Excited States of the Nucleon in 2+1 flavour QCD

Derek Leinweber
CSSM Lattice Collaboration

Key Collaborators: Selim Mahbub, Waseem Kamleh, Ben Lasscock, Peter Moran, Alan Ó Cais and Tony Williams

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Derek Leinweber  
Excited States of the Nucleon in 2+1 flavour QCD
**Roper Resonance**

- *Roper resonance* ($P_{11}$) is the first positive parity excited state of the nucleon
- Observed in 1960’s from $\pi N$ scattering
- The resonance is interesting due to its low mass (1440 MeV) relative to the nearest negative-parity ($S_{11}$) resonance (1535 MeV).
- In a constituent quark model, the Roper state is $\approx 100$ MeV *above* the $S_{11}$ (1535 MeV) state.
- The Roper state appeared very high in all previous lattice simulations using the variational method.
Two point correlation function:

\[ G_{ij}(t, \vec{p}) = \sum_{\vec{x}} e^{-i\vec{p} \cdot \vec{x}} \langle \Omega | T\{\chi(x)\bar{\chi}(0)\} | \Omega \rangle. \]

Inserting completeness

\[ \sum_{B, \vec{p}', s} |B, \vec{p}', s\rangle \langle B, \vec{p}', s| = I \]

Then

\[ G_{ij}(t, \vec{p}) = \sum_{B^+} \lambda_{B^+} \bar{\lambda}_{B^+} e^{-E_{B^+} t} \left( \frac{\gamma \cdot p_{B^+} + M_{B^+}}{2E_{B^+}} \right) \]

\[ + \sum_{B^-} \lambda_{B^-} \bar{\lambda}_{B^-} e^{-E_{B^-} t} \left( \frac{\gamma \cdot p_{B^-} - M_{B^-}}{2E_{B^-}} \right) \]
\( \lambda_{B^\pm}, \bar{\lambda}_{B^\pm} \) are the couplings of \( \chi(0) \) and \( \bar{\chi}(0) \) with \( |B^\pm\rangle \) defined by

\[
\langle \Omega | \chi(0) | B^+, \vec{p}, s \rangle = \lambda_{B^+} \sqrt{\frac{M_{B^+}}{E_{B^+}}} \ u_{B^+}(\vec{p}, s),
\]

\[
\langle B^+, \vec{p}, s | \bar{\chi}(0) | \Omega \rangle = \bar{\lambda}_{B^+} \sqrt{\frac{M_{B^+}}{E_{B^+}}} \ \bar{u}_{B^+}(\vec{p}, s),
\]

and for the negative parity states,

\[
\langle \Omega | \chi(0) | B^-, \vec{p}, s \rangle = \lambda_{B^-} \sqrt{\frac{M_{B^-}}{E_{B^-}}} \ \gamma_5 \ u_{B^-}(\vec{p}, s),
\]

\[
\langle B^-, \vec{p}, s | \bar{\chi}(0) | \Omega \rangle = -\bar{\lambda}_{B^-} \sqrt{\frac{M_{B^-}}{E_{B^-}}} \ \bar{u}_{B^-}(\vec{p}, s) \ \gamma_5.
\]
At $\vec{p} = 0$

$$G_{ij}^{\pm}(t, \vec{0}) = \text{Tr}_{sp}[\Gamma_{\pm} G_{ij}(t, \vec{0})]$$

$$= \sum_{B^{\pm}} \lambda_{i}^{\pm} \bar{\lambda}_{j}^{\pm} e^{-M_{B^{\pm}} t}.$$ 

Parity projection operator,

$$\Gamma_{\pm} = \frac{1}{2} (1 \pm \gamma_{0}).$$

And

$$G_{ij}^{\pm}(t, \vec{0}) \overset{t \to \infty}{\longrightarrow} \lambda_{i0}^{\pm} \bar{\lambda}_{j0}^{\pm} e^{-M_{0^{\pm}} t} \quad \text{or} \quad M_{0^{\pm}} = \ln \left( \frac{G_{ij}^{\pm}(t, \vec{0})}{G_{ij}^{\pm}(t + 1, \vec{0})} \right).$$
Interpolators

Consider

\[ \chi_1(x) = \epsilon^{abc}(u^T a(x) C \gamma_5 d^b(x)) u^c(x), \]
\[ \chi_2(x) = \epsilon^{abc}(u^T a(x) C d^b(x)) \gamma_5 u^c(x), \]
\[ \chi_4(x) = \epsilon^{abc}(u^T a(x) C \gamma_5 \gamma_4 d^b(x)) u^c(x). \]
Consider $N$ interpolating fields, then

$$\bar{\phi}^\alpha = \sum_{i=1}^N u_i^\alpha \bar{\chi}_i,$$

$$\phi^\alpha = \sum_{i=1}^N v_i^\alpha \chi_i,$$

such that,

$$\langle B_\beta, p, s | \bar{\phi}^\alpha | \Omega \rangle = \delta_{\alpha\beta} \bar{z}^\alpha \bar{u}(\alpha, p, s),$$

$$\langle \Omega | \phi^\alpha | B_\beta, p, s \rangle = \delta_{\alpha\beta} z^\alpha u(\alpha, p, s),$$
Then a two point correlation function matrix for $\vec{p} = 0$, 

$$ G_{ij}^{\pm}(t) u_j^\alpha = \left( \sum_{\vec{x}} \text{Tr}_{sp}\{ \Gamma_{\pm} \langle \Omega | \chi_i \bar{\chi}_j | \Omega \rangle \} \right) u_j^\alpha $$

$$ = \lambda_i^{\alpha} \bar{z}^{\alpha} e^{-m_\alpha t}.$$ 

(no sum over $\alpha$)

t dependence only in the exponential term
Then one can have a recurrence relation at time \((t_0 + \Delta t)\),

\[ G_{ij}(t_0 + \Delta t) u_j^\alpha = e^{-m_\alpha \Delta t} G_{ij}(t_0) u_j^\alpha. \]

Multiplying by \([G_{ij}(t_0)]^{-1}\) from left,

\[ [(G(t_0))^{-1} G(t_0 + \Delta t)]_{ij} u_j^\alpha = c_\alpha u_i^\alpha, \]

where \(c_\alpha = e^{-m_\alpha \Delta t}\) is the eigenvalue.

Similarly, it can also be solved for the left eigenvalue equation for \(v^\alpha\) eigenvector,

\[ v_i^\alpha [G(t_0 + \Delta t)(G(t_0))^{-1}]_{ij} = c_\alpha v_j^\alpha. \]
The vectors $u_j^\alpha$ and $v_i^\alpha$ diagonalize the correlation matrix at time $t_0$ and $t_0 + \triangle t$ making the projected correlation function

$$v_i^\alpha \, G_{ij}(t) \, u_j^\beta = \delta^{\alpha\beta} \, z^\alpha \, \bar{z}^\beta \, e^{-m_\alpha t}.$$ 

The projected correlator, is then analyzed to obtain masses of different states,

$$v_i^\alpha \, G_{ij}^{\pm}(t) \, u_j^\alpha \equiv G_{\pm}^\alpha,$$

We construct the effective mass

$$M_{\text{eff}}(t) = \ln \left( \frac{G_{\pm}^\alpha(t, \vec{0})}{G_{\pm}^\alpha(t + 1, \vec{0})} \right).$$
Projected Mass Vs Mass From Eigenvalue

- $t_{\text{start}} = t_0$ is shown in major tick marks
- $\Delta t$ is shown in minor tick marks

2 $\times$ 2 correlation matrix of $\chi_1 \chi_2$ for a point source
Eigenvectors - Point Source, for $\chi_1\chi_2$

**Left Eigenvectors**

**Right Eigenvectors**

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**Excited States of the Nucleon in 2+1 flavour QCD**
Roper state: Compilation of existing results in QQCD

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{roper_state}
\caption{Comparison of Roper state results from various studies.}
\end{figure}

- Mathur et al. (3.2 fm)
- Sasaki et al. (3.0 fm)
- Guadagnoli et al. (1.86 fm)
- Brommel et al. (2.4 fm)
- Lasscock et al. (2.56 fm)
- Melnitchok et al. (1.95 fm)
- Burch et al. (2.38 fm)
- Basak et al. (2.34 fm)
- Mahbub et al. (2.03 fm)
- G.S. N^{1/2} (exp.)
- Roper (exp.)

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Excited States of the Nucleon in 2+1 flavour QCD

[arXiv:0905.3616 [hep-lat]]
Roper state: Compilation of existing results in QQCD

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- Roper (exp.)

$m_{\pi}^2$ (GeV$^2$) vs. M (GeV)
Source Smearing

Correlation matrices are built from a variety of source and sink smearings.

\[ \psi_i(x, t) = \sum_{x'} F(x, x') \psi_{i-1}(x', t), \]

where,

\[ F(x, x') = (1 - \alpha) \delta_{x,x'} + \frac{\alpha}{6} \sum_{\mu=1}^{3} \left[ U_{\mu}(x) \delta_{x',x+\hat{\mu}} + U_{\mu}^\dagger(x - \hat{\mu}) \delta_{x',x-\hat{\mu}} \right], \]

Fixing \( \alpha = 0.7 \), the procedure is repeated \( N_{sm} \) times.
Consider smeared–smeared correlation functions

Variety of smearing sweeps used to form basis interpolators

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Introduction

Variational Method

Lattice Simulation Results

Summary of Results

Methodology and Status

$N1/2^-$ State and the Level Crossing

Roper State in Dynamical-Fermion QCD

$N1/2^-$ State in Dynamical-Fermion QCD

Smeared Source - Point Sink Correlators

$M_{\text{eff}}$ (GeV) vs. $t$

- point
- so16
- so1
- so26
- so3
- so35
- so7
- so48
- so12
- so65

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Excited States of the Nucleon in 2+1 flavour QCD
$4 \times 4$ correlation matrix for the $4^{th}$ basis (3, 12, 26, 35)

Projected Mass Vs Mass From Eigenvalue

$t_{\text{start}} = t_0$ is shown in major tick marks

$\triangle t$ is shown in minor tick marks
Effective Mass of Roper: 5th Basis

\[ \chi^2 / \text{dof} = 0.51 \]
### 4 \times 4 bases of $\chi_1 \bar{\chi}_1$

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**Bases**

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Excited States of the Nucleon in 2+1 flavour QCD
Projected correlator masses from $4 \times 4$ analysis
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Excited States of the Nucleon in 2+1 flavour QCD
### 6 × 6 bases of $\chi_1 \bar{\chi}_1$

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### 6 × 6 bases of $\chi_1 \chi_2$

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### 8 × 8 bases of $\chi_1 \chi_2$

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Review of excited “states”

Mass (GeV) vs Various Matrices chart with points labeled g.s and 1st e.s.
Review of excited “states”
Review of excited “states”

Mass (GeV)

Various Matrices

2x2, xX
3x3, xXx
4x4, xX

g.s
1st e.s
2nd e.s
3rd e.s
Review of excited “states”

- g.s
- 1st e.s
- 2nd e.s
- 3rd e.s
- 4th e.s
- 5th e.s

Various Matrices

Mass (GeV)

N1/2− State and the Level Crossing
Roper State in Dynamical-Fermion QCD
N1/2− State in Dynamical-Fermion QCD
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Methodology and Status
$N1/2^−$ State and the Level Crossing
Roper State in Dynamical-Fermion QCD
$N1/2^−$ State in Dynamical-Fermion QCD

Review of excited “states”

![Graph showing excited states in QCD](image-url)
Review of excited “states”

The figure shows a graph with various matrix representations and mass values in GeV. Different states are represented by various markers such as circles, triangles, squares, and others, each corresponding to different mass values. The x-axis represents various matrices, and the y-axis shows the mass in GeV. The graph includes data points for different states, indicating their mass values across the matrices.
Projected masses from $8 \times 8$ analysis of $\chi_1 \chi_2$
Positive Parity Results

Mahbub et al., [arXiv:hep-lat/1004.5455].

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Excited States of the Nucleon in 2+1 flavour QCD
Positive Parity Results

CSSM red, LHP blue, BGR green.
### 8 × 8 bases of $\chi_1 \chi_2$ for $N1/2^-$ Analysis

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Projected $N1/2^-$ masses from $8 \times 8$ bases
Eigenvectors: state 1
Eigenvectors: state 2
Excited States of the Nucleon in 2+1 flavour QCD

Eigenvectors: state 3
Eigenvectors: state 4
Roper and $N1/2^-$ states


- Lattice volume: $32^3 \times 64$
- Non-perturbative $O(a)$-improved Wilson quark action
- Iwasaki gauge action
- 2 + 1 flavour dynamical-fermion QCD
- $\beta = 1.9$ providing $a = 0.0907$ fm
- $K_{ud} = \{ 0.13700, 0.13727, 0.13754, 0.13770, 0.13781 \}$
- $K_s = 0.13640$
- Lightest pion mass is 156 MeV.
4 × 4 bases of $\chi_1 \bar{\chi}_1$

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<td>70</td>
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<td>125</td>
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<td>400</td>
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</tr>
</tbody>
</table>
For second lightest quark: 50 cfgs
For all $4 \times 4$ bases: $K_{ud} = 0.137700$
Smeared Source - Point Sink Effective Masses

For the heaviest quark: 50 cfgs

For the heaviest quark: 50 cfgs
Even Parity Nucleon Spectrum in full QCD

Configs: Lightest = 750 configs, rest are 350 configs.
Ground and Roper states (fixed lattice spacing)

\begin{center}
\begin{tikzpicture}
\begin{axis}[
    title={Ground and Roper states (fixed lattice spacing)},
    xlabel={$m_\pi^2 \text{ (GeV}^2\text{)}$},
    ylabel={$M \text{ (GeV)}$},
    xmin=0, xmax=0.8,
    ymin=0, ymax=4,
    xtick={0,0.2,0.4,0.6,0.8},
    ytick={0,1,2,3,4},
    grid=major,
    legend pos=north east
]
\addplot[green,mark=*] coordinates {
(0,2)
(0.2,2.5)
(0.4,3)
(0.6,3.5)
(0.8,4)
};
\addlegendentry{g.s}
\addplot[red,mark=triangle] coordinates {
(0,1.5)
(0.2,2)
(0.4,2.5)
(0.6,3)
(0.8,3.5)
};
\addlegendentry{Roper}
\end{axis}
\end{tikzpicture}
\end{center}
Ground and Roper states (Sommer scale sets $a$)
Ground and Roper states (Sommer scale sets $a$)

![Graph showing the relationship between $m_\pi^2$ and $M$ for P-wave $N+\pi$ states and g.s. and Roper states.]

- **P-wave $N+\pi$**: Dashed line connecting data points.
- **g.s.**: Green circle markers.
- **Roper**: Red triangle markers.

Derek Leinweber
Excited States of the Nucleon in 2+1 flavour QCD
Ground and Roper states (Sommer scale sets $a$)

![Graph showing the relationship between $m_\pi^2$ and $M$ for P-wave N+π and S-wave N+π+π states, with data points for ground state (g.s) and Roper state.](image-url)
Quenched Vs Dynamical (Sommer scale)
Quenched Vs Dynamical (Sommer scale)

\[
\begin{align*}
M &\quad \text{(GeV)} \\
m^2_\pi &\quad (\text{GeV}^2)
\end{align*}
\]
$N^{1-}_{\frac{1}{2}}$ (1535) (Sommer scale)
$N_{1/2}^{{1^-}} (1535)$ (Sommer scale)

![Graph showing the mass $M$ versus $m_\pi^2$ for different states: $P$-wave $N+\pi$, $S$-wave $N+\pi$, $S$-wave $N+\pi+\pi$, ground state (g.s.), Roper, and $N^-$. The graph includes error bars for experimental data.]
$N_{1/2}^{-} (1535)$ (Sommer scale)
Quenched Vs Dynamical, $N^-$ states (Sommer scale)

The graph shows a comparison between quenched and dynamical states in $2+1$ flavour QCD. The states are plotted against the Sommer scale ($m^2$).

- Solid line with squares: $S$-wave $N^+$ (dynamical)
- Dotted line with triangles: $S$-wave $N^+$ (quenched)
- Solid line with circles: ground state (dynamical)
- Open circles: ground state (quenched)
- Open triangles: $N^-$ (quenched)

The graph illustrates the differences in energy levels between quenched and dynamical states for $N^-$ states in 2+1 flavour QCD.
Summary

- Several fermion-source and -sink smearing levels have been used to construct correlation matrices.
- A variety of $4 \times 4$, $6 \times 6$, and $8 \times 8$ matrices were considered to demonstrate the independence of the eigenstate energies from the basis interpolators.
- A low-lying Roper state has been identified in both quenched and full QCD using this correlation-matrix based method.
- The approach to the chiral limit is significantly different.
- The two heaviest quark masses considered in the dynamical case provide states consistent with $P$-wave $\pi N$ scattering states.
Quenched Vs Dynamical (Sommer scale)

- P-wave N+π (dyn.)
- S-wave N+π+π (dyn.)
- g.s (dyn.)
- Roper (dyn.)
- g.s (quen.)
- Roper (quen.)

- P-wave N+π (quen.)
- S-wave N+π+π (quen.)
The $N1/2^-$ results in quenched and dynamical QCD reveal significant differences in the approach to the physical point. A level crossing between the Roper and $N1/2^-$ states is observed in quenched QCD at $m_\pi \simeq 400$ MeV. A level crossing between the Roper and $N1/2^-$ states is anticipated in full QCD at $m_\pi \simeq 150$ MeV, just above the physical pion mass. The approach to the experimentally measured masses is encouraging. The effects of the finite volume and the role of scattering states remains to be resolved.
Quenched Vs Dynamical $N^{1-}_{1/2}$ (1535) (Sommer scale)
Future Plans

- Extend to a comprehensive analysis of all low-lying baryons.
  - See Ben Menadue’s Poster on the $\Lambda(1405)$.
- Examine the nature of the Roper wave function.
  - See Dale Roberts’ Poster on the proton in a magnetic field.
- Explore chiral curvature via chiral effective field theory.
  - Knowledge of meson-baryon couplings to nearby states.
  - See Jonathan Hall’s talk on intrinsic scales in $\chi$EFT.
- Resolve excited-state electromagnetic properties.
  - Three-point function and background field approaches.
  - See Thom Primer’s Poster on the background field approach to magnetic properties.