Weak charge of proton: loop corrections to parity-violating electron scattering

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Why proton weak charge $Q_W^p = 1 - 4 \sin^2 \theta_W$?

**electromagnetic**

$$G_E^\gamma = \sum_q e_q G_E^q$$

**weak**

$$G_E^Z = \sum_q g_q^V G_E^q$$

PDG convention

($= 1/2 \times$ nuclear physics convention)

$$g_q^V = I_q^w - 2e_q \sin^2 \theta_W$$
Why proton weak charge $Q^p_W = 1 - 4 \sin^2 \theta_W$?

**electromagnetic**

\[
G^{\gamma E} = \sum_q e_q G^q_E
\]

**weak**

\[
G^{Z E} = \sum_q g^V_q G^q_E
\]

at $Q^2 = 0$

\[
G^{u/p}_E \equiv G^{d/n}_E = 2, \quad G^{d/p}_E \equiv G^{u/n}_E = 1
\]

\[
G^{\gamma p}_E = 1
\]

\[
G^{\gamma n}_E = 0
\]

\[
G^{\gamma n}_E \ll G^{\gamma p}_E
\]

\[
G^{Z p}_E = \frac{1}{2} Q^p_W
\]

\[
G^{Z n}_E = -\frac{1}{2}
\]

\[
|G^{Z p}_E| \ll |G^{Z n}_E|
\]

$\rightarrow$ $G^{Z p}_E$ small but fundamental quantity!

$\rightarrow$ measured in $Q_{\text{weak}}$ experiment at JLab
Why proton weak charge $Q_W^p = 1 - 4 \sin^2 \theta_W$?

$\sin^2 \theta_W$ vs $Q$ (GeV)

- Standard Model
- Completed Experiments
- Future Experiments

SLAC E158
APV (Cs)
Møller [JLab]
$\nu$-DIS
PV-DIS [JLab]

Z-pole
D0
CDF

* 4% measurement of $Q_W^p$

Bentz, Cloet, Londergan, Thomas
PLB 693, 462 (2010)
Outline

- **Background**: two-photon exchange in elastic $ep$ scattering
  
  - electric/magnetic form factor ratio: Rosenbluth separation vs. polarization transfer

- Parity-violating electron scattering
  
  - effect of $\gamma Z$ exchange on strange form factors
  
  - dispersive corrections to proton’s weak charge: “Qweak” experiment at Jefferson Lab

- Summary
Two-photon exchange in elastic $e-p$ scattering
Proton $G_E/G_M$ ratio

**LT method**

$$\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{T} G_E^2(Q^2)$$

→ $G_E$ from slope in $\varepsilon$ plot

→ suppressed at large $Q^2$

Rosenbluth (Longitudinal-Transverse) Separation

Arrington et al., PRC 68, 034325 (2003)
Proton $G_E/G_M$ ratio

Rosenbluth (Longitudinal-Transverse) Separation
Arrington et al., PRC 68, 034325 (2003)

Polarization Transfer
Jones et al., PRL 84, 1398 (2000)
Gayou et al., PRL 88, 092301 (2002)

LT method

$$
\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)
$$

$G_E$ from slope in $\varepsilon$ plot
$P_{T,L}$ recoil proton polarization in $\vec{e} \ p \rightarrow e \ \vec{p}$

PT method

$$
\frac{G_E}{G_M} = -\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}} \frac{P_T}{P_L}
$$

$\rightarrow$ suppressed at large $Q^2$
Proton $G_E/G_M$ ratio

\[ \sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2) \]

**LT method**

- $G_E$ from slope in $\varepsilon$ plot
- suppressed at large $Q^2$

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**PT method**

- $P_{T,L}$ recoil proton polarization in $\bar{e} p \rightarrow e \bar{p}$

Puckett et al., PRL **104**, 242301 (2010)
QED radiative corrections

- cross section modified by $1\gamma$ loop effects

\[ d\sigma = d\sigma_0 (1 + \delta) \]

- \( \delta \) contains additional \( \varepsilon \) dependence, mostly from box diagrams (most difficult to calculate)

* IR divergences cancel
Two-photon exchange

- interference between Born and TPE amplitudes

\[ M_0 \quad \times \quad M_{\gamma\gamma} \]

- contribution to cross section:

\[ \delta^{(2\gamma)} = \frac{2 \text{Re} \left\{ M_0^\dagger M_{\gamma\gamma} \right\}}{|M_0|^2} \]

- “soft photon approximation” (used in all previous data analyses)
  - approximate integrand in \( M_{\gamma\gamma} \) by values at \( \gamma^* \) poles
  - neglect nucleon structure (no form factors)

*Mo, Tsai (1969)*
Two-photon exchange

\[ \mathcal{M}_{\gamma\gamma} = e^4 \int \frac{d^4k}{(2\pi)^4} \frac{N(k)}{D(k)} \]

where

\[ N(k) = \bar{u}(p_3) \gamma_\mu (\not{p}_1 - \not{k} + m_e) \gamma_\nu u(p_1) \times \bar{u}(p_4) \Gamma^\mu (q - k) (\not{p}_2 + \not{k} + M) \Gamma^\nu (k) u(p_2) \]

and

\[ D(k) = (k^2 - \lambda^2) ((k - q)^2 - \lambda^2) \times ((p_1 - k)^2 - m^2) ((p_2 + k)^2 - M^2) \]

with \( \lambda \) an IR regulator, and e.m. current is

\[ \Gamma^\mu (q) = \gamma^\mu F_1(q^2) + \frac{i \sigma^{\mu\nu} q_\nu}{2M} F_2(q^2) \]

on-shell approximation
Two-photon exchange

- “exact” calculation of loop diagram (including hadron structure)

\[ \delta_{\text{full}}^{(2\gamma)} - \delta_{\text{Mo-Tsai}}^{(2\gamma)} \]

\[ \Delta (\varepsilon, Q^2) \]

- few % magnitude, non-linear in \( \varepsilon \), positive slope
- will reduce Rosenbluth ratio
- does not depend strongly on vertex form factors

Blunden, Melnitchouk, Tjon
PRL 91 (2003) 142304;
PRC 72 (2005) 034612
Higher-mass intermediate states

- lowest mass excitation is $P_{33}$ $\Delta(1232)$ resonance

\[ \Gamma_{\gamma\Delta \rightarrow N}(p, q) \equiv iV_{\Delta in}(p, q) = i\frac{eF_{\Delta}(q^2)}{2M_{\Delta}^2} \left\{ g_1 \left[ g^{\nu\alpha}\phi\frac{1}{\mu} - p^\nu\gamma^\alpha\phi - \gamma^\nu\gamma^\alpha p \cdot q + \gamma^\nu\phi q^\alpha \right] 
+ g_2 \left[ p^\nu q^\alpha - g^{\nu\alpha} p \cdot q \right] + \left( g_3/M_{\Delta} \right) \left[ q^2(p^\nu\gamma^\alpha - g^{\nu\alpha}\phi) + q^\nu(q^\alpha\phi - \gamma^\alpha p \cdot q) \right] \right\} \gamma_5 T_3 \]

- coupling constants

\[ g_1 \text{ magnetic } \rightarrow 7 \]
\[ g_2 - g_1 \text{ electric } \rightarrow 9 \]
\[ g_3 \text{ Coulomb } \rightarrow -2 \ldots 0 \]
higher-mass intermediate states

more model dependent, since couplings & form factors not as well known (especially at high $Q^2$)

$\Delta$ partially cancels $N$ contribution
Global analysis

Arrington, Melnitchouk, Tjon
PRC 76 (2007) 035205
final form factor results from global analysis including TPE corrections

\[
\left\{ G_E, \frac{G_M}{\mu_p} \right\} = \frac{1 + \sum_{i=1}^{n} a_i \tau^i}{1 + \sum_{i=1}^{n+2} b_i \tau^i}
\]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$G_M/\mu_p$</th>
<th>$G_E$</th>
</tr>
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<tr>
<td>$a_1$</td>
<td>-1.465</td>
<td>3.439</td>
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<tr>
<td>$a_2$</td>
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<td>$b_4$</td>
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</tr>
<tr>
<td>$b_5$</td>
<td>13.245</td>
<td>8.650</td>
</tr>
</tbody>
</table>

Arrington, Melnitchouk, Tjon
PRC 76 (2007) 035205
$e^+/e^-$ comparison

- $1\gamma$ ($2\gamma$) exchange changes sign (invariant) under $e^+ \leftrightarrow e^-$
- ratio of $e^+p/e^-p$ cross sections sensitive to $\Delta(\varepsilon, Q^2)$

$1 - 2\Delta$

- simultaneous $e^+p/e^-p$ measurement using tertiary $e^+/e^-$ beam to $Q^2 \sim 1-2$ GeV$^2$
  (Hall B experiment E04-116)

Data at various $\varepsilon$
$e^+/e^-$ comparison

1γ (2γ) exchange changes sign (invariant) under $e^+ \leftrightarrow e^-$

Very preliminary Novosibirsk data

Arrington, Holt et al. (2010)
$e^+/e^-$ comparison

1$\gamma$ (2$\gamma$) exchange changes sign (invariant) under $e^+ \leftrightarrow e^-$

→ strong indication of *inadequacy* of one-photon exchange approximation in $ep$ scattering

→ significant role played by *hadron structure dependent* two-photon exchange corrections
Parity-violating electron scattering
Parity-violating $e$ scattering

- Left-right polarization asymmetry in $\bar{e}p \rightarrow ep$ scattering

$$A_{PV} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4\sqrt{2}\alpha}\right) (A_V + A_A + A_s)$$

- Measure interference between e.m. and weak currents

Born (tree) level
Parity-violating $e$ scattering

- **Left-right polarization asymmetry in $\vec{e} \ p \rightarrow e \ p$ scattering**

\[
A_{PV} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4 \sqrt{2} \alpha}\right) (A_V + A_A + A_s)
\]

→ measure interference between e.m. and weak currents

\[
A_V = g_A^e \rho \left[ (1 - 4\kappa \sin^2 \theta_W) - (\epsilon G_{E}^{\gamma p} G_{E}^{\gamma n} + \tau G_{M}^{\gamma p} G_{M}^{\gamma n}) / \sigma^{\gamma p} \right]
\]

radiative corrections, including TBE

→ using relations between weak and e.m. form factors

\[
G_{E,M}^{Z,p} = (1 - 4 \sin^2 \theta_W) G_{E,M}^{\gamma p} - G_{E,M}^{\gamma n} - G_{E,M}^s
\]
Parity-violating $e$ scattering

- Left-right polarization asymmetry in $\bar{e} p \rightarrow e p$ scattering

\[
A_{PV} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left( \frac{G_F Q^2}{4\sqrt{2}\alpha} \right) (A_V + A_A + A_s)
\]

\(\rightarrow\) measure interference between e.m. and weak currents

\[
A_A = g^e_V \sqrt{\tau (1 + \tau)(1 - \varepsilon^2)} \tilde{G}_A^Z p G_{\gamma p}^M / \sigma^{\gamma p}
\]

includes axial RCs + anapole term

\[
A_s = -g^e_A \rho (\varepsilon G_{\gamma p}^{\gamma p} G_{s E}^{s} + \tau G_{\gamma p}^{\gamma p} G_{s M}^{s}) / \sigma^{\gamma p}
\]

strange electric & magnetic form factors
Two-boson exchange corrections

- current PDG estimates computed at $Q^2 = 0$
  - Marciano, Sirlin (1980)

- do not include hadron structure effects
Two-boson exchange corrections

- parameterize corrections to asymmetry as

\[ A_{PV} = (1 + \delta)A_{PV}^0 \equiv \left( \frac{1 + \delta_{Z(\gamma\gamma)} + \delta_{\gamma(Z\gamma)}}{1 + \delta_{\gamma(\gamma\gamma)}} \right) A_{PV}^0 \]

- Born asymmetry

\[
\delta_{Z(\gamma\gamma)} = \frac{2 \text{Re}(M_Z^* M_{\gamma\gamma})}{2 \text{Re}(M_Z^* M_{\gamma})}
\]

\[
\delta_{\gamma(Z\gamma)} = \frac{2 \text{Re}(M_{\gamma Z}^* M_{\gamma Z} + M_{\gamma}^* M_{Z\gamma})}{2 \text{Re}(M_Z^* M_{\gamma})}
\]

\[
\delta_{\gamma(\gamma\gamma)} = \frac{2 \text{Re}(M_{\gamma\gamma}^* M_{\gamma\gamma})}{|M_{\gamma}|^2}
\]

→ total TBE correction

\[
\delta \approx \delta_{Z(\gamma\gamma)} + \delta_{\gamma(Z\gamma)} - \delta_{\gamma(\gamma\gamma)}
\]
Two-boson exchange corrections

- nucleon intermediate states

\[ \delta_{\nu}(\varepsilon, Q^2) \]

\[ Q^2 = 0.01 \text{ GeV}^2 \]

\[ Q^2 = 1 \text{ GeV}^2 \]

\[ \rightarrow \text{cancellation between } Z(\gamma\gamma) \text{ and } \gamma(\gamma\gamma) \text{ corrections, especially at low } Q^2 \]

\[ \rightarrow \text{dominated by } \gamma(Z\gamma) \text{ contribution} \]

Tjon, Melnitchouk, PRL 100 (2008) 082003
Tjon, Blunden, Melnitchouk, PRC 79 (2009) 055201
Effects on strange form factors

- global analysis of all PVES data at $Q^2 < 0.3 \text{ GeV}^2$

$$G_E^s = 0.0023 \pm 0.0182$$
$$G_M^s = -0.020 \pm 0.254$$

at $Q^2 = 0.1 \text{ GeV}^2$

Young et al., PRL 97 (2006) 102002

- including TBE corrections:

$$G_E^s = 0.0025 \pm 0.0182$$
$$G_M^s = -0.011 \pm 0.254$$

at $Q^2 = 0.1 \text{ GeV}^2$
Effects on strange form factors

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\[ G_E^s = 0.0023 \pm 0.0182 \]
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at $Q^2 = 0.1 \text{ GeV}^2$

fixed mainly by $^4\text{He}$ data ...

... TBE for $^4\text{He}$ not yet included
Correction to proton weak charge

In forward limit $A_{PV}$ measures weak charge of proton $Q_W^p$

\[ A_{PV} \to \frac{G_F Q_W^p}{4\sqrt{2}\pi\alpha} t \]

\[ k \quad \text{\textgamma}\quad q \quad Z \quad k' \approx k \quad \gamma^* \]

Forward limit
\[ t = (k - k')^2 \to 0 \]
\[ s = (k + p)^2 \]
\[ = M(M + 2E) \]

At tree level $Q_W^p$ gives weak mixing angle

\[ Q_W^p = 1 - 4\sin^2\theta_W \]
Correction to proton weak charge

including higher order radiative corrections

\[ Q^p_W = (1 + \Delta \rho + \Delta_e)(1 - 4 \sin^2 \theta_W(0) + \Delta'_e) \]

“standard” electroweak vertex & other corrections
Correction to proton weak charge

including higher order radiative corrections

\[ Q_p^W = (1 + \Delta \rho + \Delta_e)(1 - 4 \sin^2 \theta_W(0) + \Delta'_e) \]
\[ + \begin{array}{c}
\Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z}
\end{array} \quad \text{box diagrams} \]

\[ = 0.0713 \pm 0.0008 \]

*Erler et al., PRD 72, 073003 (2005)*
Correction to proton weak charge

- including higher order radiative corrections

\[
Q_W^p = (1 + \Delta \rho + \Delta_e)(1 - 4 \sin^2 \theta_W(0) + \Delta'_e) \\
+ \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z}
\]

\[
= 0.0713 \pm 0.0008
\]

*Erler et al., PRD 72, 073003 (2005)*

→ *WW* and *ZZ* box diagrams dominated by short distances, evaluated perturbatively

→ *γZ* box diagram sensitive to long distance physics, has two contributions

\[
\Box_{\gamma Z} = \Box^{A}_{\gamma Z} + \Box^{V}_{\gamma Z}
\]

vector *e* – axial *h* (finite at *E*=0)  
axial *e* – vector *h* (vanishes at *E*=0)
Vector $h$ correction

- What is energy dependence of vector $h$ correction $\square^V_Z$?

→ Computed in forward limit using dispersion relations

\[ \Re \left[ \square^V_Z(E) \right] = \frac{2E}{\pi} \int_0^\infty \frac{1}{E'^2 - E^2} \, dE' \, \Im \left[ \square^V_Z(E') \right] \]

* Integration over $E' < 0$ corresponds to crossed-box, vector $h$ contribution symmetric under $E' \leftrightarrow -E'$

* Vanishes as $E \rightarrow 0$ (e.g. atomic parity violation) but what about at $\mathcal{O}(1 \text{ GeV})$ of $Qweak$ experiment?
Vector $h$ correction

imaginary part given by $\gamma Z$ interference structure functions

$$\Im m \square^{\gamma Z}(E) = \frac{\alpha}{(s - M^2)^2} \int_{W^2}^{s} dW^2 \int_{0}^{Q^2_{\text{max}}} dQ^2 \frac{1 + Q^2/M^2_Z}{1 + Q^2/M^2_Z}$$

$$\times \left( F_{1}^{\gamma Z} + F_{2}^{\gamma Z} \frac{s (Q^2_{\text{max}} - Q^2)}{Q^2(W^2 - M^2 + Q^2)} \right)$$

little direct data on interference structure functions
(neutral currents at HERA at very small $x$)

in parton model $F_{2}^{\gamma Z} = 2x \sum_{q} e_{q} g_{V}^{q} (q + \bar{q}) = 2x F_{1}^{\gamma Z}$

$F_{2}^{\gamma Z} \approx F_{2}^{\gamma}$ good approximation at low $x$

provides upper limit at large $x$ ($F_{2}^{\gamma Z} \lesssim F_{2}^{\gamma}$)
Vector $h$ correction

- in resonance region use phenomenological input for $F_2$, empirical SLAC fit for $R = \sigma_L/\sigma_T = (1 + 4M^2x^2/Q^2)F_2/(2xF_1) - 1$

  $\Rightarrow$ for transitions to $I = 3/2$ states (e.g. $\Delta$), CVC and isospin symmetry give $F_i^{\gamma Z} = (1 + Q^p_W)F_i^\gamma$

  $\Rightarrow$ for transitions to $I = 1/2$ states, SU(6) wave functions predict $Z$ & $\gamma$ transition couplings equal to few percent

  $\Rightarrow$ include contributions from prominent resonances:

  $P_{33}(1232), D_{13}(1520), F_{15}(1680), F_{37}(1950)$
Vector $h$ correction

- in resonance region use phenomenological input for $F_2$, empirical SLAC fit for

$$R = \sigma_L/\sigma_T = (1 + 4M^2x^2/Q^2)F_2/(2xF_1) - 1$$

GVMD model (used as input by Gorchtein & Horowitz)
Vector $h$ correction

- total $\Re e [\gamma Z] = 0.0047^{+0.0011}_{-0.0004}$

or 6.6$^{+1.5}_{-0.6}$% of uncorrected $Q_p^W$ 

$Q_p^W = 0.0713(8) \rightarrow 0.0760^{+0.0014}_{-0.0009}$

→ significant shift in central value, errors within projected experimental uncertainty $\Delta Q_p^W = \pm 0.003$
Axial $h$ correction

- axial $h$ correction $\Box_{\gamma Z}^A$ dominant $\gamma Z$ correction in atomic parity violation at very low (zero) energy

$\rightarrow$ computed by Marciano & Sirlin as sum of two parts:

- low-energy part approximated by Born contribution (elastic intermediate state)

- high-energy part (above scale $\Lambda \sim 1$ GeV) computed in terms of scattering from free quarks

\[
\Box_{\gamma Z}^A = \frac{5\alpha}{2\pi} (1 - 4\sin^2 \theta_W) \left[ \ln \frac{M_Z^2}{\Lambda^2} + C_{\gamma Z}(\Lambda) \right]
\]

$\approx 0.0028$


Erler et al., PRD 68, 016006 (2003)
Axial $h$ correction

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\]

- Repeat calculation for realistic (structure function) input
Axial $h$ correction

→ imaginary part given by interference $F_3^{\gamma Z}$ structure function

$$\Im m \Box_A^{\gamma Z}(E) = \frac{\alpha}{(s - M^2)^2} \int_{W^2_s}^s dW^2 \int_0^{Q_{\max}^2} \frac{dQ^2}{1 + Q^2/M^2_Z} \times \frac{g^e_V}{2g^e_A} \left( \frac{4ME}{W^2 - M^2 + Q^2} - 1 \right) F_3^{\gamma Z}$$

with $g^e_A = -\frac{1}{2}$, $g^e_V = -\frac{1}{2}(1 - 4\sin^2\theta_W)$

★ axial $h$ contribution antisymmetric under $E' \leftrightarrow -E'$:

$$\Re \Box_A^{\gamma Z}(E') = \frac{2}{\pi} \int_0^\infty dE' \frac{E'}{E'^2 - E^2} \Im m \Box_A^{\gamma Z}(E')$$

★ imaginary part can only grow as $\log E'/E'$
Axial $h$ correction

- $F_3^{\gamma Z}$ structure function
  - elastic part given by $G_M^p G_A^Z$
  - resonance part from parametrization of $\nu$ scattering data (Lalakulich-Paschos)
  - DIS part dominated by leading twist PDFs at small $x$ (MSTW, CTEQ, Alekhin)
Axial $h$ correction

- $F_3^{\gamma Z}$ structure function
  - elastic part given by $G_M^p G_A^Z$
  - resonance part from parametrization of $\nu$ scattering data
    (Lalakulich-Paschos)
  - DIS part dominated by leading twist PDFs at small $x$
    (MSTW, CTEQ, Alekhin)

- real part of $\Box^{A \gamma Z}$ from dispersion relation
  \[
  \Re \Box^{A \gamma Z}(0) = 0.0006 + 0.0002 + 0.0025 = 0.0033
  \]
  \[
  \uparrow \quad \uparrow \quad \uparrow \quad \text{elastic} \quad \text{resonance} \quad \text{DIS}
  \]

  → additional $+ 0.7\%$ correction $Q_W^p = 0.0760 \rightarrow 0.0765$

Blunden et al. (2010)
Summary

- Two-photon exchange corrections resolve most of Rosenbluth / polarization transfer $G_E^p / G_M^p$ discrepancy

  \[ \rightarrow \text{striking demonstration of limitation of one-photon exchange approximation in } ep \text{ scattering} \]

- Dramatic effect of $\gamma(Z\gamma)$ corrections at forward angles on proton weak charge, $\Delta Q_W^p \sim 6\text{--}7\%$

  \[ \rightarrow \text{would shift extracted weak angle by } \Delta \sin^2 \theta_W \approx 0.0013 \]

  \[ \rightarrow \text{will be better constrained by direct measurement of } F_1^{\gamma,Z}, F_2^{\gamma,Z}, F_3^{\gamma,Z} \text{ (e.g. in PVDIS at JLab)} \]
The End
• **Mo-Tsai:** soft $\gamma$ approximation
  $\rightarrow$ integrand most singular when $k = 0$
  and $k = q$
  $\rightarrow$ replace $\gamma$ propagator which is
  not at pole by $1/q^2$
  $\rightarrow$ approximate numerator $N(k) \approx N(0)$
  $\rightarrow$ neglect all structure effects

• **Maximon-Tjon:** improved loop calculation
  $\rightarrow$ exact treatment of propagators
  $\rightarrow$ still evaluate $N(k)$ at $k = 0$
  $\rightarrow$ first study of form factor effects
  $\rightarrow$ additional $\varepsilon$ dependence

• **Blunden-WM-Tjon:** exact loop calculation
  $\rightarrow$ no approximation in $N(k)$ or $D(k)$
  $\rightarrow$ include form factors
Our formula for \( \Im m \Box_{\gamma Z}^V \) factor 2 larger

(incorrect definition of parton model structure functions:
"nuclear physics" vs. "particle physics" weak charges!)

\( \Re \delta_{\gamma Z} = \Re \Box_{\gamma Z}^V / Q_W^p \approx 6\% \)

mostly from high-\(W\)
("Regge") contribution

\[ \Re \delta_{\gamma Z} = \Re \Box_{\gamma Z}^V / Q_W^p \approx 6\% \]

\( \Box_{\gamma Z}^V = eO_{\gamma Z}^V / Q_W^p \approx 6\% \)

Numerical agreement purely coincidental!

\[ \Box_{\gamma Z}^V = eO_{\gamma Z}^V / Q_W^p \approx 6\% \]

GH omit factor \((1-x)\) in definition of \(F_{1,2}\)
(spurious \(~30\%\) enhancement)

GH use \(Q_W^p \sim 0.05\) \(cf.\sim 0.07\)
(spurious \(~40\%\) enhancement)

Numerical agreement purely coincidental!