Dynamics behind the Quark Mass Hierarchy and Electroweak Symmetry Breaking

M. Hashimoto and V.M., PRD 80, 013004 (2009); PRD 81, 055014 (2010)
The masses of quarks are

\[ m_t = 171.2^{+2.1}_{-2.1} \text{ GeV}, \quad m_b = 4.20^{+0.17}_{-0.07} \text{ GeV}, \]
\[ m_c = 1.27^{+0.07}_{-0.11} \text{ GeV}, \quad m_s = 104^{+26}_{-34} \text{ MeV}, \]
\[ m_u = 1.5 - 3.3 \text{ MeV}, \quad m_d = 3.5 - 6.0 \text{ MeV}. \]

The quark spectrum is characterized by the following striking features: (1) There is a large hierarchy between quark masses from different families,

\[ m_u / m_t \sim 10^{-5}, \quad m_u / m_c \sim 10^{-3}, \quad m_c / m_t \sim 10^{-2}, \]
\[ m_d / m_b \sim 10^{-3}, \quad m_d / m_s \sim 10^{-2}, \quad m_s / m_b \sim 10^{-1}. \]

(2) The isospin violation is also hierarchical: It is very strong in the third family, strong (although essentially weaker) in the second family, and mild in the first one:

\[ m_t / m_b \sim 40.8, \quad m_c / m_s \sim 11.5, \quad m_u / m_d = 0.35 - 0.60. \]
Our basic assumption is the separation of the dynamics triggering the strong isospin violation in the third and second families from that responsible for the generation of the $W$ and $Z$ masses, i.e., electroweak symmetry breaking (EWSB). The latter could be provided by one of the following known mechanisms:

(a) An elementary Higgs field (or fields).
(b) A modern version of the technicolor (TC) scenario.
(c) Higgless dynamics.
(d) At last, it could be a dynamical Higgs mechanism with a Higgs doublet (or doublets) composed of $t'$ and $b'$ quarks of the fourth family.
We assume that the dynamics primarily responsible for the EWSB leads to the mass spectrum of quarks with no (or weak) isospin violation. Moreover, we assume that the values of these masses are of the order of the observed masses of the down-type quarks. In the case of an elementary Higgs field (or fields), they are provided by the conventional yukawa interactions. In the case of the dynamical Higgs mechanism, in order to generate these masses, one should use flavor-changing-neutral (FCN) interactions: the extended technicolor (ETC) in the case of the TC scenario, and the horizontal interactions between the 4th family and the first three ones in the case of the scenario with the fourth family.
FCN interactions of the up- and down-quark sectors. Here \( u^{(1,2,3)} = u, c, t \) and \( d^{(1,2,3)} = d, s, b \), respectively. \( \Lambda^{(i4)} \) are masses of exchange vector particles.

\[
 m_0^{(i)} \simeq \frac{C_2 g_{t' u^{(i)}}^2}{4\pi^2} \frac{(\Lambda^{(4)})^2}{(\Lambda^{(i4)})^2} m_{t'} \simeq \frac{C_2 g_{b' d^{(i)}}^2}{4\pi^2} \frac{(\Lambda^{(4)})^2}{(\Lambda^{(i4)})^2} m_{b'}, 
\]

\[
 \left(\frac{\Lambda^{(14)}}{\Lambda^{(24)}}\right)^2 \gg \left(\frac{\Lambda^{(34)}}{\Lambda^{(4)}}\right)^2
\]

\[
 \Rightarrow m_0^{(1)} \ll m_0^{(2)} \ll m_0^{(3)} \ll m_{t'} \sim m_{b'}. 
\]
The second (central) stage is introducing the horizontal interactions for the quarks in the first three families (this stage is essentially the same for all EWSB mechanisms mentioned above.) First, we utilize strong (although subcritical) diagonal horizontal interactions for the top quark which lead to the observed ratio $\frac{m_t}{m_b} \simeq 40.8$. The second step is introducing the equal strengths horizontal FCN interactions between the $t$ and $c$ quarks and the $b$ and $s$ ones in order to get the observed ratio $m_c/m_s \simeq 11.5$ in the second family.
Isospin symmetric quark masses

Let us now describe this stage in the scenario of the dynamical EWSB with the fourth family. The masses of the 4th family quarks are constrained as

\[ m_{b'} > 325 \text{ GeV}, \quad m_{t'} > 311 \text{ GeV}. \]

At the composite scale \( \Lambda^{(4)} \), the 4th family quarks \( t' \) and \( b' \) condense and thereby they break the electroweak symmetry. By using the Pagels-Stokar formula, we can estimate the corresponding decay constants,

\[ v_{t'}^2(b') = \frac{N}{8\pi^2} m_{t'}^2(b') \ln \left( 1 + \frac{(\Lambda^{(4)})^2}{m_{t'}^2(b')} \right), \]

with \( v_{t'}^2 + v_{b'}^2 = v^2 \), where \( N = 3 \) and \( v = 246 \text{ GeV} \). The constraint of the \( T \)-parameter suggests that \( m_{t'} \simeq m_{b'} \) is favorable and thereby \( v_{t'} \simeq v_{b'} \) follows.
To obtain almost correct masses for the down-type quarks,

\[ m_0^{(3)} \sim 1 \text{ GeV}, \quad m_0^{(2)} \sim 100 \text{ MeV}, \quad m_0^{(1)} \sim 1 \text{ MeV}, \]

we introduce the following horizontal FCN interactions

\[
\begin{align*}
\Lambda^{(i4)} & \quad t' \quad u
\end{align*}
\]

\[
\begin{align*}
\Lambda^{(i4)} & \quad b' \quad d
\end{align*}
\]

In order to obtain the hierarchical masses \( m_0^{(1,2,3)} \), we assume

\[
(\Lambda^{(14)})^2 \gg (\Lambda^{(24)})^2 \gg (\Lambda^{(34)})^2 \gg (\Lambda^{(4)})^2.
\]

We may expect \( C_2 g_{t'u(i)}^2 \simeq C_2 g_{b'd(i)}^2 \sim \mathcal{O}(1) \). Then, at this stage, the mass spectrum of quarks is isospin symmetric. The running masses are essentially equal to the constants \( m_0^{(i)} \) up to the scale of \( \Lambda^{(i4)} (i = 1, 2, 3) \). Above \( \Lambda^{(i4)} \), they rapidly, as \( 1/q^2 \), decrease (\( q \) is the momentum of the running masses).
Horizontal interactions as a source of isospin violation in quark mass spectrum

At energy scales less than the mass of a horizontal vector boson $\Lambda^{(3)} \sim \Lambda^{(34)}$, the corresponding horizontal interactions can be presented by the four-fermion Nambu-Jona-Lasinio (NJL) ones. We apply strong (although subcritical) dynamics for the horizontal diagonal interactions for the $t$ quark. The isospin symmetric mass $m_0^{(3)}$, plays the role of a bare mass with respect to these interactions.
The solution of the Schwinger-Dyson equation for the $t$ quark propagator leads to the following mass $m_t$

$$m_t \simeq \frac{1}{\Delta g_t}m_0^{(3)},$$

where $\Delta g_q$ denotes the difference of the critical coupling and the (normalized) dimensionless NJL one for a $q$ quark, so that

$$\Delta g_t \simeq \frac{m_0^{(3)}}{m_t} \sim 6 \times 10^{-3},$$

where we used $m_t = 171.2$ GeV and $m_0^{(3)} = 1$ GeV. For the bottom quark, it should be $\Delta g_b \sim \mathcal{O}(1)$.

$$\Delta g_b - \Delta g_t \simeq \frac{m_0^{(3)}}{m_b}.$$
Let us now turn to the generation of the realistic masses for the second family.

\[
\begin{align*}
\Lambda^{(23)} \\
\begin{array}{cccc}
C_L & t_L & t_R & C_R \\
| & | & | & |
\end{array} & \begin{array}{cccc}
C_L & t_L & t_R & C_R \\
| & | & | & |
\end{array} \\
\Lambda^{(23)} \\
\begin{array}{cccc}
s_L & b_L & b_R & s_R \\
| & | & | & |
\end{array} & \begin{array}{cccc}
s_L & b_L & b_R & s_R \\
| & | & | & |
\end{array}
\end{align*}
\]

\[
m_c = m_0^{(2)} + \eta^{(23)}_t m_t, \quad m_s = m_0^{(2)} + \eta^{(23)}_b m_b,
\]

where \(m_0^{(2)} \sim 100\) MeV is the isospin symmetric mass for the second family, and \(\eta^{(23)}_{t,b}\) are

\[
\eta^{(23)}_t \equiv \frac{C_2 g_{tc(bs)}^2}{4\pi^2} \frac{(\Lambda^{(3)})^2}{(\Lambda^{(23)})^2}
\]

for \(\Lambda^{(23)} \gg \Lambda^{(3)}\).
Taking $m_0^{(2)} = 100$ MeV and $\eta_t^{(23)} = \eta_b^{(23)} = 1/100$, we get

\begin{align*}
m_c &= 100 \text{ MeV} + m_t/100 \sim 1 \text{ GeV}, \\
m_s &= 100 \text{ MeV} + m_b/100 \sim 140 \text{ MeV}.
\end{align*}

with $m_b/m_t \approx 1/40$. Let us emphasize that the presence of the isospin symmetric mass $m_0^{(2)} \sim 100$ MeV $\sim m_s$ is crucial here: with $m_0^{(2)} \ll 100$ MeV, the ratio $m_s/m_c$ would be close to $m_b/m_t$. As to the horizontal FCN gauge bosons which couple to the quarks of the 1st and 2nd families, we assume that they are very heavy,

\begin{align*}
c - u - \Lambda^{(12)}, \\
s - d - \Lambda^{(12)},
\end{align*}

with $\Lambda^{(12)} \sim \mathcal{O}(1000 \text{ TeV})$. As a result, their contributions to the masses of the $u$ and $d$ quarks are very small.
Mixing terms and CKM structure

\[ -\mathcal{L}_Y = \sum_{i,j} \bar{\psi}_L^{(i)} Y_D^{ij} d_R^{(j)} \tilde{\Phi}_b' + \sum_{i,j} \bar{\psi}_L^{(i)} Y_U^{ij} u_R^{(j)} \tilde{\Phi}_{t'} + y_t \bar{\psi}_L^{(3)} t_R \Phi_t, \]

\[ \mathcal{Y}_D \equiv \frac{\sqrt{2}}{v_{b'}} M_D, \quad \mathcal{Y}_U \equiv \frac{\sqrt{2}}{v_{t'}} M_U. \]

\[ M_D = \begin{pmatrix}
  m_d & \xi_1 m_d & \xi_1 m_d & \xi_1 m_d \\
  \xi_1 m_d & m_s & \xi_2 m_s & \xi_2 m_s \\
  \xi_1 m_d & \xi_2 m_s & m_b & \xi_2 m_s \\
  \xi_1 m_d & \xi_2 m_s & \xi_2 m_s & m_{b'}
\end{pmatrix} \]

Let us consider only two parameters:

- \( \xi_1 \) is fixed by \( |V_{us}| = 0.23 \);
- \( \xi_2 \) is fixed by \( |V_{cb}| = 0.04 \).

CKM matrix elements

\[ |V_{ud}| \sim |V_{cs}| \sim 1 - \frac{\xi_1^2}{2} \left( \frac{m_d}{m_s} \right)^2, \quad |V_{us}| \sim |V_{cd}| \sim \xi_1 \frac{m_d}{m_s}, \quad V_{\text{CKM}}^{4 \times 4} = \begin{pmatrix}
  0.97 & 0.23 & -0.006 & 0.00009 \\
  -0.23 & 0.97 & -0.04 & -0.008 \\
  -0.003 & 0.04 & 1.0 & 0.02 \\
  -0.002 & 0.007 & -0.02 & 1.0
\end{pmatrix} \]

\[ |V_{ub}| \sim |V_{td}| \sim \xi_1 \frac{m_d}{m_b}, \quad |V_{cb}| \sim |V_{ts}| \sim \xi_2 \frac{m_s}{m_b}, \]

\[ |V_{tb}| \sim 1, \]

\[ |V_{t'd}| \sim \xi_1 \frac{m_d}{m_s} \cdot \frac{m_c}{m_{t'}} \sim 0.23 \times \xi_2 \frac{m_c}{m_{t'}} \sim O(10^{-3}), \]

\[ |V_{t's}| \sim |V_{t'b}| = \xi_2 \frac{m_c}{m_{t'}} \sim O(10^{-2}). \]
Since we consider the condensation both of the $t'$ and $b'$, there appear at least two composite Higgs doublets. For the 3rd family, we may estimate the mass of the top-Higgs doublet (resonance) $\phi_t$ via the NJL relation:

$$M_{\phi_t} \sim \Lambda^{(3)} \left( \frac{2\Delta g_t}{\ln \frac{1}{2\Delta g_t}} \right)^{1/2} \sim 0.05 \Lambda^{(3)},$$

where we used $\Delta g_t \sim 6 \times 10^{-3}$. For the bottom-Higgs resonance $\phi_b$, it should be $M_{\phi_b} \sim \Lambda^{(3)}$, i.e., it is very heavy and unstable. Note that the quark structures of the composites $\phi_t$ and $\phi_b$ are $\phi_t \sim (\Lambda^{(3)})^{-2} \bar{t}_R(t, b)_L$ and $\phi_b \sim (\Lambda^{(3)})^{-2} \bar{b}_R(b, -t)_L$, respectively.
Superheavy quarks and multi-Higgs doublets

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The Yukawa couplings have the Landau pole, so that the theory is effectively only up to the scale $\mathcal{O}(10 \text{ TeV})$.
The Nambu-Jona-Lasinio description is applicable at low energy.

The masses of $t'$, $b'$ and $t$ are $\mathcal{O}(\nu = 246 \text{ GeV})$.

$$m_{t'}, m_{b'} \gtrsim 300 \text{ GeV} \quad m_t = 175 \text{ GeV}$$

\[\Downarrow\]

The $t'$ and $b'$ condensations can dynamically trigger the EWSB and also the top may contribute but not much.
Three Higgs doublet model

Model: low energy effective theory at composite scale

\[ \mathcal{L} = \mathcal{L}_f + \mathcal{L}_g + \mathcal{L}_{NJL} \]

- \( \mathcal{L}_f \) : kinetic term for the fermions
  We consider only \( t', \ b' \) and \( t \).

- \( \mathcal{L}_g \) : kinetic term for the gauge bosons

- \( \mathcal{L}_{NJL} \) : Nambu-Jona-Lasinio couplings
effectively induced at low energy
\[ \mathcal{L}_{\text{NJL}} = G_{t'}(\bar{\psi}^{(4)}_L t'_R)(\bar{t}'_R \psi^{(4)}_L) + G_{b'}(\bar{\psi}^{(4)}_L b'_R)(\bar{b}'_R \psi^{(4)}_L) + G_t(\bar{\psi}^{(3)}_L t_R)(\bar{t}_R \psi^{(3)}_L) \\
+ G_{t'b'}(\bar{\psi}^{(4)}_L t'_R)(\bar{b}'_R c i \tau_2 (\psi^{(4)}_L)^c) + (\text{h.c.}) \\
+ G_{t't}(\bar{\psi}^{(4)}_L t'_R)(\bar{t}_R \psi^{(3)}_L) + (\text{h.c.}) \\
+ G_{b't}(\bar{\psi}^{(3)}_L t_R)(\bar{b}'_R c i \tau_2 (\psi^{(4)}_L)^c) + (\text{h.c.}). \]

How to get them:

- \( G_{t'}, G_{b'}, G_t \): Topcolor gauge boson exchange
- \( G_{t'b'} \): Topcolor instanton
- \( G_{t't} \): Flavor changing neutral interaction between \( t' \) and \( t \)
- \( G_{b't} \): We do not know a natural candidate of the origin. \( G_{b't} = 0 \).
Auxiliary Field Method

Let us introduce the auxiliary field, $\Phi^{(0)}_t = G_t^{-1}(\bar{t}_R \psi_L^{(3)})$, etc.

After Yukawa integration, we have

$$\mathcal{L}_{\text{NJL}} \rightarrow$$

$$\bar{\psi}^{(4)}_L t'_R \Phi^{(0)}_{t'} + \bar{\psi}^{(4)}_L b'_R \tilde{\Phi}^{(0)}_{b'} + \bar{\psi}^{(3)}_L t_R \Phi^{(0)}_t + (\text{h.c.})$$

$$+ M^2_{\Phi_{t'}} \left( \Phi^{(0)}_{t'} \right)^\dagger \Phi^{(0)}_{t'} + M^2_{\Phi_{b'}} \left( \Phi^{(0)}_{b'} \right)^\dagger \Phi^{(0)}_{b'} + M^2_{\Phi_{t}} \left( \Phi^{(0)}_{t} \right)^\dagger \Phi^{(0)}_{t}$$

$$+ M^2_{\Phi_{t'}^{(0)} \Phi^{(0)}_{b'}} \left( \Phi^{(0)}_{t'} \right)^\dagger \Phi^{(0)}_{b'} + (\text{h.c.})$$

$$+ M^2_{\Phi_{b'}^{(0)} \Phi^{(0)}_{t}} \left( \Phi^{(0)}_{b'} \right)^\dagger \Phi^{(0)}_{t} + (\text{h.c.})$$

Higgs mass terms

$$+ M^2_{\Phi_{b'}^{(0)} \Phi^{(0)}_{t}} \left( \Phi^{(0)}_{b'} \right)^\dagger \Phi^{(0)}_{t} + (\text{h.c.}),$$

If $G_{b't} = 0 \Rightarrow M^2_{\Phi_{b'}^{(0)} \Phi^{(0)}_{t}} \approx 0$. 
The low energy effective theory at EWSB scale

Higgs quartic couplings at $1/N_c$ leading approximation
When we ignore the EW 1-loop effect, the (2+1)-Higgs structure is safely kept. The quartic term is then written as

\[ V^{(4)}_{2+1} = \lambda_1 (\Phi_{b'}^\dagger \Phi_{b'})^2 + \lambda_2 (\Phi_{t'}^\dagger \Phi_{t'})^2 + \lambda_3 |\Phi_{t'}|^2 |\Phi_{b'}|^2 + \lambda_4 |\Phi_{t'}^\dagger \Phi_{b'}|^2 + \lambda_t (\Phi_{t}^\dagger \Phi_{t})^2. \]

Higgs quartic coupling — 2 Higgs part + 1 Higgs part

(2+1)-Higgs doublet model

(The mass terms are general one.)

\[ |\Phi_{t'}|^2 |\Phi_{t}|^2 \] is absent.
The mass terms are,

\[ V_2 = M_{\Phi b'}^2 (\Phi_{b'}^\dagger \Phi_{b'}) + M_{\Phi t'}^2 (\Phi_{t'}^\dagger \Phi_{t'}) + M_{\Phi t}^2 (\Phi_{t}^\dagger \Phi_{t}) \\
+ M_{\Phi t'}^2 (\Phi_{t'}^\dagger \Phi_{b'}) + M_{\Phi b'}^2 (\Phi_{b'}^\dagger \Phi_{t}) + M_{\Phi t'}^2 (\Phi_{t'}^\dagger \Phi_{t}) + (h.c.). \]

While \( M_{\Phi b'}^2 \) and \( M_{\Phi t'}^2 \) are negative, the mass square \( M_{\Phi t}^2 \) is positive, which reflects a subcritical dynamics of the \( t \) quark. The top-Higgs \( \Phi_t \) acquires a vacuum expectation value only due to its mixing with \( \Phi_{t'} \) (as was already indicated above, we assume that its mixing with \( \Phi_{b'} \) is negligible).
Numerical Analysis

We have 8 theoretical parameters

\[ G_{t'} \quad G_{b'} \quad G_t \quad G_{t'b'} \quad G_{t't} \quad G_{b't} \quad \Lambda_4 \quad \Lambda_3. \]

\( \Lambda_4 \): composite scale (Landau pole) of \( t' \) and \( b' \)
\( \Lambda_3 \): composite scale (Landau pole) of the top.

The physical quantities are

3 Higgs doublets: CP even Higgs – 3

\[ H_1, \ H_2, \ H_3 \]
\[ (M_{H_1} < M_{H_2} < M_{H_3}) \]

CP odd Higgs – 2

\[ A_1, \ A_2 \]
\[ (M_{A_1} < M_{A_2}) \]

charged Higgs – 2+2

\[ H^\pm_1, \ H^\pm_2 \]

VEV – 3 etc.

\[ \nu_{b'}, \ \nu_{t'}, \ \nu_t \]
It is convenient to take the following parameters:

\[ v(= 246 \text{ GeV}), \quad m_t(= 171.2 \text{ GeV}) \]

\[ \tan(\beta_4)(\simeq 1) \quad M_{A_1} \quad M_{A_2} \quad \Lambda_4 \quad \Lambda_3 \]

\[ M^2_{\Phi_{b',\Phi_t}} \approx 0. \]

The outputs are

\[ m_{t'} \quad m_{b'} \quad M_{H_1} \quad M_{H_2} \quad M_{H_3} \quad M_{H_1^\pm} \quad M_{H_2^\pm} \quad \tan \beta_{34}, \]

decay widths of \( H_{1,2,3} \rightarrow WW, ZZ \) etc.,
Yukawa couplings between the fermions and the Higgs bosons.
Definition of the angles of the VEVs

\[ \nu_{b'} \equiv \langle \Phi_{b'} \rangle = \nu \cos \beta_4 \cos \beta_{34} \]
\[ \nu_{t'} \equiv \langle \Phi_{t'} \rangle = \nu \sin \beta_4 \cos \beta_{34} \]
\[ \nu_t \equiv \langle \Phi_t \rangle = \nu \sin \beta_{34} \]

\[ \nu = 246 \text{ GeV} \]

It is natural to take similar composite scales.

\[ \Lambda_4 \simeq \Lambda_3 \quad \Lambda_4 \leftarrow y_{t'}, \ y_{b'} \quad \Lambda_3 \leftarrow y_t. \]

Owing to \( y_{t'} = y_{b'} \), the \( T \) parameter constraint implies

\[ m_{t'} \simeq m_{b'} \Rightarrow \nu_{t'} \simeq \nu_{b'} \quad \tan \beta_4 \simeq 1. \]

Also, \( \tan^2 \beta_{34} \simeq \frac{m_t^2}{m_{t'}^2 + m_{b'}^2} \sim 0.1 - 0.2 \ll 1. \)
We calculate the mass spectrum by using the RGE:

RGE for the (2+1)-Higgs doublets + compositeness conditions
(Bardeen-Hill-Lindner approach)

\[ m_{t'} \text{ and } m_{b'} \text{ for various } \Lambda_4 \]

The bold curves are for \( \Lambda_3/\Lambda_4 = 1 \)
The dashed curves are for \( \Lambda_3/\Lambda_4 = 2 \)
The mass spectrum of the Higgs bosons for various $M_{A_1}$

We also used $\tan \beta_4 = 1$ and $\Lambda_3/\Lambda_4 = 1.5$.

$M_{A_2} \sim M_{H_3} \sim M_{H_2^\pm} \gg M_{H_1,H_2,H_1^\pm} \rightarrow (2+1)$ Higgs structure
How about the Higgs contribution to the S,T-parameter

\[ S_H = 0 \sim 0.04, \quad T_H = -0.02 \sim -0.1 \]

\[ \Lambda_4 = 2 - 10 \text{ TeV} \quad \Lambda_3/\Lambda_4 = 1 - 2 \]

\[ 0.2 \text{ TeV} < M_{A_1} < 0.6 \text{ TeV} \quad 0.5 \text{ TeV} < M_{A_2} < 0.8 \text{ TeV} \]

\[ \rightarrow \text{If } M_{\tau'} - M_{\nu'} \sim 150 \text{ GeV, the model is within 95\%CL limit of } S, T. \]

\[ R_b \text{ constraint is potentially dangerous.} \]

\[ R_{b}^{obs} = 0.21629 \pm 0.00066 \quad R_{b}^{SM} = 0.21584. \]

\[ 2\sigma \text{ bounds yield the constraint to } M_{H_{2}^{\pm}} \text{ and it corresponds to} \]

\[ M_{A_2} = 0.70, \ 0.58, \ 0.50 \text{ TeV} \]

\[ \text{for } \Lambda_4 = 2, \ 5, \ 10 \text{ TeV}. \]
What is the signature?

An example data for the scenario with $M_{H_1} > 2m_t$

Inputs:

$$\Lambda_4 = 3\ \text{TeV},\quad \Lambda_3/\Lambda_4 = 1.5,\quad \tan \beta_4 = 1$$

$$M_{A_1} = 0.4\ \text{TeV},\quad M_{A_2} = 0.8\ \text{TeV}.$$ 

Outputs:

$$m_{t'} = m_{b'} = 0.33\ \text{TeV},$$
$$M_{H_1^\pm} = 0.48\ \text{TeV},$$
$$M_{H_2^\pm} = 0.82\ \text{TeV},$$
$$M_{H_1} = 0.45\ \text{TeV},$$
$$M_{H_2} = 0.53\ \text{TeV},$$
$$M_{H_3} = 0.86\ \text{TeV}.$$
Yukawa couplings

\[ \bar{t} t H_1 = \frac{m_t}{v} \cdot 1.73 \]
\[ \bar{t}' t' H_1 = \frac{m_{t'}}{v} \cdot 1.16 \]
\[ \bar{b}' b' H_1 = \frac{m_{b'}}{v} \cdot 0.183 \]
\[ \bar{t} t H_2 = \frac{m_t}{v} \cdot 0.243 \]
\[ \bar{t}' t' H_2 = \frac{m_{t'}}{v} \cdot 0.127 \]
\[ \bar{b}' b' H_2 = \frac{m_{b'}}{v} \cdot 1.51 \]
\[ \bar{t} t H_3 = \frac{m_t}{v} \cdot 2.11 \]
\[ \bar{t}' t' H_3 = \frac{m_{t'}}{v} \cdot 0.967 \]
\[ \bar{b}' b' H_3 = \frac{m_{b'}}{v} \cdot 0.024 \]

Decay width into WW, ZZ

\[ \Gamma(H_1 \rightarrow WW/ZZ) = 0.66\Gamma_{SM} \]
\[ \Gamma(H_2 \rightarrow WW/ZZ) = 0.32\Gamma_{SM} \]
\[ \Gamma(H_3 \rightarrow WW/ZZ) = 0.02\Gamma_{SM} \]

Enhancement of Higgs production of \( H_1 \)

\[ \Gamma(H_1 \rightarrow gg) = 7.4\Gamma_{SM} \]
\[ \frac{\Gamma_{ZZ}}{\Gamma_{WW} + \Gamma_{ZZ} + \Gamma_{tt}} = 0.51 \times SM. \]
We may have a ttbar resonance of $H_1$

The heavier Higgs $H_2$ resonance may exist in the ZZ mode.

Also, in the ttbar channel, there may appear a scalar resonance $H_3$.

Higgs Phenomenology is quite rich!
The two crucial ingredients in the present class of models are (i) the assumption that the EWSB dynamics leads to the isospin symmetric quark mass spectrum, with the masses of the order of the down-type quarks, and (ii) the existence of strong (although subcritical) horizontal diagonal interactions for the $t$ quark plus horizontal flavor-changing neutral interactions between different families. The signature of such dynamics is the presence of composite Higgs bosons. It is noticeable that this dynamics can be build into the scenarios with different EWSB mechanisms.

The $2 + 1$ composite Higgs model with the 4th family shows that these two ingredients quite naturally lead to the realistic masses for quarks. It is also noticeable that by using a simple extension of the present mechanism for producing the quark masses, the essential features of the CKM matrix can be extracted.

It is quite nontrivial that this model passes the electroweak precision data constraints.

The model has a clear signature: the $2 + 1$ structure of the composite Higgs bosons. Its phenomenology is quite rich.

The generation of lepton (in particular, neutrino) masses is certainly one of the most important problems which should still be resolved in the present approach.