The Large Nc Limit**s** of QCD

Co-conspirators:
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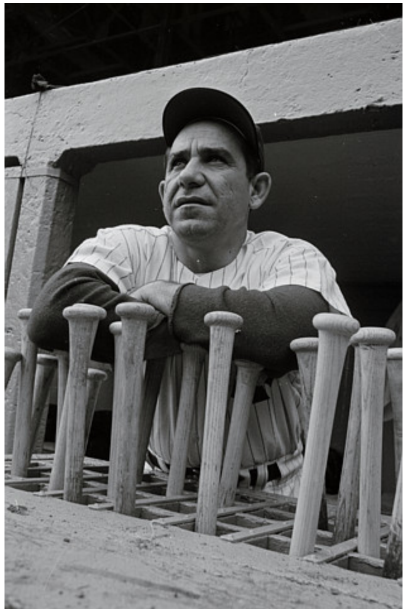
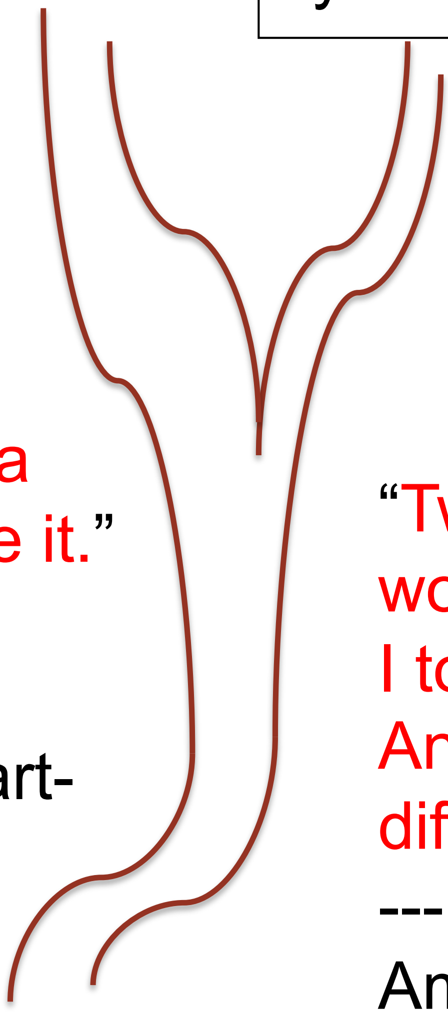


Quarks in Fundamental

Quarks in 2-index anti-symmetric



"When you come to a fork in the road, take it."
---Yogi Berra,
American baseball player, coach and part-time philosopher

"Two roads diverged in a wood, and I—
I took the one less traveled by
And that has made all the difference."
---Robert Frost,
American poet

Large Nc QCD

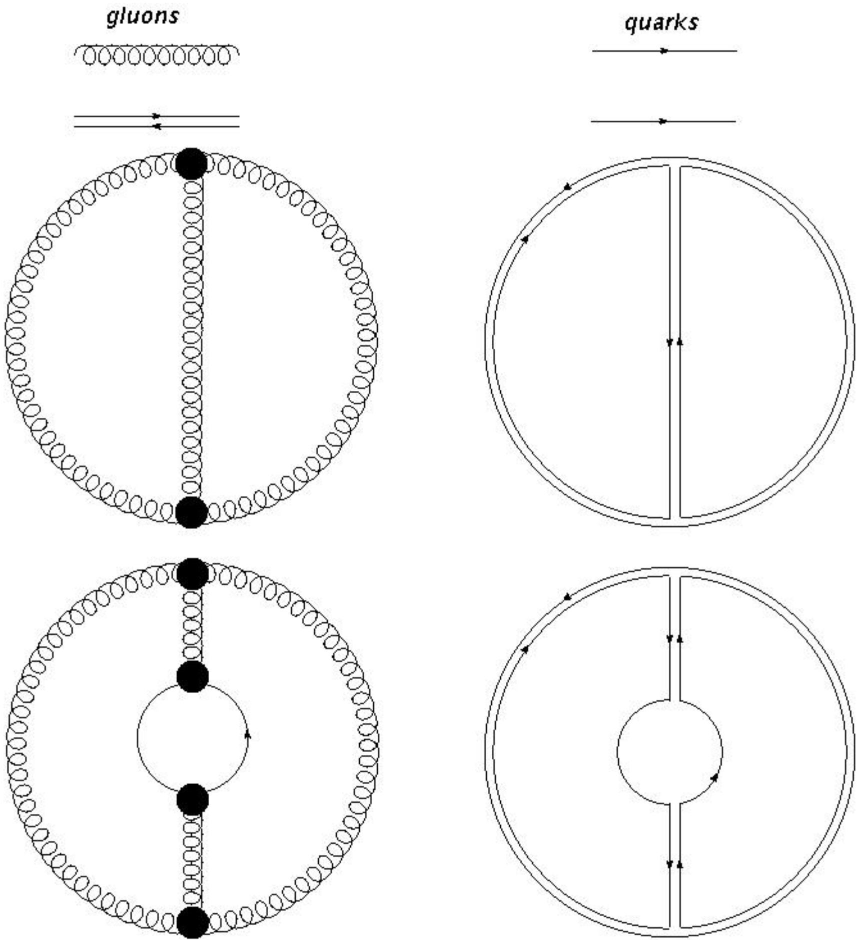
QCD and its large Nc limits:

- The large Nc limit of QCD is not unique
  - For gluons there is a unique prescription SU(3)➔SU(Nc)
  - However for quarks, we can choose different representations of the gauge group
  - Asymptotic freedom restricts the possibilities to the fundamental (F), adjoint (Adj), two index symmtetric (S), two index anti-symmtetric (S),
    - Adj transforms like gluons (traceless fundamental color-anticolor); dimension Nc^2-1; 8 for Nc=3 (unlike our world).
    - S transforms like two colors (eg fundamental quarks) with indices symmetrized; dimension Nc^2-Nc; 6 for Nc=3 (unlike our world).
    - AS transforms like two colors (eg fundamental quarks) with indices antisymmetrized; dimension ½Nc(Nc-1); 3 for Nc=3 (just like our world).

- Note that Nc=3 quarks in the AS representation are indistinguishable from the (anti-) fundamental.
- However quarks in the AS and F extrapolate to large Nc in different ways.
  - The large Nc limits are physically different
  - The 1/Nc expansions are different.
  - A priori it is not obvious which expansion is better
  - It may well depend on the observable in question
- The idea of using QCD (AS) at large Nc is old
  - Corrigan &Ramond (1979)
  - Idea was revived in early part of this decade by Armoni, Shifman and Veneziano who discovered a remarkable duality that emerges at large Nc.

Principal difference between QCD(AS) and QCD(F) at large Nc is in the role of quarks loops

Easy to see this using `t Hooft color flow diagrams



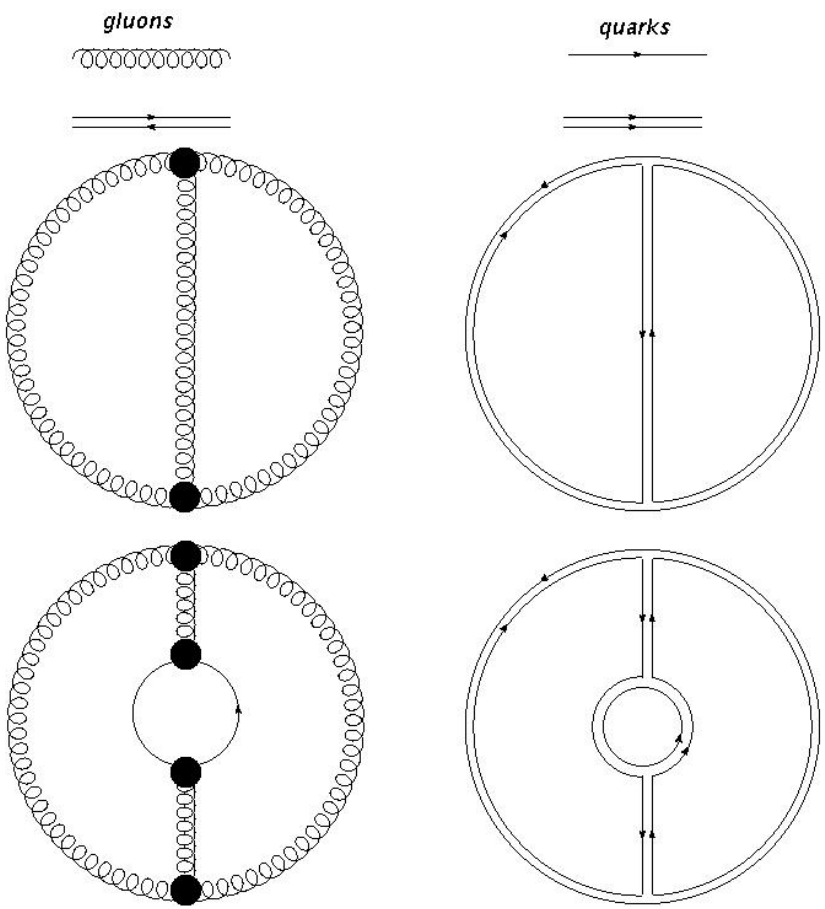
$$\left.\begin{array}{r} g^2 \sim 1/N_c \\ \text{3 color loops } N_c^3 \end{array}\right\} N_c^2$$

$$\left.\begin{array}{r} g^4 \sim 1/N_c^2 \\ \text{3 color loops } N_c^3 \end{array}\right\} N_c$$

QCD(F)

Insertion of a planar quark loops yields a 1/Nc suppression.

Leading order graphs are made of planar gluons



QCD(AS)

$$\left.\begin{array}{r} g^2 \sim 1/N_c \\ 3 \text{ color loops } N_c^3 \end{array}\right\} N_c^2$$

Insertion of a planar quark loops does not lead to a 1/Nc suppression.

$$\left.\begin{array}{r} g^4 \sim 1/N_c^2 \\ 4 \text{ color loops } N_c^4 \end{array}\right\} N_c^2$$

Leading order graphs are made of planar gluons and quarks

Principal phenomenological difference between the two is the inclusion of quark loop effects at leading order in QCD (AS)

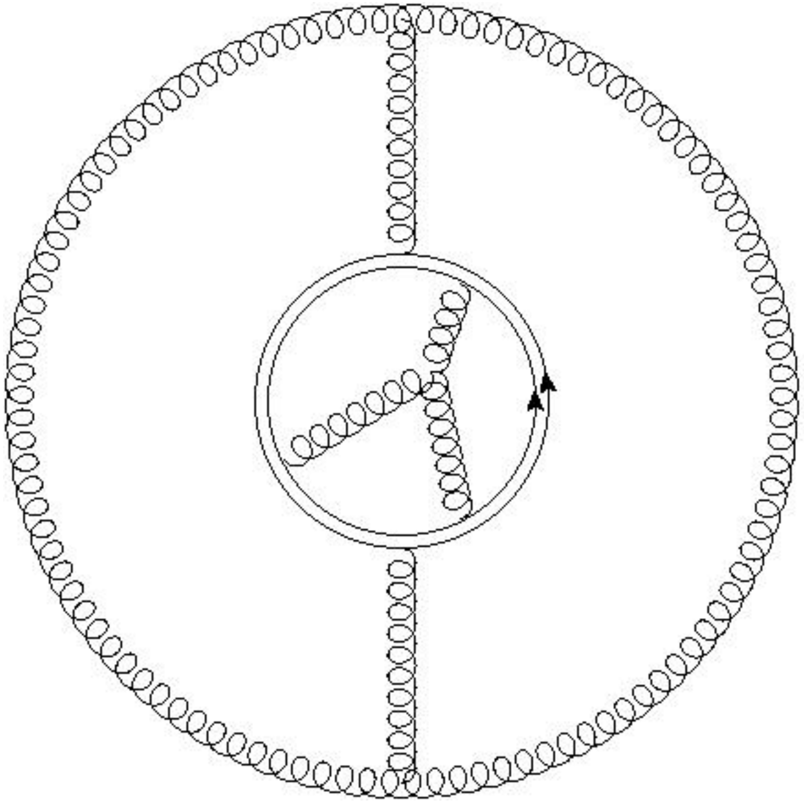
A remarkable fact about QCD(AS):

At large Nc, QCD(AS) with Dirac fermions becomes equivalent to QCD(Adj) with Majorona fermions for a certain class of observables. These "neutral sector" observables include $\langle \bar{q} q \rangle$.

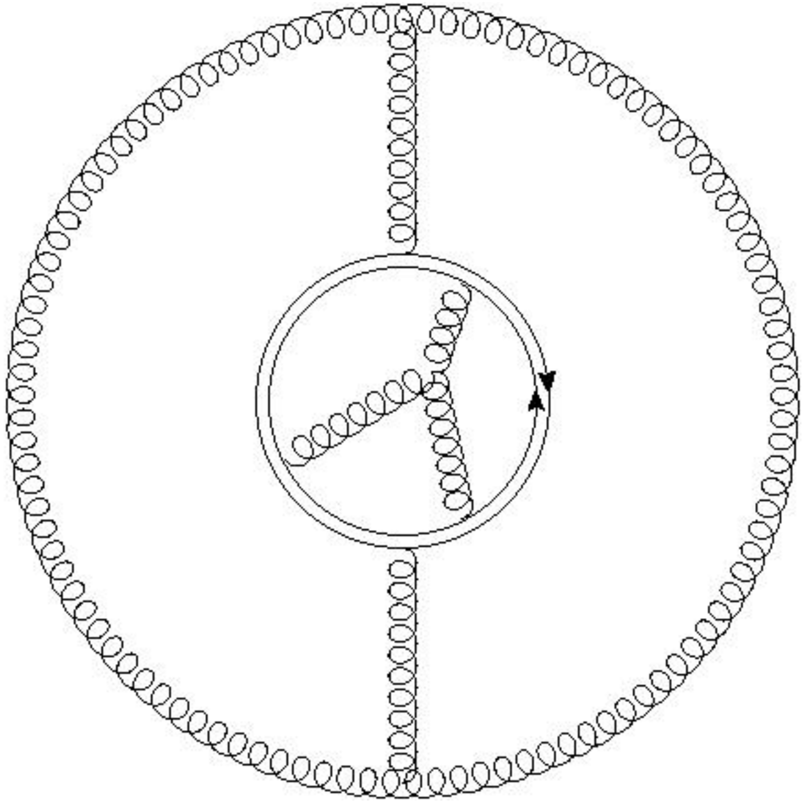
The full nonperturbative demonstration of this by Armoni, Shifman and Venziano (ASV)is quite beautiful and highly nontrivial. There is a simple hand waving argument which gets to the guts of it

Due to large Nc planarity, any fermion loops divide any gluons in a diagram into those inside and those outside.

With two index representations the "inside" gluons couple to the inner color line of the quark and "outside" gluons to the outer ones



QCD(AS)



QCD(Adj)

Since the inside gluons don’t know about what happens outside, one can flip the direction of color flow on the outside without changing the dynamics.

This equivalence is pretty but can you make any money on it?

If all you can do is relate one intractable theory to another, it would be of limited utility.

However: QCD(Adj) with a single massless quark is $\mathcal{N}$=1 SUSY Yang-Mills. Thus, at large Nc a non-Supersymmetric theory (QCD(AS) with one flavor) is equivalent to a supersymmetric theory. Thus one can use all the power of SUSY to compute observables in $\mathcal{N}$=1 SYM and at large Nc one has predicted observables in QCD(AS) !

Can you make any *phenomenological* money on it?

Real QCD has more than one flavor!!!

ASV scheme: Suppose you put the quarks one flavor in the AS representation and the other flavor(s) in the F. For example put up quarks in AS and down quarks in F. The ones in the F are dynamically suppressed at large Nc and the theory again becomes equivalent to $\mathcal{N}$=1 SYM.

My view: the scheme is likely not be viable phenomenologically. It **BADLY** breaks flavor symmetry for any Nc≠3.

Accordingly in the remainder of this talk I will focus entirely on the cases where all flavors are either AS or F.

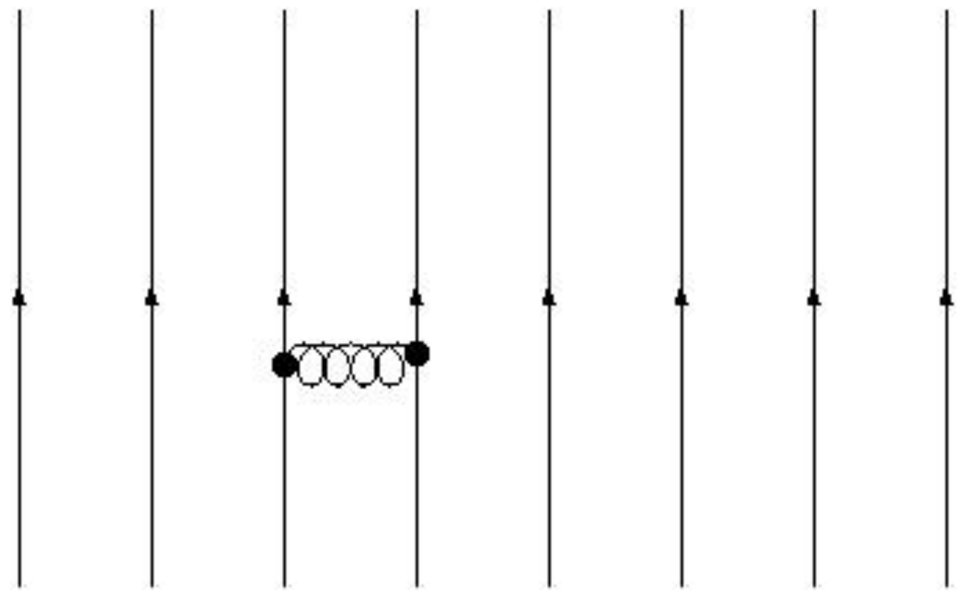
Generic Virtues and Vices of QCD(AS) and QCD(F) at large Nc

		
QCD(F)	Explains the success of the OZI rule in a natural way	Fails to explain effects involving the anomaly (eg. η’)
QCD(AS)	Naturally includes effects involving the anomaly	Fails to explains the success of the OZI rule

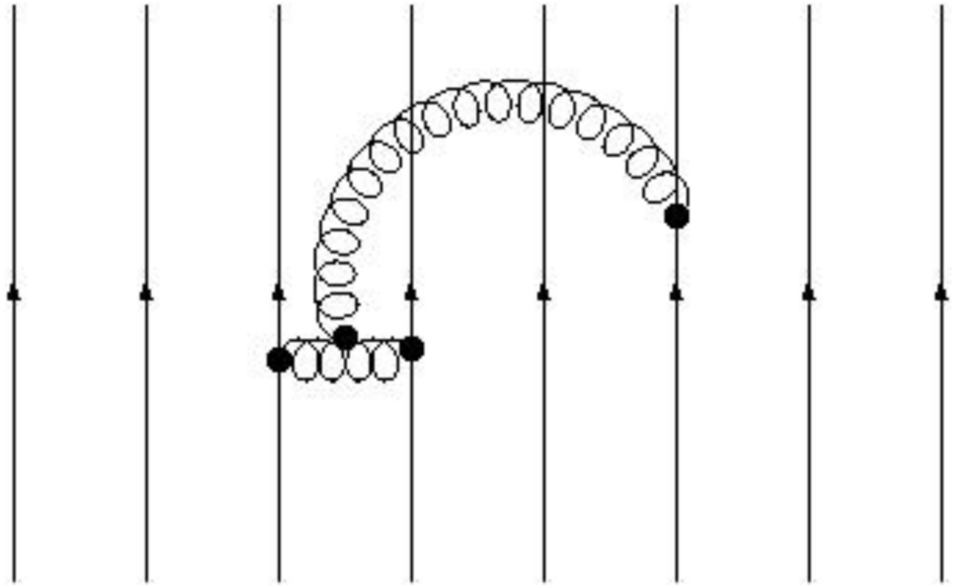
Implication for Baryons

- Baryons are heavy
  - QCD(F) $M_N \sim Nc$ (Consistency shown by Witten 1979)
  - QCD(AS) $M_N \sim Nc^2$ (Consistency shown by Cherman&TDC 2006, Bolognesi 2006; TDC, Lebed, Schafer 2010)

QCD(F): There are Nc quarks each of which contributes to the energy as it propagates. The interactions between between quarks also contribute of order Nc.



$$\left.\begin{array}{c} g^2 \sim 1/N_c \\ \text{Combinatoric Factor } N_c^2 \\ \text{(each vertex has } N_c \text{ possibilities} \end{array}\right\} N_c$$



$$\left.\begin{array}{c} g^4 \sim 1/N_c^2 \\ \text{Combinatoric Factor } N_c^3 \\ \text{(each vertex has } N_c \text{ possibilities)} \end{array}\right\} N_c$$

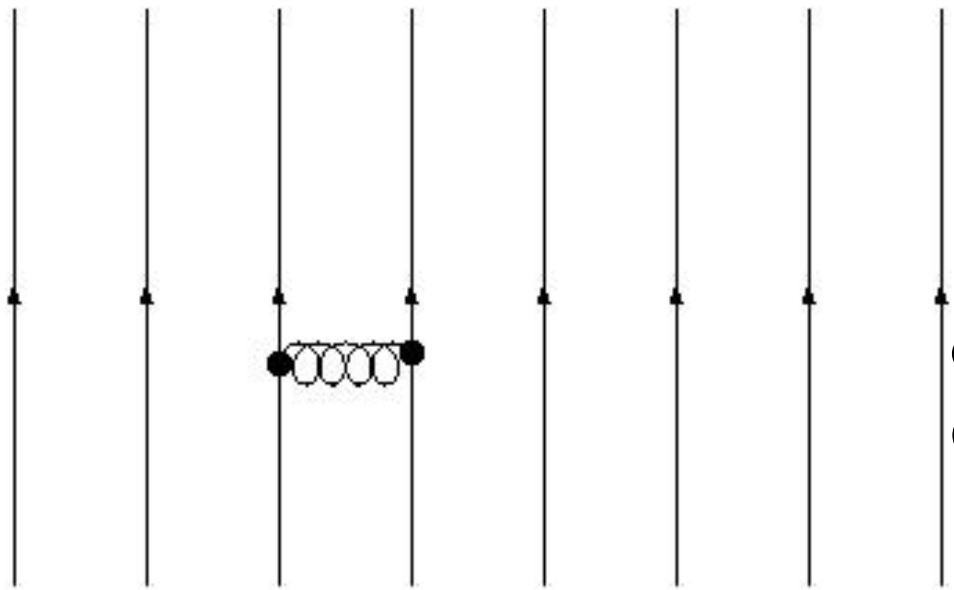
Relatively easy to see that all classes of connected diagram contribute at order Nc or less to the mass in QCD(F).

What about QCD(AS)?

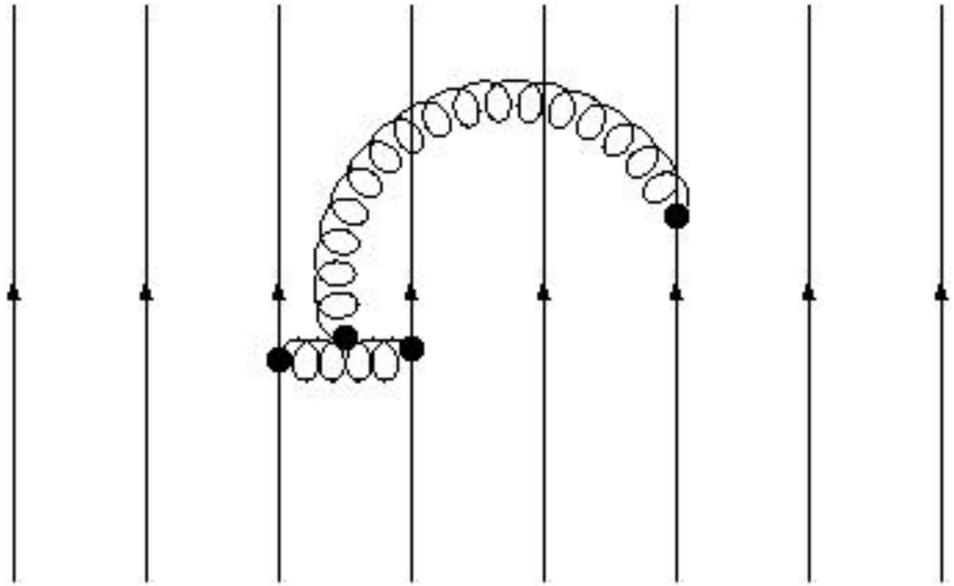
Bolognesi showed that a color singlet baryon had each kind quark color once and only once: Nc(Nc-1)/2 quarks. Thus one expects baryon mass to scale as Nc^2

- There is a problem: apply Witten's reasoning and there is an inconsistency---the interactions don't appear to scale as N_c^2

Look at the one-gluon contribution



$$\left.\begin{array}{c} g^2 \sim 1/N_c \\ \text{Combinatoric Factor } \sim N_c^4 \\ (\text{each vertex has } \sim N_c^2 \text{ possibilities}) \end{array}\right\} N_c^3 \neq N_c^2$$



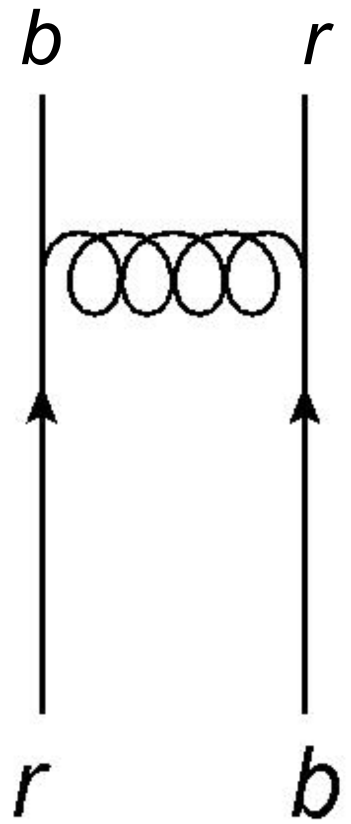
$$\left.\begin{array}{l} g^4 \sim 1/N_c^2 \\ \text{Combinatoric Factor } \sim N_c^6 \\ \text{(each vertex has } \sim N_c^2 \text{ possibilities)} \end{array}\right\} N_c^4 \neq N_c^2$$

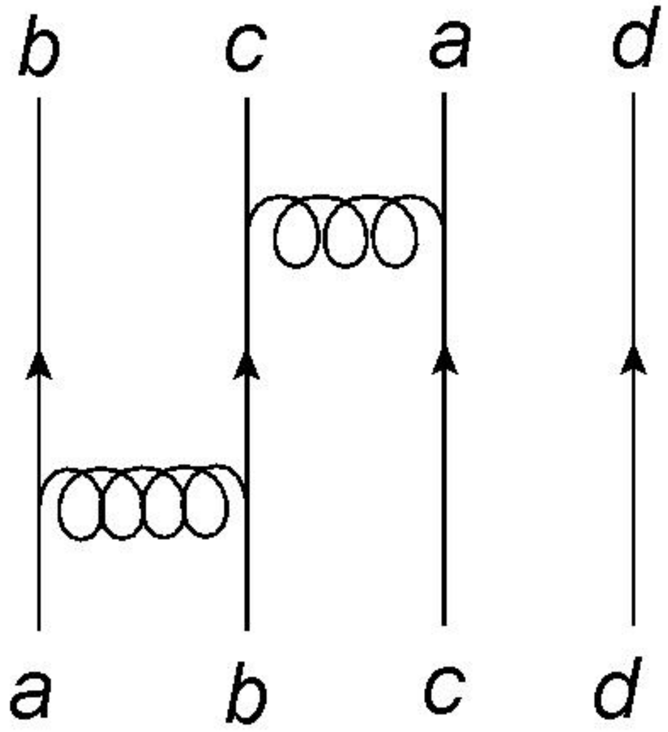
Even worse!!

What's going on?

The combinatorics are wrong. There is a subtlety which does not arise in the case of QCD(F)

Examples (in QCD_F)





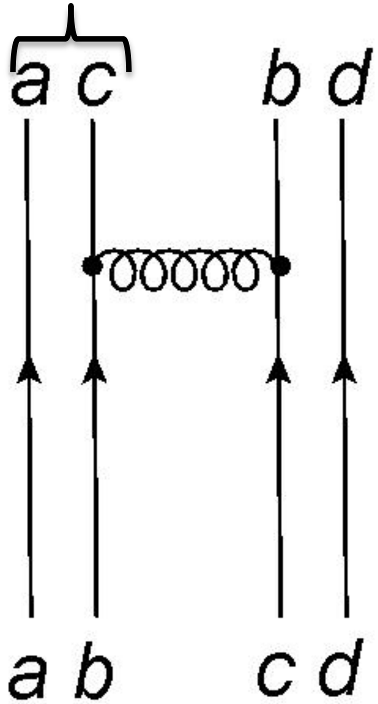
Gluon exchange simply flips colors of quarks

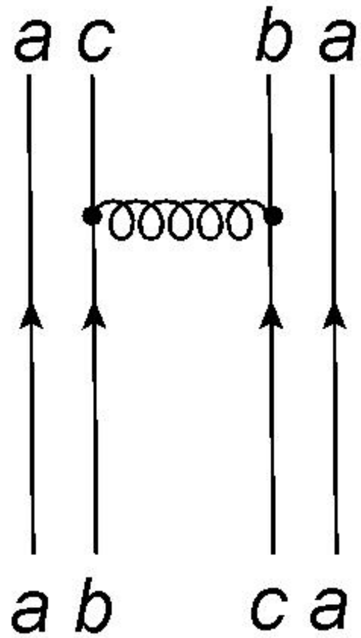
Final quark colors are same as initial ones; all such exchanges are allowed for color singlets.

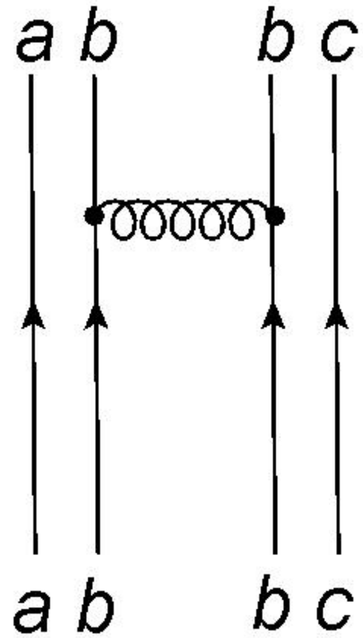
The Case of QCD(AS)

Not all exchanges contribute in a color singlet, (which requires each color combination once and only once).

Each quark has 2 color indices







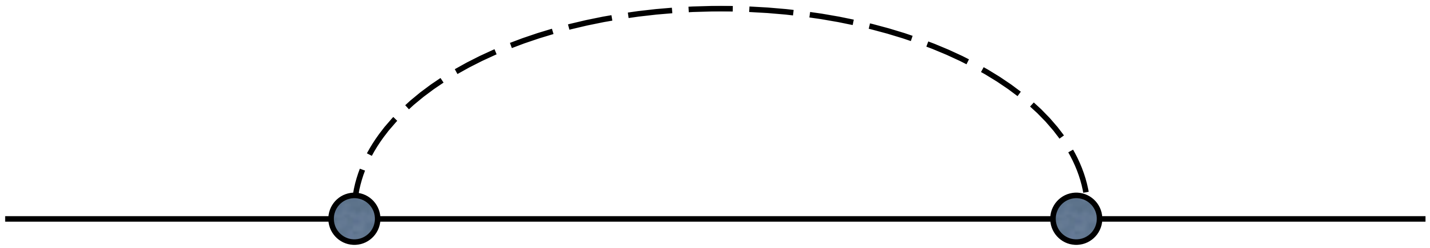
Naively, $O(Nc^3)$ but No contribution

Contributes $O(Nc^2)$

Contributes $O(Nc^2)$

- This fact suppresses many of the combinatoric factors.
- A Cherman & TDC(2006) showed that for a wide a class of diagrams the total contributions are ~Nc^2 as needed.
  - However general proof was lacking due to the complexity of the general case
- Recently, some new diagrammatic tools were developed which allowed for a full proof. Even with these tools the demonstration is rather intricate TDC, RF Lebed and D.L. Shafer(2010).
  - The scaling of the baryon mass as Nc^2 for QCD(AS) is now on as solid ground as Witten's demonstration that it scales as Nc in QCD(F)

- Generic meson-baryon coupling is strong
  - QCD(F) $g_{Nm} \sim Nc^{1/2}$ (Witten 1979)
  - QCD(AS) $g_{Nm} \sim Nc$ (Cherman&TDC 2006)
- In both the case of QCD(F) and QCD(AS) baryons include effects which at the hadronic level appear to be due to meson loops
  - This fact is often not fully appreciated but is clearly true for both QCD(AS) and QCD(F)



$\sqrt{N_c}$ QCD(F) $\qquad$ $\sqrt{N_c}$ QCD(F)

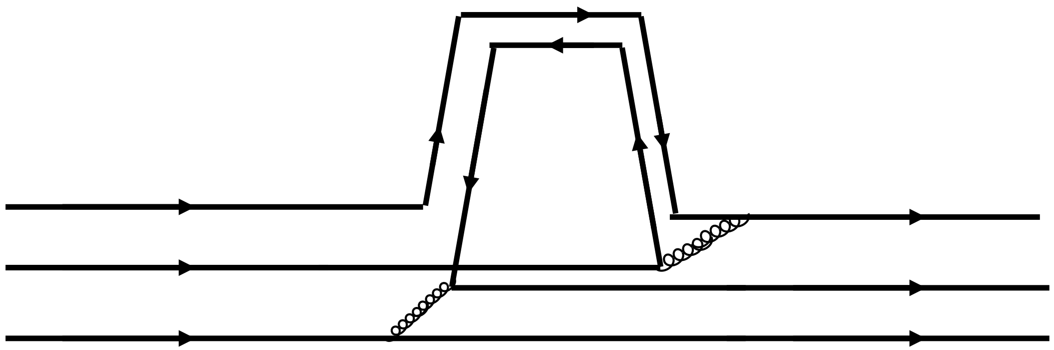
N_c QCD(AS) $\qquad$ N_c QCD(AS)

Meson loop contribution to the nucleon self-energy is order N_c (QCD(F)) or N_c^2 (QCD(As)). This is leading order since $M_N \sim N_c$ (QCD(F)) or N_c^2 (QCD(As)). .

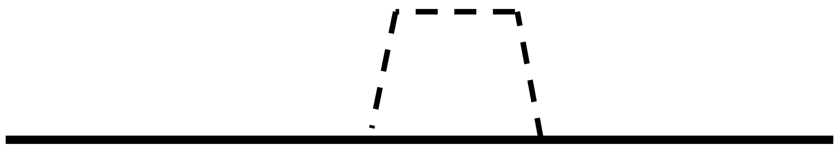
How can this be? Quark loops are suppressed at large Nc for QCD(F) and surely meson loops involve quark loops.

Actually this is not true.

While meson loops in meson do involve quark loops for baryons they need not (TDC & D.B. Leinweber 1992):consider "z-graphs" in "old fashioned" perturbation theory for quarks in a nucleon



At hadronic level this looks like



Very strong evidence for this: Skyrme and other large Nc chiral soliton models exactly reproduce the non-analytic dependence on m_π which emerge from pion loops in chiral perturbation theory(TDC& W. Broniowski 1992)

QCD(AS) also has meson contribution at leading order from internal quark loops. This yields some qualitative differences:

Eg. strange quark form factors in the nucleon:

Leading order---QCD(AS)

Suppressed at leading order---QCD(F)

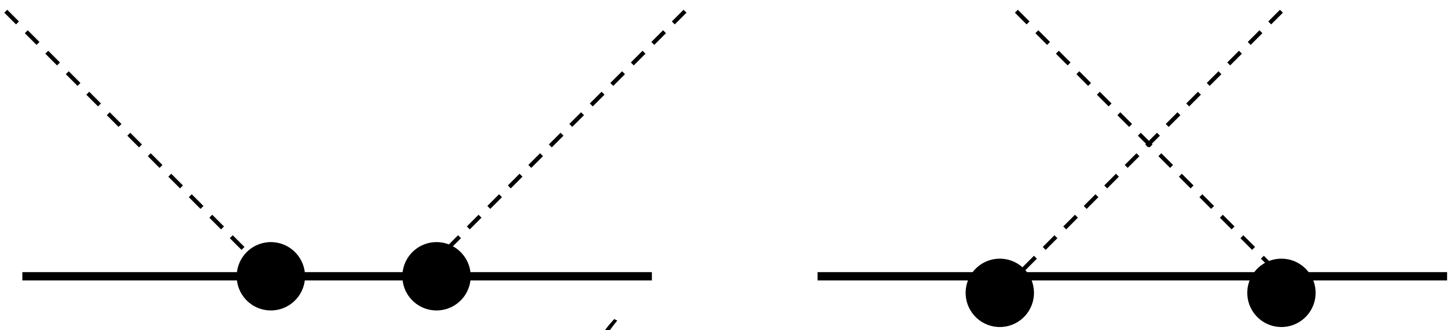
(Cherman&TDC 2007)

- If pion coupling to the nucleon g_A/f_π has a generic strength ($g_A/f_\pi \sim Nc^{1/2}$ for QCD(F); $g_A/f_\pi \sim Nc$ for QCD(AS)) then an $S(2N_f)$ spin-flavor symmetry emerges at large Nc.

- This is a consequence of demanding “large Nc consistency” in which the π-N scattering amplitude is Nc^0 while the Born and cross-born contributions are Nc^1 (F) or Nc^2 (AS) (Gervais& Sakita 1984; Dashen&Manohar 1993)

- Spin-Flavor (Gervais&Sakita84, Dashen&Manohar92)

Consider pion-nucleon scattering



$$A = i g_A^2 p_i p_j \left(\frac{\sigma_i \tau_a \sigma_j \tau_b}{f_\pi^2(-\omega)} + \frac{\sigma_i \tau_a \sigma_j \tau_b}{f_\pi^2 \omega} \right)$$

$\sim N_c^2$ QCD(F)

$\sim N_c^4$ QCD(AS)

$\sim N_c^1$ QCD(F)

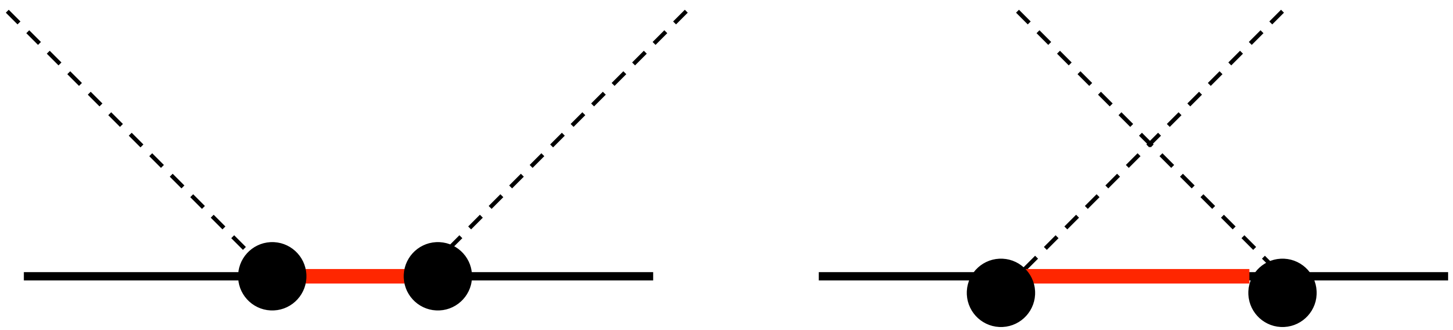
$\sim N_c^2$ QCD(AS)

$$\mathrm{A} \sim N_c [\sigma_i \tau_a, \sigma_j \tau_b]$$ QCD(F)

$$\mathrm{A} \sim N_c^2 [\sigma_i \tau_a, \sigma_j \tau_b]$$ QCD(AS)

This violates unitarity (and Witten scaling rules)

To get sensible results this needs to be canceled



Cancellations require

- Other baryons in intermediated state which are degenerate with nucleon at large N_c. (eg. Δ)

- Conspiracy between vertices

Group Theory

- Assume family of degenerate baryons at large N_c.
- Assume coupling constants X_{ia} between these baryons. Consistency requires

$$[X_{ia},X_{jb}]=0$$

- Full group structure follows from spin and flavor transformation properties; contracted SU(2 N_f)
- Scale of the corrections fixed:

$[X_{ia},X_{jb}]=N_c^{-1}$ QCD(F) $[X_{ia},X_{jb}]=N_c^{-2}$ QCD(AS)

Contracted $SU(2N_f)$ Symmetry

$$[J_i, J_j] = i\varepsilon_{ijk} J_k$$

$$[T_a, T_b] = if_{abc} T_c$$

$$[T_i, X_{jb}] = i\varepsilon_{ijk} X_{kb}$$

$$[T_a, X_{jb}] = if_{abc} X_{jc}$$

$$[X_{ia}, X_{jb}] = 0$$

Degenerate baryons fall in irreps of this group at large Nc

Such a symmetry implies that there is an infinite tower of baryon states with I=J which are degenerate at large Nc and with relative matrix elements fixed by CG coefficients of the group.

For Nc=3 the N& Δ are identified as members of the band. (Other states are large Nc artifacts)

Corrections to this:

$$\text{QCD(F)}: \quad M_\Delta - M_N \sim \frac{1}{N_c} \qquad \text{Fractional correction to ratio of ME's} \sim \frac{1}{N_c}$$

$$\text{Fractional correction to ratio of "Golden" ME's} \sim \frac{1}{N_c^2}$$

$$\text{QCD(AS)}: \quad M_\Delta - M_N \sim \frac{1}{N_c^2} \qquad \text{Fractional correction to ratio of ME's} \sim \frac{1}{N_c^2}$$

$$\text{Fractional correction to ratio of "Golden" ME's} \sim \frac{1}{N_c^4}$$

Phenomenologically the predictions of the contracred SU(2N_f) symmetry and the scale of its breaking do very well

Eg. Axial couplings Dashen & Manohar 1993

Baryon mass relations and SU(3) flavor breaking Jenkins &Lebed 1995

Cherman,Cohen &Lebed 2009

Isoscalar mass combinations

$$N_0 = \frac{1}{2}(p+n)\,, \quad \text{and } \Lambda$$

$$\Sigma_0 = \frac{1}{3}(\Sigma^+ + \Sigma^0 + \Sigma^-)\,,$$

$$\Xi_0 = \frac{1}{2}(\Xi^0 + \Xi^-)\,,$$

$$\Delta_0 = \frac{1}{4}(\Delta^{++} + \Delta^+ + \Delta^0 + \Delta^-)\,,$$

$$\Sigma_0^* = \frac{1}{3}(\Sigma^{*+} + \Sigma^{*0} + \Sigma^{*-})\,,$$

$$\Xi_0^* = \frac{1}{2}(\Xi^{*0} + \Xi^{*-})\,. \text{ and } \Omega$$

Scale of SU(3) flavor breaking

- One of many possible measures:

$$\epsilon \equiv \frac{1}{3}\sum_{i=1}^{3}\frac{B_i - N_0}{(B_i + N_0)/2}$$ 25

with $B_i = \Sigma_0, \Lambda, \Xi_0$

- Any other reasonable definition should give $\varepsilon \approx 0.25\text{–}0.30$

The I = 0 Mass Combinations Special to $1/N_c$

	Mass Combination	Large $N_c^{\rm F}$ suppression	Large $N_c^{\rm AS}$ suppression
M_1	$5(2N_0+3\Sigma_0+\Lambda+2\Xi_0)-4(4\Delta_0+3\Sigma_0^*+2\Xi_0^*+\Omega)$	$1/N_c$	$1/N_c^2$
M_2	$5(6N_0-3\Sigma_0+\Lambda-4\Xi_0)-2(2\Delta_0-\Xi_0^*-\Omega)$	ϵ	ϵ
M_3	$N_0-3\Sigma_0+\Lambda+\Xi_0$	ϵ/N_c	ϵ/N_c^2
M_4	$(-2N_0-9\Sigma_0+3\Lambda+8\Xi_0)+2(2\Delta_0-\Xi_0^*-\Omega)$	ϵ/N_c^2	ϵ/N_c^4
M_5	$35(2N_0-\Sigma_0-3\Lambda+2\Xi_0)-4(4\Delta_0-5\Sigma_0^*-2\Xi_0^*+3\Omega)$	ϵ^2/N_c	ϵ^2/N_c^2
M_6	$7(2N_0-\Sigma_0-3\Lambda+2\Xi_0)-2(4\Delta_0-5\Sigma_0^*-2\Xi_0^*+3\Omega)$	ϵ^2/N_c^2	ϵ^2/N_c^4
M_7	$\Delta_0-3\Sigma_0^*+3\Xi_0^*-\Omega$	ϵ^3/N_c^2	ϵ^3/N_c^4

ε Is SU(3) flavors breaking scale

Can we see evidence of large Nc behavior behavior in these relations beyond mere SU(3) flavor and its breaking?

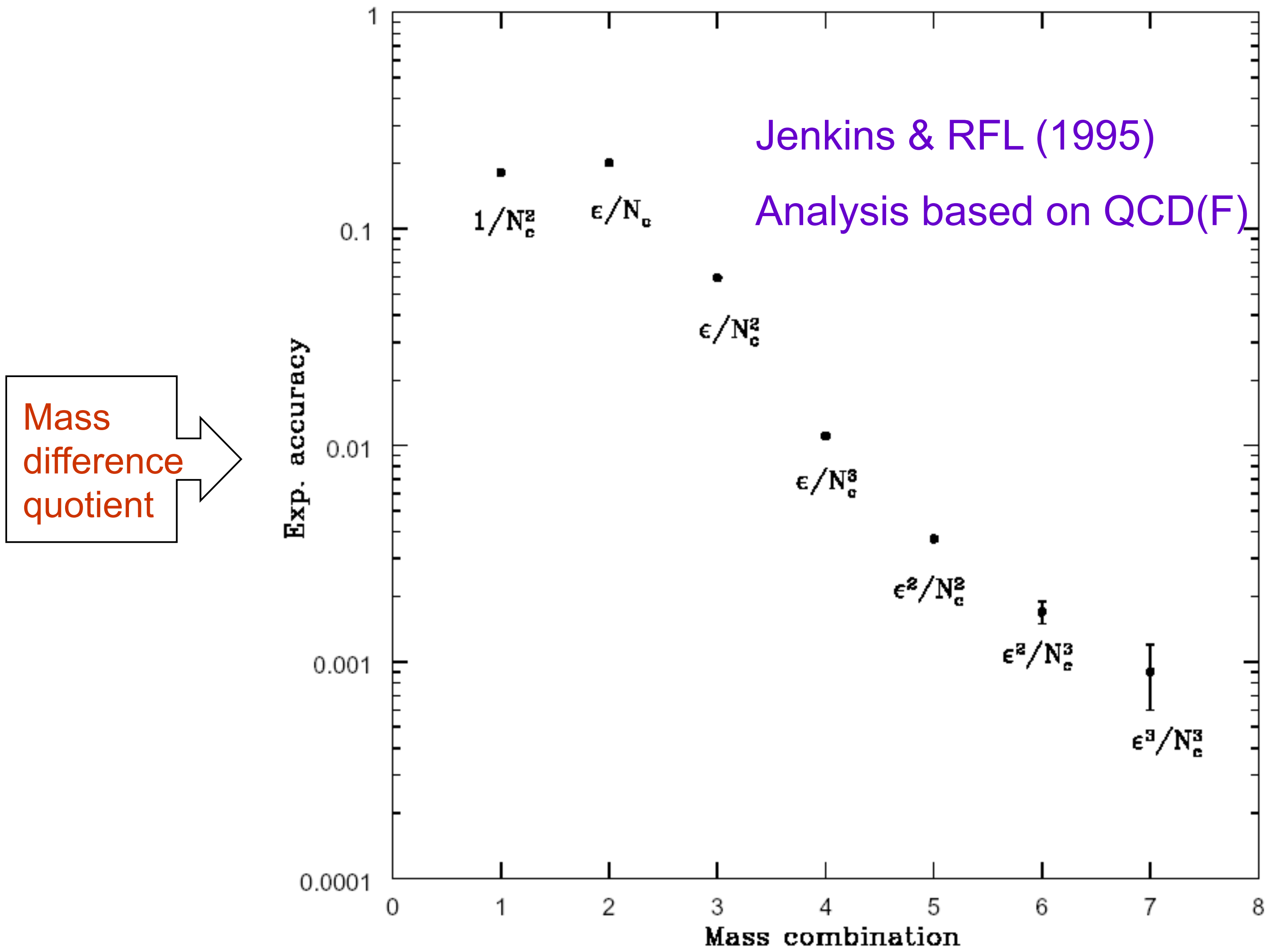
Cherman, Cohen & RFL, Phys. Rev. D **80**, 036002 [2009]:
Compare these results for N_C^F and N_C^{AS}

To test the quality of the large Nc predcitions of mass relations quantitatively we need quantitative measure of their accuracy.

- Take each M_i and form M_i', the same combination with all "–" signs turned to "+" (Note that M_i' is $O(N_C)$ [N_C^F], $O(N_C^2)$ [N_C^{AS}])
- Define the scale-independent ratios $R_i \equiv M_i /(\frac{1}{2} M_i')$

  e.g., $M_3 = N_0 - 3\Sigma_0 + \Lambda + \Xi_0$

  → $R_3 = (N_0 - 3\Sigma_0 + \Lambda + \Xi_0)/[\frac{1}{2} (N_0 + 3\Sigma_0 + \Lambda + \Xi_0)]$



Large Nc QCD(F) has real predictive power: the relations are MUCH better than pure SU(3)!!

You might think that there's *no way* N_C^{AS} can give results that good. And yet, …

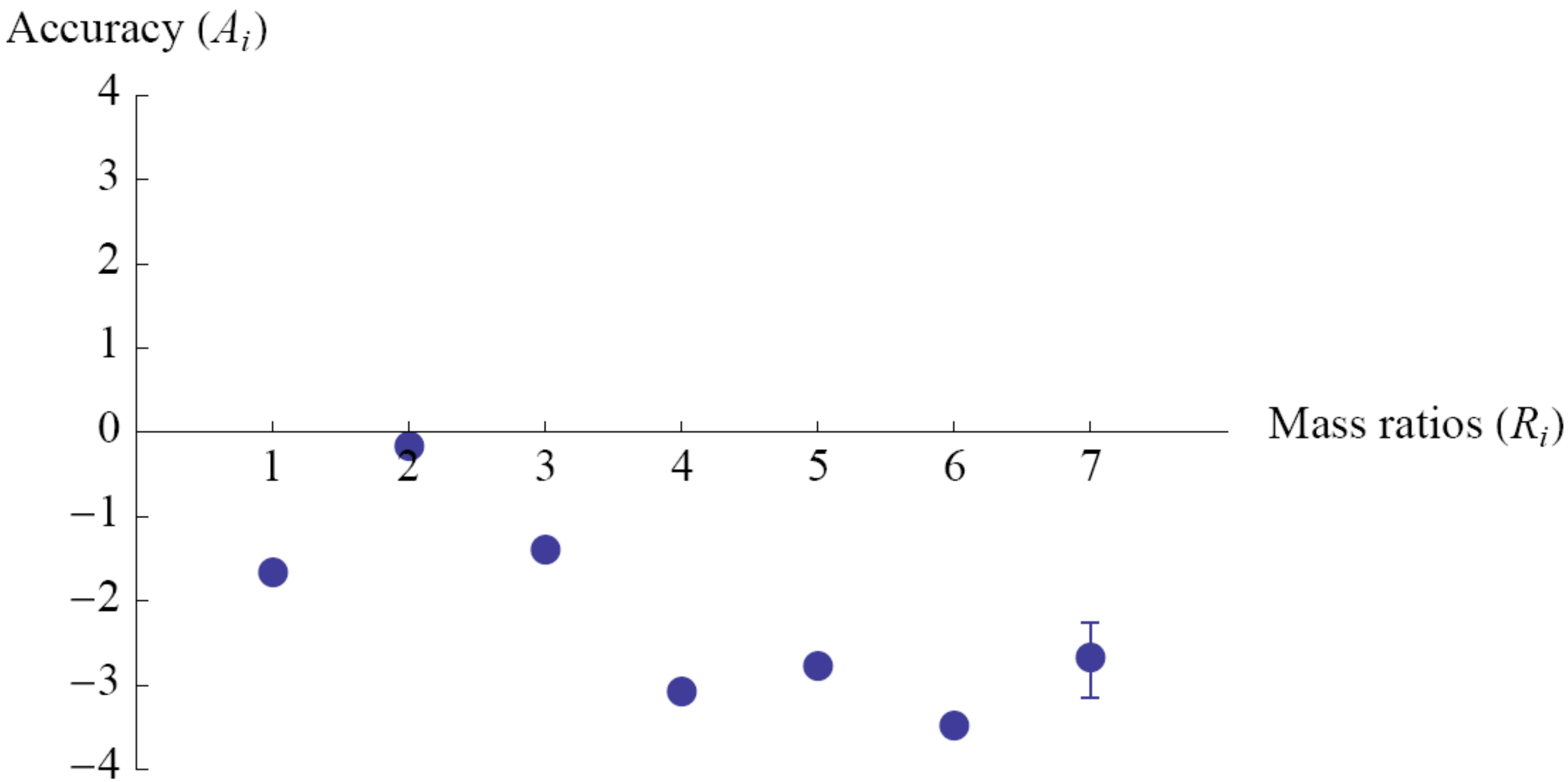
- Start with the ratio R_i introduced above
- Compute the corresponding suppression factors S_i by replacing the masses in M_i and M_i', with their N_C and ε scalings

  e.g., in N_C^F, $M_3 \sim \varepsilon N_C^0$, $M_3' \sim N_C \rightarrow S_3 = \varepsilon/N_C$
- How good is the expansion? Define *accuracy* $A_i \equiv \ln(|R_i|/S_i)$. A perfect prediction has $|R_i| = S_i \rightarrow A_i = 0$

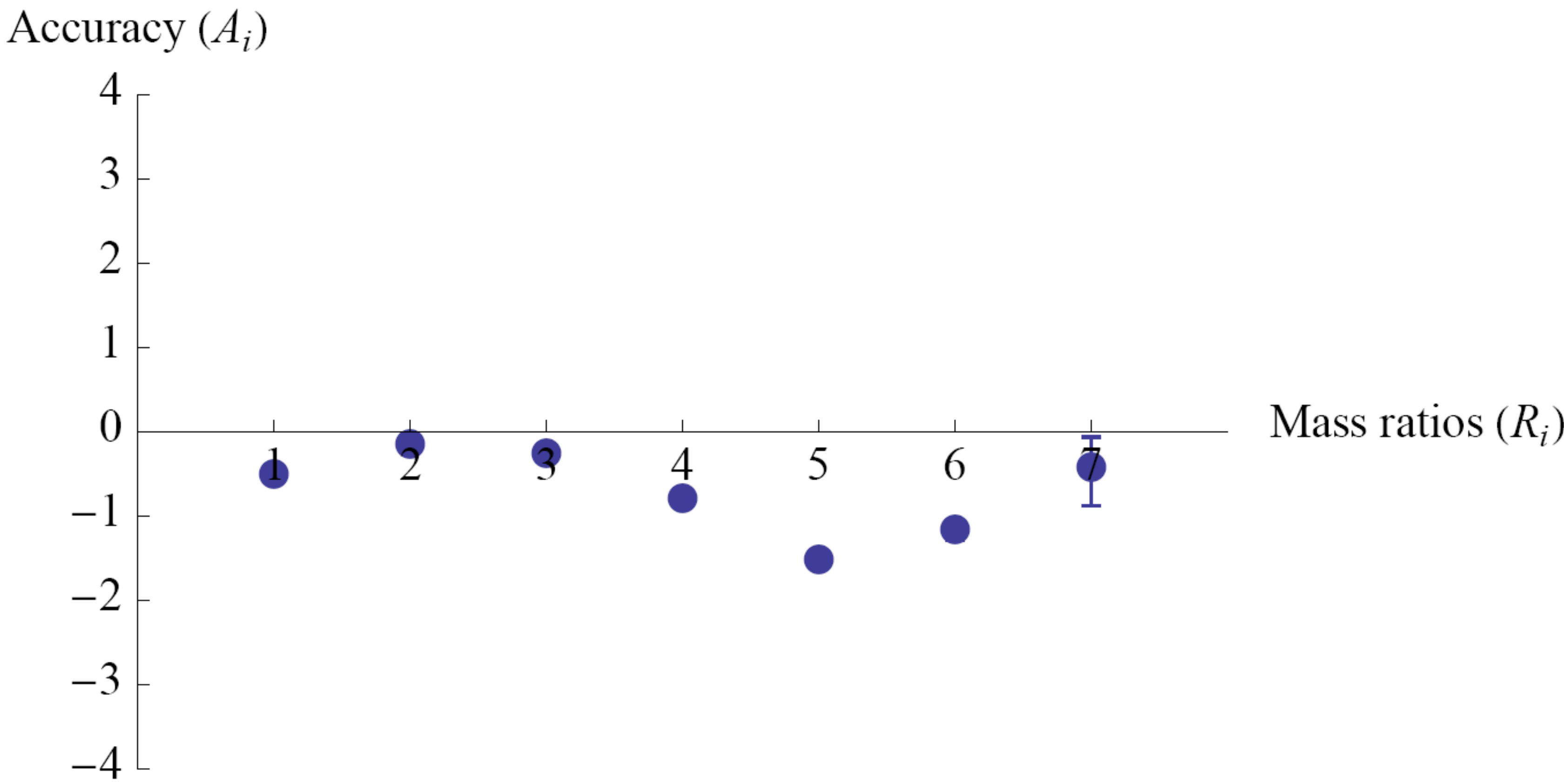
  A poor prediction has $|R_i|/S_i > N_C$ or $< 1/N_C$

  Since $\ln(3) \approx 1$, the figure of merit is whether all A_i turn out to lie in a band of < 2 units wide around zero

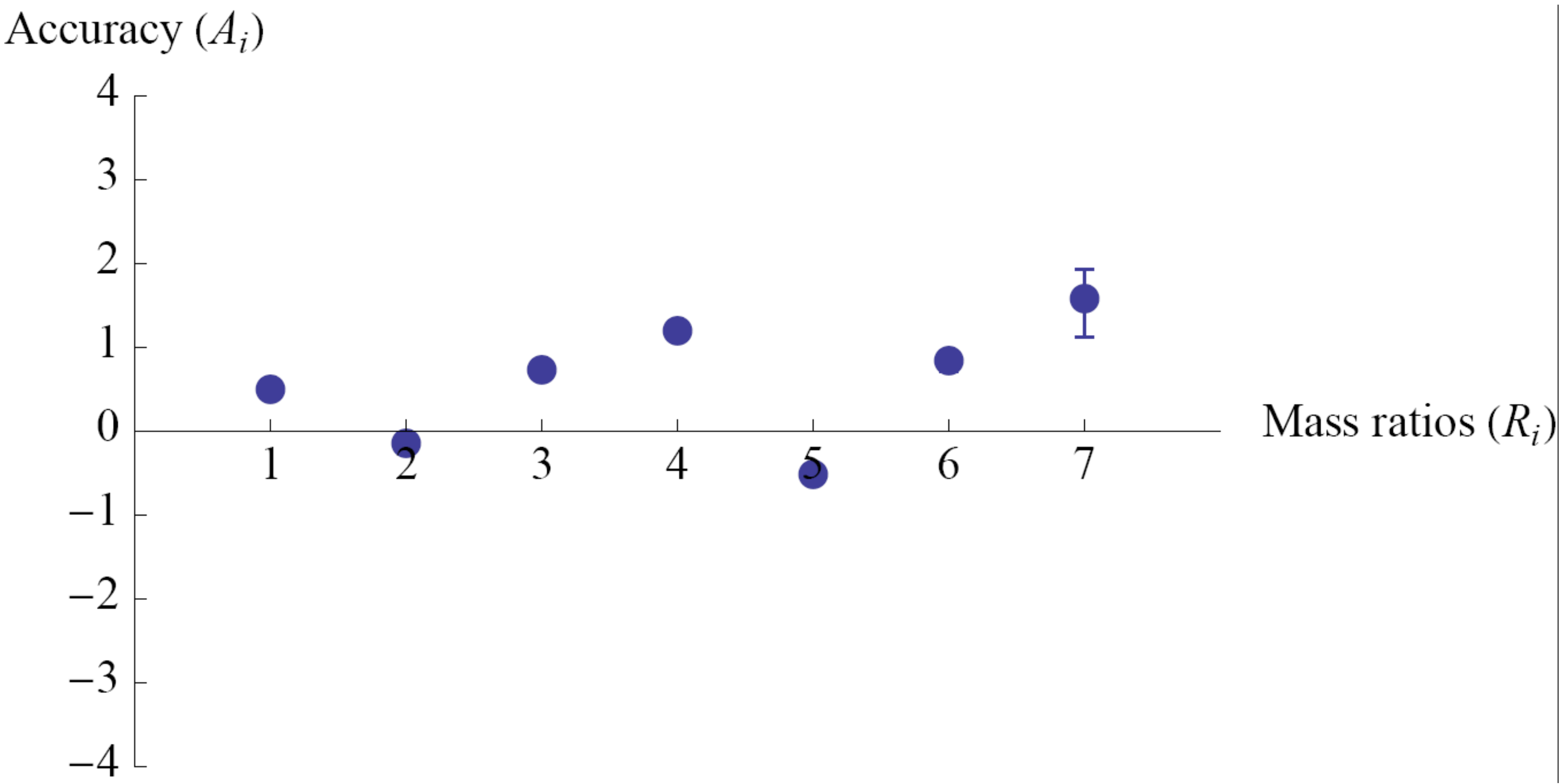
SU(3) Breaking Only, **ε** = 0.25



Large N_c^F Limit, **ε** = 0.25



Large N_c^{AS} Limit, **ε** = 0.25



Both large Nc expansions work well---as well as can be expected and much better than pure SU(3)

An optimist *might* take this success in Baryon spectroscopy to validate QCD(AS). A real optimist might hope at QCD (AS) will be a useful starting point to understand cold dense baryonic matter---a regime where QCD(F) is known to fail. (See M. Buchoff, A Cherman, TDC 2009)

Perhaps with enough good Australian wine I could be convinced of this

But it would take **a lot** of good Australian wine!!

Summary

- QCD(AS) is an alternative way to extrapolate to large Nc.
- QCD(AS) included quark loop effects at leading order.
- Both variants have a contracted $SU(2N_f)$ symmetry
  - The pattern of symmetry breaking in baryons has predictive power for either limit.