Excited-State Hadrons using the Stochastic LapH Method

Colin Morningstar

(Carnegie Mellon University)

Cairns, Australia

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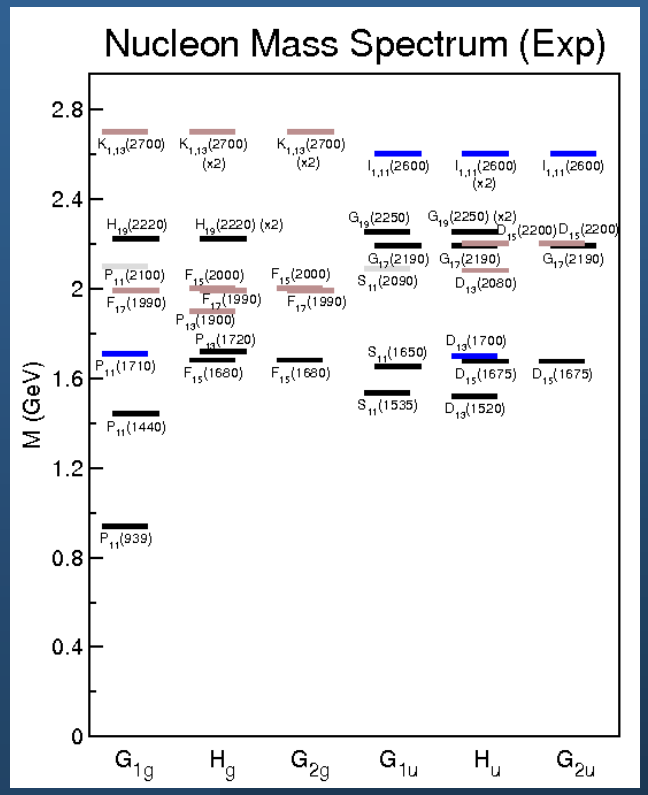
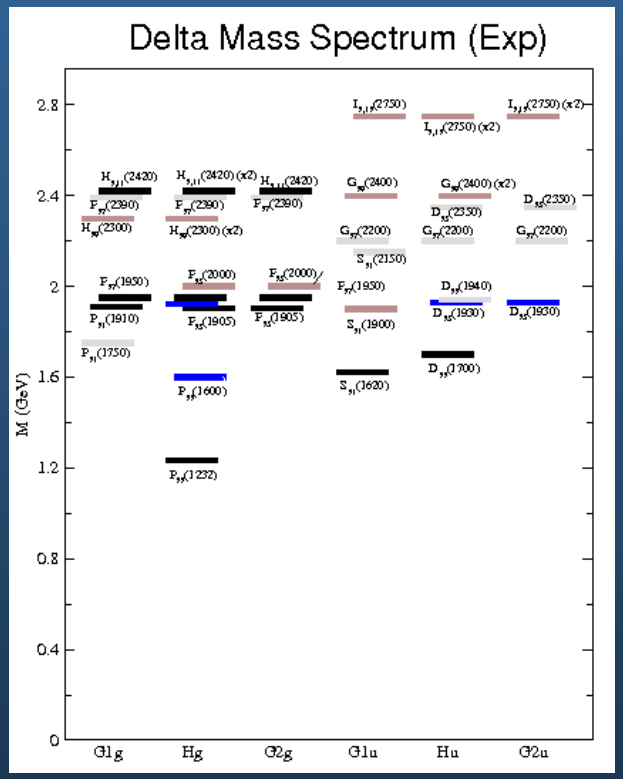
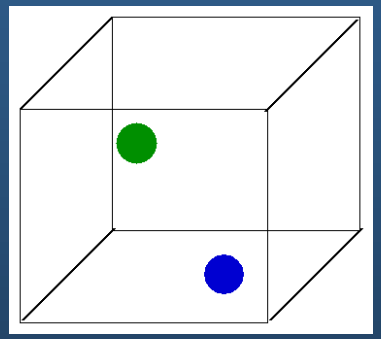
Cairns Colonial Club Resort

Objective

- Goal: compute energies of low-lying stationary states of QCD in a box using Monte Carlo method on space-time lattice







Key issues

- most excited hadrons are unstable (resonances)
- excited states more difficult to extract in Monte Carlo calculations
  - correlation matrices needed
  - operators with very good overlaps onto states of interest
- must extract all states lying below a state of interest
  - as pion get lighter, more and more multi-hadron states
- best multi-hadron operators made from constituent hadron operators with well-defined relative momenta
  - need for "slice-to-slice" quark propagators
- disconnected diagrams

Hadron Spectrum Collaboration

- spin-off from the Lattice Hadron Physics Collaboration which was spear-headed by Nathan Isgur and John Negele
- current members:
  - David Lenkner, Colin Morningstar, Ricky Wong (CMU)
  - Justin Foley (Utah)
  - John Bulava (DESY, Zeuthen)
  - Keisuke Jimmy Juge (U. of Pacific)
  - Mike Peardon (Trinity Coll. Dublin)
  - Steve Wallace (U. Maryland)
  - R. Edwards, B. Joo, D. Richards, C. Thomas (Jefferson Lab.)
  - H.W. Lin (U. Washington)
  - J. Dudek (Old Dominion)

Stochastic LapH team

Outline

- Operator selection
  - Single hadrons at rest
  - Single hadrons with momentum
- Slice-to-slice quark propagators
  - Stochastic LapH method (arXiv:1002.0818 [hep-lat])

Quark propagation

- quark propagator is inverse of Dirac matrix $M^{-1}[x,\alpha,a\,|\,y,\beta,b]$
  - rows/columns involve lattice site, spin, color
  - very large $N_{tot} \times N_{tot}$ matrix for each flavor, where
    $$N_{tot} = N_{sites} N_{spin} N_{color}$$
  - for $24^3 \times 128$ lattice, one has $N_{tot} = 21.2\,\text{million}$
- not feasible to compute (or store) all elements of inverse
- solve linear systems $Mx = y$ for variable source vectors y
  - more ill-conditioned as quark mass decreases
  - results first obtained for unphysically heavy quark masses
- key issue: find method requiring as few source vectors as possible
- our method:
  - Laplacian Heaviside (LapH) quark smearing
  - stochastic estimation with noise dilutions

Excited-state energies from Monte Carlo

- extracting excited-state energies requires matrix of correlators
- for a given $N \times N$ correlator matrix $C_{\alpha\beta}(t) = \langle 0 | O_\alpha(t) O_\beta^\dagger(0) | 0 \rangle$ one defines the N *principal correlators* $\lambda_\alpha(t, t_0)$ as the eigenvalues of

$$C(t_0)^{-1/2} C(t)\, C(t_0)^{-1/2}$$

  where t_0 (the time defining the "metric") is small
- can show that $\lim_{t\to\infty} \lambda_\alpha(t, t_0) = e^{-(t-t_0)E_\alpha}(1 + e^{-t\Delta E_\alpha})$
- N principal effective masses defined by $m_\alpha^{\text{eff}}(t) = \ln\left(\dfrac{\lambda_\alpha(t, t_0)}{\lambda_\alpha(t+1, t_0)}\right)$ now tend (plateau) to the N lowest-lying stationary-state energies
- standard analysis:
  - fits to "rotated" matrix when off-diagonals zero
  - matrix fits to "rotated" matrix

Operator design issues

- statistical noise increases with temporal separation t
- use of very good operators is <u>crucial</u> or noise swamps signal
- recipe for making better operators
  - crucial to construct operators using *smeared* fields
    - link variable smearing
    - quark field smearing
  - spatially extended operators
  - use large *set* of operators (variational coefficients)

Quantum numbers in toroidal box

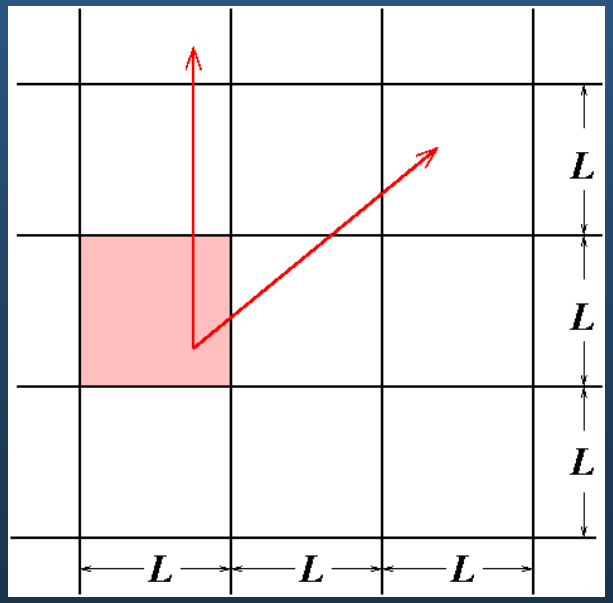
- Periodic boundary conditions in cubic box
  - not all directions equivalent
- Label stationary states of QCD in a periodic cubic box by irreps of cubic space group *even in the continuum limit*
  - zero momentum states: little group O_h

    $A_{1a}, A_{2a,} E_a, T_{1a}, T_{2a}, \quad G_{1a}, G_{2a}, H_a, \quad a = g, u$
  - on-axis momenta: little group C_{4v}

    $A_1, A_2, B_1, B_2, E, \quad G_1, G_2$
  - planar-diagonal momenta: C_{2v}

    $A_1, A_2, B_1, B_2, \quad G$
  - cubic-diagonal momenta: C_{3v}

    $A_1, A_2, E, \quad F_1, F_2, G$



- include G-parity in some meson sectors

Three stage approach (**PRD72:094506,2005**)

- (1) basic building blocks: smeared, covariant-displaced quark fields

$$(\tilde{D}_j^{(p)}\tilde{\psi}(\vec{x},t))_{Aa\alpha} \quad p\text{-link displacement } (j=0,\pm1,\pm2,\pm3)$$

- (2) construct elemental operators

$$B^F(t)=\sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}}\,\phi^F_{ABC}\,\varepsilon_{abc}(\tilde{D}_i^{(p)}\tilde{\psi}(\vec{x},t))_{Aa\alpha}(\tilde{D}_j^{(p)}\tilde{\psi}(\vec{x},t))_{Bb\beta}(\tilde{D}_k^{(p)}\tilde{\psi}(\vec{x},t))_{Cc\gamma}$$

$$M^F(t)=\sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}}\,\phi^F_{AB}\,\delta_{ab}(\bar{\tilde{\psi}}(\vec{x},t)\overleftarrow{\tilde{D}}_i^{(p)})_{Aa\alpha}(\tilde{D}_j^{(p)}\tilde{D}_k^{(p)}\tilde{\psi}(\vec{x},t))_{Bb\beta}$$

  - flavor structure from isospin
  - color structure from gauge invariance

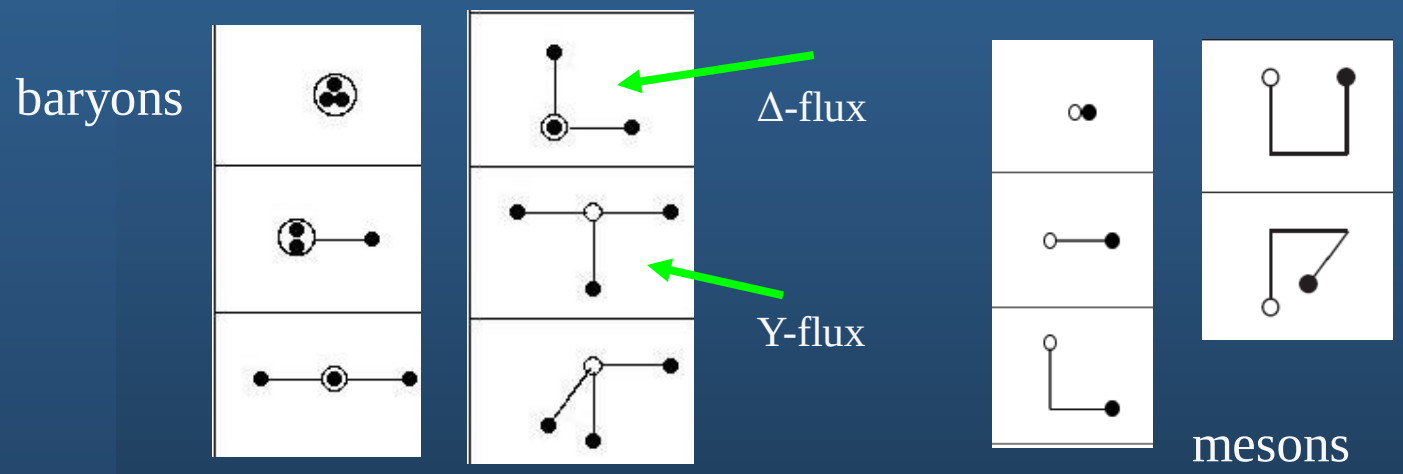
- (3) group-theoretical projections onto irreps of $G=O_h, C_{4v}, C_{3v}, C_{2v}$

$$H_i^{\Lambda\lambda F}(t)=\frac{d_\Lambda}{g_G}\sum_{R\in G} D^{(\Lambda)}_{\lambda\lambda}(R)^* U_R H_i^F(t)\, U_R^+$$

- (4) momenta: zero, on-axis, planar-diagonal, cubic-diagonal

Single-hadron operators

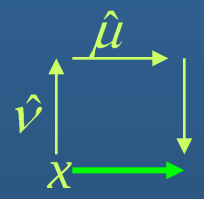
- covariantly-displaced quark fields as building blocks
- group-theoretical projections onto irreps of lattice symmetry group
- displacements of different lengths build up radial structure
- displacements in different directions build up orbital structure



- reference: PRD72, 094506 (2005)
- similar for *nonzero-momentum* operators, but must distinguish transverse and longitudinal displacements

Quark- and gauge-field smearing

- smeared quark and gluon fields fields → dramatically reduced coupling with short wavelength modes
- link-variable smearing (stout links PRD69, 054501 (2004))
  - define $C_\mu(x) = \sum_{\pm\nu\neq\mu} \rho_{\mu\nu} U_\nu(x) U_\mu(x+\hat{\nu}) U_\nu^+(x+\hat{\mu})$
  - spatially isotropic $\rho_{jk} = \rho, \quad \rho_{4k} = \rho_{k4} = 0$



  - exponentiate traceless Hermitian matrix

$$\Omega_\mu = C_\mu U_\mu^+ \qquad Q_\mu = \frac{i}{2}\left(\Omega_\mu^+ - \Omega_\mu\right) - \frac{i}{2N}\mathrm{Tr}\left(\Omega_\mu^+ - \Omega_\mu\right)$$

  - iterate $U_\mu^{(n+1)} = \exp\left(iQ_\mu^{(n)}\right) U_\mu^{(n)}$

$$U_\mu \to U_\mu^{(1)} \to \cdots \to U_\mu^{(n)} \equiv \tilde{U}_\mu$$

- initial quark-field smearing (Laplacian using smeared gauge field)

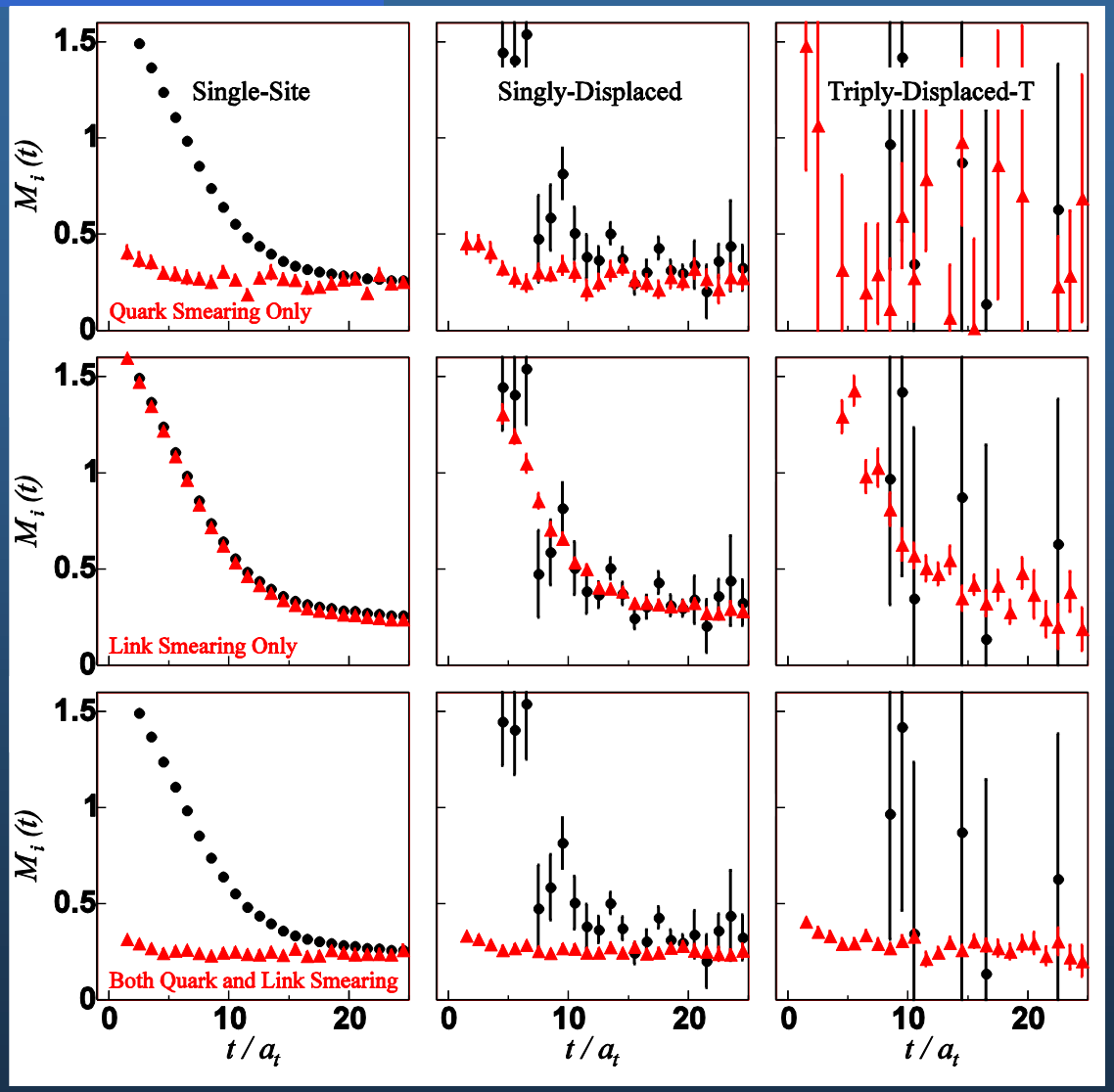
$$\tilde{\psi}(x) = \left(1 + \frac{\sigma_s}{4n_\sigma}\tilde{\Delta}\right)^{n_\sigma} \psi(x)$$

Importance of smearing

- Nucleon G_{1g} channel
- effective masses of 3 selected operators
- noise reduction from link variable smearing, especially for displaced operators
- quark-field smearing reduces couplings to high-lying states

$$\sigma_s = 4.0, \quad n_\sigma = 32$$
$$n_\rho \rho = 2.5, \quad n_\rho = 16$$

- less noise in excited states using $\sigma_s = 3.0$



Operator selection

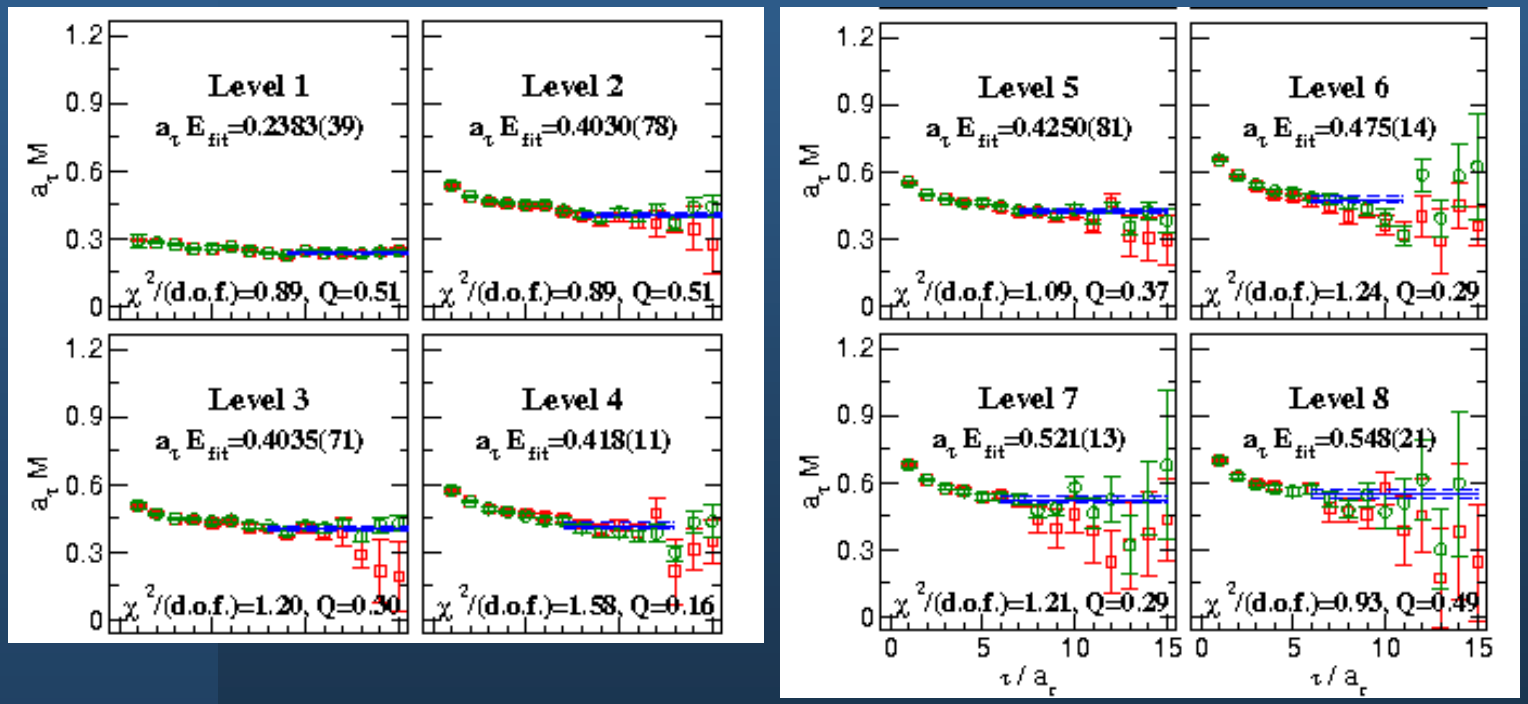
- operator construction leads to very large number of operators
- rules of thumb for "pruning" operator sets
  - noise is the enemy!
  - prune first using intrinsic noise (diagonal correlators)
  - prune next within operator *types* (single-site, singly-displaced, *etc.*) based on condition number of
  - prune across all operators based on condition number
- best to keep a variety of different types of operators, as long as condition numbers maintained

$$\hat{C}_{ij}(t) = \frac{C_{ij}(t)}{\sqrt{C_{ii}(t)C_{jj}(t)}}, \qquad t = 1$$

- typically use 16 operators to get 8 lowest lying levels

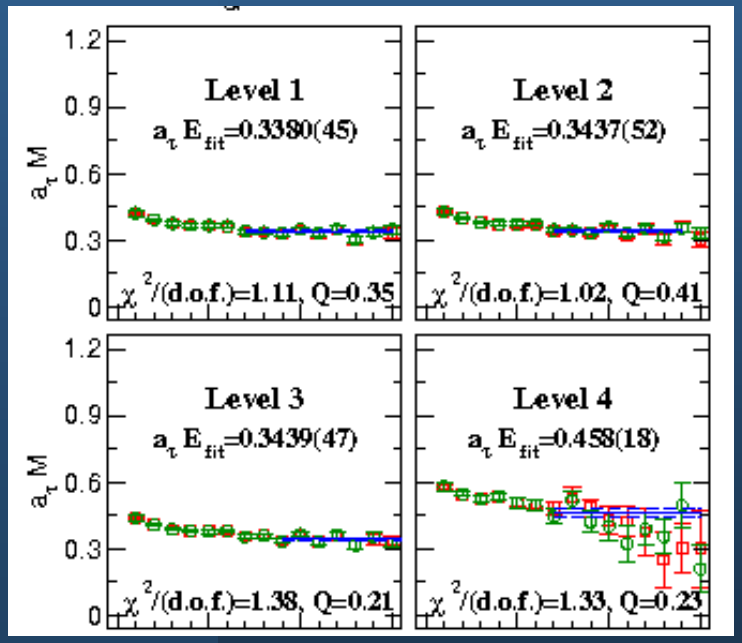
Nucleon G_{1g} effective masses

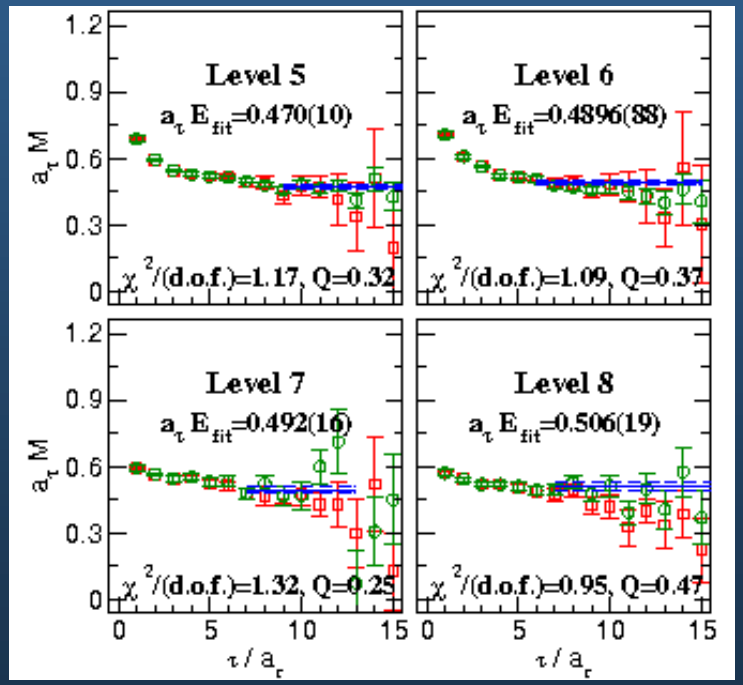
- 200 quenched configs, $12^3 \times 48$ anisotropic Wilson lattice, $a_s \sim 0.1$ fm, $a_s/a_t \sim 3$, $m_\pi \sim 700$ MeV
- nucleon G_{1g} channel
- green=fixed coefficients, red=principal



Nucleon H_u effective masses

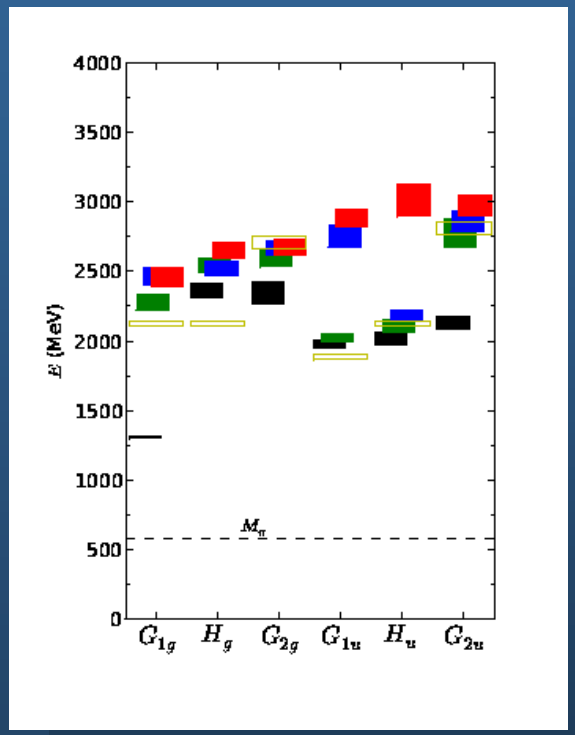
- 200 quenched configs, $12^3 \times 48$ anisotropic Wilson lattice, $a_s \sim 0.1$ fm, $a_s/a_t \sim 3$, $m_\pi \sim 700$ MeV
- nucleon H_u channel
- green=fixed coefficients, red=principal

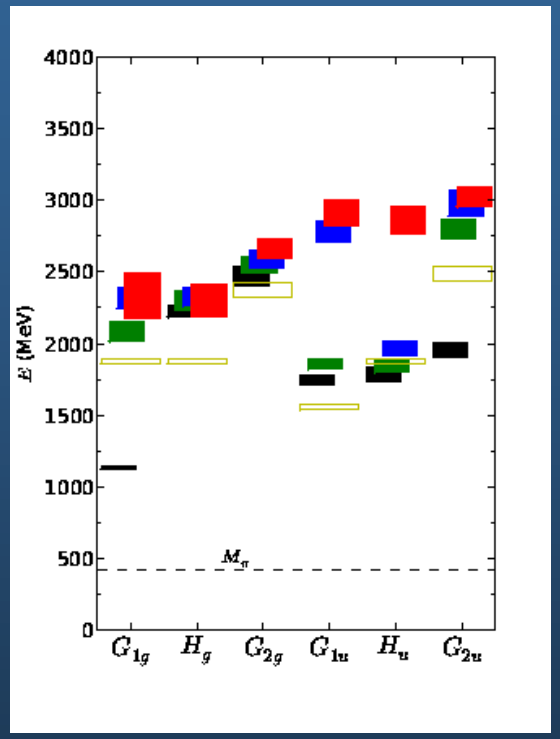




Nucleons

- N_f=2 on $24^3\times64$ anisotropic clover lattice, a_s~0.11 fm, a_s/a_t~3
- Left: m_π=578 MeV Right: m_π=416 MeV PRD 79, 034505 (2009)





- multi-hadron thresholds above show need for multi-hadron operators to go to lower pion masses!!

Spatial summations

- baryon at rest is operator of form

$$B(\vec{p}=0,t)=\frac{1}{V}\sum_{\vec{x}}\varphi_B(\vec{x},t)$$

- baryon correlator has a double spatial sum

$$\left\langle 0\left|\bar{B}(\vec{p}=0,t)\,B(\vec{p}=0,0)\right|0\right\rangle=\frac{1}{V^2}\sum_{\vec{x},\vec{y}}\left\langle 0\left|\bar{\varphi}_B(\vec{x},t)\,\varphi_B(\vec{y},0)\right|0\right\rangle$$

- computing all elements of propagators exactly not feasible
- translational invariance can limit summation over source site to a single site for local operators

$$\left\langle 0\left|\bar{B}(\vec{p}=0,t)\,B(\vec{p}=0,0)\right|0\right\rangle=\frac{1}{V}\sum_{\vec{x}}\left\langle 0\left|\bar{\varphi}_B(\vec{x},t)\,\varphi_B(0,0)\right|0\right\rangle$$

Slice-to-slice quark propagators

- good baryon-meson operator of total zero momentum has form

$$B(\vec{p},t)M(-\vec{p},t) = \frac{1}{V^2}\sum_{\vec{x},\vec{y}} \varphi_B(\vec{x},t)\varphi_M(\vec{y},t)e^{i\vec{p}\cdot(\vec{x}-\vec{y})}$$

- cannot limit source to single site for multi-hadron operators
- disconnected diagrams (scalar mesons) will also need many-to-many quark propagators
- quark propagator elements from all spatial sites to all spatial sites are needed!

Laplacian Heaviside quark-field smearing

- new quark-field smearing method PRD80, 054506 (2009)
- judicious choice of quark-field smearing makes exact computations with all-to-all quark propagators possible (on small volumes)
- to date, quark field smeared using covariant Laplacian

$$\tilde{\psi}(x) = \left(1 + \frac{\sigma_s}{4n_\sigma}\tilde{\Delta}\right)^{n_\sigma} \psi(x)$$

- express in term of eigenvectors/eigenvalues of Laplacian

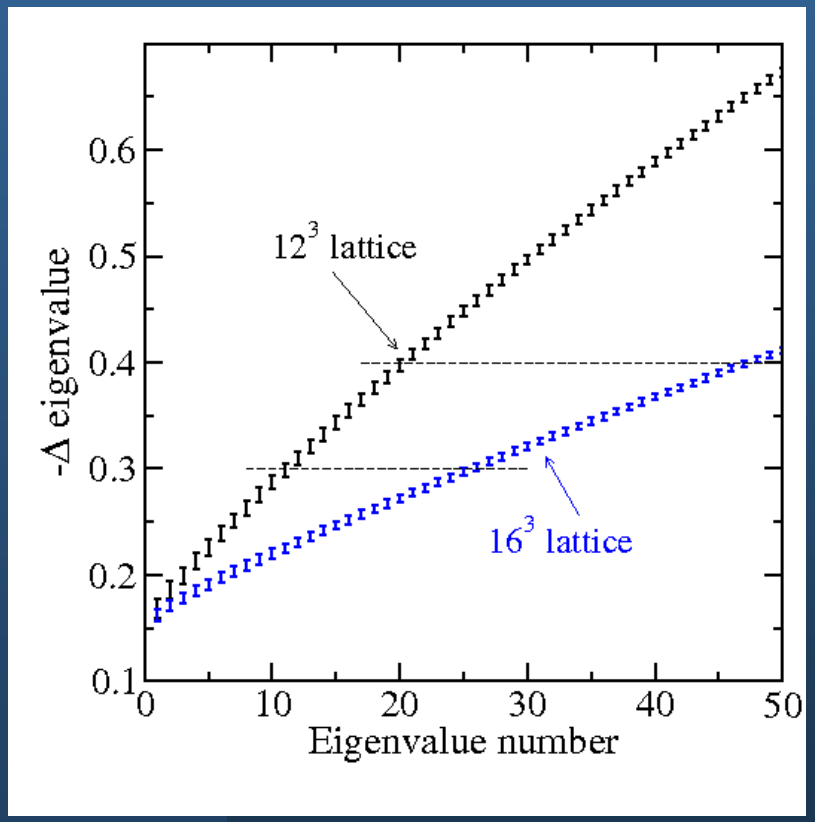
$$\tilde{\psi}(x) = \left(1 + \frac{\sigma_s}{4n_\sigma}\tilde{\Delta}\right)^{n_\sigma} \sum_k |\varphi_k\rangle\langle\varphi_k|\psi(x)$$

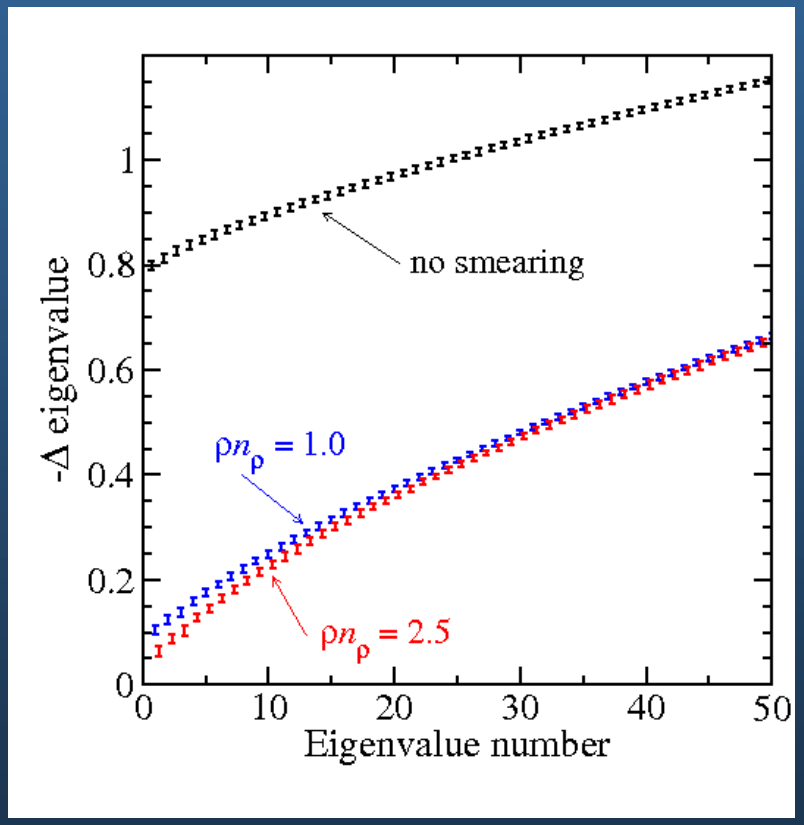
$$= \sum_k \left(1 + \frac{\sigma_s \lambda_k}{4n_\sigma}\right)^{n_\sigma} |\varphi_k\rangle\langle\varphi_k|\psi(x)$$

- truncate sum and set weights to unity → Laplacian Heaviside

Getting to know the Laplacian

- spectrum of the covariant Laplacian
- *left:* dependence on lattice size; *right:* dependence on link smearing





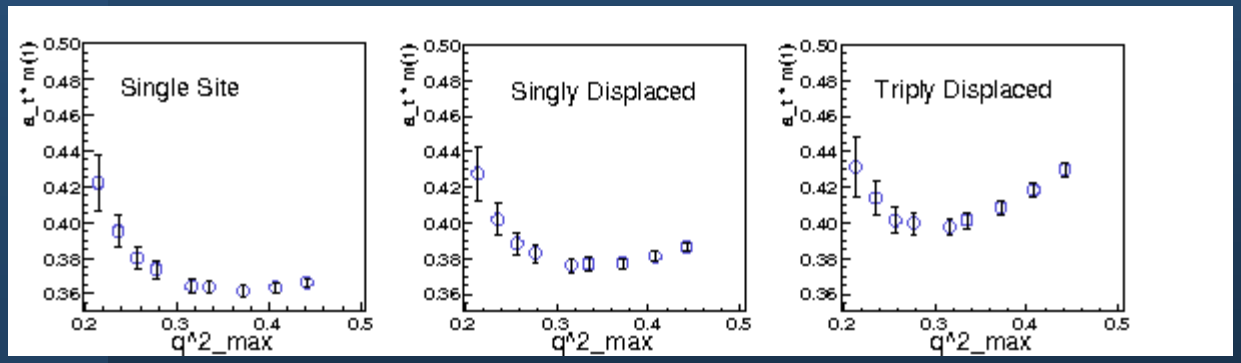
Choosing the smearing cut-off

- Laplacian Heaviside (Laph) quark smearing

$$\tilde{\psi}(x) = \Theta\ \ \sigma_s^2 + \tilde{\Delta}\ \ \psi(x)$$

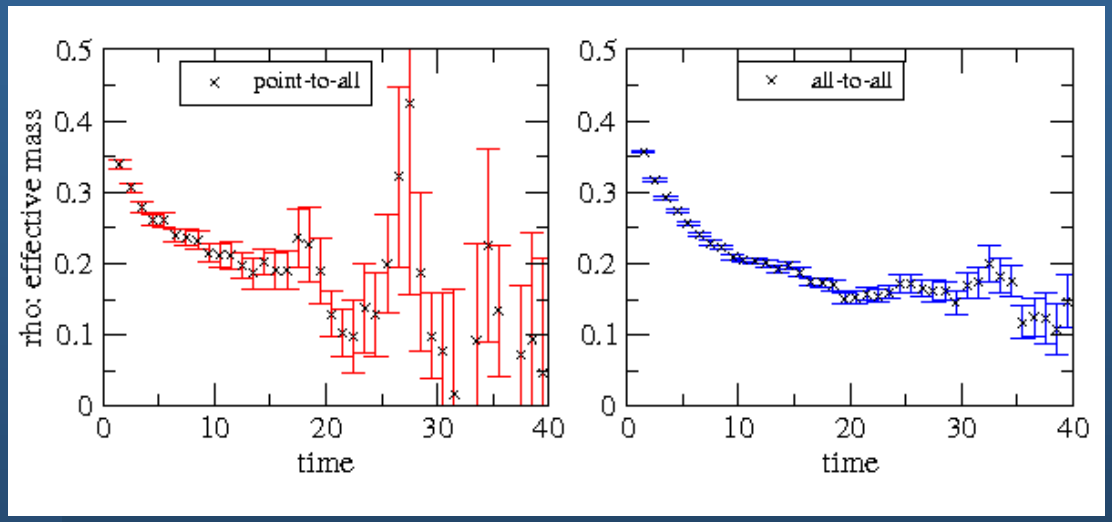
$$\approx \sum_{k=1}^{N_{\max}} |\varphi_k\rangle\langle\varphi_k|\psi(x)$$

- choose smearing cut-off based on minimizing excited-state contamination, keep noise small
  - behavior of nucleon t=1 effective masses



Tests of Laplacian Heaviside smearing

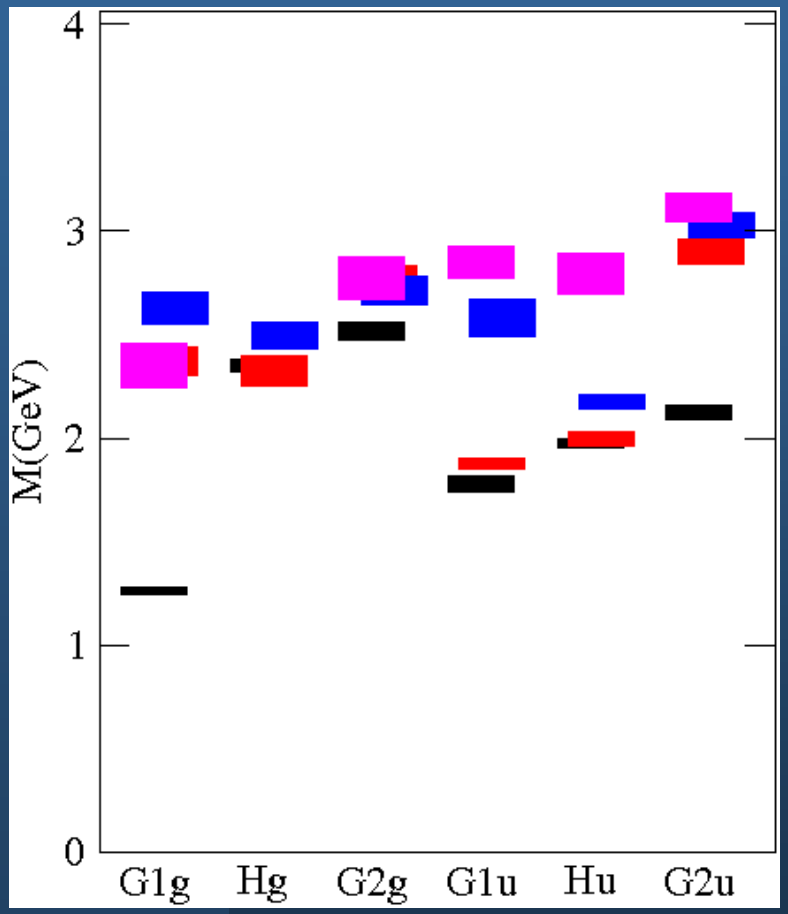
- comparison of ρ-meson effective masses using same number of gauge-field configurations

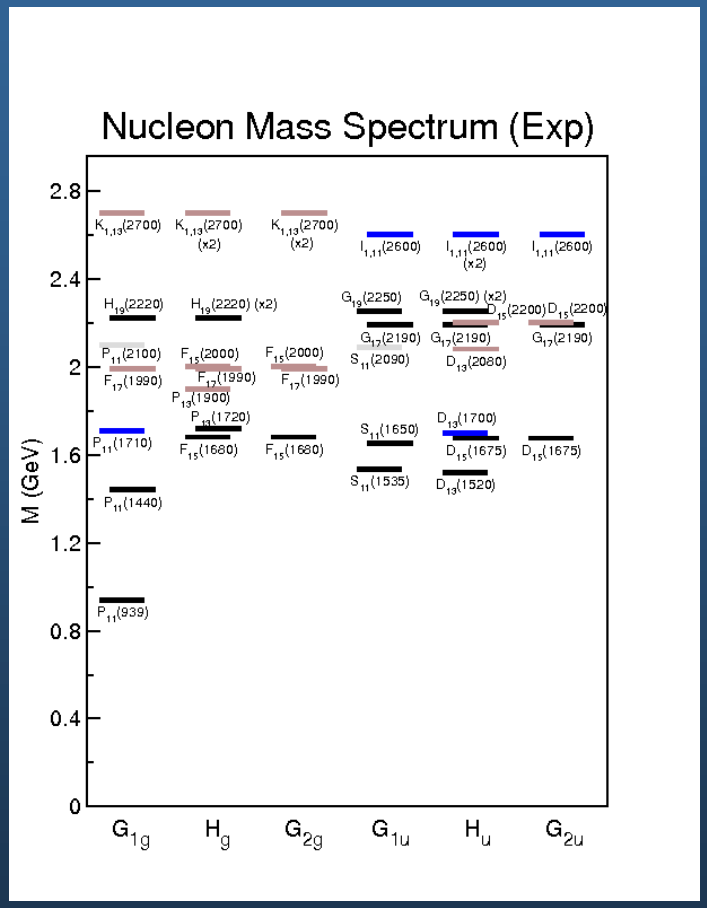


- typically need about 32 modes on 16^3 lattice
- about 112 modes on 24^3 lattice, 256 modes on 32^3 lattice

Nucleon operator pruning

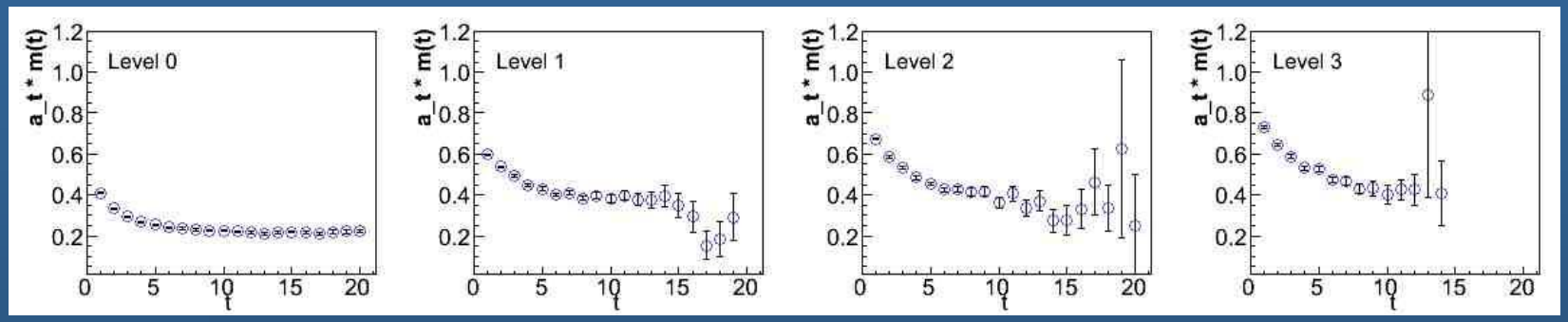
- N_f=2+1 on $16^3 \times 128$ lattice, m_π= 380 MeV (100 configs, 32 eigvecs)



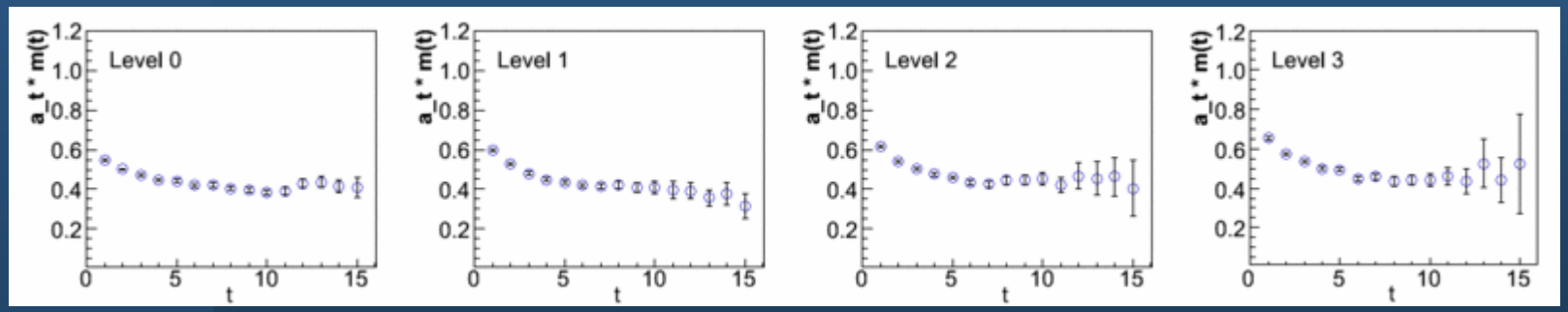


Nucleon rotated effective masses

- G_{1g} using 10x10 matrix: off-diagonals are zero

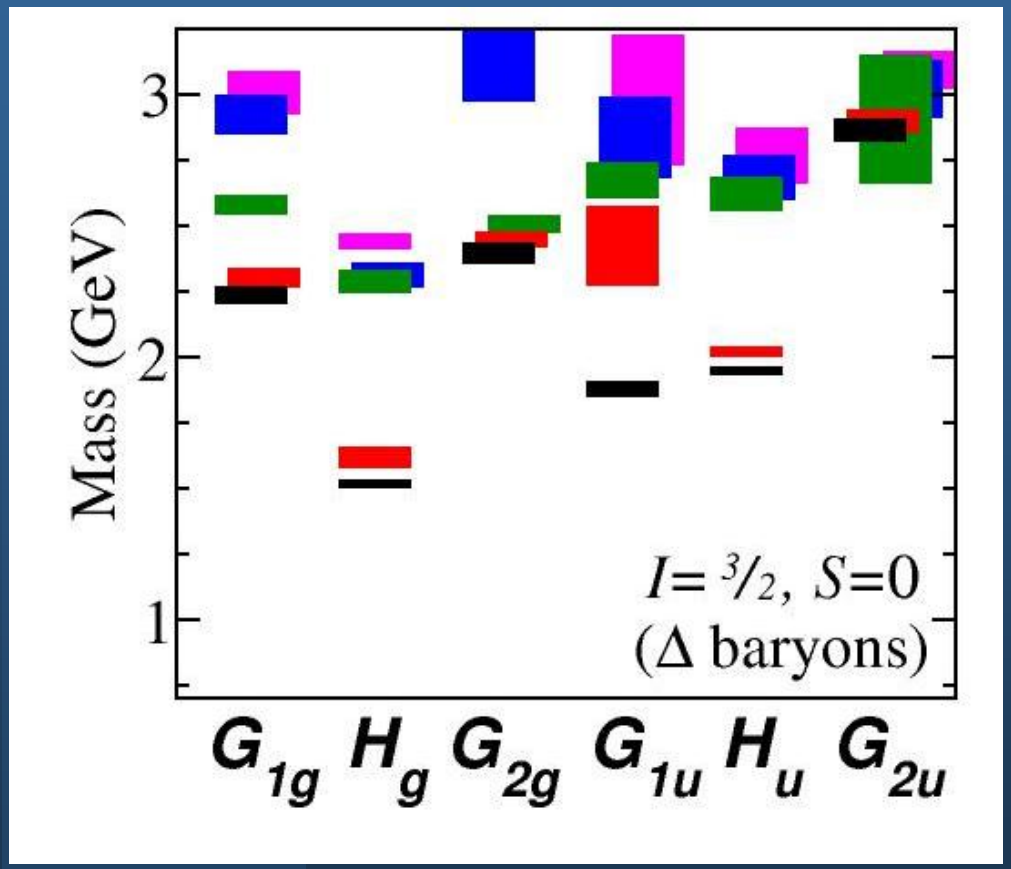


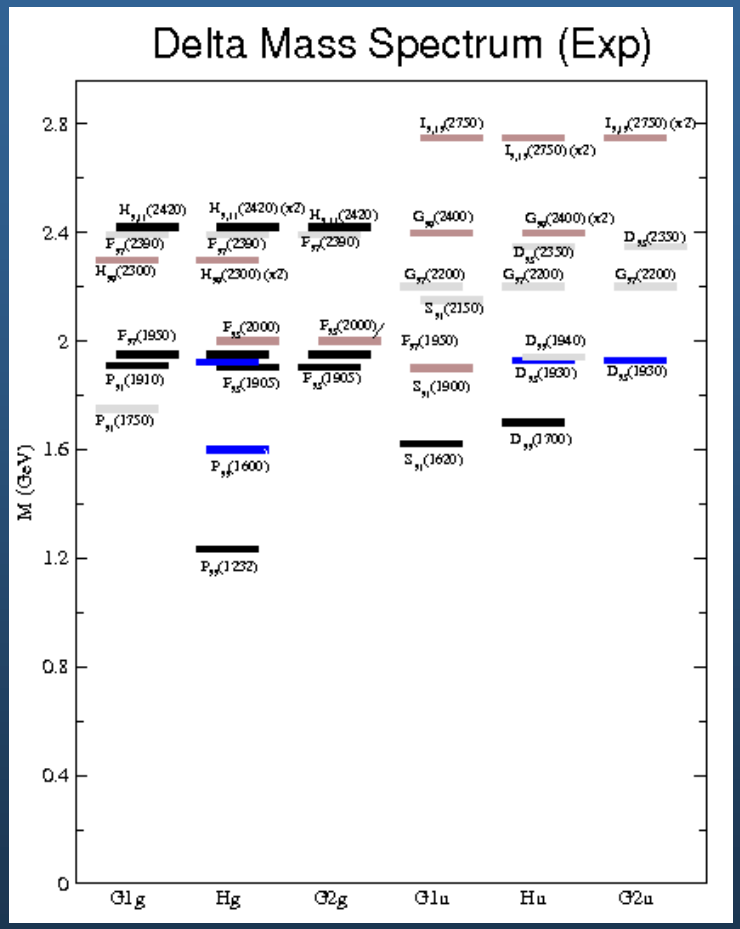
- H_g using 10x10 matrix: off-diagonals are zero



Delta operator pruning

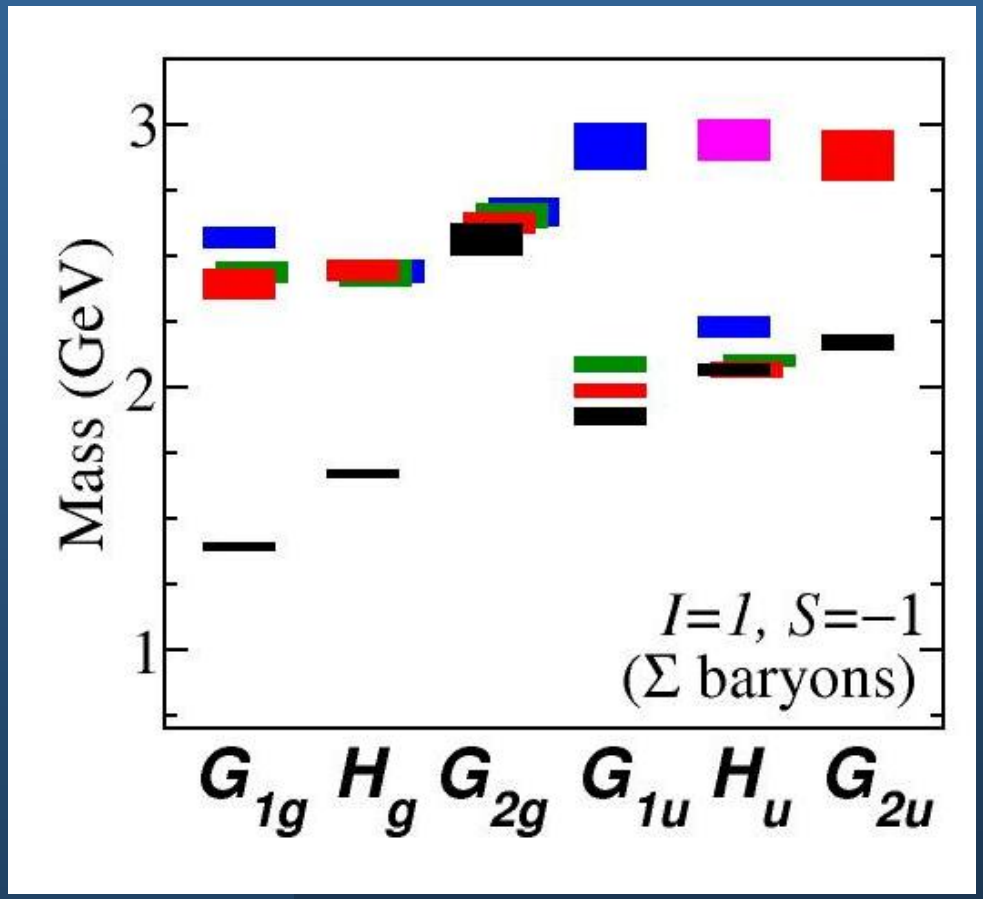
- N_f=2+1 on $16^3 \times 128$ lattice, m_π= 380 MeV (481 configs, 32 eigvecs)

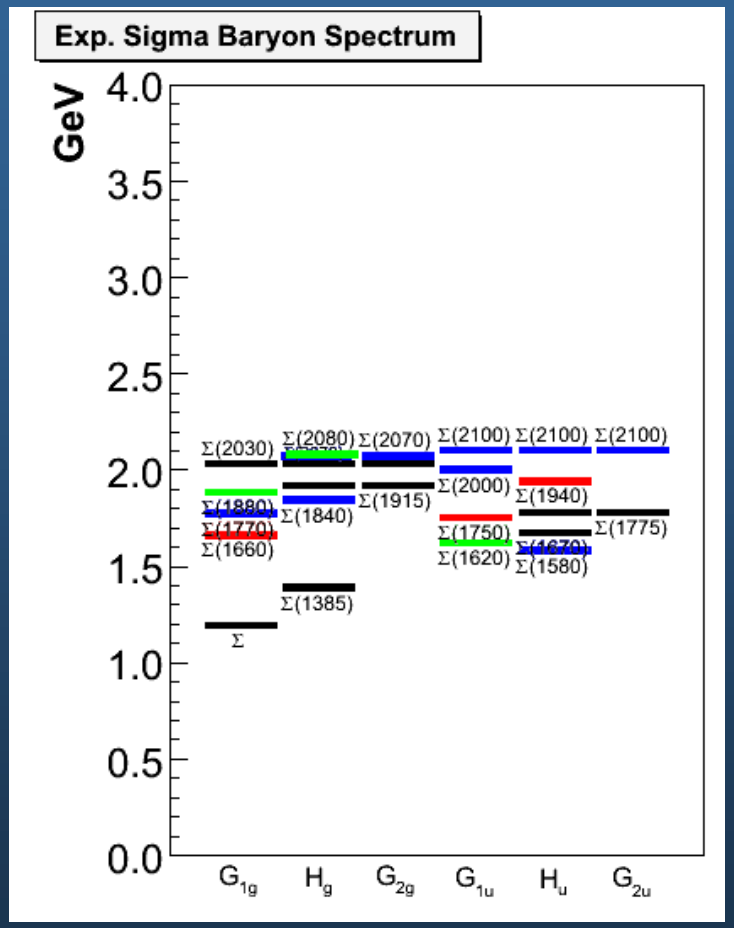




Sigma operator pruning

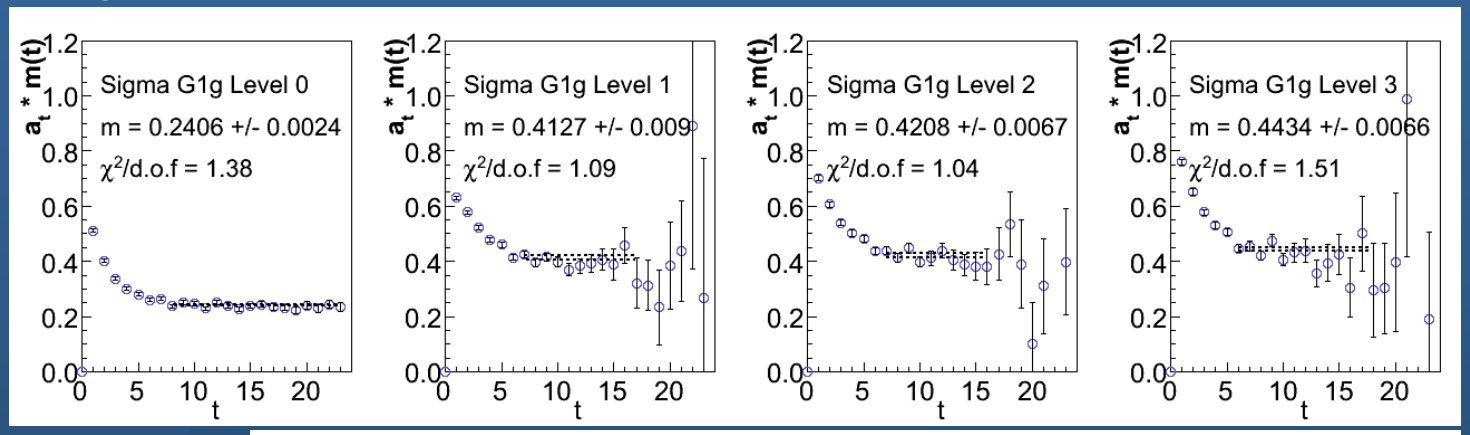
- N_f=2+1 on $16^3 \times 128$ lattice, m_π= 380 MeV (100 configs, 32 eigvecs)



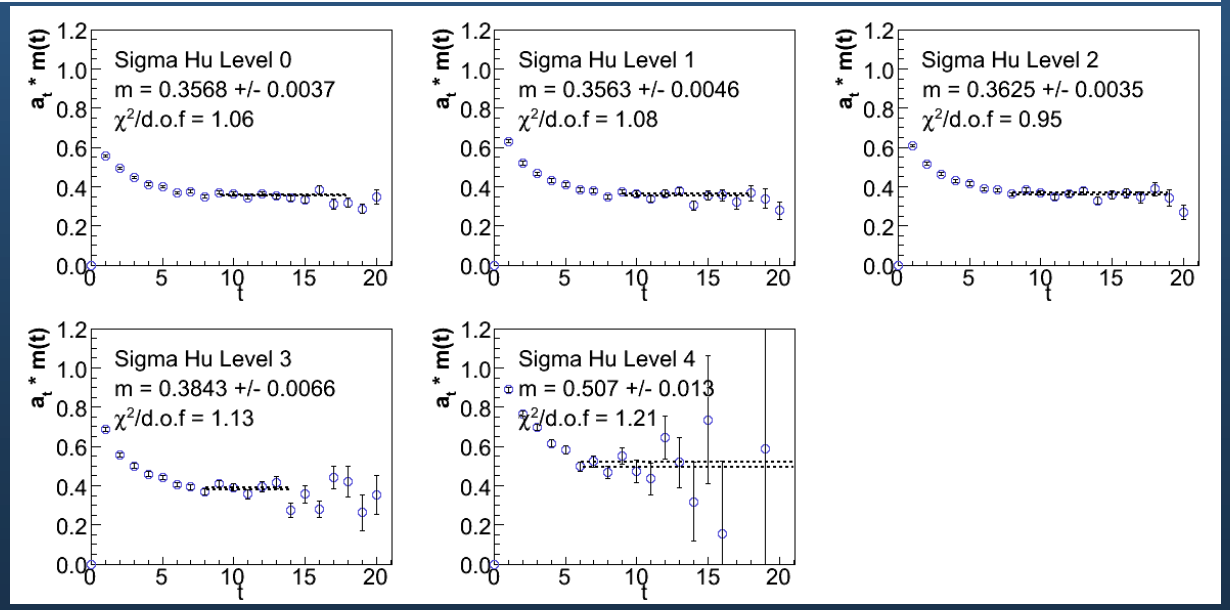


Sigma rotated effective masses

- G_{1g}

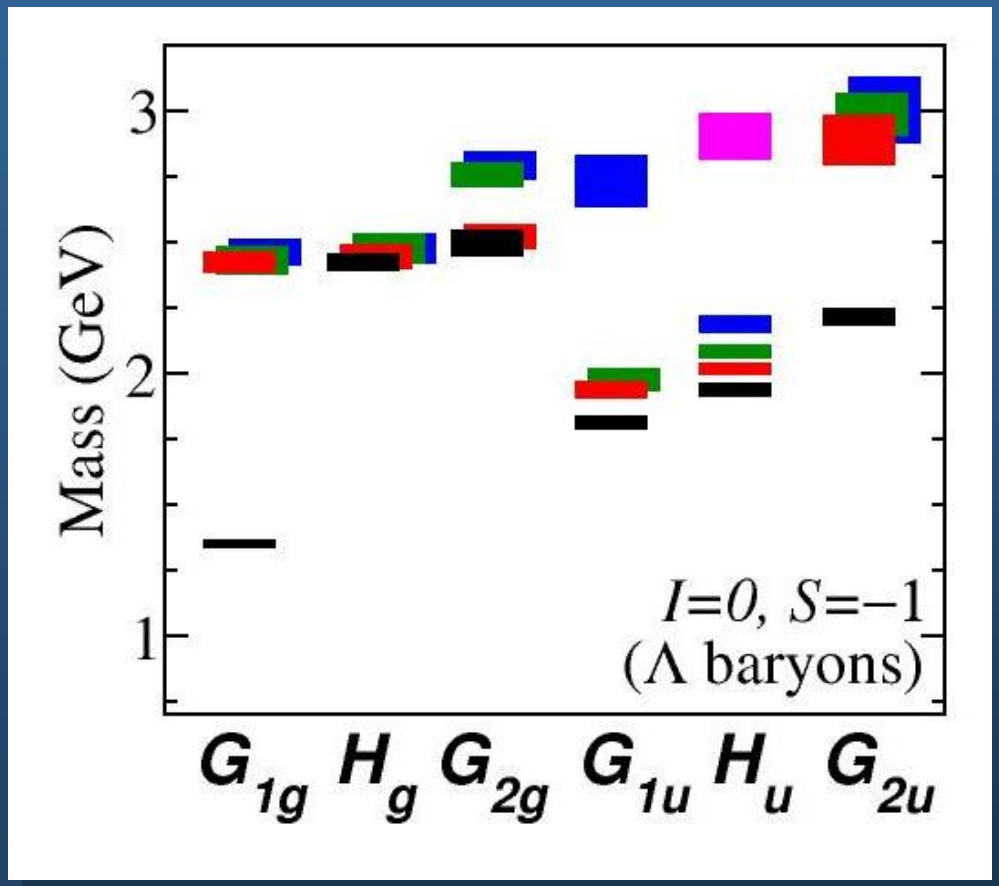


- H_u



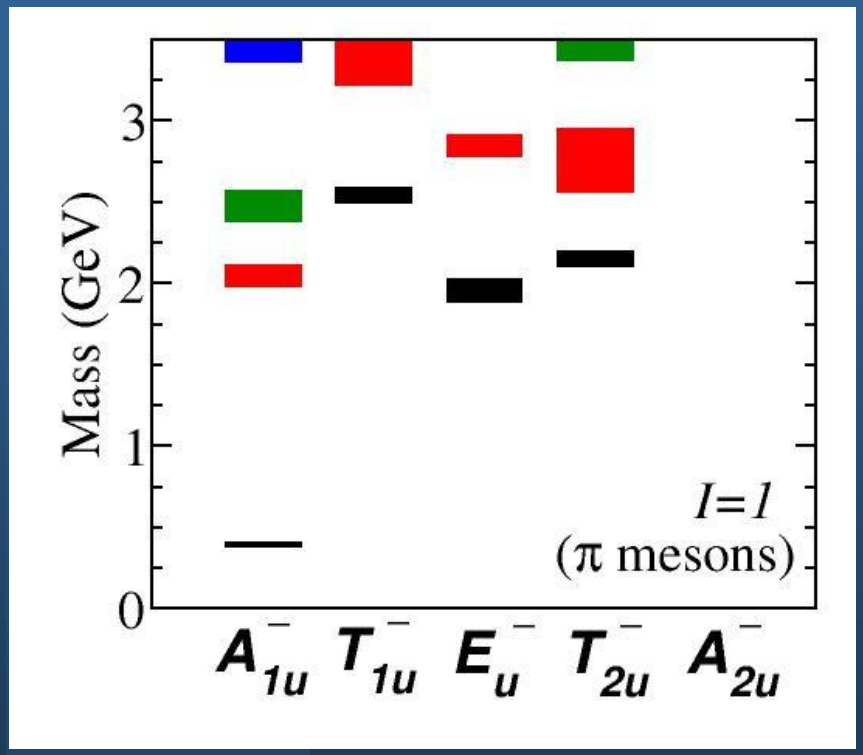
Lambda operator pruning

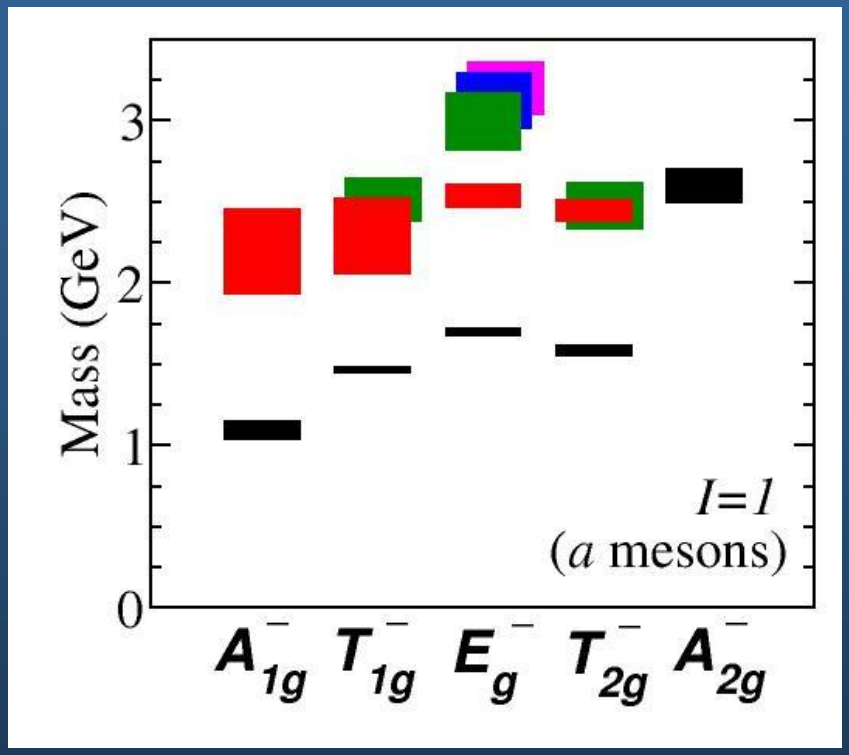
- N_f=2+1 on $16^3 \times 128$ lattice, m_π= 380 MeV (100 configs, 32 eigvecs)



Isovector G-parity odd mesons

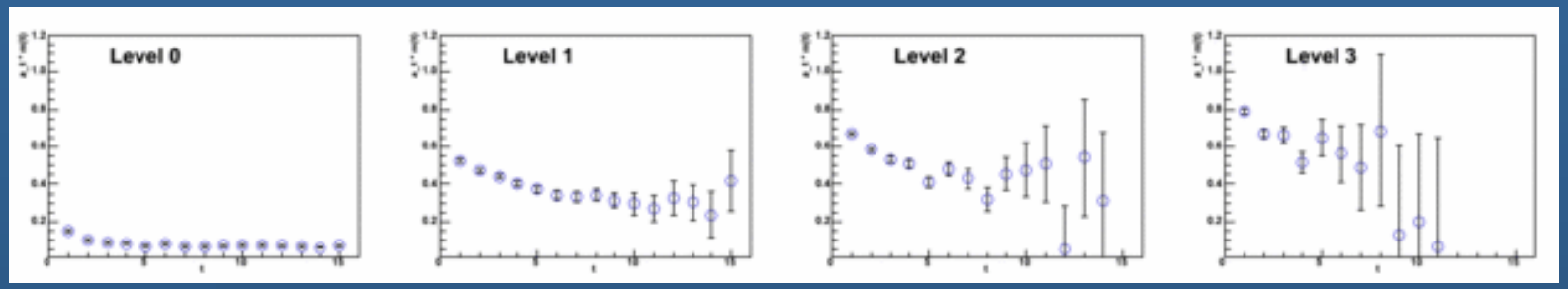
- N_f=2+1 on $16^3 \times 128$ lattice, m_π= 380 MeV (100 configs, 32 eigvecs)



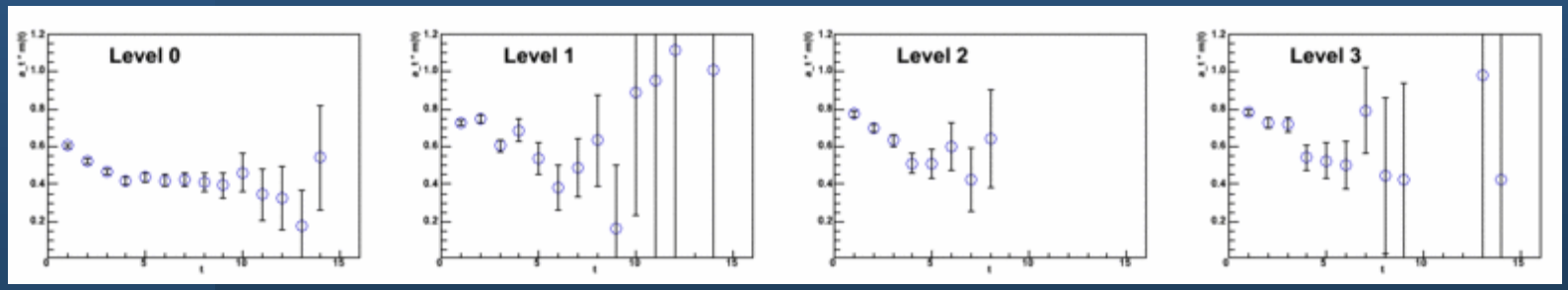


Isovector G-parity odd rotated effective masses

- A_{1um}

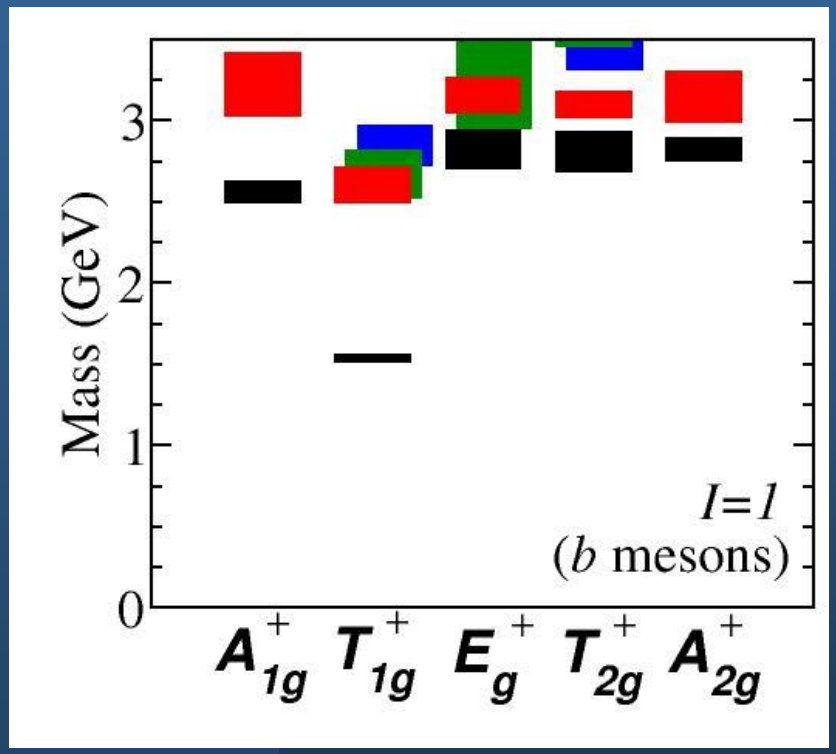


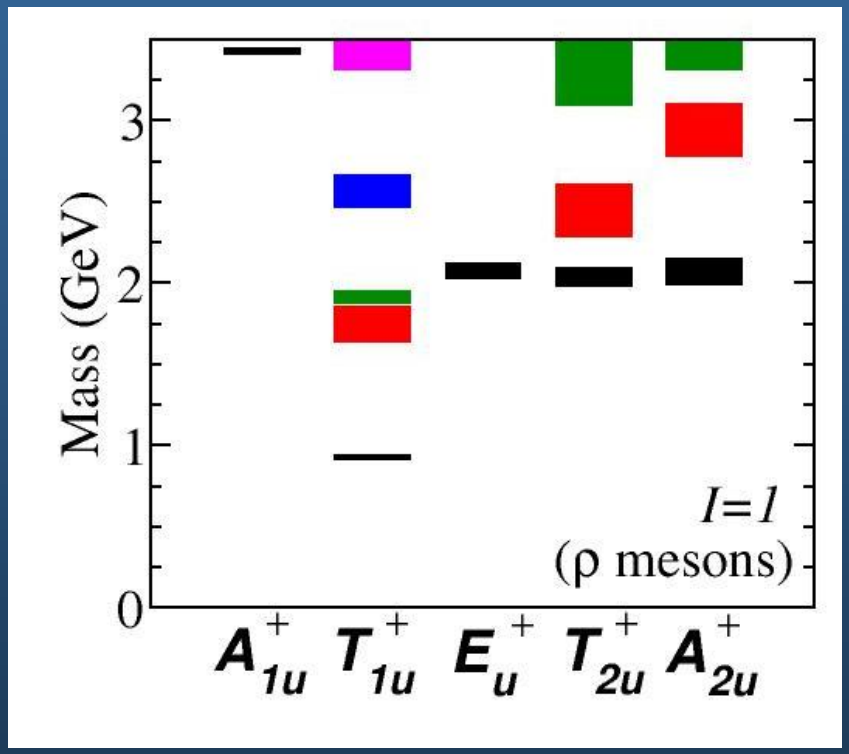
- T_{1um}



Isovector G-parity even mesons

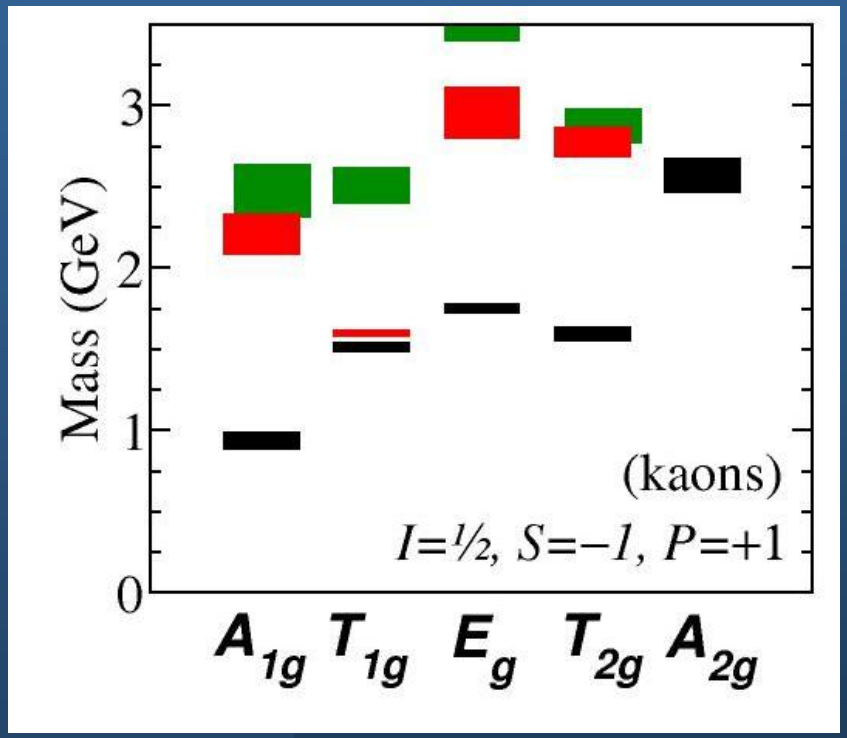
- N_f=2+1 on $16^3 \times 128$ lattice, m_π= 380 MeV (100 configs, 32 eigvecs)

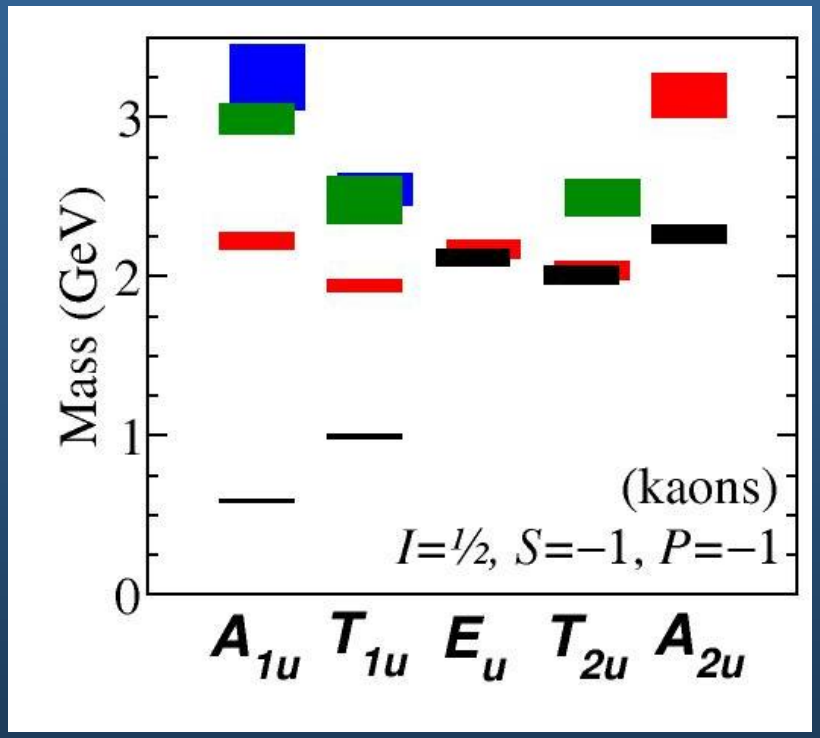




Kaons

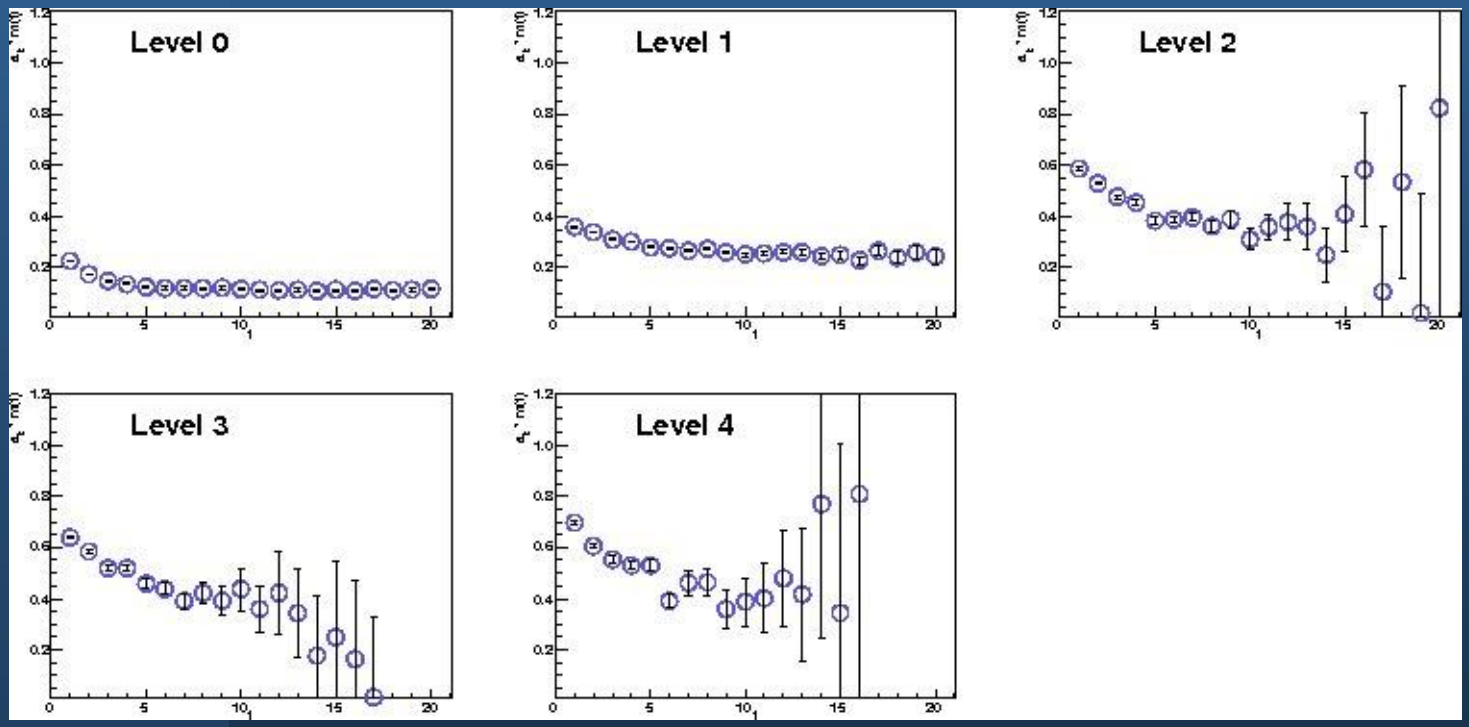
- N_f=2+1 on $16^3 \times 128$ lattice, m_π= 380 MeV (100 configs, 32 eigvecs)



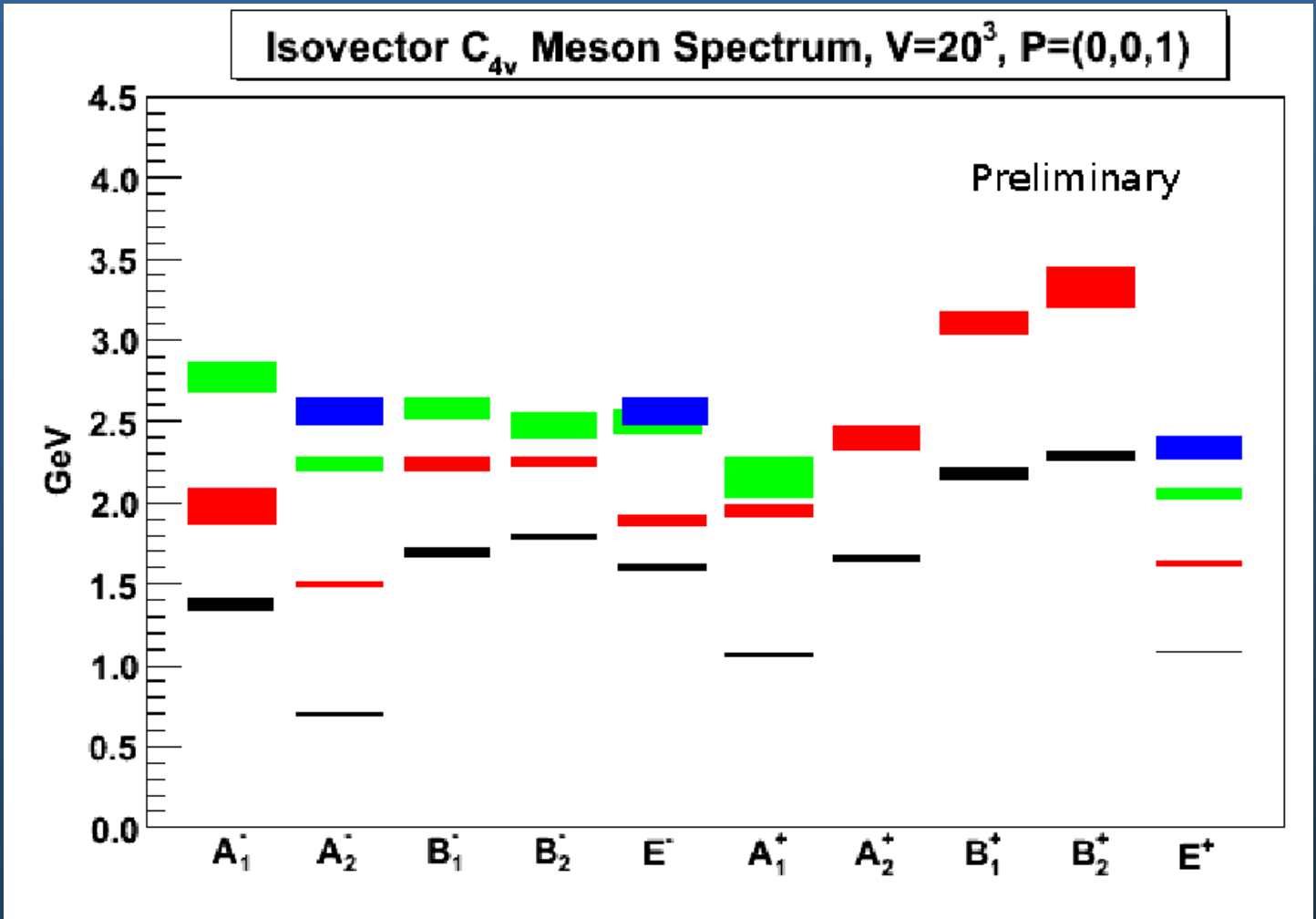


Moving π and *a* mesons

- first step towards including multi-hadron operators:
  - moving single hadrons
  - results below have one unit of on-axis momentum
  - projections onto space group irreps



Moving isovector mesons



Stochastic estimation of quark propagators

- new Laph quark smearing method allows exact computation of all-to-all quark propagators
- *but* number of Laplacian eigenvectors needed becomes prohibitively large on large lattices
  - 112 modes needed on 24^3 lattice, 256 modes on 32^3 lattice
- computational method is rather cumbersome, too
- need to resort to stochastic estimation
  - much more efficient
  - better source-sink factorization

Stochastic estimation

- quark propagator is just inverse of Dirac matrix M
- noise vectors η satisfying $E(\eta_i)=0$ and $E(\eta_i \eta_j^*)=\delta_{ij}$ are useful for stochastic estimates of inverse of a matrix M
- Z_4 noise is used $1, i, -1, -i$

- solve $M\, X^{(r)} = \eta^{(r)}$ for each of N_R noise vectors $\eta^{(r)}$ for Monte Carlo estimate of all elements of M^{-1}:

$$M_{ij}^{-1} \approx \frac{1}{N_R} \sum_{r=1}^{N_R} X_i^{(r)} \eta_j^{(r)*}$$

- variances in above estimates usually unacceptably large
- introduce variance reduction using source *dilution*

Source dilution for single matrix inverse

- dilution introduces a complete set of projections:

$$P^{(a)}P^{(b)} = \delta^{ab}P^{(a)}, \qquad \sum_a P^{(a)} = 1, \qquad P^{(a)\dagger} = P^{(a)}$$

- define

$$\eta_k^{[a]} = P_{kk'}^{(a)}\eta_{k'}, \qquad \eta_j^{[a]*} = \eta_{j'}^{*}P_{j'j}^{(a)}, \qquad X_k^{[a]} = M_{kj}^{-1}\eta_j^{[a]}$$

- Monte Carlo estimate is now

$$M_{ij}^{-1} \approx \frac{1}{N_R}\sum_{r=1}^{N_R}\sum_a X_i^{(r)[a]}\eta_j^{(r)[a]*}$$

- dramatically reduced variance

Earlier schemes

- Introduce Z_N noise in color, spin, space, time

$$\eta_{a\alpha}\ \ \vec{x},t$$

- Time dilution (particularly effective)

$$P^{(B)}_{a\alpha;b\beta}\ \ \vec{x},t;\vec{y},t' \ = \delta_{ab}\delta_{\alpha\beta}\delta\ \ \vec{x},\vec{y}\ \ \delta_{Bt}\delta_{Bt'}, \qquad B=0,1,\ldots,N_t-1$$

- Spin dilution

$$P^{(B)}_{a\alpha;b\beta}\ \ \vec{x},t;\vec{y},t' \ = \delta_{ab}\delta_{B\alpha}\delta_{B\beta}\delta\ \ \vec{x},\vec{y}\ \ \delta_{tt'}, \qquad B=0,1,2,3$$

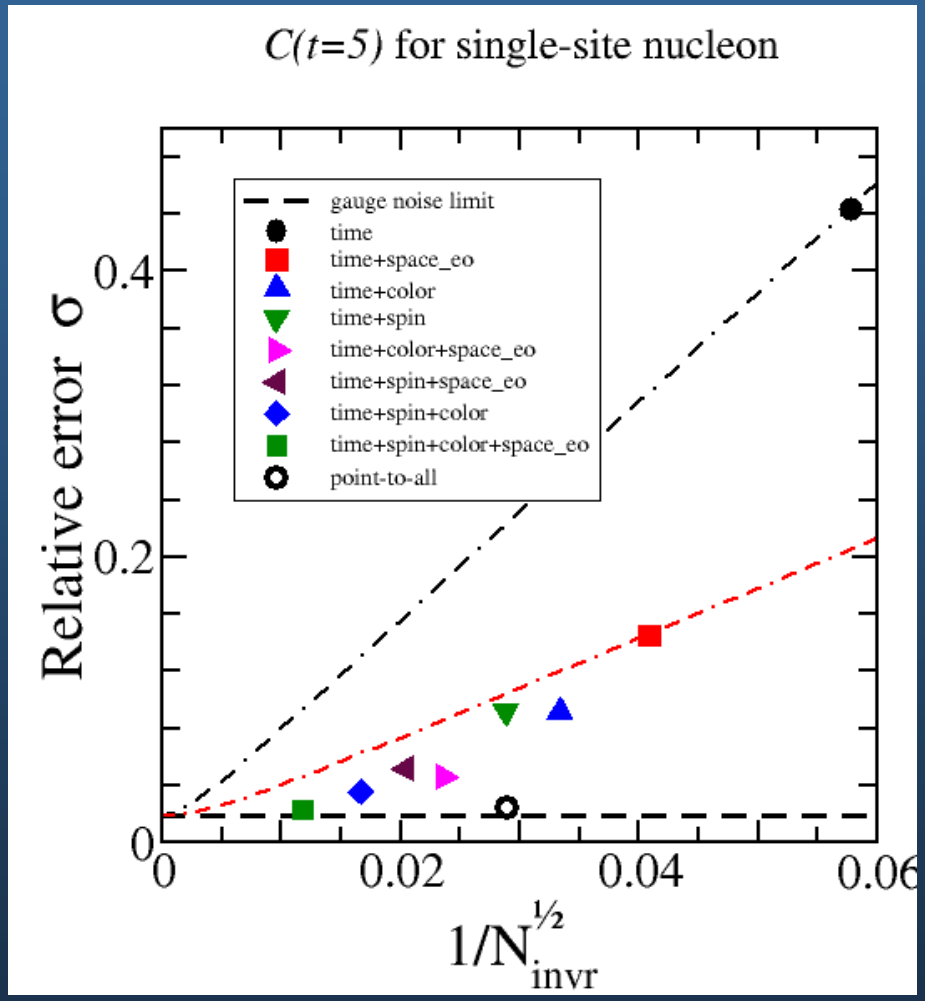
- Color dilution

$$P^{(B)}_{a\alpha;b\beta}\ \ \vec{x},t;\vec{y},t' \ = \delta_{Ba}\delta_{Bb}\delta_{\alpha\beta}\delta\ \ \vec{x},\vec{y}\ \ \delta_{tt'}, \qquad B=0,1,2$$

- Spatial dilutions?
  - even-odd

Dilution tests (old method)

- 100 quenched configs, $12^3 \times 48$ anisotropic Wilson lattice



New stochastic Laph method

- Introduce Z_N noise *in Laph subspace*

$$\rho_{\alpha k}(t) \quad t = \text{time}, \alpha = \text{spin}, k = \text{eigenvector number}$$

- Four dilution schemes:

$$P_{ij}^{(a)} = \delta_{ij}, \qquad a = 0 \qquad \text{(none)}$$

$$P_{ij}^{(a)} = \delta_{ij}\delta_{ai}, \qquad a = 0,1,\ldots,N-1 \quad \text{(full)}$$

$$P_{ij}^{(a)} = \delta_{ij}\delta_{a,\,Ki/N}, \qquad a = 0,1,\ldots,K-1 \quad \text{(interlace-}K\text{)}$$

$$P_{ij}^{(a)} = \delta_{ij}\delta_{a,\,i \bmod K}, \qquad a = 0,1,\ldots,K-1 \quad \text{(block-}K\text{)}$$

- Apply dilutions to
  - Time indices (full for <u>fixed</u> src, interlace-16 for <u>relative</u> src)
  - Spin indices (full)
  - Laplacian eigenvector indices (interlace-8)

Dilution projectors

- Full dilution

$$P_1 = \begin{bmatrix} 1 & & & & & \\ & 0 & & & & \\ & & 0 & & & \\ & & & 0 & & \\ & & & & 0 & \\ & & & & & 0 \end{bmatrix} \quad P_2 = \begin{bmatrix} 0 & & & & & \\ & 1 & & & & \\ & & 0 & & & \\ & & & 0 & & \\ & & & & 0 & \\ & & & & & 0 \end{bmatrix} \quad P_3 = \begin{bmatrix} 0 & & & & & \\ & 0 & & & & \\ & & 1 & & & \\ & & & 0 & & \\ & & & & 0 & \\ & & & & & 0 \end{bmatrix} \quad P_4 = \begin{bmatrix} 0 & & & & & \\ & 0 & & & & \\ & & 0 & & & \\ & & & 1 & & \\ & & & & 0 & \\ & & & & & 0 \end{bmatrix} \quad P_5 = \begin{bmatrix} 0 & & & & & \\ & 0 & & & & \\ & & 0 & & & \\ & & & 0 & & \\ & & & & 1 & \\ & & & & & 0 \end{bmatrix} \quad P_6 = \begin{bmatrix} 0 & & & & & \\ & 0 & & & & \\ & & 0 & & & \\ & & & 0 & & \\ & & & & 0 & \\ & & & & & 1 \end{bmatrix}$$

- Block-3 dilution

$$P_1 = \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 0 & & & \\ & & & 0 & & \\ & & & & 0 & \\ & & & & & 0 \end{bmatrix} \quad P_2 = \begin{bmatrix} 0 & & & & & \\ & 0 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 0 & \\ & & & & & 0 \end{bmatrix} \quad P_3 = \begin{bmatrix} 0 & & & & & \\ & 0 & & & & \\ & & 0 & & & \\ & & & 0 & & \\ & & & & 1 & \\ & & & & & 1 \end{bmatrix}$$

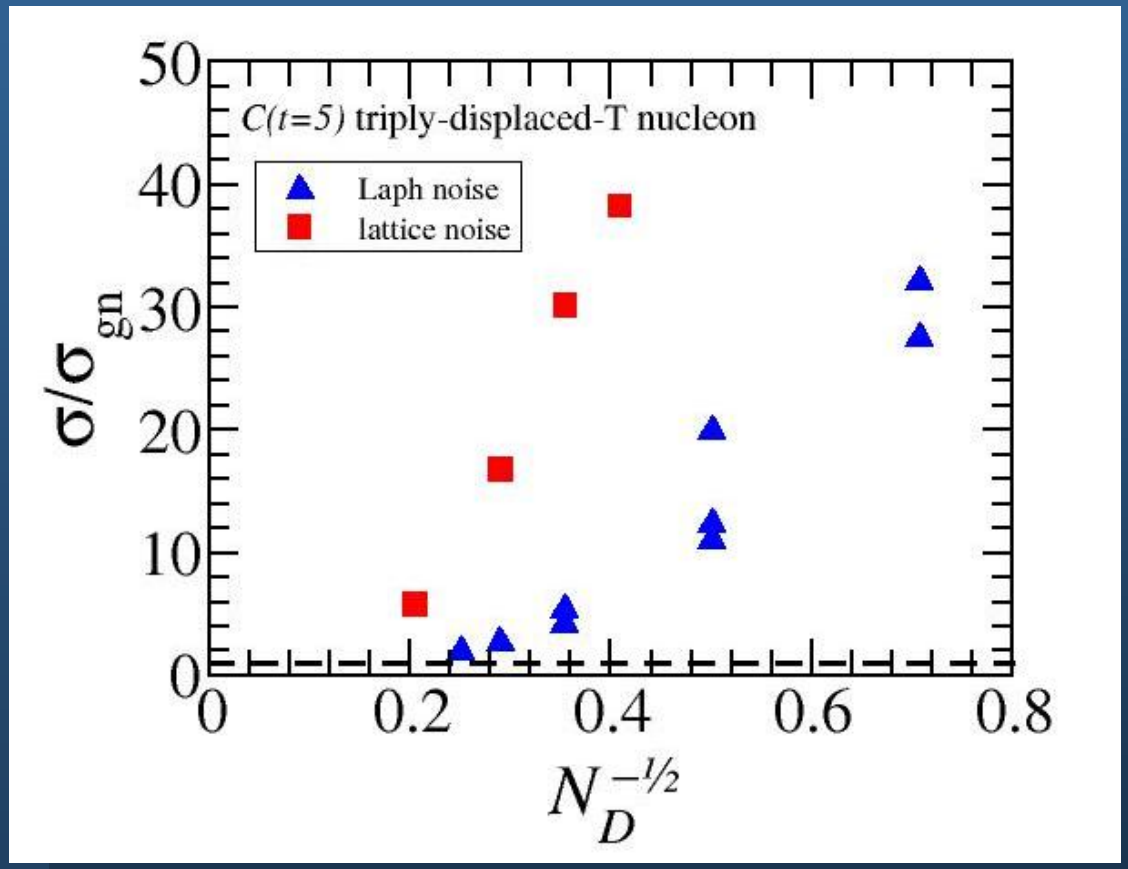
- Interlace-3 dilution

$$P_1 = \begin{bmatrix} 1 & & & & & \\ & 0 & & & & \\ & & 0 & & & \\ & & & 1 & & \\ & & & & 0 & \\ & & & & & 0 \end{bmatrix} \quad P_2 = \begin{bmatrix} 0 & & & & & \\ & 1 & & & & \\ & & 0 & & & \\ & & & 0 & & \\ & & & & 1 & \\ & & & & & 0 \end{bmatrix} \quad P_3 = \begin{bmatrix} 0 & & & & & \\ & 0 & & & & \\ & & 1 & & & \\ & & & 0 & & \\ & & & & 0 & \\ & & & & & 1 \end{bmatrix}$$

  - useful for evaluating sources on all time slices

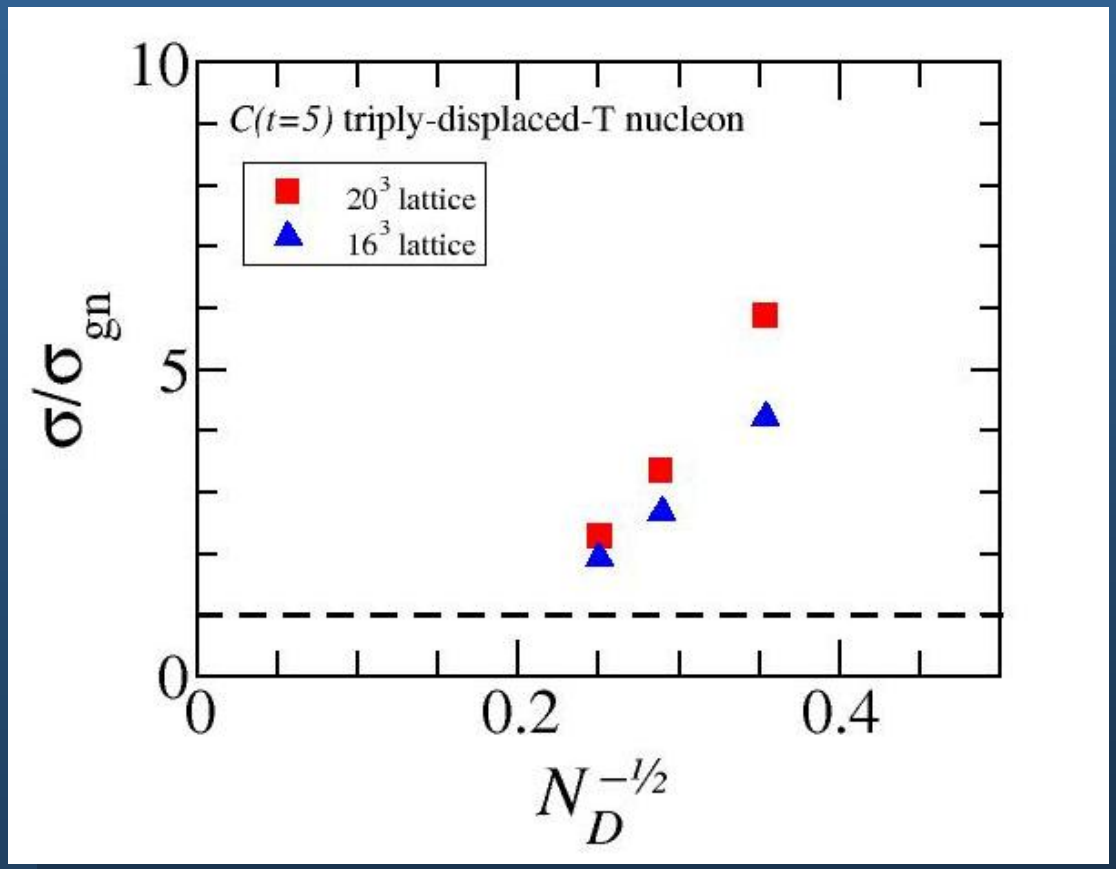
Old stochastic versus new stochastic

- new method has dramatically decreased variance
- test using a triply-displaced-T nucleon operator



Mild volume dependence

- 16^3 lattice versus 20^3 lattice, both old and new stochastic methods
- test using triply-displaced-T nucleon operator



Source-sink factorization

- baryon correlator has form

$$C_{l\bar{l}} = c_{ijk}^{(l)} c_{\bar{i}\bar{j}\bar{k}}^{(\bar{l})*} Q_{i\bar{i}}^{A} Q_{j\bar{j}}^{B} Q_{k\bar{k}}^{C}$$

- stochastic estimates with dilution

$$C_{l\bar{l}} \approx \frac{1}{N_R} \sum_r \sum_{d_A d_B d_C} c_{ijk}^{(l)} c_{\bar{i}\bar{j}\bar{k}}^{(\bar{l})*} \quad \varphi_i^{(Ar)[d_A]} \eta_{\bar{i}}^{(Ar)[d_A]*}$$

$$\times \; \varphi_j^{(Br)[d_B]} \eta_{\bar{j}}^{(Br)[d_B]*} \quad \varphi_k^{(Cr)[d_C]} \eta_{\bar{k}}^{(Cr)[d_C]*}$$

- define

$$\Gamma_l^{(r)[d_A d_B d_C]} = c_{ijk}^{(l)} \, \varphi_i^{(Ar)[d_A]} \varphi_j^{(Br)[d_B]} \varphi_k^{(Cr)[d_C]}$$

$$\Omega_l^{(r)[d_A d_B d_C]} = c_{ijk}^{(l)} \, \eta_i^{(Ar)[d_A]} \eta_j^{(Br)[d_B]} \eta_k^{(Cr)[d_C]}$$

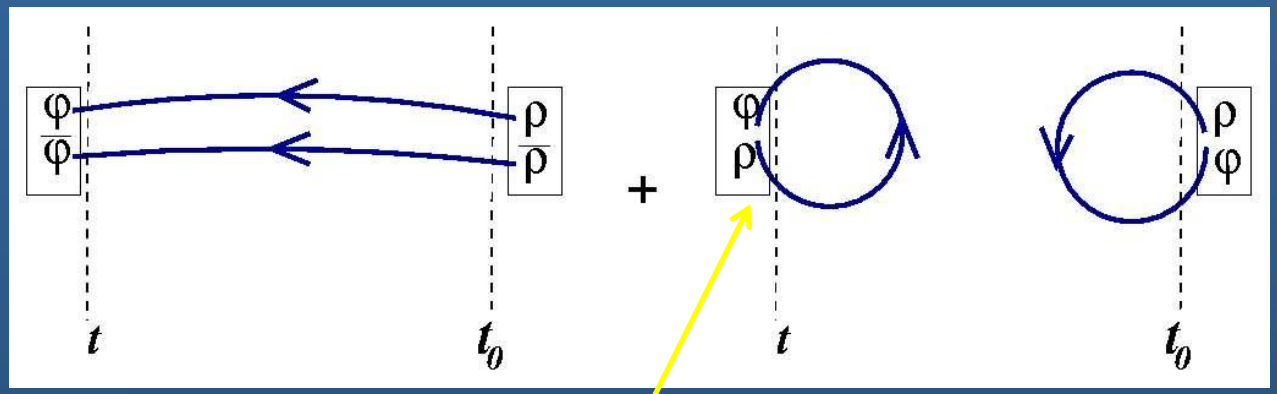
- correlator becomes dot product of source vector with sink vector

$$C_{l\bar{l}} \approx \frac{1}{N_R} \sum_r \sum_{d_A d_B d_C} \Gamma_l^{(r)[d_A d_B d_C]} \, \Omega_{\bar{l}}^{(r)[d_A d_B d_C]*}$$

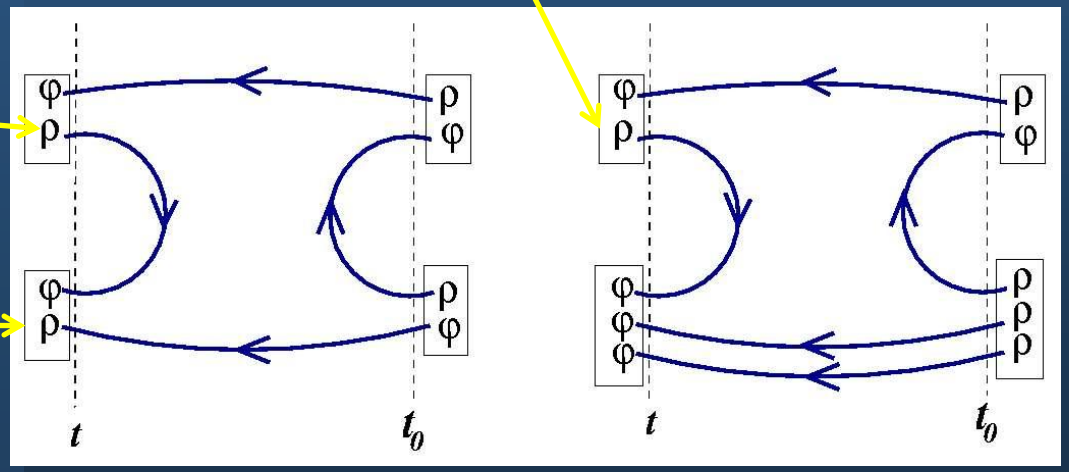
- store *ABC* permutations to handle Wick orderings

Same-time quark lines

- Last step to finite-box spectra: same time *t*-to-*t* quark lines

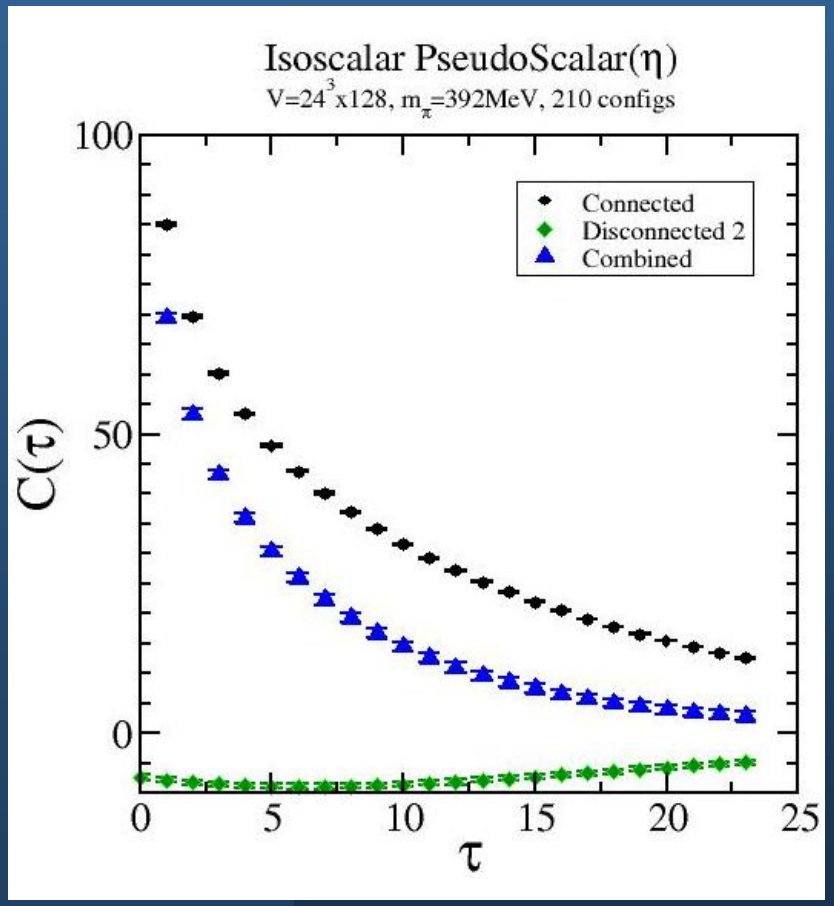


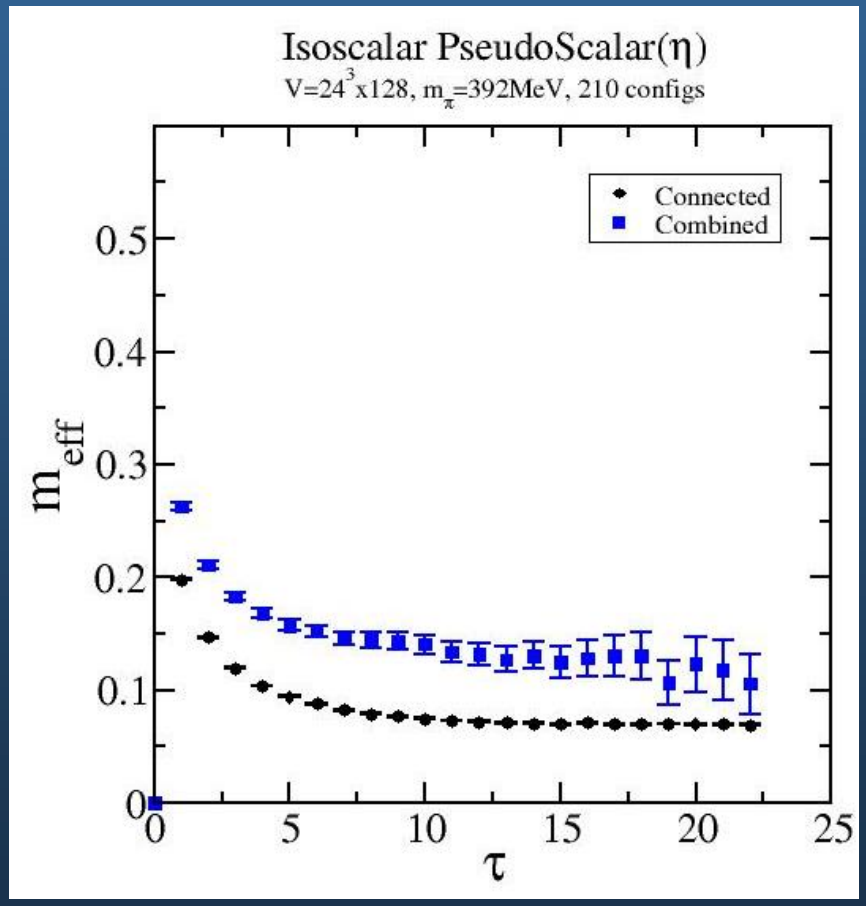
interlace-16 time dilution (relative sources)



First results with t-to-t diagrams

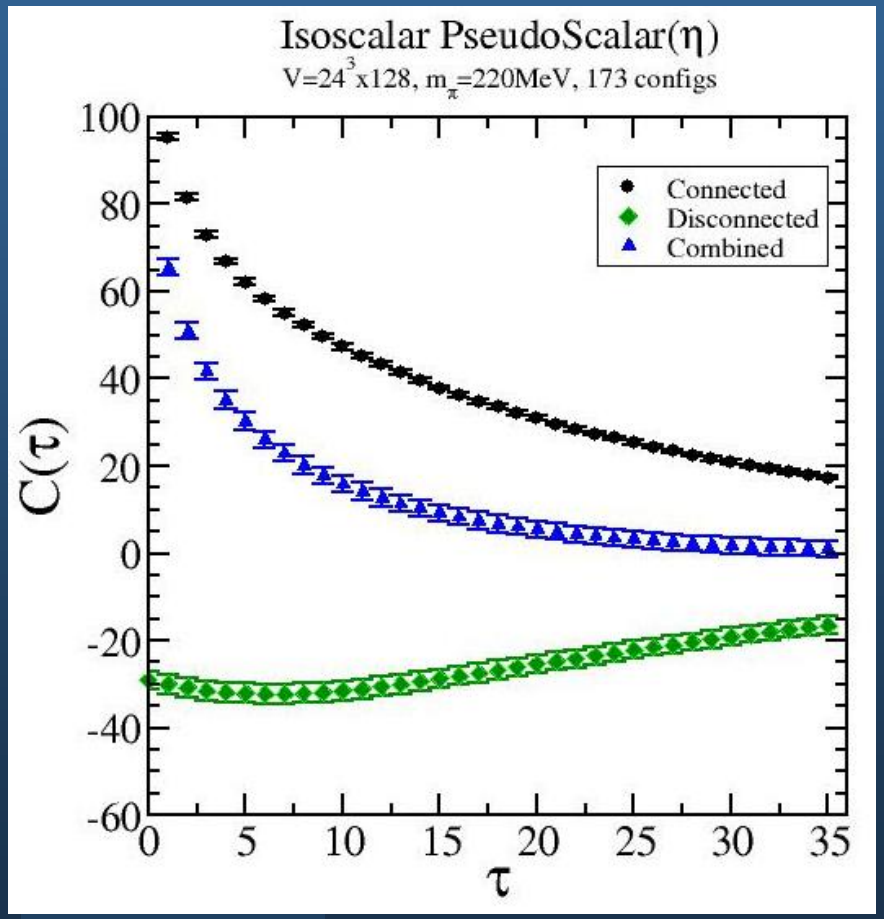
- 24^3x128 lattice: dilution schemes (TF,SF,LI8) (TI16,SF,LI8)
- 112 eigenvectors, local operators only

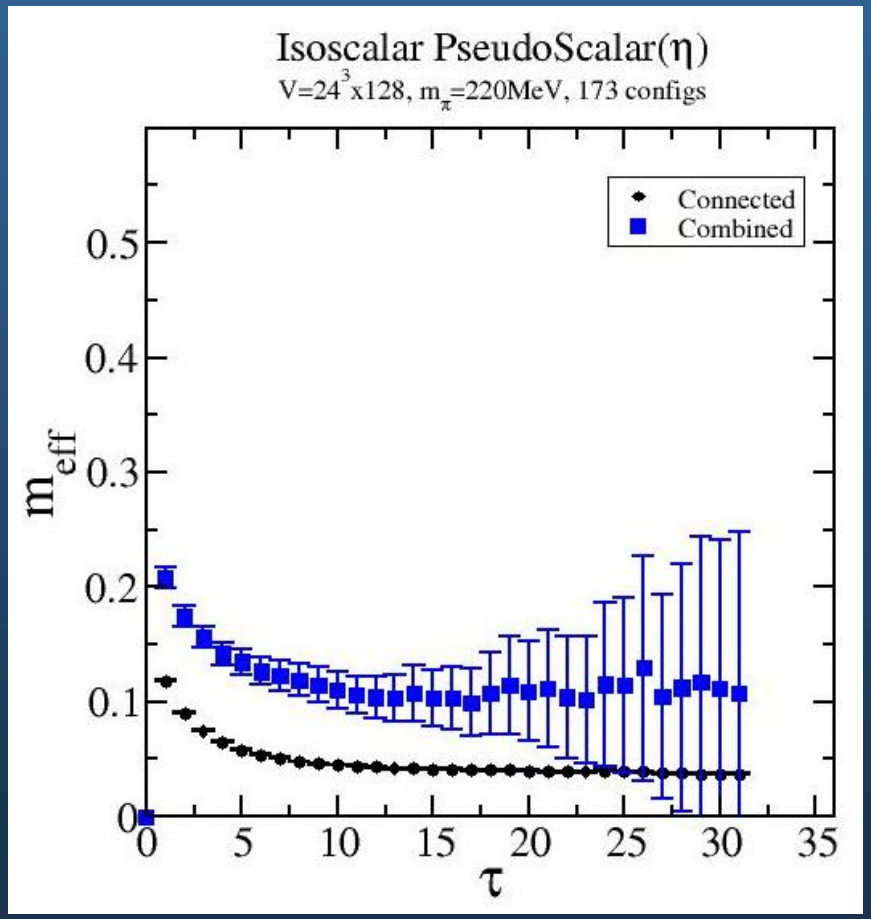




Results at lighter pion mass

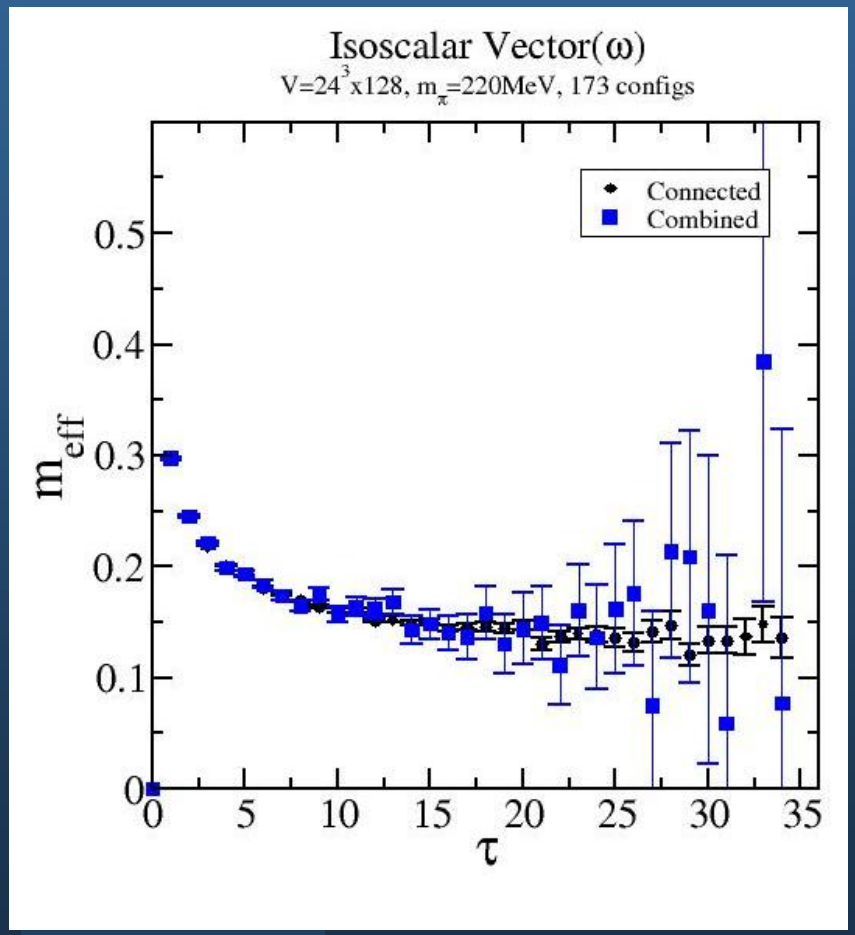
- 24^3x128 lattice: dilution schemes (TF,SF,LI8) (TI16,SF,LI8)
- 112 eigenvectors, local operators only

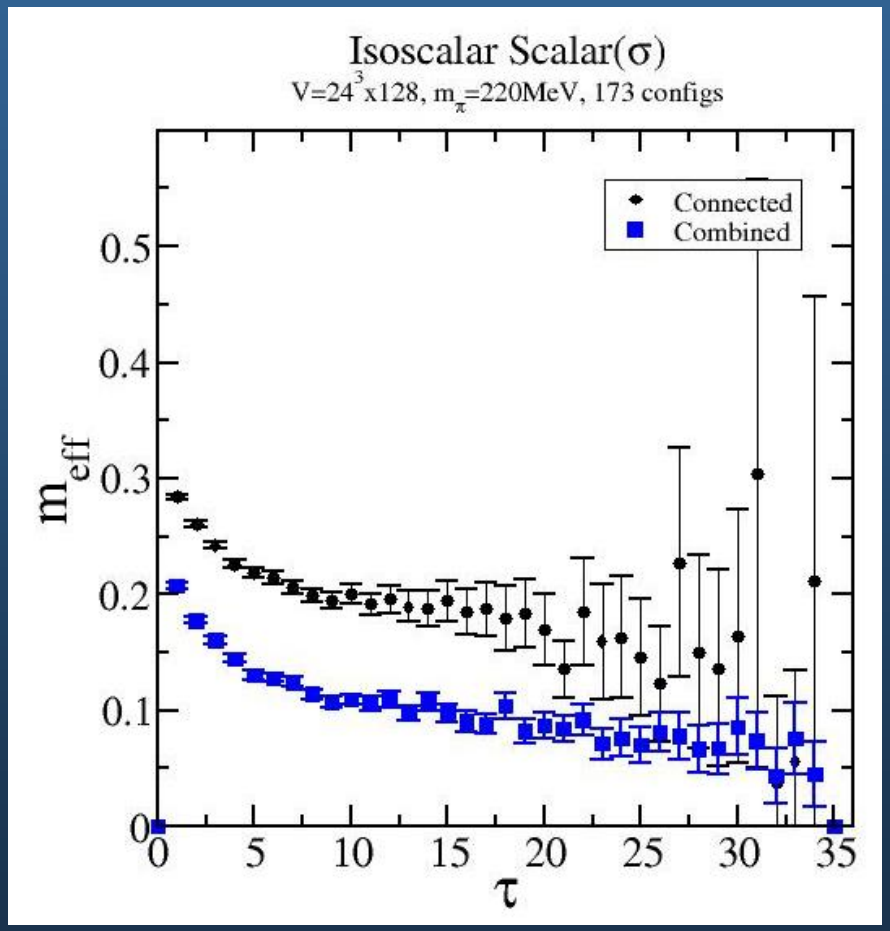




Results at lighter pion mass (cont'd)

- 24^3x128 lattice: dilution schemes (TF,SF,LI8) (TI16,SF,LI8)
- 112 eigenvectors, local operators only





Multi-hadron diagrams

- Two-pion energy levels currently in progress
  - all needed quark propagators evaluated
  - moving pion sources/sink in progress
  - formation of correlators is final step
- Will then proceed to other moving mesons and baryons

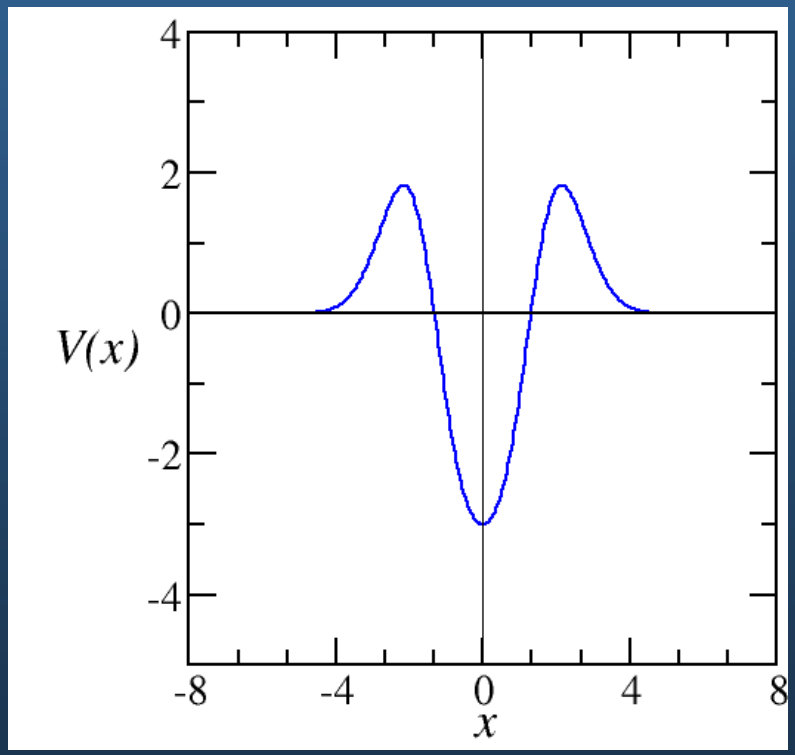
Configuration generation

- significant time on USQCD (DOE) and NSF computing resources
- anisotropic clover fermion action (with stout links) and anisotropic improved gauge action
  - tunings of couplings, aspect ratio, lattice spacing done
- anisotropic Wilson configurations generated during clover tuning
- goal:
  - three lattice spacings: a = 0.125 fm, 0.10 fm, 0.08 fm
  - three volumes: $V = (3.2\text{ fm})^4, (4.0\text{ fm})^4, (5.0\text{ fm})^4$
  - 2+1 flavors, $m_\pi \sim$ 390 MeV, 220 MeV, 180 MeV
- USQCD Chroma software suite
- Progress
  - 24^3x128 lattice for $m_\pi \sim$ 390 MeV, 220 MeV done
  - 32^3x256 lattice for $m_\pi \sim$ 390 MeV, 220 MeV in progress

Resonances in a box: an example

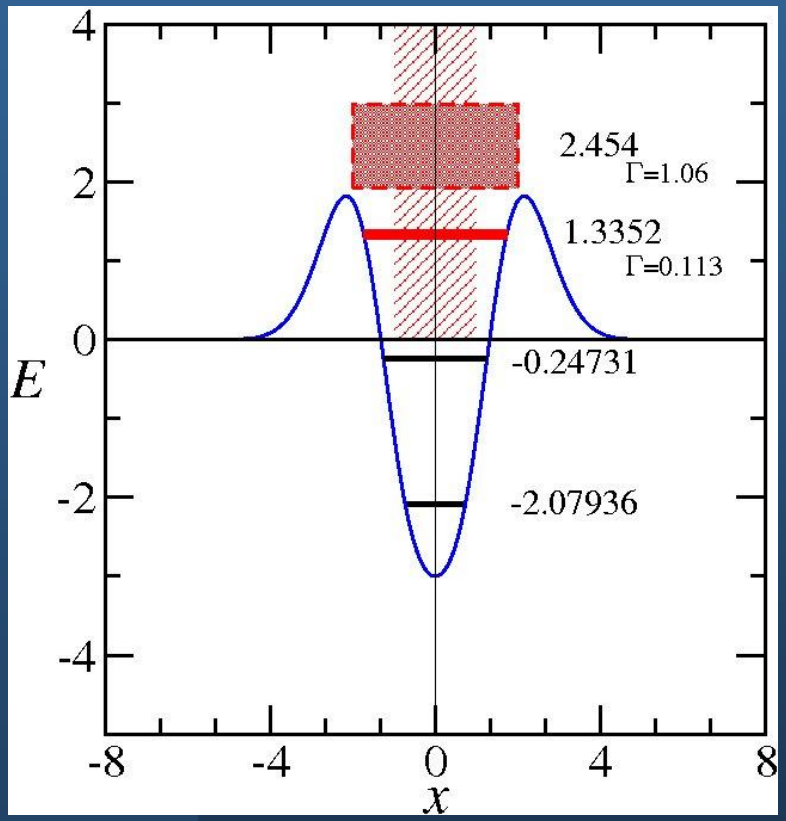
- Consider simple 1D quantum mechanics example
- Hamiltonian

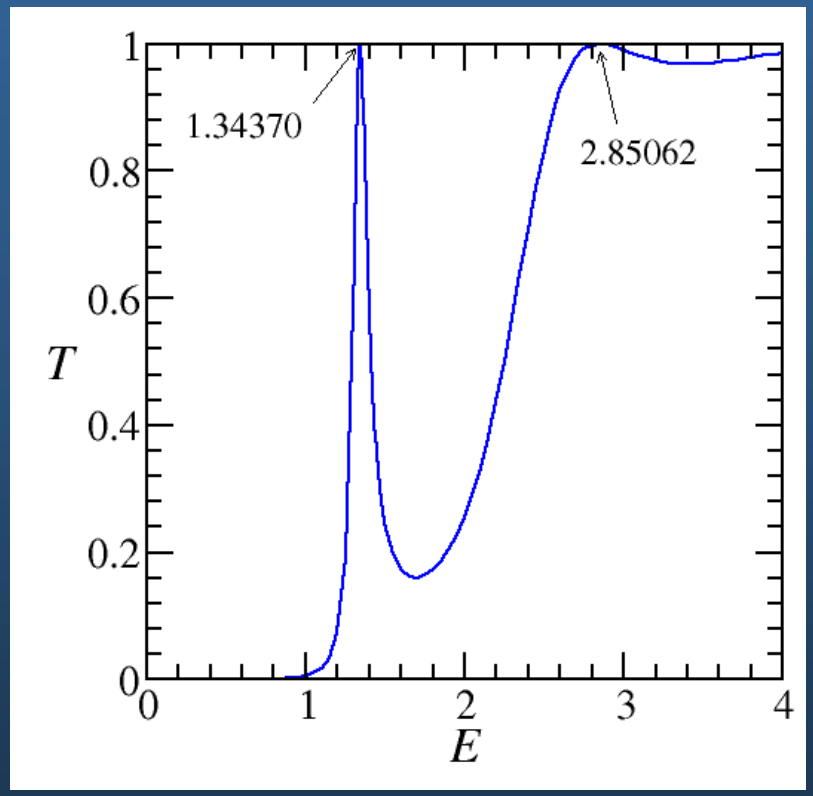
$$H = \frac{1}{2} p^2 + V(x) \qquad V(x) = (x^4 - 3)\, e^{-x^2/2}$$



1D example spectrum

- Spectrum has two bound states, two resonances for $E<4$

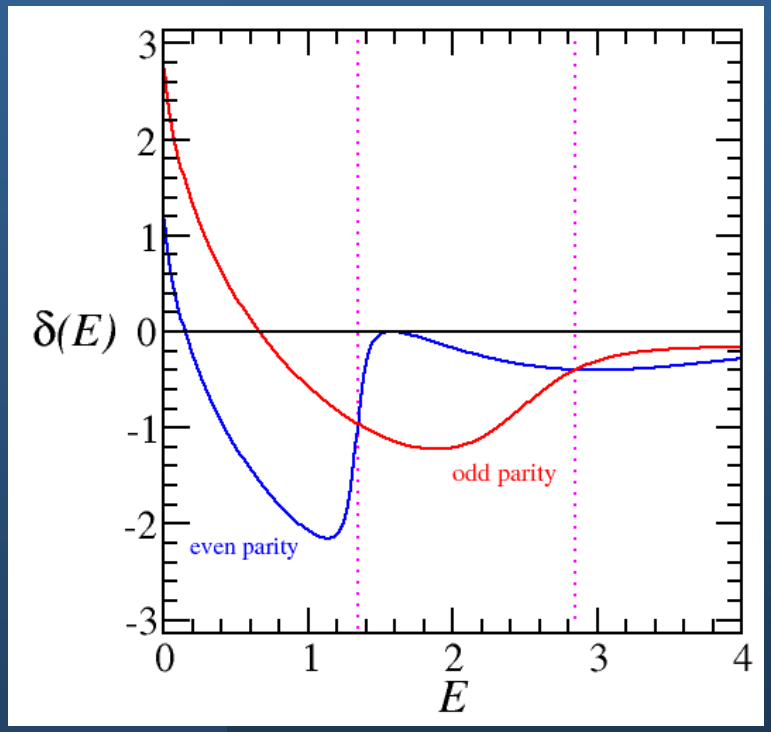


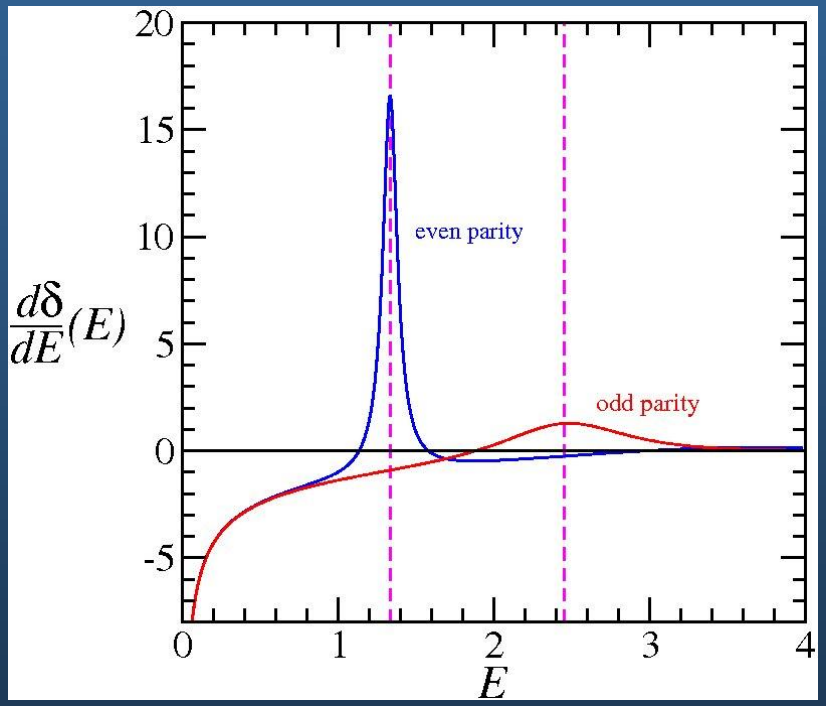


transmission coefficient

Scattering phase shifts

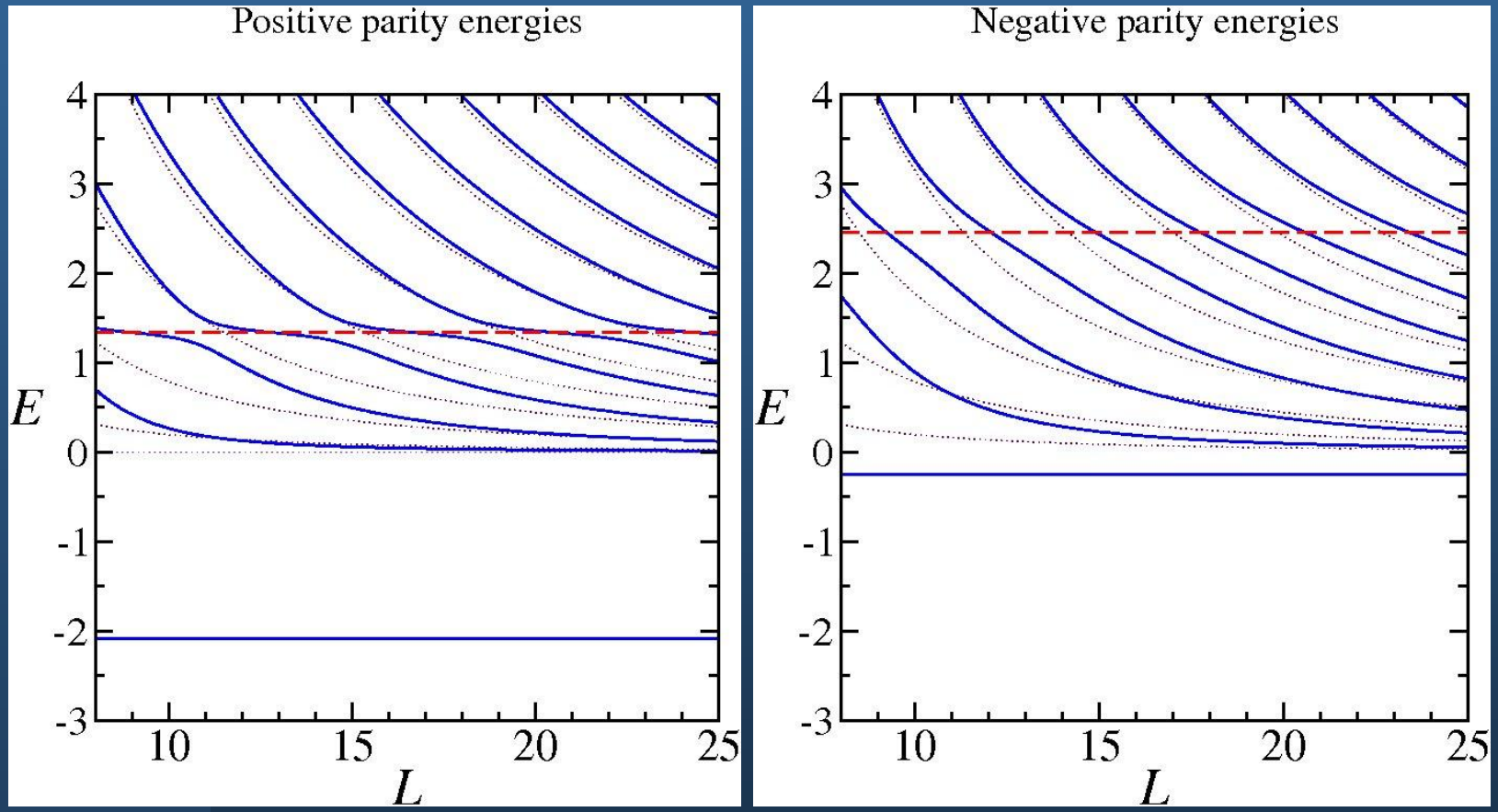
- define even- and odd-parity phase shifts $\delta_\pm$
  - phase between transmitted and incident wave





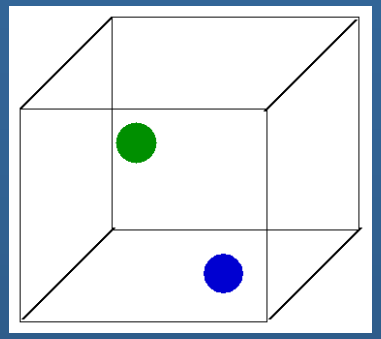
Spectrum in box (periodic b.c.)

- spectrum is discrete in box (momentum quantized)
- narrow resonance is avoided level crossing, broad resonance?



Dotted curves are V=0 spectrum

Unstable particles (resonances)



- our computations done in a periodic box
  - momenta quantized
  - discrete energy spectrum of stationary states → single hadron, 2 hadron, …
- how to extract resonance info from box info?
  - phase-shift methods
  - construct effective theory of hadrons?

Summary

- goal: compute energies of low-lying stationary states of QCD in a box using Monte Carlo method on space-time lattice
- single hadron operator construction
- need for multi-hadron operators as pion gets lighter
- slice-to-slice quark propagators
- new stochastic Laph method

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