Excited States of the Nucleon in 2+1 flavour QCD

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CSSM Lattice Collaboration

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Outline

Roper Resonance

- *Roper resonance* (P_{11}) is the first positive parity excited state of the nucleon
- Observed in 1960's from πN scattering
- The resonance is interesting due to its low mass (1440 MeV) relative to the nearest negative-parity (S_{11}) resonance (1535 MeV).
- In a constituent quark model, the Roper state is $\approx$100 MeV *above* the S_{11} (1535 MeV) state.
- The Roper state appeared very high in all previous lattice simulations using the variational method.

- Two point correlation function:

$$G_{ij}(t,\vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}.\vec{x}} \langle \Omega | T\{\chi_i(x)\bar{\chi}_j(0)\} | \Omega \rangle.$$

- Inserting completeness

$$\sum_{B,\vec{p'},s} |B,\vec{p'},s\rangle\langle B,\vec{p'},s| = I$$

- Then

$$G_{ij}(t,\vec{p}) = \sum_{B^+} \lambda_{B^+}\bar{\lambda}_{B^+}\, e^{-E_{B^+}t}\, \frac{\gamma.p_{B^+} + M_{B^+}}{2E_{B^+}} + \sum_{B^-} \lambda_{B^-}\bar{\lambda}_{B^-}\, e^{-E_{B^-}t}\, \frac{\gamma.p_{B^-} - M_{B^-}}{2E_{B^-}}$$

- $\lambda_{B^\pm}$, $\bar{\lambda}_{B^\pm}$ are the couplings of $\chi(0)$ and $\bar{\chi}(0)$ with $|B^\pm\rangle$ defined by

$$\langle\Omega|\chi(0)|B^+,\vec{p},s\rangle = \lambda_{B^+}\sqrt{\frac{M_{B^+}}{E_{B^+}}}\,u_{B^+}(\vec{p},s),$$

$$\langle B^+,\vec{p},s|\bar{\chi}(0)|\Omega\rangle = \bar{\lambda}_{B^+}\sqrt{\frac{M_{B^+}}{E_{B^+}}}\,\bar{u}_{B^+}(\vec{p},s),$$

and for the negative parity states,

$$\langle\Omega|\chi(0)|B^-,\vec{p},s\rangle = \lambda_{B^-}\sqrt{\frac{M_{B^-}}{E_{B^-}}}\,\gamma_5\,u_{B^-}(\vec{p},s),$$

$$\langle B^-,\vec{p},s|\bar{\chi}(0)|\Omega\rangle = -\bar{\lambda}_{B^-}\sqrt{\frac{M_{B^-}}{E_{B^-}}}\,\bar{u}_{B^-}(\vec{p},s)\,\gamma_5.$$

- At $\vec{p} = 0$

$$G_{ij}^{\pm}(t,\vec{0}) = \mathrm{Tr}_{\mathrm{sp}}[\Gamma_{\pm}\, G_{ij}(t,\vec{0})] = \sum_{B^{\pm}} \lambda_i^{\pm} \bar{\lambda}_j^{\pm}\, e^{-M_{B^{\pm}}t}.$$

- Parity projection operator,

$$\Gamma_{\pm} = \frac{1}{2}(1 \pm \gamma_0).$$

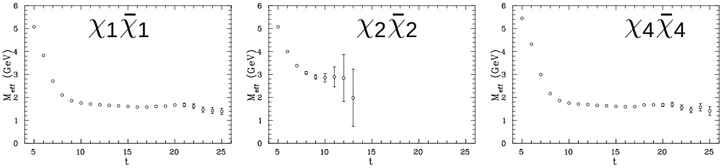
- And

$$G_{ij}^{\pm}(t,\vec{0}) \stackrel{t\to\infty}{=} \lambda_{i0}^{\pm}\bar{\lambda}_{j0}^{\pm}\, e^{-M_{0^{\pm}}t} \quad \text{or} \quad M_0^{\pm} = \ln\left(\frac{G_{ij}^{\pm}(t,\vec{0})}{G_{ij}^{\pm}(t+1,\vec{0})}\right).$$

Interpolators

- Consider

$$\chi_1(x) = \epsilon^{abc}(u^{Ta}(x)\, C\gamma_5\, d^b(x))\, u^c(x)\,,$$
$$\chi_2(x) = \epsilon^{abc}(u^{Ta}(x)\, C\, d^b(x))\, \gamma_5\, u^c(x)\,,$$
$$\chi_4(x) = \epsilon^{abc}(u^{Ta}(x)\, C\gamma_5\gamma_4\, d^b(x))\, u^c(x).$$



Variational Method

- Consider N interpolating fields, then

$$\bar{\phi}^{\alpha} = \sum_{i=1}^{N} u_i^{\alpha} \, \bar{\chi}_i ,$$

$$\phi^{\alpha} = \sum_{i=1}^{N} v_i^{\alpha} \, \chi_i ,$$

such that,

$$\langle B_\beta, p, s | \bar{\phi}^{\alpha} | \Omega \rangle = \delta_{\alpha\beta} \, \bar{z}^{\alpha} \, \bar{u}(\alpha, p, s),$$

$$\langle \Omega | \phi^{\alpha} | B_\beta, p, s \rangle = \delta_{\alpha\beta} \, z^{\alpha} \, u(\alpha, p, s),$$

- Then a two point correlation function matrix for $\vec{p} = 0$,

$$G^{\pm}_{ij}(t)u^{\alpha}_{j} = \Big(\sum_{\vec{x}} \mathrm{Tr}_{\mathrm{sp}}\{\Gamma_{\pm}\langle\Omega|\chi_i\bar{\chi}_j|\Omega\rangle\}\Big)\, u^{\alpha}_{j}$$
$$= \lambda^{\alpha}_{i}\, \bar{z}^{\alpha}\, e^{-m_{\alpha}t}.$$

  (no sum over α)
- t dependence only in the exponential term

- Then one can have a recurrence relation at time $(t_0 + \triangle t)$,

$$G_{ij}(t_0 + \triangle t)\, u_j^{\alpha} = e^{-m_\alpha \triangle t}\, G_{ij}(t_0)\, u_j^{\alpha}.$$

- Multiplying by $[G_{ij}(t_0)]^{-1}$ from left,

$$[(G(t_0))^{-1} G(t_0 + \triangle t)]_{ij}\, u_j^{\alpha} = c^{\alpha}\, u_i^{\alpha},$$

- where $c^{\alpha} = e^{-m_\alpha \triangle t}$ is the eigenvalue.
- Similarly, it can also be solved for the left eigenvalue equation for v^{α} eigenvector,

$$v_i^{\alpha}\, [G(t_0 + \triangle t)(G(t_0))^{-1}]_{ij} = c^{\alpha}\, v_j^{\alpha}.$$

- The vectors u_j^α and v_i^α diagonalize the correlation matrix at time t_0 and $t_0 + \triangle t$ making the projected correlation function

$$v_i^\alpha \, G_{ij}(t) \, u_j^\beta = \delta^{\alpha\beta} \, z^\alpha \, \bar{z}^\beta \, e^{-m_\alpha t} \, .$$

- The projected correlator, is then analyzed to obtain masses of different states,

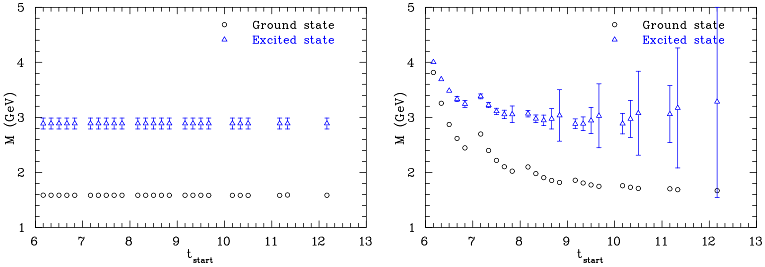
$$v_i^\alpha \, G_{ij}^\pm(t) \, u_j^\alpha \equiv G_\pm^\alpha ,$$

- We construct the effective mass

$$M_{\rm eff}^\alpha(t) = \ln\left(\frac{G_\pm^\alpha(t,\vec{0})}{G_\pm^\alpha(t+1,\vec{0})}\right) .$$

2×2 correlation matrix of $\chi_1 \chi_2$ for a point source

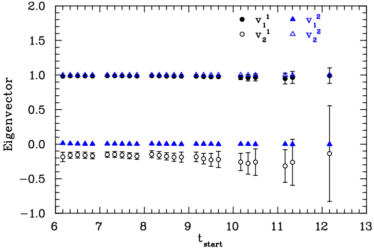
Projected Mass Vs Mass From Eigenvalue



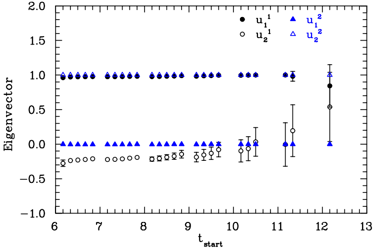
- $t_{\mathrm{start}} = t_0$ is shown in major tick marks
- $\triangle t$ is shown in minor tick marks

Eigenvectors - Point Source, for $\chi_1\chi_2$

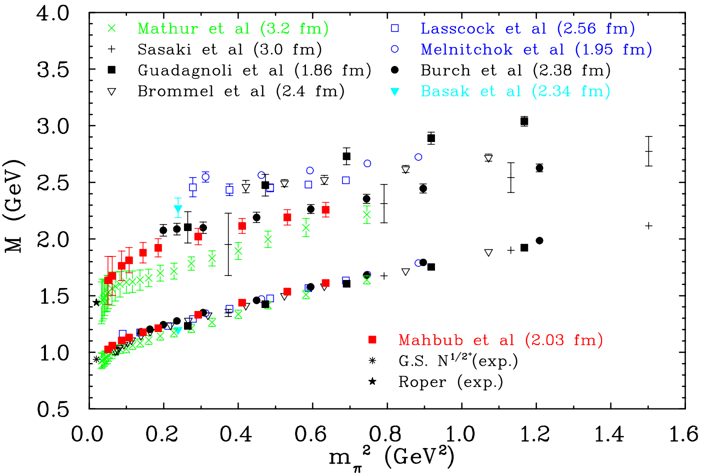
Left Eigenvectors



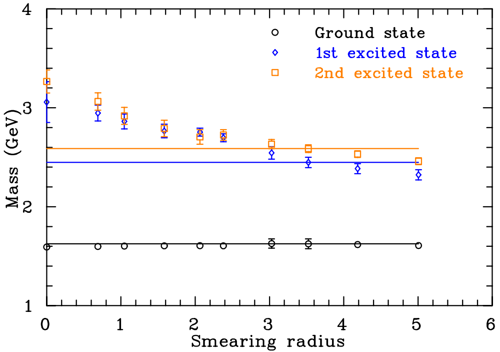
Right Eigenvectors



Roper state: Compilation of existing results in QQCD



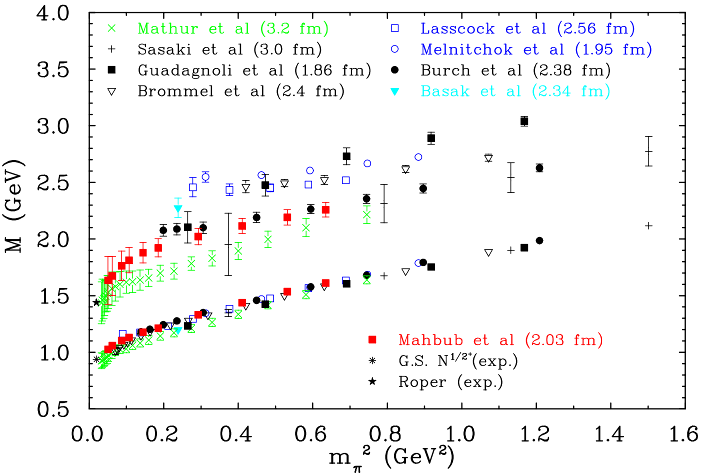
Smeared Source Problem



Mahbub, *et al.*, Phys. Rev. D **80**, 054507 (2009)

[arXiv:0905.3616 [hep-lat]]

Roper state: Compilation of existing results in QQCD



Source Smearing

Correlation matrices are built from a variety of source and sink smearings.

$$\psi_i(x,t) = \sum_{x'} F(x,x')\, \psi_{i-1}(x',t),$$

where,

$$F(x,x') = (1-\alpha)\,\delta_{x,x'} + \frac{\alpha}{6}\sum_{\mu=1}^{3}\left[U_\mu(x)\,\delta_{x',x+\hat\mu} + U_\mu^\dagger(x-\hat\mu)\,\delta_{x',x-\hat\mu}\right],$$

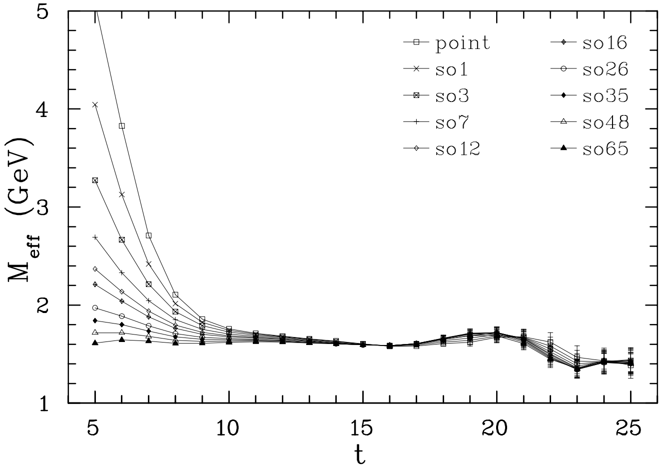
Fixing $\alpha = 0.7$, the procedure is repeated $N_{\rm sm}$ times.

4×4 bases of $\chi_1\bar{\chi}_1$

- Consider smeared–smeared correlation functions
- Variety of smearing sweeps used to form basis interpolators

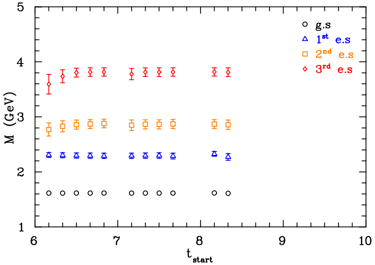
Sweeps $\rightarrow$	1	3	7	12	16	26	35	48
Basis No. $\downarrow$	Bases							
1	1	-	7	-	16	-	35	-
2	-	3	7	-	16	-	35	-
3	1	-	-	12	-	26	-	48
4	-	3	-	12	-	26	35	-
5	-	3	-	12	-	26	-	48
6	-	-	-	12	16	26	35	-
7	-	-	7	-	16	-	35	48

Smeared Source - Point Sink Correlators



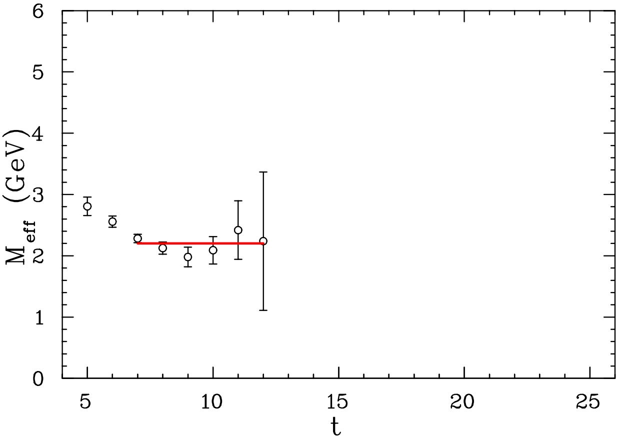
4×4 correlation matrix for the 4^{th} basis (3, 12, 26, 35)

Projected Mass Vs Mass From Eigenvalue



- $t_{\text{start}} = t_0$ is shown in major tick marks
- $\triangle t$ is shown in minor tick marks

Effective Mass of Roper: 5$^{\text{th}}$ Basis

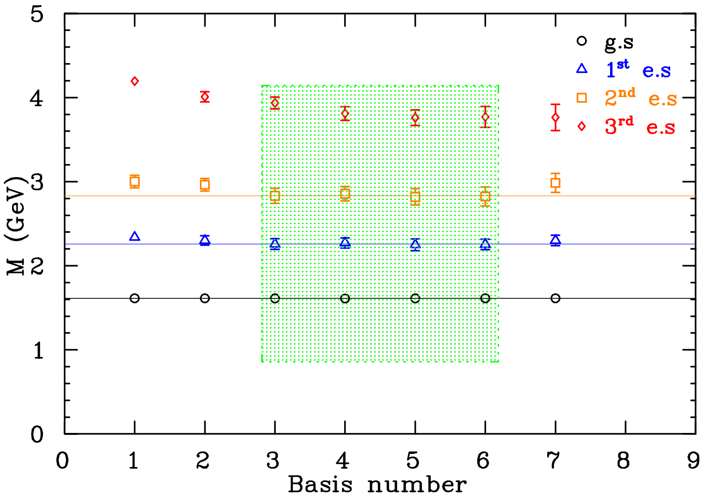


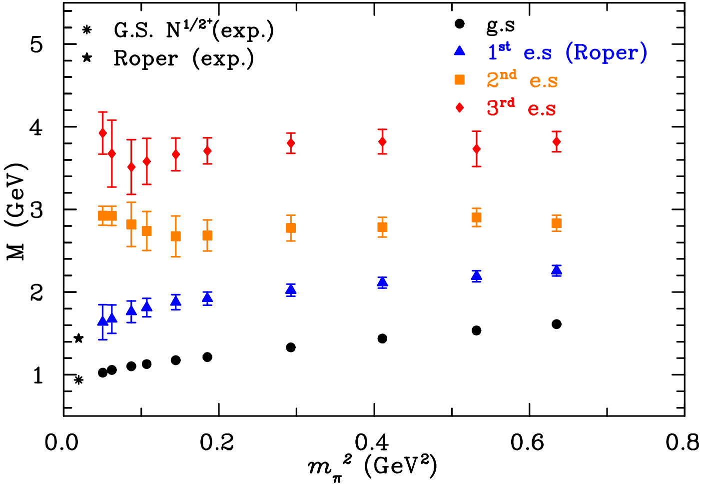
$\chi^2/dof = 0.51$

4×4 bases of $\chi_1\bar{\chi}_1$

Sweeps →	1	3	7	12	16	26	35	48
Basis No. ↓	Bases							
1	1	-	7	-	16	-	35	-
2	-	3	7	-	16	-	35	-
3	1	-	-	12	-	26	-	48
4	-	3	-	12	-	26	35	-
5	-	3	-	12	-	26	-	48
6	-	-	-	12	16	26	35	-
7	-	-	7	-	16	-	35	48

Projected correlator masses from 4 × 4 analysis





Mahbub *et al.*, Phys. Lett. B **679**, 418 (2009), [arXiv:hep-lat/0906.5433].

6×6 bases of $\chi_1\bar{\chi}_1$

Sweeps $\rightarrow$	1	3	7	12	16	26	35	48
Basis No. $\downarrow$	Bases							
1	1	3	7	12	16	26	-	-
2	1	3	7	12	16	-	35	-
3	1	3	7	-	16	26	35	-
4	1	3	-	12	16	26	-	48
5	1	-	7	12	16	26	35	-
6	-	3	7	12	16	26	35	-

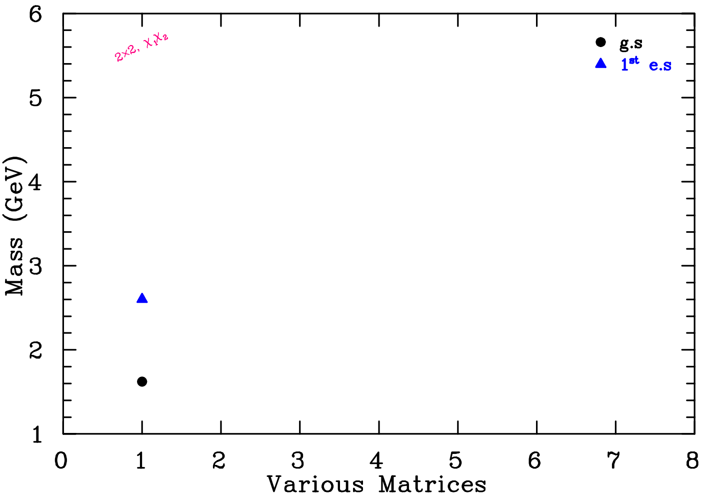
6×6 bases of $\chi_1\,\chi_2$

Sweeps $\rightarrow$	1	3	7	12	16	26	35	48
Basis No. $\downarrow$	Bases							
1	1	-	-	-	16	-	-	48
2	-	3	-	12	-	26	-	-
3	-	3	-	-	16	-	-	48
4	-	-	7	-	16	-	35	-
5	-	-	-	12	16	26	-	-
6	-	-	-	-	16	26	35	-

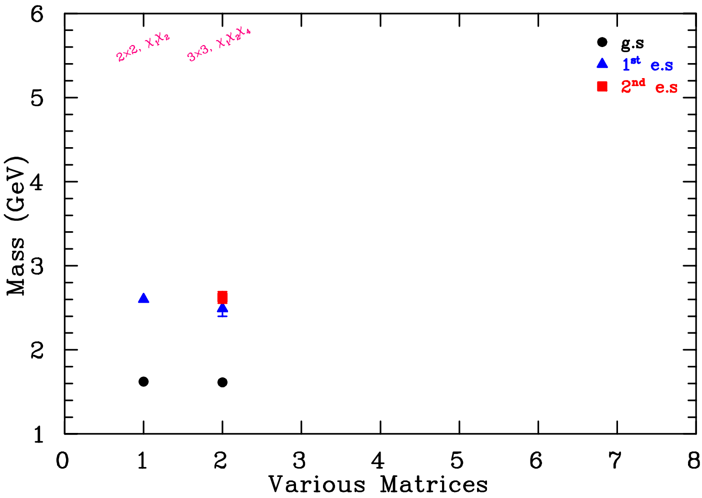
8×8 bases of $\chi_1\,\chi_2$

Sweeps $\rightarrow$	1	3	7	12	16	26	35	48
Basis No. $\downarrow$	Bases							
1	1	-	7	-	16	-	35	-
2	-	-	7	12	16	26	-	-
3	-	3	-	12	-	26	-	48
4	-	-	7	12	-	26	35	-
5	-	-	7	-	16	26	35	-
6	-	-	7	-	16	-	35	48
7	-	-	-	12	16	26	35	-

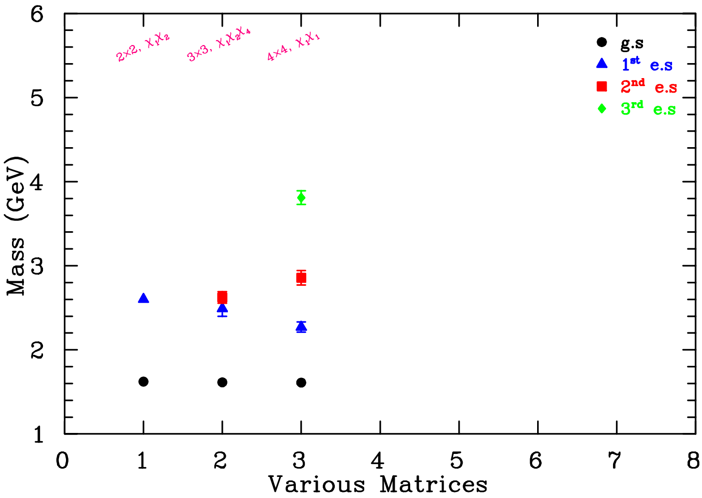
Review of excited “states”



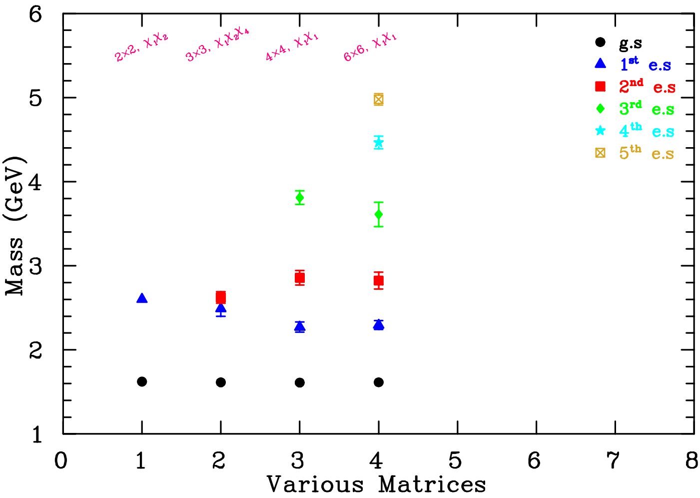
Review of excited “states”



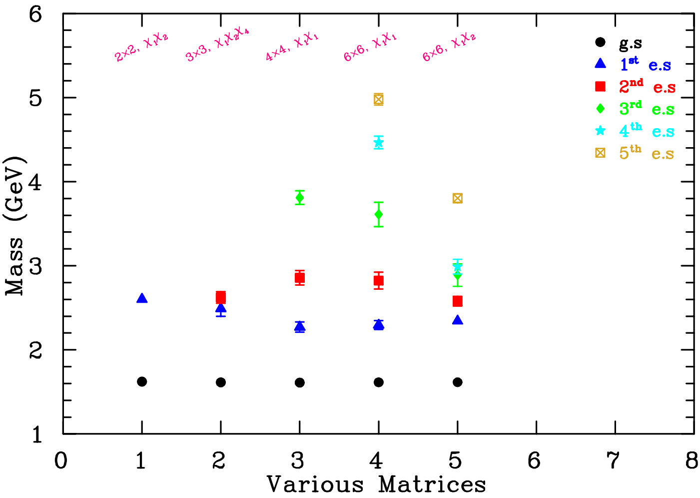
Review of excited "states"



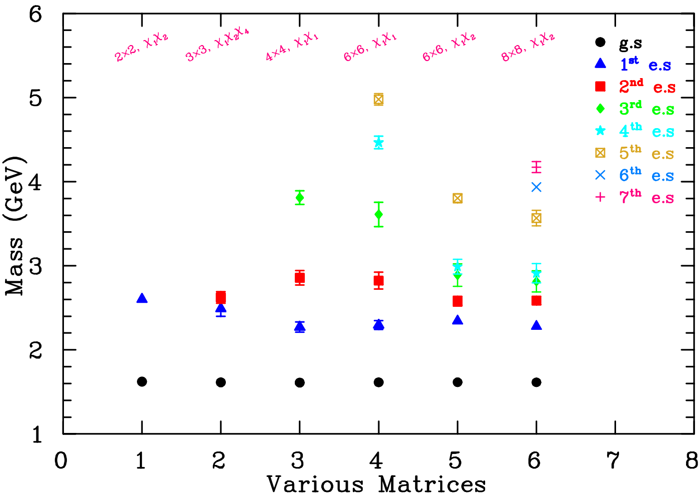
Review of excited “states”



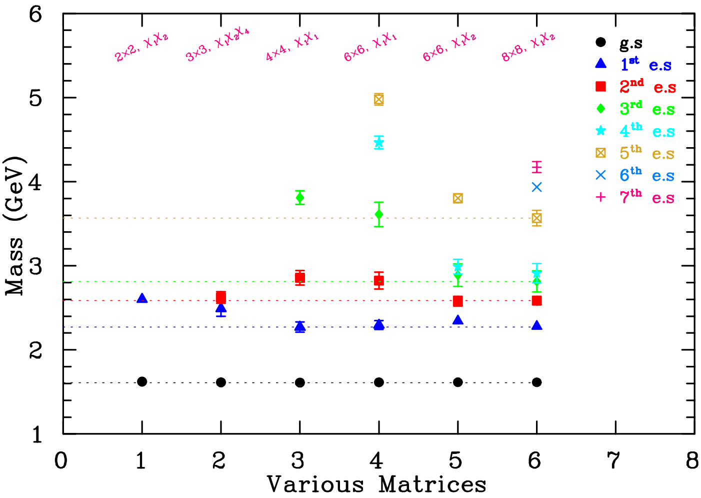
Review of excited “states”



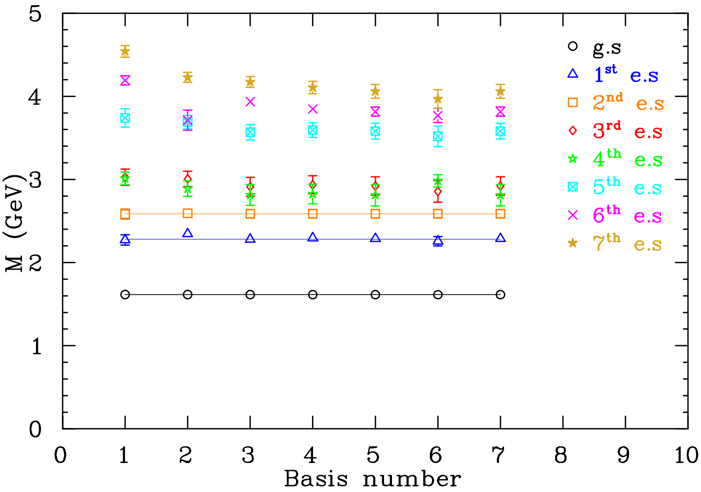
Review of excited “states”



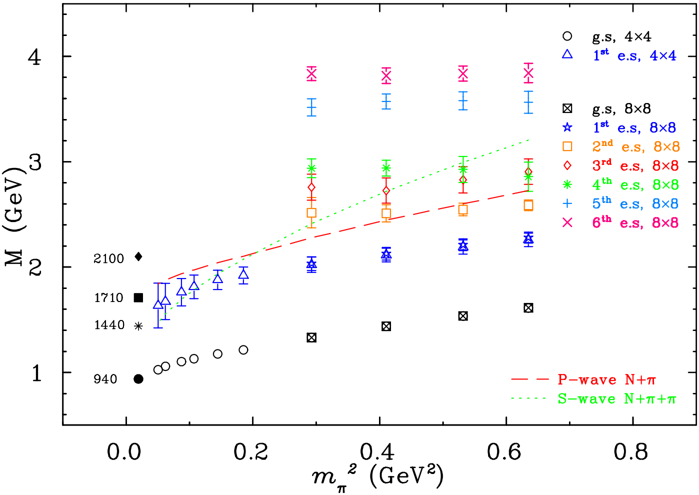
Review of excited “states”



Projected masses from 8 × 8 analysis of $\chi_1 \chi_2$

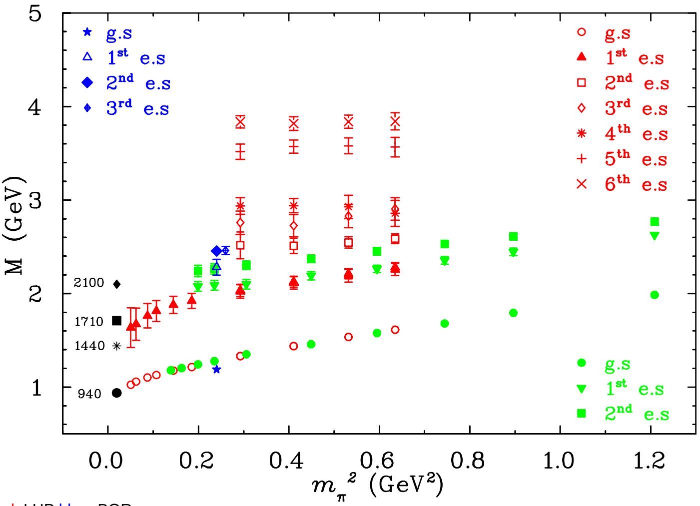


Positive Parity Results



Mahbub *et al.*, [arXiv:hep-lat/1004.5455].

Positive Parity Results

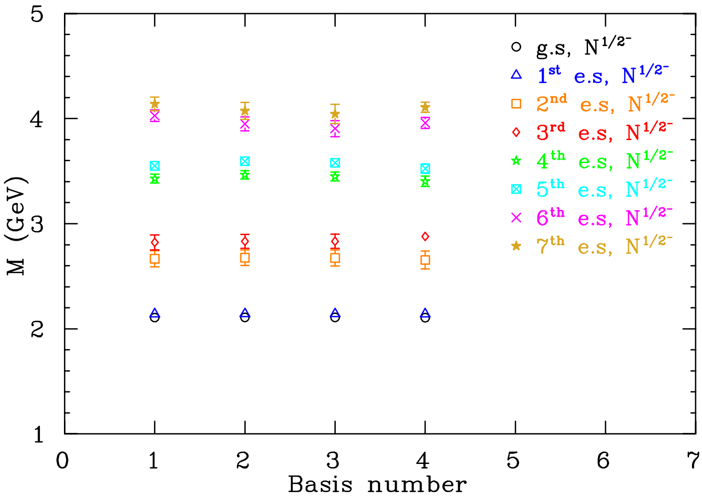


CSSM red, LHP blue, BGR green.

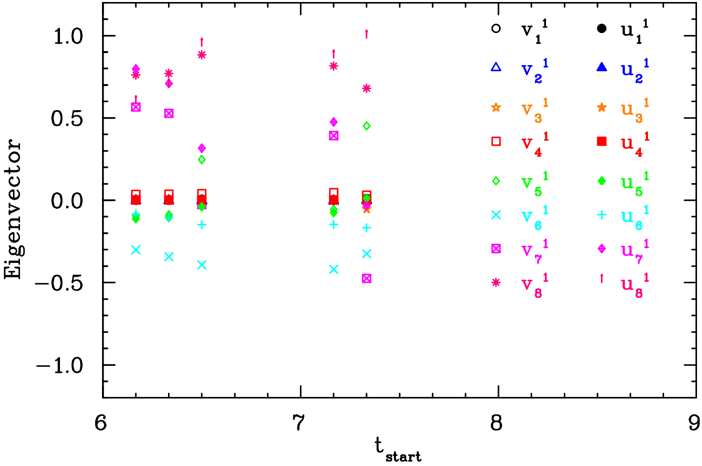
8×8 bases of $\chi_1\,\chi_2$ for $N1/2^-$ Analysis

Sweeps $\rightarrow$	1	3	7	12	16	26	35	48
Basis No. $\downarrow$	Bases							
1	-	3	-	12	-	26	-	48
2	-	-	7	12	-	26	35	-
3	-	-	7	-	16	26	35	-
4	-	-	7	-	16	-	35	48

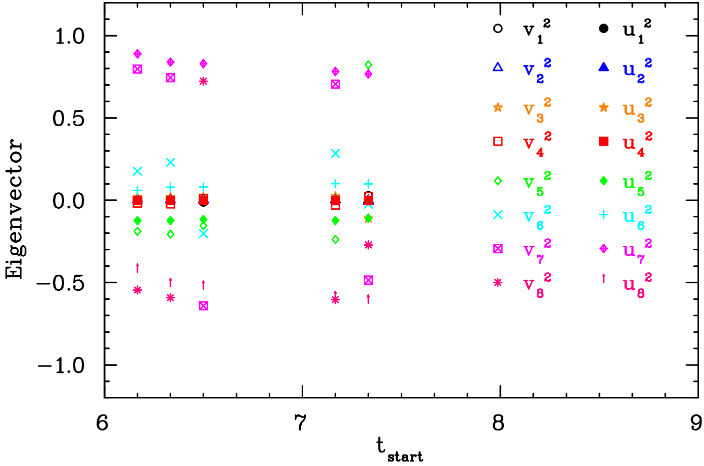
Projected $N1/2^-$ masses from 8×8 bases



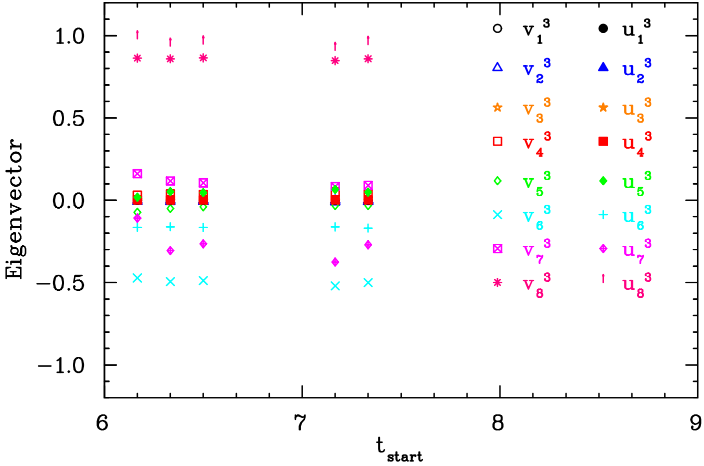
Eigenvectors: state 1



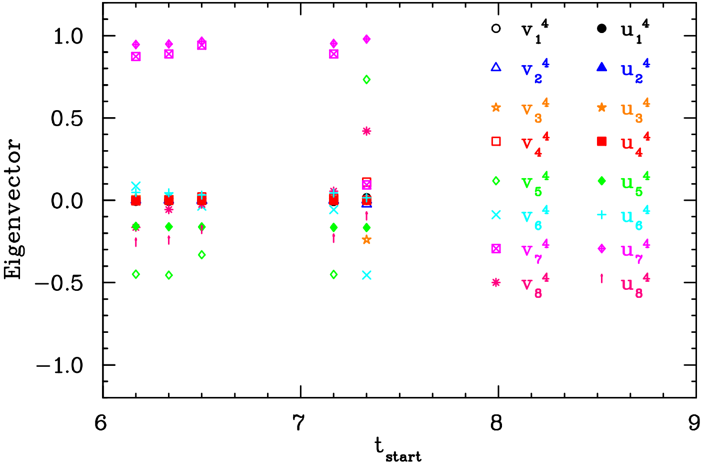
Eigenvectors: state 2



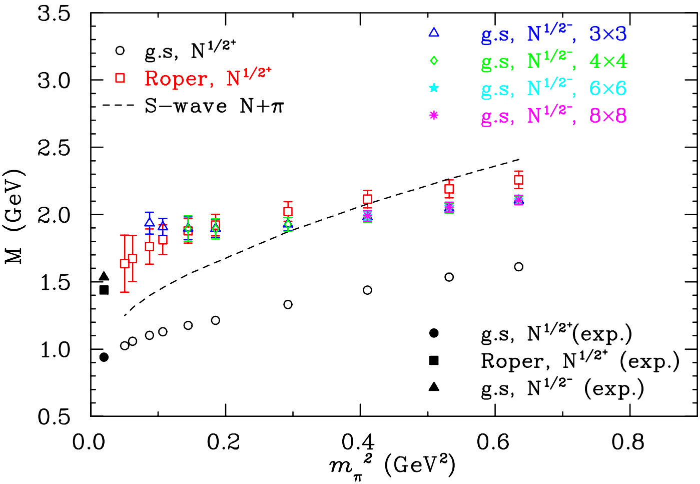
Eigenvectors: state 3



Eigenvectors: state 4



Roper and $N1/2^-$ states



Mahbub *et al.*, Phys. Lett. B **693**, 351 (2010), [arXiv:hep-lat/1007.4871].

PACS-CS lattice: Simulation details

PACS-CS Collaboration: S. Aoki, et al., Phys. Rev. **D79** (2009) 034503.

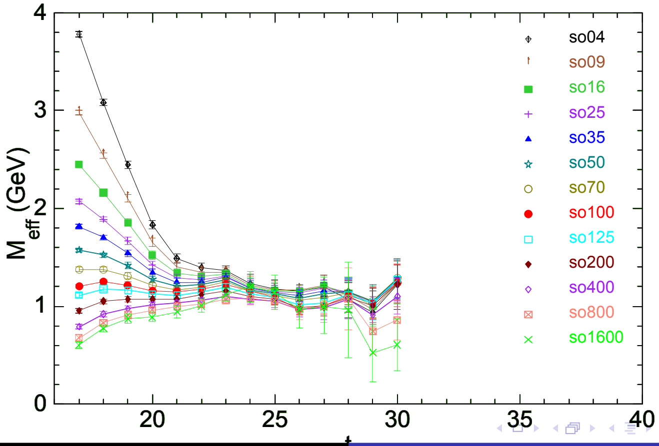
- Lattice volume: $32^3 \times 64$
- Non-perturbative $\mathcal{O}(a)$-improved Wilson quark action
- Iwasaki gauge action
- $2 + 1$ flavour dynamical-fermion QCD
- $\beta = 1.9$ providing $a = 0.0907$ fm
- $K_{ud} = \{$ 0.13700, 0.13727, 0.13754, 0.13770, 0.13781 $\}$
- $K_s = 0.13640$
- Lightest pion mass is 156 MeV.

4×4 bases of $\chi_1\bar{\chi}_1$

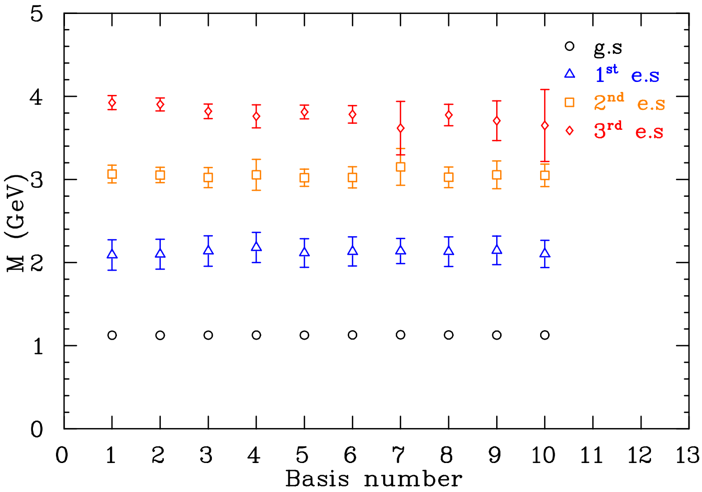
Sweeps $\rightarrow$	16	25	35	50	70	100	125	200	400	800
Basis No. $\downarrow$	Bases									
1	16	-	35	-	70	100	-	-	-	-
2	16	-	35	-	70	-	125	-	-	-
3	16	-	35	-	-	100	-	200	-	-
4	16	-	35	-	-	100	-	-	400	-
5	16	-	-	50	-	100	125	-	-	-
6	16	-	-	50	-	100	-	200	-	-
7	16	-	-	50	-	-	125	-	-	800
8	-	25	-	50	-	100	-	200	-	-
9	-	25	-	50	-	100	-	-	400	-
10	-	-	35	-	70	-	125	-	400	-

Smeared Source - Point Sink Effective Masses

For second lightest quark : 50 cfgs

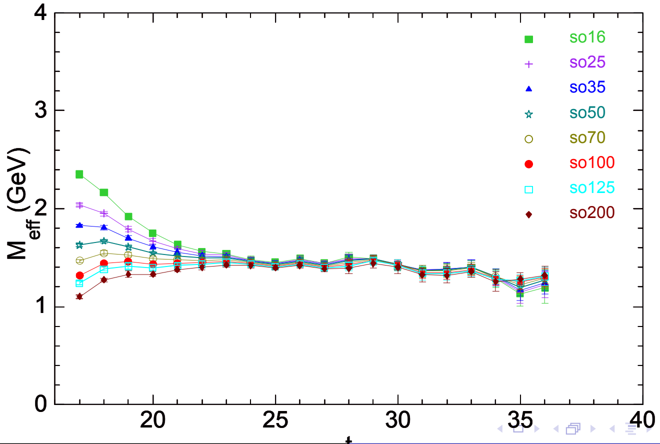


For all 4×4 bases: $K_{ud} = 0.137700$

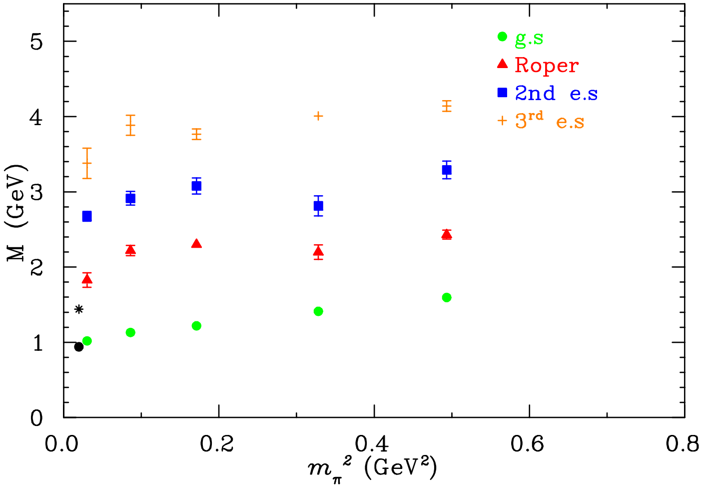


Smeared Source - Point Sink Effective Masses

For the heaviest quark: 50 cfgs

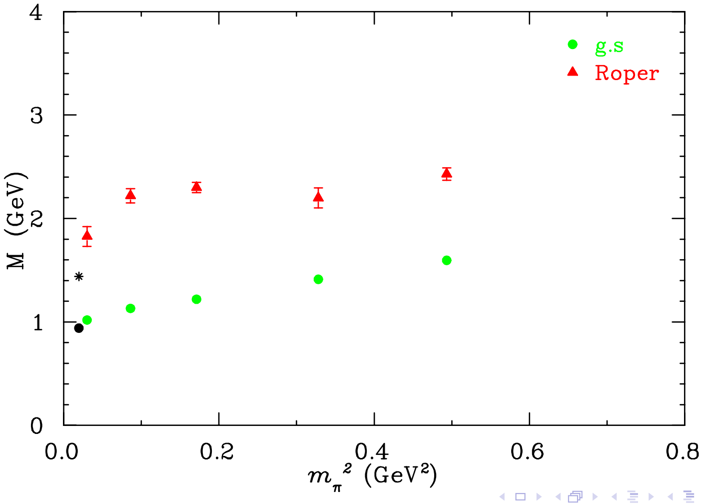


Even Parity Nucleon Spectrum in full QCD

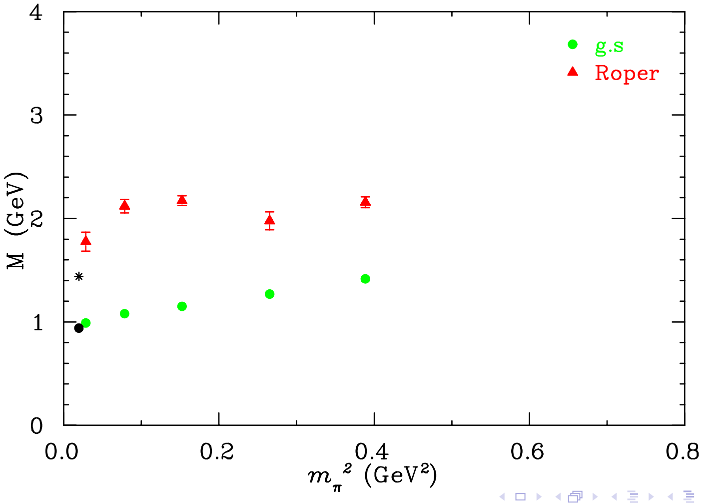


Configs: Lightest = 750 cfgs, rest are 350 cfgs.

Ground and Roper states (fixed lattice spacing)



Ground and Roper states (Sommer scale sets *a*)



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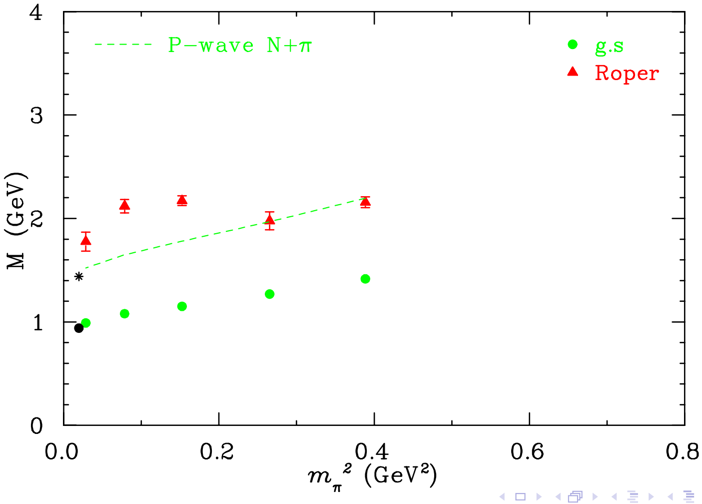
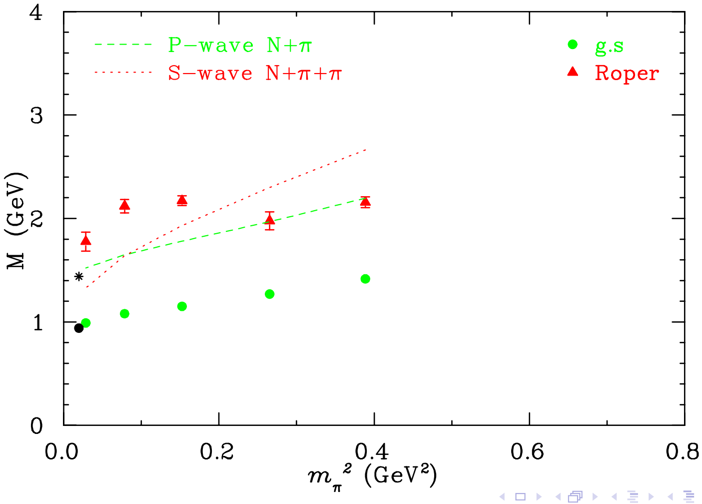
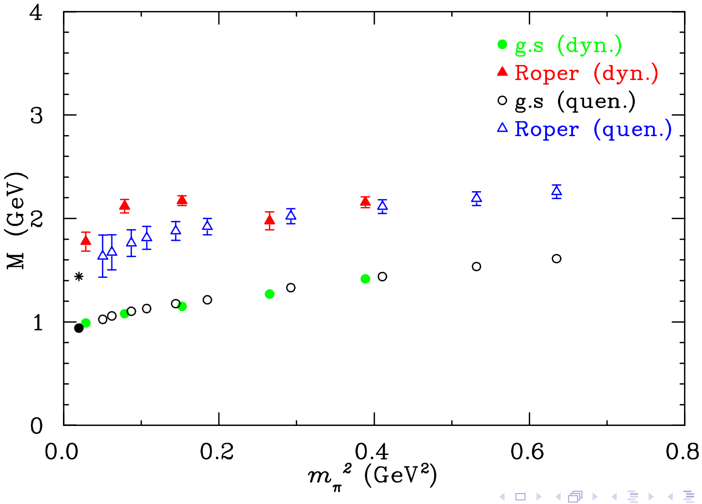


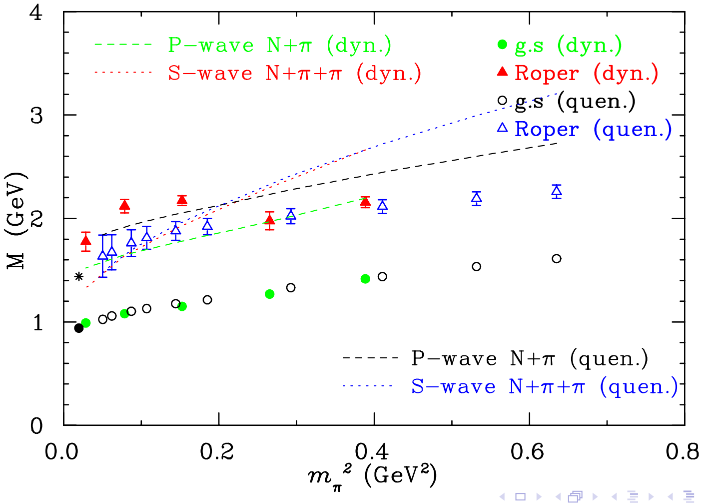
Ground and Roper states (Sommer scale sets *a*)



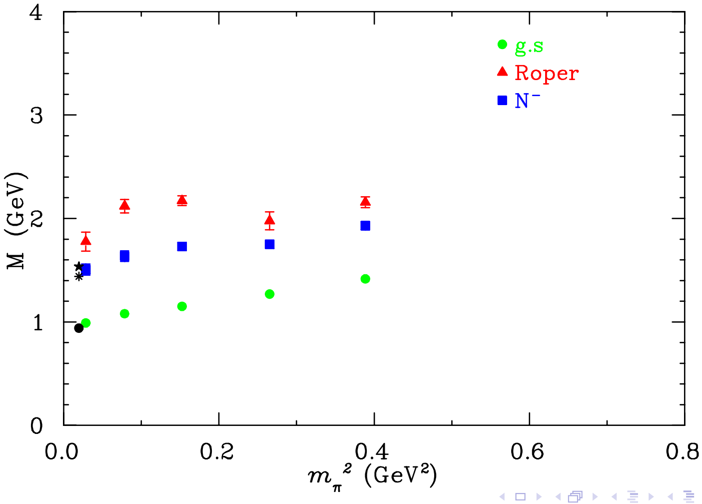
Quenched Vs Dynamical (Sommer scale)



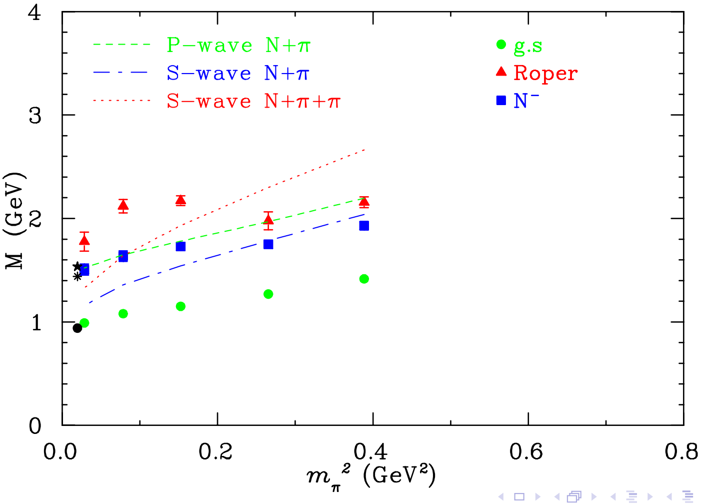
Quenched Vs Dynamical (Sommer scale)



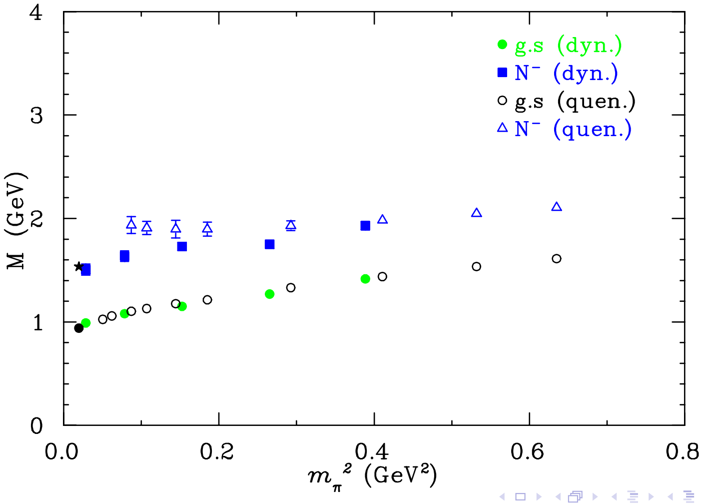
$N\frac{1}{2}^-$ (1535) (Sommer scale)



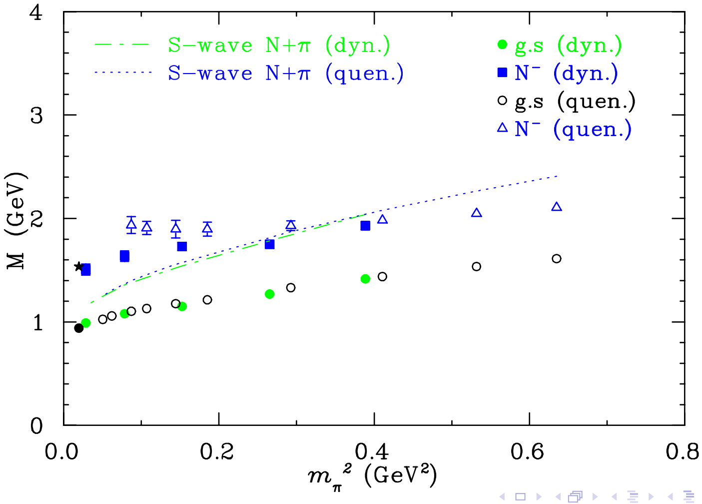
$N\frac{1}{2}^-$ (1535) (Sommer scale)



$N\frac{1}{2}^-$ (1535) (Sommer scale)



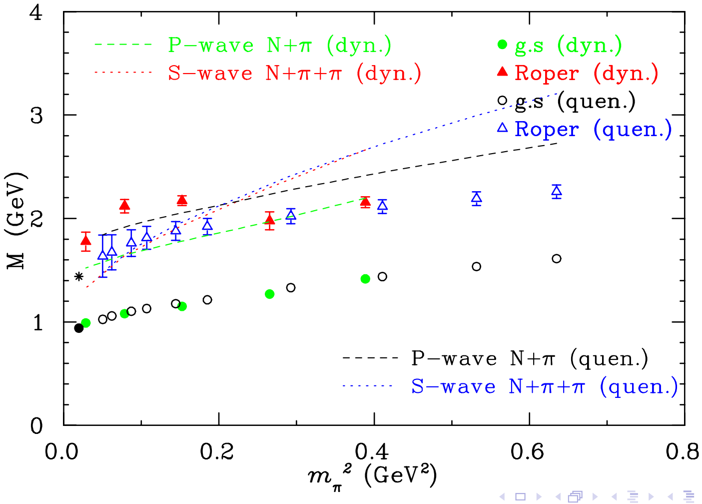
Quenched Vs Dynamical, N^- states (Sommer scale)



Summary

- Several fermion-source and -sink smearing levels have been used to construct correlation matrices.
- A variety of 4×4, 6×6, and 8×8 matrices were considered to demonstrate the independence of the eigenstate energies from the basis interpolators.
- A low-lying Roper state has been identified in both quenched and full QCD using this correlation-matrix based method.
- The approach to the chiral limit is significantly different.
- The two heaviest quark masses considered in the dynamical case provide states consistent with P-wave πN scattering states.

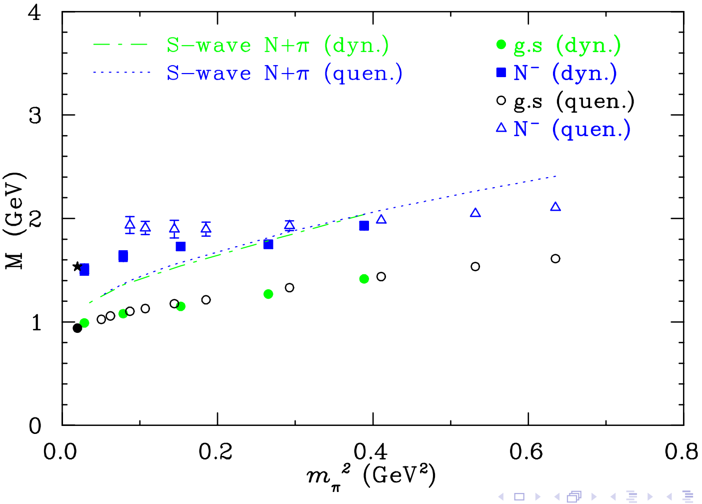
Quenched Vs Dynamical (Sommer scale)



Summary continued...

- The $N1/2^-$ results in quenched and dynamical QCD reveal significant differences in the approach to the physical point.
- A level crossing between the Roper and $N1/2^-$ states is observed in quenched QCD at $m_\pi \simeq 400$ MeV.
- A level crossing between the Roper and $N1/2^-$ states is anticipated in full QCD at $m_\pi \simeq 150$ MeV, just above the physical pion mass.
- The approach to the experimentally measured masses is encouraging.
- The effects of the finite volume and the role of scattering states remains to be resloved.

Quenched Vs Dynamical $N\frac{1}{2}^-$ (1535) (Sommer scale)



Future Plans

- Extend to a comprehensive analysis of all low-lying baryons.
  - See Ben Menadue's Poster on the $\Lambda(1405)$.
- Examine the nature of the Roper wave function.
  - See Dale Roberts' Poster on the proton in a magnetic field.
- Explore chiral curvature via chiral effective field theory.
  - Knowledge of meson-baryon couplings to nearby states.
  - See Jonathan Hall's talk on intrinsic scales in χEFT.
- Resolve excited-state electromagnetic properties.
  - Three-point function and background field approaches.
  - See Thom Primer's Poster on the background field approach to magnetic properties.