First-principles Calculation of Excited State Spectrum in QCD

David Richards

Hadron Spectrum Collaboration

Justin Foley, Colin Morningstar
Carnegie Mellon University
Jozef Dudek, Robert Edwards, Balint Joo,
David Richards, Christopher Thomas
Jefferson Lab
Jimmy Juge
University of the Pacific
Stephen Wallace
University of Maryland
Huey-Wen Lin
University of Washington
International:
John Bulava
NIC, DESY-Zeuthen, Germany
Nilmani Mathur
Tata Institute, Mumbai
Mike Peardon, Sinead Ryan
Trinity College, Dublin

Thursday

Meson spectrum

T(r)opical QCD
Cairns, Australia
Sept 26 - Oct 1, 2010

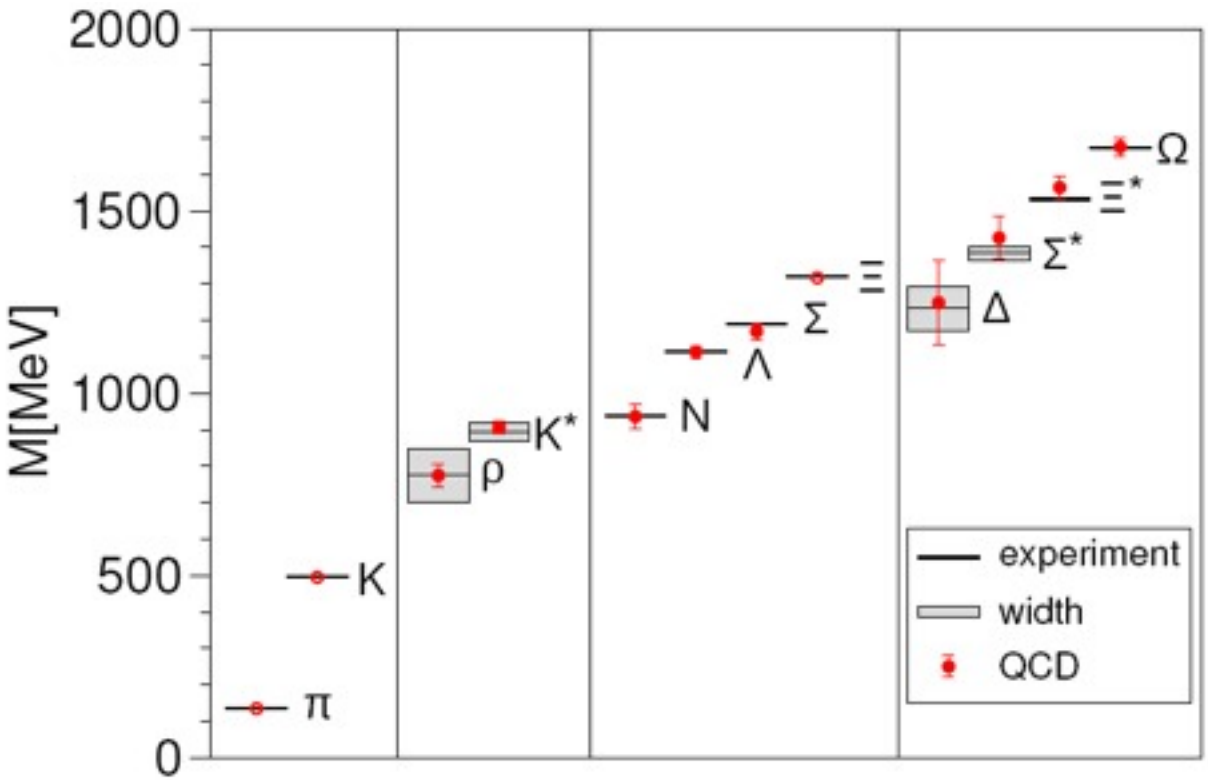
Outline

- Motivation
- Excited States in Lattice QCD
  - Methodology
  - Light meson spectrum
  - Unstable resonances - pi-pi scattering length
  - Light baryon spectrum
- Electromagnetic Properties
  - Radiative Transitions
- Summary

Low-lying Hadron Spectrum

$$C(t) = \sum_{\vec{x}} \langle 0 \mid N(\vec{x},t)\bar{N}(0) \mid 0 \rangle = \sum_{n,\vec{x}} \langle 0 \mid e^{ip\cdot x} N(0) e^{-ip\cdot x} \mid n \rangle \langle n \mid \bar{N}(0) \mid 0 \rangle$$

$$= \mid \langle n \mid N(0) \mid 0 \rangle \mid^2 e^{-E_n t} = \sum_n A_n e^{-E_n t}$$



Durr et al., BMW Collaboration

Science 2008

Control over:

- ***Quark-mass dependence***
- ***Continuum extrapolation***
- ***finite-volume effects (pions, resonances)***

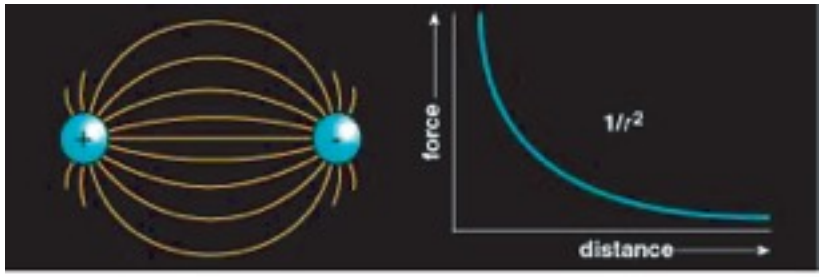
Benchmark calculation of QCD - *enabling us to do something else!*

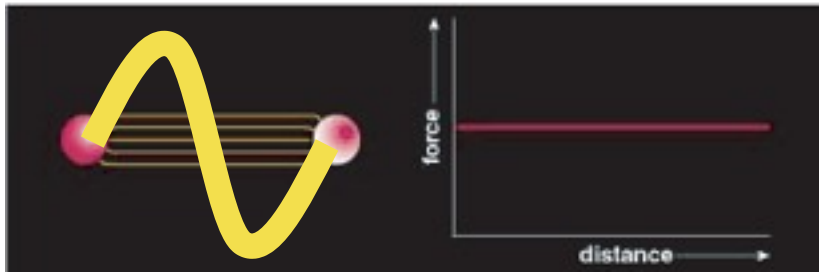
Goals - I

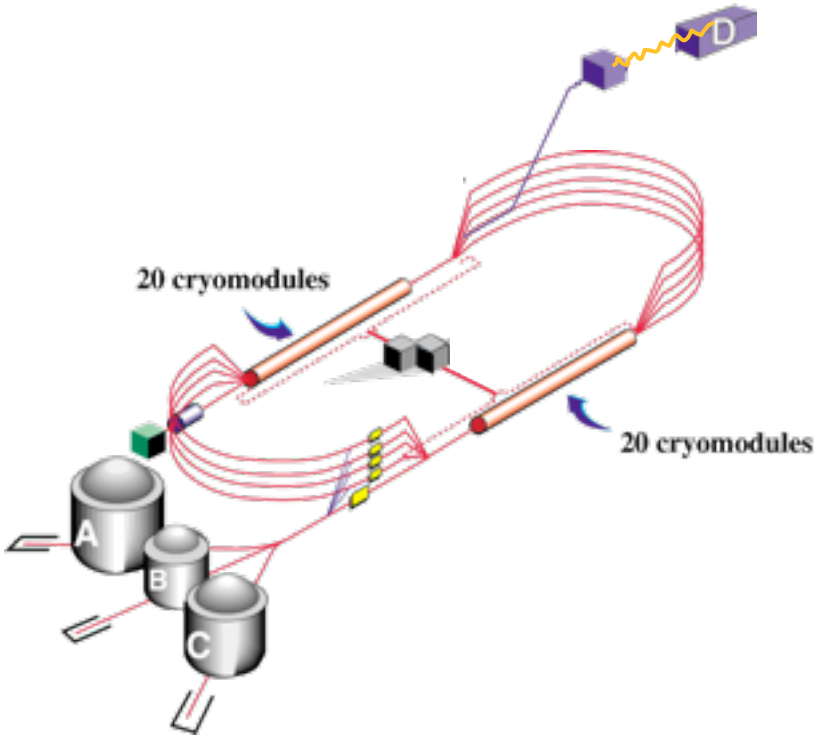
.....but a quantitative understanding of the spectrum is important *in its own right...*

- *Why is it important?*
  - *What are the key degrees of freedom describing the bound states?*
    - *How do they change as we vary the quark mass?*
  - *What is the role of the gluon in the spectrum – search for exotics?*
  - *What is the origin of confinement, describing 99% of observed matter?*
  - *If QCD is correct and we understand it, expt. data must confront ab initio calculations*

Goals - II



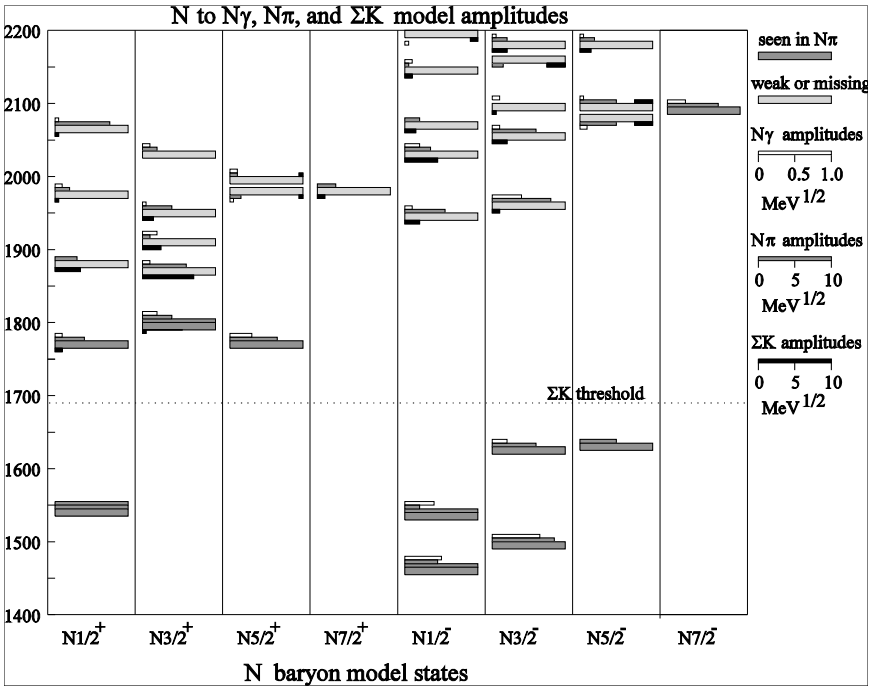




- Exotic Mesons are those whose values of J^{PC} are in accessible to quark model
  - Multi-quark states:
  - Hybrids with *excitations of the flux-tube*
- Study of hybrids: revealing gluonic and flux-tube degrees of freedom of QCD.

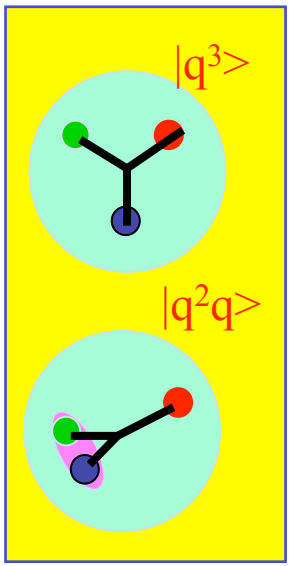
Goals - III

- **Nucleon Spectroscopy: Quark model masses and amplitudes – states classified by isospin, parity and spin.**



Capstick and Roberts, PRD58 (1998) 074011

- Missing, because our pictures do not capture correct degrees of freedom?
- Do they just not couple to probes?



CLAS at JLab

Excited Lattice Spectroscopy

Variational Method

C. Michael, NPB 259, 58 (1985), Luscher and Wolff, NPB 339, 222 (1990)

- **Given N x N correlator matrix** $C_{\alpha\beta} = \langle 0 \mid \mathcal{O}_\alpha(t)\mathcal{O}_\beta(0) \mid 0 \rangle$**, one defines the N *principal correlators* $\lambda_i(t,t_0)$ as the eigenvalues of**

$$C^{-1/2}(t_0)C(t)C^{-1/2}(t_0)$$

- **Principal effective masses defined from correlators plateau to lowest-lying energies**

$$\lambda_i(t,t_0) \to e^{-E_i(t-t_0)}\left(1 + O(e^{-\Delta E(t-t_0)})\right)$$

Eigenvectors, with metric $C(t_0)$, are orthonormal and project onto the respective states

➡ Resolve energy dependence - *anisotropic lattice*
➡ Efficient computation of correlators - *distillation*
➡ Judicious construction of interpolating operators - *cubic symmetry*

Anisotropic Clover Generation - I

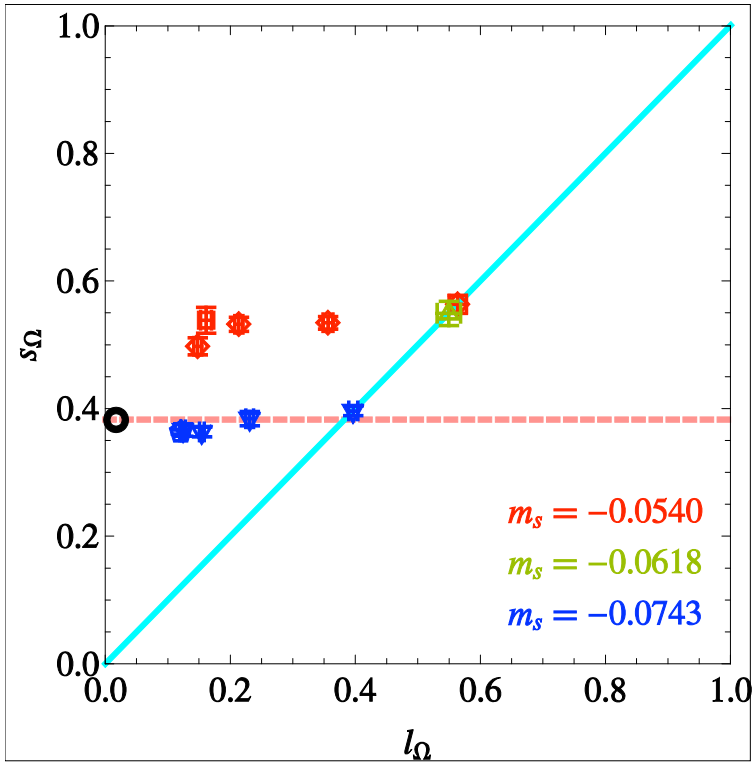
- Interested in excited states: $M_H a \simeq 1$
- "Clover" Anisotropic lattices $a_t < a_s$

Challenge: setting scale and strange-quark mass

Lattice coupling *fixed*

$$s_X = (9/4)[2m_K^2 - m_\pi^2]/m_X^2$$

Omega

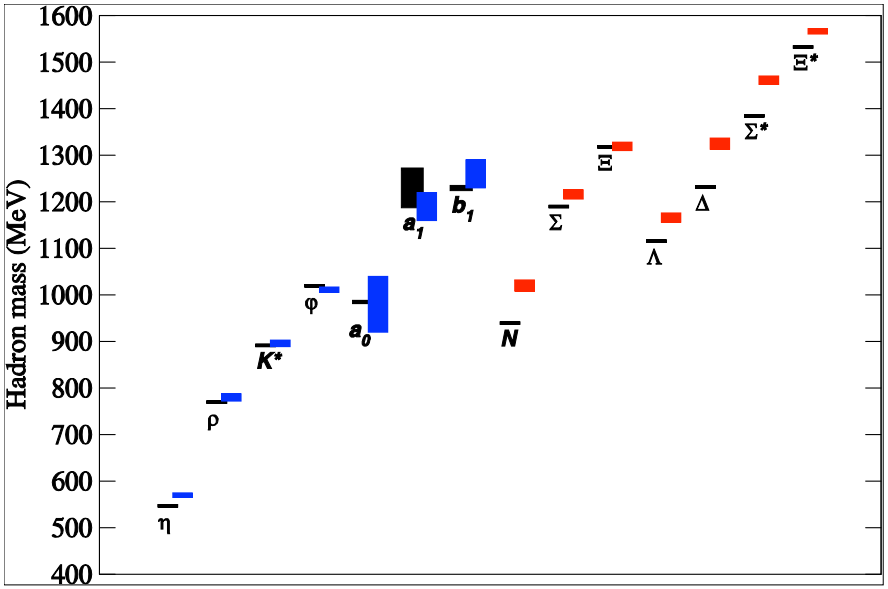


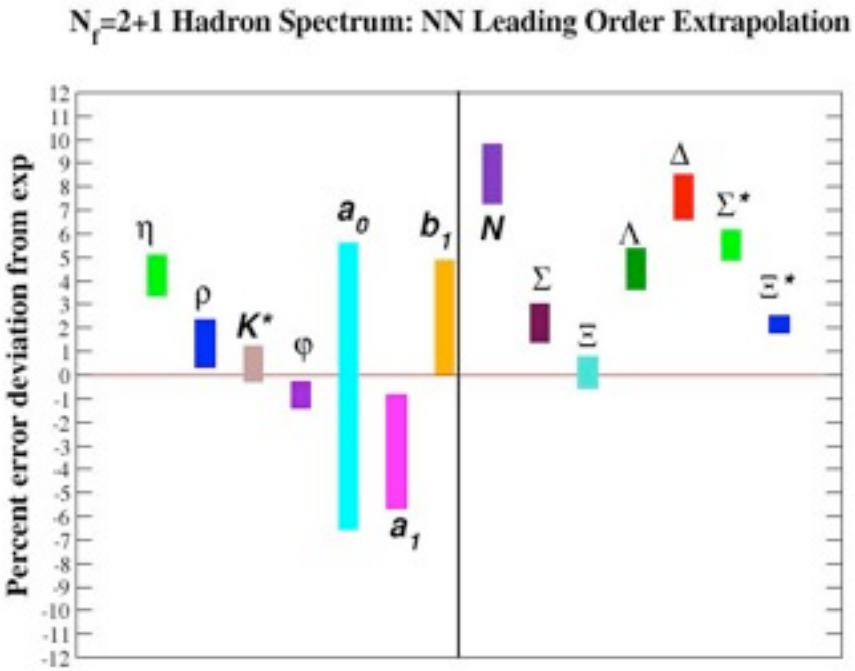
$$l_X = (9/4)m_\pi^2/m_X^2$$

Express physics in (dimensionless) (l,s) coordinates

H-W Lin et al (Hadron Spectrum Collaboration), PRD79, 034502 (2009)

Anisotropic Clover – II





Low-lying spectrum: *agrees with experiment to 10%*

Correlation functions: Distillation

- Use the new "distillation" method.
- Observe

Eigenvectors of Laplacian

$$L^{(J)} \equiv (1 - \frac{\kappa}{n}\Delta)^n = \sum_{i=1} f(\lambda_i) v^{(i)} \otimes v^{*(i)}$$

- Truncate sum at sufficient i to capture relevant physics modes – we use 64: set "weights" f to be unity
- Meson correlation function

Includes displacements

$$C_M(t,t') = \langle 0 \mid \bar{d}(t')\Gamma^B(t')u(t')\bar{u}(t)\Gamma^A(t)d(t)|0\rangle$$

- Decompose using "distillation" operator as

M. Peardon *et al.*, PRD80,054506 (2009)

$$C_M(t,t') = \mathrm{Tr}\langle \phi^A(t')\tau(t',t)\Phi^B(t)\tau^\dagger(t',t), \rangle$$

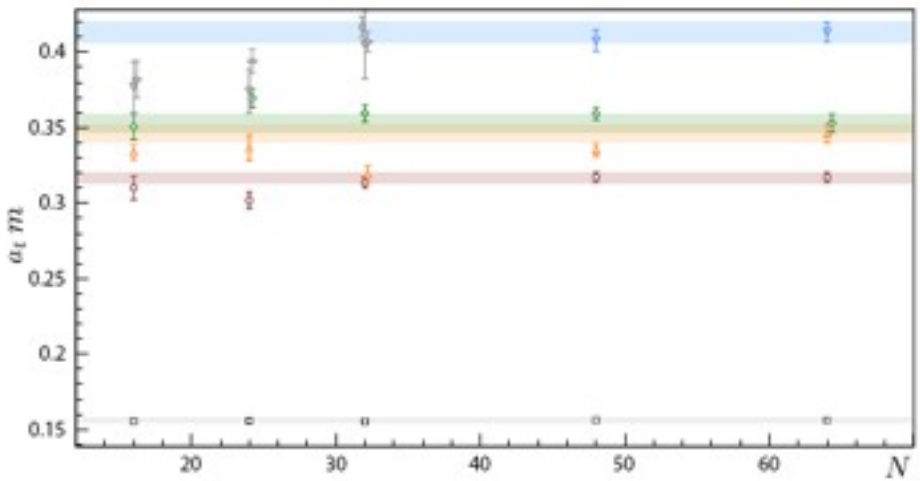
where

Perambulators →

$$\Phi^{A,ij}_{\alpha\beta} = v^{*(i)}(t)[\Gamma^A(t)\gamma_5]_{\alpha\beta} v^{(j)}(t')$$

$$\tau^{ij}_{\alpha\beta}(t,t') = v^{*(i)}(t') M^{-1}_{\alpha\beta}(t',t) v^{(j)}(t).$$

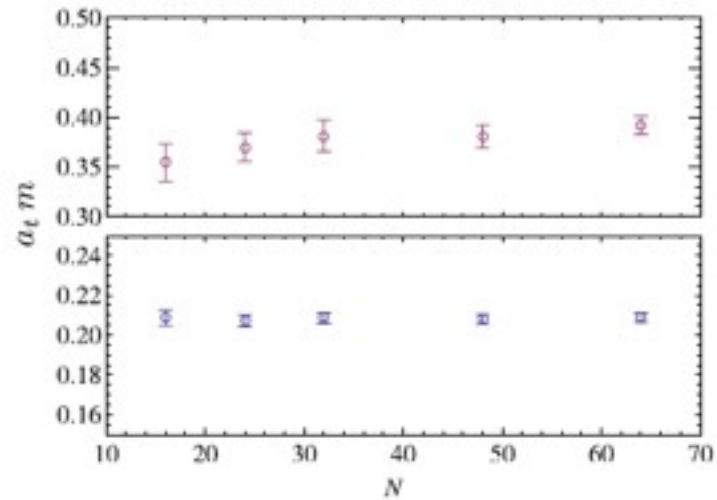
Distillation Results

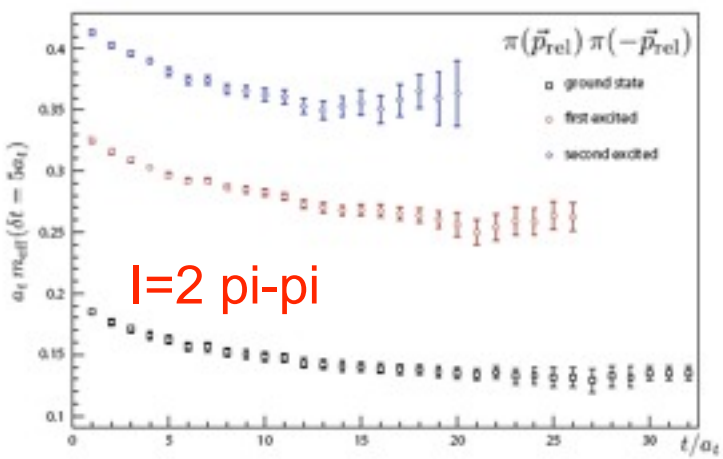


Nucleon Variational Analysis

ρ Variational Analysis

Errors < 3%





I=2 pi-pi

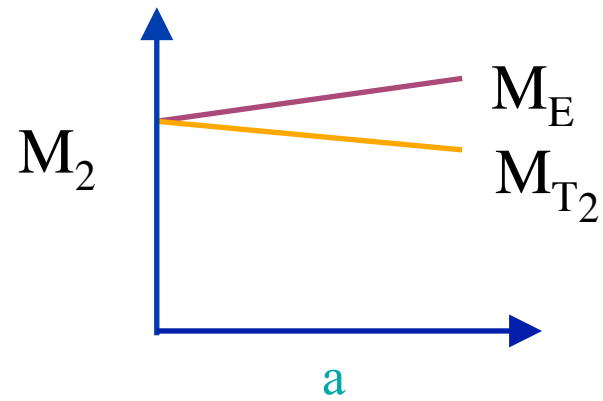
Overall momentum 0
Basis: pairs of back-to-back operators at momentum ***p***

Identification of Spin - I

- **Lattice does not respect symmetries of continuum: *cubic symmetry for states at rest***

Problem: requires data at several Lattice spacings – density of states in each irrep large.

Solution: exploit known continuum behavior of overlaps



- Construct interpolating operators of *definite* (continuum) JM: O^{JM}

$$\langle 0 \mid O^{JM} \mid J', M' \rangle = Z^J \delta_{J,J'} \delta_{M,M'}$$

- Use projection formula to find subduction under irrep. of cubic group

$$\mathcal{O}^{[J]}_{\Lambda,\lambda} \equiv (\Gamma \times D^{[n_D]}_{\dots})^J_{\Lambda,\lambda} = \sum_M \mathcal{S}^{J,M}_{\Lambda,\lambda} \, (\Gamma \times D^{[n_D]}_{\dots})^{J,M} \equiv \sum_M \mathcal{S}^{J,M}_{\Lambda,\lambda} \mathcal{O}^{J,M}$$

Lattice ops. retain memory of their continuum ancestors

Identification of Spin - MESONS

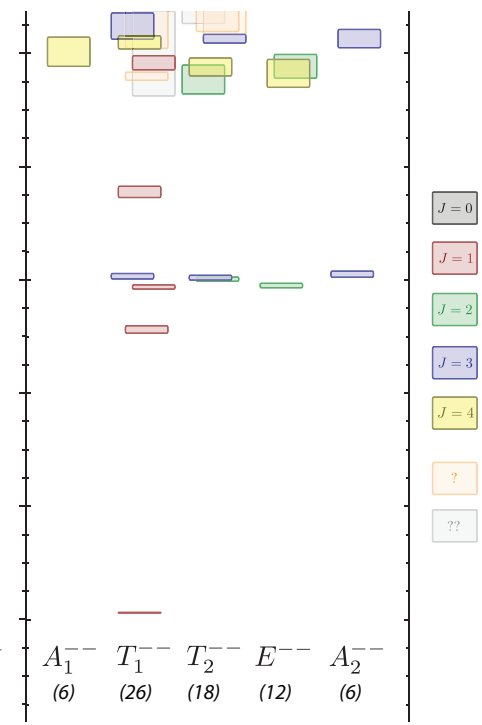
Hadspec collab. (dudek et al), 0909.0200, PRL

Overlap of state onto subduced operators

$$\langle 0 \mid O^{J,M} \mid J', M' > \;=\; Z_J \delta_{J,J'} \delta_{M,M'}$$

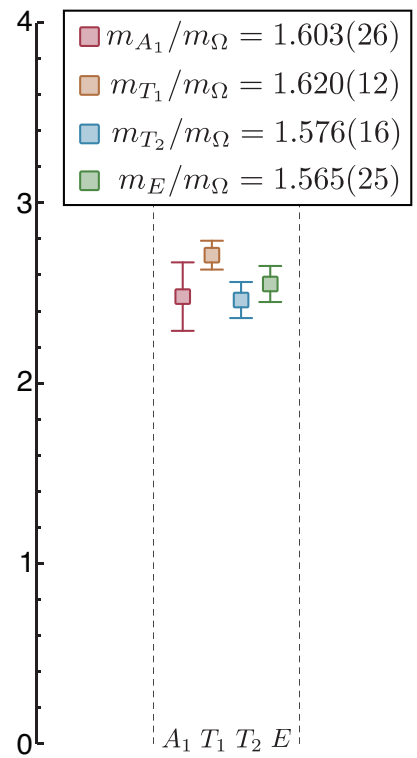
$$\langle 0 \mid O^{J}_{\Lambda,\lambda} \mid J', M' \rangle \;=\; S^{J,M'}_{\Lambda,\lambda} Z_J \delta_{J,J'}$$

Common across irreps.

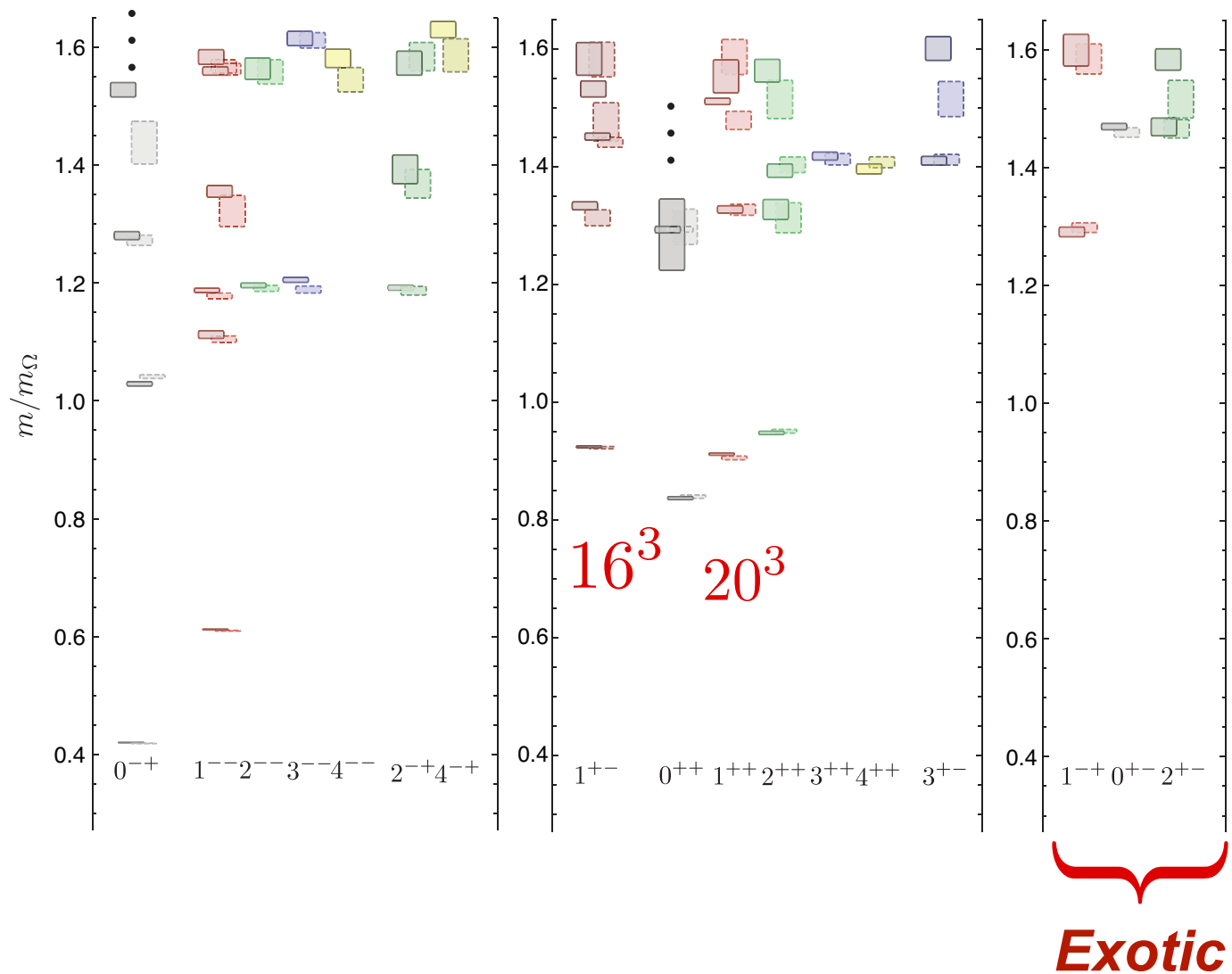


J	irreps
0	$A_1(1)$
1	$T_1(3)$
2	$T_2(3) \oplus E(2)$
3	$T_1(3) \oplus T_2(3) \oplus A_2(1)$
4	$A_1(1) \oplus T_1(3) \oplus T_2(3) \oplus E(2)$

$$N_f = 3$$



Isovector Meson Spectrum - I

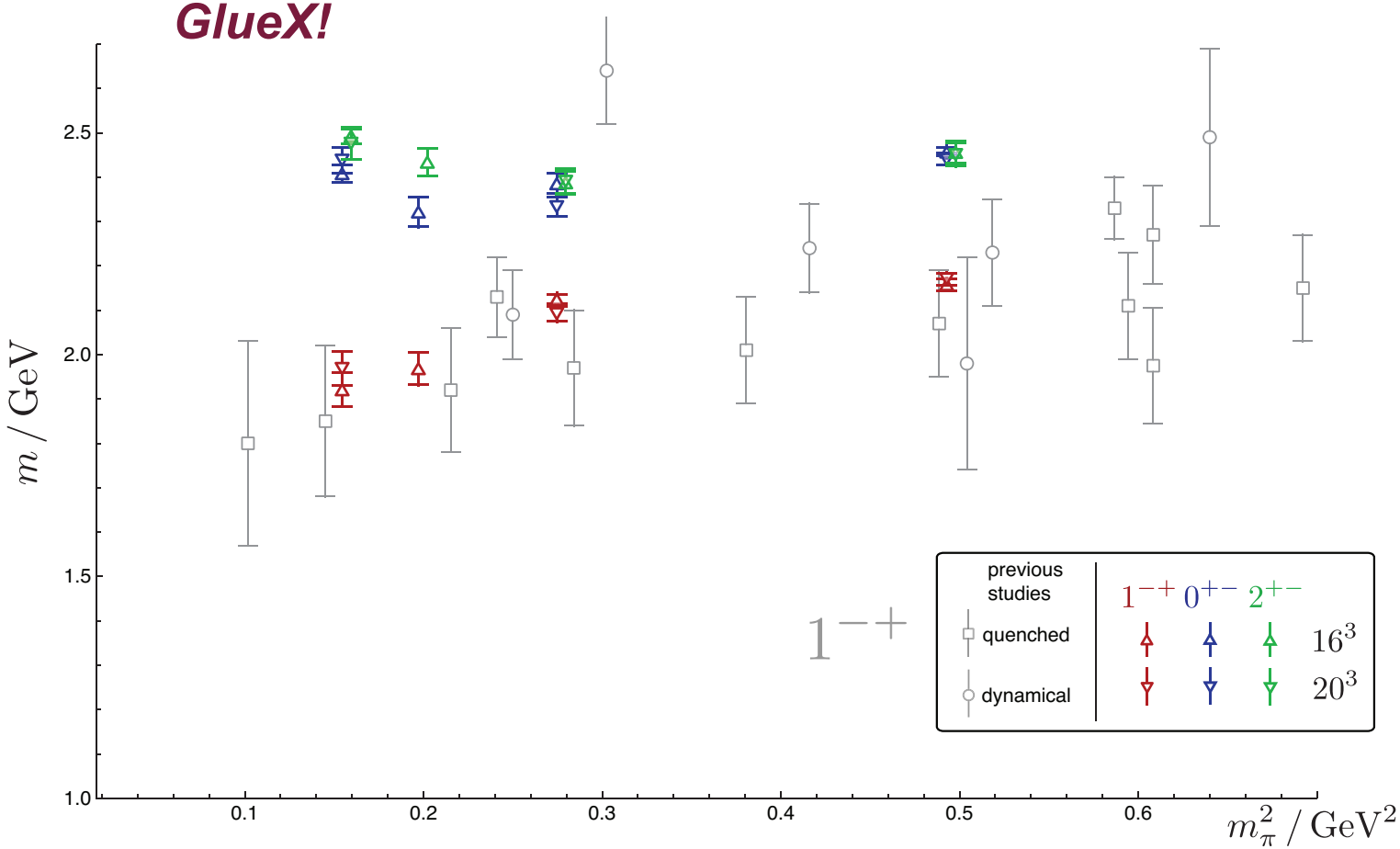


PRL 103:262001,2009

Isovector spectrum with quantum numbers reliably identified

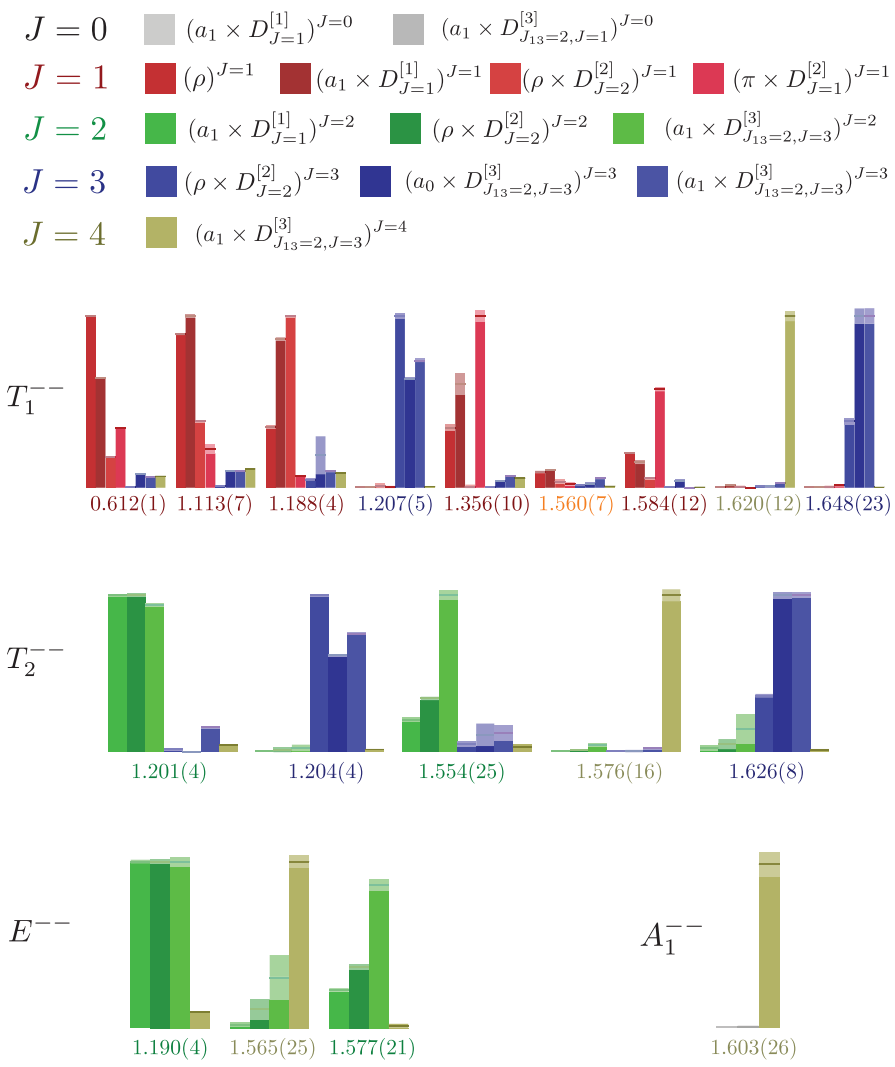
Isovector Meson Spectrum - II

States of exotic quantum numbers in region accessible to GlueX!



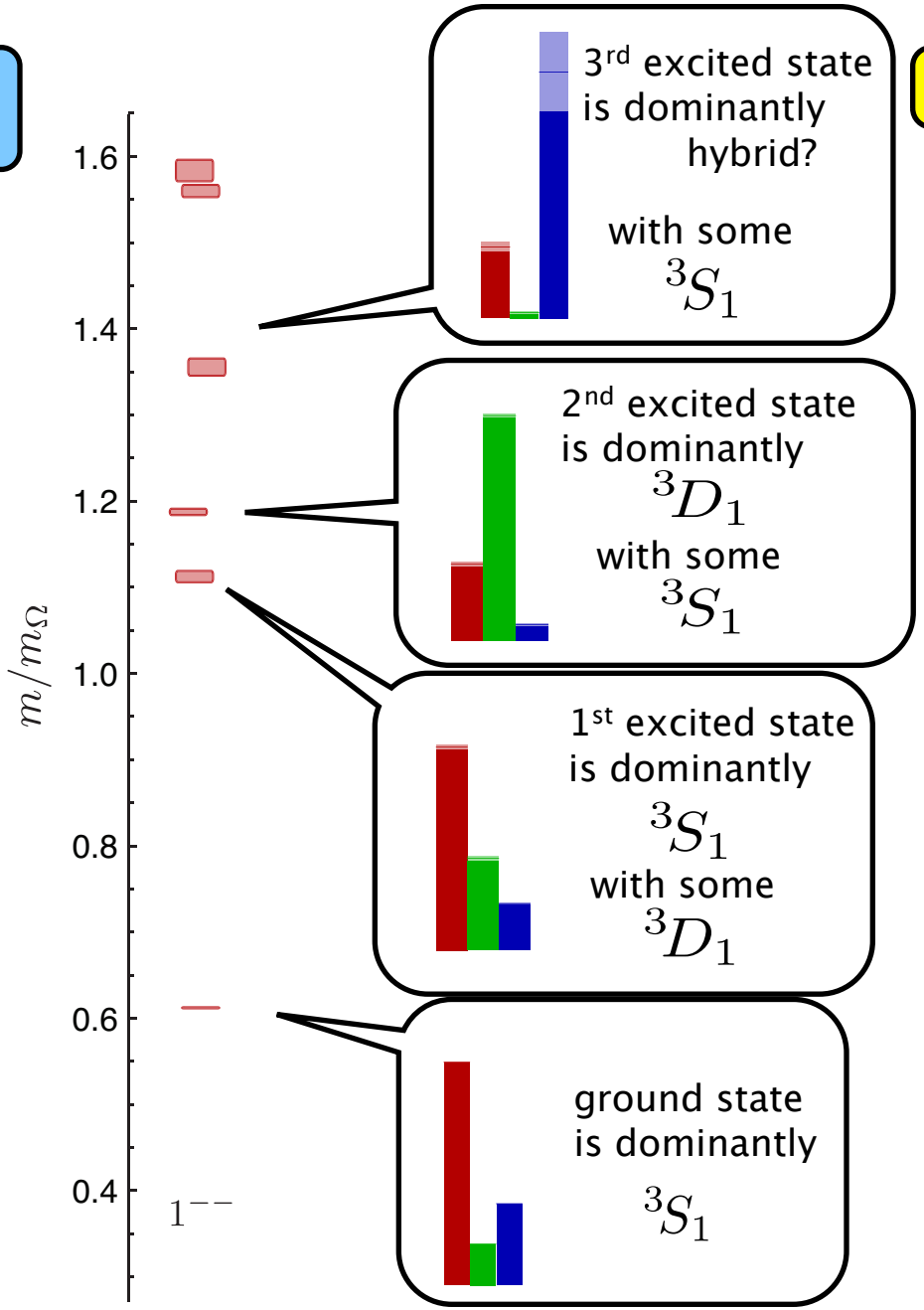
Dudek, Edwards, DGR, Thomas, PRD82, 034508 (2010)

Interpretation of Meson Spectrum



Overlaps: *information about quark and gluon structure*

1^{--}



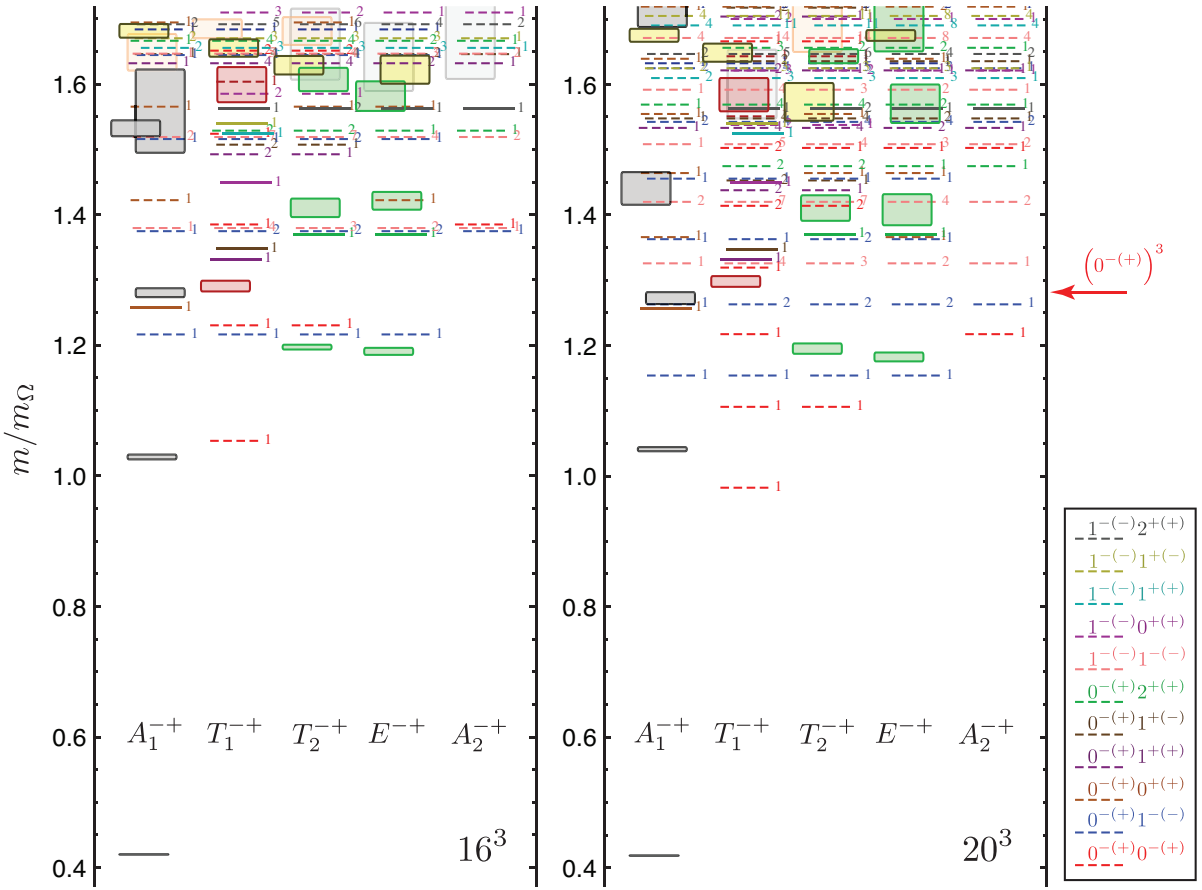
look at the 'overlaps' $Z_n^{\mathbf{\Gamma}} = \langle n | \bar{\psi} \mathbf{\Gamma} \psi | 0 \rangle$

ρ	$^3S_1$
$\left(\rho \times D^{[2]}_{J=2}\right)^{J=1}$	$^3D_1$
$\left(\pi \times D^{[2]}_{J=1}\right)^{J=1}$	hybrid?

build a **bound state model** phenomenology comparable to the quark model using non-perturbative **QCD** calculations

Non-exotic hybrid?

Where are the multi-hadrons?



Meson spectrum on two volumes: dashed lines denote expected (non-interacting) multi-particle energies.

Calculation is incomplete.

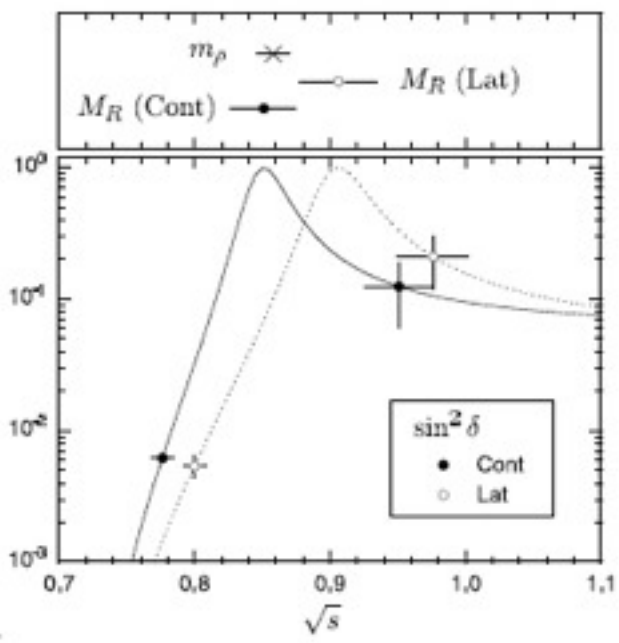
Allowed two-particle states - symmetries of volume

Strong Decays

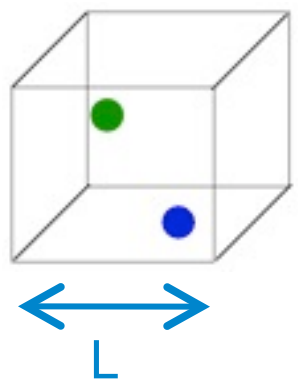
- For interacting particles, energies are shifted from their free-particle values, by an amount that depends on the energy.
- Luscher: relates shift in the free-particle energy levels to the phase shift at the corresponding E.

Breit-Wigner fit

CP-PACS, arXiv:0708.3705



$$\delta E(L) \leftrightarrow \delta(E)$$



$$E_n = 2\sqrt{m_\pi^2 + (\frac{2n\pi}{L})^2}$$

Talk of Gerrit Schierholz

I=2 Pi-Pi Scattering - I

- Simpler - no bound state - but illustrates method
- Use a operator basis back-to-back pions

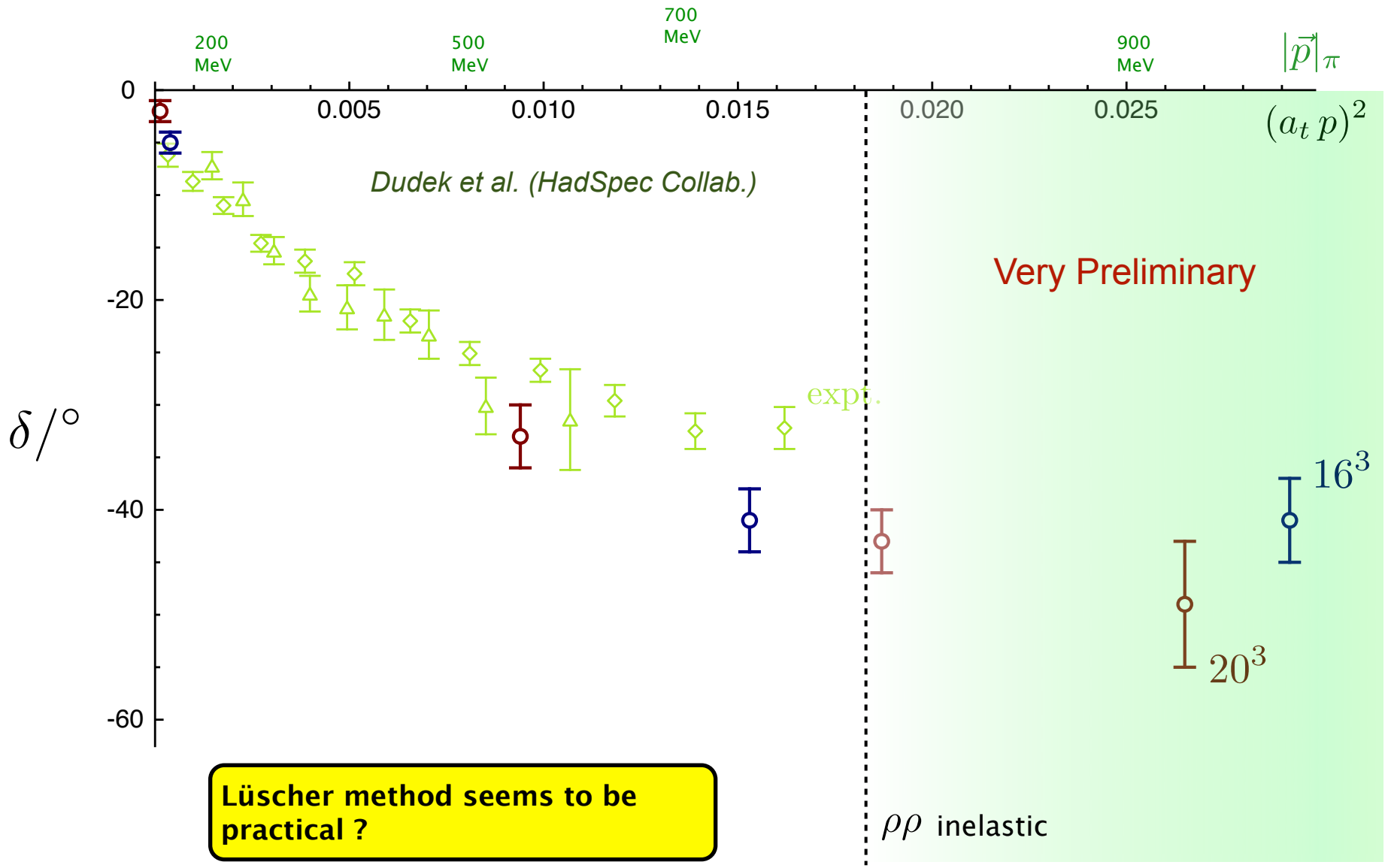
$$\sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}}[\bar{\psi}\gamma_5\psi](\vec{x}) \cdot \sum_{\vec{y}} e^{-i(-\vec{p})\cdot\vec{y}}[\bar{\psi}\gamma_5\psi](\vec{y})$$

Non-interacting states are:

$$|\,\pi(0,0,0)\pi(0,0,0)\rangle, |\,\pi(-1,0,0)\pi(1,0,0)\rangle, |\,\pi(-1,-1,0)\pi(1,1,0)\rangle \ldots$$

Gives non-interacting energy levels - depend on volume
Method applies below inelastic threshold

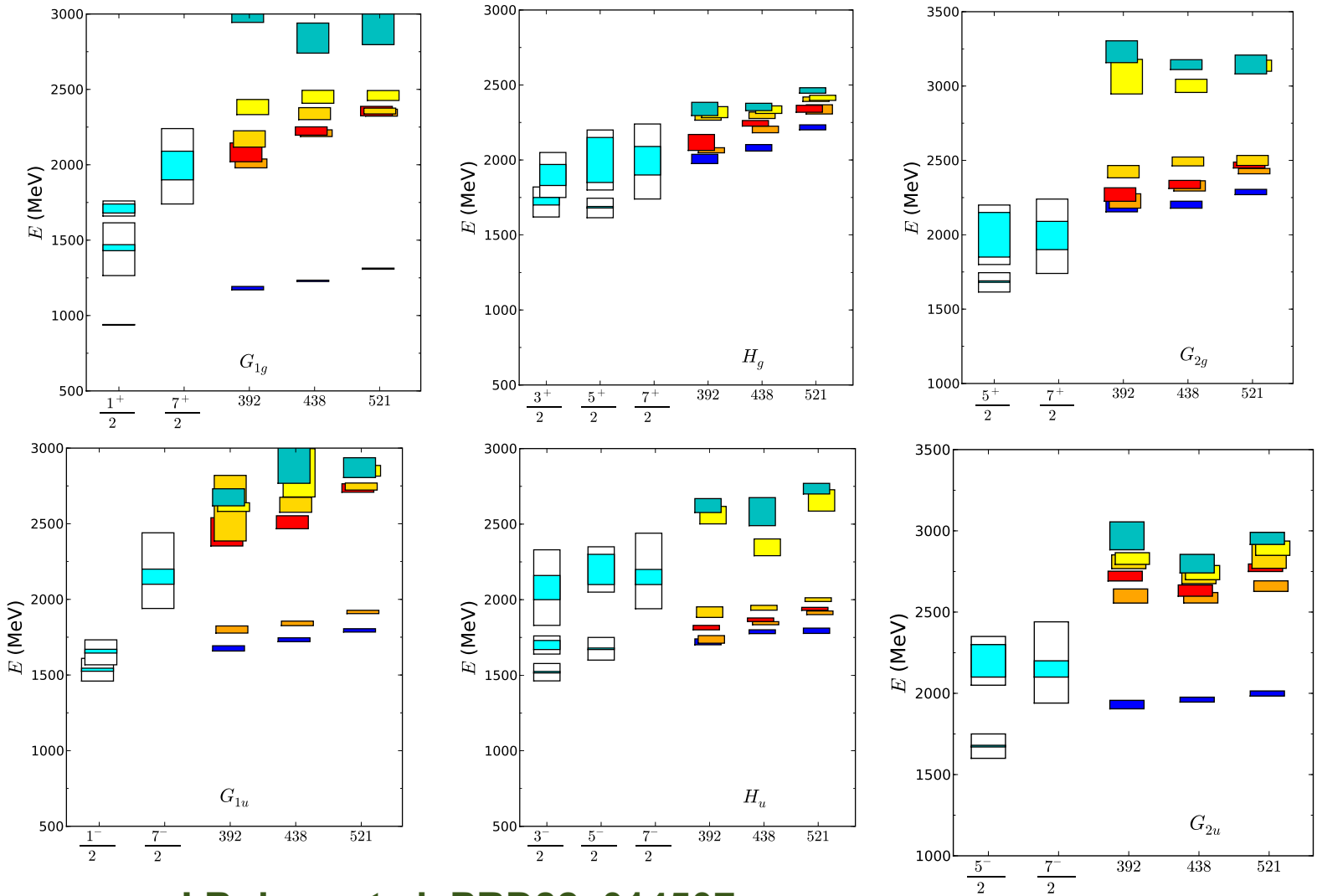
N_F = 2+1 (u,d,s) $m_\pi \sim 400$ MeV



Lüscher method seems to be practical ?

Excited Baryon Spectrum

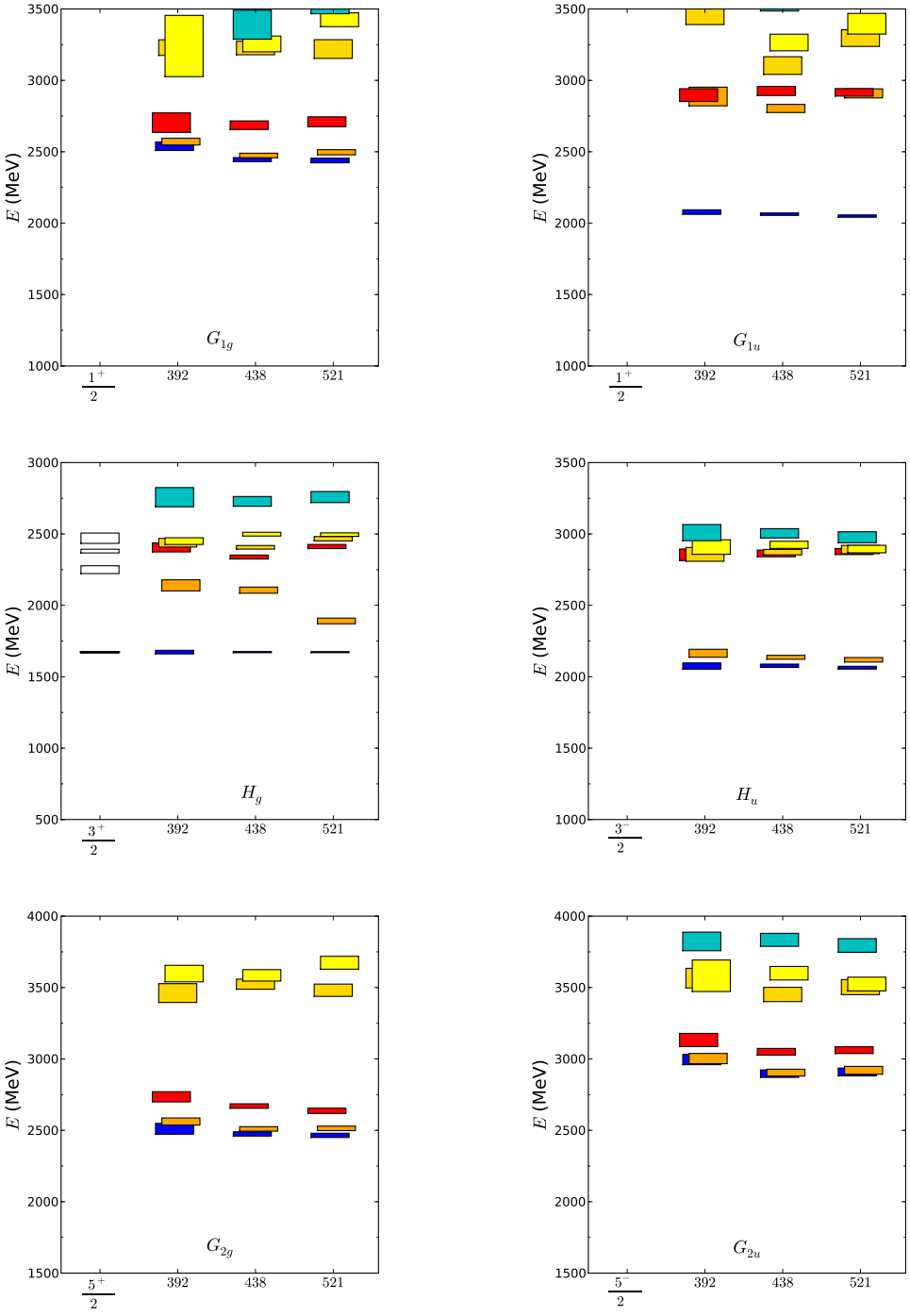
Excited Baryon Spectrum - I



$16^3.128$

Scaling to larger volumes - Morningstar..

J Bulava et al, PRD82, 014507

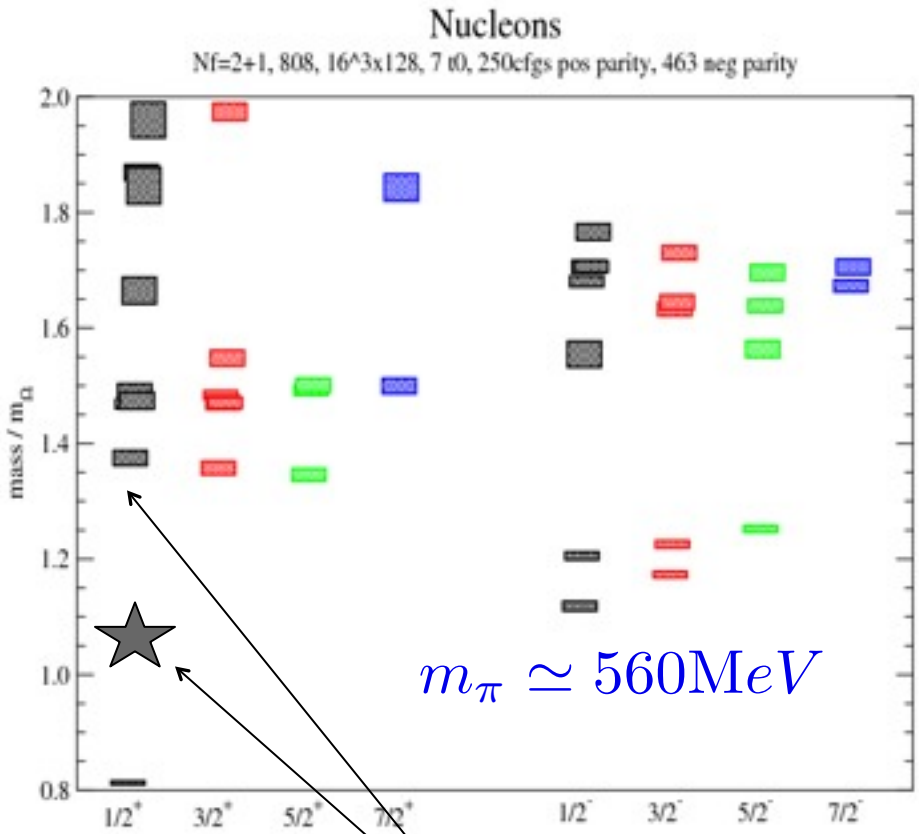


Omega spectrum - *only ground-state quantum assignments*

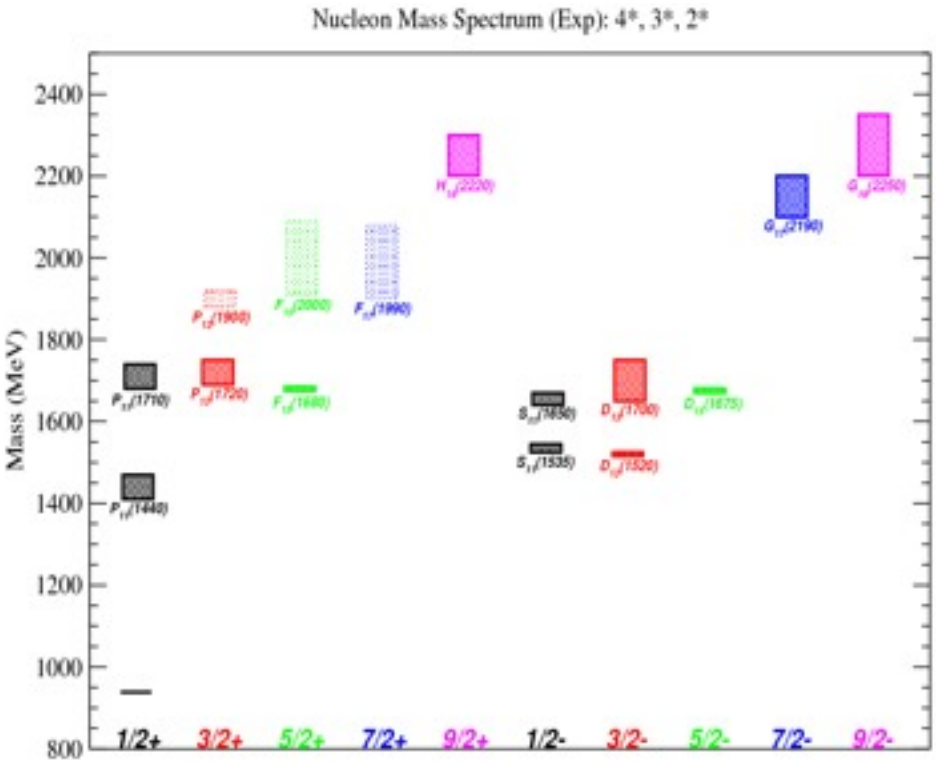
Lattice QCD Predictions

Excited Baryon Spectrum - II

Subduction of continuum operators - reliable determination of baryon spins



$m_\pi \simeq 560 MeV$



Where is the "Roper"?

R. Edwards, Hadron 2009

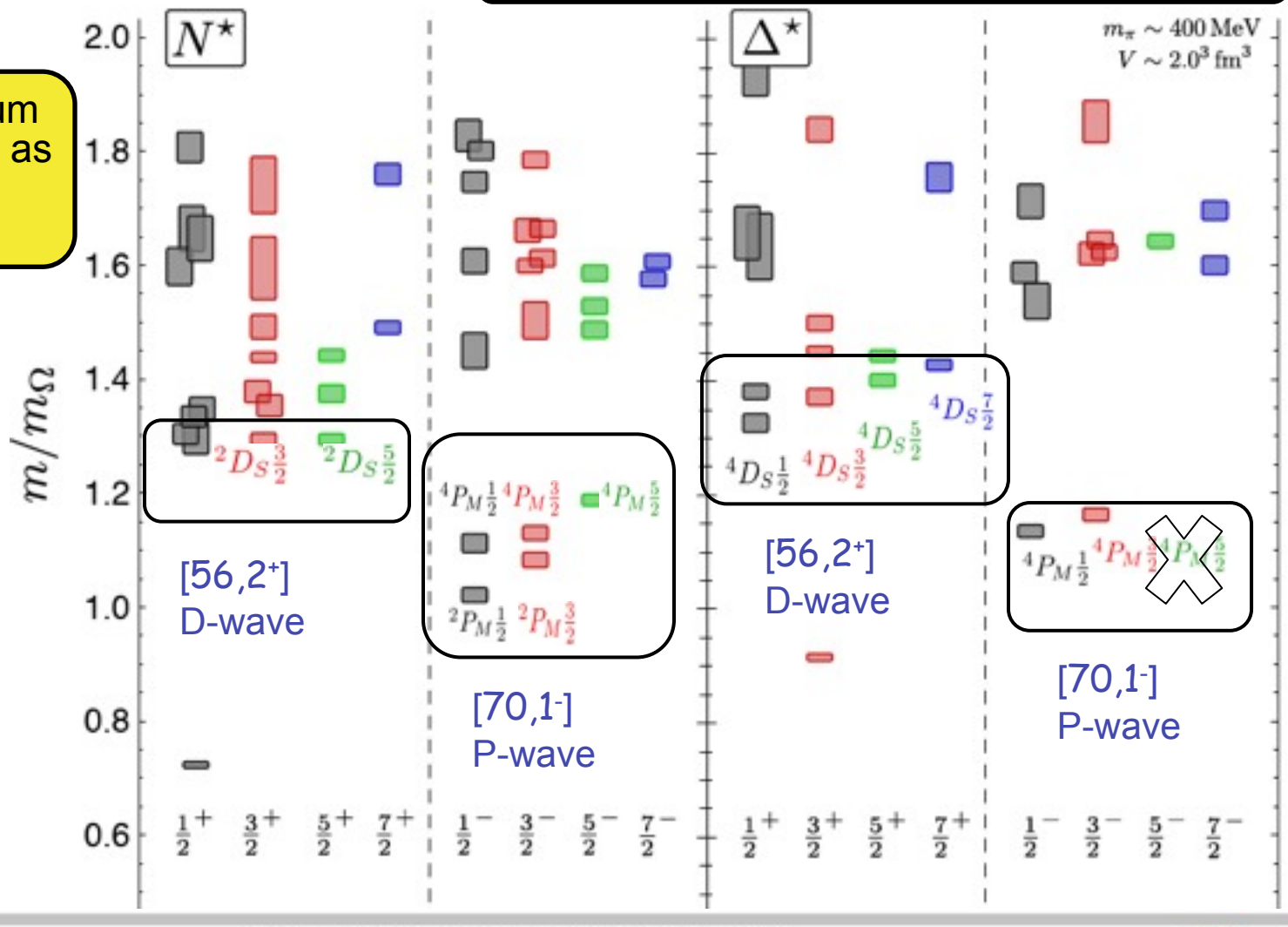
Excited Baryon Spectrum -III

Leonid - please close your eyes..

Discern structure: wave-function overlaps

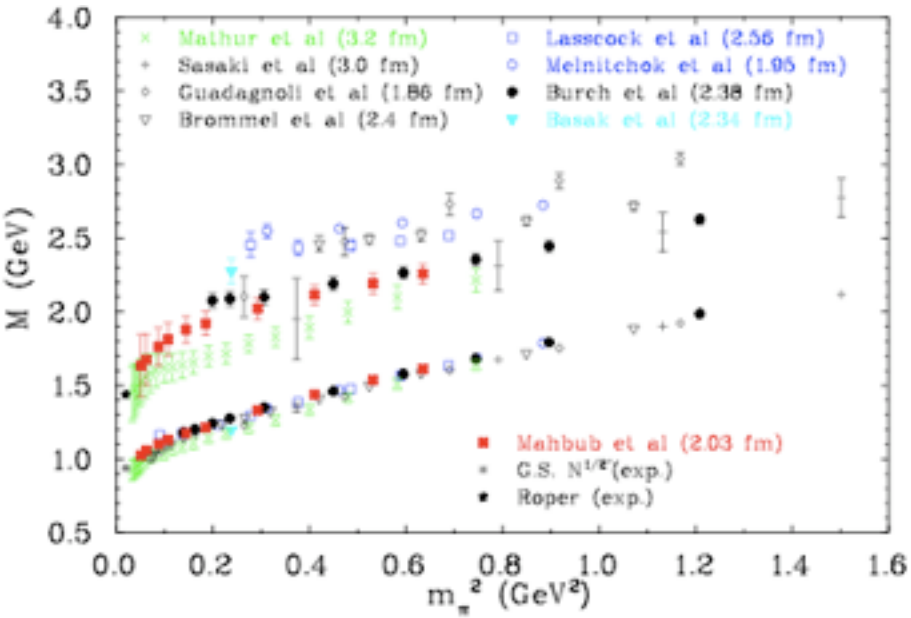
Suggests spectrum at least as dense as quark model

< 2% error bars

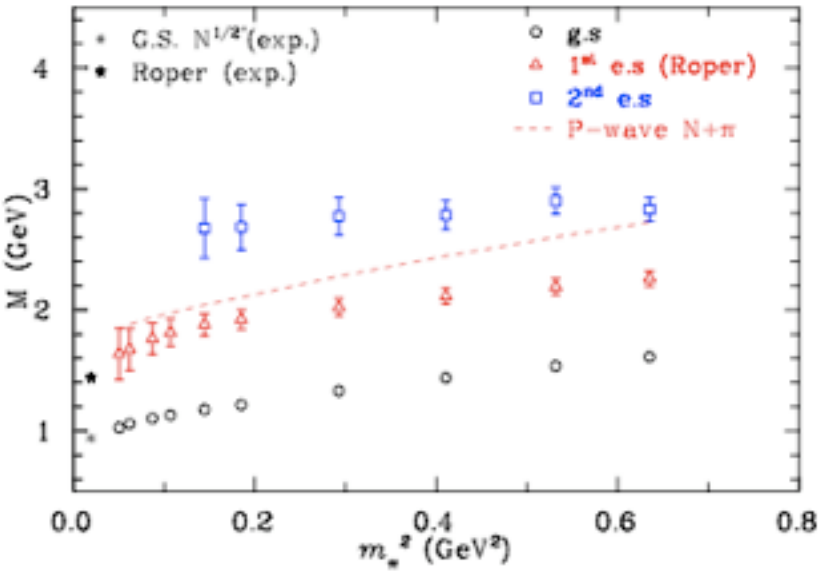


Roper Resonance - I

Roper (1440): lightest positive parity excitation of the nucleon – lighter than the N(1535) negative-parity excitation. Hard to reconcile with constituent quark model.



Two quenched calculations observe light Roper



Mahbub et al., arXiv:0910:2789

Derek Leinweber - PM

Radiative Transitions

Testbed - Charmonium

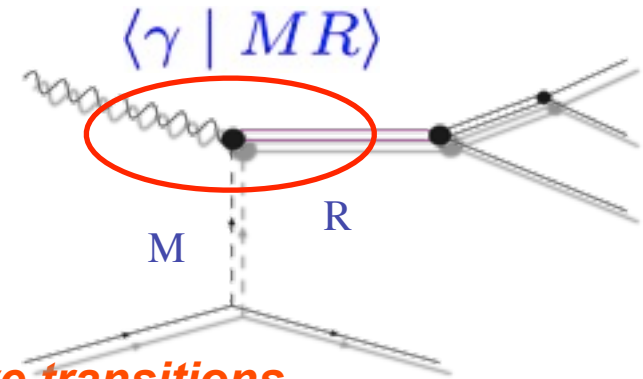
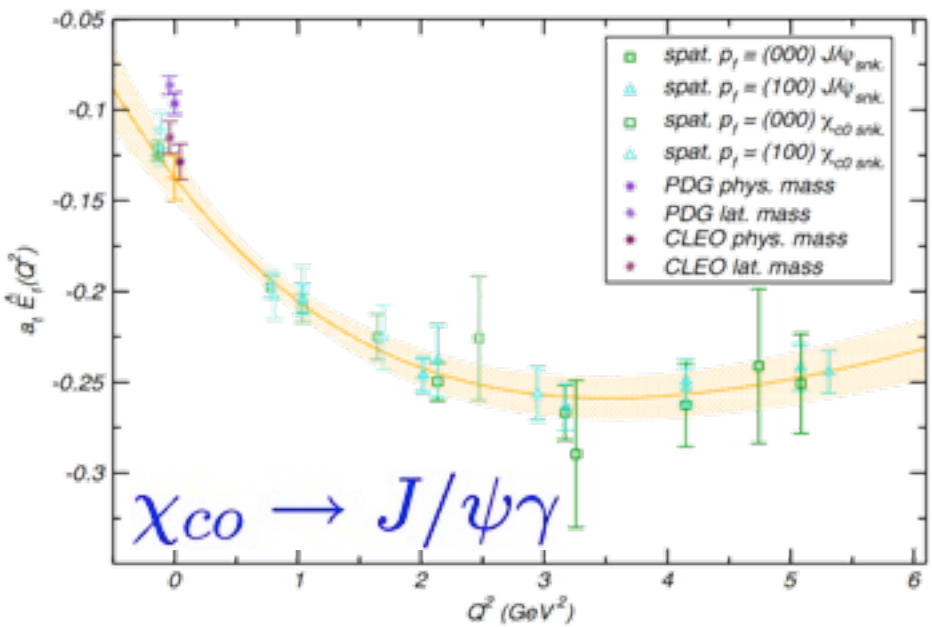
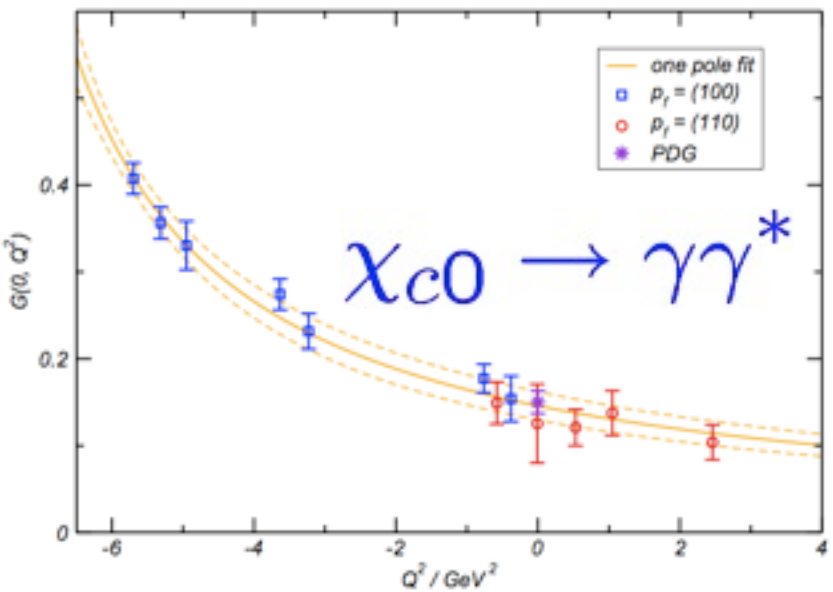


Photo-production

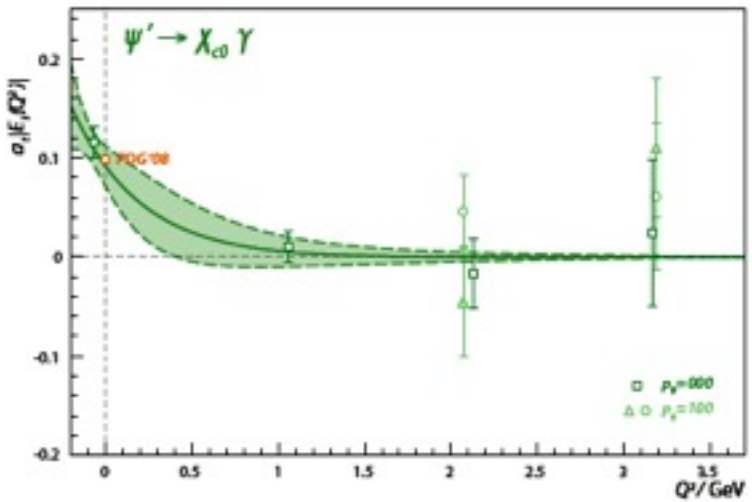
Radiative transitions,
Dudek, Edwards, DGR, PRD73, 074507

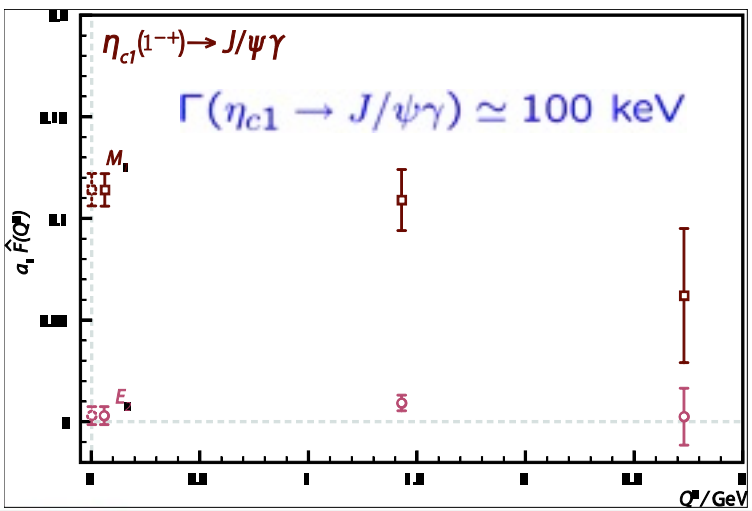


Two-photon widths,
Dudek, Edwards, PRL97, 172001



Radiative Transitions in Charmonium - II





J Dudek, R Edwards, C Thomas, Phys. Rev. D79:094504 (2009).

Use of variational method, and the optimized meson operators, to compute radiative transitions between excited states and exotics.

Radiative width of hybrid comparable to conventional meson – important for GlueX

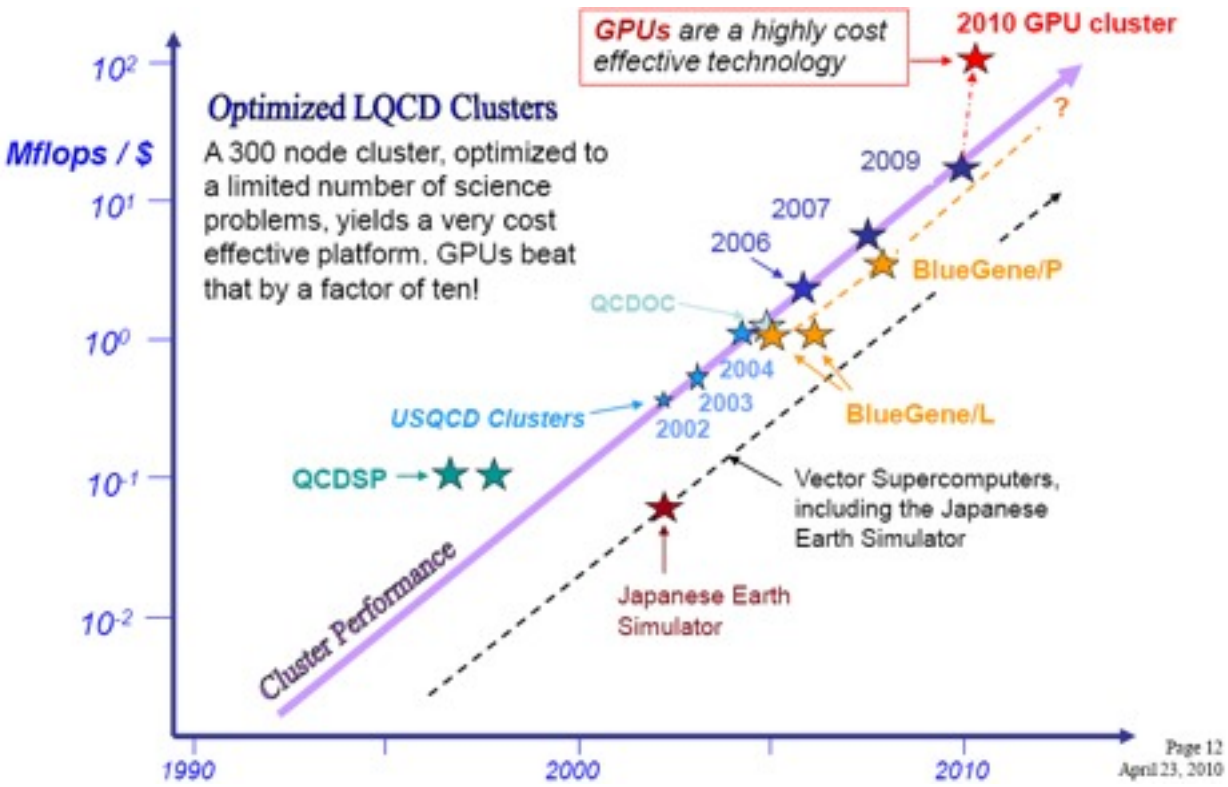
Summary

- Spectroscopy of excited states affords an excellent theatre in which to study QCD in low-energy regime.
- Major progress at reliable determinations of the spectrum, *with quantum numbers identified.* *Major advance!*
- Lattice calculations used to construct new "phenomenology" of QCD.
- Beyond spectrum - electromagnetic properties
- Next step for lattice QCD:
  - Isoscalars...
  - Complete the calculation: where are the multi-hadrons?
  - Determine the *phase shifts* - model dependent extraction of resonance parameters

- Calculations at physical light-quark masses

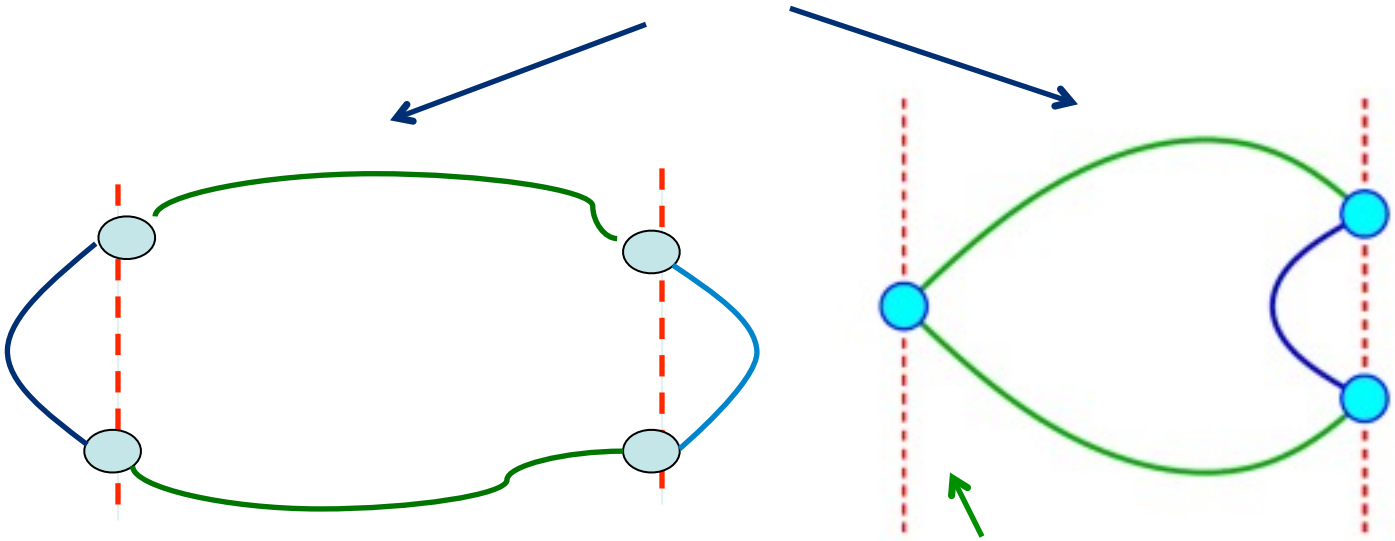
Accelerator "setup"

- Crays/BlueGene for Gauge Generation - capability
- GPUs for physics measurements - capacity



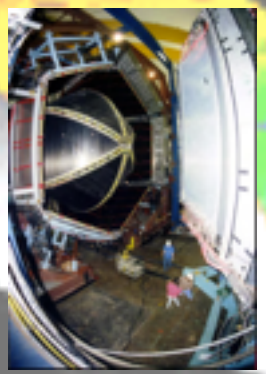
Multi-hadron Operators

Need "all-to-all"



Usual methods give "point-to-all"

The road to exascale for Spectroscopy



panda

Spectrum and properties of mesons, in particular with exotic quantum number

N-N* transition form factors

N* Spectrum

GlueX

Photocouplings in charmonium

Cascade Spectrum

Spectrum and photoproduction of isovector mesons

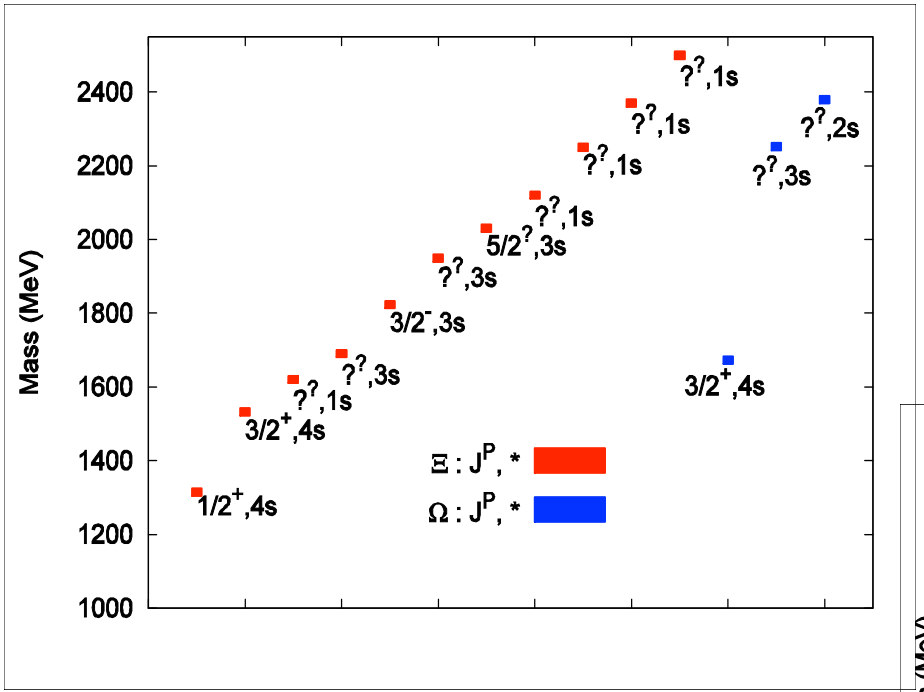
Meson and baryon spectrum with m_π ~180 MeV

Precise computations of ground-states

10x tera | 100x tera | peta | 10x peta | 100x peta | exa

Discovery Potential: Cascade

Cascades (uss) are largely *terra incognita*



Thanks to N. Mathur