T®opical Dyson Schwinger Equations

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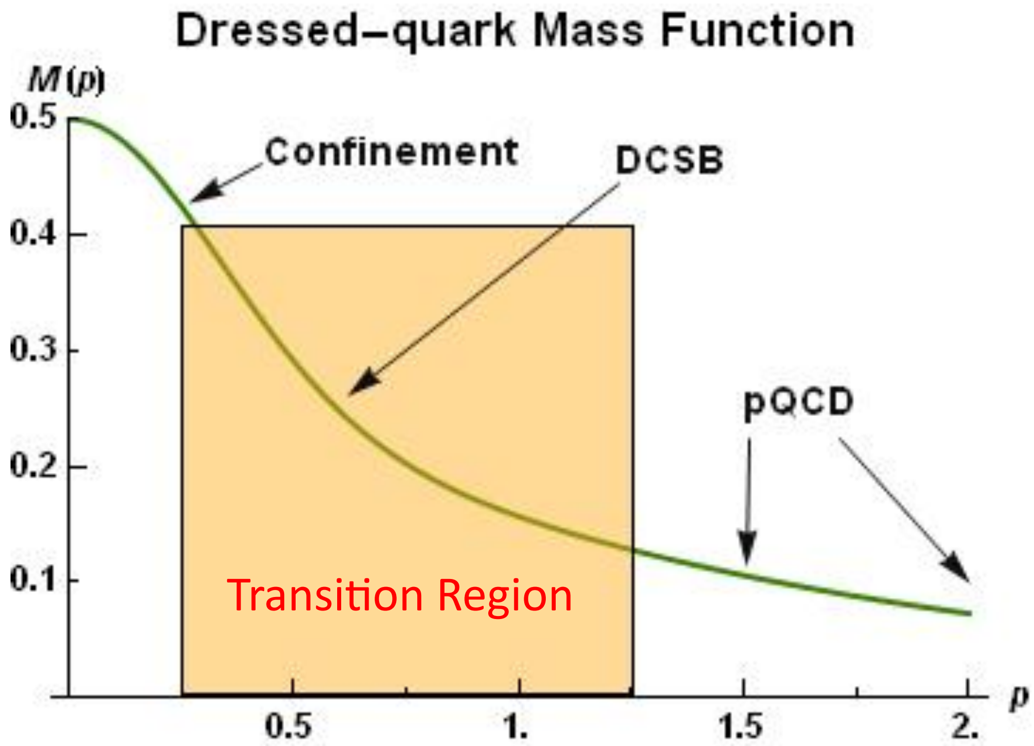

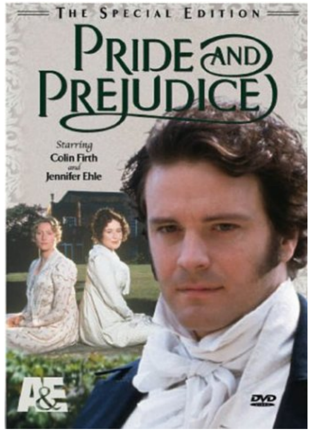

Universal Truths

- **Spectrum of hadrons (ground, excited and exotic states), and hadron elastic and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron's characterising properties amongst its QCD constituents.**
- **Dynamical Chiral Symmetry Breaking (DCSB) is most important mass generating mechanism for visible matter in the Universe.**

  Higgs mechanism is (*almost*) irrelevant to light-quarks.
- **Running of quark mass entails that calculations at even modest Q^2 require a Poincaré-covariant approach.**

  Covariance requires existence of quark orbital angular momentum in hadron's rest-frame wave function.
- **Confinement is expressed through a violation of reflection positivity; and can almost be read-off from a plot of a states' dressed-propagator.**

  It is intimately connected with DCSB.

Universal Truths

- **Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons.**

In-Hadron Condensates

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- One problem: DCSB - an established keystone of low-energy QCD and the origin of constituent-quark masses - has not yet been realised in the light-front formulation.

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- **Resolution**
  - **Whereas it might sometimes be convenient in computational truncation schemes to imagine otherwise, "condensates" do not exist as spacetime-independent mass-scales that fill all spacetime.**
  - ***So-called* vacuum condensates can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.**

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Brodsky, Roberts, Shrock, Tandy, Phys. Rev. C**82** (Rapid Comm.) (2010) 022201

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  - **No qualitative difference between f_π and ρ_π**

$$\begin{aligned} i f_\pi P_\mu &= \langle 0|\bar{q}\gamma_5\gamma_\mu q|\pi\rangle \\ &= Z_2(\zeta,\Lambda)\,\mathrm{tr}_{\mathrm{CD}}\int^\Lambda \frac{d^4q}{(2\pi)^4} i\gamma_5\gamma_\mu S(q_+)\Gamma_\pi(q;P)S(q_-), \end{aligned} \tag{5}$$

$$\begin{aligned} i\rho_\pi &= -\langle 0|\bar{q}i\gamma_5 q|\pi\rangle \\ &= Z_4(\zeta,\Lambda)\,\mathrm{tr}_{\mathrm{CD}}\int^\Lambda \frac{d^4q}{(2\pi)^4}\gamma_5 S(q_+)\Gamma_\pi(q;P)S(q_-). \end{aligned} \tag{6}$$

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$$-\langle \bar{q} q \rangle_\zeta^\pi \equiv -f_\pi \langle 0 | \bar{q} \gamma_5 q | \pi \rangle = f_\pi \rho_\pi(\zeta) =: \kappa_\pi(\hat{m}; \zeta).$$

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Chiral limit

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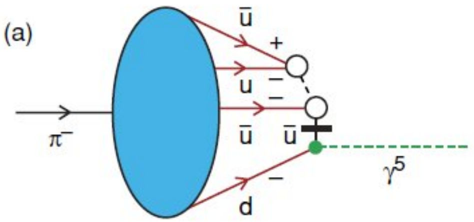
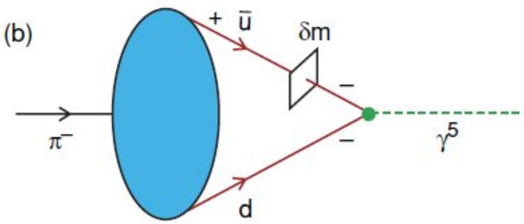


  - ***Conjecture*****: Light-Front DCSB obtained via coherent contribution from countable infinity of higher Fock-state components in LF-wavefunction mediated by LF-instantaneous interaction.**



In-Hadron Condensates

“Void that is truly empty
solves dark energy puzzle”
Rachel Courtland, New Scientist 1st Sept. 2010

“EMPTY space may really be empty. Though quantum theory suggests that a vacuum should be fizzing with particle activity, it turns out that this paradoxical picture of nothingness may not be needed. A calmer view of the vacuum would also help resolve a nagging inconsistency with dark energy, the elusive force thought to be speeding up the expansion of the universe.”

- **Cosmological Constant:**
  - **Putting QCD condensates back into hadrons reduces the mismatch between experiment and theory by a factor of 10^{45}**
  - **Possibly by far more, if technicolour-like theories are the correct paradigm for extending the Standard Model**

Charting the interaction between light-quarks

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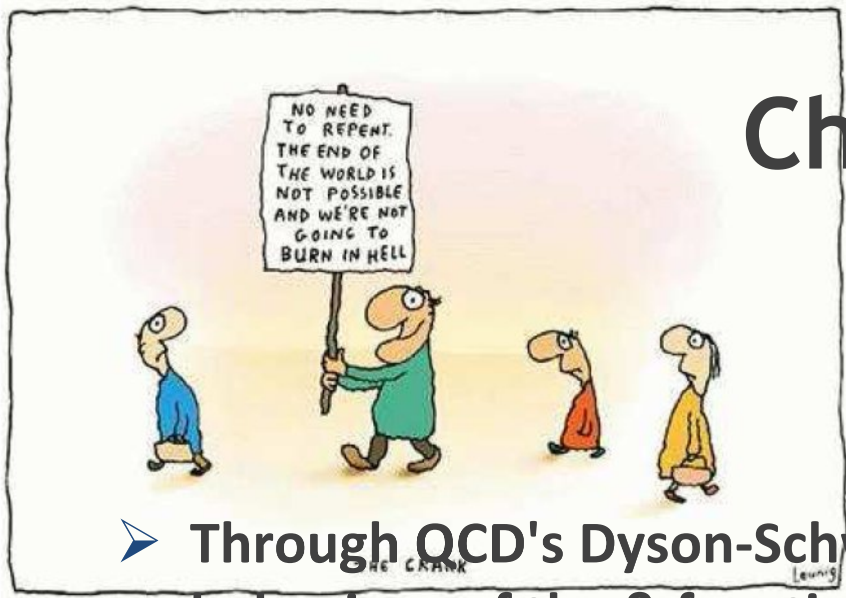
Charting the interaction between light-quarks

- **Confinement can be related to the analytic properties of QCD's Schwinger functions.**
- **Question of light-quark confinement can be translated into the challenge of charting the infrared behavior of QCD's universal β-function**
  - **This function may depend on the scheme chosen to renormalise the quantum field theory but it is unique within a given scheme.**
- **Behaviour of the β-function on the perturbative domain is well known.**
- **This is a well-posed problem, whose solution is an elemental goal of modern hadron physics.**

Charting the interaction between light-quarks



- **Through QCD's Dyson-Schwinger equations (DSEs) the pointwise behaviour of the β-function determines pattern of chiral symmetry breaking.**

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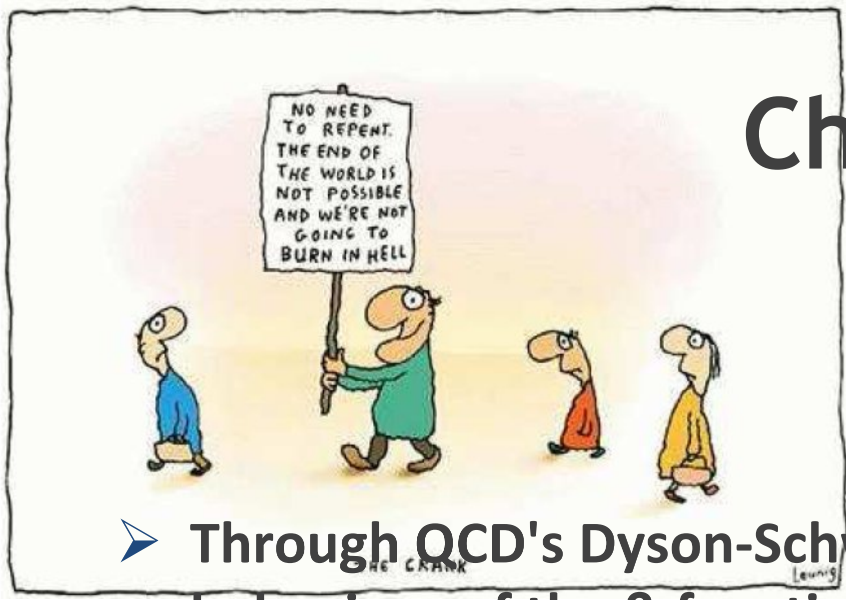
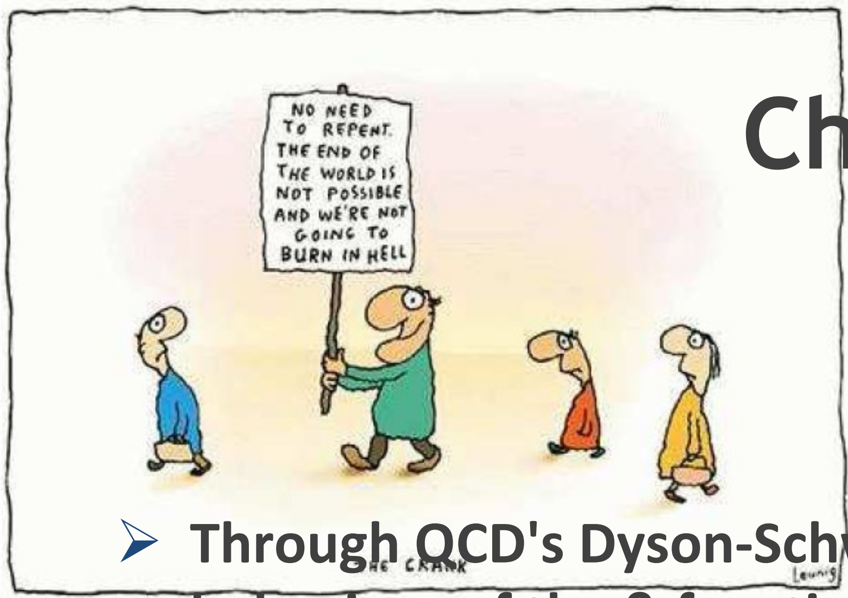


- Through QCD's Dyson-Schwinger equations (DSEs) the pointwise behaviour of the β-function determines pattern of chiral symmetry breaking.
- DSEs connect β-function to experimental observables. Hence, comparison between computations and observations of
  - Hadron mass spectrum
  - Elastic and transition form factors

  can be used to chart β-function's long-range behaviour.



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  - **Hadron mass spectrum**
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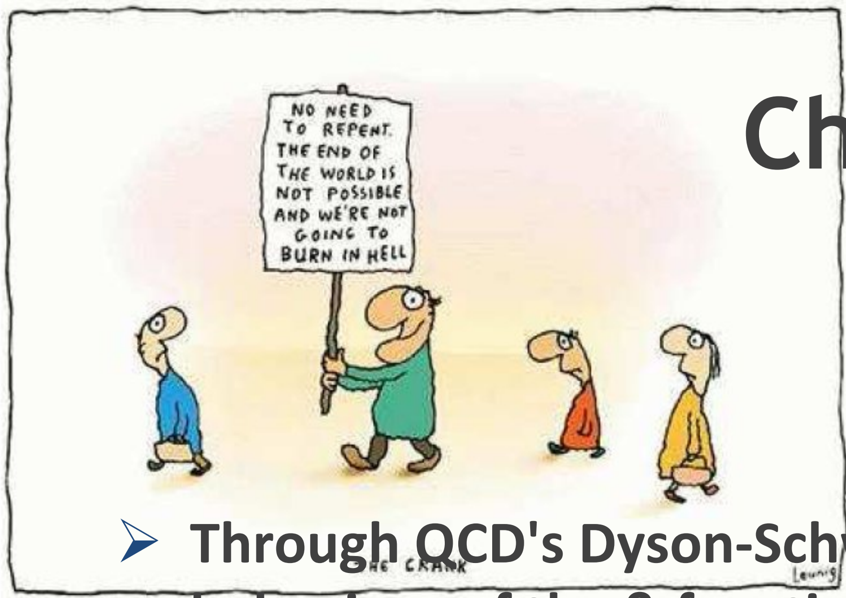
  can be used to chart β-function's long-range behaviour.
- **Extant studies of mesons show that the properties of hadron excited states are a great deal more sensitive to the long-range behaviour of the β-function than those of the ground states.**

Charting the interaction between light-quarks



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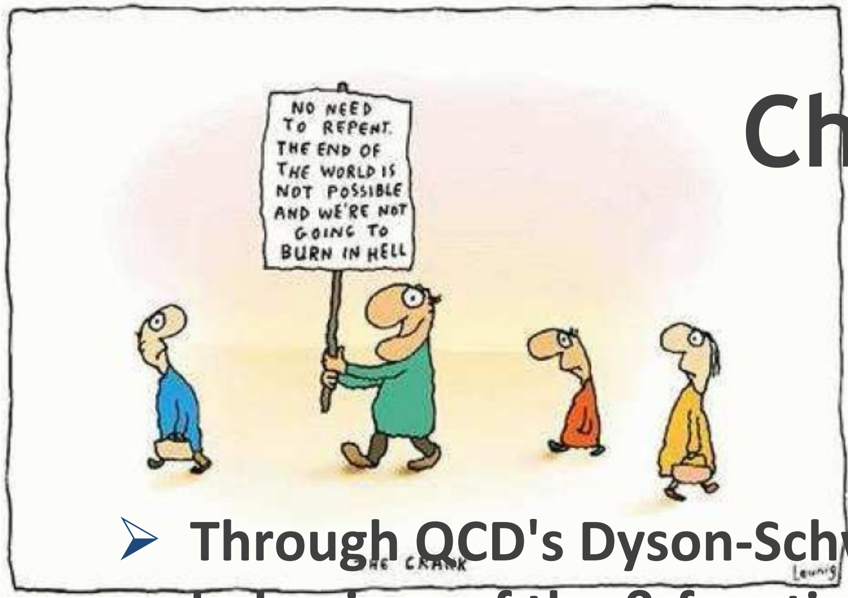
- **DSEs connect β-function to experimental observables. Hence, comparison between computations and observations can be used to chart β-function's long-range behaviour.**
- **To realise this goal, a nonperturbative symmetry-preserving DSE truncation is necessary:**
  - **Steady quantitative progress is being made with a scheme that is systematically improvable (Bender, Roberts, von Smekal – nucl-th/9602012)**



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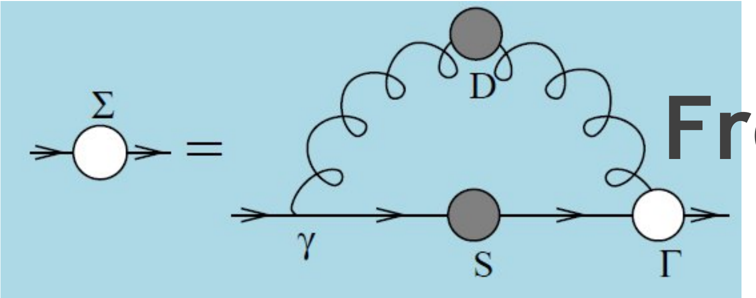
- Through QCD's Dyson-Schwinger equations (DSEs) the pointwise behaviour of the β-function determines pattern of chiral symmetry breaking.
- DSEs connect β-function to experimental observables. Hence, comparison between computations and observations can be used to chart β-function's long-range behaviour.
- To realise this goal, a nonperturbative symmetry-preserving DSE truncation is necessary:
  - On the other hand, at significant qualitative advances are possible with symmetry-preserving kernel Ansätze that express important additional nonperturbative effects – $M(p^2)$ – *difficult/impossible to capture in any finite sum of contributions*.

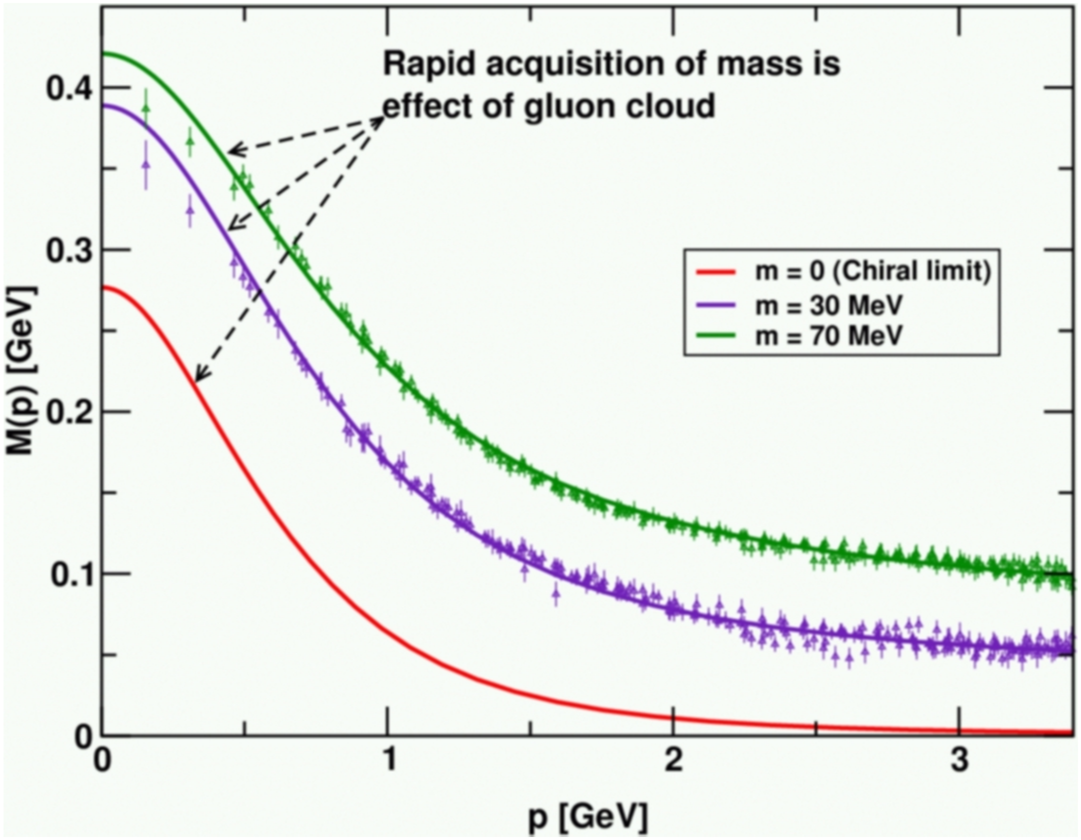
Can't walk beyond the rainbow, but must leap!

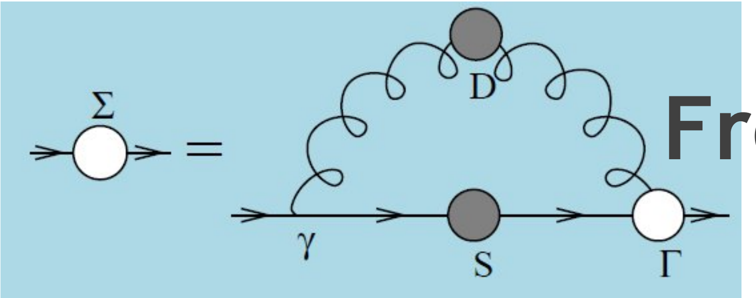


Frontiers of Nuclear Science: Theoretical Advances

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies (m =0, red curve) acquires a large constituent mass at low energies.





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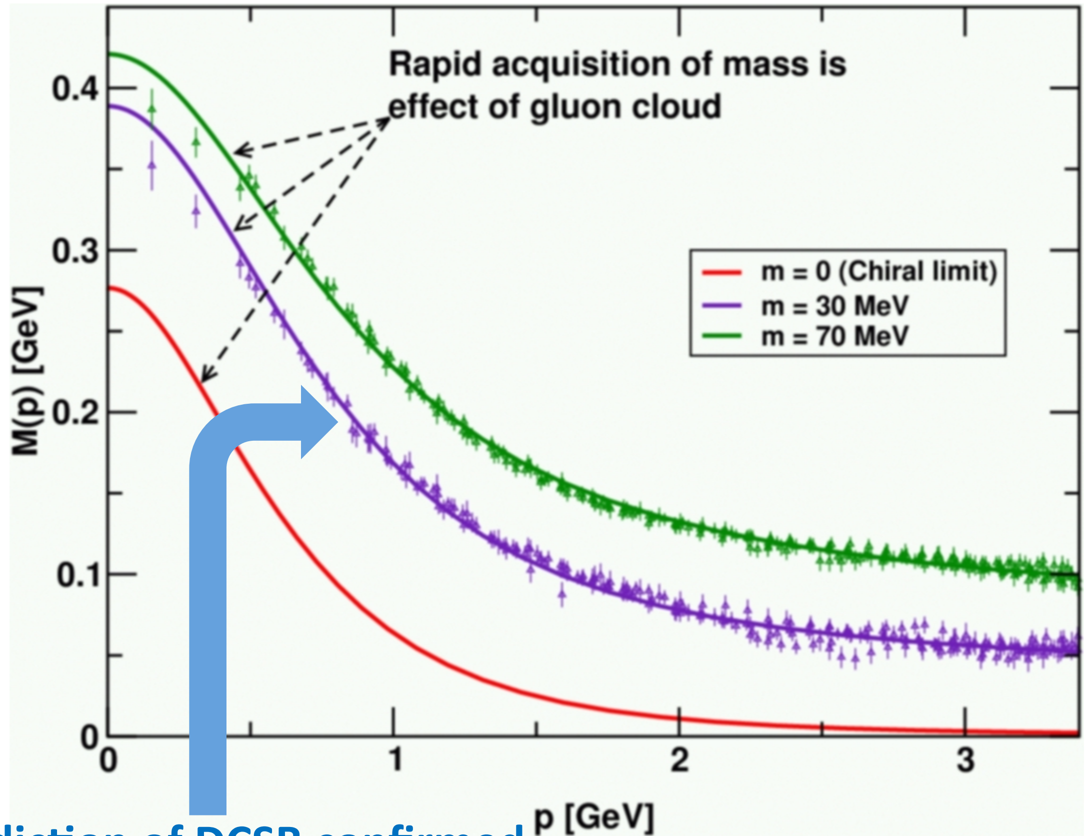
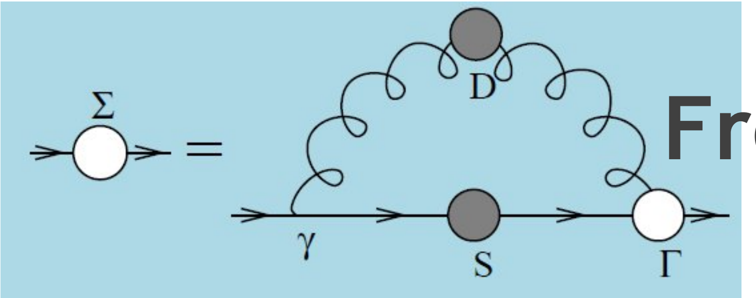


DSE prediction of DCSB confirmed



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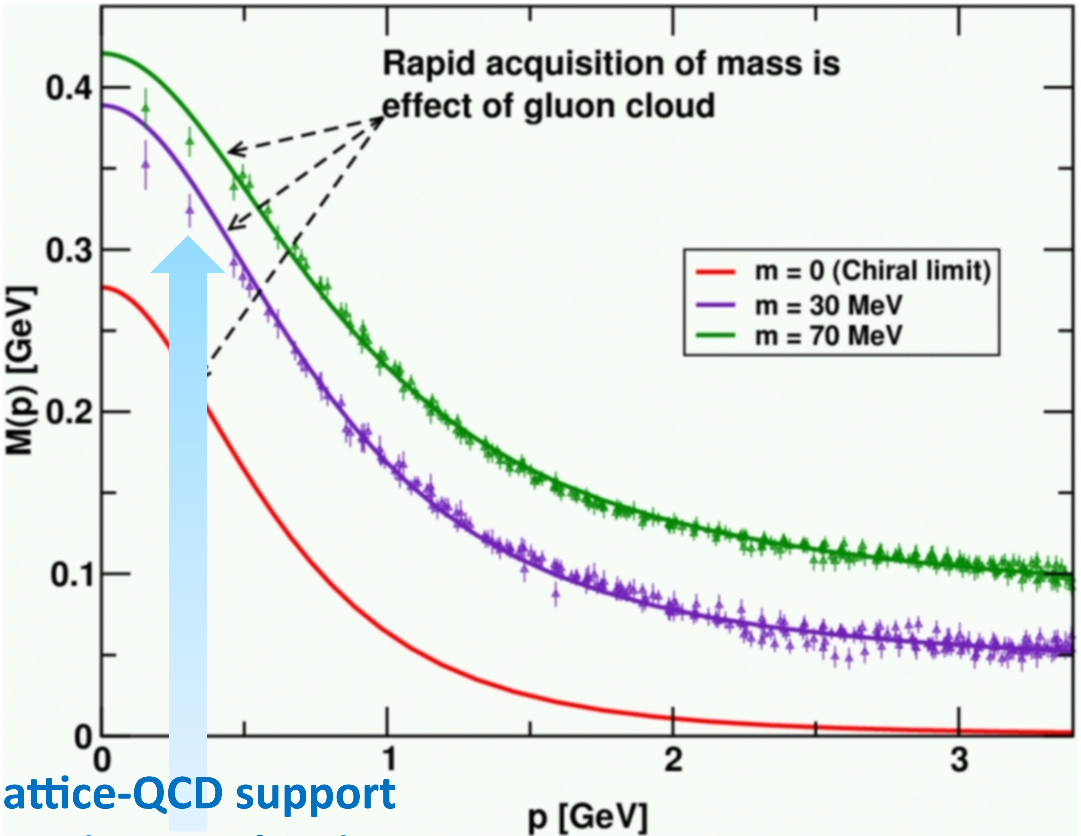
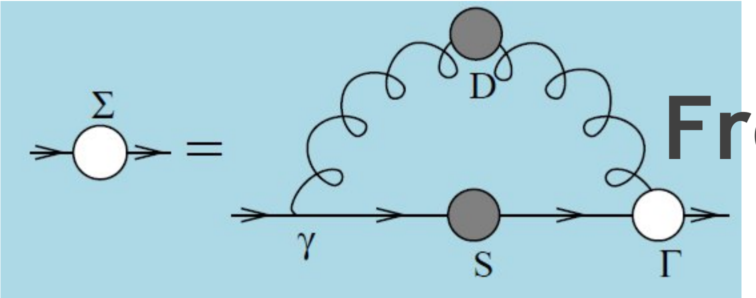


Hint of lattice-QCD support for DSE prediction of violation of reflection positivity



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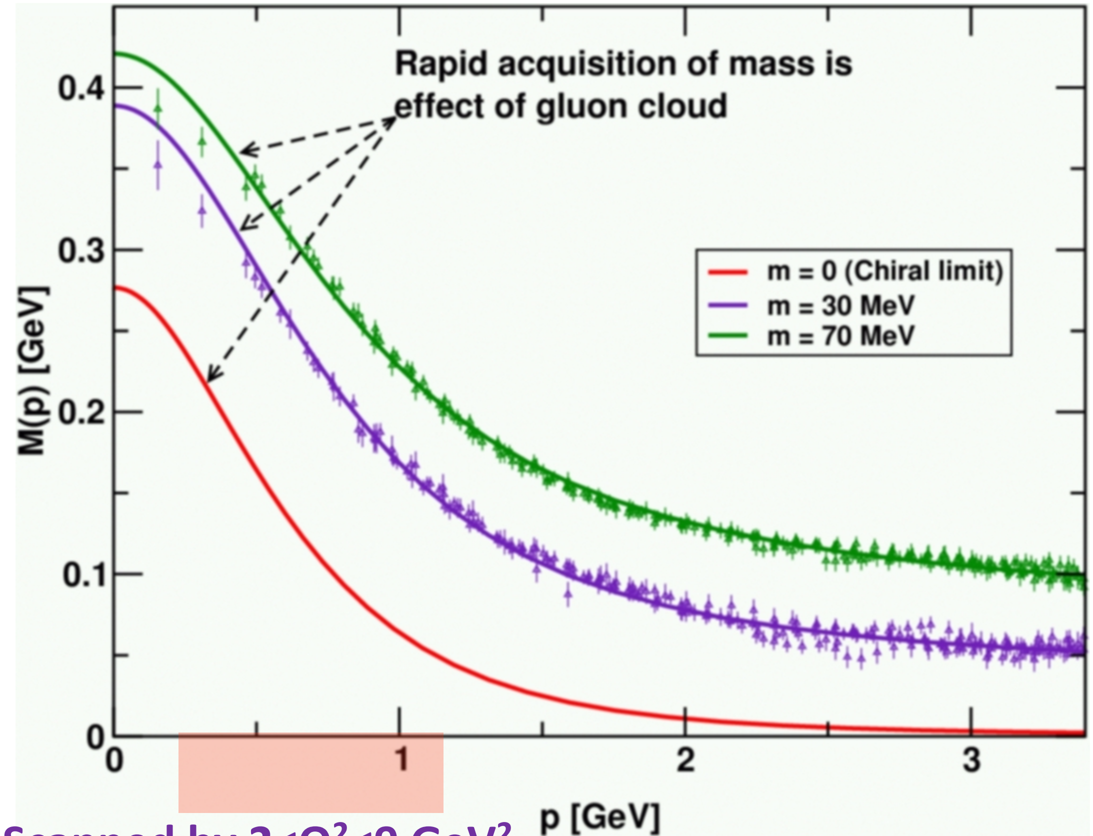
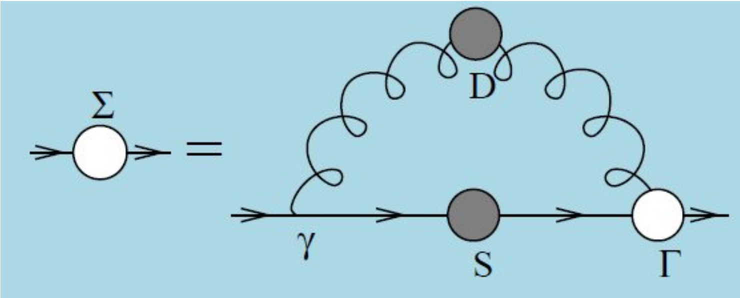


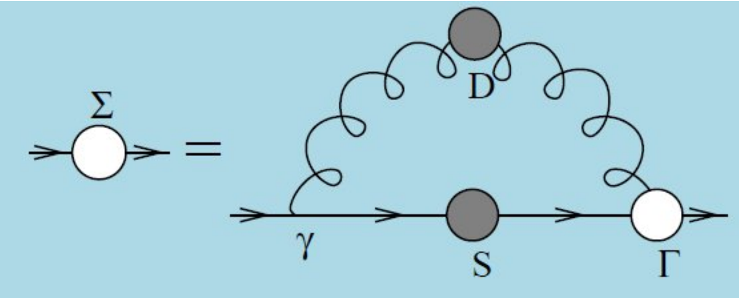
Jlab 12GeV: Scanned by $2<Q^2<9$ GeV^2 elastic & transition form factors.



Gap Equation General Form

$$S_f(p)^{-1} = Z_2\,(i\gamma\cdot p + m_f^{\rm bm}) + \Sigma_f(p)\,,$$

$$\Sigma_f(p) = Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S_f(q)\frac{\lambda^a}{2}\Gamma^f_\nu(q,p)$$



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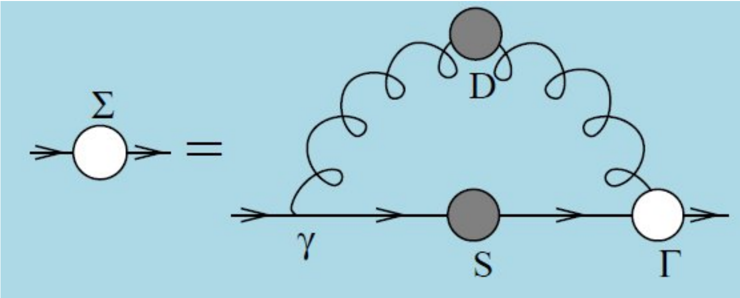
- ***$\Gamma_\nu(q,p)$ – dressed-quark-gluon vertex***



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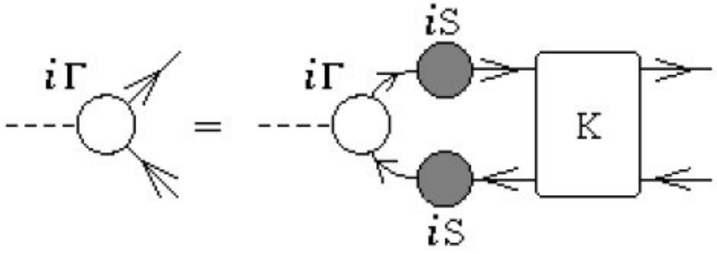
- ***$D_{\mu\nu}(k)$ – dressed-gluon propagator***
- ***$\Gamma_\nu(q,p)$ – dressed-quark-gluon vertex***
- **Suppose one has in hand – from anywhere – the exact form of the dressed-quark-gluon vertex**

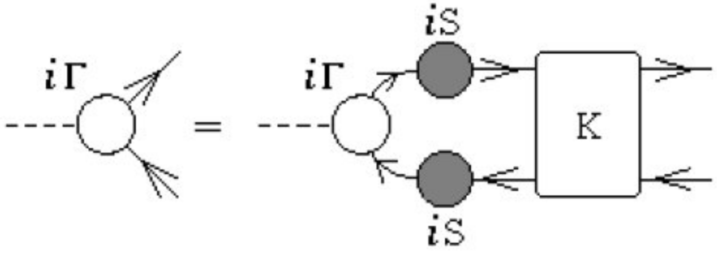
What is the associated symmetry-preserving Bethe-Salpeter kernel?!



Bethe-Salpeter Equation
Bound-State DSE

$$\left[\Gamma_\pi^j(k;P)\right]_{tu} = \int_q^\Lambda \left[S(q+P/2)\Gamma_\pi^j(q;P)S(q-P/2)\right]_{sr} K_{tu}^{rs}(q,k;P)$$

- ***K(q,k;P) – fully amputated, two-particle irreducible, quark-antiquark scattering kernel***
- **Textbook material.**
- **Compact. Visually appealing. Correct**



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Blocked progress for more than 60 years.

Bethe-Salpeter Equation
General Form

Lei Chang and C.D. Roberts
0903.5461 [nucl-th]
Phys. Rev. Lett. 103 (2009) 081601

$$\Gamma^{fg}_{5\mu}(k;P) = Z_2 \gamma_5 \gamma_\mu$$
$$- \boxed{\int_q g^2 D_{\alpha\beta}(k-q)\, \frac{\lambda^a}{2}\, \gamma_\alpha S_f(q_+) \Gamma^{fg}_{5\mu}(q;P) S_g(q_-) \frac{\lambda^a}{2}\, \Gamma^g_\beta(q_-,k_-)}$$
$$+ \int_q g^2 D_{\alpha\beta}(k-q)\, \frac{\lambda^a}{2}\, \gamma_\alpha S_f(q_+) \frac{\lambda^a}{2} \Lambda^{fg}_{5\mu\beta}(k,q;P),$$

- **Equivalent exact bound-state equation *but* in this form**

  K(q,k;P) → Λ(q,k;P)

  which is *completely determined by dressed-quark self-energy*
- **Enables derivation of a Ward-Takahashi identity for Λ(q,k;P)**

Ward-Takahashi Identity Bethe-Salpeter Kernel

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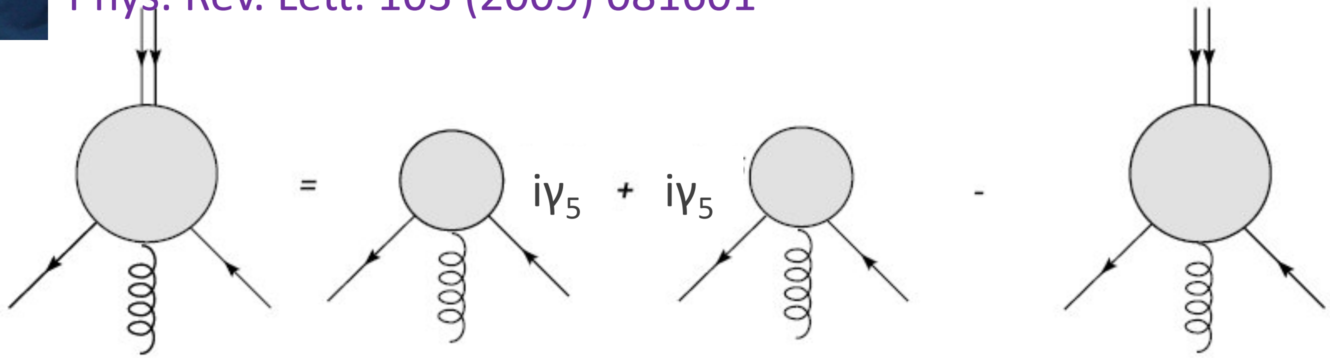

$$P_\mu \Lambda^{fg}_{5\mu\beta}(k,q;P) = \Gamma^f_\beta(q_+,k_+)\, i\gamma_5 + i\gamma_5\, \Gamma^g_\beta(q_-,k_-) - i[m_f(\zeta) + m_g(\zeta)]\Lambda^{fg}_{5\beta}(k,q;P),$$

- **Now, for first time, it's possible to formulate an *Ansatz* for Bethe-Salpeter kernel given *any form* for the dressed-quark-gluon vertex by using this identity**

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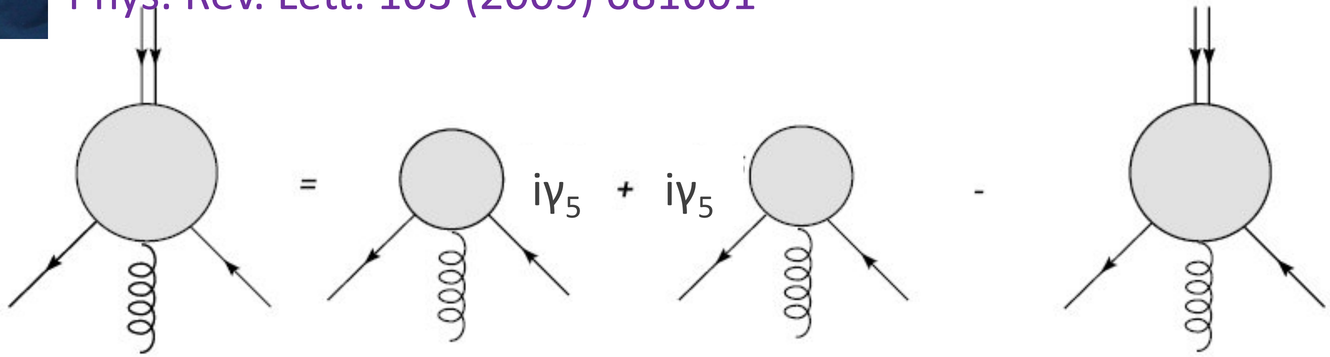

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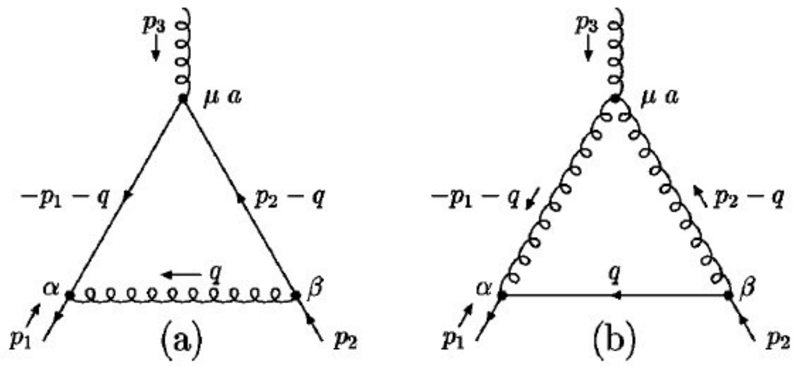
- **This enables the identification and elucidation of a wide range of novel consequences of DCSB**

Dressed-quark anomalous magnetic moments

- **Schwinger's result for QED:** $\frac{q}{2m} \to \left(1 + \frac{\alpha}{2\pi}\right)\frac{q}{2m}$



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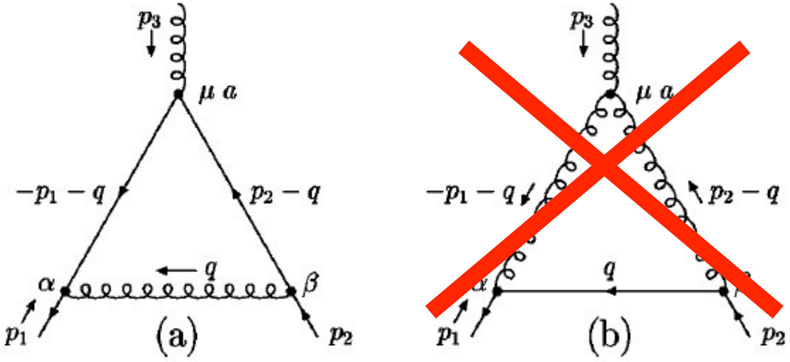
  - **(a) is QED-like**
  - **(b) is only possible in QCD – involves 3-gluon vertex**



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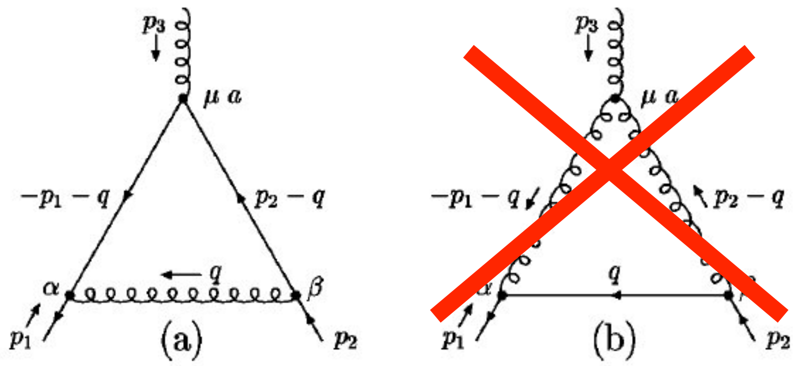
    ~ (– ⅙) QED-result



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  - **(a) Produces a finite result: " – ⅙ $\alpha_s/2\pi$ "**
    ~ (– ⅙) QED-result
- **But, in QED and QCD, the *anomalous chromo- and electro-magnetic moments vanish identically in the chiral limit*!**

Dressed-quark anomalous magnetic moments

$$\int d^4x \, \tfrac{1}{2} q \, \bar{\psi}(x) \sigma_{\mu\nu} \psi(x) F_{\mu\nu}(x)$$

- **Interaction term that describes magnetic-moment coupling to gauge field**
  - **Straightforward to show that it mixes left ↔ right**
  - **Thus, explicitly violates chiral symmetry**

Dressed-quark anomalous magnetic moments

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  - **Straightforward to show that it mixes left ↔ right**
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  No Gordon Identity
  - **Hence *massless fermions cannot not possess a measurable chromo- or electro-magnetic moment***
- **But what if the chiral symmetry is dynamically broken, strongly, as it is in QCD?**

Lei Chang, Yu-Xin Liu and Craig D. Roberts
arXiv:1009.3458 [nucl-th]

Dressed-quark anomalous magnetic moments

- DCSB Three strongly-dressed and essentially-nonperturbative contributions to dressed-quark-gluon vertex:

$$\lambda^3_\mu(p,q) = 2(p+q)_\mu \Delta_B(p,q)$$

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$\mathcal{F}(z) = (1-\exp(-z))/z$, $z = (p_i^2 + p_f^2 - 2M_E^2)/\Lambda_{\mathcal{F}}^2$, $\Lambda_{\mathcal{F}} = 1\,\mathrm{GeV}$, Simplifies numerical analysis;

$M_E = \{s|s>0, s = M^2(s)\}$ is the Euclidean constituent-quark mass

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Simplifies numerical analysis;

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Role and importance is
Novel discovery
•Essential to recover pQCD
•Constructive interference with Γ^5

Dressed-quark anomalous magnetic moments

Lei Chang, Yu-Xin Liu and Craig D. Roberts
arXiv:1009.3458 [nucl-th]

- Formulated and solved general Bethe-Salpeter equation
- Obtained dressed electromagnetic vertex
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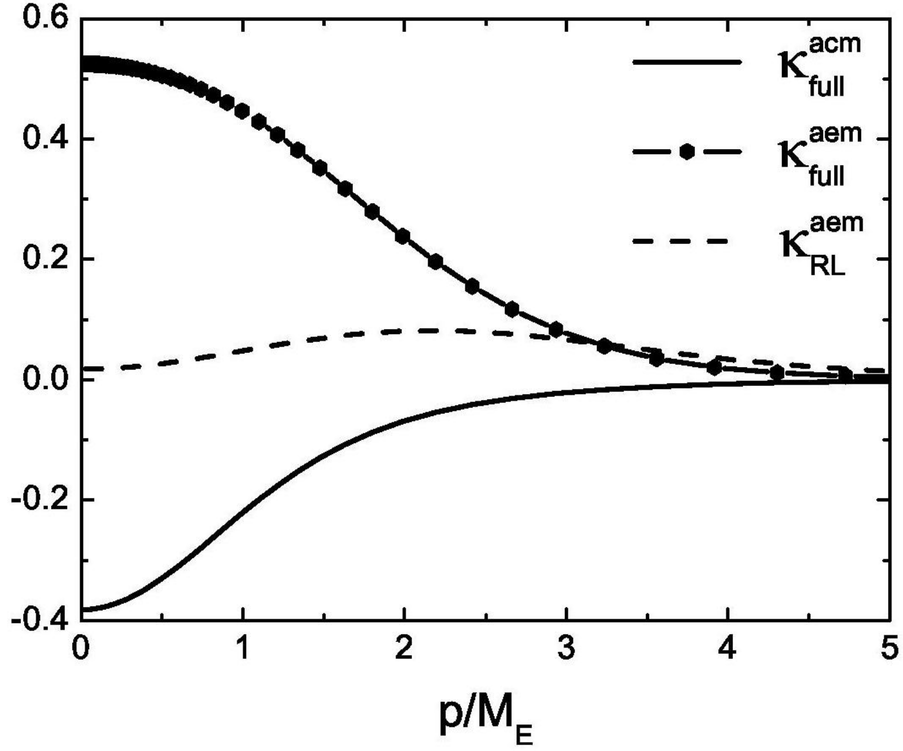

- AEM is opposite in sign but of roughly equal magnitude as ACM
  - Potentially important for transition form factors, etc.
  - Muon *g-2* ?

	M^E	κ^{ACM}	κ^{AEM}
Full vertex	0.44	-0.22	0.45
Rainbow-ladder	0.35	0	0.048

Dressed Vertex & Meson Spectrum

	Experiment	Rainbow-ladder	One-loop corrected	Ball-Chiu	Full vertex
a1	1230				
ρ	770				
Mass splitting	455				

- **Splitting known experimentally for more than 35 years**
- **Hitherto, no explanation**

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- **Hitherto, no explanation**
- **Systematic symmetry-preserving, Poincaré-covariant DSE truncation scheme of nucl-th/9602012.**
  - **Never better than ~ ¼ of splitting**
- **Constructing kernel skeleton-diagram-by-diagram, DCSB cannot be faithfully expressed:**

Full impact of $M(p^2)$ cannot be realised!

Dressed Vertex & Meson Spectrum

	Experiment	Rainbow-ladder	One-loop corrected	Ball-Chiu	Full vertex
a1	1230	759	885	1066	
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- **Fully consistent treatment of Ball-Chiu vertex**
  - **Retain λ_3 – term but ignore Γ^4 & Γ^5**
  - **Some effects of DCSB built into vertex & Bethe-Salpeter kernel**
    - **Big impact on σ – π complex**
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- **Fully-consistent treatment of complete vertex *Ansatz***
- ***Subtle interplay between competing effects, which can only now be explicated***
- **Promise of first reliable prediction of light-quark hadron spectrum, including the so-called hybrid and exotic states.**

Maris, Roberts and Tandy
nucl-th/9707003

Pion's Golderberger -Treiman relation

- Pion's Bethe-Salpeter amplitude

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\Big]$$

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Exact in Chiral QCD

$$\begin{aligned}
f_\pi E_\pi(k;P=0) &= B(p^2)\\
F_R(k;0) + 2\, f_\pi F_\pi(k;0) &= A(k^2)\\
G_R(k;0) + 2\, f_\pi G_\pi(k;0) &= 2A'(k^2)\\
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\end{aligned}$$

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Pseudovector components necessarily nonzero. Cannot be ignored!

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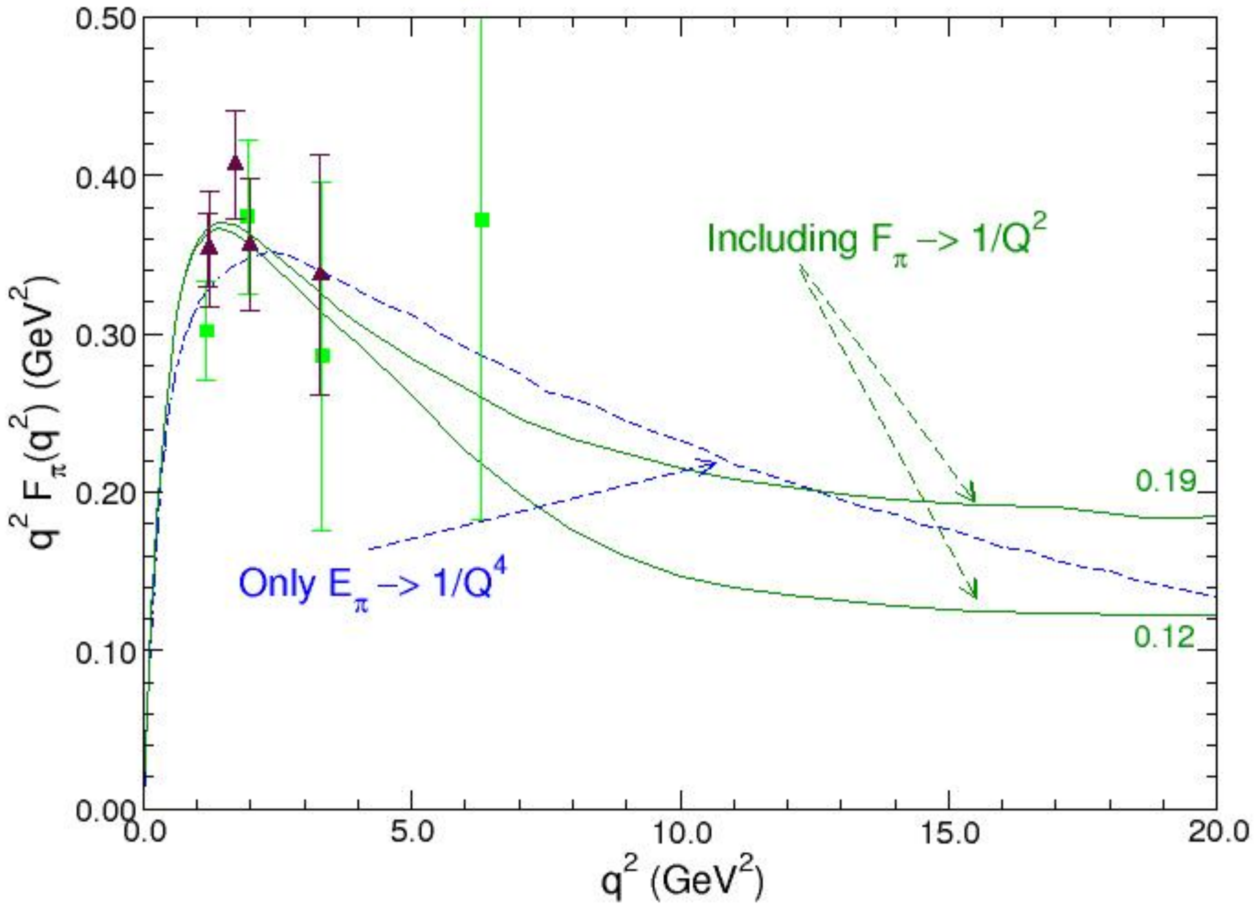
Maris and Roberts
nucl-th/9804062

Pion's GT relation
Implications for observables?



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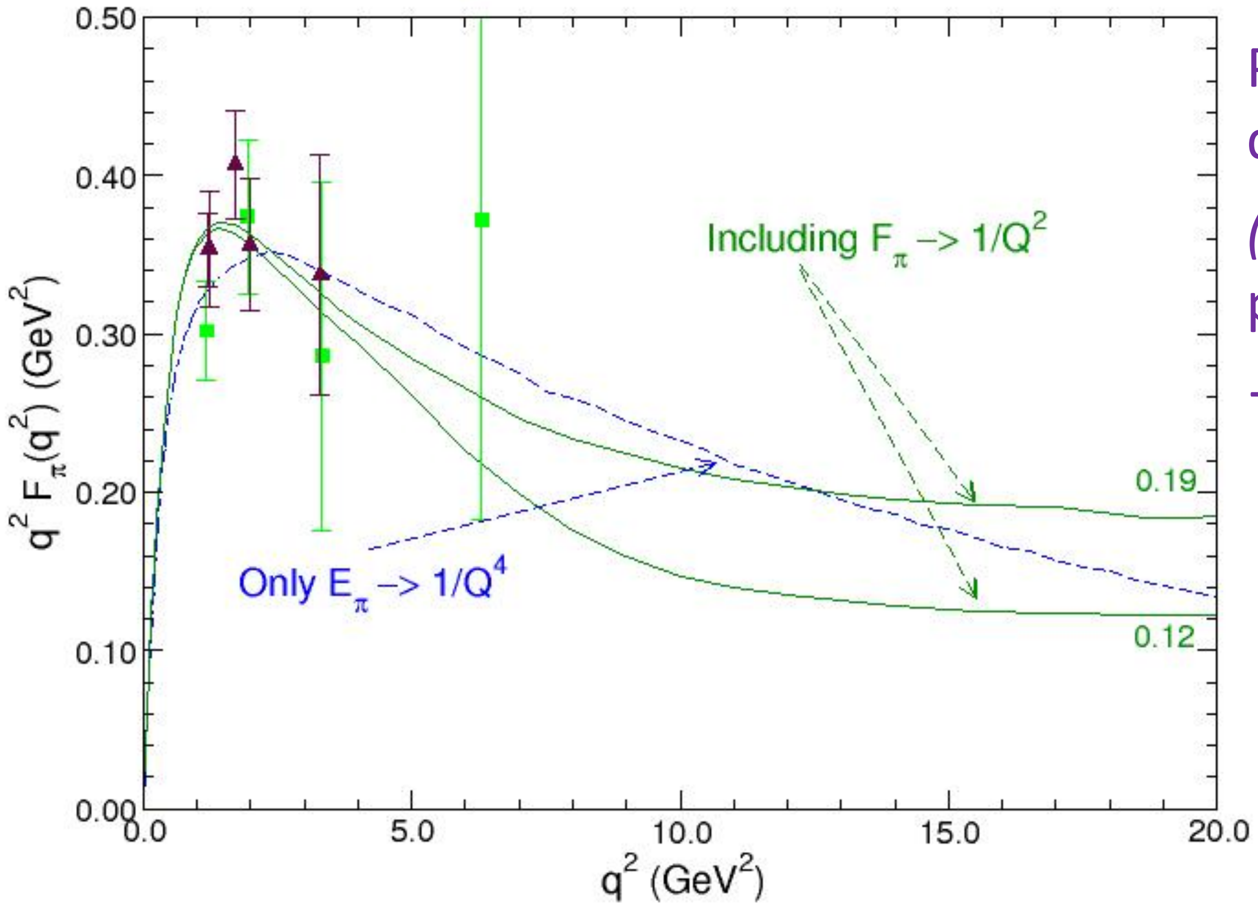

Pseudovector components dominate in ultraviolet:

$(Q/2)^2 = 2\ GeV^2$
pQCD point for $M(p^2)$

→ pQCD at $Q^2 = 8GeV^2$

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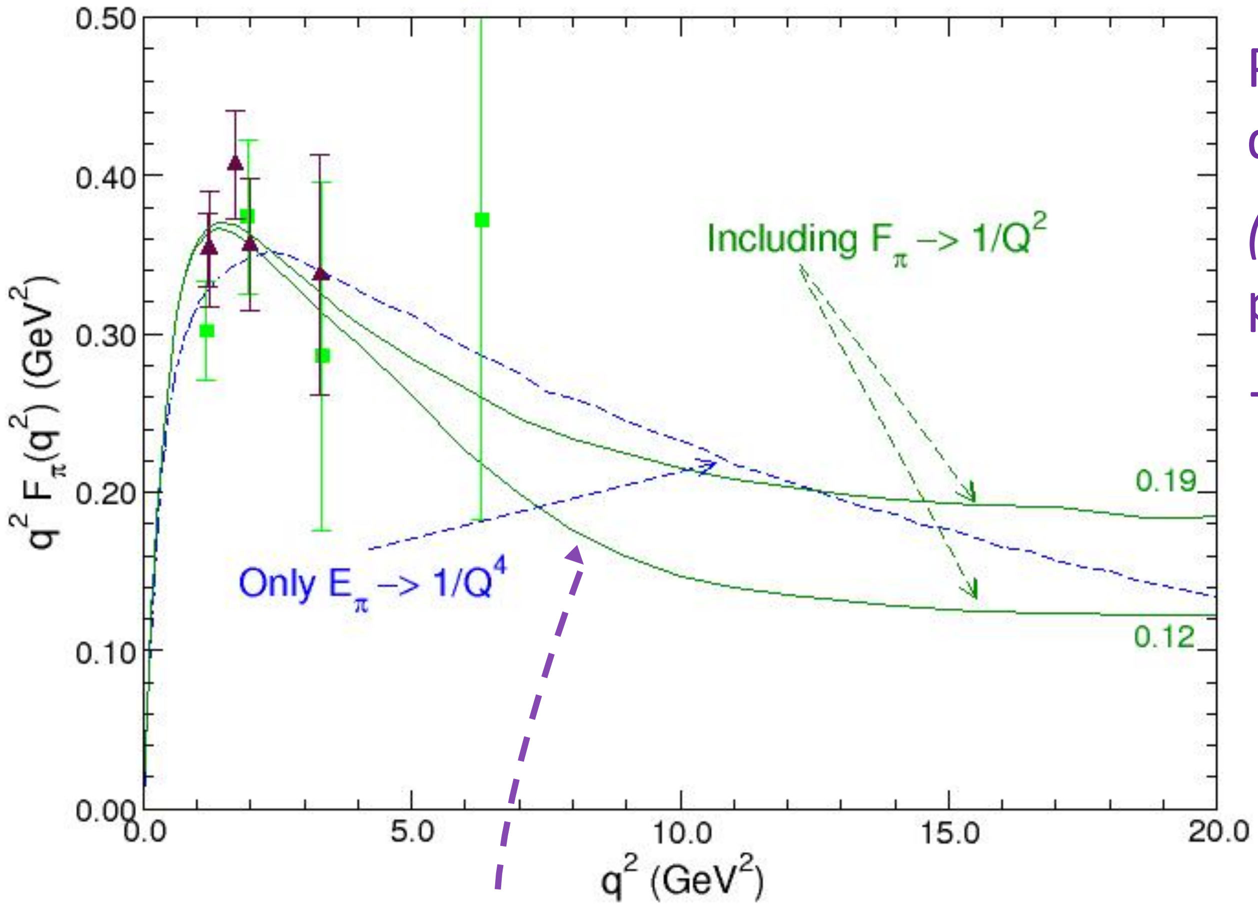

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$E_\pi(P)\, F_\pi(P)$ – *cross-term*

$\rightarrow F_\pi^{em}(Q^2) = (Q^2/M_Q^2) * [E_\pi(P)/F_\pi(P)] * E_\pi^2(P)\text{-term} =$ ***CONSTANT!***

Guttierez, Bashir, Cloët, Roberts
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Pion's Electromagnetic Form Factor

- QCD-based DSE prediction: $D(x-y) = \dfrac{1}{(x-y)^2}$ produces $M(p^2) \sim 1/p^2$
- cf. contact-interaction: $D(x-y) \sim \delta^4(x-y)$ produces $M(p^2)$=constant

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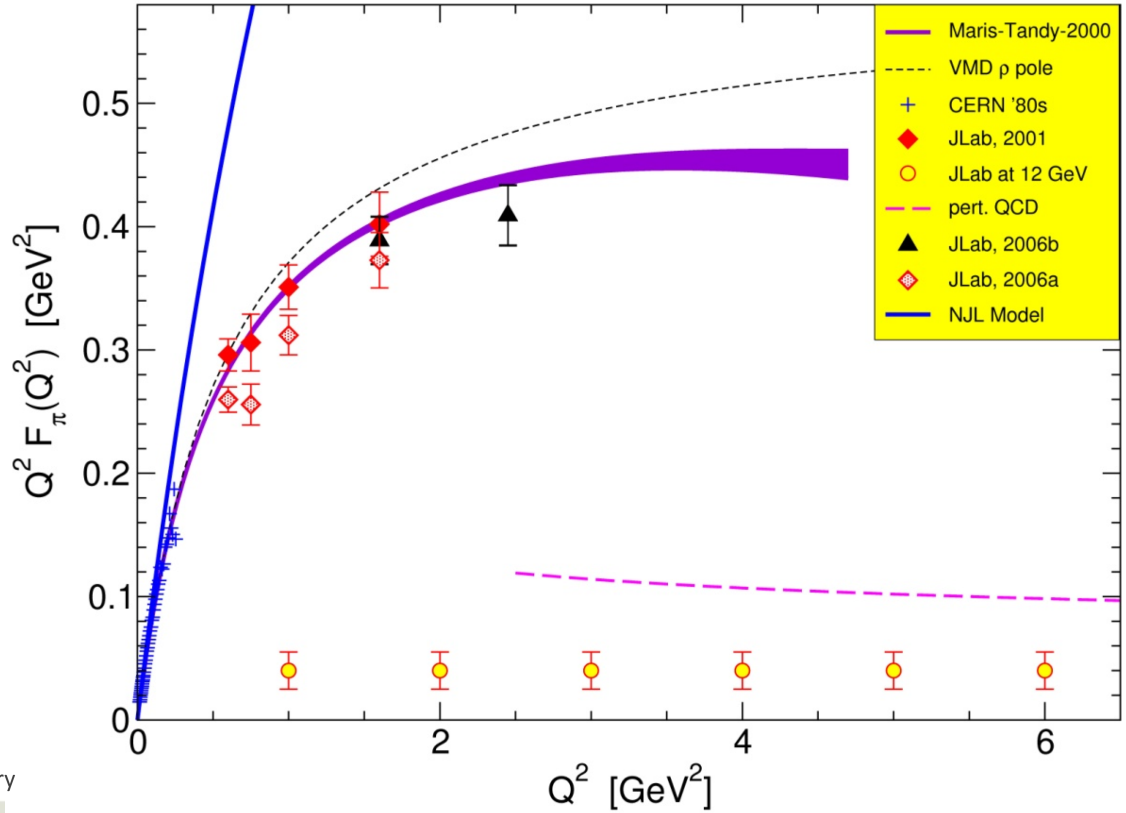

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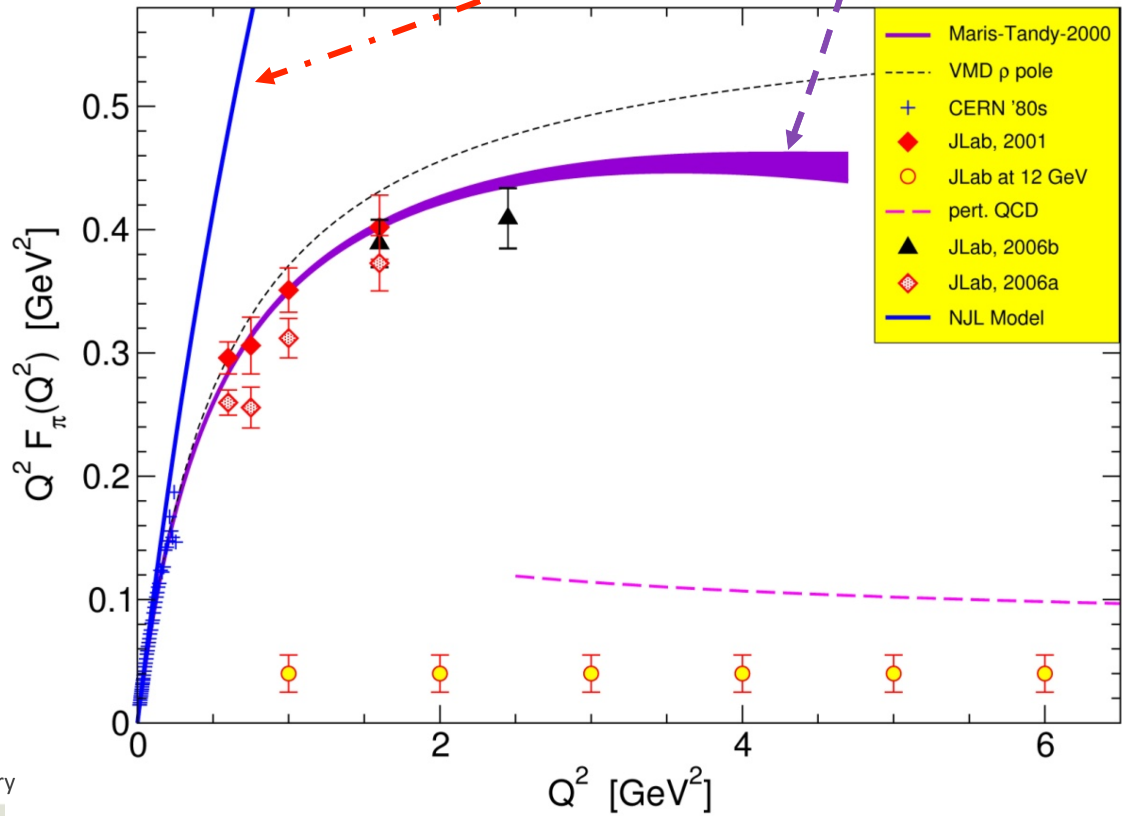

- ❖ Single mass parameter in both studies
- ❖ Same predictions for $Q^2=0$ observables
- ❖ *Disagreement >20% for $Q^2>M_Q^2$*

H.L.L. Roberts, C.D. Roberts, Bashir, Guttierez, Tandy
arXiv:1009.0067 [nucl-th]

BaBar *Anomaly*
$\gamma^* \gamma \to \pi^0$

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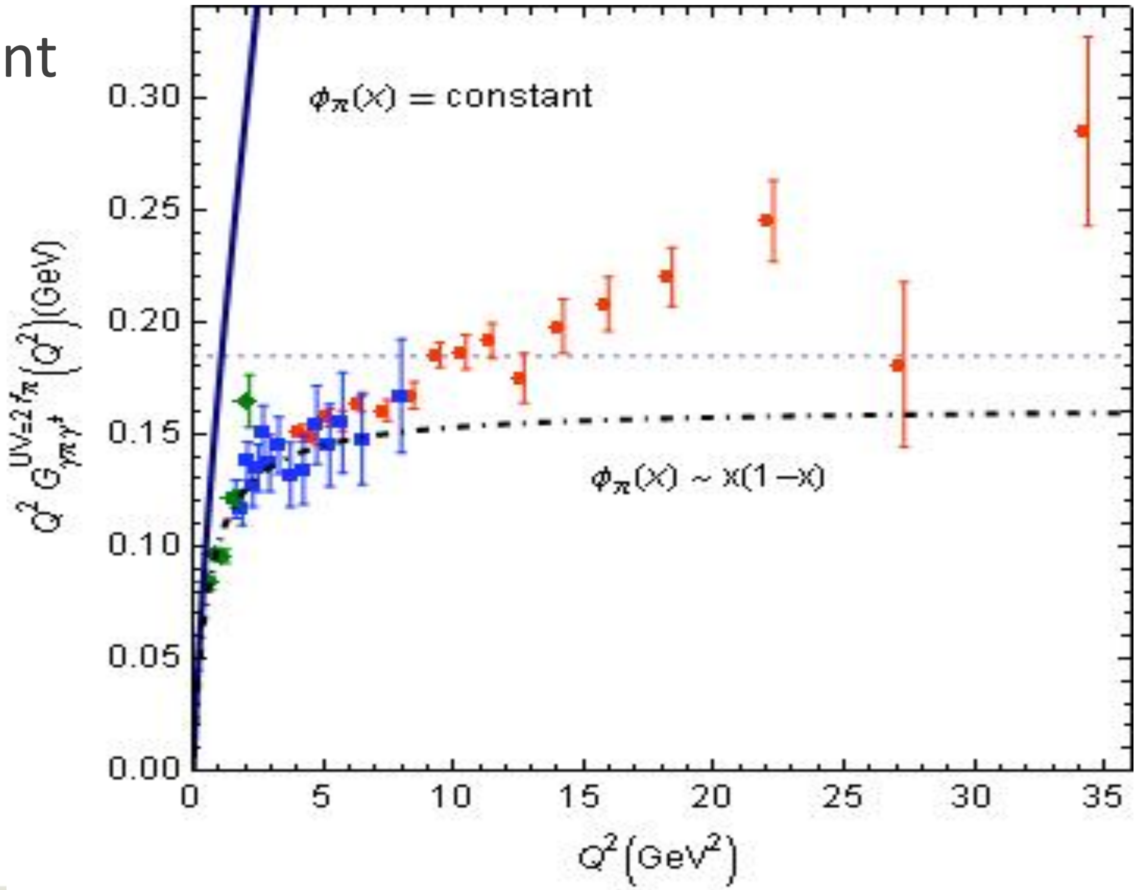

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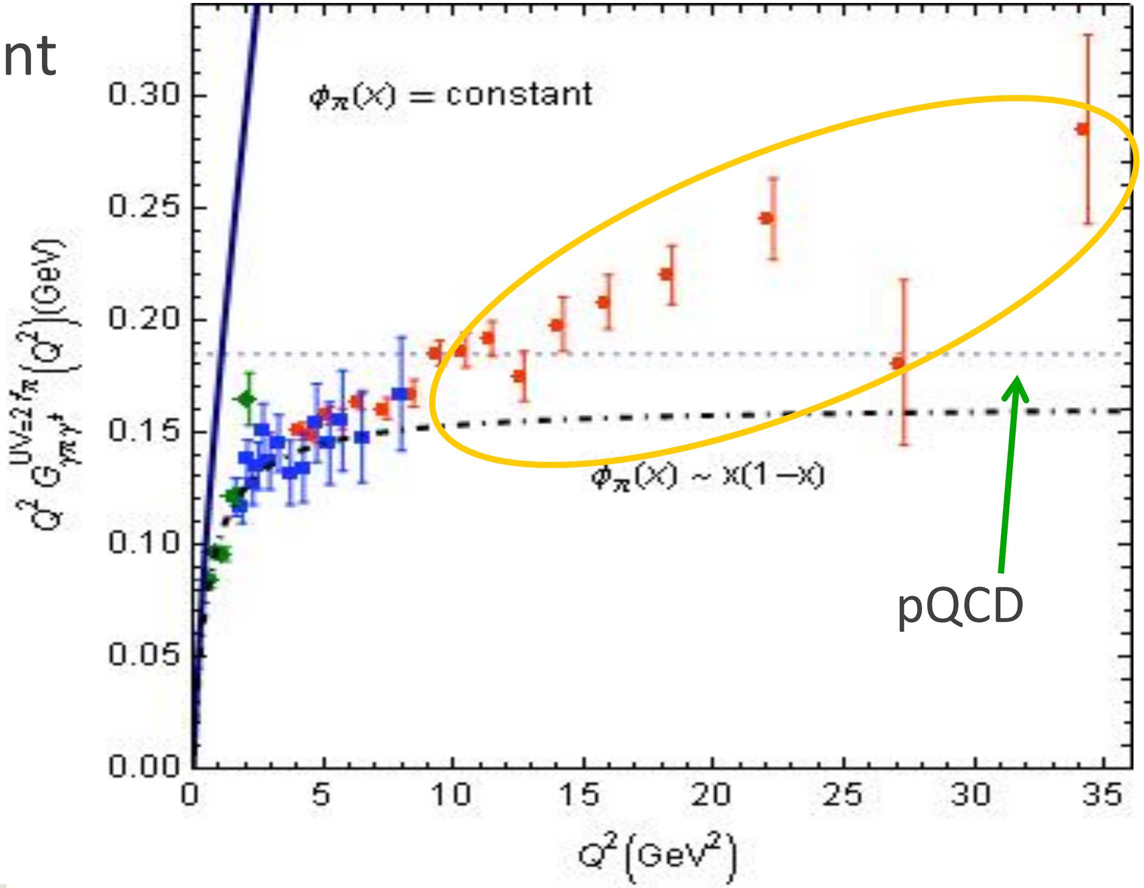

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γ γ → π⁰*

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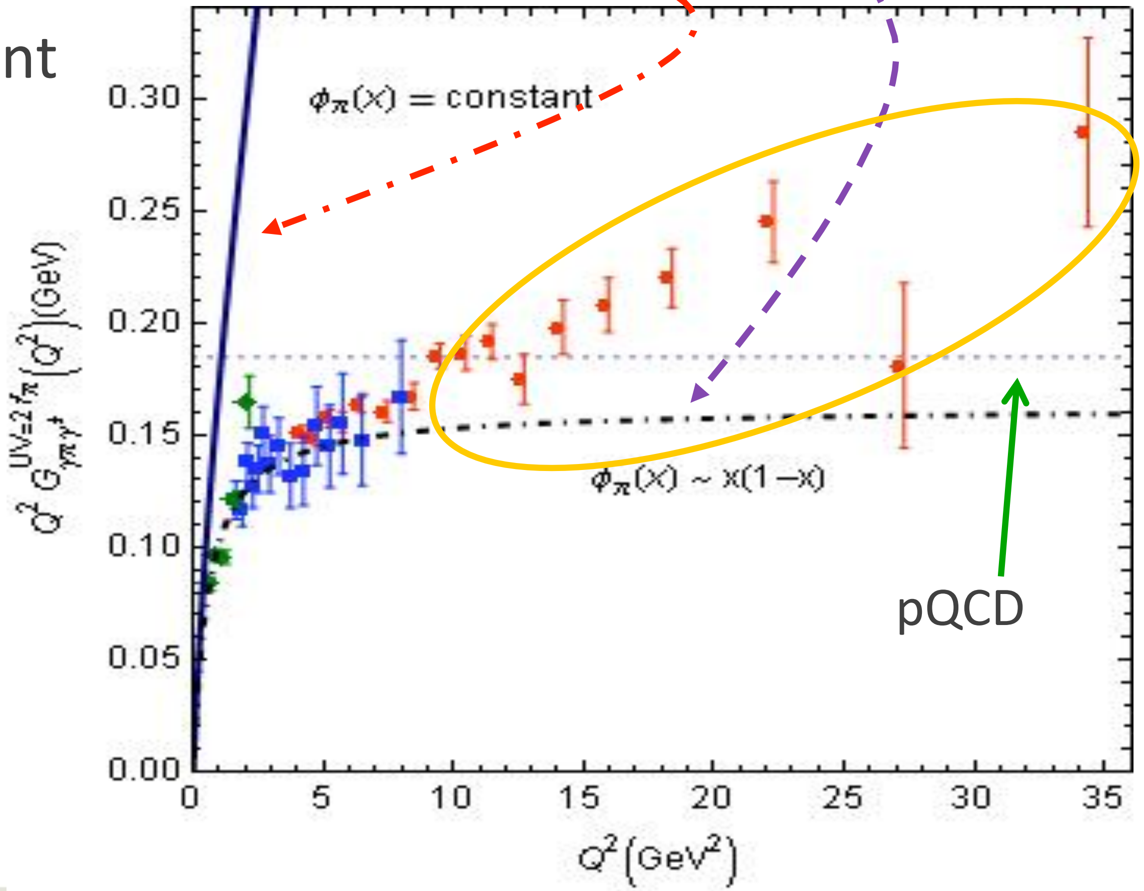

- No fully-self-consistent treatment of the pion can reproduce the BaBar data.
- All produce monotonically-increasing concave functions.
- *BaBar data not a true measure of $\gamma^* \gamma \to \pi^0$*
- Likely source of error is misidentification of $\pi^0\,\pi^0$ events where 2nd π^0 isn't seen.

Unifying Baryons and Mesons

- $M(p^2)$ – effects have enormous impact on meson properties.
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- $M(p^2)$ is essentially a quantum field theoretical effect. In quantum field theory
  - Meson appears as pole in four-point quark-antiquark Green function → Bethe-Salpeter Equation
  - *Nucleon appears as a pole in a six-point quark Green function → Faddeev Equation*.

R.T. Cahill *et al.*,
Austral. J. Phys. 42 (1989) 129-145

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- Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks
- Tractable equation is founded on observation that an interaction which describes colour-singlet mesons also generates *nonpointlike* quark-quark (diquark) correlations in the colour-antitriplet channel

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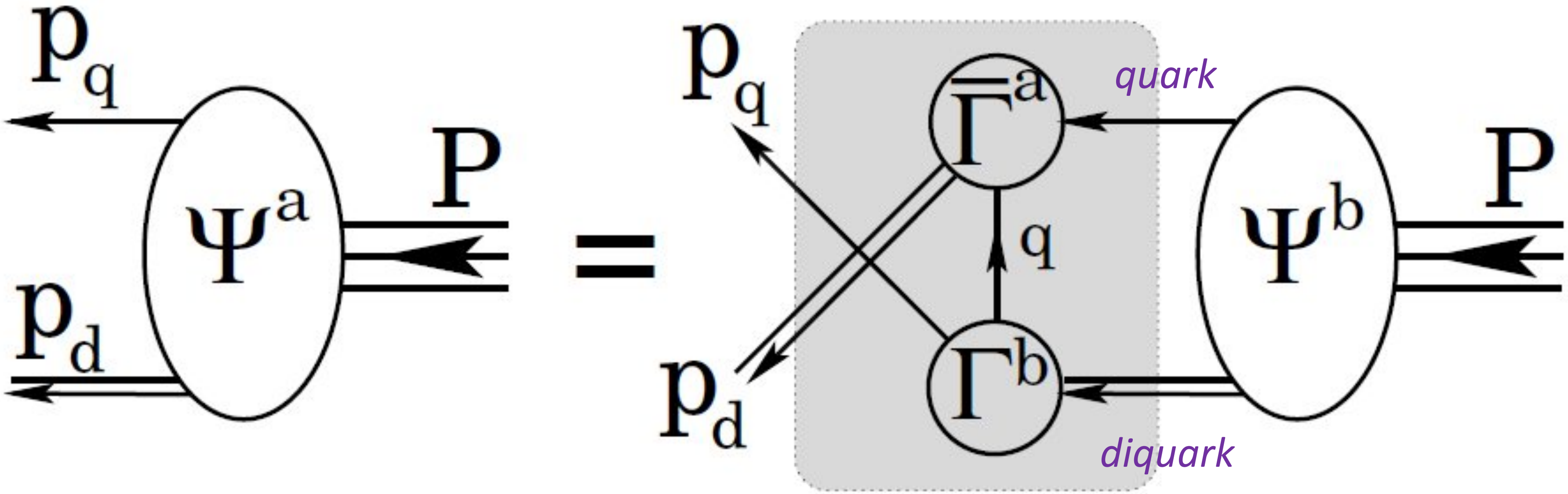
[Austral. J. Phys. 42 (1989) 129-145]

Faddeev Equation



- Linear, Homogeneous Matrix equation

Faddeev Equation

R.T. Cahill *et al.*,
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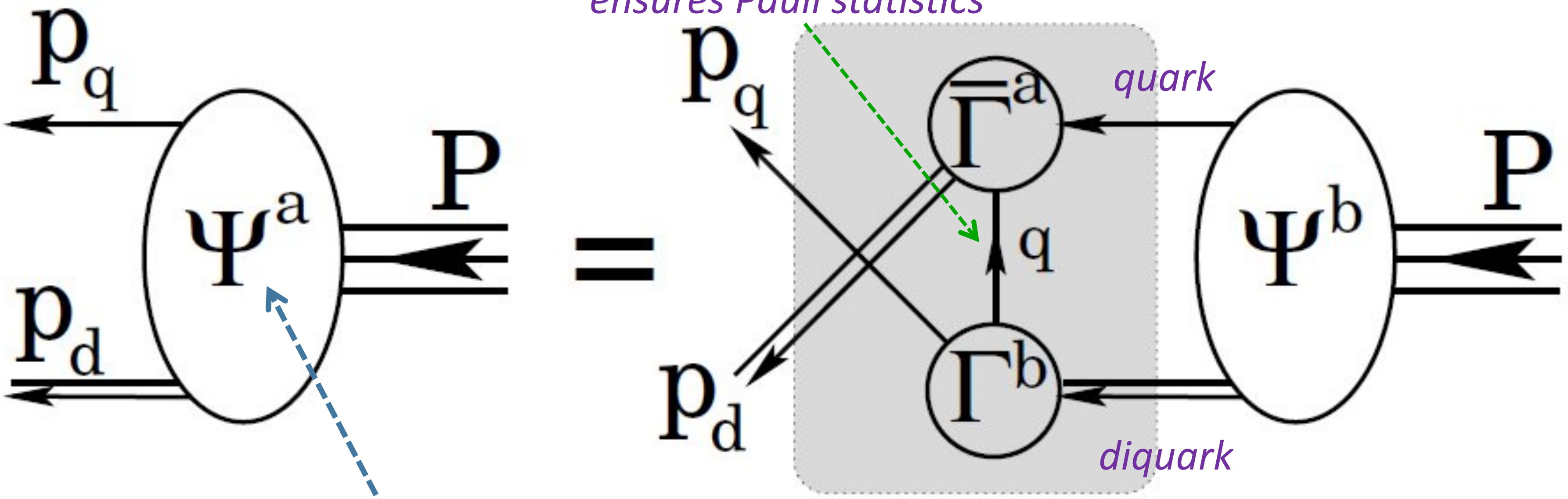

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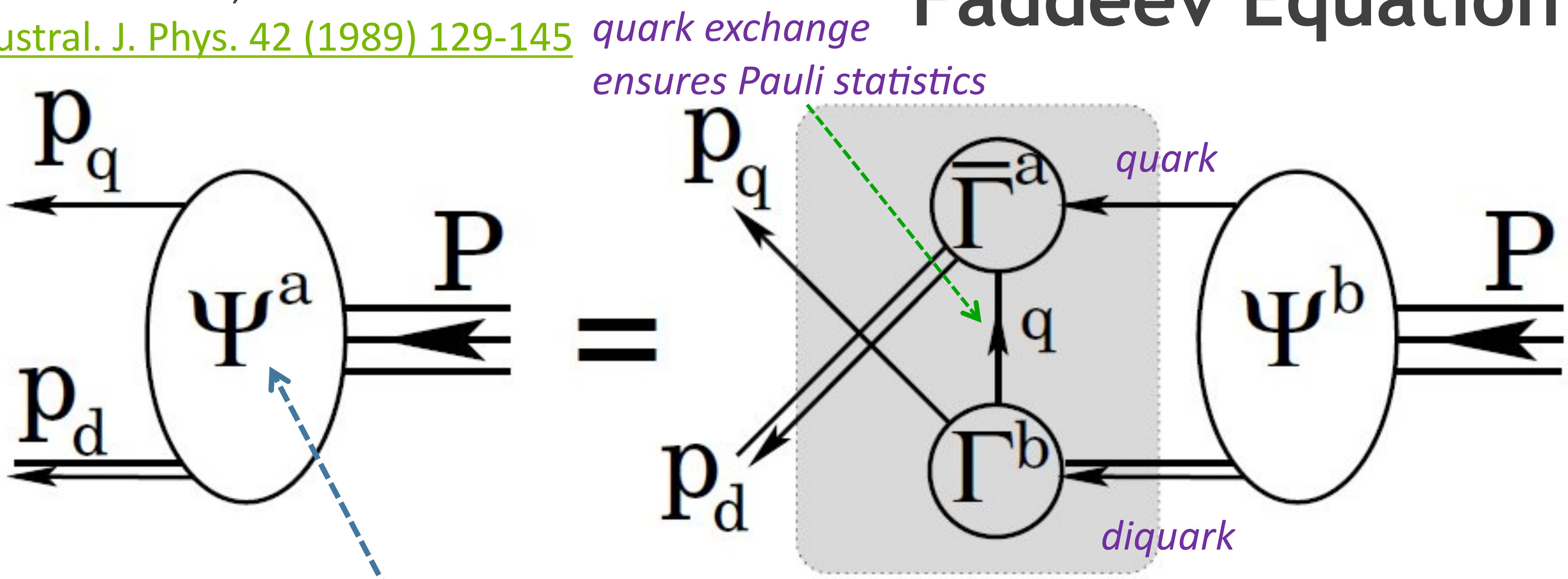

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  - Yields *wave function (Poincaré Covariant Faddeev* Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks . . .
  - Both have "*correct*" parity and "*right*" masses
  - In Nucleon's Rest Frame Amplitude has

    s−, p− & d−wave correlations

H.L.L. Roberts, L. Chang and C.D. Roberts
arXiv:1007.4318 [nucl-th]
H.L.L. Roberts, L. Chang, I.C. Cloët and C.D. Roberts
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Spectrum of some known *u*- & *d*-quark baryons

- Mesons & Diquarks

m_{0^+}	m_{1^+}	m_{0^-}	m_{1^-}	m_π	m_ρ	m_σ	m_{a1}
0.72	1.01	1.17	1.31	0.14	0.80	1.06	1.23

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	m_N	m_{N^*}	$m_N(\frac{1}{2}^-)$	$m_{N^*}(\frac{1}{2}^-)$	m_Δ	m_{Δ^*}	$m_\Delta(\frac{3}{2}^-)$	$m_{\Delta^*}(\frac{3}{2}^-)$
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EBAC		1.76	1.80		1.39		1.98	

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1st radial Excitation of N(1535)?

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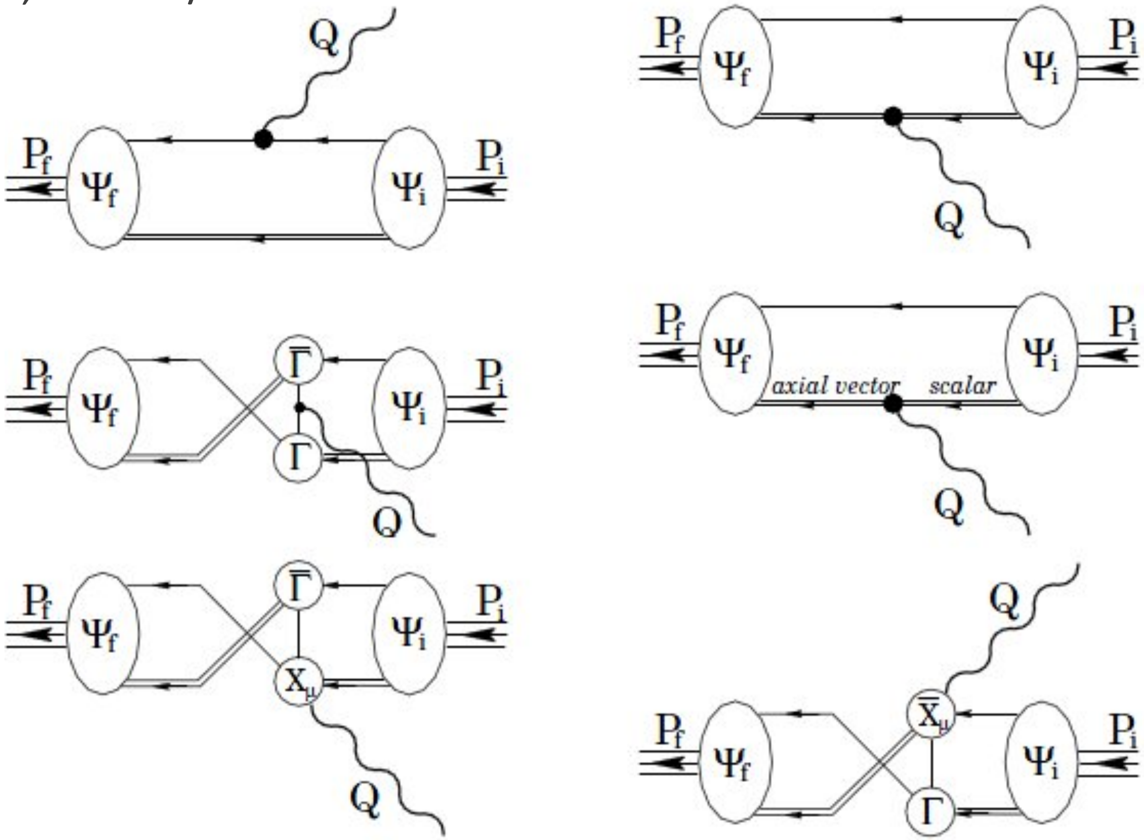
arXiv:0812.0416 [nucl-th]

Nucleon Elastic Form Factors

- Photon-baryon vertex

Oettel, Pichowsky and von Smekal, nucl-th/9909082



I.C. Cloët, C.D. Roberts, *et al.*
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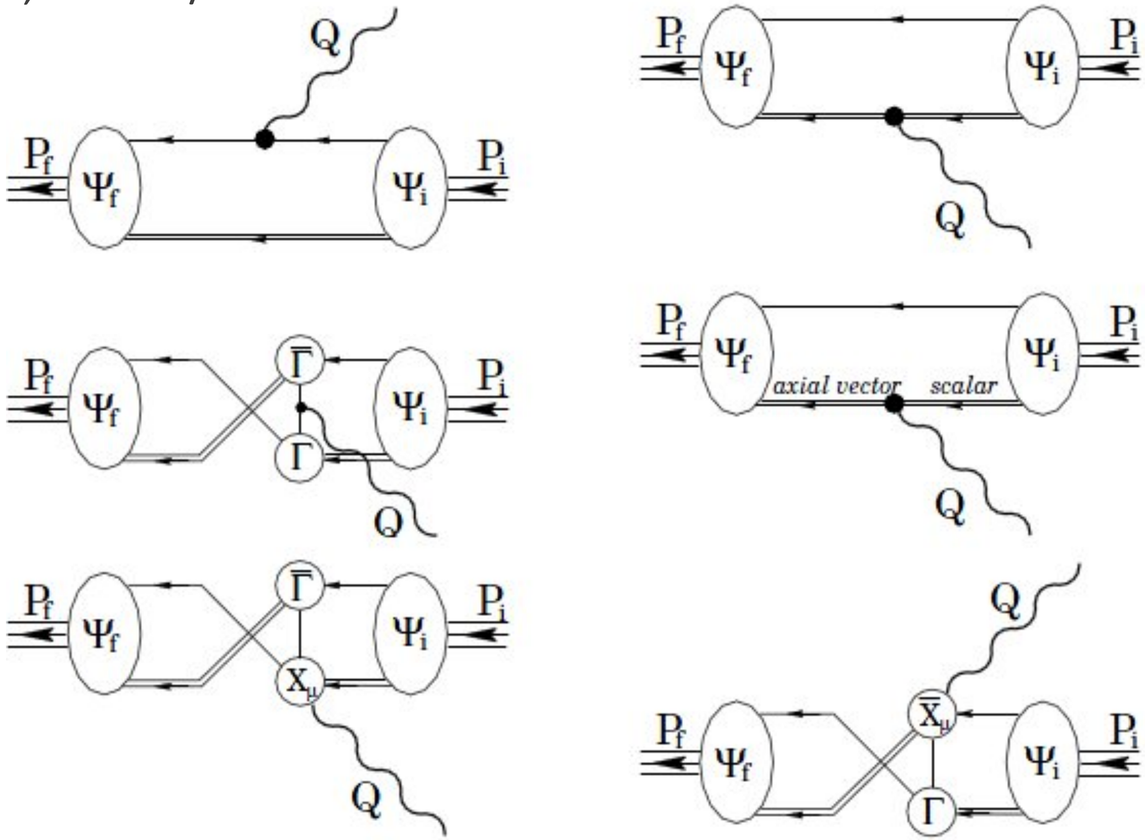
- "*Survey of nucleon electromagnetic form factors*"

  – unification of meson and baryon observables; and prediction of nucleon elastic form factors to 15 GeV^2



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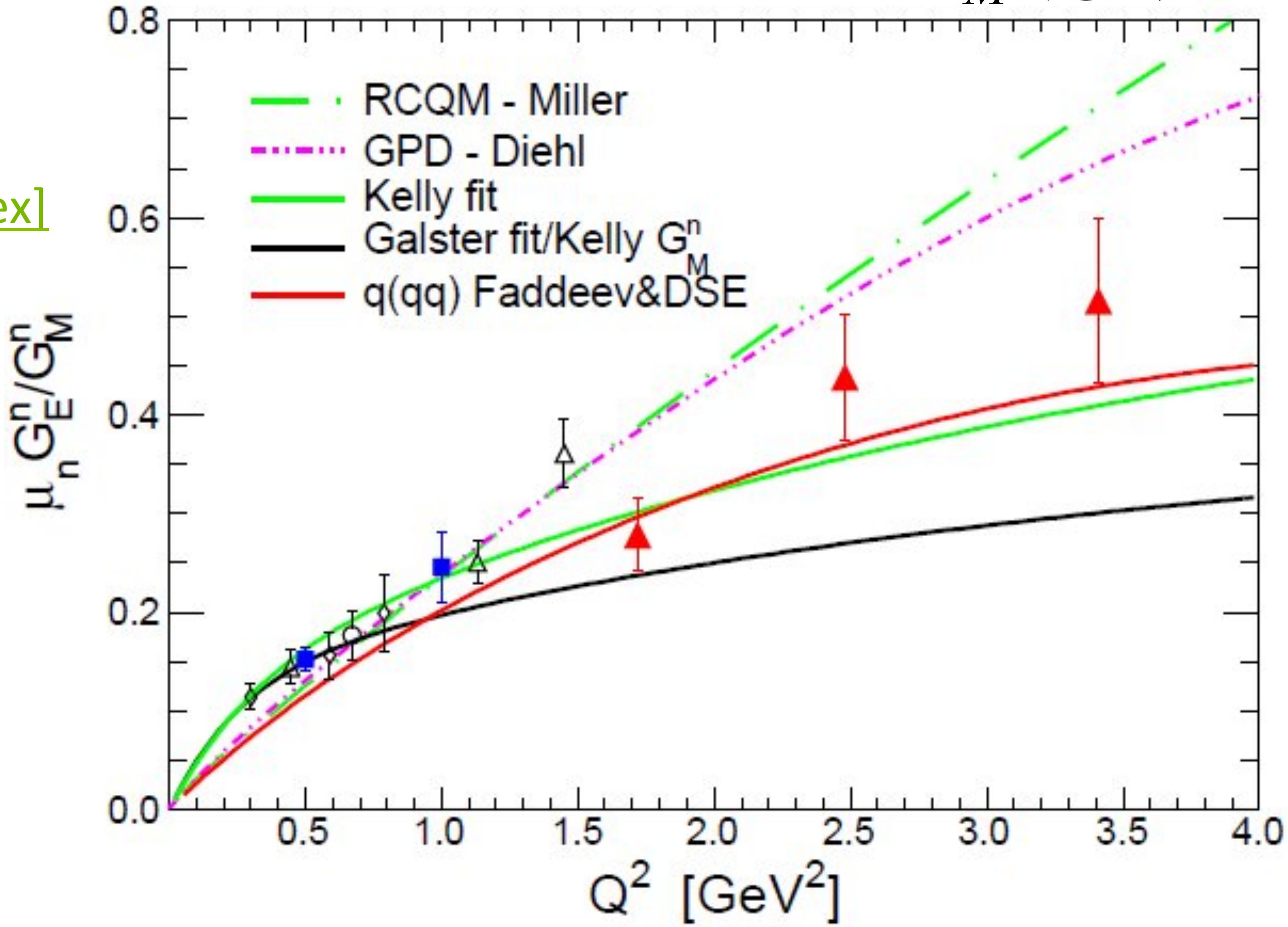

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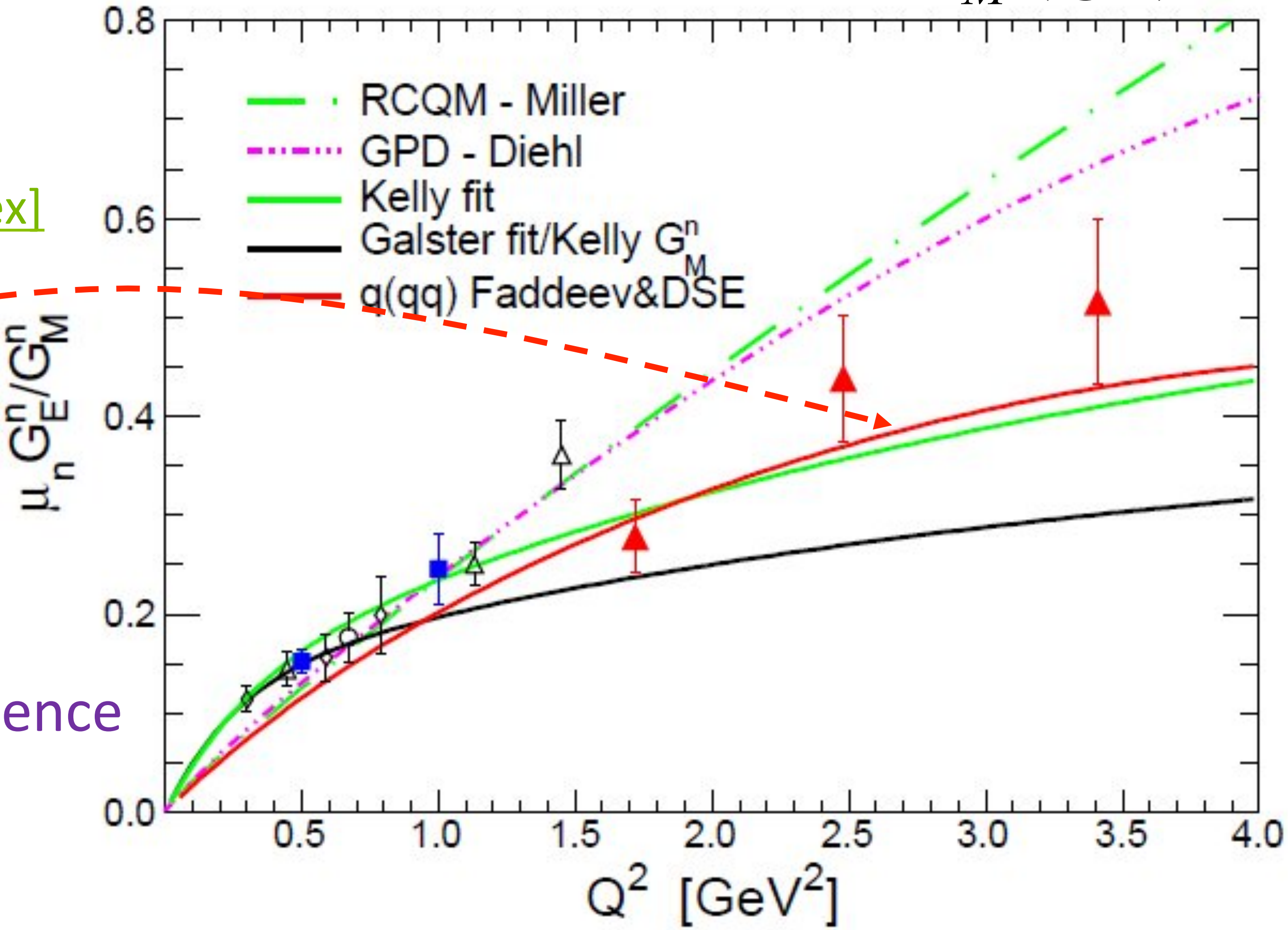
- DSE-prediction

- This evolution is very sensitive to momentum-dependence of dressed-quark propagator



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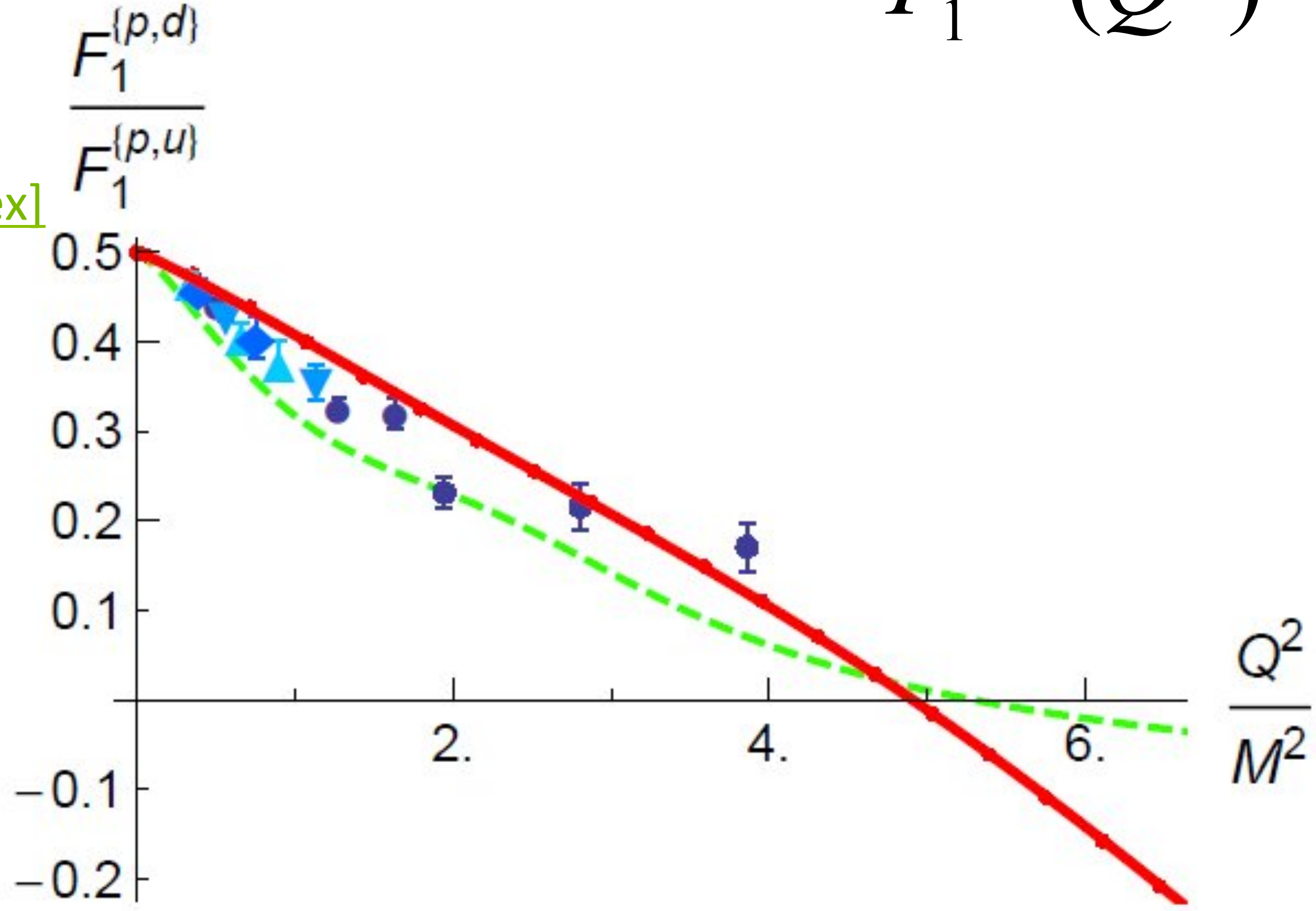

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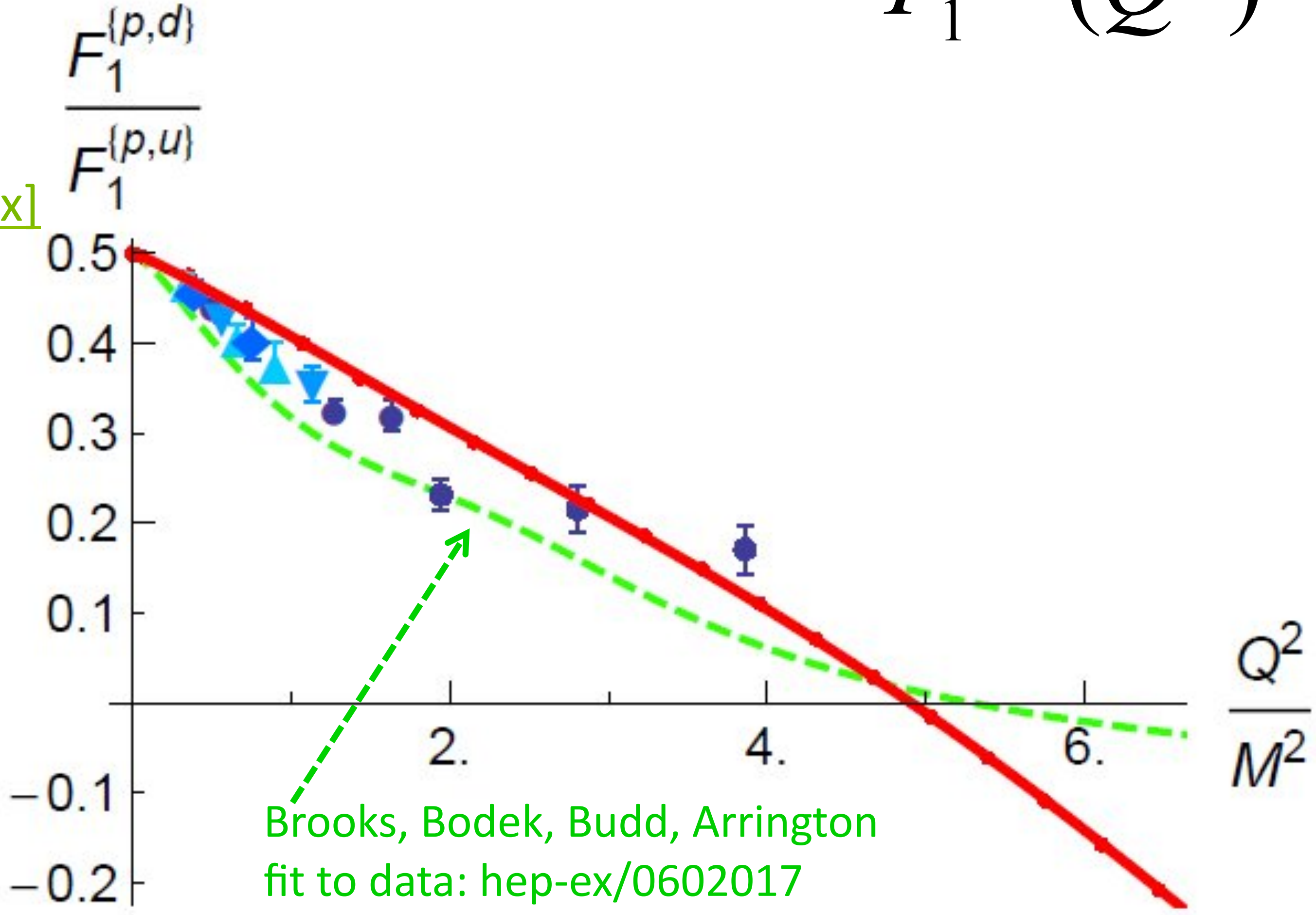

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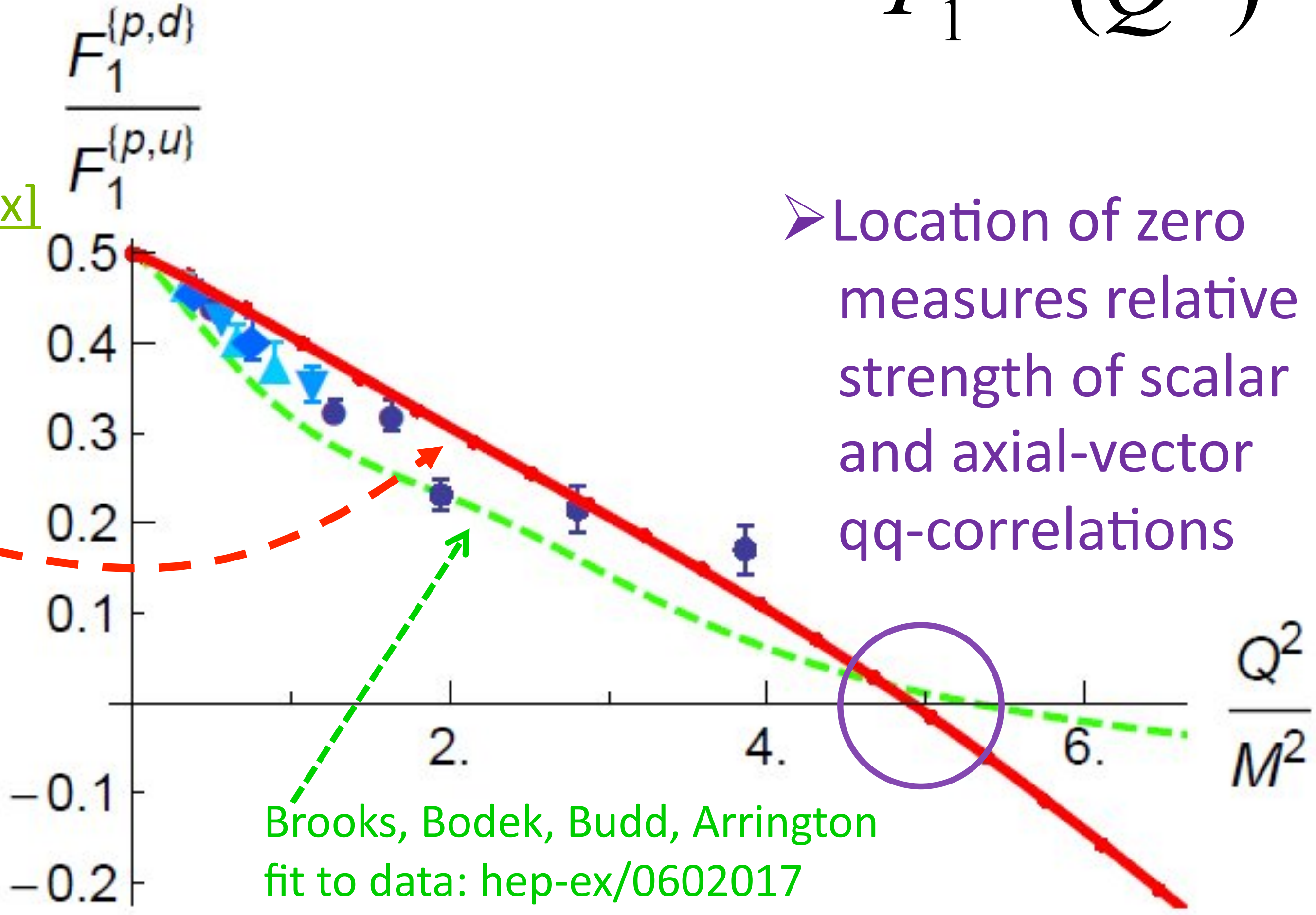

- Location of zero measures relative strength of scalar and axial-vector qq-correlations

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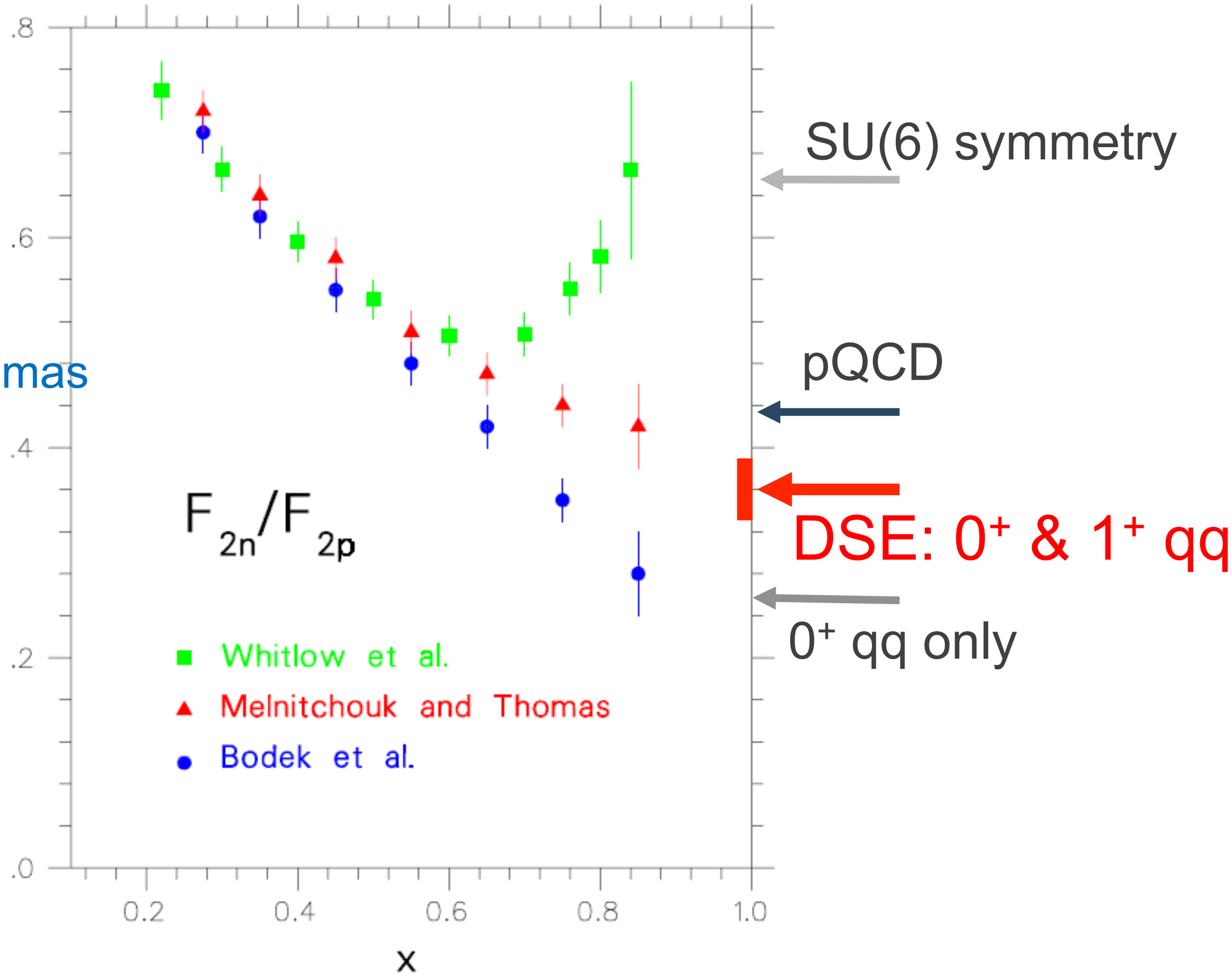
arXiv:0812.0416 [nucl-th]

Neutron Structure Function at high x

Reviews:
- S. Brodsky *et al.* NP B441 (1995)
- W. Melnitchouk & A.W.Thomas PL B377 (1996) 11
- N. Isgur, PRD 59 (1999)
- R.J. Holt & C.D. Roberts RMP (2010)



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- Dyson-Schwinger equations:
  - single framework, with IR model-input turned to advantage, *"almost unique in providing unambiguous path from a defined interaction → Confinement & DCSB → Masses → radii → form factors → distribution functions → etc."* McLerran & Pisarski arXiv:0706.2191 [hep-ph]