Phase Transitions of Strong Interaction System in Dyson-Schwinger Equation Approach

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Outline

I. Introduction
II. The Approach
III. P.Ts. in Intrinsic Space & in Medium
IV. Possible Observables
V. Summary

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1. Introduction

The Evolution Process of the Universe

History of the Universe

- BIG BANG
- Inflation
- Quarks and gluons
- Non-confined quarks and gluons and leptons (F.S.) (F.S.B.)
- Hadrons
- Nuclear synthesis
- Nuclei

Key:
- $w, z$ bosons
- $q$ quark
- $g$ gluon
- $e$ electron
- $\mu$ muon
- $\nu$ neutrino
- photon
- meson
- baryon
- star
- ion
- galaxy
- atom
- black hole
The Phase Transitions and Schematic PD

Phase Transitions involved:
Deconfinement–confinement
DCS – DCSB
Flavor Sym. – FSB

Items Influencing the Phase Transitions:
Medium: Temperature $T$, Density $\mu$ (or $\mu$)
Size
Intrinsic: Current mass, Coupling Strength, Color-flavor structure,
The primary goal of the RHIC scientific program in the coming years is to progress from qualitative statements to rigorous quantitative conclusions. Quantitative conclusions require sophisticated modeling of relativistic heavy-ion collisions and rigorous comparison of such models with lattice QCD studies and phenomenological modeling, and an increase of funding to support new initiatives enabled by experimental and theoretical breakthroughs. The success of this effort mandates significant additional investment in theoretical resources in terms of focused collaborative initiatives.

The approach should manifest simultaneously:

1. DCSB & its Restoration,
2. Confinement & Deconfinement.

Lattice QCD:
- Running coupling behavior,
- Vacuum Structure,
- Temperature effect,
- “Small chemical potential”; ...

Continuum:
1. Phenomenological models NJL, QMC, QMF,
2. Field Theoretical Chiral perturbation,
   Renormalization Group, OCD sum rules.
A comment on the DSE approach in QCD

Phases of dense quarks at large $N_c$

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One way of computing the properties of a quarkyonic phase is to use approximate solutions of Schwinger–Dyson equations [23]. These are, almost uniquely, the one approximation scheme which includes both confinement and chiral symmetry breaking. They do have features reminiscent of large $N_c$: at low momentum, if chiral symmetry breaking occurs, the gluon propagator for $N_f = 3$ is numerically close to that for $N_f = 0$. At present, solutions at $\mu \neq 0$ assume a Fermi surface dominated by quarks; if quark screening is not too large at moderate $\mu$, these models should exhibit a quarkyonic phase.
II. The Dyson-Schwinger Equation Approach

**Dyson-Schwinger Equations**

**Slavnov-Taylor Identity**

\[ k_\lambda \Gamma^{\lambda\mu\nu}(k, p, q) = \Pi^{\mu\nu}(p) - \Pi^{\mu\nu}(q) \]

**axial gauges BBZ**

**covariant gauges**

\[ k_\lambda \Gamma^{\lambda\mu\nu}(k, p, q) = H(k^2) \left[ G_{\mu,\sigma}(q, -k) \Pi^{T}_{\sigma,\nu}(p) - G_{\nu\sigma}(p, -k) \Pi^{T}_{\sigma,\mu}(q) \right] \]

---

Practical Algorithm at Present Stage

- Quark equation at zero chemical potential

\[
G^{-1}(p) = Z_2(i\gamma \cdot p + m_{\text{bar}}) + \frac{4}{3} \int_q^A 4\pi\alpha(p - q)D_{\mu\nu}^{\text{free}}(p - q)\gamma_\mu G(q)\Gamma_\nu, \tag{1}
\]

where \(D_{\mu\nu}^{\text{free}}(p - q)\) is the effective gluon propagator, \(G^{-1}(p)\) can be conventionally decomposed as

\[
G^{-1}(p) = i\gamma \cdot pA(p^2) + B(p^2),
\]

- Quark equation in medium

\[
G^{-1}(p) \implies \mathcal{G}^{-1}(p, \omega_n, \mu)
\]

- with

\[
\mathcal{G}^{-1}(p, \omega_n, \mu) = iA(p, \omega_n, \mu)\gamma \cdot \vec{p} + iC(p, \mu)\gamma_4(\omega_n + i\mu) + B(\vec{p}) + \cdots \tag{3}
\]

Meeting the requirements!
❄ Models of Vertex

\[ \Gamma^{a}_{\mu}(q,p) = t^{a} \Gamma_{\mu}(q,p) \]

(1) **Bare Vertex**

\[ \Gamma_{\mu}(q,p) = \gamma_{\mu} \]

(Rainbow-Ladder Approx.)

(2) **Ball-Chiu Vertex**

\[
\Gamma^{BC}_{\mu}(p,q) = \frac{A(p^2) + A(q^2)}{2} \gamma_{\mu} + \left(\frac{p+q}{p^2-q^2}\right) \gamma_{\mu} \left\{ \frac{(A(p^2) - A(q^2))}{2} \right\} \left(\gamma \cdot p + \gamma \cdot q\right) \\
- i[B(p^2) - B(q^2)]\}
\]

(3) **Curtis-Pennington Vertex**

\[
\Gamma^{CP}_{\mu}(p,q) = \Gamma^{BC}_{\mu}(p,q) + \frac{1}{2} (A(p^2) - A(q^2)) \gamma_{\mu} \left( \frac{p^2 - q^2}{d(p,q)} \right) - (k+p)_{\mu} \gamma \cdot (p+q) \\
d(p,q) = \frac{(p^2 - q^2)^2 + [M^2(p^2) + M^2(q^2)]^2}{p^2 + q^2}.
\]
\( g^2 D_{\rho\sigma}(k) = 4\pi \frac{G(k^2)}{k^2} \left( \delta_{\rho\sigma} - \frac{k_{\rho}k_{\sigma}}{k^2} \right) \)

(1) MN Model \[ g^2 D(p - q) = \frac{3}{16} \eta^2 \delta(p - q), \]

(2) \((q^4 + \Delta)^{-1}\) Model

(3) More Realistic model

(4) An Analytical Expression of the Realistic Model: Maris-Tandy Model

\[ \frac{G(t)}{t} = \frac{4\pi^2}{\omega^8} D e^{-t/\omega^2} + \frac{8\pi^2 \gamma_m}{\ln \left[ \tau + \left( 1 + t/\Lambda^2_{QCD} \right)^2 \right]} \left[ 1 - \exp\left(-t/[4m_F^2]\right) \right] / t \]

(5) Point Interaction: (P) NJL Model
III. The Phase Transitions in I.-S. & in Medium

Chiral Susceptibility ($\chi_S$ & $\chi_{SB}$ phases simultaneously): Signature of the Chiral Phase Transition

$$\chi = \frac{\partial M}{\partial m} = \frac{1}{1 - 4GN_cN_f\frac{\partial}{\partial M}\left\{\int \frac{d^3p}{(2\pi)^3} \frac{M}{E_p}[1 - n_p(T, \mu) - \bar{n}_p(T, \mu)]\right\}} = \frac{1}{G^2\frac{\partial^2\Omega}{\partial M^2}}\left|\frac{\partial\Omega}{\partial M}\right| = 0$$

---

Effect of the Running Coupling Strength on the Chiral Phase Transition


\[
\alpha_{\text{tue}}(k^2) = g_k^2 \frac{k^2}{4\pi} \mathcal{D}(k^2) = \pi \frac{k^4}{\omega^2} e^{-k^2/\omega^2}
\]

\[
V(r) = \frac{\sqrt{\pi} \xi \omega^3}{8} (\omega^2 r^2 - 6) e^{-\frac{\omega^2 r^2}{4}}
\]

Parameters are taken from Phys. Rev. D 65, 094026 (1997), with \( f_\pi \) fitted as \( f_\pi = 93\,\text{MeV} \)

Lattice QCD result
PRD 72, 014507 (2005)

(Bare vertex)
CS phase

(CSB phase)

Chiral observables (in units of \( a^-1 \))

\[ \frac{\mathcal{G}(t)}{t} = \frac{4 \pi^2}{\omega^6} D t e^{-t/\omega^2} + \frac{8 \pi^2 \gamma_m}{\ln \left[ \tau + \left(1 + t/\Lambda_{\text{QCD}}^2\right)^2 \right]} \mathcal{F}(t), \]

with \( t = k^2 \), \( \mathcal{F}(t) = [1 - \exp(-t/[4m_f^2])/t] \), \( m_f^2 = 0.5 \text{ GeV}^2 \), \( \tau = e^2 - 1 \), \( \gamma_m = 12/25 \) and \( \Lambda_{\text{QCD}} = \Lambda_{\text{MS}}^{(\mu)} = \)

\( \ldots \)

Solutions of the DSE with \( \omega = 0.4 \text{ GeV} \), \( D = 16 \text{ GeV}^2 \), \( \omega = 0.4 \text{ GeV} \).
$\bar{q}q$ condensate for light quarks beyond the chiral limit

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confirms the existence of the 3rd solution, and give the 4th solution.

Part of the QCD Phase Diagram in terms of the Current Mass and Coupling Strength

The one with multi-node solutions is more complicated and more interesting.
Special Topic (1) of the P.T. in Medium: Critical EndPoint (CEP)

The Position of CEP is a highly debated problem!

- (p)NJL model & others give quite large $\mu_E/T_E$ (> 3.0)
  - Sasaki, et al., PRD 77, 034024 (2008); Costa, et al., PRD 77, 096001 (2008);
  - Fu, et al., PRD 77, 014006 (2008); Ciminale, et al., PRD 77, 054023 (2008);

- Lattice QCD gives smaller $\mu_E/T_E$ (0.4 ~ 1.1)
  - Fodor, et al., JHEP 4, 050 (2004); Gavai, et al., PRD 71, 114014 (2005);

- RHIC Exp. Estimate hints quite small $\mu_E/T_E$ (~ 0.33)
  - R.A. Lacey, et al., nucl-ex/0708.3512; …

- Simple DSE Calculations with Different Effective Gluon Propagators Generate Different Results (0.0, 1.3)

- What can sophisticated DSE calculation produce?

- Why different models give distinct results?
Special topic (2) of the P.T. in Medium:
Coexistence region (Quarkyonic ?)

Lattice QCD Calculation
de Forcrand, et al.,

and General (large-N_c) Analysis
McLerran, et al., NPA 796, 83 (‘07);
NPA 808, 117 (‘08);
NPA 824, 86 (‘09), …

claim that there exists a quarkyonic phase.

Inconsistent with Coleman-Witten Theorem!!

Can sophisticated continuous field approach of QCD give the coexistence (quarkyonic) phase?

What can we know more for the coexistence phase?
Special Topic (3) of the P.T. in Medium: QM at T above but near $T_c$

- **HTL Cal.** (Pisarski, PRL 63, 1129(‘89); Blaizot, PTP S168, 330(’07)), **Lattice QCD** (Karsch, et al., NPA 830, 223 (‘09); PRD 80, 056001 (’09)) & **NJL Cal.** (Wambach, et al., PRD 81, 094022(2010)) show: there exists thermal & Plasmino excitations in hot QM.

- **Other Quenched Lattice QCD Simulations** (Hamada, et al., Phys. Rev. D 81, 094506 (2010)) claims: NO qualitative difference between the quark propagators in the deconfined and confined phases near the $T_c$.

- **RHIC experiments** (Gyulassy, et al., NPA 750, 30 (2005); Shuryak, PPNP 62, 48 (2009); Song, et al., JPG 36, 064033 (2009);) indicate: the matter is in sQGP state.

What is the nature of the matter in DSE?
Phase Diagram of Strong Interaction Matter in Present DSE Approach of QCD

Result in bare vertex

Result in Ball-Chiu vertex

Chiral Sym.

Chiral Sym. Broken

Co-existence

S.X. Qin, L. Chang, Y.X. Liu, C.D. Roberts, to be published.
**Model Parameter Dependence of the CEP**

<table>
<thead>
<tr>
<th>model</th>
<th>vertex</th>
<th>$D$(GeV$^2$)</th>
<th>$\omega$(GeV)</th>
<th>$\Delta_c$</th>
<th>$T_c$</th>
<th>$(T_E, \mu_E)$(GeV)</th>
<th>$\mu_E/T_E$</th>
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<tr>
<td>BC</td>
<td>0.50</td>
<td>0.50</td>
<td></td>
<td>0.026</td>
<td>0.124</td>
<td>(0.110, 0.140)</td>
<td>1.273</td>
</tr>
<tr>
<td>BC</td>
<td>0.50</td>
<td>0.45</td>
<td></td>
<td>0.075</td>
<td>0.142</td>
<td>(0.130, 0.125)</td>
<td>0.962</td>
</tr>
<tr>
<td>Bare</td>
<td>1.00</td>
<td>0.50</td>
<td></td>
<td>0.220</td>
<td>0.133</td>
<td>(0.120, 0.130)</td>
<td>1.083</td>
</tr>
<tr>
<td>Bare</td>
<td>1.04</td>
<td>0.45</td>
<td></td>
<td>0.250</td>
<td>0.138</td>
<td>(0.125, 0.120)</td>
<td>0.960</td>
</tr>
</tbody>
</table>

Small $\omega \rightarrow$ short range in momentum space

$\rightarrow$ long range in coordinate space

MN model $\rightarrow$ infinite range in r-space

NJL model $\rightarrow$ “zero” range in r-space

Stronger & Longer range Int. $\rightarrow$ Smaller $\mu_E/T_E$
At high temperature (e.g., $T = 3.0T_c$), there exists Normal thermal mode & Plasmino mode excitations. At high momentum, the N.T. mode plays the main role & behaves like a free particle.

At the temperature near but above the $T_c$ (e.g., $T = 1.1T_c$), there exists a zero mode, besides the N.T. mode and the P. mode.

The zero mode exists at low momentum ($<7.0T_c$), and is long-range correlated (i.e., $\lambda_{sN}$).

The quark at the $T$ where $\chi_S$ is restored involves still rich phases. And the matter is sQGP.
IV. Possible Observables

“QCD” Phase Transitions may Happen

General idea, Phenom. Calc., Sophist. Calc., quark may get deconfined (QCD PT) at high T and/or ρ

QCD Phase Transitions

Signals for QCD Phase Transitions:
In Lab. Expt.
Jet Q., $v_2$, Viscosity, CC Fluct. & Correl., Hadron Prop., ...

In Astron. Observ.
M-R Rel., R.S., Rad., Inst. R. Oscil., Freq. G. Oscil., ...
Density & Temperature Dependence of some Properties of Nucleon in DSE Soliton Model

(Y. X. Liu, et al., NPA 695, 353 (2001); NPA 725, 127 (2003); NPA 750, 324 (2005))

(Y. Mo, S.X. Qin, and Y.X. Liu, Phys. Rev. C 82, 025206 (2010))
Temperature dependence of some properties of $\Phi$ mesons in the model with contact interaction

(Wei-jie Fu, and Yu-xin Liu, Phys. Rev. D 79, 074011 (2009))

Fluctuation & Correlation of Conserved Charges

(W.J. Fu, Y.X. Liu, & Y.L. Wu, Phys. Rev. D 79, 014028 (2010))
Distinguishing Newly Born Strange Quark Stars from Neutron Stars

**Neutron Star:** RMF,  **Quark Star:** Bag Model

**Frequency of g-mode pulsation**

<table>
<thead>
<tr>
<th>Radial or of g-mode</th>
<th>$f$ (Hz)</th>
<th>Quark Star</th>
</tr>
</thead>
</table>
| $n = 1$             |          | $l=2$  
|                     |          | $n=1$  
| $n = 2$             |          | $= 200$  
|                     |          | $t = 300$  
| $n = 3$             |          | $= 78.0$  
|                     |          | $= 63.1$  
|                     |          | $= 45.5$  
|                     |          | $= 40.0$  
|                     |          | $= 30.8$  
|                     |          | $= 27.8$  |

Taking into account the DCSB effect

Newly obtained results for QS in NJL Model

<table>
<thead>
<tr>
<th>Radial order of g-mode</th>
<th>Neutron Star</th>
<th>Strange Quark Star</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$t = 100$</td>
<td>$t = 200$</td>
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<tr>
<td></td>
<td>$t = 100$</td>
<td>$t = 200$</td>
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<tr>
<td>$n = 1$</td>
<td>717.6</td>
<td>774.6</td>
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<tr>
<td></td>
<td>100.2</td>
<td>115.4</td>
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<tr>
<td>$n = 2$</td>
<td>443.5</td>
<td>467.3</td>
</tr>
<tr>
<td></td>
<td>60.1</td>
<td>57.0</td>
</tr>
<tr>
<td>$n = 3$</td>
<td>323.8</td>
<td>339.0</td>
</tr>
<tr>
<td></td>
<td>42.9</td>
<td>40.9</td>
</tr>
</tbody>
</table>
Ott et al. have found that these g-mode pulsation of supernova cores are very efficient as sources of g-waves (PRL 96, 201102 (2006)).

DS Cheng, R. Ouyed, T. Fischer, ….

The g-mode pulsation frequency can be a signal to distinguish the newly born strange quark stars from neutron stars, i.e., an astronomical signal of QCD phase transition.

FIG. 4 (color). Characteristic strain spectra contrasted with initial and advanced LIGO (optimal) rms noise curves.
V. Summary & Remarks

♦ Discussed some aspects of QCD phase transitions in the DS equation approach of QCD
  • Coexistence Phase;
  • CEP ($\mu_E/T_E$ comparable with Lattice D & EE)
• Effects of the C.-Strength & Current Mass
♦ Far from Well Established!

♣ Observables ?!
♣ Mechanism ?! Process ?!

Thanks !!
Color Superconductivity (CSC)
Analytic Continuation from Euclidean Space to Minkowskian Space

\[ g_{\mu\nu}(\epsilon) = \begin{pmatrix} e^{i\epsilon} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \]

\[ \epsilon = 0, \ e^{i\epsilon} = 1, \Rightarrow \text{E.S.} \]

\[ \epsilon = \pi, \ e^{i\epsilon} = -1, \Rightarrow \text{M.S.} \]

(W. Yuan, S.X. Qin, H. Chen, & YXL, PRD 81, 114022 (2010))
Dynamical chiral symmetry breaking (DCSB) generates the mass of Fermion

\[ M(p) \simeq m_0 \left[ \ln \frac{p}{\Lambda_{QCD}} \right]^d + C \frac{-\langle \bar{q}q \rangle}{p^2 \left[ \ln \frac{p}{\Lambda_{QCD}} \right]^d} \]

\[ \langle \bar{q}q \rangle_0 \sim -(240 \, \text{MeV})^3 \]

\[ \text{Dynamical Mass} \]

Maris & Roberts
Hadron Structure

Meson Bethe-Salpeter Eqn

Quantum field theory bound states: BSE

\[ \Gamma_M(p; P) = \int_k^\Lambda K(p, k; P) \, S(k_+) \, \Gamma_M(k; P) \, S(k_-) \]

Light quark propagator \( \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)} \)

Heavy quark propagator \( \frac{1}{i\gamma \cdot p + M_{\text{cons}}} \)

Fit \( M_{\text{cons}} \) to lightest ps meson
Some Numerical Results

DSE and Lattice results for $M_V$ and $M_{ps}$

Pion electromagnetic form factor

Axial anomaly and $\eta - \eta'$ states

Ch symm: $\partial_\mu (z) \langle \bar{q}(z) q(x) \bar{q}(y) \rangle$ involves $2 \text{tr}(\mathcal{F}^\alpha) \langle Q_\tau (z) q(x) \bar{q}(y) \rangle$

Matrix elements, amputated $\Rightarrow$ AV-WTI

$$\Gamma_\mu \Gamma_5 \Gamma_5(k; P) = -2i M^{\alpha \beta} [\delta_{\alpha \beta} \Gamma_\Lambda (k; P)$$

$$+ \epsilon^{\alpha \beta \gamma \delta} \Gamma_\delta (k_+) \gamma_\gamma \bar{q}(x) f^{\alpha \beta \gamma} \Gamma_\delta (k_-)$$

Residues at PS poles $\Rightarrow$ PS mass formula for arbitrary $m_q$, any flavor:

$$i \rho_5^p (\mu) = \frac{Z_A \text{tr} \int_q \mathcal{F}^\alpha \gamma_5 \bar{q}(p; q)}{2 \tau (F^0) \langle |Q_\tau| \rangle_p}$$

---

[Refs: Bhagwat, Chang, Liu, Roberts, PCT, PRC (76), 2007; arXiv:0708.1118]
Solving the 4-dimensional covariant B-S equation with the kernel being fixed by the solution of DS equation and flavor symmetry breaking, we obtain

<table>
<thead>
<tr>
<th></th>
<th>Expt. (GeV)</th>
<th>Calc. (GeV)</th>
<th>Th/</th>
<th>Expt. (GeV)</th>
<th>Calc. (GeV)</th>
<th>Th/Ex-1 (%)</th>
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<td>&quot;ρ^0&quot;</td>
<td>0.7755</td>
<td>0.7704</td>
<td>π^0</td>
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<td>0.13460</td>
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<tr>
<td>ρ^±</td>
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<td>0.7755</td>
<td>π^±</td>
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<tr>
<td>ω^±</td>
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<td>K^±</td>
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<tr>
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<td>D_s^±</td>
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<td>3.0969</td>
<td>η_c</td>
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<td>B^±</td>
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<td>B^±</td>
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<td>B^0</td>
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<td>B_s^0</td>
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<td>B_c^±</td>
<td>6.286</td>
<td>6.1505</td>
<td>-2.2</td>
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DSE Soliton Description of Nucleon

Maris-Tandy 模型有效胶子传播子

\[ g^2 D(q) = \frac{4\pi^2 d}{\omega^6} q^2 e^{-q^2/\omega^2} + \frac{8\pi^2 \gamma_m \pi}{\ln[1 + \frac{q^2}{\Lambda_{QCD}^2}]} \frac{1 - \exp(-\frac{q^2}{4m_t^2})}{q^2}, \]

\( \gamma_m = \frac{12}{25}, \tau = \epsilon^2 - 1, \Lambda_{QCD} = 0.234 \text{ GeV} \) 和 \( m_t = 0.5 \text{ GeV} \)。介子性质对于参数的约束 \( \omega d = (0.72 \text{ GeV})^3 \)。

\[ \begin{array}{c|c|c|c|c|c|c}
\hline
\omega (\text{GeV}) & 2T & 2F & 3T & 3F & 4T & 4F \\
\hline
\omega (\text{GeV}) & 0.401 & 0.450 & 0.472 \\
\rho (\text{GeV}^2) & 0.930 & 0.830 & 0.790 \\
\varphi_{\text{val}} (\text{MeV}) & 107 & 162 & 189 & 237 & 270 & 300 \\
E_p (\text{MeV}) & 109 & 123 & 126 & 106 & 93 & 15 \\
E_k (\text{MeV}) & 766 & 608 & 527 & 355 & 224 & 18 \\
E_{\text{tot}} (\text{MeV}) & 1196 & 1217 & 1220 & 1172 & 1127 & 933 \\
M_{\text{per}} (\text{MeV}) & 1060 & 1094 & 1084 & 1044 & 996 & 890 \\
M_{\text{rec}} (\text{MeV}) & 956 & 909 & 892 & 814 & 761 & 618 \\
R_{\text{per}} (\text{fm}) & 0.67 & 0.71 & 0.79 & 0.93 & 1.15 & 2.74 \\
R_{\text{rec}} (\text{fm}) & 0.60 & 0.61 & 0.68 & 0.77 & 0.95 & 2.22 \\
\hline
\end{array} \]


Compositions and Phase Structure of Compact Stars and their Identification

Radio Pulses $\Rightarrow$ “Neutron” Stars

Composition & Structure of NS are Still Under Study!

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Fig. 3. The major regions and possible composition inside a normal-matter neutron star. The top bar illustrates expected geometric transi-
Possible Composition of Compact Stars

Fig. 1. Competing structures and novel phases of subatomic matter predicted by theory to make their appearance in the cores ($R \lesssim 8$ km) of neutron stars [1].
Property of the matter above but near the T_c

Solving quark's DSE $\rightarrow$ Quark's Propagator

Maximum Entropy Method


$\rightarrow$ Spectral Function
Disperse Relation and Momentum Dependence of the Residues of the Quasi-particles’ poles at $T = 3.0T_c$ and $T = 1.1T_c$