The first Two Fermion Generations in Twisted Mass Lattice QCD

Karl Jansen

for the **European Twisted Mass Collaboration**

- **Maximally twisted mass fermions, $N_f = 2$ results**
- **Introducing $N_f = 2 + 1 + 1$ flavours**
- **Using $N_f = 2 + 1 + 1$ flavours of quarks**
  - light meson physics
  - Osterwalder-Seiler valence quarks
  - non-perturbative renormalization

Why the first two generations?

- want to include as many quarks as possible: charm still realistic
- charm quark mass
- needed for charmed mesons, e.g. η, η', η_c and baryons
  yesterdays talk by M. Shepherd
- decay constants f_D, f_{D_s}
- heavy quark effects in operator matrix elements
- running of α_s with $N_f = 4$
- very natural to include strange/charm doublet for twisted mass

European Twisted Mass Collaboration

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- **France (Orsay, Grenoble)**
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- **Netherlands (Groningen)**
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Wilson (Frezzotti, Rossi) twisted mass QCD (Frezzotti, Grassi, Sint, Weisz)

Fermion action of twisted mass fermions

$$S_l = \sum_x^l \bar{\chi}_x \left[m_q + \tfrac{1}{2}\gamma_\mu \left[\nabla_\mu + \nabla_\mu^*\right] - ar\tfrac{1}{2}\nabla_\mu^*\nabla_\mu + i\mu_{\rm tm}\tau_3\gamma_5\right] \chi_x^l$$

$$S_h = \sum_x \bar{\chi}_x^h \left[m_q + \tfrac{1}{2}\gamma_\mu \left[\nabla_\mu + \nabla_\mu^*\right] - ar\tfrac{1}{2}\nabla_\mu^*\nabla_\mu i\gamma_5\tau_1\mu_\sigma + \tau_3\mu_\delta\right] \chi_x^h$$

- quark mass parameter m_q , twisted mass parameter $\mu_{\rm tm}$
- strange and charm quark masses

$$m_{s,c} = Z_P^{-1}\left(\mu_\sigma \pm Z_P/Z_S\mu_\delta\right)$$

simulation: $Z_P/Z_S \approx 0.65$

- note, m_q the same in S_l and S_h

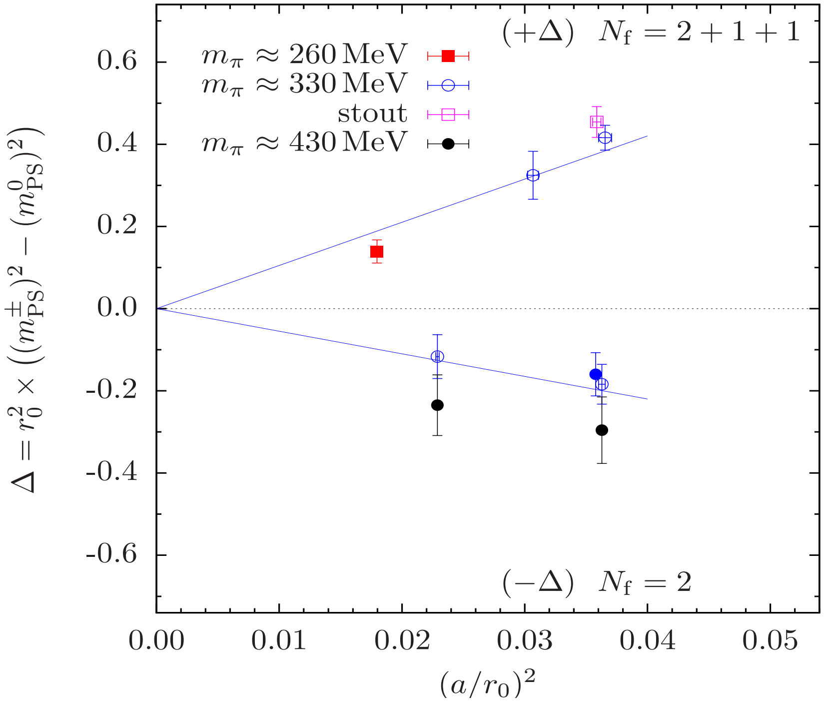
Pros and Cons of a generic fermion action

Pro maximal twisted mass:

- $O(a)$-improvement for all physical quantities *automatically*
- helps to simplify mixing patterns in non-perturbative renormalization
- explicit infrared regularization through $\mu_{\rm tm}$

Con twisted mass:

- isospin violation at any $a \neq 0$
- observe large $O(a^2)$ effect in neutral pion mass (a similar large $O(a^2)$ effect expected for Wilson fermions in another quantity)



Controlling the effect theoretically:

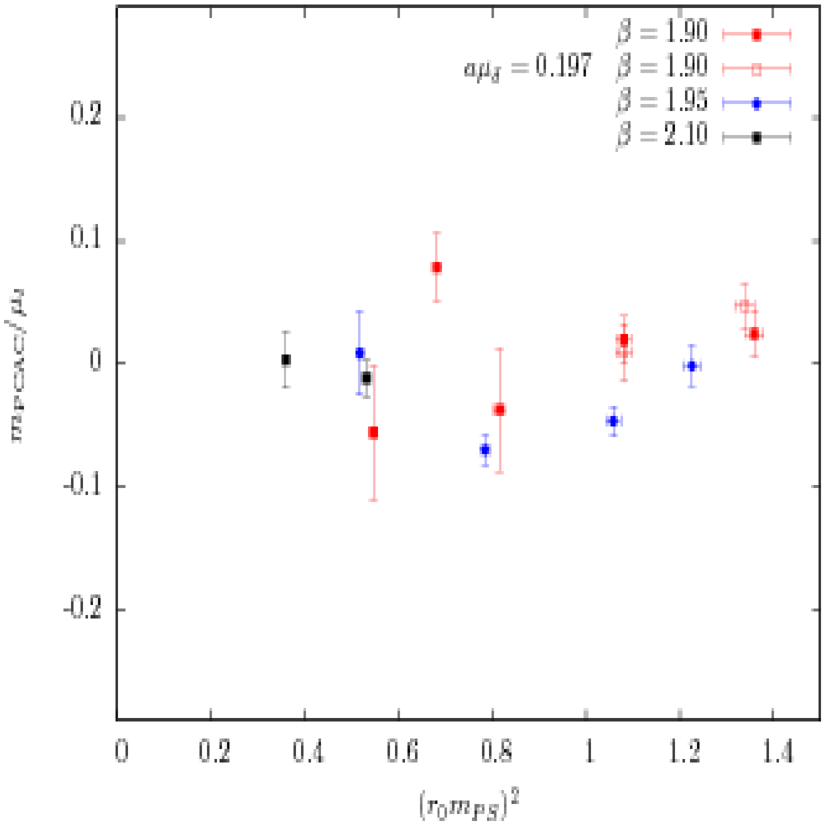
Frezzotti, Rossi (2007), Dimopoulos et.al (2010), Colangelo, Wenger, Wu (2010), Bär (2010)

what counts in the end: Universality

Tuning to maximal twist

Maximal twist: tune m_q such that

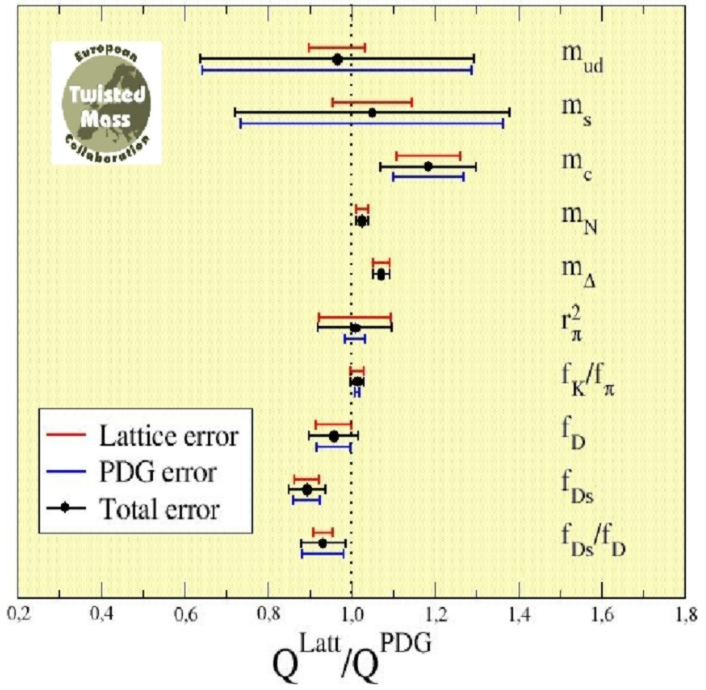
$$m_{\mathrm{PCAC}} = \frac{\sum_{\mathbf{x}} \langle \partial_0 A_0^a(x) P^a(0) \rangle}{2 \sum_{\mathbf{x}} \langle P^a(x) P^a(0) \rangle} = 0$$



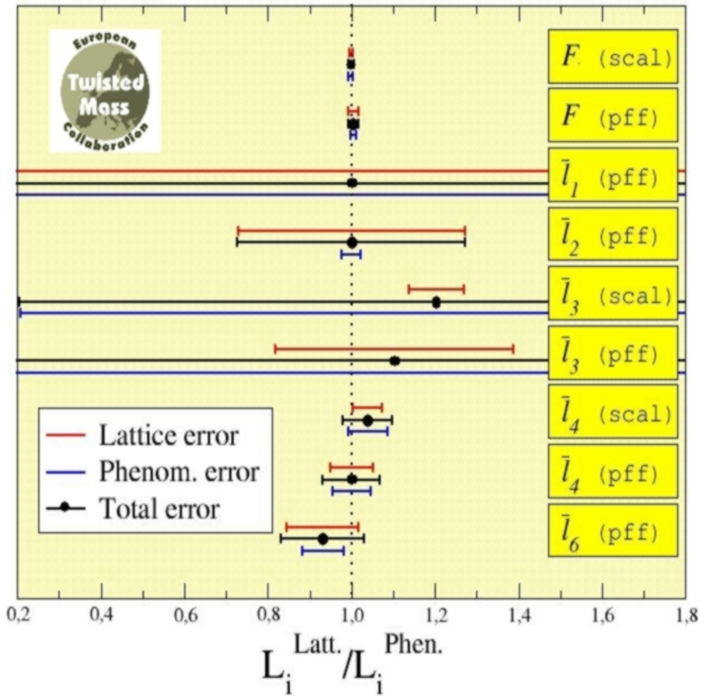
- tuning of m_q at *each* μ_{tm} used
- demand $m_{\mathrm{PCAC}} \lesssim 0.1\mu_{\mathrm{tm}}$
- demand $\Delta(m_{\mathrm{PCAC}}) \lesssim 0.1\mu_{\mathrm{tm}}$

Selected results for $N_f = 2$

Simulation results versus PDG

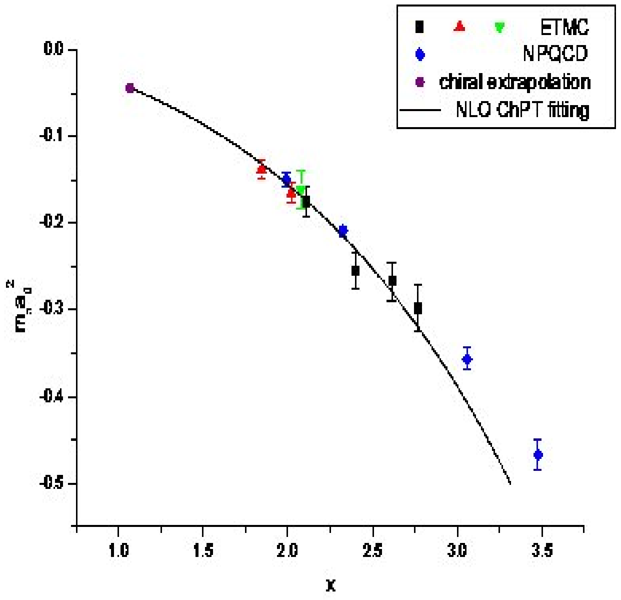


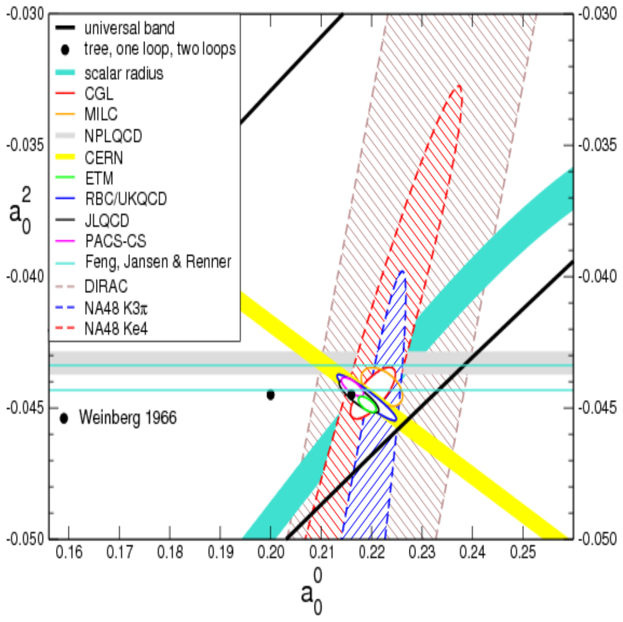
Low energy constants



I=2 Pion scattering length

(X. Feng, D. Renner, K.J.)





energy determined from

$$R(t) = \langle(\pi^+\pi^+)^\dagger(t+t_s)(\pi^+\pi^+)(t_s)\rangle / \langle(\pi^+)^\dagger(t+t_s)\pi^+(t_s)\rangle^2$$

$$\rightarrow \quad \Delta E = c/L^3 \cdot a_{\pi\pi}^{I=2}(1+O(1/L))$$

E865 (BNL) $m_\pi a_{\pi\pi}^{I=0} = 0.203\,(33)$ and $m_\pi a_{\pi\pi}^{I=2} = -0.055\,(23)\,.$

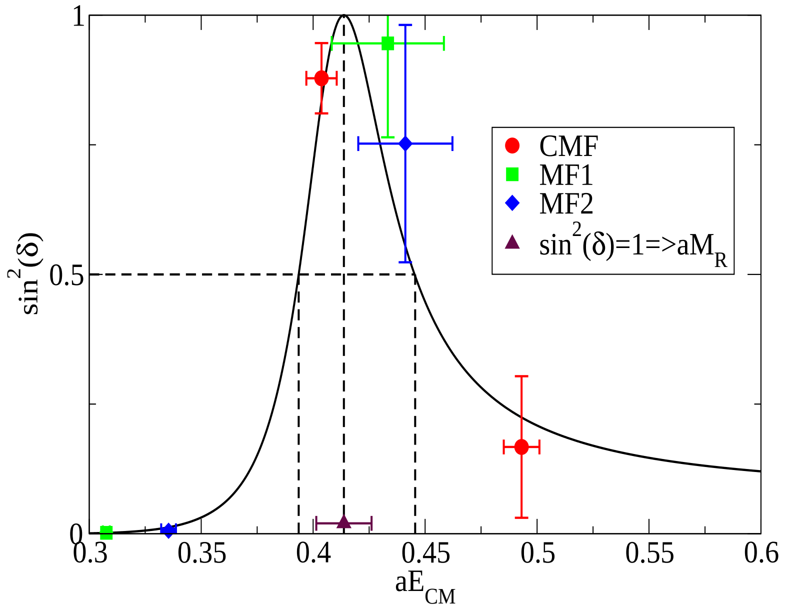
NA48/2 (CERN) $m_\pi a_{\pi\pi}^{I=0} = 0.221\,(5)$ and $m_\pi a_{\pi\pi}^{I=2} = -0.0429\,(47).$

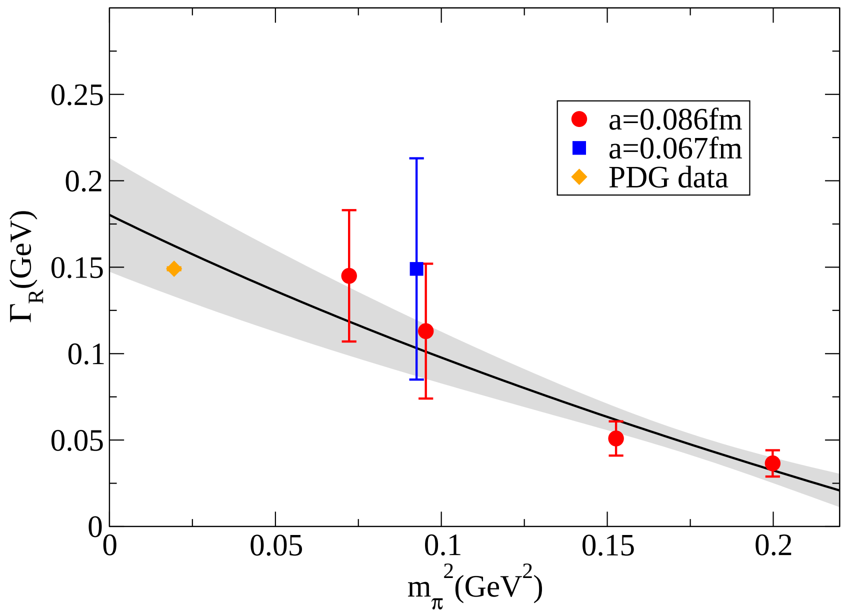
our work $m_\pi a_{\pi\pi}^{I=2} = -0.04385\,(28)(38)$

The ρ-meson resonance: dynamical quarks at work

(X. Feng, D. Renner, K.J.)

- usage of three Lorentz frames





$m_{\pi^+} = 330$ MeV, $a = 0.079$ fm, $L/a = 32$

fitting $z = (M_\rho + i\frac{1}{2}\Gamma_\rho)^2$

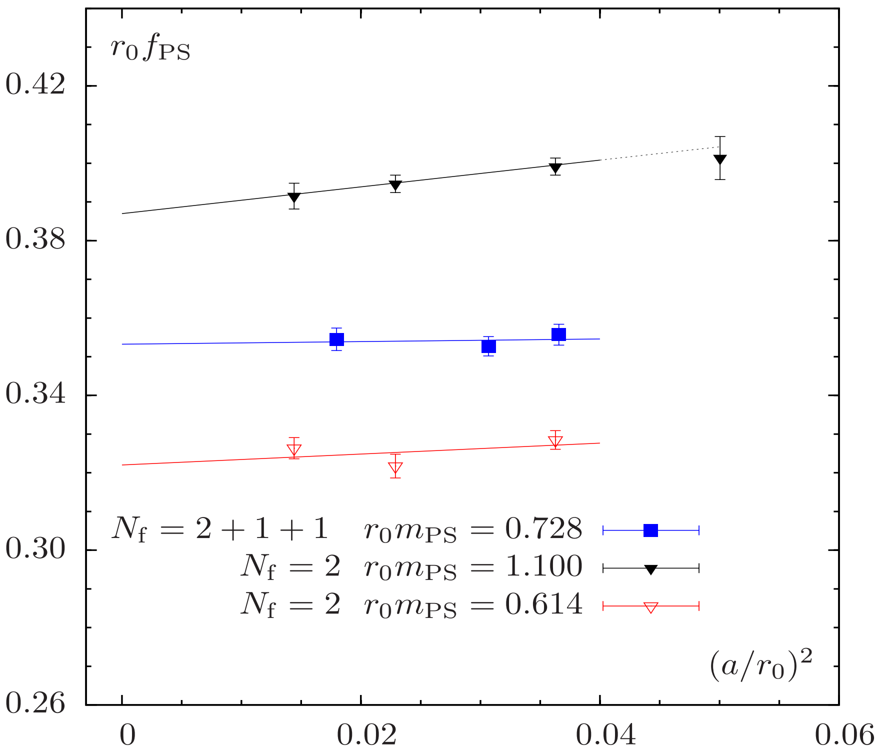
$m_\rho = 1033(31)$ MeV, $\Gamma_\rho = 123(43)$ MeV

Simulation setup for $N_f = 2+1+1$
Configurations available through ILDG

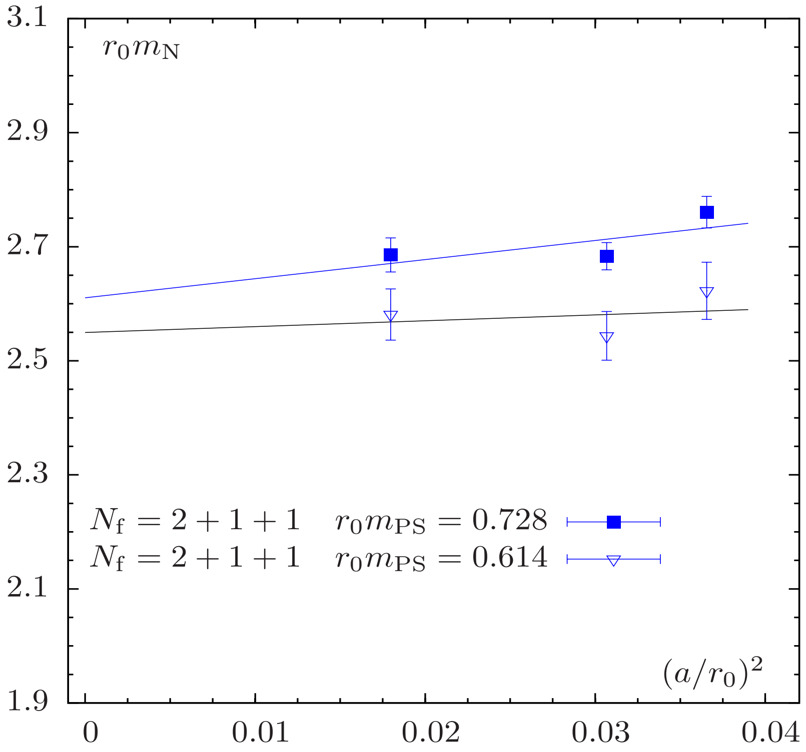
β	a[fm]	L^3T/a^4	m_π[MeV]	status
1.9	≈ 0.085	$24^3 48$	300 – 500	ready
1.95	≈ 0.075	$32^3 64$	300 –500	ready
2.0	≈ 0.065	$32^3 64$	300	ready
2.1	≈ 0.055	$48^3 96$	300 – 500	running/ready
		$64^3 128$	230	thermalizing
		$64^3 128$	200	planned
		$96^3 192$	160	planned

- trajectory length always one
- 1000 trajectores for thermalization
- $\geq$ 5000 trajectores for measurements

$N_f = 2+1+1$ light quark sector: scaling



pseudoscalar decay constant $f_{\rm PS}$

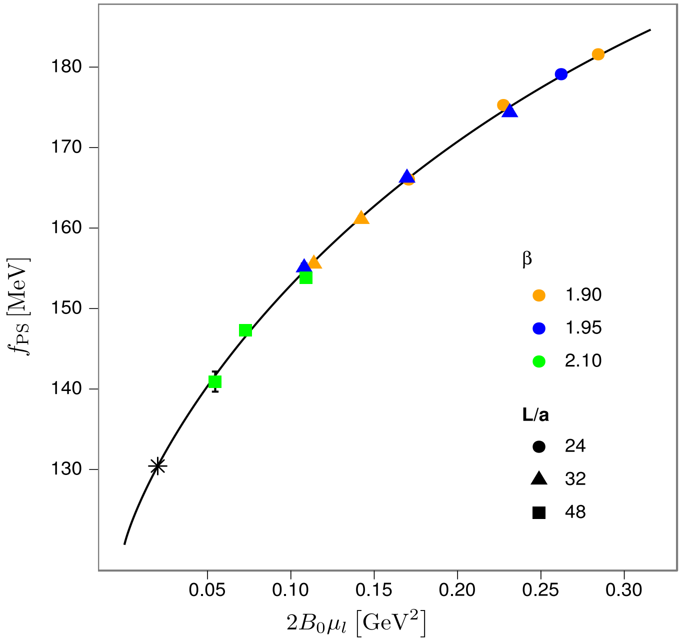


nucleon mass

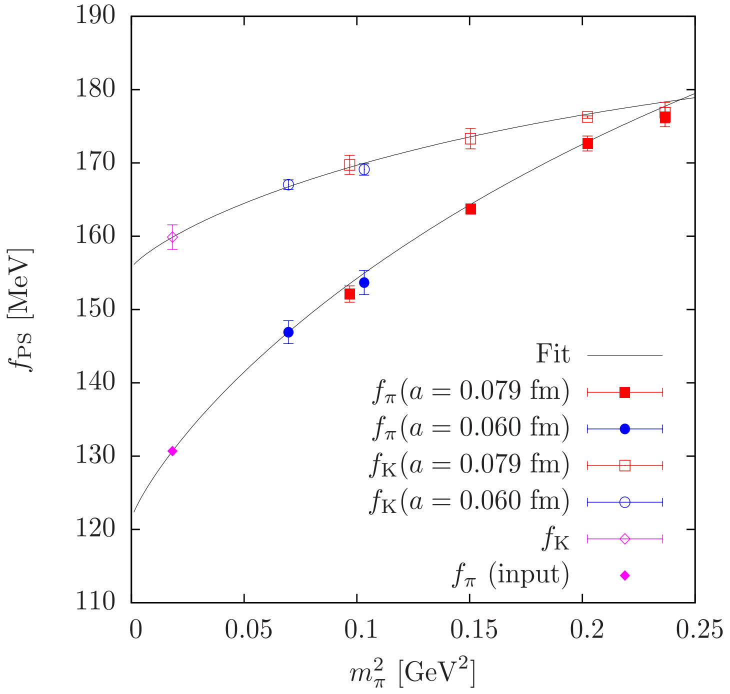
$N_f = 2+1+1$ **light quark sector:** χ**PT fit**

- central values + stat. error : $f_\pi = 130.4(2)$ MeV $\rightsquigarrow$ scale
- estimate systematic effects : lattice artifacts, FSE

	$N_f = 2$	$N_f = 2+1+1$
$\bar{\ell}_3$	3.70(27)	3.50(31)
$\bar{\ell}_4$	4.67(10)	4.66(33)
f_π/f_0	1.076(3)	1.076(9)
B_0 [MeV]	2437(120)	2638(200)
$\langle r^2\rangle_s^{\text{NLO}}$ [fm^2]	0.710(28)	0.715(77)



$N_f = 2+1+1$ light quark sector: adding strange quark



- fit $\beta = 1.95$ and $\beta = 2.10$ simultaneously
- from setting $m^2_{\mathrm{PS}}(\mu_\ell, \mu_s, \mu_s) = 2m_K^2 - m_\pi^2$
- $m_\pi = 135$ MeV, $\quad f_\pi = 130.7$ MeV, $m_K = 497.7$ MeV

prelinimary fit results:

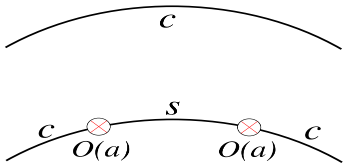
- $f_K/f_\pi = 1.224(13), \qquad f_K = 160(2)$ MeV, $\qquad \bar{\ell}_4 = 4.78(2)$
- errors statistical only

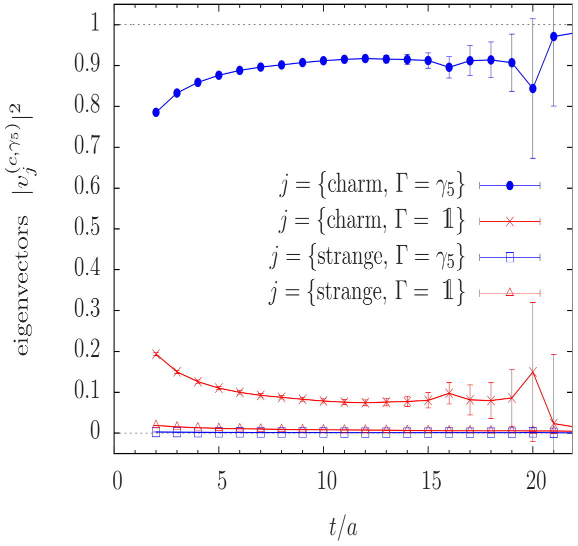
$N_f = 2+1+1$ **heavy quark sector**

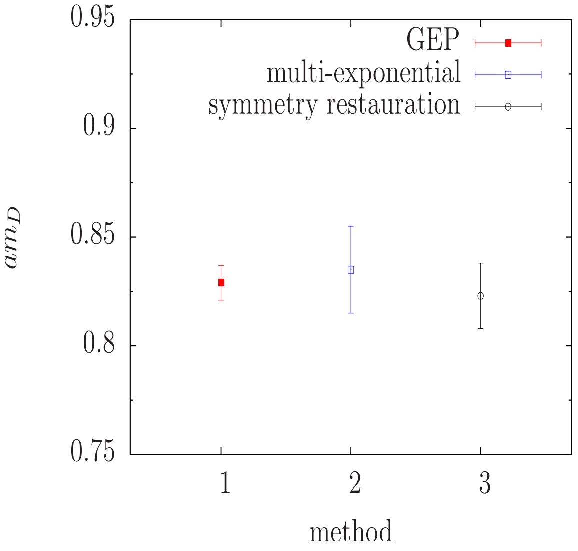
Wilson twisted mass Dirac operator for (c,s) pair:

$$D_h = \begin{pmatrix} \gamma_\mu \tilde{\nabla}_\mu + \mu_\sigma + \mu_\delta & i\gamma_5 \left(\frac{a}{2}\nabla^*_\mu \nabla_\mu - m_q\right) \\ i\gamma_5 \left(\frac{a}{2}\nabla^*_\mu \nabla_\mu - m_q\right) & \gamma_\mu \tilde{\nabla}_\mu + \mu_\sigma - \mu_\delta \end{pmatrix}$$

- mixing of c and s flavour and of parity
- Kaon is the ground state : good precision
- D meson appears as an excited state
- three independent methods:
  - generalised eigenvalue problem
  - multi-exponential fits
  - imposing parity and flavour restoration at finite a
- they provide consistent results for m_D
- overcome mixing of flavour $\rightsquigarrow$ mixed action







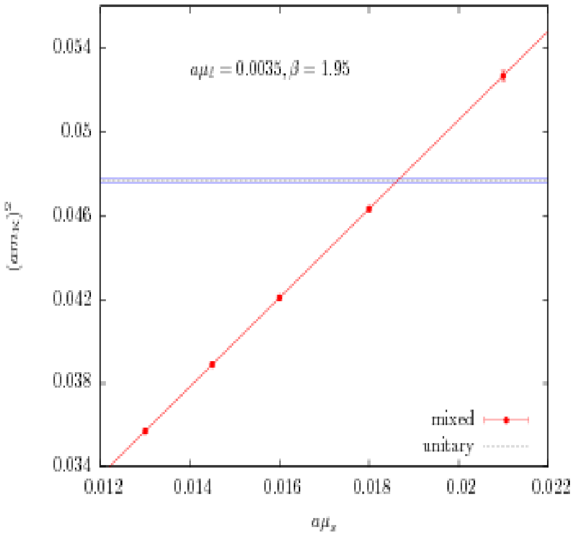
$N_f = 2+1+1$ approaching the charm quark

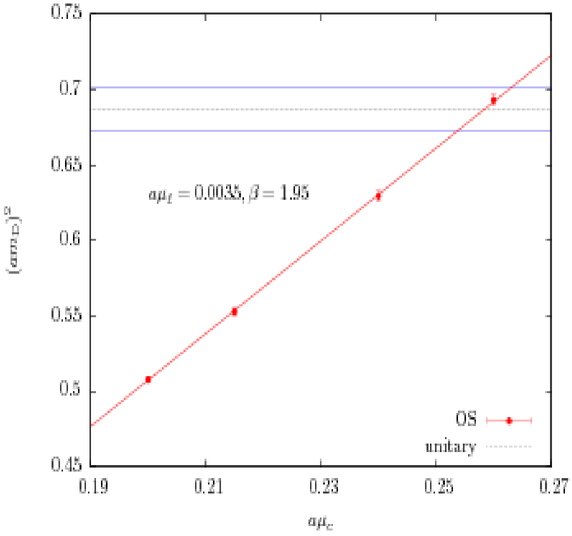
- introduce Wilson twisted mass doublets in the valence sector

$$D_{tm}(\mu_{val}) = D + m_{\mathrm{crit}} + i\ \mu_{val}\gamma_5\tau^3$$

(Osterwalder, Seiler (1990), Pena et al. (2004); Frezzotti, Rossi (2004))

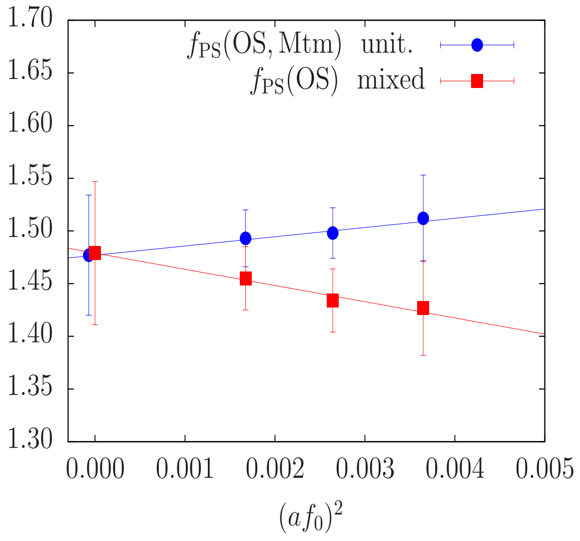
- m_{crit} from unitary set-up
- $4-6$ values for μ_{val} in the strange μ_s and the charm μ_c region
  inversions with multi-mass solver
- matching to unitary set-up using m_K and m_D
  $\Rightarrow$ obtain simulated μ_s and μ_c

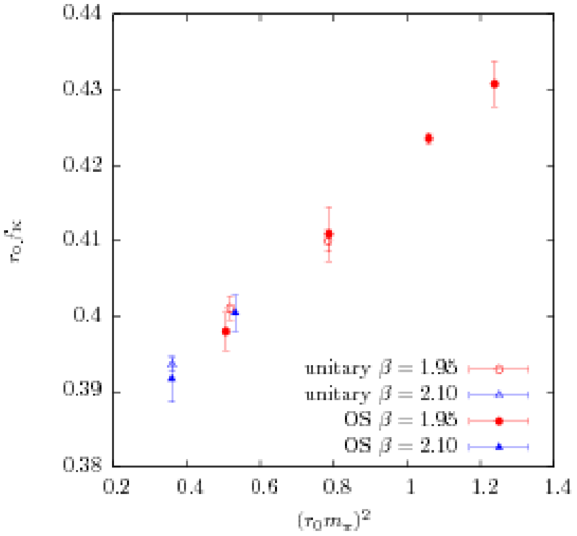




Unitary versus Osterwalder-Seiler: f_K

- the unitary f_K can be computed from: $f_K = (m_\ell + m_s)\frac{\langle 0|P_K|K\rangle}{m_K^2}$
  with $m_s = \mu_\sigma - (Z_\mathrm{P}/Z_\mathrm{S})\mu_\delta$
- similar formula for f_D
- P_K is the physical Kaon projecting operator
- the mixed action f_K computed from: $f_\mathrm{PS} = \left(\mu_{val}^{(1)} + \mu_{val}^{(2)}\right) \frac{|\langle 0|P|PS\rangle|}{m_\mathrm{PS}\sinh m_\mathrm{PS}}$,





Test for $N_f = 2$

situation for $N_f = 2+1+1$

Projection operator

- unitary kaon decay constant

$$f_K = \frac{\mu_\ell + \mu_\sigma - (Z_P/Z_S)\mu_\delta}{2m_K^2} \cdot \langle 0|(P_K - P_D) + i(Z_S/Z_P)(S_K + S_D)|K\rangle$$

- Kaon is lowest state, so flavour mixings should play no role

- mixing of scalar and pseudoscalar

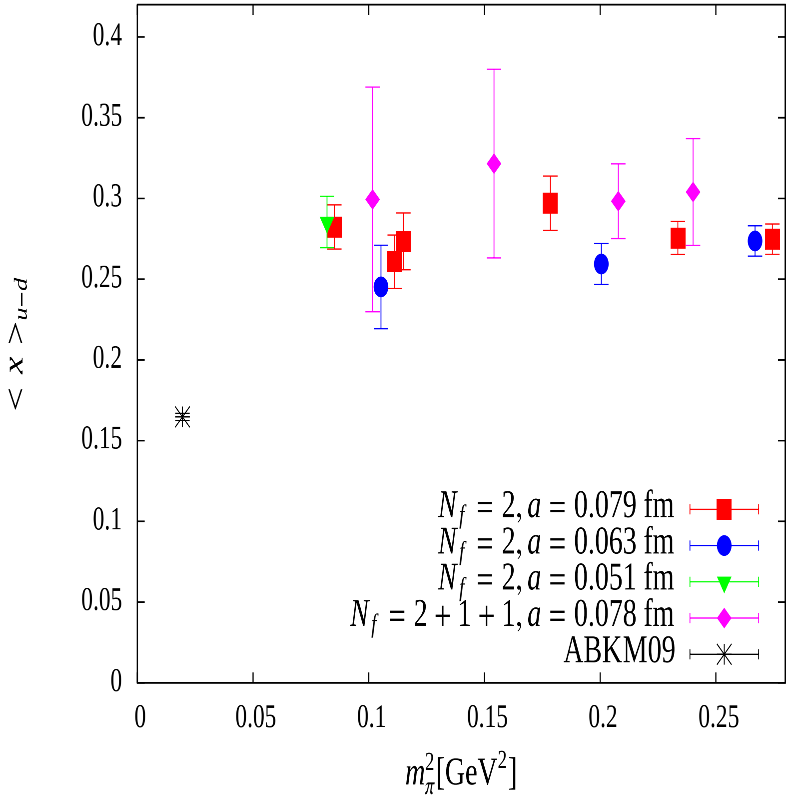
Preliminary analysis of f_D and f_{D_s} in MA set-up

- $SU(2)$ heavy meson χPT fit to our data for $f_{D_s}\sqrt{m_{D_s}}$ and $f_{D_s}\sqrt{m_{D_s}}/(f_D\sqrt{m_D})$
  (ETMC, Blossier et al. (2009))
- including terms proportional to $a^2 m_{D_s}^2$ and $1/m_{D_s}$
- results very encouraging
  $f_{D_s} = 250(3)\ \mathrm{MeV}, \quad f_D = 204(3)\ \mathrm{MeV}, \quad f_{D_s}/f_D = 1.230(6)$
- very preliminary but very first results from $N_f = 2+1+1$!

Nucleon structure for $N_f = 2+1+1$

(C. Alexandrou, M. Constantinou, S. Dinter, V. Drach, D. Renner, K.J.)

First calculation for $\langle x \rangle$ comparison to $N_f = 2$



same effect as for $N_f = 2$: need to explore smaller quark mass region

simulations are underway

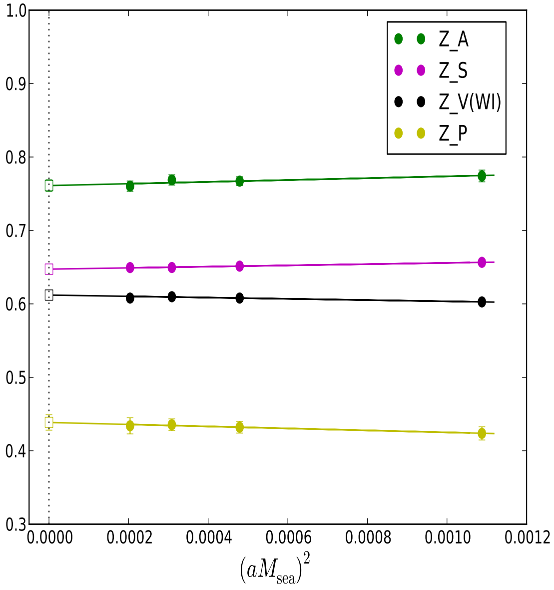
Non-perturbative renormalization for $N_f = 2+1+1$

renormalisation factors computed from dedicated $N_f = 4$ flavour simulations of Wilson fermions

- RI-MOM scheme at non zero values of both the standard and twisted mass parameters

$$M_R = \frac{1}{Z_{\mathrm{P}}} \sqrt{(Z_{\mathrm{A}} m_{\mathrm{PCAC}})^2 + \mu_q^2} \quad \rightarrow \quad 0$$

- $O(a)$ improvement via average of simulations with $+m_{\mathrm{PCAC}}$ and $-m_{\mathrm{PCAC}}$

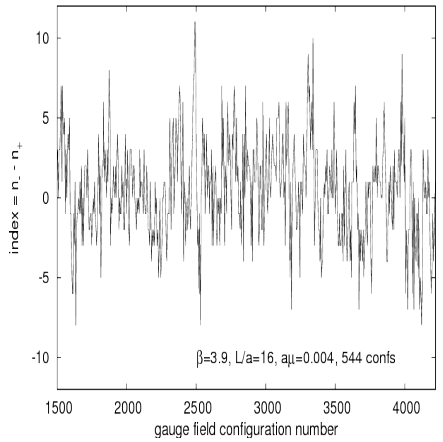


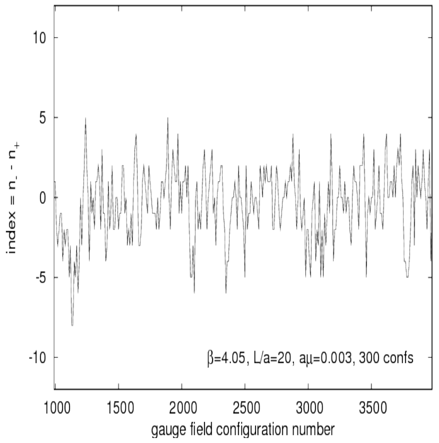
study at $\beta = 1.95$

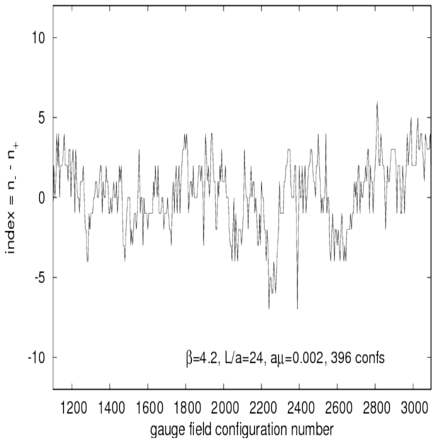
$a = 0.08$ fm, $L = 1.9$ fm

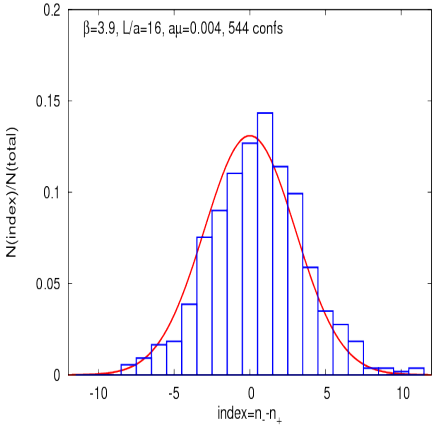
- linear mass dependence
- allows for chiral extrapolation

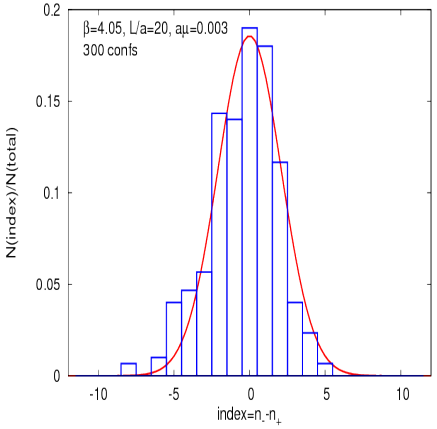
Topology

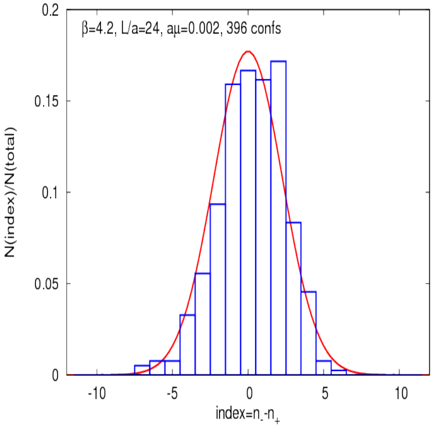

index = $n_- - n_+$
10
5
0
-5
-10
1500
2000
2500
3000
3500
4000
gauge field configuration number
β=3.9, L/a=16, aμ=0.004, 544 confs


index = $n_- - n_+$
10
5
0
-5
-10
1000
1500
2000
2500
3000
3500
gauge field configuration number
β=4.05, L/a=20, aμ=0.003, 300 confs


index = $n_- - n_+$
10
5
0
-5
-10
1200
1400
1600
1800
2000
2200
2400
2600
2800
3000
gauge field configuration number
β=4.2, L/a=24, aμ=0.002, 396 confs


N(index)/N(total)
0.2
0.15
0.1
0.05
0
-10
-5
0
5
10
index=n_--n_+
β=3.9, L/a=16, aμ=0.004, 544 confs


N(index)/N(total)
0.2
0.15
0.1
0.05
0
-10
-5
0
5
10
index=n_--n_+
β=4.05, L/a=20, aμ=0.003
300 confs


N(index)/N(total)
0.2
0.15
0.1
0.05
0
-10
-5
0
5
10
index=n_--n_+
β=4.2, L/a=24, aμ=0.002, 396 confs

Summary

- successful simulations with $N_f = 2$ flavours
  - using maximally twisted mass fermions
  - *automatic* $O(a)$- *improvement*

- First simulations with $N_f = 2 + 1 + 1$ flavours
  - using maximally twisted mass fermions in the sea
  - use Osterwalder-Seiler fermions in the heavy valence sector
  - already precise results for f_K/f_π, f_D, f_{D_s}
  - non-perturbative renormalization under way

- our conclusion: adding stange and charm as dynamical degrees of freedom perfectly feasible