Transverse (Spin) Structure of Hadrons

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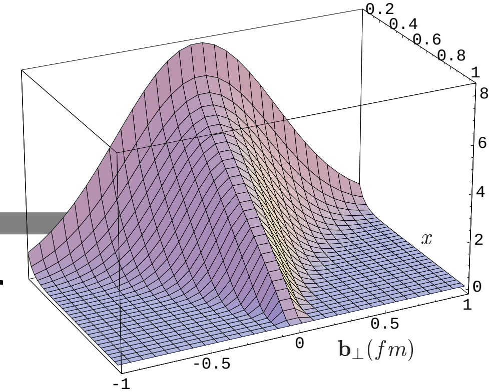
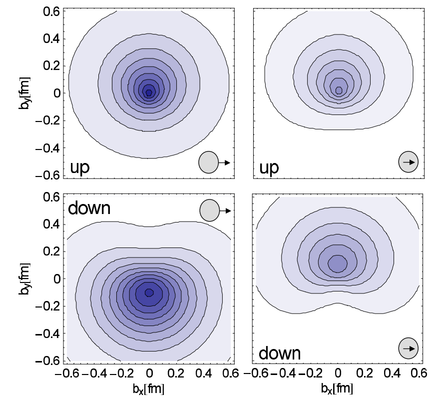
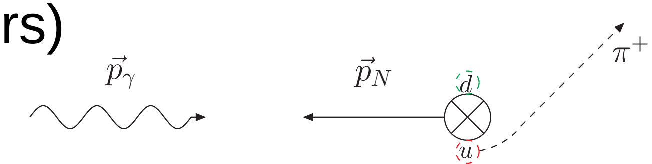
Outline



- Probabilistic interpretation of GPDs as Fourier trafo of impact parameter dependent PDFs
  - $H(x,0,-\mathbf{\Delta}_\perp^2) \longrightarrow q(x,\mathbf{b}_\perp)$
  - $E(x,0,-\mathbf{\Delta}_\perp^2) \longrightarrow$ $\perp$ distortion of PDFs when the target is $\perp$ polarized
  - $\bar{E}_T(x,0,-\mathbf{\Delta}_\perp^2) \longrightarrow$ transversity distribution in unpol. target



$\hookrightarrow$ SSA in SIDID/DY (Sivers & Boer-Mulders)



$\hookrightarrow$ twist-3 quark-gluon correlations: $\int dx\, x^2 \bar{g}_2(x)$ & $\int dx\, x^2 \bar{e}(x)$

- Summary

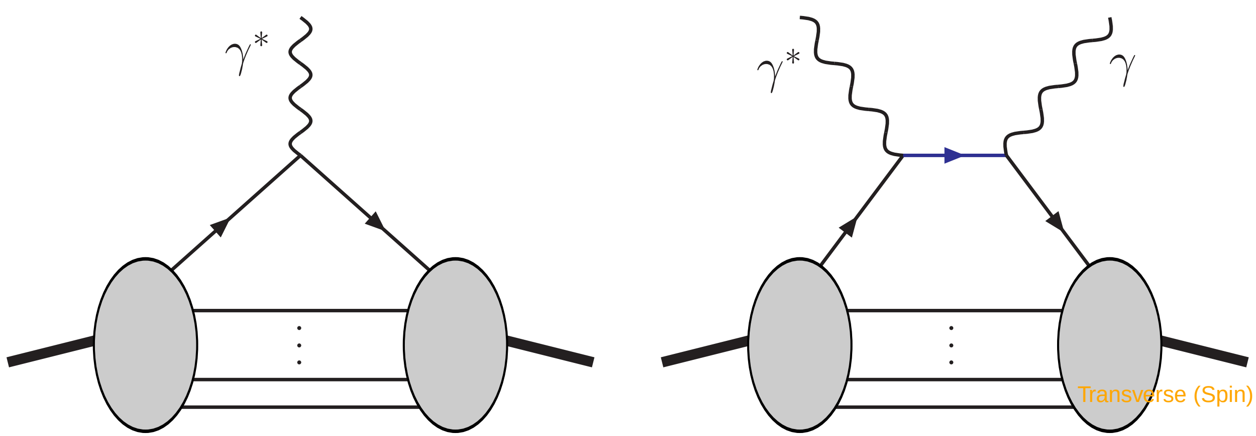
Generalized Parton Distributions (GPDs)

- GPDs: **decomposition of form factors** at a given value of t, w.r.t. the average momentum fraction $x = \frac{1}{2}(x_i + x_f)$ of the active quark

$$\int dx H_q(x,\xi,t) = F_1^q(t) \qquad \int dx \tilde{H}_q(x,\xi,t) = G_A^q(t)$$
$$\int dx E_q(x,\xi,t) = F_2^q(t) \qquad \int dx \tilde{E}_q(x,\xi,t) = G_P^q(t),$$

  - x_i and x_f are the momentum fractions of the quark before and after the momentum transfer; $2\xi = x_f - x_i$

- GPDs can be probed in deeply virtual Compton scattering (DVCS) as well as deeply virtual meson production (DVMP)



Impact parameter dependent PDFs

- define $\perp$ localized state [D.Soper,PRD**15**, 1141 (1977)]

$$\left|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \lambda\right\rangle \equiv \mathcal{N} \int d^2\mathbf{p}_\perp \left|p^+, \mathbf{p}_\perp, \lambda\right\rangle$$

  Note: $\perp$ boosts in IMF form Galilean subgroup $\Rightarrow$ this state has
  $\mathbf{R}_\perp \equiv \frac{1}{P^+} \int dx^- d^2\mathbf{x}_\perp \, \mathbf{x}_\perp T^{++}(x) = \sum_i x_i \mathbf{r}_{i,\perp} = \mathbf{0}_\perp$
  (cf.: working in CM frame in nonrel. physics)

- define impact parameter dependent PDF

$$q(x, \mathbf{b}_\perp) \equiv \int \frac{dx^-}{4\pi} \left\langle p^+, \mathbf{R}_\perp = \mathbf{0}_\perp \right| \bar{q}(-\frac{x^-}{2}, \mathbf{b}_\perp) \gamma^+ q(\frac{x^-}{2}, \mathbf{b}_\perp) \left| p^+, \mathbf{R}_\perp = \mathbf{0}_\perp \right\rangle e^{ixp^+x^-}$$

$\hookrightarrow$

$$\begin{aligned} q(x, \mathbf{b}_\perp) &= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp} H(x, 0, -\mathbf{\Delta}_\perp^2), \\ \Delta q(x, \mathbf{b}_\perp) &= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp} \tilde{H}(x, 0, -\mathbf{\Delta}_\perp^2), \end{aligned}$$

Impact parameter dependent PDFs

- No relativistic corrections (Galilean subgroup!)

$\hookrightarrow$ corrolary: interpretation of 2d-FT of $F_1(Q^2)$ as charge density in transverse plane also free from relativistic corrections (Soper 1977; MB 2003)

- $q(x, \mathbf{b}_\perp)$ has probabilistic interpretation as number density
- $\xi = 0$ essential for probabilistic interpretation

$$\left\langle p^{+\prime}, 0_\perp \right| b^\dagger(x, \mathbf{b}_\perp) b(x, \mathbf{b}_\perp) \left| p^+, 0_\perp \right\rangle \sim \left| b(x, \mathbf{b}_\perp) \rangle | p^+, 0_\perp \right|^2$$

works only for $p^+ = p^{+\prime}$

- Reference point for IPDs is transverse center of (longitudinal) momentum $\mathbf{R}_\perp \equiv \sum_i x_i \mathbf{r}_{i,\perp}$

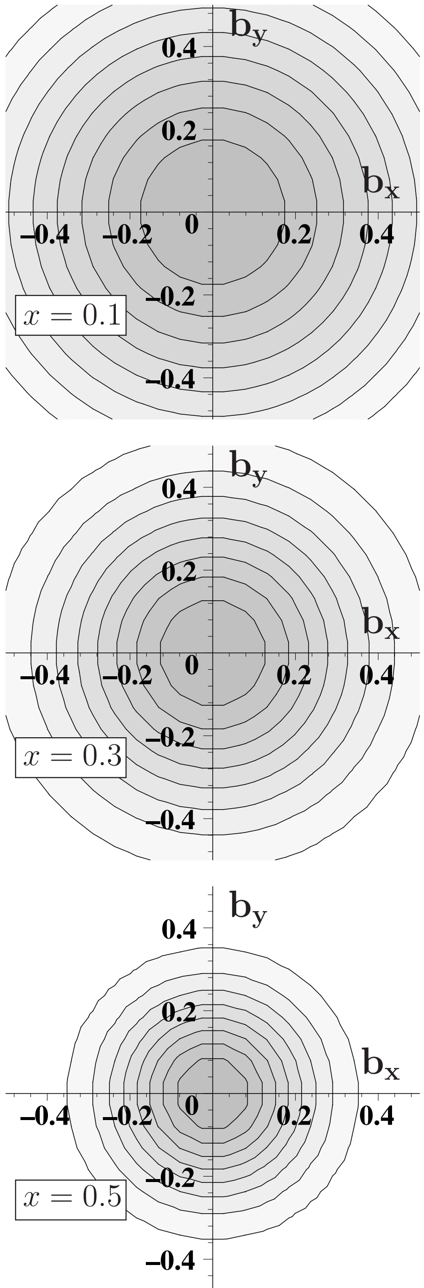
$\hookrightarrow$ for $x \to 1$, active quark 'becomes' COM, and $q(x, \mathbf{b}_\perp)$ must become very narrow (δ-function like)

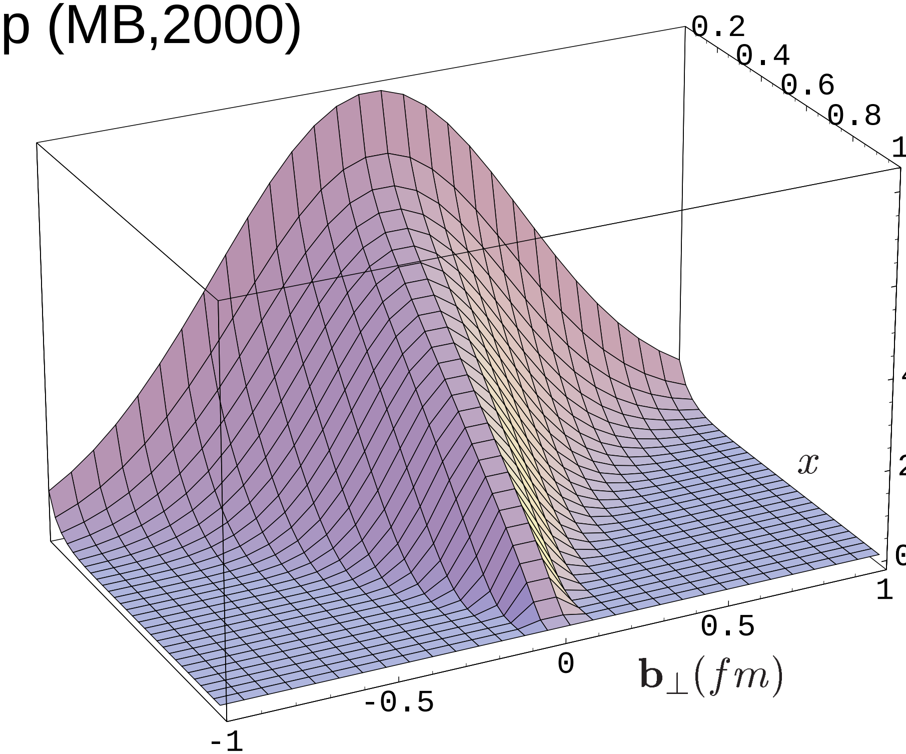
$\hookrightarrow$ $H(x, 0, -\mathbf{\Delta}_\perp^2)$ must become $\mathbf{\Delta}_\perp$ indep. as $x \to 1$ (MB, 2000)

$\hookrightarrow$ consistent with lattice results for first few moments

unpolarized p (MB,2000)

$q(x, \mathbf{b}_\perp)$ for unpol. p





$x =$ momentum fraction of the quark

$\vec{b} = \perp$ position of the quark

Transversely Deformed Distributions and $E(x, 0, -\mathbf{\Delta}_\perp^2)$

M.B., Int.J.Mod.Phys.A**18**, 173 (2003)

- So far: only unpolarized (or long. pol.) nucleon! In general ($\xi = 0$):

$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \langle P+\Delta,\uparrow|\bar{q}(0)\,\gamma^+ q(x^-)|\,P,\uparrow\rangle = H(x,0,-\mathbf{\Delta}_\perp^2)$$

$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \langle P+\Delta,\uparrow|\bar{q}(0)\,\gamma^+ q(x^-)|\,P,\downarrow\rangle = -\frac{\Delta_x - i\Delta_y}{2M} E(x,0,-\mathbf{\Delta}_\perp^2).$$

- Consider nucleon polarized in x direction (in IMF)
  $|X\rangle \equiv |p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \uparrow\rangle + |p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \downarrow\rangle$.

$\hookrightarrow$ unpolarized quark distribution for this state:

$$q(x,\mathbf{b}_\perp) = \mathcal{H}(x,\mathbf{b}_\perp) - \frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} E(x, 0, -\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp}$$

- Physics: $j^+ = j^0 + j^3$, and left-right asymmetry from j^3 !
  [X.Ji, PRL **91**, 062001 (2003)]

Transversely Deformed PDFs and $E(x, 0, -\mathbf{\Delta}_\perp^2)$

- $q(x, \mathbf{b}_\perp)$ in $\perp$ polarized nucleon is deformed compared to longitudinally polarized nucleons !

$$q(x,\mathbf{b}_\perp) = \mathcal{H}(x,\mathbf{b}_\perp) - \frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} E(x,0,-\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp}$$

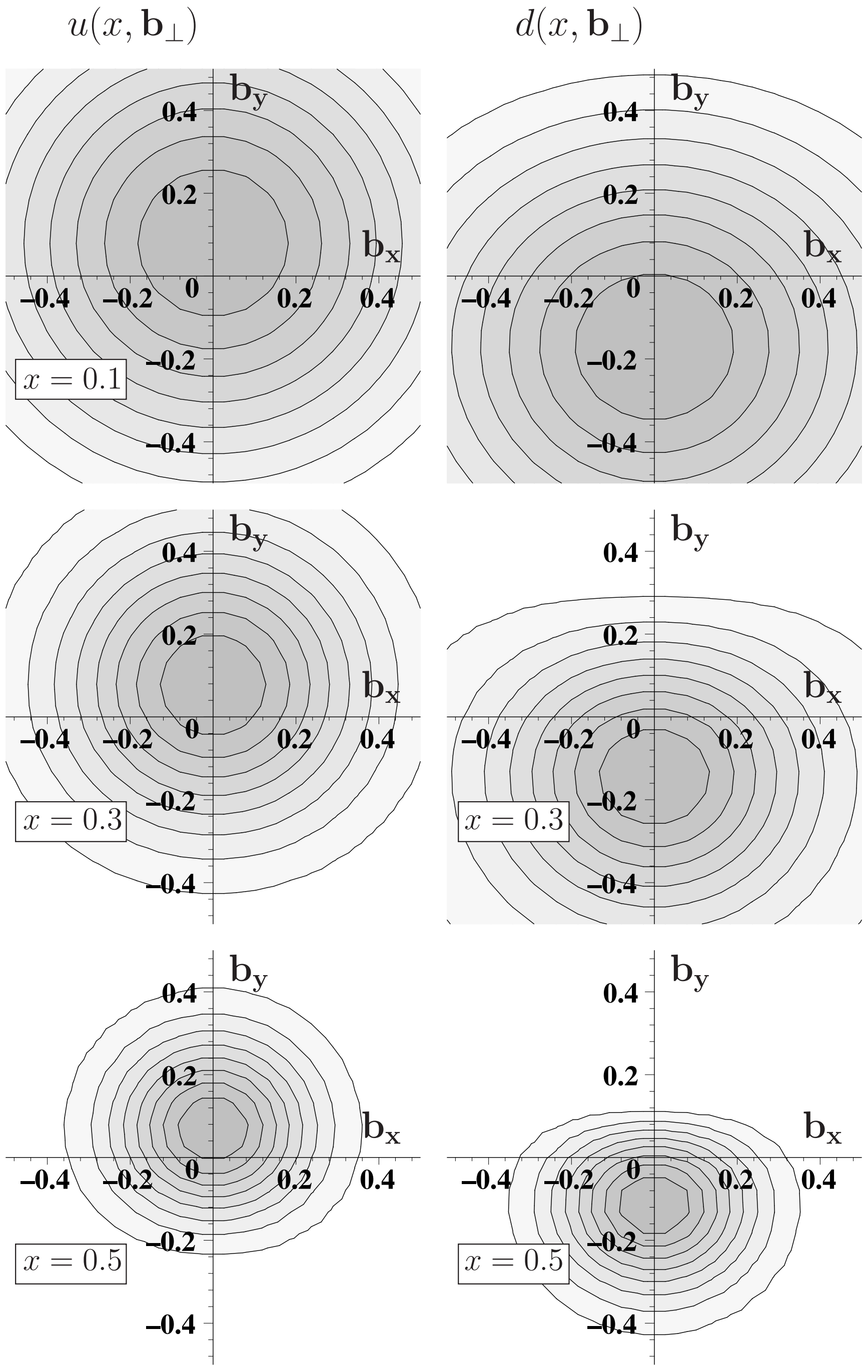
- mean $\perp$ deformation of flavor q ($\perp$ flavor dipole moment)

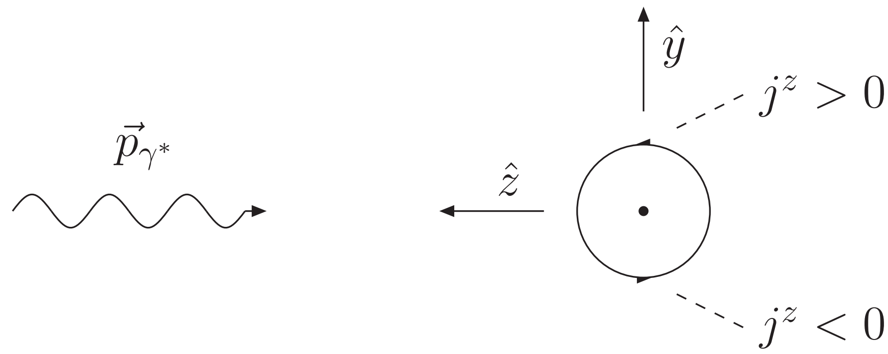
$$d_y^q \equiv \int dx \int d^2\mathbf{b}_\perp q(x,\mathbf{b}_\perp) b_y = \frac{1}{2M}\int dx E_q(x,0,0) = \frac{\kappa_q^p}{2M}$$

- $\kappa^p = 1.913 = \frac{2}{3}\kappa_u^p - \frac{1}{3}\kappa_d^p + \dots$

$\hookrightarrow$ neglecting strange (and heavier) quarks:
  - $\kappa_u^p = 2\kappa_p + \kappa_n = 1.673 \quad \Rightarrow$ shift in $+\hat{y}$ direction
  - $\kappa_d^p = 2\kappa_n + \kappa_p = -2.033 \quad \Rightarrow$ shift in $-\hat{y}$ direction
  - for proton polarized in $+\hat{x}$ direction
  - $d_y^q = \mathcal{O}(\pm 0.2 fm)$

p polarized in $+\hat{x}$ direction (MB,2003)



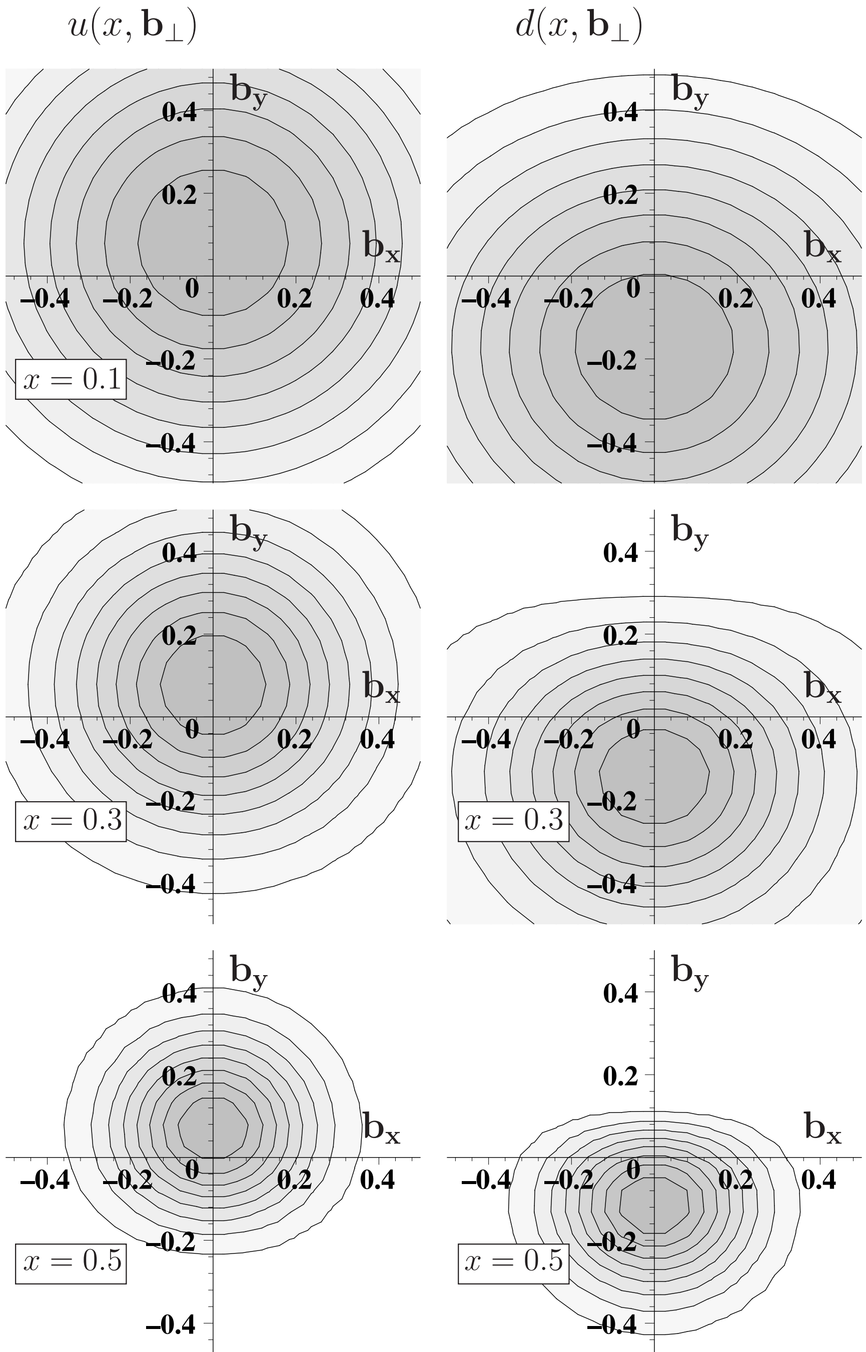


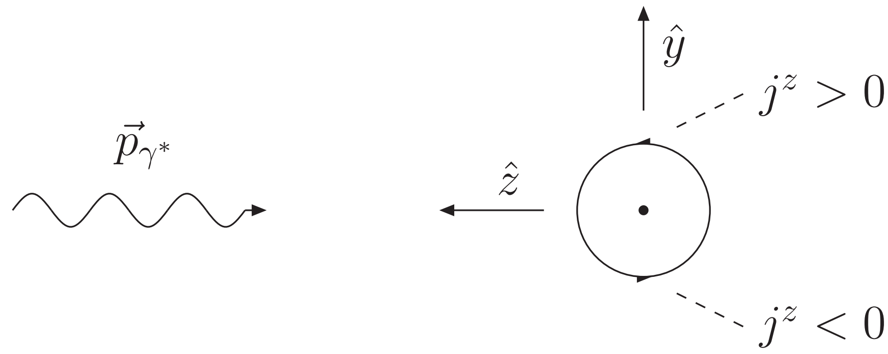
- virtual photon 'sees' enhancement when quark currents point in direction opposite to photon momentum

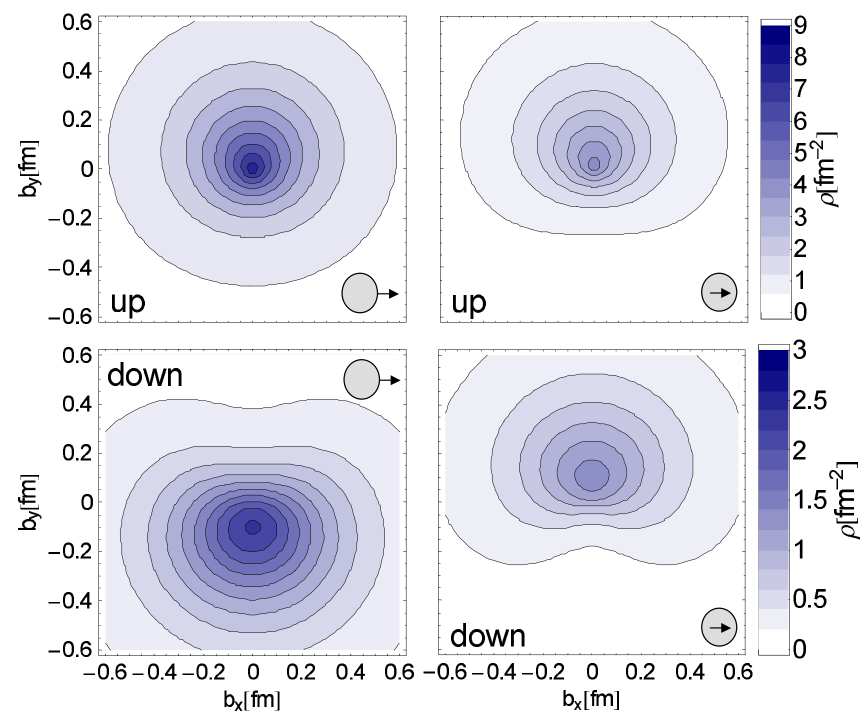
$\hookrightarrow$ sideways shift of quark distributions

- sign & magnitude of shift (model-independently) predicted to be related to the proton/neutron anomalous magnetic moment!

p polarized in $+\hat{x}$ direction



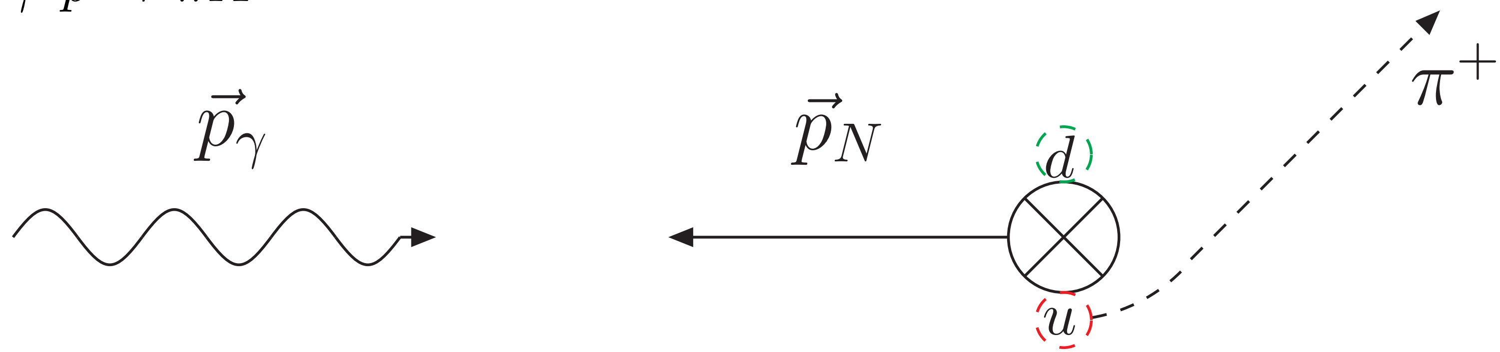




lattice results (→ QCDSF)

GPD $\longleftrightarrow$ SSA (Sivers)

- example: $\gamma^* p \to \pi X$



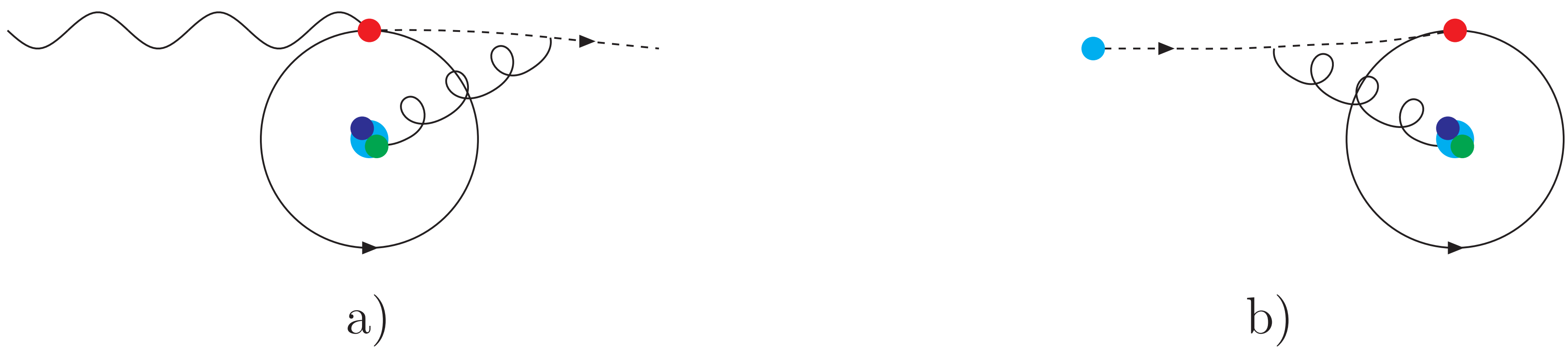
- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign "determined" by κ_u & κ_d
- attractive FSI deflects active quark towards the center of momentum

$\hookrightarrow$ FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction

$\hookrightarrow$ correlation between sign of κ_q^p and sign of SSA: $f_{1T}^{\perp q} \sim -\kappa_q^p$

- $f_{1T}^{\perp q} \sim -\kappa_q^p$ confirmed by HERMES data (also consistent with COMPASS deuteron data $f_{1T}^{\perp u} + f_{1T}^{\perp d} \approx 0$)

$f_{1T}^{\perp}(x, \mathbf{k}_\perp)_{DY} = -f_{1T}^{\perp}(x, \mathbf{k}_\perp)_{SIDIS}$



- time reversal: FSI $\leftrightarrow$ ISI

SIDIS: compare FSI for ‘red’ q that is being knocked out with ISI for an anti-red $\bar{q}$ that is about to annihilate that bound q

$\hookrightarrow$ FSI for knocked out q is attractive

DY: nucleon is color singlet $\rightarrow$ when to-be-annihilated q is ‘red’, the spectators must be anti-red

$\hookrightarrow$ ISI with spectators is repulsive

- test of this relation is a test of TMD factorization

Quark-Gluon Correlations (Introduction)

- (longitudinally) polarized polarized DIS at leading twist $\longrightarrow$ 'polarized quark distribution' $g_1^q(x) = q^\uparrow(x) + \bar{q}^\uparrow(x) - q_\downarrow(x) - \bar{q}_\downarrow(x)$
- $\frac{1}{Q^2}$-corrections to X-section involve 'higher-twist' distribution functions, such as $g_2(x)$

$$\sigma_{LL} \propto g_1 - \frac{2Mx}{\nu} g_2$$

- $g_2(x)$ involves quark-gluon correlations and does not have a parton interpretation as difference between number densities
- for $\perp$ polarized target, g_1 and g_2 contribute equally to σ_{LT}

$$\sigma_{LT} \propto g_T \equiv g_1 + g_2$$

$\hookrightarrow$ 'clean' separation between higher order corrections to leading twist (g_1) and higher twist effects (g_2)

- what can one learn from g_2?

Quark-Gluon Correlations (QCD analysis)

- $g_2(x) = g_2^{WW}(x) + \bar{g}_2(x)$, with $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$
- $\bar{g}_2(x)$ involves quark-gluon correlations, e.g.

$$\int dx x^2 \bar{g}_2(x) = \frac{1}{3} d_2 = \frac{1}{6MP^{+2}S^x} \left\langle P,S \left| \bar{q}(0) g G^{+y}(0) \gamma^+ q(0) \right| P,S \right\rangle$$

- $\sqrt{2} G^{+y} \equiv G^{0y} + G^{zy} = -E^y + B^x$
- sometimes called color-electric and magnetic polarizabilities
  $2M^2 \vec{S} \chi_E = \left\langle P,S \left| \vec{j}_a \times \vec{E}_a \right| P,S \right\rangle$ & $2M^2 \vec{S} \chi_B = \left\langle P,S \left| j_a^0 \vec{B}_a \right| P,S \right\rangle$
  with $d_2 = \frac{1}{4}(\chi_E + 2\chi_M)$ — but these names are misleading!

Quark-Gluon Correlations (Interpretation)

- $\bar{g}_2(x)$ involves quark-gluon correlations, e.g.

$$\int dx x^2 \bar{g}_2(x) = \frac{1}{3} d_2 = \frac{1}{6MP^{+2}S^x} \left\langle P,S \left| \bar{q}(0) g G^{+y}(0) \gamma^+ q(0) \right| P,S \right\rangle$$

- QED: $\bar{q}(0) e F^{+y}(0) \gamma^+ q(0)$ correlator between quark density $\bar{q}\gamma^+ q$ and ($\hat{y}$-component of the) Lorentz-force

$$F^y = e\left[\vec{E} + \vec{v} \times \vec{B}\right]^y = e\left(E^y - B^x\right) = -e\left(F^{0y} + F^{zy}\right) = -e\sqrt{2} F^{+y}.$$

for charged paricle moving with $\vec{v} = (0,0,-1)$ in the $-\hat{z}$ direction

↪ matrix element of $\bar{q}(0) e F^{+y}(0) \gamma^+ q(0)$ yields γ^+ density (density relevant for DIS in Bj limit!) weighted with the Lorentz force that a charged particle with $\vec{v} = (0,0,-1)$ would experience at that point

↪ d_2 a measure for the **color Lorentz force** acting on the struck quark in SIDIS in the instant **after being hit by the virtual photon**

$$\langle F^y(0) \rangle = -2M^2 d_2 \qquad \text{(rest frame; } S^x = 1\text{)}$$

Quark-Gluon Correlations (Interpretation)

- x^2-moment of twist-4 polarized PDF $g_3(x)$
  $\int dx x^2 g_3(x) \rightsquigarrow \left\langle P,S \left| \bar{q}(0) g\tilde{G}^{\mu\nu}(0)\gamma_\nu q(0) \right| P,S \right\rangle \sim f_2$

$\hookrightarrow$ different linear combination $f_2 = \chi_E - \chi_B$ of χ_E and χ_M

$\hookrightarrow$ combine with $d_2 \Rightarrow$ disentangle electric and magnetic force

- What should one expect (sign/magnitude)?
  - $\kappa_q^p \longrightarrow$ signs of deformation (u/d quarks in $\pm\hat{y}$ direction for proton polarized in $+\hat{x}$ direction $\longrightarrow$ expect force in $\mp\hat{y}$

  $\hookrightarrow$ d_2 positive/negative for u/d quarks in proton

  - large N_C: $d_2^{u/p} = -d_2^{d/p}$ (consistent with $f_{1T}^{\perp u} + f_{1T}^{\perp d} \approx 0$)
  - $F^y = -2M^2 d_2 = -10\frac{\text{GeV}}{fm} d_2$ $\Rightarrow$ expect $|d_2| \ll 1$

- lattice (Göckeler et al.): $d_2^u \approx 0.010$ and $d_2^d \approx -0.0056$

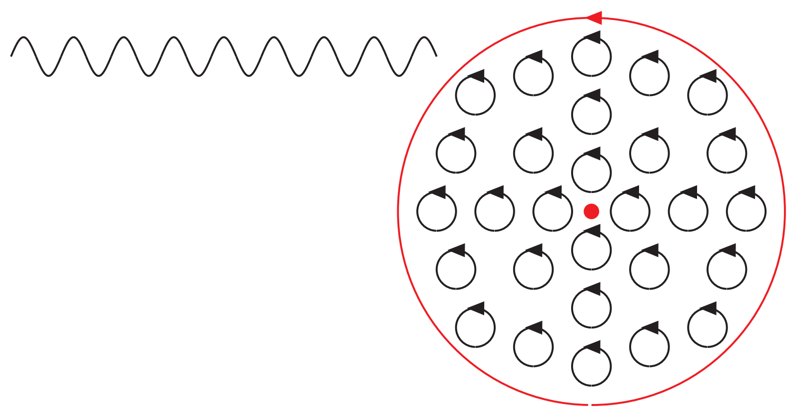
$\hookrightarrow$ $\langle F_u^y(0)\rangle \approx -100\frac{\text{MeV}}{fm}$ $\langle F_d^y(0)\rangle \approx 56\frac{\text{MeV}}{fm}$

- x^2-moment of chirally odd twist-3 PDF $e(x) \longrightarrow$ **transverse force on transversely polarized quark in unpolarized target** ($\leftrightarrow$ Boer-Mulders $h_1^\perp$)

Transversity Distribution in Unpolarized Target (sign)

- Consider quark in ground state hadron polarized out of the plane

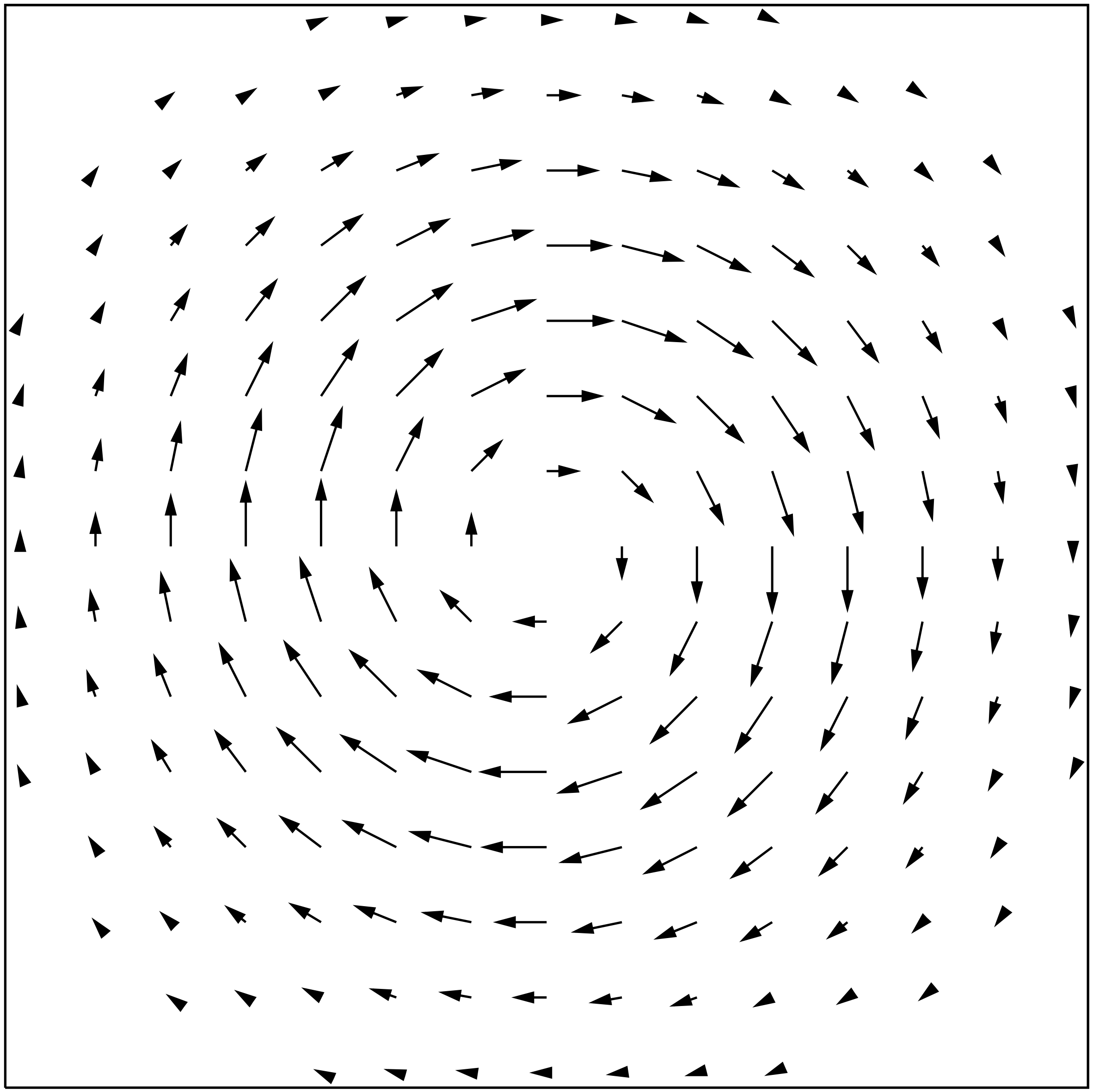
↪ expect counterclockwise net current $\vec{j}$ associated with the magnetization density in this state



- virtual photon 'sees' enhancement of quarks (polarized out of plane) at the top, i.e.

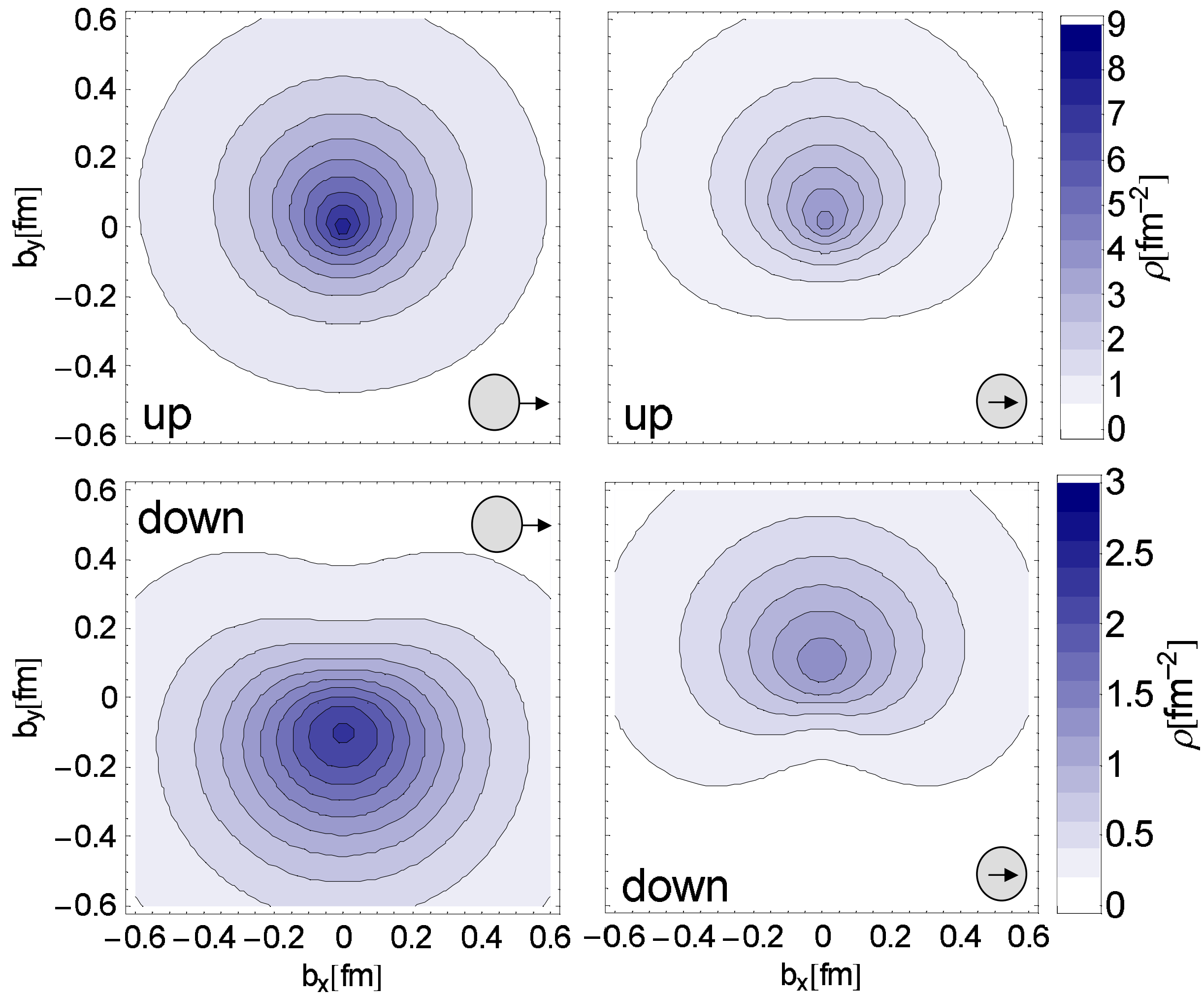
↪ virtual photon 'sees' enhancement of quarks with polarization up (down) on the left (right) side of the hadron

Transversity Distribution in Unpolarized Target



IPDs on the lattice (QCDSF)

- lowest moment of distribution $q(x, \mathbf{b}_\perp)$ for unpol. quarks in $\perp$ pol. proton (left) and of $\perp$ pol. quarks in unpol. proton (right):



Boer-Mulders Function

- SIDIS: attractive FSI expected to convert position space asymmetry into momentum space asymmetry

$\hookrightarrow$ e.g. quarks at negative b_x with spin in $+\hat{y}$ get deflected (due to FSI) into $+\hat{x}$ direction

$\hookrightarrow$ (qualitative) connection between Boer-Mulders function $h_1^\perp(x, \mathbf{k}_\perp)$ and the chirally odd GPD $\bar{E}_T$ that is similar to (qualitative) connection between Sivers function $f_{1T}^\perp(x, \mathbf{k}_\perp)$ and the GPD E.

- Boer-Mulders: distribution of $\perp$ **pol.** quarks in **unpol.** proton

$$f_{q^\uparrow/p}(x, \mathbf{k}_\perp) = \frac{1}{2}\left[f_1^q(x, \mathbf{k}_\perp^2) - h_1^{\perp q}(x, \mathbf{k}_\perp^2)\frac{(\hat{\mathbf{P}} \times \mathbf{k}_\perp) \cdot S_q}{M}\right]$$

- $h_1^{\perp q}(x, \mathbf{k}_\perp^2)$ can be probed in Drell-Yan (RHIC, J-PARC, GSI) and tagged SIDIS (JLab, eRHIC), using Collins-fragmentation

probing BM function in tagged SIDIS

how to measure the transversity distribution of quarks without measuring the transversity of a quark?

- consider semi-inclusive pion production off unpolarized target
- spin-orbit correlations in target wave function provide correlation between (primordial) quark transversity and impact parameter

$\hookrightarrow$ (attractive) FSI provides correlation between quark spin and $\perp$ quark momentum $\Rightarrow$ BM function

- Collins effect: left-right asymmetry of π distribution in fragmentation of $\perp$ polarized quark $\Rightarrow$ 'tag' quark spin

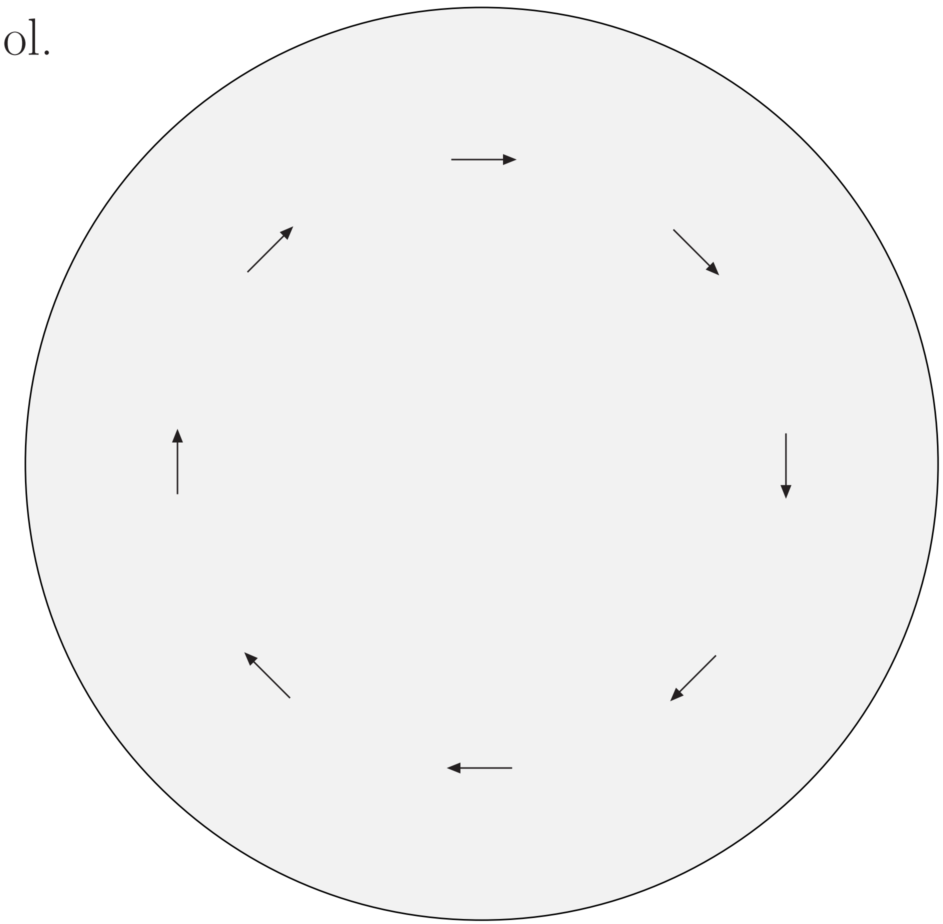
$\hookrightarrow$ $\cos(2\phi)$ modulation of π distribution relative to lepton scattering plane

$\hookrightarrow$ $\cos(2\phi)$ asymmetry proportional to: Collins $\times$ BM

probing BM function in tagged SIDIS

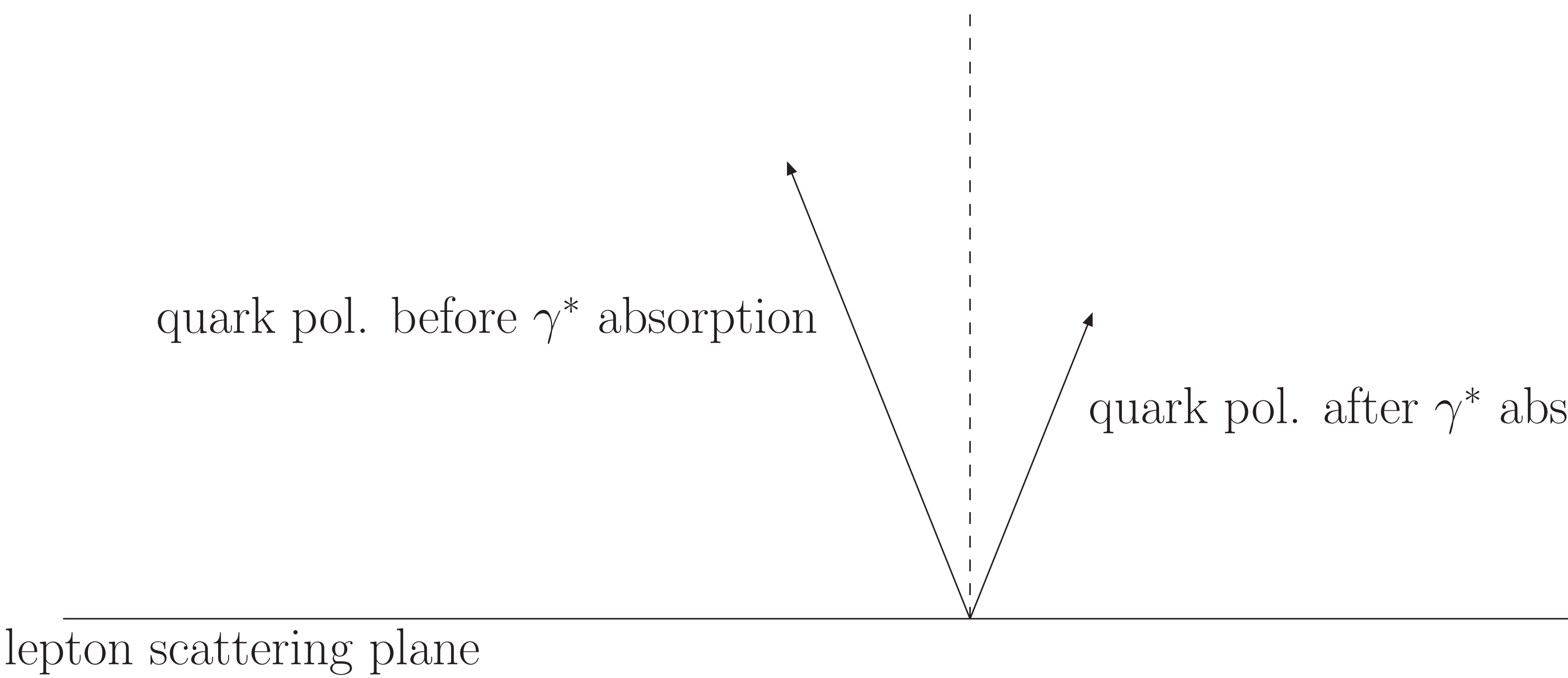
Primordial Quark Transversity Distribution

→ $\perp$ quark pol.



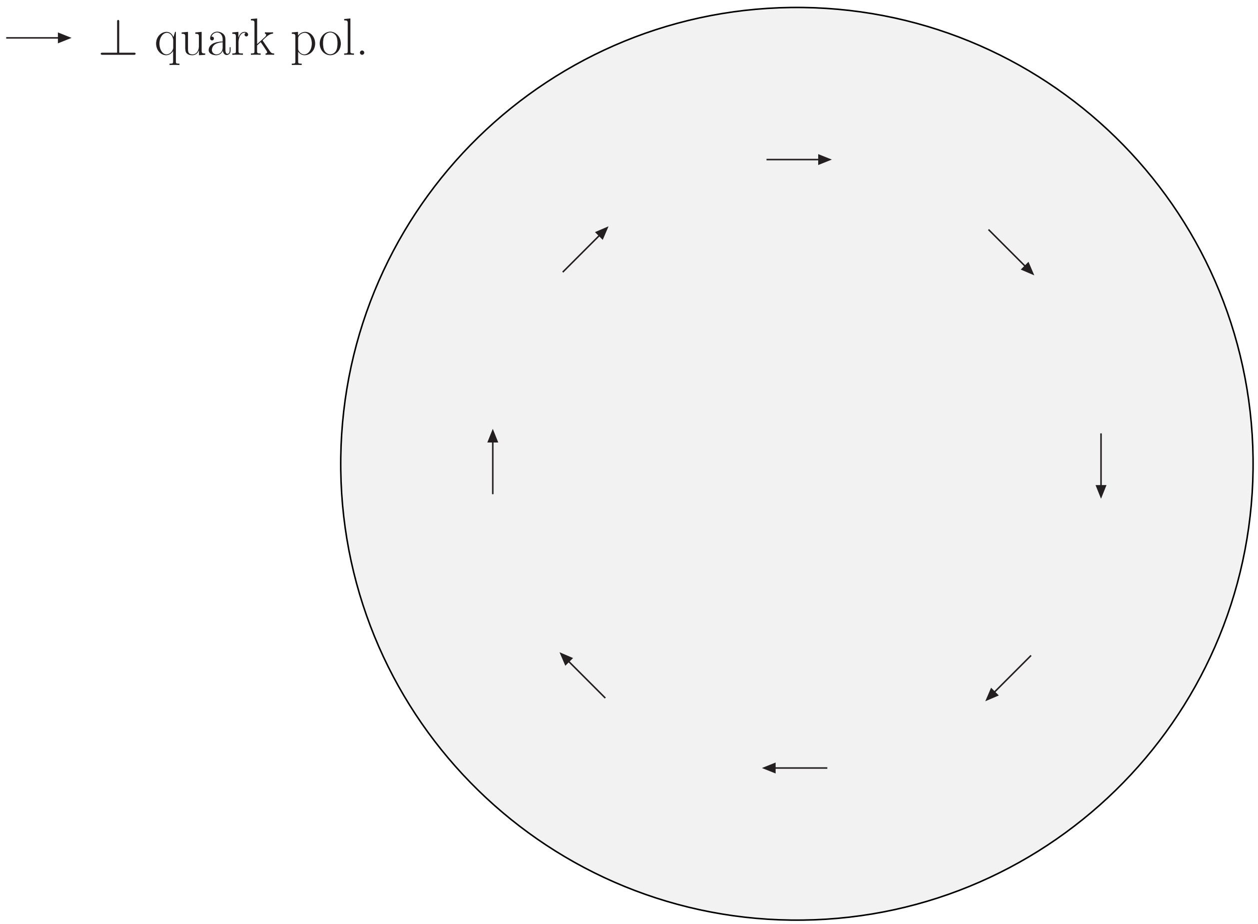
$\perp$ polarization and γ^* absorption

- QED: when the γ^* scatters off $\perp$ polarized quark, the $\perp$ polarization gets modified
  - gets reduced in size
  - gets tilted symmetrically w.r.t. normal of the scattering plane



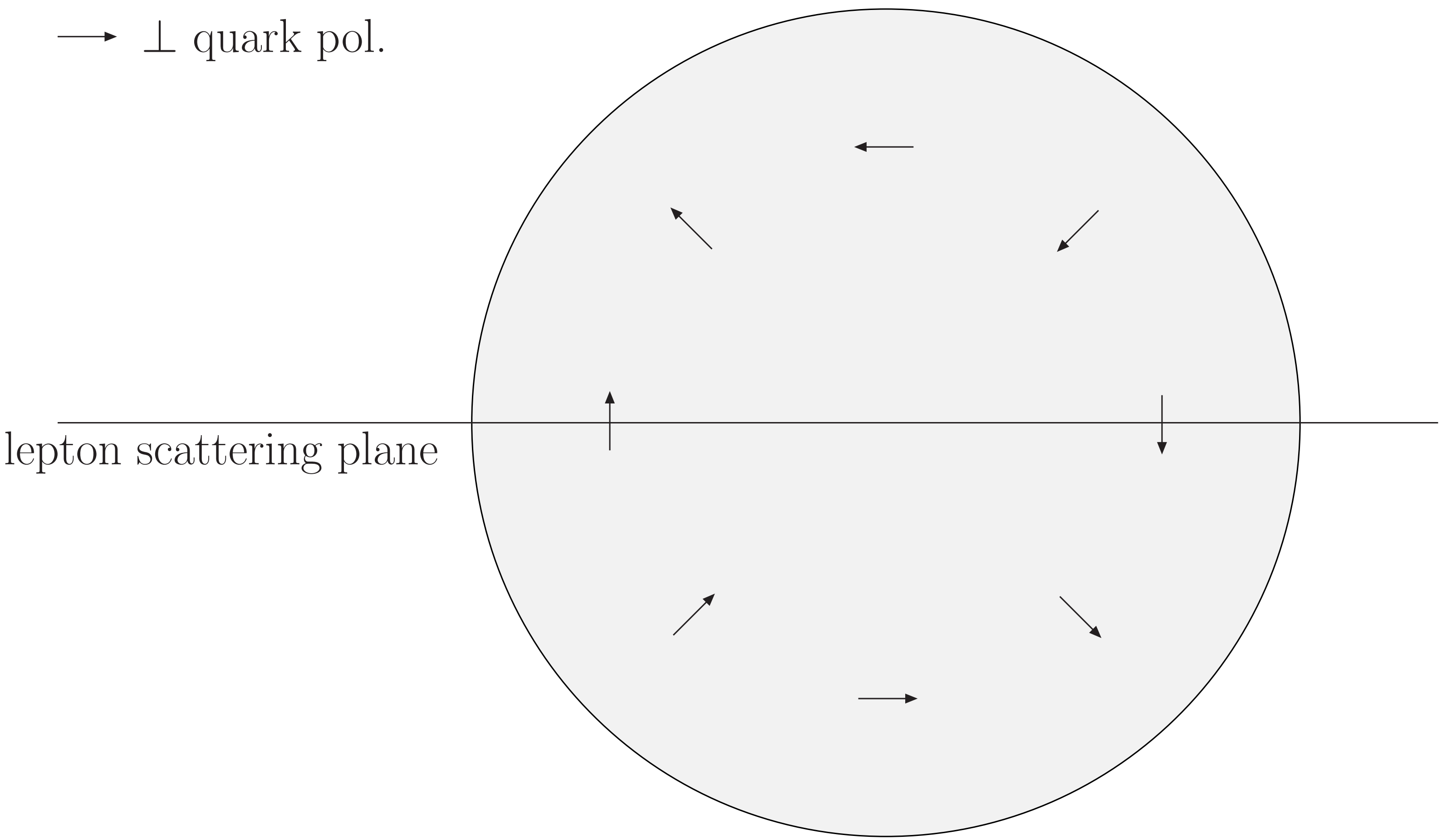
probing BM function in tagged SIDIS

Primordial Quark Transversity Distribution



probing BM function in tagged SIDIS

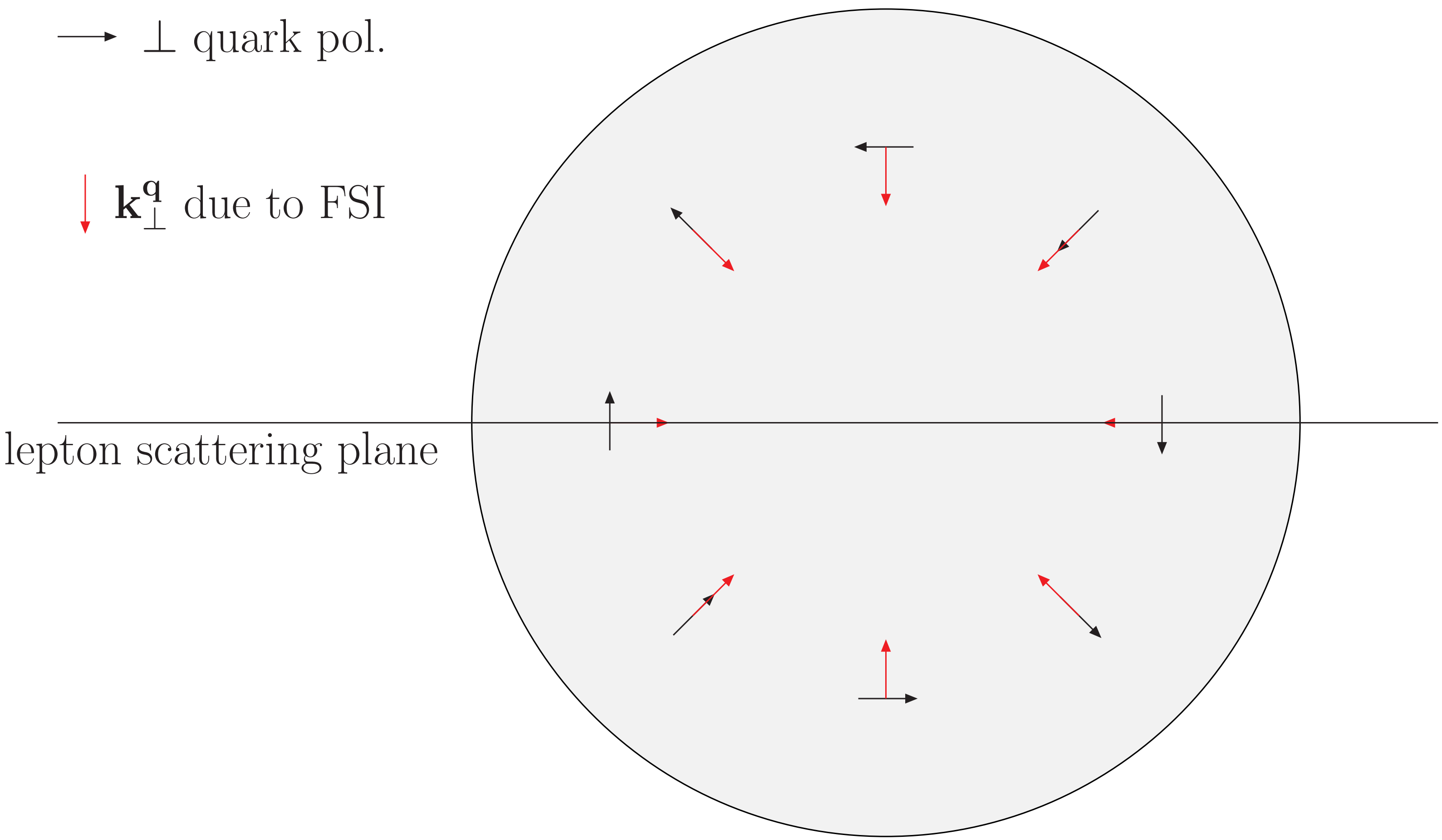
Quark Transversity Distribution after γ^* absorption



quark transversity component in lepton scattering plane flips

probing BM function in tagged SIDIS

$\perp$ momentum due to FSI

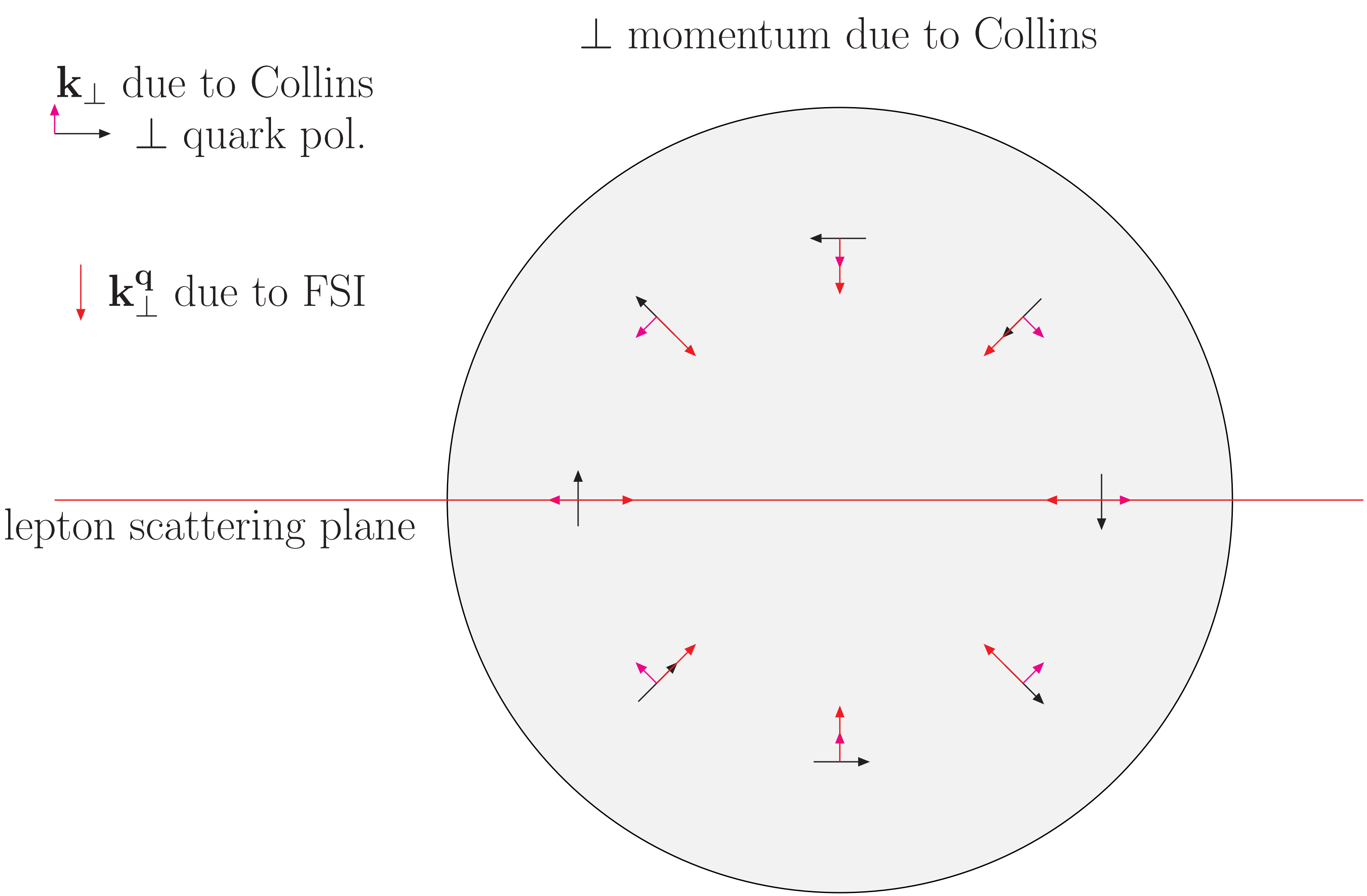


on average, FSI deflects quarks towards the center

Collins effect

- When a $\perp$ polarized struck quark fragments, the strucure of jet is sensitive to polarization of quark
- distribution of hadrons relative to $\perp$ polarization direction may be left-right asymmetric
- asymmetry parameterized by **Collins fragmentation function**
- Artru model:
  - struck quark forms pion with $\bar{q}$ from $q\bar{q}$ pair with $^3P_0$ 'vacuum' quantum numbers
  - $\hookrightarrow$ pion 'inherits' OAM in direction of $\perp$ spin of struck quark
  - $\hookrightarrow$ produced pion preferentially moves to left when looking into direction of motion of fragmenting quark with spin up
- Artru model confirmed by HERMES experiment
- more precise determination of Collins function under way (KEK)

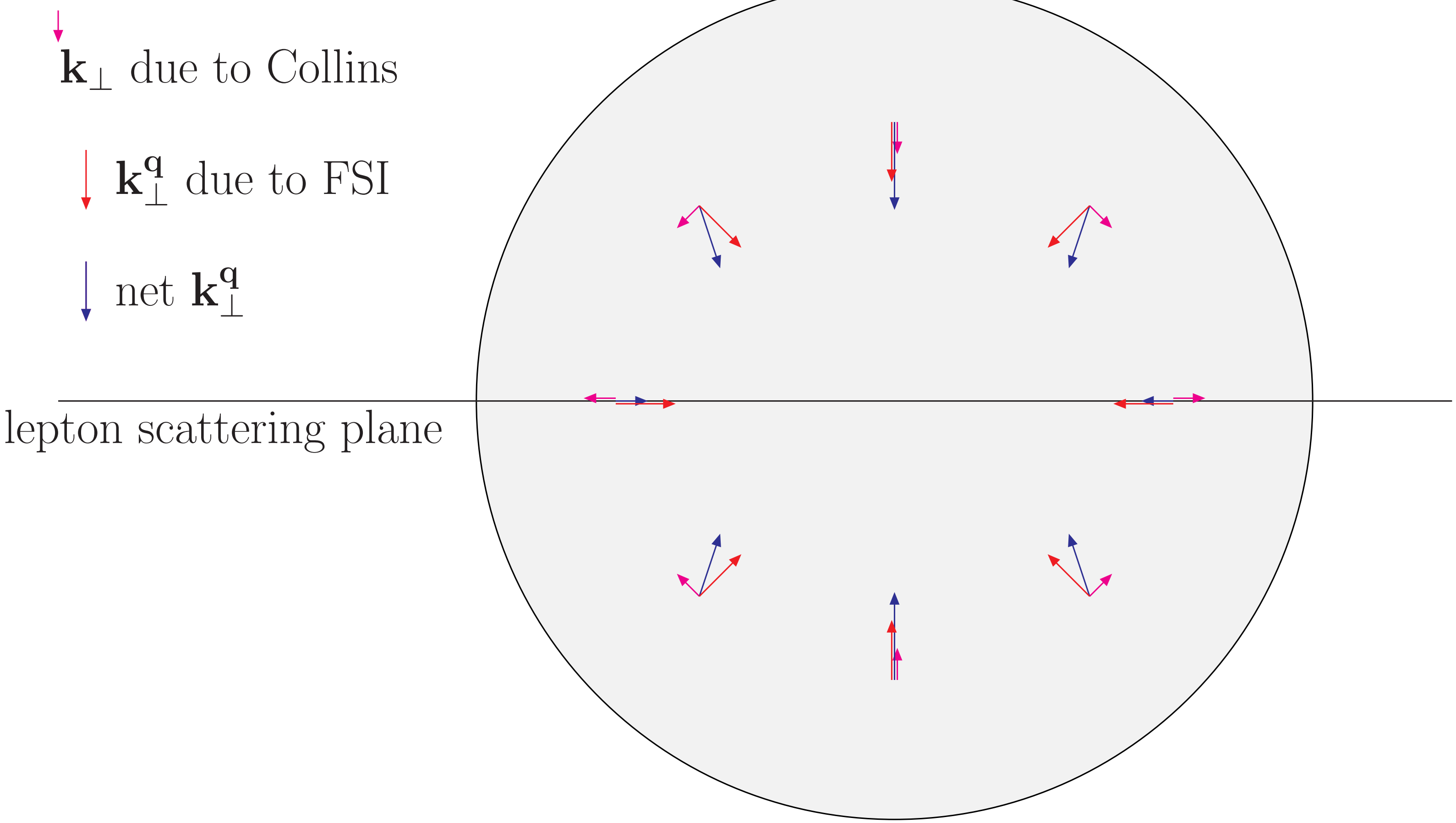
probing BM function in tagged SIDIS



SSA of π in jet emanating from $\perp$ pol. q

probing BM function in tagged SIDIS

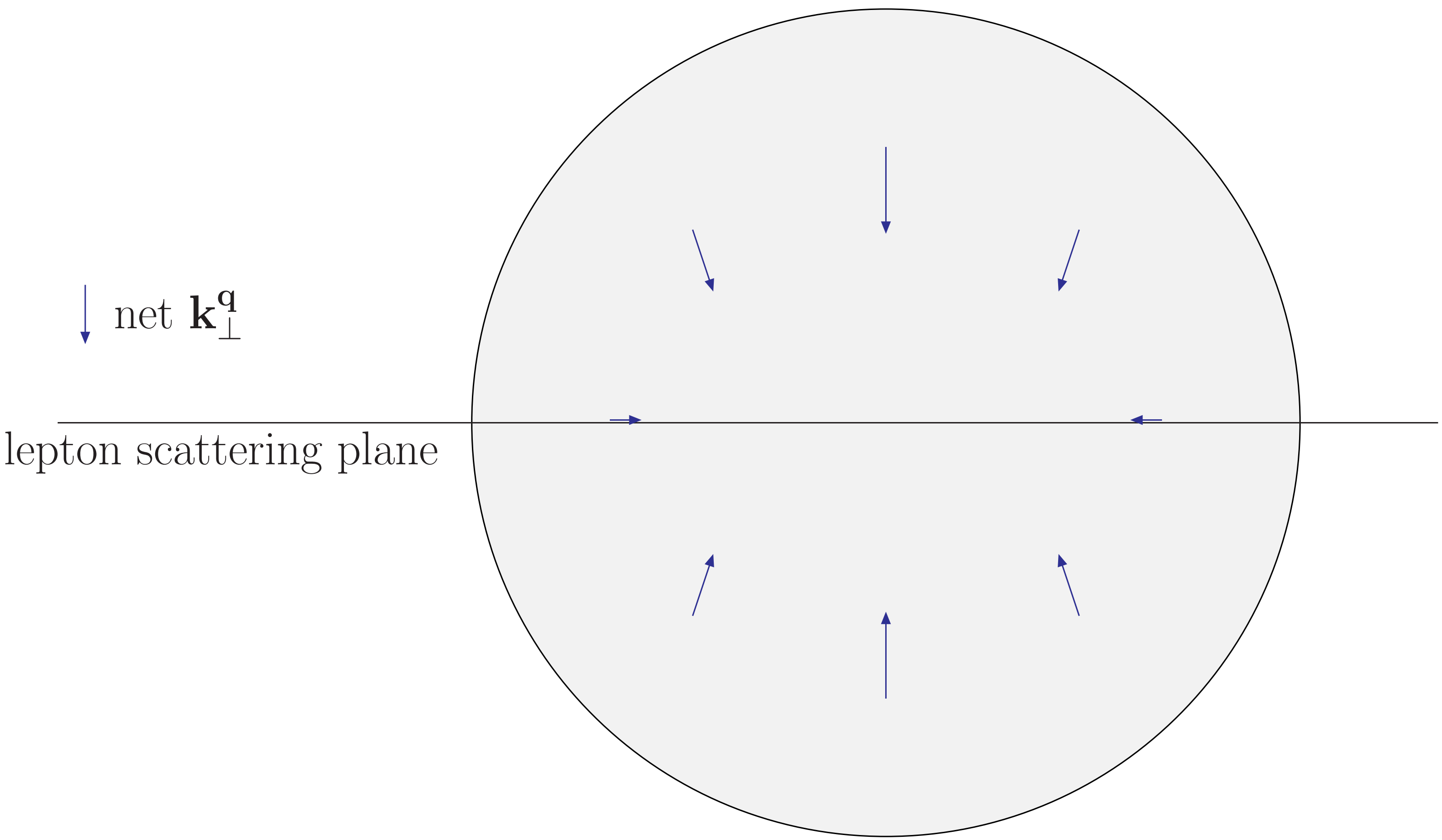
net $\perp$ momentum (FSI+Collins)



$\hookrightarrow$ in this example, enhancement of pions with $\perp$ momenta $\perp$ to lepton plane

probing BM function in tagged SIDIS

net $k_\perp^\pi$ (FSI + Collins)



$\hookrightarrow$ expect enhancement of pions with $\perp$ momenta $\perp$ to lepton plane

Quark-Gluon Correlations (chirally odd)

- $\perp$ momentum for quark polarized in $+\hat{x}$-direction (unpolarized target)

$$\langle k_\perp^y \rangle = \frac{g}{2p^+} \left\langle P, S \left| \bar{q}(0) \int_0^\infty dx^- G^{+y}(x^-) \sigma^{+y} q(0) \right| P, S \right\rangle$$

- compare: interaction-dependent twist-3 piece of $e(x)$ (scalar twist-3 PDF)

$$\int dx x^2 \bar{e}(x) \equiv \bar{e}_2 = \frac{g}{4MP^{+2}} \left\langle P, S \left| \bar{q}(0) G^{+y}(0) \sigma^{+y} q(0) \right| P, S \right\rangle$$

$\hookrightarrow$ $\langle F^y \rangle = M^2 \bar{e}_2$

$\hookrightarrow$ (chromodynamic lensing) $\bar{e}_2 < 0$

Summary

- GPDs $\overset{FT}{\longleftrightarrow}$ IPDs (impact parameter dependent PDFs)
- $E^q(x, 0, -\mathbf{\Delta}_\perp^2) \leftrightarrow \kappa_{q/p}$ (contribution from quark flavor q to anom magnetic moment)
- $E^q(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow \perp$ deformation of PDFs for $\perp$ polarized target
- $\perp$ deformation $\leftrightarrow$ (sign of) SSA (Sivers; Boer-Mulders)
- $\perp$ deformation $\leftrightarrow$ (sign of) quark-gluon correlations ($\int dx\, x^2 \bar{g}_2(x)$, $\int dx\, x^2 \bar{e}(x)$)