Chiral Effective Field Theory Beyond the Power Counting Regime

Jonathan Hall
Supervisors: Derek Leinweber & Ross Young

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Overview

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- Introduction
- Effective field theory for nucleons
  - Loop integrals
  - Renormalization
- Ideal 'pseudodata'
- Intrinsic energy scale
- Quenched ρ meson mass
- Nucleon magnetic moment
- Nucleon electric charge radius
- Conclusion

Aims

- Chiral Perturbation Theory describes the low energy region, but is limited to use over a very small range of quark masses. How can we overcome this?
- Lattice QCD is difficult to evaluate at physical quark mass, large volumes and small lattice spacings. We want to be able to extrapolate current results to the physical point.
- Using more of the available data often entails scheme-dependence. But the lattice data themselves provide guidance on the choice of scheme.
- This will lead us to realizing the presence of an 'intrinsic energy scale', embedded in lattice QCD data.

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Why use χEFT?

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- Chiral Effective Field Theory (χEFT) complements lattice QCD.
- It assists in understanding the consequences of dynamical chiral symmetry breaking.
- It provides a scheme-independent approach for investigating the properties of hadrons.
- In particular, it can be used in conjunction with lattice QCD data to extrapolate results:
  - to physical quark mass,
  - to infinite lattice volume and the continuum limit.

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- χPT provides a formal expansion in terms of low energy momenta and quark masses.
- The expansion is convergent if the quark mass is small so that higher order terms are negligible. This is called the Power Counting Regime (PCR).
- Within the PCR, χPT is scheme-independent, and can be used to connect lattice simulations to the real world.
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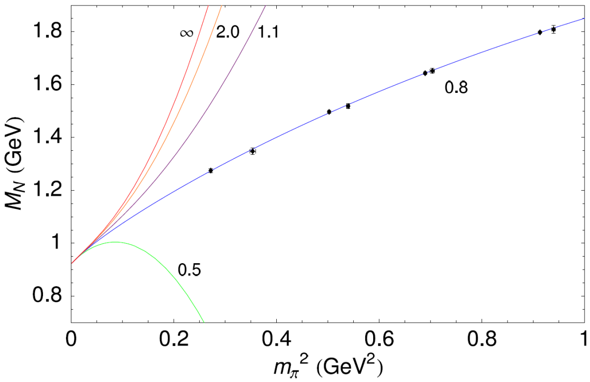
The PCR: Nucleon Mass

- The PCR is small; lattice results often extend outside the PCR.
- Example: The leading order, low energy coefficients are held fixed for different schemes.



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- Using the Gell-Mann–Oakes–Renner Relation, $m_q \propto m_\pi^2$:

$$\begin{aligned} M_N &= \{\text{terms analytic in } m_\pi^2\} + \{\text{chiral loop corrections}\} \\ &= \{a_0 + a_2 m_\pi^2 + a_4 m_\pi^4 + \mathcal{O}(m_\pi^6)\} + \{\Sigma_{\text{loops}}\}\,. \end{aligned}$$

- The analytic coefficients a_i of the 'residual series' will be determined by fitting to lattice QCD data.
- The chiral loops have known, scheme-independent coefficients, but given rise to non-analytic behaviour.

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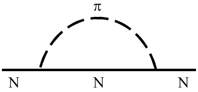
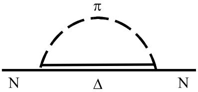
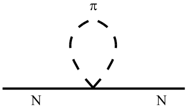
Chiral Loops: Heavy Baryon Limit



$$= \Sigma_N = \frac{\chi_N}{2\pi^2} \int \mathrm{d}^3k \, \frac{k^2 u^2(k;\Lambda)}{\omega^2(k)}$$



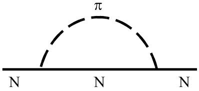
$$= \Sigma_\Delta = \frac{\chi_\Delta}{2\pi^2} \int \mathrm{d}^3k \, \frac{k^2 u^2(k;\Lambda)}{\omega(k)(\Delta + \omega(k))}$$



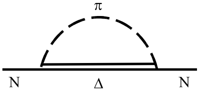
$$= \Sigma_{tad} = m_\pi^2 \frac{\chi_t'}{4\pi} \int \mathrm{d}^3k \, \frac{2u^2(k;\Lambda)}{\omega(k)}$$

- Δ is the mass splitting $M_\Delta - M_N$ and $\omega(k) \equiv \sqrt{k^2 + m_\pi^2}$.

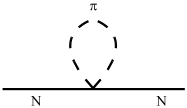
'Taylor' Expansion of Chiral Loops



$$= \Sigma_N = b_0^N + b_2^N m_\pi^2 + \chi_N m_\pi^3 + b_4^N m_\pi^4 + \mathcal{O}(m_\pi^5)$$



$$= \Sigma_\Delta = b_0^\Delta + b_2^\Delta m_\pi^2 + b_4^\Delta m_\pi^4 + \frac{3}{4\pi\Delta}\chi_\Delta m_\pi^4 \log \frac{m_\pi}{\mu} + \mathcal{O}(m_\pi^5)$$



$$= \Sigma_{tad} = b_2^{t'} m_\pi^2 + b_4^{t'} m_\pi^4 + \chi_t' m_\pi^4 \log \frac{m_\pi}{\mu} + \mathcal{O}(m_\pi^5)$$

- Note: each integral expansion has an analytic polynomial, involving $b_i(\Lambda)$, and a non-analytic term.

Renormalization

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- The coefficients χ_N , χ_Δ & χ'_t are known, scheme-independent parameters (related to g_A, f_π, etc).
- The coefficients $b_i(\Lambda)$ however, are scheme-dependent, but they occur at the relevant chiral orders to renormalize the residual series:

$$\begin{aligned} c_0 &= a_0 + b_0^N + b_0^\Delta \,, \\ c_2 &= a_2 + b_2^N + b_2^\Delta + b_2^{t'} \,, \\ c_4 &= a_4 + b_4^N + b_4^\Delta + b_4^{t'} \,, \text{ etc.} \end{aligned}$$

- These renormalized coefficients c_i are scheme-independent.

$$\begin{aligned} M_N &= c_0 + c_2 m_\pi^2 + \chi_N m_\pi^3 + c_4 m_\pi^4 \\ &+ \left(-\frac{3}{4\pi\Delta}\chi_\Delta + \chi'_t\right) m_\pi^4 \log\frac{m_\pi}{\mu} + \mathcal{O}(m_\pi^5) \,. \end{aligned}$$

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Finite-Range Regulators

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- All forms of $u(k;\Lambda)$ are equivalent within the PCR.
- Consider the family of smooth $n-$tuple dipole attenuators:

$$u_n(k;\Lambda) = \left(1 + \frac{k^{2n}}{\Lambda^{2n}}\right)^{-2}.$$

- The dipole corresponds to $n = 1$. We shall also consider the cases $n = 2, 3$, the double and triple dipole forms, respectively.

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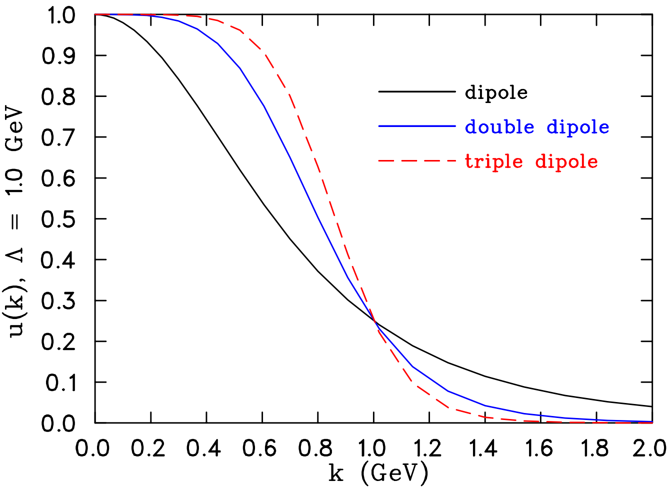
- Here are the three dipole-like forms at $\Lambda = 1.0$ GeV:



Lattice QCD Data

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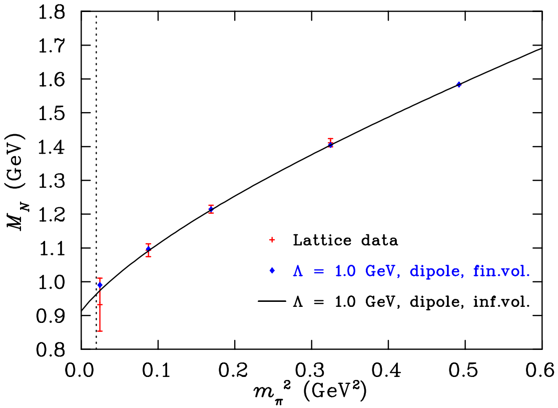
- In this investigation, we use Lattice QCD data, for the nucleon mass, from:
  - PACS-CS (2009), arXiv:0807.1661v1.
  - JLQCD (2008), arXiv:0806.4744v3.
  - CP-PACS (2002), arXiv:hep-lat/0105015v1

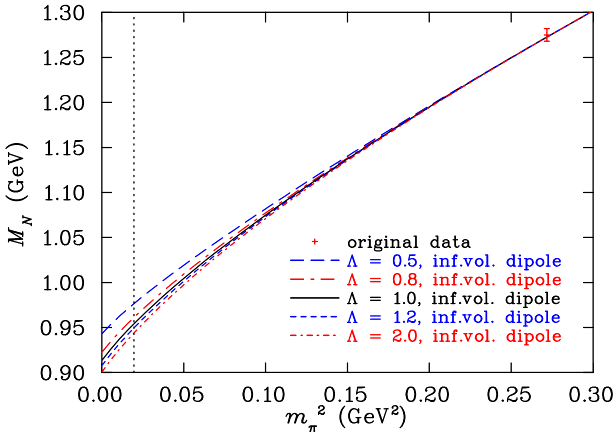
Trial Extrapolations

- Consider an extrapolation of data from PACS-CS, using a dipole regulator with $\Lambda_{\rm dip} = 1.0$ GeV.
- (PACS-CS: non-perturbatively $\mathcal{O}(a)$-improved Wilson quarks, $L = 2.9$ fm).



Trial Extrapolations

- What happens to the extrapolation as $\Lambda_{\rm dip}$ is changed?



Pseudodata

- Different choices of regulator give different results! But is there an optimal choice?
- If we want to stay close to the PCR, how many data points should we use? Does it matter?
- Let's do a test: generate some ideal 'pseudodata' (infinite volume), at $\Lambda^{\text{created}} = 1.0$ GeV.
- As we increase the fit window, ie. increase the maximum m_π^2, does the regulator dependence change?

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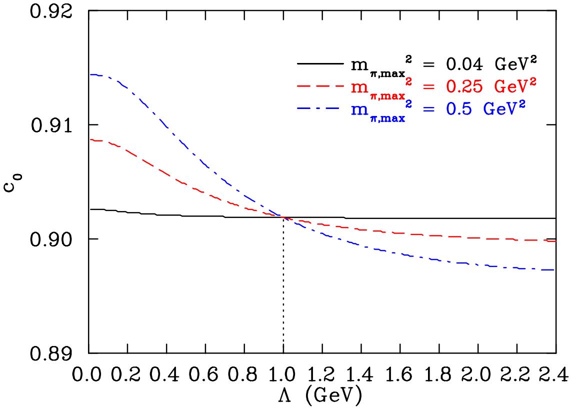
Pseudodata: Renormalization Flow

- Here is the best fit c_0(GeV) renormalization flow.
- Notice that the correct value of c_0 is recovered exactly when $\Lambda_{\text{dip}} = \Lambda_{\text{dip}}^{\text{created}}$.
- Though it is tempting to read off the value of any c_i as $\Lambda \to \infty$, it is wrong (unless constrained to the PCR).



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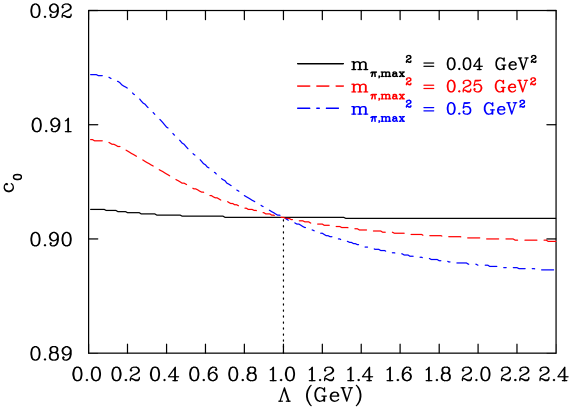
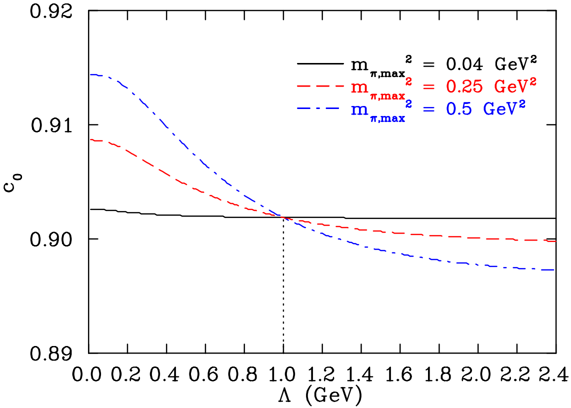


Pseudodata: Renormalization Flow

- Here is the best fit c_0(GeV) renormalization flow.
- Notice that the correct value of c_0 is recovered exactly when $\Lambda_{\text{dip}} = \Lambda_{\text{dip}}^{\text{created}}$.
- Though it is tempting to read off the value of any c_i as $\Lambda \to \infty$, it is wrong (unless constrained to the PCR).



Jonathan Hall
Supervisors: Derek Leinweber & Ross Young

- In the pseudodata test example, the optimal cutoff (by construction) was recovered from the pseudodata themselves.
- But do actual lattice QCD data have an intrinsic scale embedded in them?
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Evidence for an Intrinsic Scale

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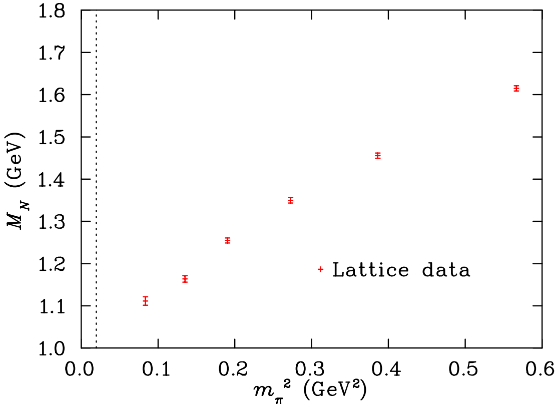
Evidence for an Intrinsic Scale

- Let us repeat our analysis for real lattice QCD results, eg. JLQCD data:



Evidence for an Intrinsic Scale

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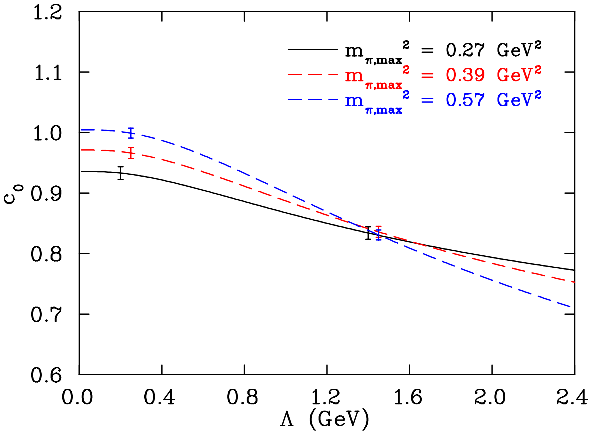
- Here is the renormalization flow for best fit c_0(GeV) using JLQCD data, working to chiral order $\mathcal{O}(m_\pi^3)$ and using a dipole regulator:



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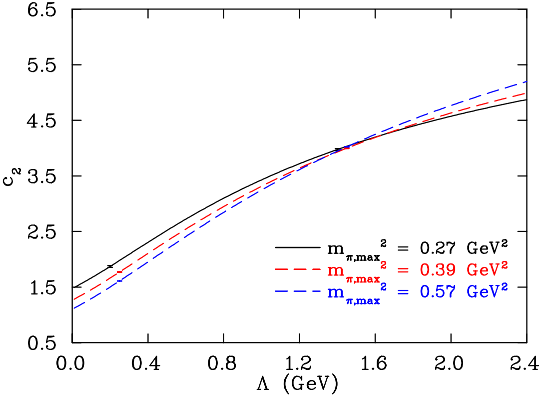
Evidence for an Intrinsic Scale

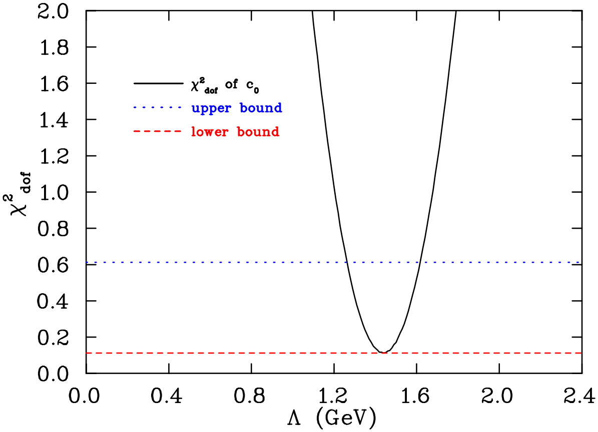
- The intersection occurs at the same value of Λ for both c_0 and c_2.
  This is a highly significant result:



- To obtain a quantitative measure of the intrinsic scale, apply a χ^2_{dof}-style analysis...

Statistical Uncertainty

- Example plot: here is the result for χ^2_{dof} obtained from c_0 using JLQCD data, working to chiral order $\mathcal{O}(m_\pi^3)$ and using a dipole regulator:



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Results

- The intrinsic scales Λ^{scale} (GeV) are tabulated for three different regulators and three different data sets:

	regulator form		
optimal scale	dipole	double	triple
$\Lambda^{\text{scale}}_{c_0,\text{JLQCD}}$	1.44	1.08	0.96
$\Lambda^{\text{scale}}_{c_2,\text{JLQCD}}$	1.40	1.05	0.94
$\Lambda^{\text{scale}}_{c_0,\text{PACS-CS}}$	1.21	0.93	0.83
$\Lambda^{\text{scale}}_{c_2,\text{PACS-CS}}$	1.21	0.93	0.83
$\Lambda^{\text{scale}}_{c_0,\text{CP-PACS}}$	1.20	0.98	0.88
$\Lambda^{\text{scale}}_{c_2,\text{CP-PACS}}$	1.19	0.97	0.87

Nucleon Mass Summary

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- The renormalization curves for different m_π^2 fit windows intersect at a well-defined point.
- This is true for a variety of regulators.
- In each case, c_0 and c_2 agree on the intrinsic scale Λ^{scale}.
- This indicates that lattice QCD data provides guidance in selecting an optimal scheme for χEFT.

'The Challenge'

Jonathan Hall
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- Consider the quenched ρ meson.
- *Challenge:* We want to predict the mass of the quenched ρ meson at physical pion mass ($m_{\pi,\text{phys}} = 140$ MeV).
- We have quenched lattice QCD (QQCD) results from the Kentucky Group, but we are blinded to the lowest energy data.
- QQCD observables are an important testing ground, since there are no experimentally known values that can introduce a prejudice in the final result.

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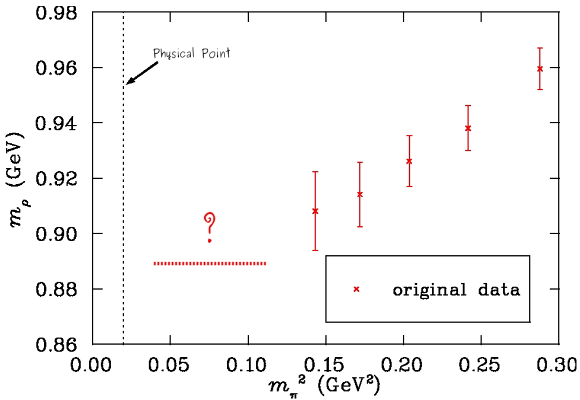
QQCD Data from the Lattice

- The following data from Kentucky Group ($L = 3.06$ fm) are missing points close to the chiral limit ($m_q = 0$).
- The available data lie in the range $380 < m_\pi < 720$ MeV,
- The unavailable data lie in the range $200 < m_\pi < 380$ MeV.



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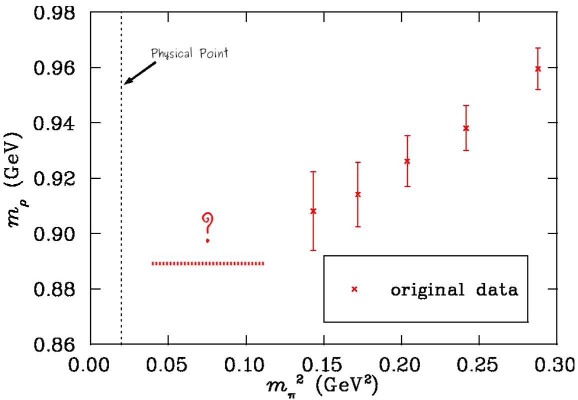

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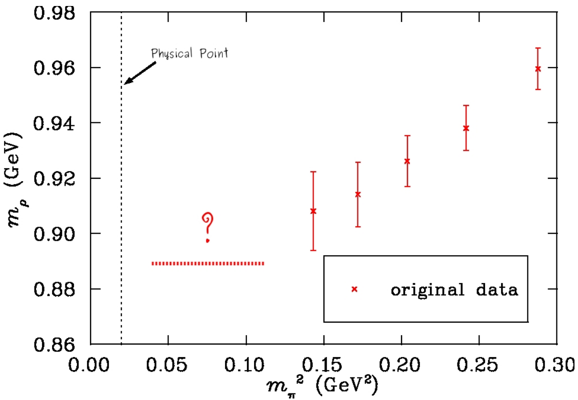

Chiral Extrapolation Formulae

Jonathan Hall
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- The quenched ρ meson mass expansion similarly contains a residual series and loop integrals.
- We will work to chiral order $\mathcal{O}(m_\pi^4)$.
- The renormalization of the low energy constants takes place just as before. The fit parameters are a_0, a_2 and a_4.

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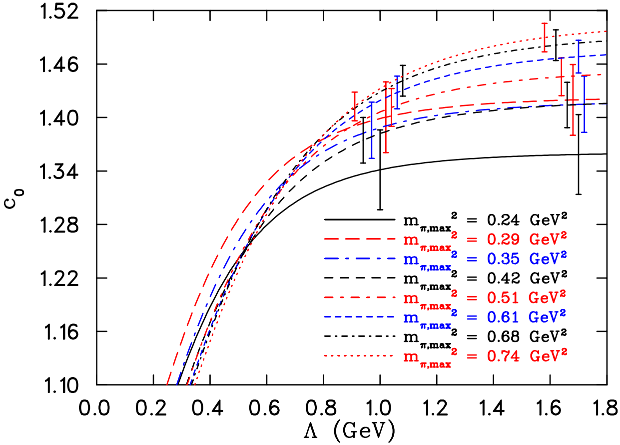
- Here is the renormalization flow for c_0 using Kentucky Group data, working to chiral order $\mathcal{O}(m_\pi^4)$ and using a triple dipole regulator:



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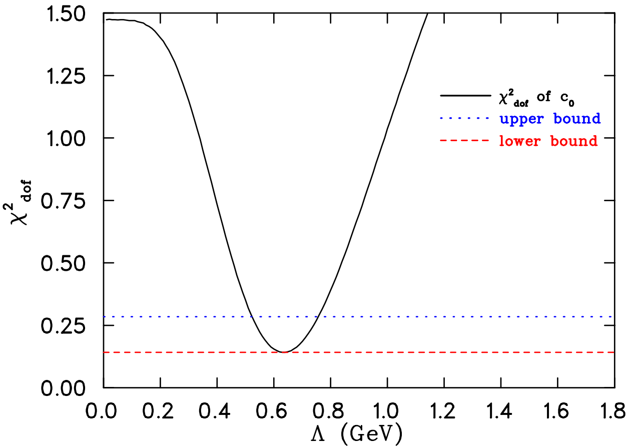
Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Magnetic Moment
Electric Charge Radius
Conclusion

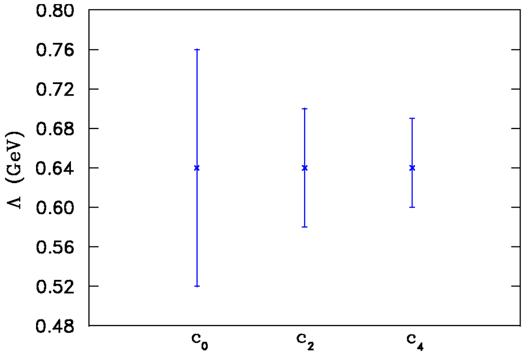
Test for an Intrinsic Scale

- The crossings are much harder to identify, so we will rely on our χ^2_{dof} method:



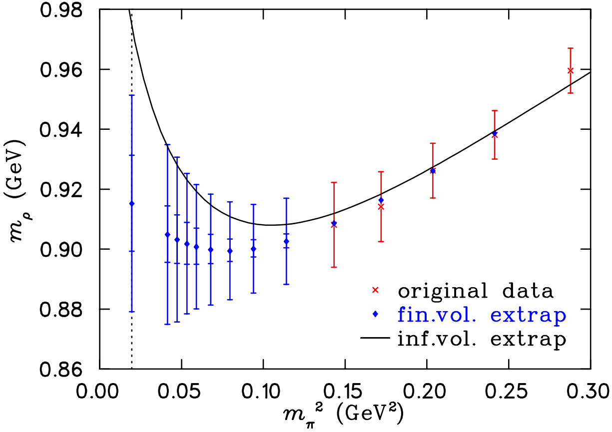
Test for an Intrinsic Scale

- The intrinsic scales for the low energy constants c_i:



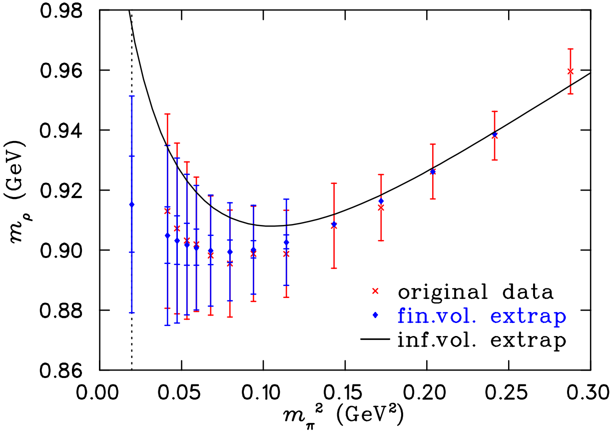
Completing 'The Challenge'

- Inner error bar: systematic error from parameters.
- Outer error bar: systematic and statistical errors in quadrature.



Completing 'The Challenge'

- Now, we 'unblind' the lattice results (red):



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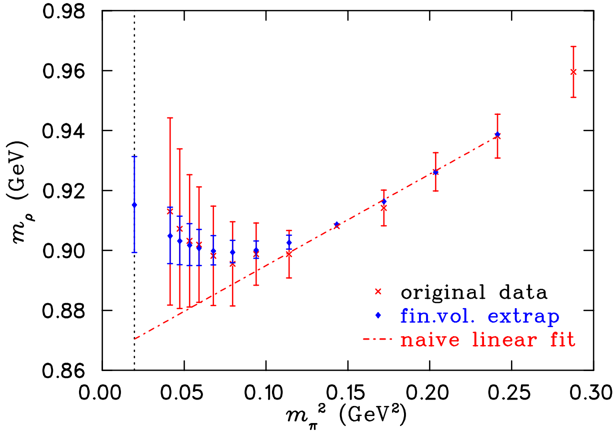
Completing 'The Challenge'

- Here, the error bars are correlated relative to the lightest data point in the original set, $m_\pi^2 = 0.143$ GeV^2.
- A factor of 10 times the computing time would be required for further comparison.



Nucleon Isovector Magnetic Moment

Jonathan Hall
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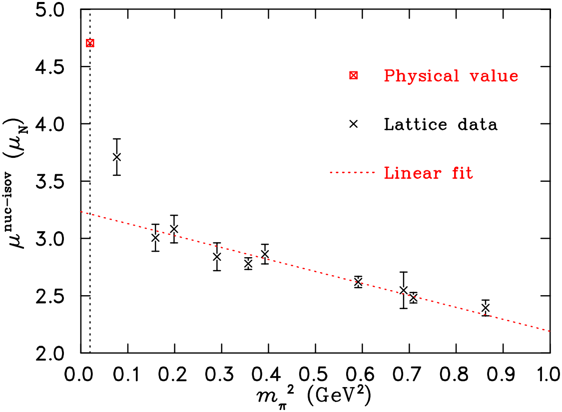
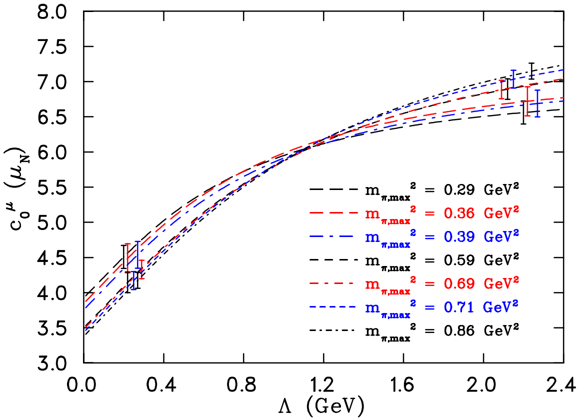
- Here is some lattice QCD data for μ^{isov} from QCDSF (Zanotti):

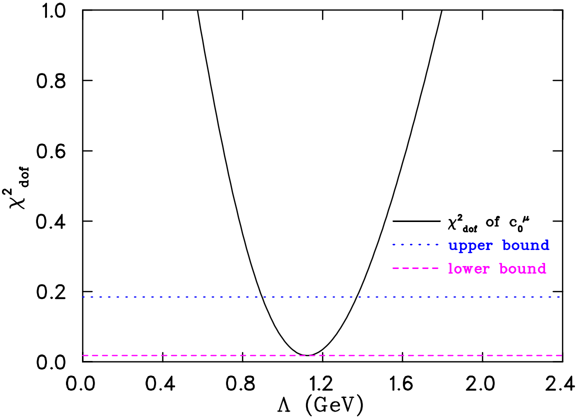


- The renormalization flow of c_0 is obtained using a dipole regulator:



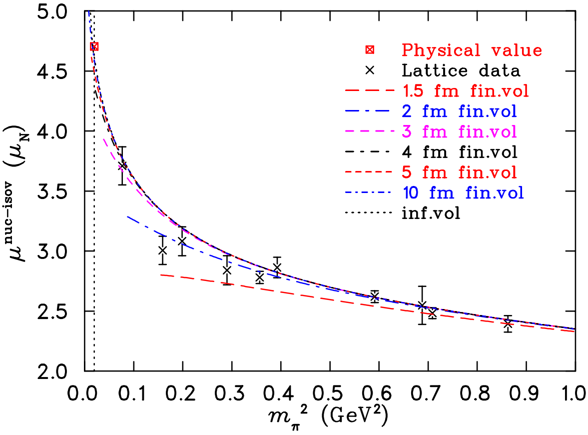
Test for an Intrinsic Scale

- The χ^2_{dof} analysis shows a distinct intrinsic scale of $\Lambda^{\text{scale}} = 1.1$ GeV (± 0.2) GeV :



Magnetic Moment: Extrapolations

- Extrapolations at different finite volumes, or infinite volume, are now possible.
- We constrain $m_\pi L > 3$.



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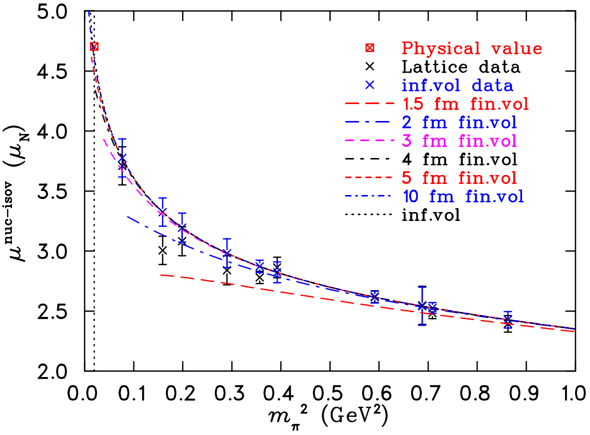
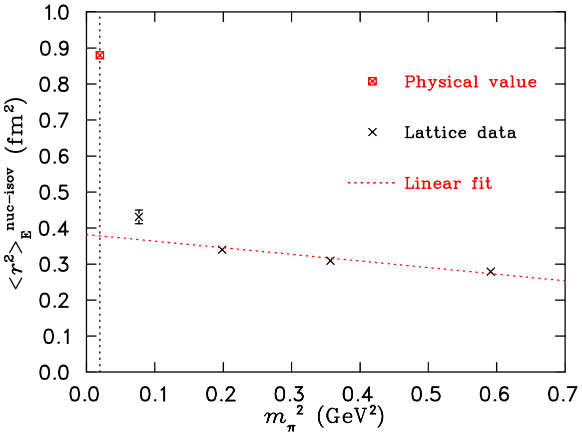


Nucleon Isovector Electric Charge Radius

- Here is some lattice QCD data for $\langle r^2 \rangle_E^{\text{isov}}$ from QCDSF (Zanotti):



Test for an Intrinsic Scale

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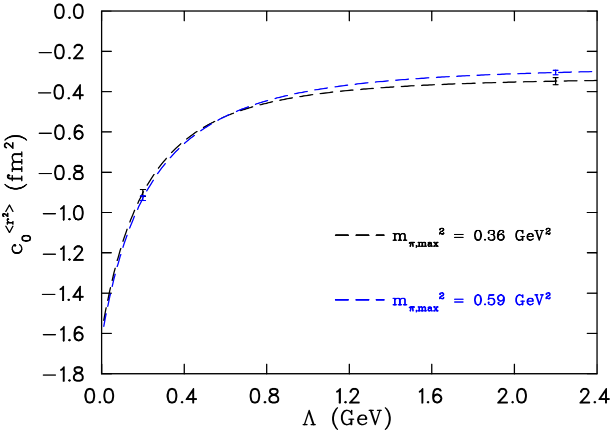
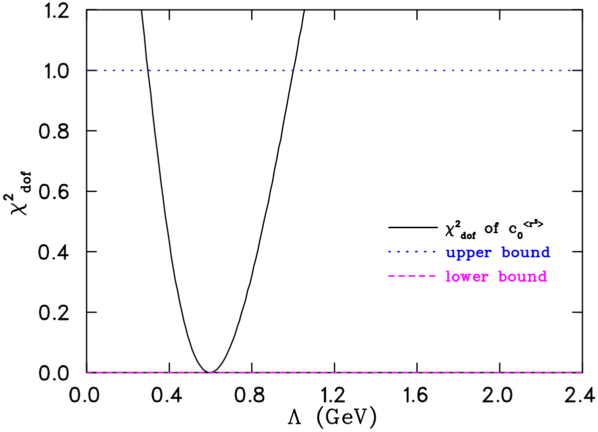


Test for an Intrinsic Scale

- The χ^2_{dof} analysis shows an intrinsic scale of $\Lambda^{\text{scale}} = 0.60$ GeV $(+0.40 - 0.30)$ GeV :



Electric Charge Radius: Extrapolations

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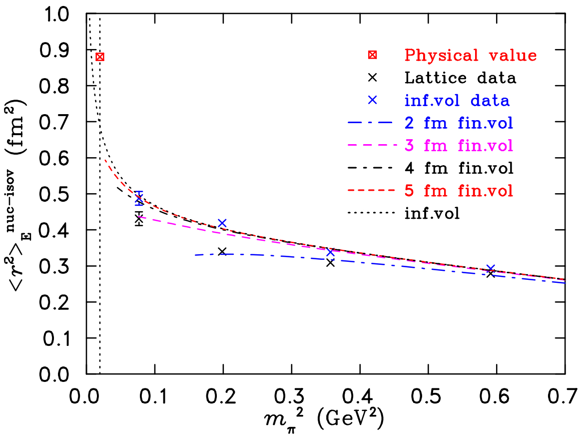

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Conclusion

- We have been able to extrapolate current lattice QCD results to the physical point, using Chiral Effective Field Theory.
- We have discovered that Finite-Range Regularized Chiral Effective Field Theory is instrumental for the analysis of data extending outside the chiral Power Counting Regime.
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Thank You

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Gary Larson (1995), *The Far Side Gallery 2*.