}	elastic	$I = 1$
$\Delta(1232)$	$\longrightarrow$	$N\pi$	}		$I = \frac{3}{2}$

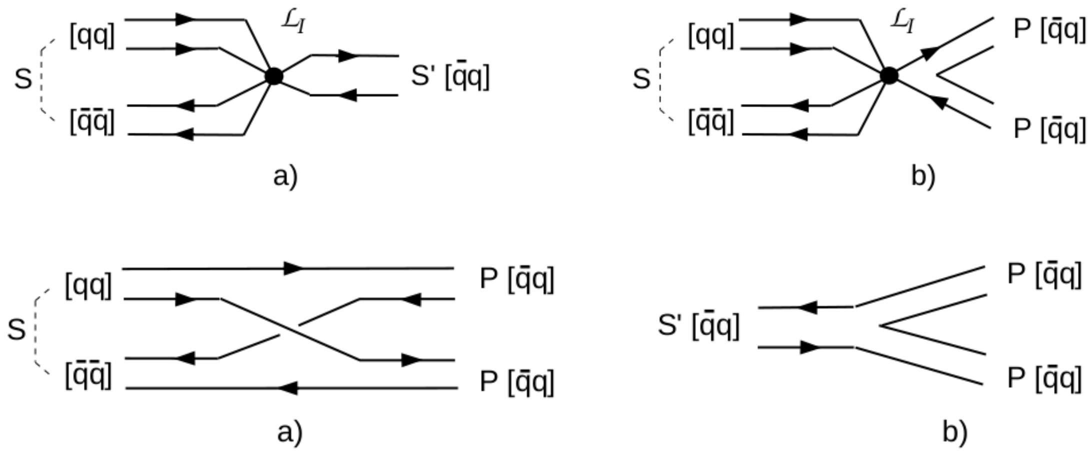
Benchmark Calculation

- $N_f = 2$ clover fermions
- Physical quark masses
- Extension of formalism to arbitrary total momenta $\vec{P}$ (representations of $H(4)$)

Challenges

$$\left.\begin{array}{rcl} \sigma(440) & \longrightarrow & \pi\pi \\ \kappa(800) & \longrightarrow & K\pi \\ f_0(965) & \longrightarrow & \pi\pi \\ & \longrightarrow & K\bar{K} \\ a_0(999) & \longrightarrow & \pi\eta \\ & \longrightarrow & K\bar{K} \end{array}\right\}$$

Tetraquarks ?



't Hooft et al.

Baryon Resonances

$$
\left.
\begin{array}{lll}
N(1440) \longrightarrow & N\pi & \text{Roper} \\
& \Delta\pi & \\
& N\eta & \\
N^*(1535) \longrightarrow & N\pi & \\
& N\eta & \\
\vdots & &
\end{array}
\right\}
$$

Shortcut

- Compute energy levels on the lattice
- Match with energy levels computed from HBChPT in finite volume

Meißner et al.

Extension of Lüscher formalism to multichannel resonances in progress

Action

$$S = S_G + S_F$$

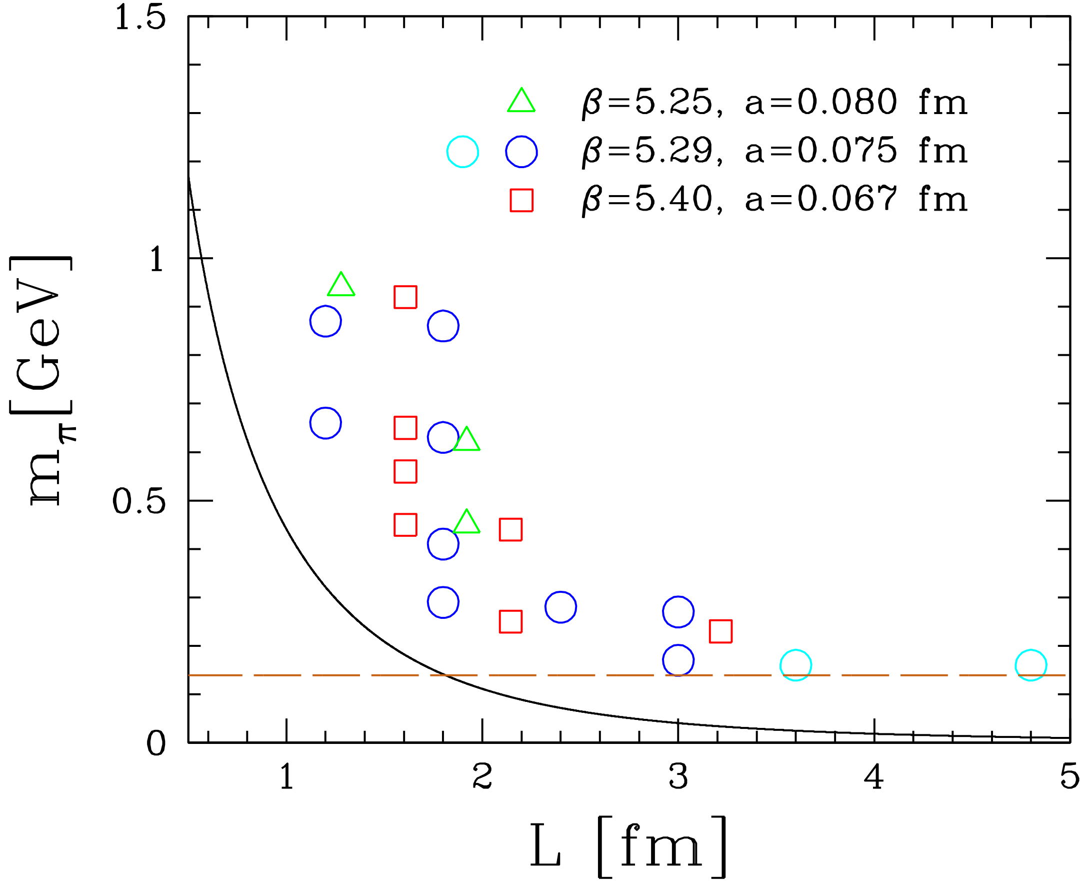
$$S_G = \beta \sum_{x,\mu<\nu} \left(1 - \frac{1}{3}\mathrm{Re}\,\mathrm{Tr}\,U_{\mu\nu}(x)\right)$$

$$S_F = \sum_x \Big\{ \bar\psi(x)\psi(x) - \kappa\,\bar\psi(x)U_\mu^\dagger(x-\hat\mu)[1+\gamma_\mu]\psi(x-\hat\mu) - \kappa\,\bar\psi(x)U_\mu(x)[1-\gamma_\mu]\psi(x+\hat\mu) - \frac{1}{2}\kappa\,c_{SW}\,g\,\bar\psi(x)\sigma_{\mu\nu}F_{\mu\nu}(x)\psi(x) \Big\}$$

Clover Fermions

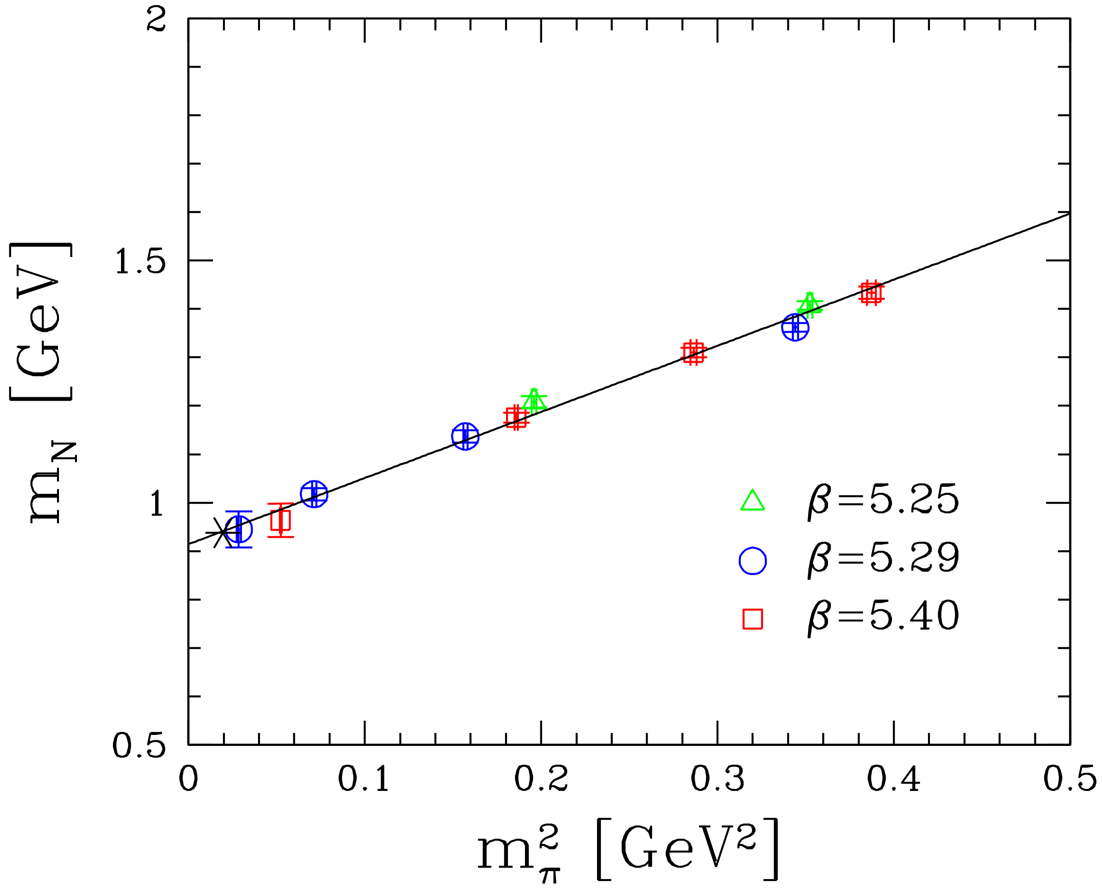
$$\boxed{N_f = 2}$$

Landscape



β=5.25, a=0.080 fm
β=5.29, a=0.075 fm
β=5.40, a=0.067 fm
m_π[GeV]
L [fm]
1.5
1
0.5
0
1
2
3
4
5

Scale



$$r^2 \left. \frac{\partial V(r)}{\partial r} \right|_{r=r_0} = 1.65 \qquad r_0 = 0.47(1)\ \mathrm{fm}$$

Rho

Interpolating fields

$$\rho_i = \quad \bar{q}\gamma_i q|0\rangle \quad \rightarrow \quad \sqrt{\frac{2}{3}}|^3S_1\rangle + \sqrt{\frac{1}{3}}|^3D_1\rangle$$

$$\rho_i^0 = \bar{q}\gamma_0\gamma_i q|0\rangle \quad \rightarrow \quad \sqrt{\frac{1}{3}}|^3S_1\rangle - \sqrt{\frac{2}{3}}|^3D_1\rangle$$

Glozman, Lang & Limmer

Green function

$$\begin{pmatrix} {\rho_i}^\dagger \rho_i & {\rho_i}^\dagger \rho_i^0 \\ \rho_i^{0\dagger} \rho_i & \rho_i^{0\dagger} \rho_i^0 \end{pmatrix}$$

$$\downarrow$$

$$G_m^P(x,k) \quad \rightarrow \quad \frac{k^2}{4\pi} M_{mm'}^P(k)\, j_1(kx)\, Y_{1m'}(\theta,\phi)$$

$$\vec{P} = \frac{2\pi}{L}\vec{n}\,, \quad \vec{n} \in \mathbb{Z}^3$$

Phase

$$\det\left[\mathrm{e}^{2i\delta_{11}}(M-i) - (M+i)\right] = 0$$

$\vec{P}/\lvert\vec{P}\rvert$	$\vec{\Gamma} = \vec{\gamma}, \gamma_0\vec{\gamma}$	$\cot\delta_{11}$	
$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$	Γ_i	w_{00}	Lüscher
$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$	$\Gamma_{1,2}$	$w_{00} - w_{20}$	Gottlieb & Rummukainen
	Γ_3	$w_{00} + 2w_{20}$	
$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$	$\Gamma_1 + \Gamma_2$	$w_{00} - w_{20} + i\sqrt{6}w_{22}$	QCDSF
	$\Gamma_1 - \Gamma_2$	$w_{00} - w_{20} - i\sqrt{6}w_{22}$	
	Γ_3	$w_{00} + 2w_{20}$	
$\frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$	$\Gamma_1 + \Gamma_2 + \Gamma_3$	$w_{00} + 2i\sqrt{6}w_{22}$	QCDSF
	$\Gamma_i - \Gamma_j$	$w_{00} - i\sqrt{6}w_{22}$	

$$w_{lm} = \frac{\gamma^{-1}\pi^{-3/2}}{\sqrt{2l+1}q^{l+1}} Z^{P}_{lm}(1;q^2)^* \,, \quad q = \frac{kL}{2\pi}$$

Hypothetical Energy Levels

Effective range formula

$$\frac{k^3}{E} \cot\, \delta_{11}(k) = \frac{24\pi}{g^2_{\rho\pi\pi}} \left(k^2_\rho - k^2\right) \qquad E = 2\sqrt{k^2 + m^2_\pi}\,, \quad k_\rho = \frac{1}{2}\sqrt{m^2_\rho - 4m^2_\pi}$$

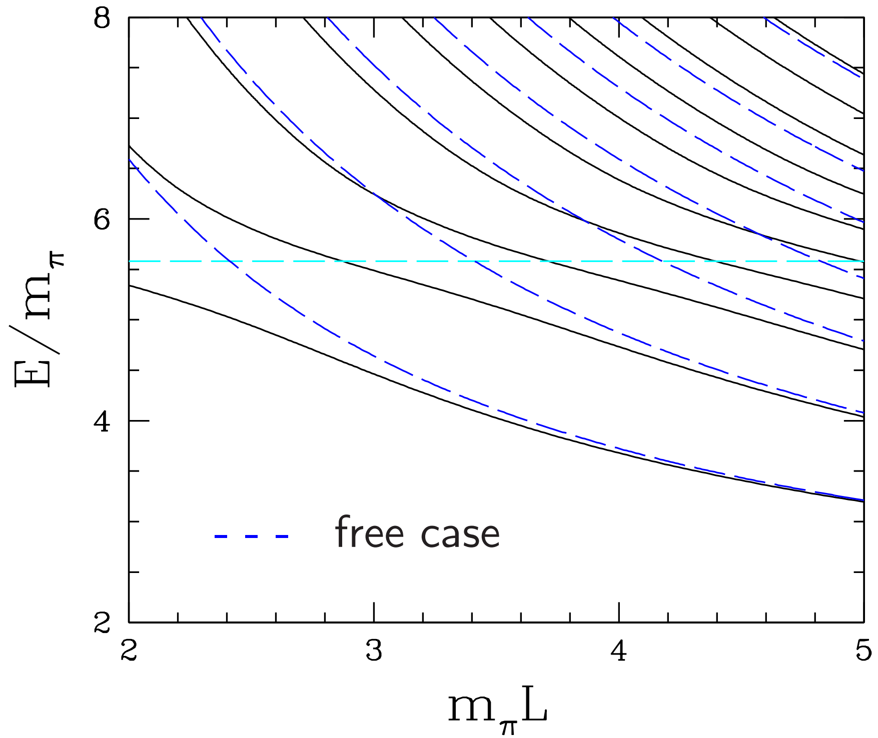
$$\Gamma_\rho = \frac{g^2_{\rho\pi\pi}}{6\pi} \frac{k^3_\rho}{m^2_\rho} \qquad \Gamma_\rho = 146\,\text{MeV} \qquad \Rightarrow \quad g_{\rho\pi\pi} = 5.9$$

Free energy levels

$$E = 2\sqrt{k^2 + m^2_\pi}\,, \quad k = \frac{2\pi|\vec{n}|}{L}\,, \quad \vec{n} \in \mathbb{N}^3$$

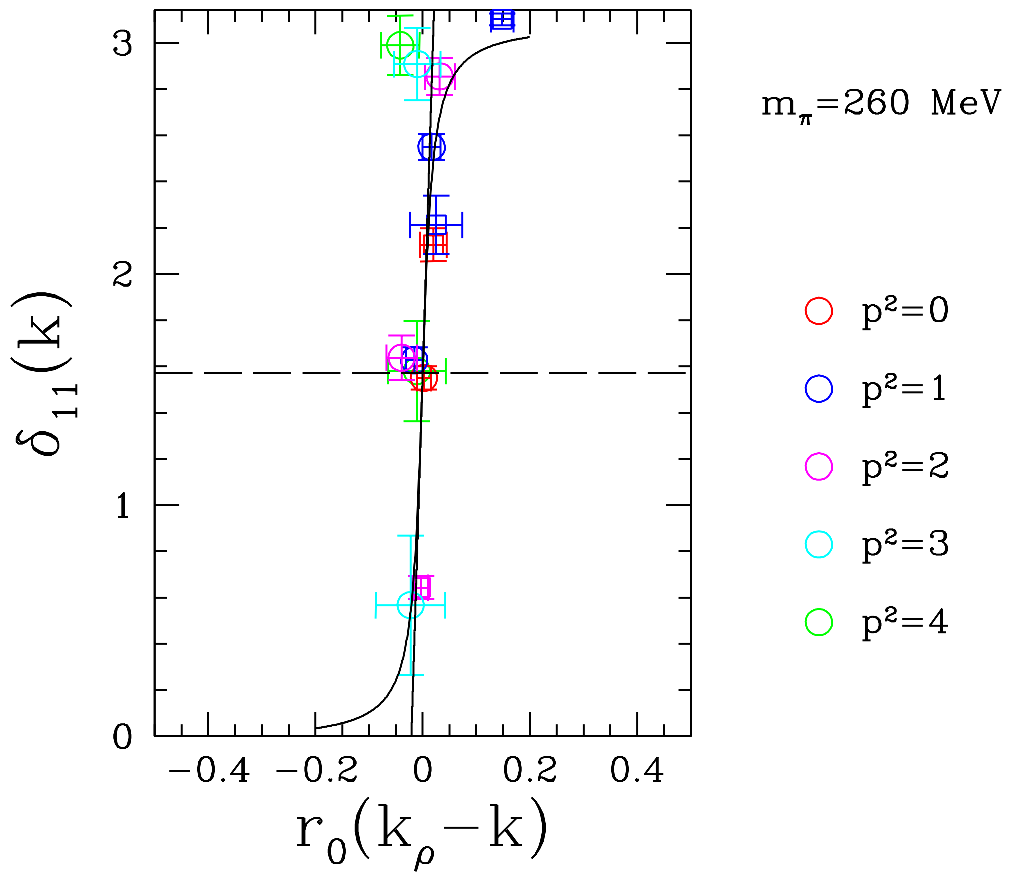
$$\frac{E}{m_\pi} = 2\sqrt{1 + \frac{(2\pi\vec{n})^2}{(m_\pi L)^2}}$$

$\vec{P} = 0$ – physical m_π, m_ρ and Γ_ρ

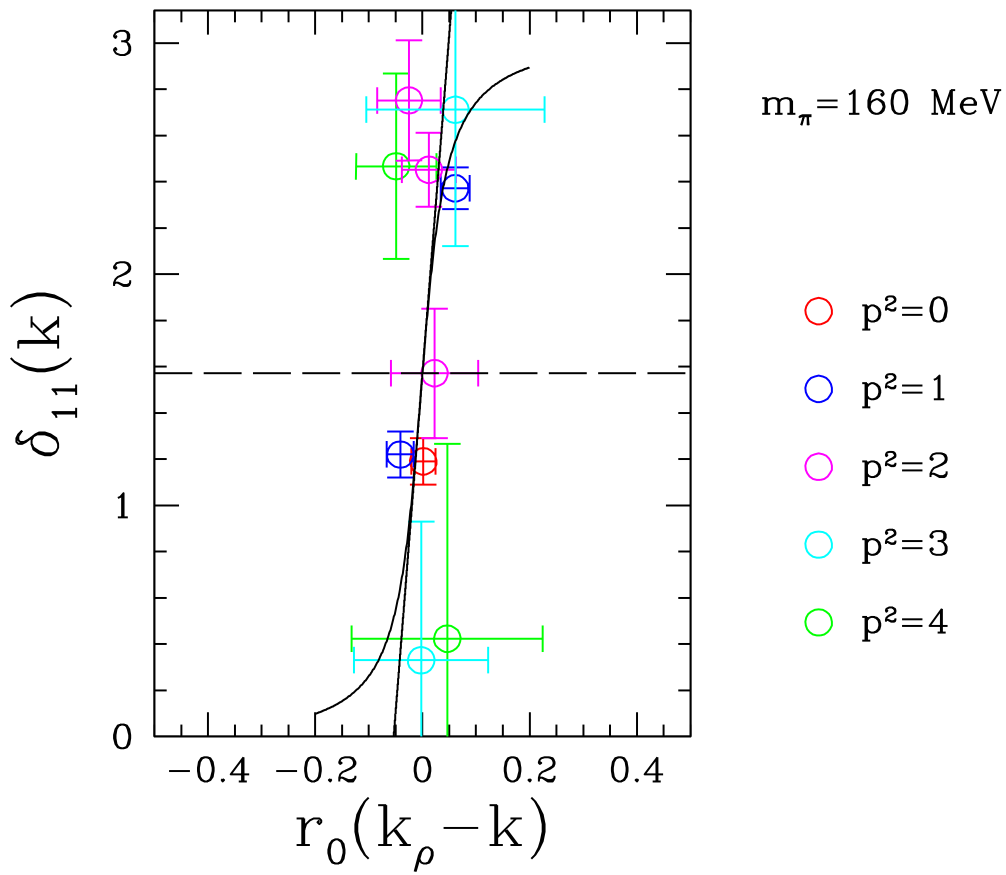


Useful region

Phases

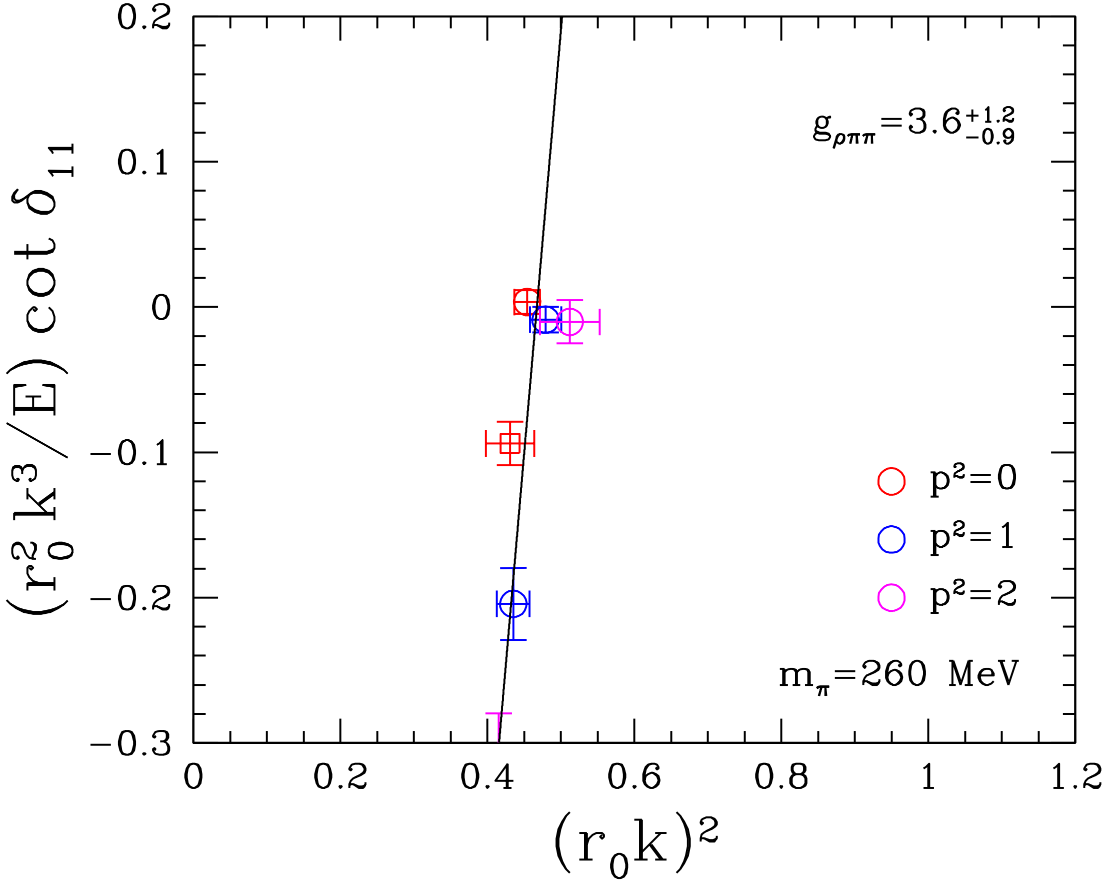


m_π=260 MeV
p^2=0
p^2=1
p^2=2
p^2=3
p^2=4
$\delta_{11}(k)$
$r_0(k_\rho - k)$
0
1
2
3
−0.4
−0.2
0
0.2
0.4

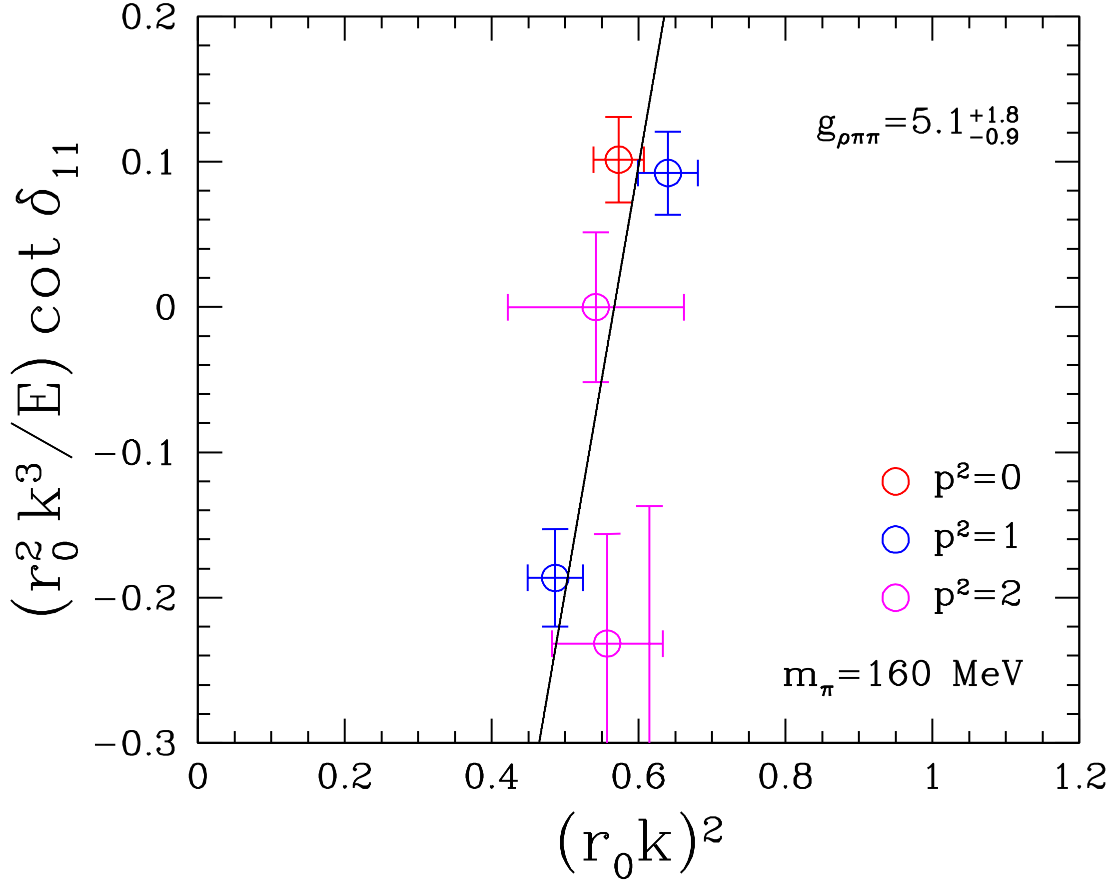


3
2
1
0
−0.4
−0.2
0
0.2
0.4
$\delta_{11}(k)$
$r_0(k_\rho - k)$
m_π=160 MeV
p^2=0
p^2=1
p^2=2
p^2=3
p^2=4

Fits

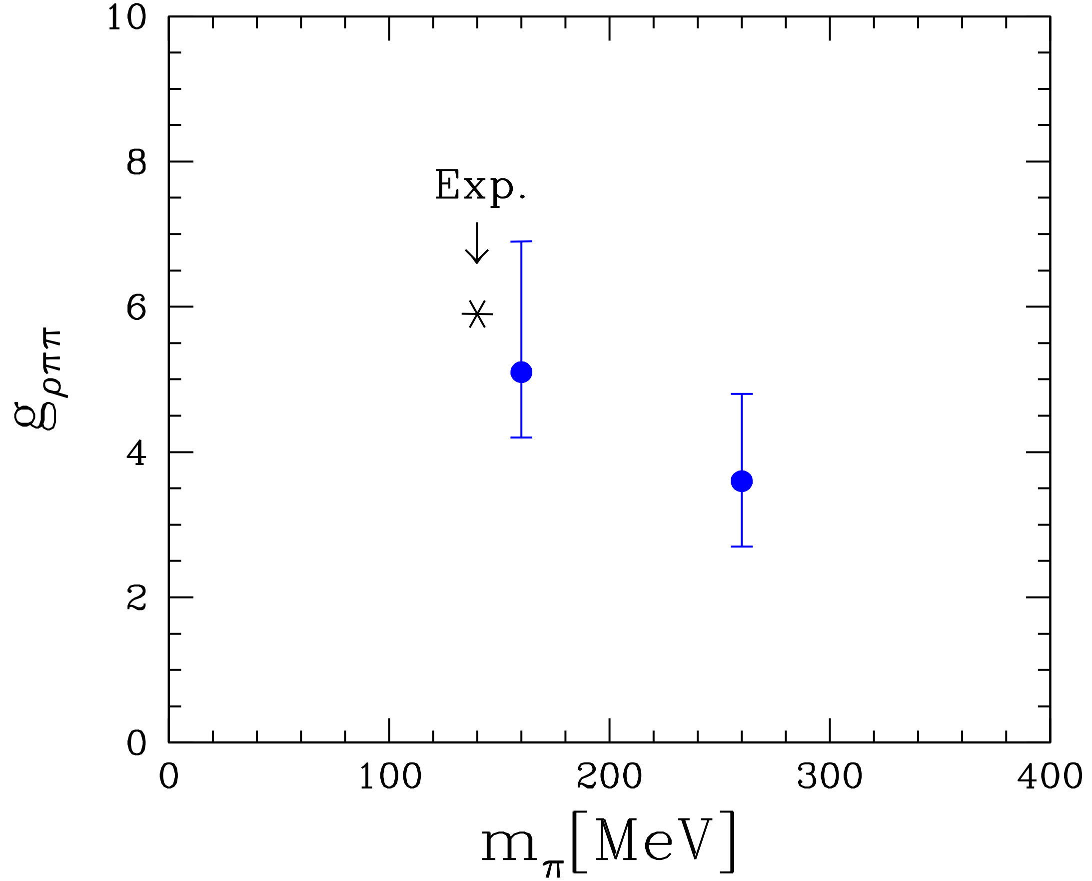


$g_{\rho\pi\pi}=3.6^{+1.2}_{-0.9}$
$p^2=0$
$p^2=1$
$p^2=2$
m_π=260 MeV
$(r_0^2 k^3/E) \cot \delta_{11}$
$(r_0 k)^2$
0.2
0.1
0
−0.1
−0.2
−0.3
0
0.2
0.4
0.6
0.8
1
1.2

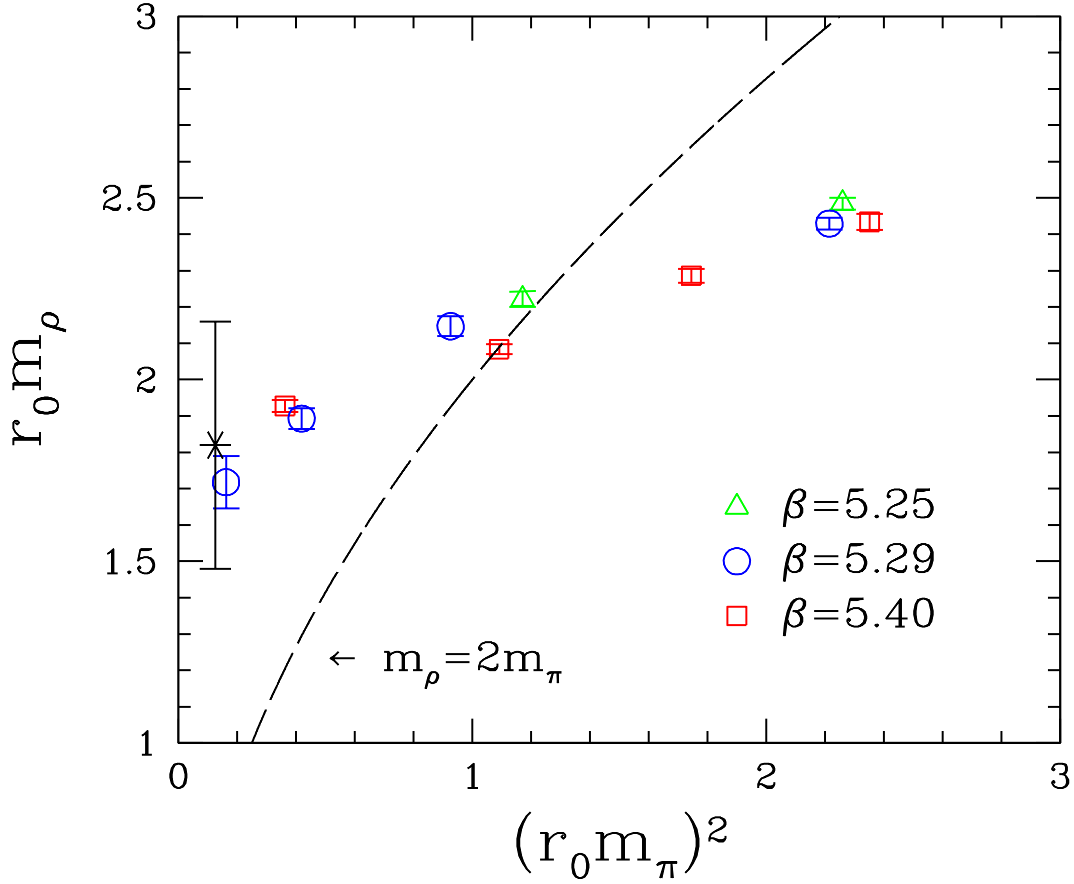


$g_{\rho\pi\pi}=5.1^{+1.8}_{-0.9}$
$p^2=0$
$p^2=1$
$p^2=2$
$m_\pi=160$ MeV
$(r_0^2 k^3/E)\cot\delta_{11}$
$(r_0 k)^2$
0.2
0.1
0
−0.1
−0.2
−0.3
0
0.2
0.4
0.6
0.8
1
1.2

Width



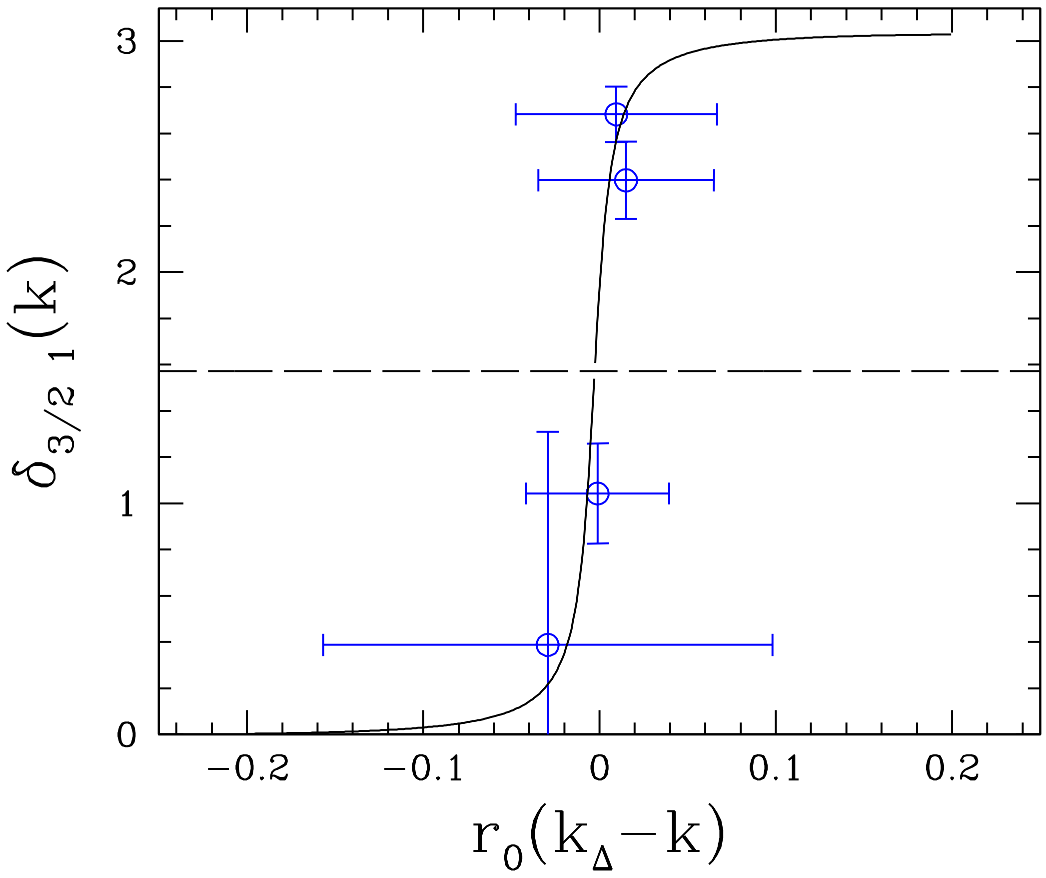
Mass



3
2.5
2
1.5
1
0
1
2
3
$r_0 m_\rho$
$(r_0 m_\pi)^2$
$\leftarrow$ $m_\rho = 2m_\pi$
β=5.25
β=5.29
β=5.40

Delta

Very preliminary



Summary

- Simulations at the physical pion mass with Wilson-type fermions progressing

- Improvement of algorithms
- Increase of computing power
- QPACE

- Benchmark calculation of ρ resonance parameters successful

  Precision of the calculation largely question of statistics

Ideal volume: $m_\pi L = 2 - 4$

$\approx 3 - 6$ fm

- Calculation of Δ resonance parameters will follow shortly

Lowest energy level E sufficient

- To go beyond elastic ρ and Δ resonances, Lüscher formalism needs to be extended to multi-channel case