DEPARTMENT OF ENERGY • UNITED STATES OF AMERICA

T(r)opical QCD - 2
Cairns, October 1, 2010

Weak charge of proton:
loop corrections to parity-violating electron scattering

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Why proton weak charge $Q_W^p = 1 - 4\sin^2\theta_W$?

electromagnetic

$$G_E^\gamma = \sum_q e_q \, G_E^q$$

weak

$$G_E^Z = \sum_q g_q^V \, G_E^q$$

PDG convention

(= $1/2$ x nuclear physics convention)

$$g_q^V = I_q^w - 2e_q \sin^2\theta_W$$

Why proton weak charge $Q_W^p = 1 - 4\sin^2\theta_W$?

electromagnetic

$$G_E^\gamma = \sum_q e_q \, G_E^q$$

weak

$$G_E^Z = \sum_q g_q^V \, G_E^q$$

at $Q^2 = 0$

$$G_E^{u/p} \equiv G_E^{d/n} = 2, \qquad G_E^{d/p} \equiv G_E^{u/n} = 1$$

$$G_E^{\gamma p} \;=\; 1$$

$$G_E^{\gamma n} \;=\; 0$$

$$G_E^{Zp} \;=\; \frac{1}{2} Q_W^p$$

$$G_E^{Zn} \;=\; -\frac{1}{2}$$

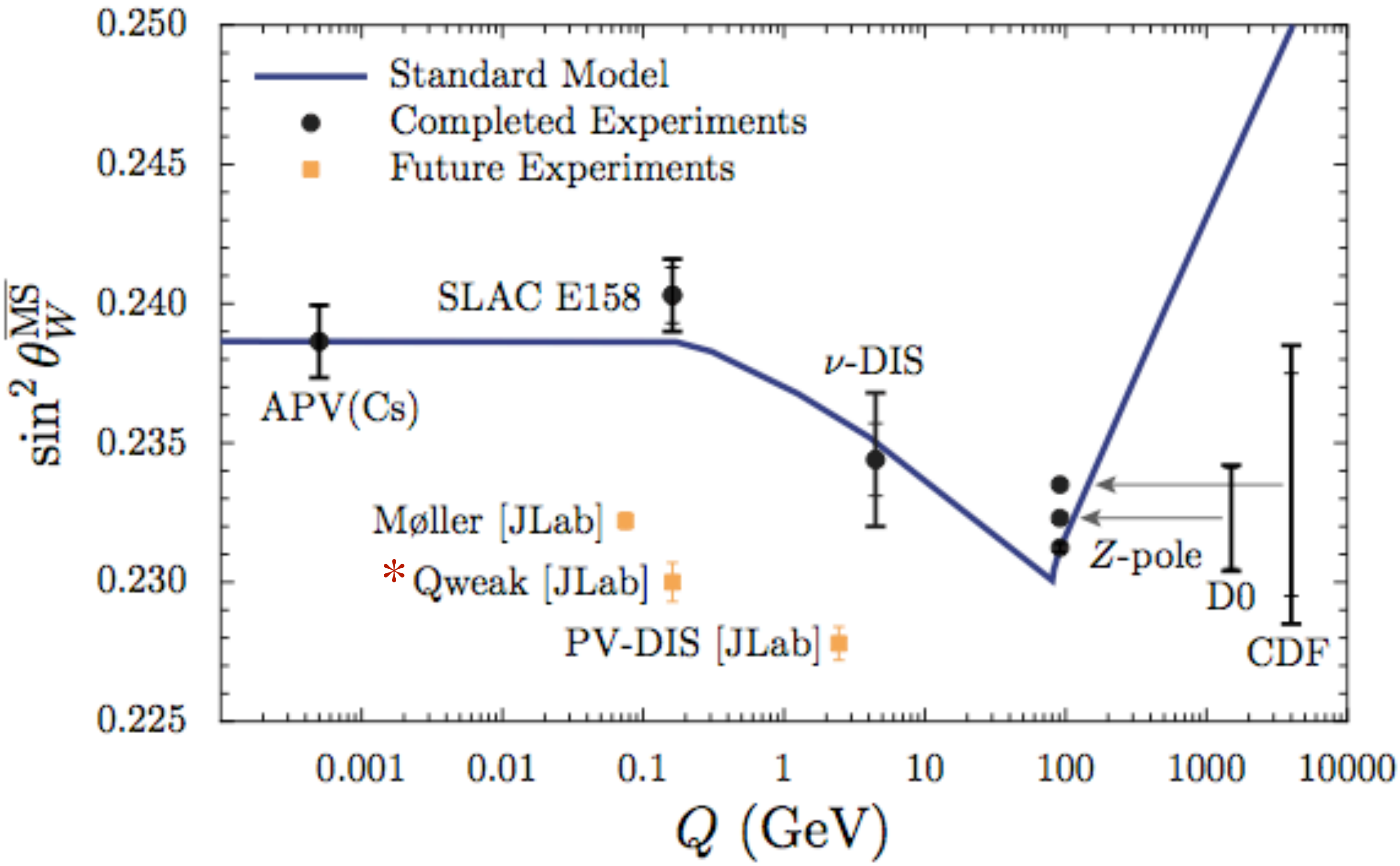
$$\boxed{G_E^{\gamma n} \;\ll\; G_E^{\gamma p}}$$

$$\boxed{|G_E^{Zp}| \;\ll\; |G_E^{Zn}|}$$

→ G_E^{Zp} small but fundamental quantity!

→ measured in *Qweak* experiment at JLab

Why proton weak charge $Q_W^p = 1 - 4\sin^2\theta_W$?



Bentz, Cloet, Londergan, Thomas
*PLB **693**, 462 (2010)*

* 4% measurement of Q_W^p

Outline

- *Background:* two-photon exchange in elastic *ep* scattering

  → electric/magnetic form factor ratio:
  Rosenbluth separation *vs.* polarization transfer

- Parity-violating electron scattering

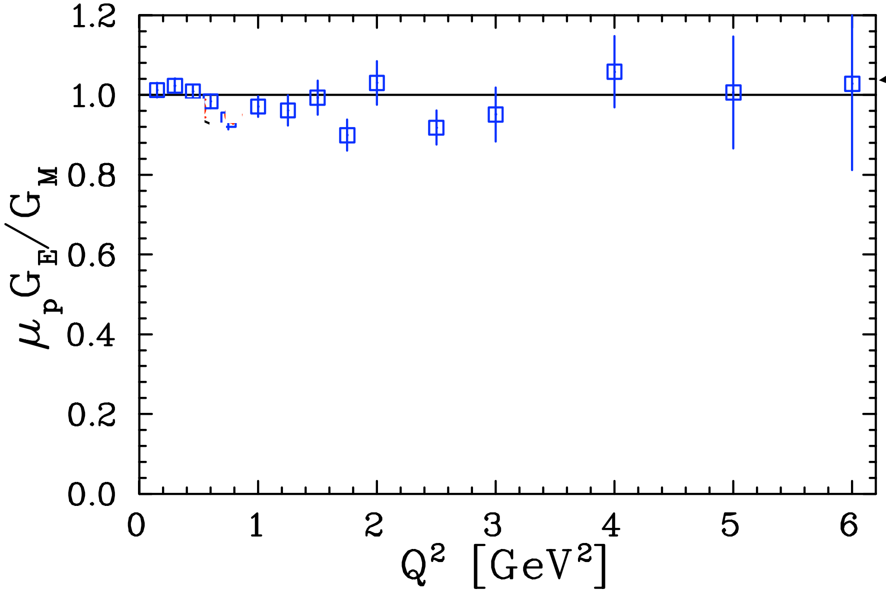
  → effect of γZ exchange on strange form factors

  → dispersive corrections to proton's weak charge:
  "Qweak" experiment at Jefferson Lab

- Summary

Two-photon exchange
in elastic *e*-*p* scattering

Proton G_E/G_M ratio



Rosenbluth (Longitudinal-Transverse) Separation

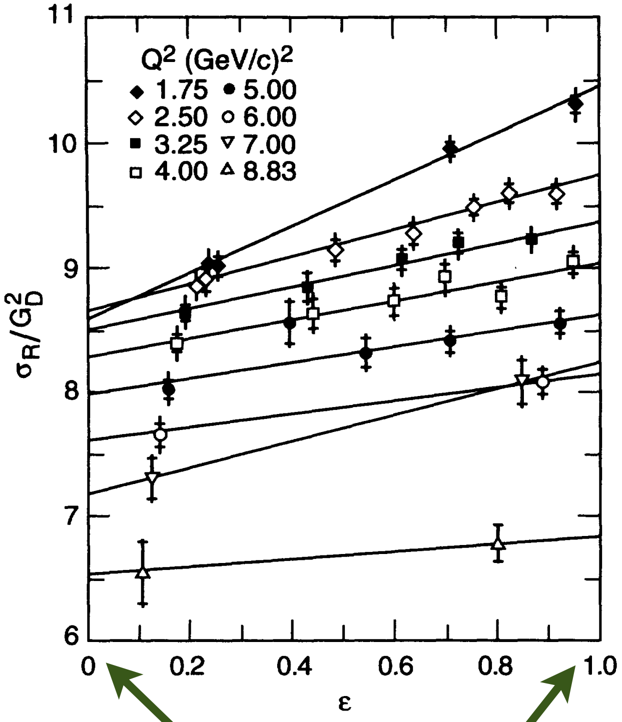
Arrington et al., PRC ***68****, 034325 (2003)*

LT method

$$\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

→ G_E from slope in ε plot

→ suppressed at large Q^2

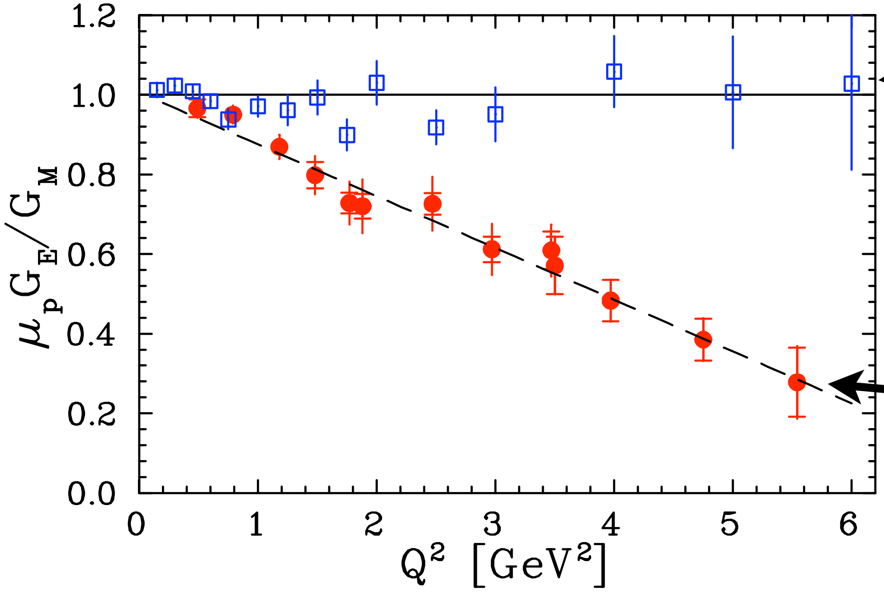


backward

forward

scattering

Proton G_E/G_M ratio



Rosenbluth (Longitudinal-Transverse) Separation

*Arrington et al., PRC **68**, 034325 (2003)*

Polarization Transfer

*Jones et al., PRL **84**, 1398 (2000)*
*Gayou et al., PRL **88**, 092301 (2002)*

LT method

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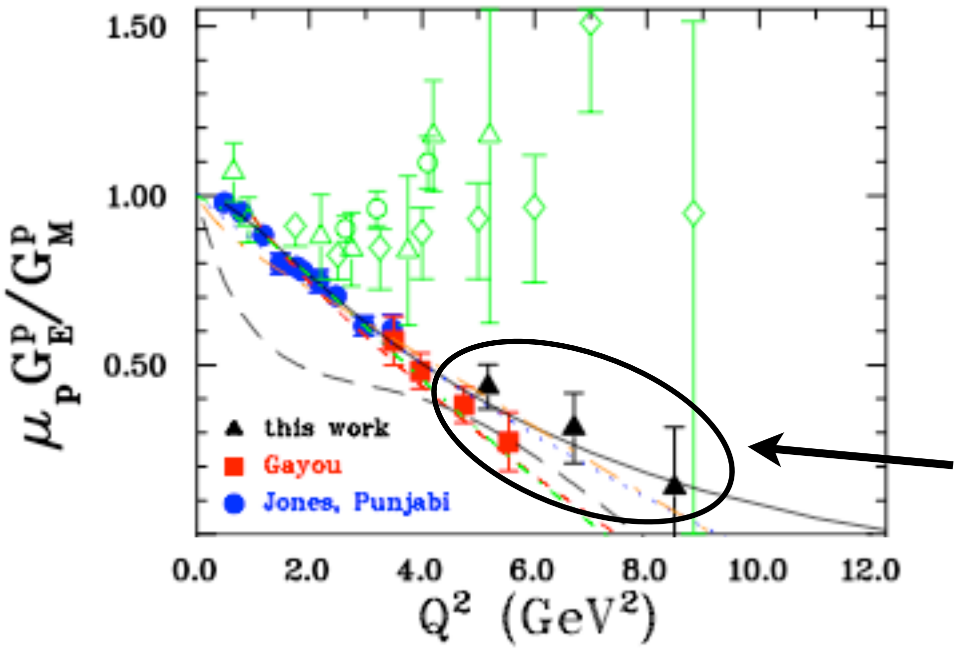
- → suppressed at large Q^2

PT method

$$\frac{G_E}{G_M} = -\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}} \frac{P_T}{P_L}$$

- → $P_{T,L}$ recoil proton polarization in $\vec{e}\, p \to e\, \vec{p}$

Proton G_E/G_M ratio



Polarization Transfer (latest from JLab)

Puckett et al., PRL ***104****, 242301 (2010)*

LT method

$$\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

→ G_E from slope in ε plot

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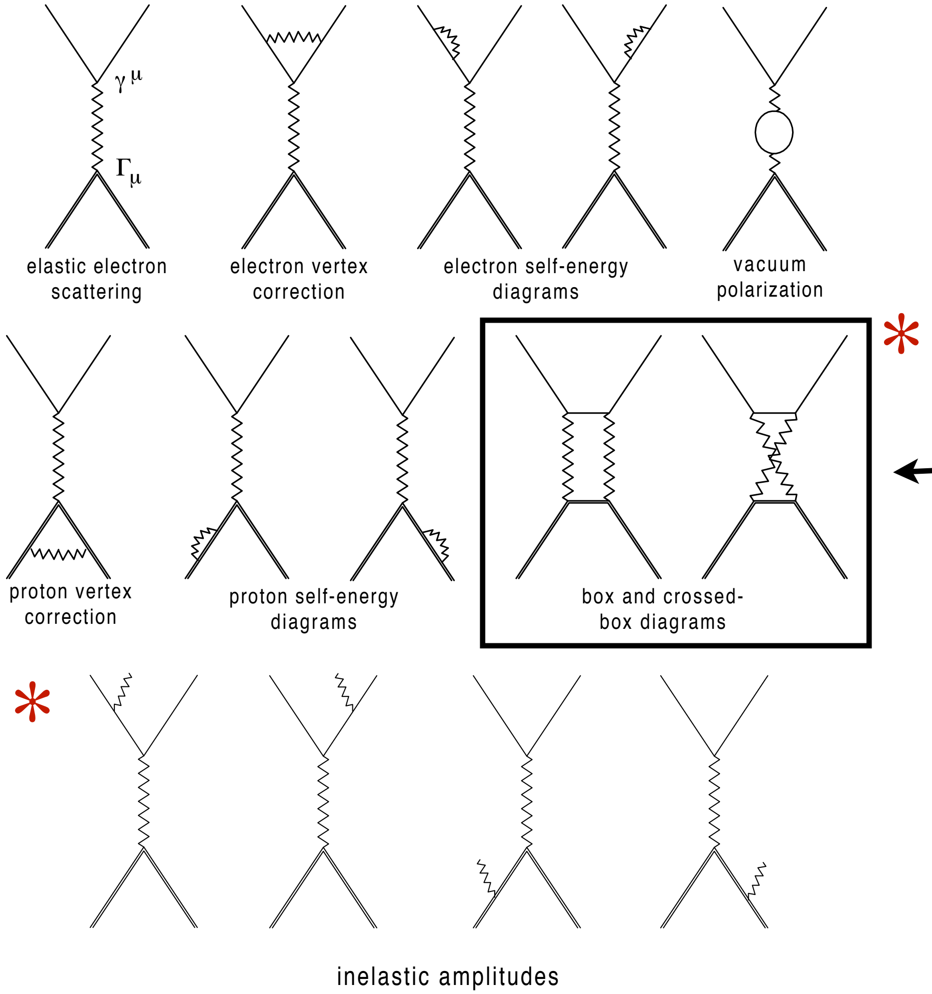
→ $P_{T,L}$ recoil proton polarization in $\vec{e}\, p \to e\, \vec{p}$

QED radiative corrections

- cross section modified by 1γ loop effects



$$d\sigma = \overset{\text{Born}}{d\sigma_0}\,(1+\overset{\text{TPE}}{\delta})$$

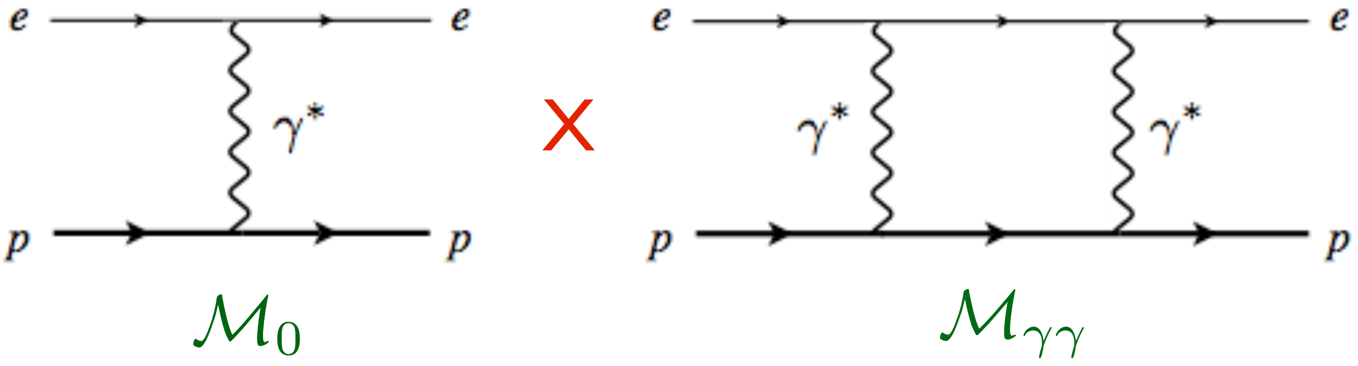
δ contains additional ε dependence, mostly from box diagrams

(most difficult to calculate)

* IR divergences cancel

Two-photon exchange

- interference between Born and TPE amplitudes



- contribution to cross section:

$$\delta^{(2\gamma)} = \frac{2\mathcal{R}e\left\{\mathcal{M}_0^\dagger\,\mathcal{M}_{\gamma\gamma}\right\}}{|\mathcal{M}_0|^2}$$

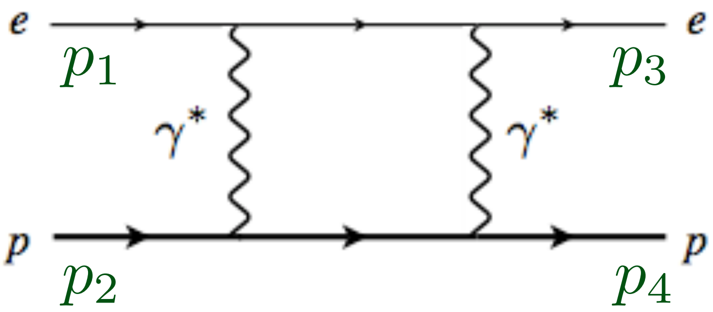
- "soft photon approximation" (used in all previous data analyses)

  ⟶ approximate integrand in $\mathcal{M}_{\gamma\gamma}$ by values at γ^* poles

  ⟶ neglect nucleon structure (no form factors)

Mo, Tsai (1969)

Two-photon exchange



$$\mathcal{M}_{\gamma\gamma} \;=\; e^4 \int \frac{d^4k}{(2\pi)^4} \frac{N(k)}{D(k)}$$

where

$$\begin{aligned} N(k) \;&=\; \bar{u}(p_3)\,\gamma_\mu(\not{p}_1 - \not{k} + m_e)\gamma_\nu\, u(p_1) \\ &\times\; \bar{u}(p_4)\,\Gamma^\mu(q-k)\,(\not{p}_2 + \not{k} + M)\,\Gamma^\nu(k)\,u(p_2) \end{aligned}$$

and

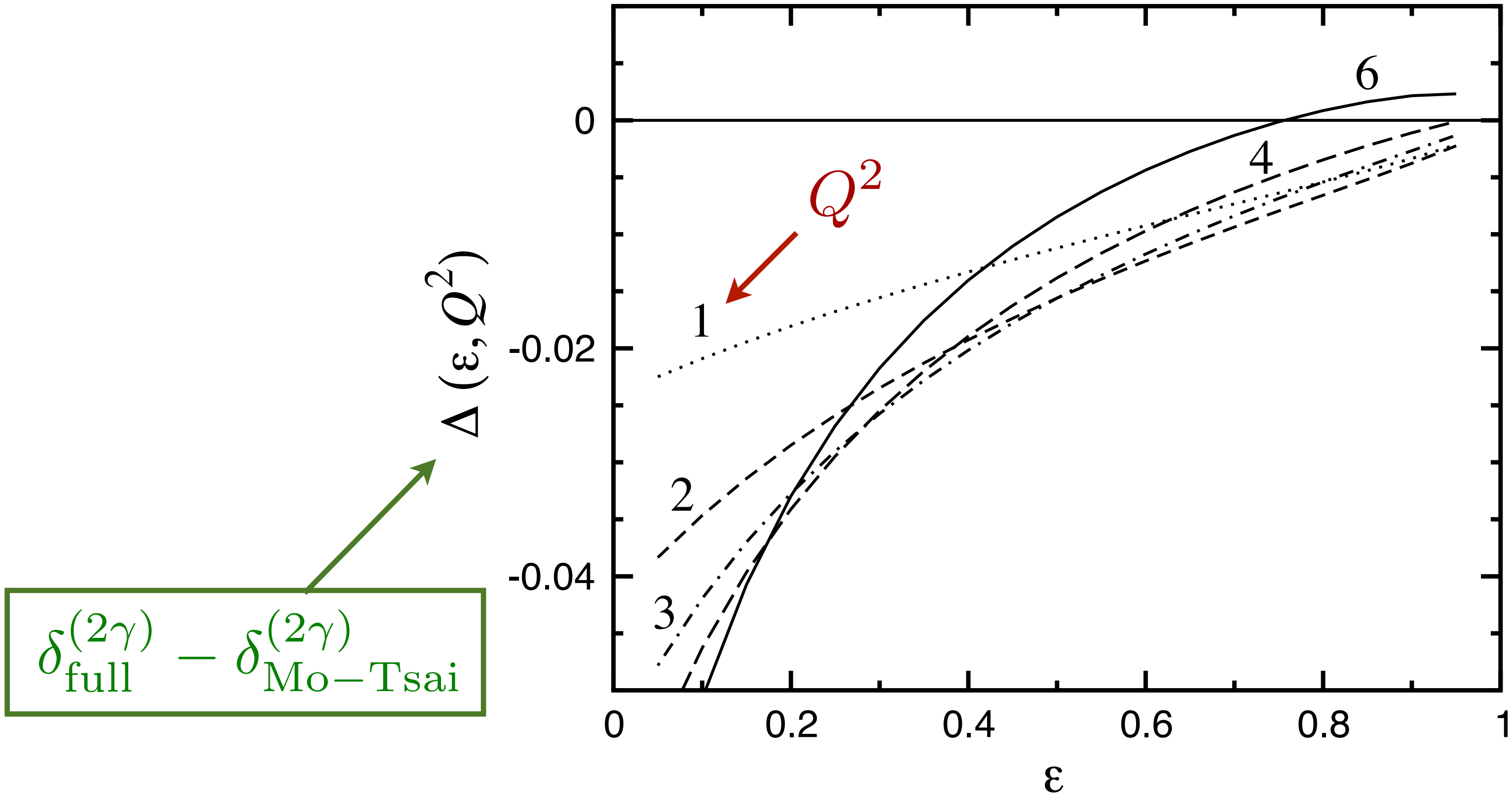
$$\begin{aligned} D(k) \;&=\; (k^2 - \lambda^2)\,((k-q)^2 - \lambda^2) \\ &\times\; ((p_1 - k)^2 - m^2)\,((p_2 + k)^2 - M^2) \end{aligned}$$

with λ an IR regulator, and e.m. current is

$$\Gamma^\mu(q) \;=\; \gamma^\mu\, F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M}\, F_2(q^2) \;\longleftarrow\; \boxed{\text{on-shell approximation}}$$

Two-photon exchange

- “exact” calculation of loop diagram (including hadron structure)



$\delta^{(2\gamma)}_{\rm full} - \delta^{(2\gamma)}_{\rm Mo-Tsai}$

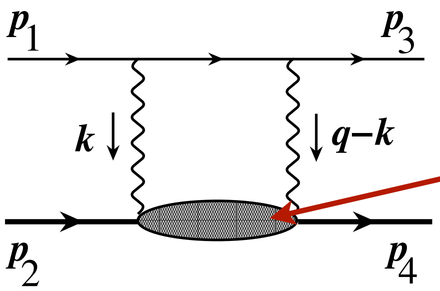
Blunden, Melnitchouk, Tjon
PRL ***91*** *(2003) 142304;*
PRC ***72*** *(2005) 034612*

→ few % magnitude, non-linear in ε, *positive* slope

→ will *reduce* Rosenbluth ratio

→ does not depend strongly on vertex form factors

Higher-mass intermediate states



$N, \ \Delta, \ P_{11}, \ S_{11}, \ S_{31}, \ldots$

- lowest mass excitation is P_{33} $\Delta(1232)$ resonance

→ relativistic $\gamma^* N\Delta$ vertex

form factor $\frac{\Lambda_\Delta^4}{\left(\Lambda_\Delta^2 - q^2\right)^2}$

$$\Gamma^{\nu\alpha}_{\gamma\Delta\to N}(p,q) \equiv iV^{\nu\alpha}_{\Delta in}(p,q) = i\frac{eF_\Delta(q^2)}{2M_\Delta^2}\Big\{ g_1\left[g^{\nu\alpha}\not{p}\not{q} - p^\nu\gamma^\alpha\not{q} - \gamma^\nu\gamma^\alpha p\cdot q + \gamma^\nu\not{p}q^\alpha \right]$$

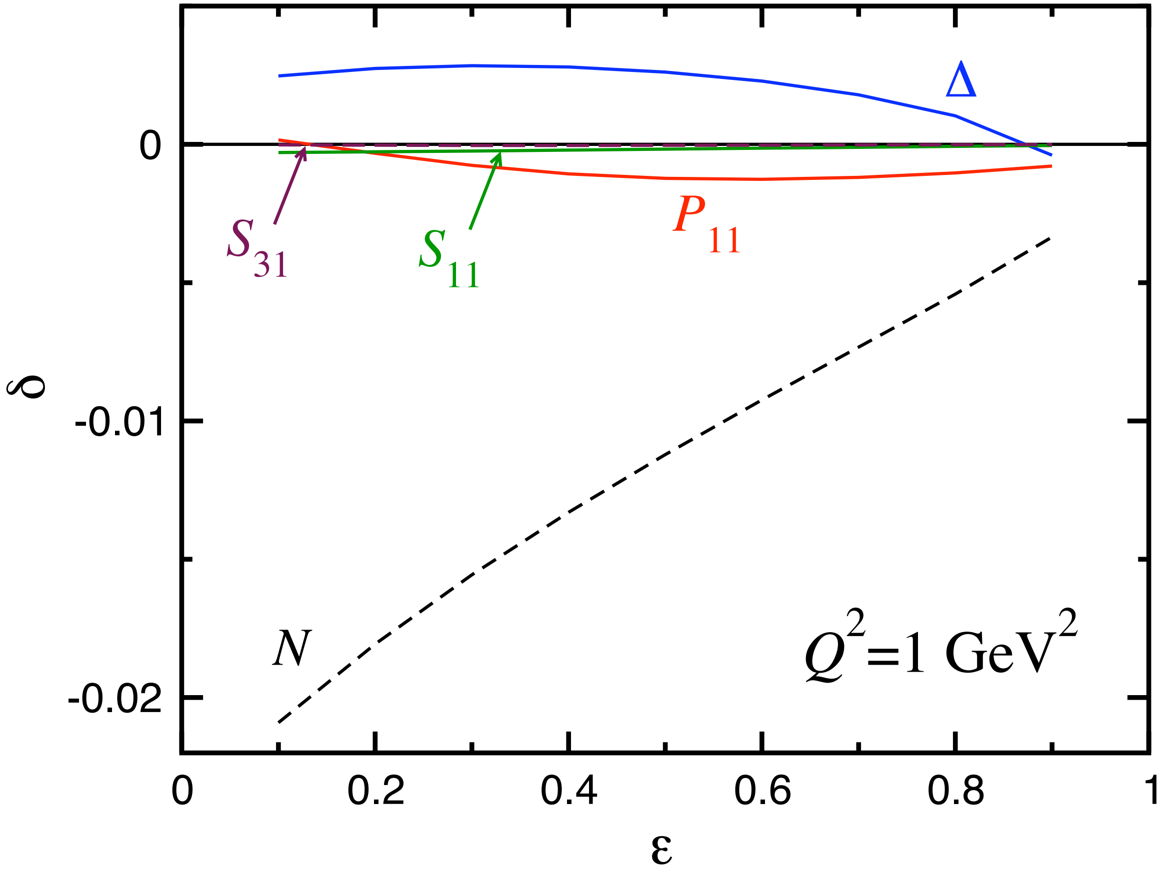
$$+g_2\left[p^\nu q^\alpha - g^{\nu\alpha}p\cdot q\right] + (g_3/M_\Delta)\left[q^2(p^\nu\gamma^\alpha - g^{\nu\alpha}\not{p}) + q^\nu(q^\alpha\not{p} - \gamma^\alpha p\cdot q)\right]\Big\}\gamma_5\, T_3$$

→ coupling constants

g_1	magnetic	➡	7
$g_2 - g_1$	electric	➡	9
g_3	Coulomb	➡	-2 ... 0

- higher-mass intermediate states

→ more model dependent, since couplings & form factors not as well known (especially at high Q^2)



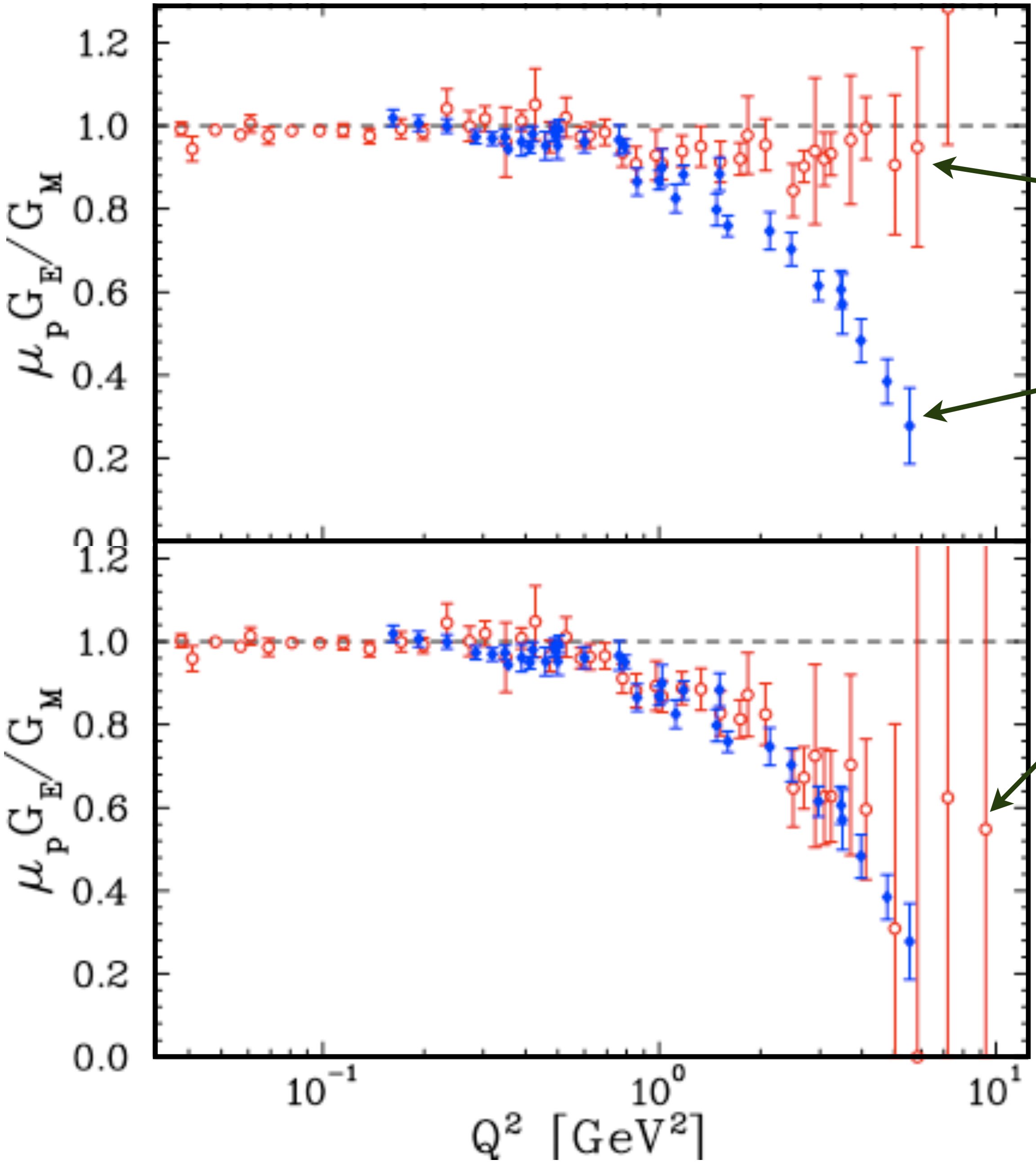
Kondratyuk, Blunden, Melnitchouk, Tjon
PRL ***95*** *(2005) 172503*

Kondratyuk, Blunden
PRC ***75*** *(2007) 038201*

→ dominant contribution from N

→ Δ partially cancels N contribution

Global analysis



LT separation

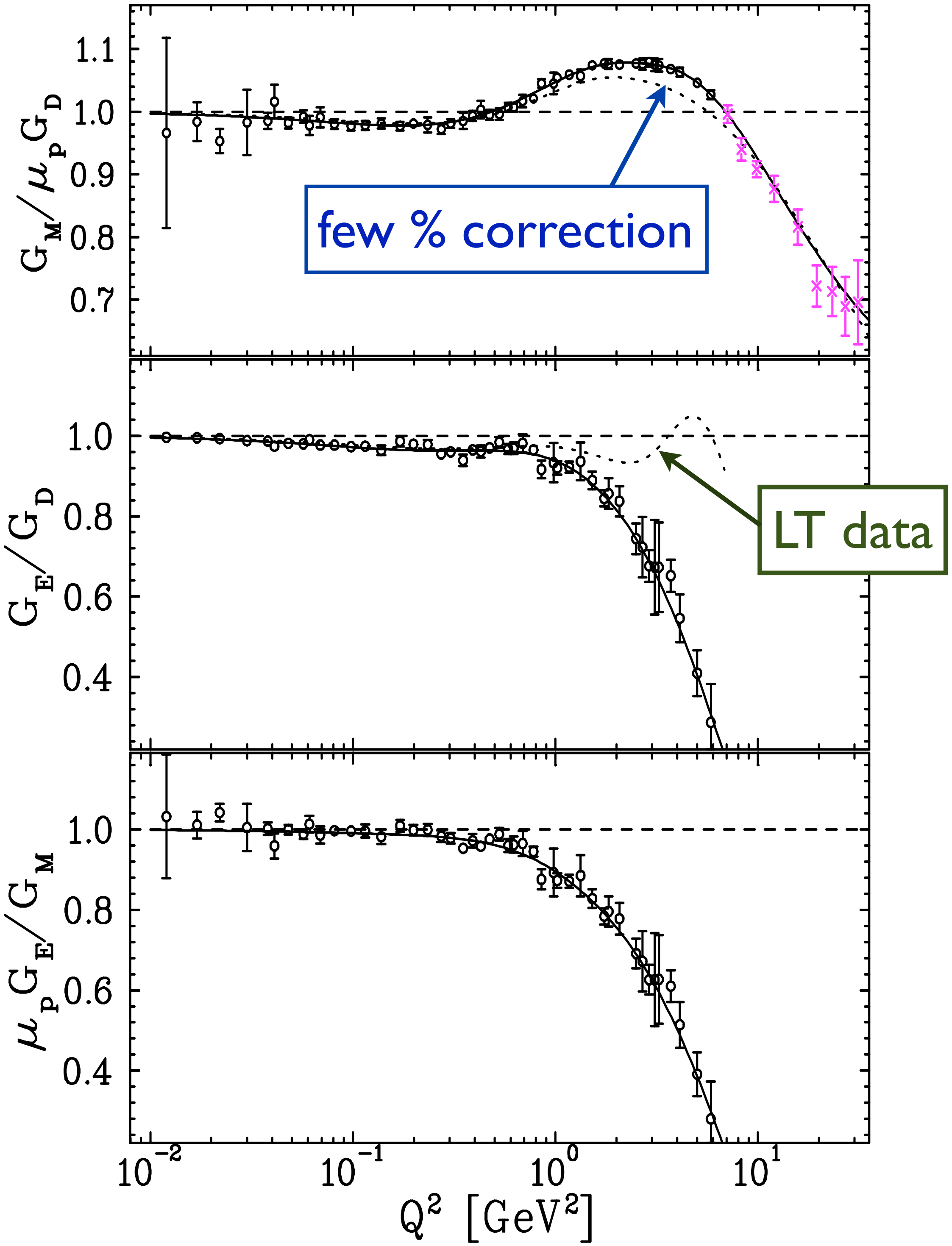
polarization transfer

with TPE correction

→ resolves discrepancy (within errors)

Arrington, Melnitchouk, Tjon
*PRC **76** (2007) 035205*

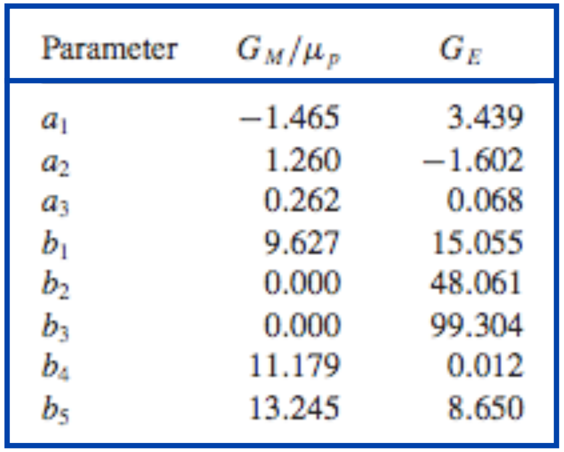
final form factor results from global analysis *including TPE corrections*



few % correction

LT data

$$\left\{G_E,\ \frac{G_M}{\mu_p}\right\} = \frac{1+\sum_{i=1}^{n} a_i\tau^i}{1+\sum_{i=1}^{n+2} b_i\tau^i}$$



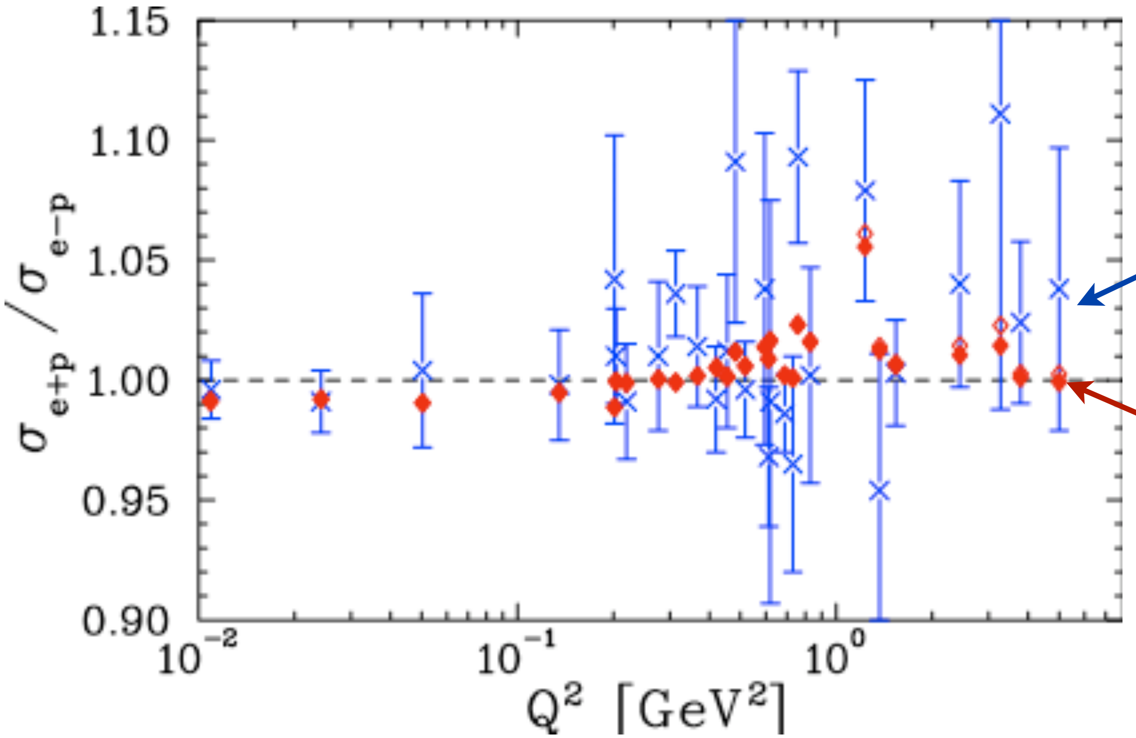
Parameter	G_M/μ_p	G_E
a_1	−1.465	3.439
a_2	1.260	−1.602
a_3	0.262	0.068
b_1	9.627	15.055
b_2	0.000	48.061
b_3	0.000	99.304
b_4	11.179	0.012
b_5	13.245	8.650

Arrington, Melnitchouk, Tjon
*PRC **76** (2007) 035205*

e^+/e^- comparison

- 1γ (2γ) exchange changes sign (invariant) under $e^+ \leftrightarrow e^-$

→ ratio of e^+p/e^-p cross sections sensitive to $\Delta(\varepsilon, Q^2)$



data at various ε

TPE calculation

$1 - 2\Delta$

→ simultaneous e^+p/e^-p measurement using tertiary e^+/e^- beam to Q^2 ~ 1-2 GeV^2

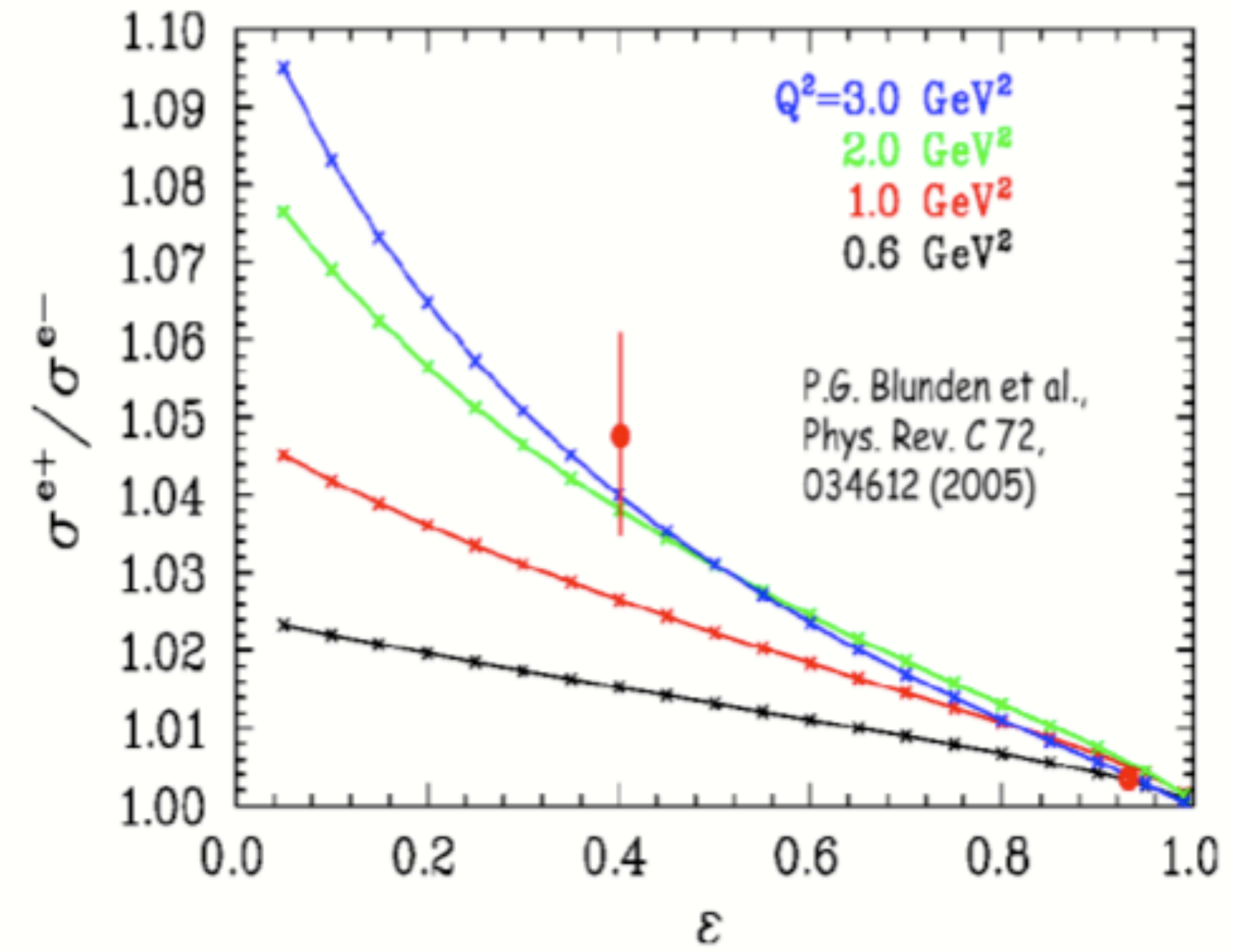
(Hall B experiment E04-116)

e^+/e^- comparison

- 1γ (2γ) exchange changes sign (invariant) under $e^+ \leftrightarrow e^-$

Very preliminary Novosibirsk data

e+-p/e- p cross section ratio



Arrington, Holt *et al*. (2010)

e^+/e^- comparison

- 1γ (2γ) exchange changes sign (invariant) under $e^+ \leftrightarrow e^-$

→ strong indication of *inadequacy* of one-photon exchange approximation in *ep* scattering

→ significant role played by *hadron structure dependent* two-photon exchange corrections

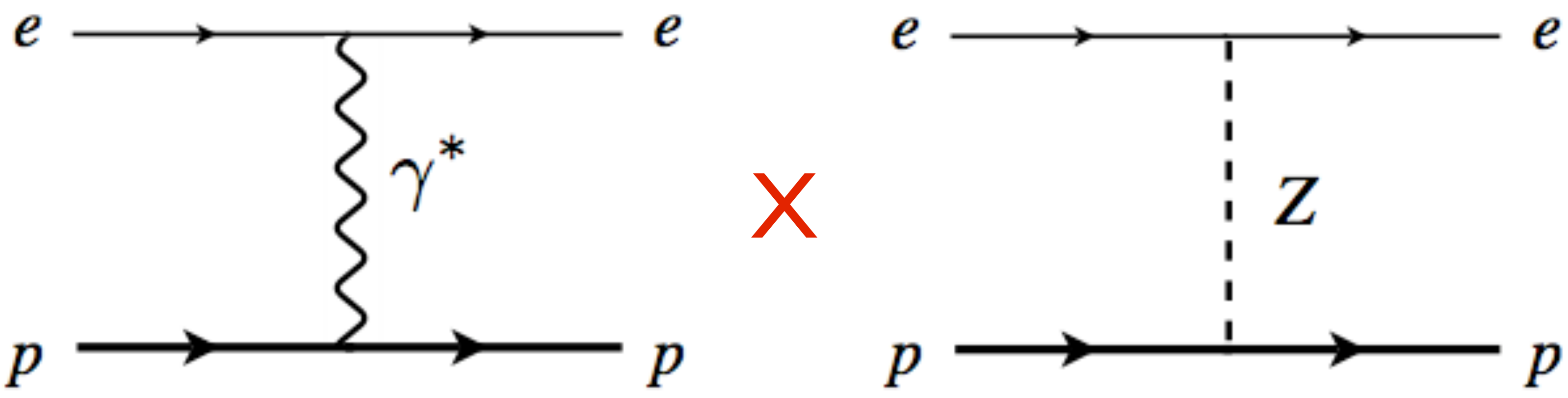
Parity-violating electron scattering

Parity-violating e scattering

- Left-right polarization asymmetry in $\vec{e}\, p \to e\ p$ scattering

$$A_{\mathrm{PV}} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4\sqrt{2}\alpha}\right)(A_V + A_A + A_s)$$

→ measure interference between e.m. and weak currents



Born (tree) level

Parity-violating e scattering

- Left-right polarization asymmetry in $\vec{e}\, p \to e\, p$ scattering

$$A_{\mathrm{PV}} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4\sqrt{2}\alpha}\right)(A_V + A_A + A_s)$$

→ measure interference between e.m. and weak currents

$$A_V = g_A^e \rho \left[(1 - 4\kappa \sin^2\theta_W) - (\varepsilon G_E^{\gamma p} G_E^{\gamma n} + \tau G_M^{\gamma p} G_M^{\gamma n})/\sigma^{\gamma p}\right]$$

radiative corrections,
including TBE

→ using relations between weak and e.m. form factors

$$G_{E,M}^{Zp} = (1 - 4\sin^2\theta_W) G_{E,M}^{\gamma p} - G_{E,M}^{\gamma n} - G_{E,M}^{s}$$

Parity-violating e scattering

- Left-right polarization asymmetry in $\vec{e}\, p \rightarrow e\, p$ scattering

$$A_{\mathrm{PV}} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4\sqrt{2}\alpha}\right)(A_V + A_A + A_s)$$

→ measure interference between e.m. and weak currents

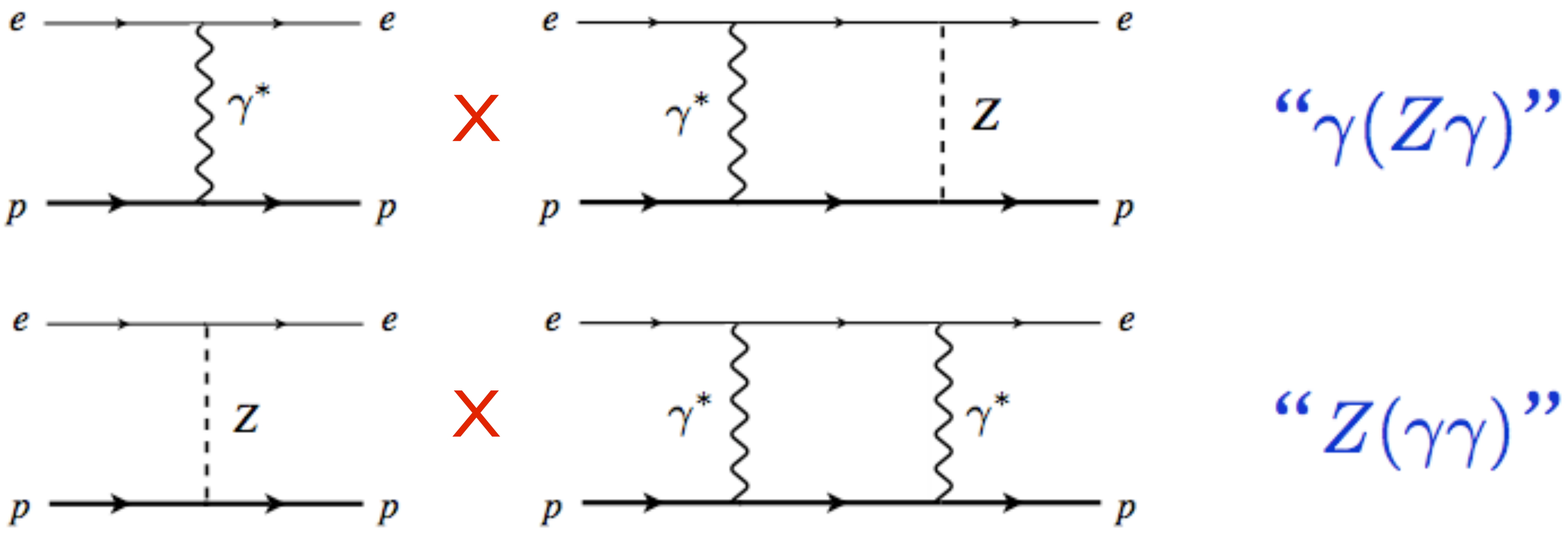
$$A_A = g_V^e \sqrt{\tau(1+\tau)(1-\varepsilon^2)}\ \widetilde{G}_A^{Zp} G_M^{\gamma p} / \sigma^{\gamma p}$$

includes axial RCs + anapole term

$$A_s = -g_A^e \rho \left(\varepsilon G_E^{\gamma p} G_E^s + \tau G_M^{\gamma p} G_M^s\right) / \sigma^{\gamma p}$$

strange electric &
magnetic form factors

Two-boson exchange corrections



- current PDG estimates computed at $Q^2 = 0$

Marciano, Sirlin (1980)
Erler, Ramsey-Musolf (2003)

- do not include hadron structure effects

Two-boson exchange corrections

- parameterize corrections to asymmetry as

$$A_{\rm PV} = (1+\delta)A^0_{\rm PV} \equiv \left(\frac{1+\delta_{Z(\gamma\gamma)}+\delta_{\gamma(Z\gamma)}}{1+\delta_{\gamma(\gamma\gamma)}}\right) A^0_{\rm PV}$$

Born asymmetry

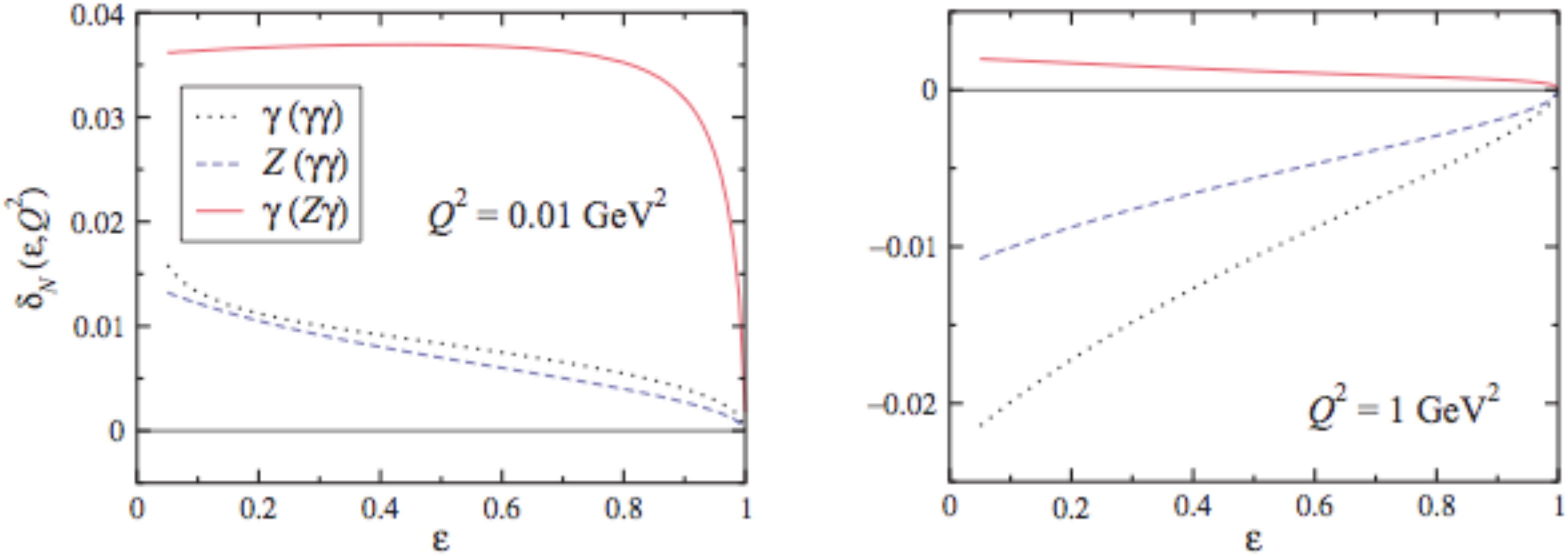
$$\begin{aligned}
\delta_{Z(\gamma\gamma)} &= \frac{2\Re e(\mathcal{M}^*_Z \mathcal{M}_{\gamma\gamma})}{2\Re e(\mathcal{M}^*_Z \mathcal{M}_{\gamma})} \\
\delta_{\gamma(Z\gamma)} &= \frac{2\Re e(\mathcal{M}^*_\gamma \mathcal{M}_{\gamma Z} + \mathcal{M}^*_\gamma \mathcal{M}_{Z\gamma})}{2\Re e(\mathcal{M}^*_Z \mathcal{M}_{\gamma})} \\
\delta_{\gamma(\gamma\gamma)} &= \frac{2\Re e(\mathcal{M}^*_\gamma \mathcal{M}_{\gamma\gamma})}{|\mathcal{M}_\gamma|^2}
\end{aligned}$$

→ total TBE correction

$$\delta \;\approx\; \delta_{Z(\gamma\gamma)} + \delta_{\gamma(Z\gamma)} - \delta_{\gamma(\gamma\gamma)}$$

Two-boson exchange corrections

- nucleon intermediate states



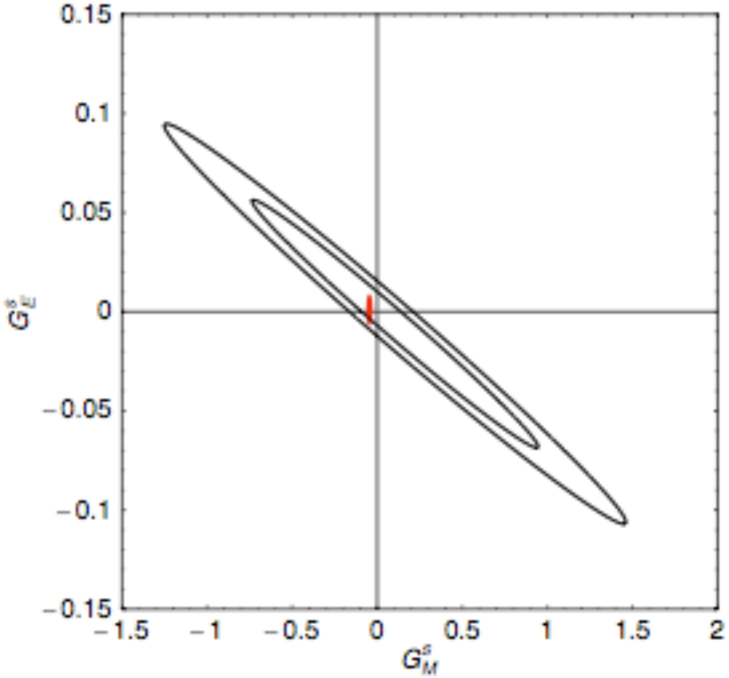
Tjon, Melnitchouk, PRL ***100*** *(2008) 082003*
Tjon, Blunden, Melnitchouk, PRC ***79*** *(2009) 055201*

→ cancellation between $Z(\gamma\gamma)$ and $\gamma(\gamma\gamma)$ corrections, especially at low Q^2

→ dominated by $\gamma(Z\gamma)$ contribution

Effects on strange form factors

- global analysis of all PVES data at $Q^2 < 0.3 \text{ GeV}^2$



$$G_E^s = 0.0025 \pm 0.0182$$

$$G_M^s = -0.011 \pm 0.254$$

$$\text{at } Q^2 = 0.1 \text{ GeV}^2$$

Young et al., *PRL* ***97*** *(2006) 102002*

- including TBE corrections:

$$\boxed{\begin{aligned} G_E^s &= 0.0023 \pm 0.0182 \\ G_M^s &= -0.020 \pm 0.254 \end{aligned}}$$

$$\text{at } Q^2 = 0.1 \text{ GeV}^2$$

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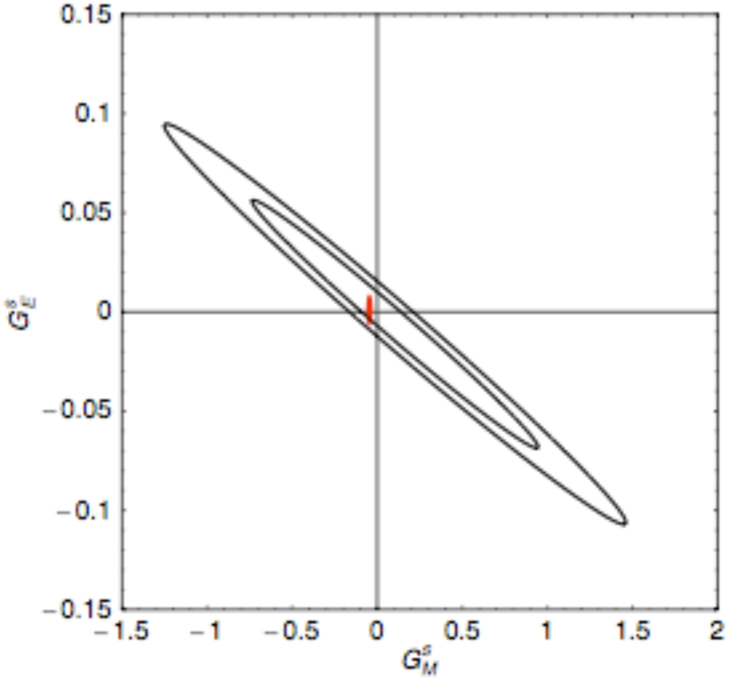

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- including TBE corrections:

$$\boxed{\begin{aligned} G_E^s &= 0.0023 \pm 0.0182 \\ G_M^s &= -0.020 \pm 0.254 \end{aligned}}$$

fixed mainly by ^{4}He data ...

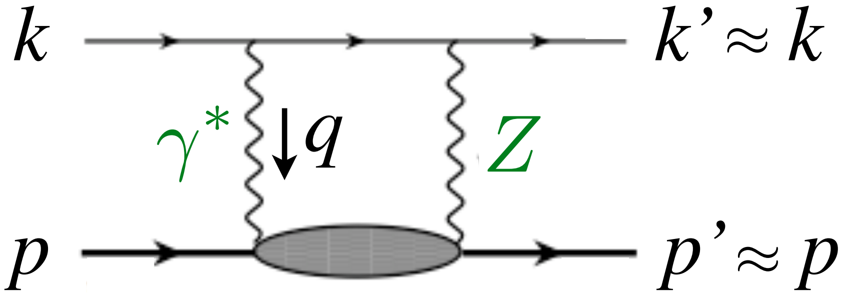
... TBE for ^{4}He not yet included

at $Q^2 = 0.1\ \mathrm{GeV}^2$

Correction to proton weak charge

- in forward limit $A_{\rm PV}$ measures weak charge of proton Q_W^p

$$A_{\rm PV} \;\to\; \frac{G_F\, Q_W^p}{4\sqrt{2}\pi\alpha}\, t$$



forward limit

$$t = (k - k')^2 \to 0$$
$$s = (k+p)^2$$
$$= M(M+2E)$$

- at <u>tree level</u> Q_W^p gives weak mixing angle

$$Q_W^p = 1 - 4\sin^2\theta_W$$

Correction to proton weak charge

- including higher order radiative corrections

$$Q_W^p \quad = \quad (1 + \Delta\rho + \Delta_e)(1 - 4\sin^2\theta_W(0) + \Delta_e')$$

"standard" electroweak vertex & other corrections

Correction to proton weak charge

- including higher order radiative corrections

$$\begin{aligned} Q_W^p &= (1 + \Delta\rho + \Delta_e)(1 - 4\sin^2\theta_W(0) + \Delta_e') \\ &\quad + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z} \longleftarrow \text{box diagrams} \\ &= 0.0713 \pm 0.0008 \end{aligned}$$

Erler et al., PRD ***72****, 073003 (2005)*

Correction to proton weak charge

- including higher order radiative corrections

$$\begin{aligned} Q_W^p &= (1+\Delta\rho+\Delta_e)(1-4\sin^2\theta_W(0)+\Delta'_e) \\ &\quad + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z} \longleftarrow \text{box diagrams} \\ &= 0.0713 \pm 0.0008 \end{aligned}$$

Erler et al., PRD ***72***, *073003 (2005)*

→ WW and ZZ box diagrams dominated by short distances, evaluated perturbatively

→ γZ box diagram sensitive to long distance physics, has two contributions $\Box_{\gamma Z} = \Box_{\gamma Z}^{A} + \Box_{\gamma Z}^{V}$

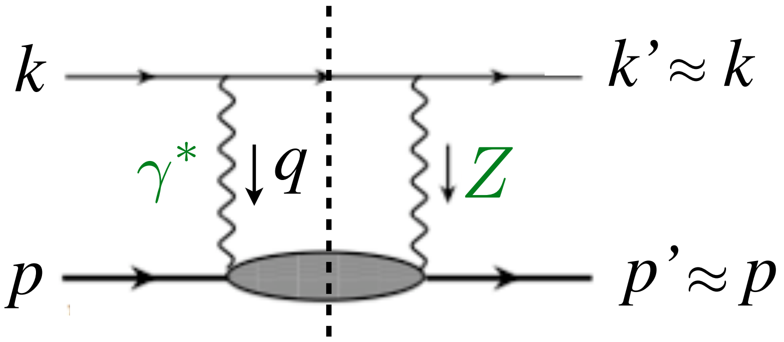
vector e – axial h (finite at E=0) ↗ $\Box_{\gamma Z}^{A}$

axial e – vector h (vanishes at E=0) ↖ $\Box_{\gamma Z}^{V}$

Vector h correction

- what is energy dependence of vector h correction $\Box^{V}_{\gamma Z}$?

→ computed in forward limit using dispersion relations



Gorchtein, Horowitz
PRL ***102****, 091806 (2009)*

- ★ $\Re e\, \Box^{V}_{\gamma Z}(E) = \frac{2E}{\pi} \int_0^\infty dE' \frac{1}{E'^2 - E^2}\, \Im m\, \Box^{V}_{\gamma Z}(E')$
- ★ integration over $E' < 0$ corresponds to crossed-box, vector h contribution symmetric under $E' \leftrightarrow -E'$
- ★ vanishes as $E \rightarrow 0$ ($e.g.$ atomic parity violation) but what about at $\mathcal{O}(1\ \mathrm{GeV})$ of *Qweak* experiment?

Vector h correction

→ imaginary part given by γZ interference structure functions

$$\Im m\,\Box^{V}_{\gamma Z}(E) = \frac{\alpha}{(s-M^2)^2}\int_{W_\pi^2}^{s} dW^2 \int_0^{Q^2_{\max}} \frac{dQ^2}{1+Q^2/M_Z^2}$$

$$\times\left(F_1^{\gamma Z} + F_2^{\gamma Z}\,\frac{s\,(Q^2_{\max}-Q^2)}{Q^2(W^2-M^2+Q^2)}\right)$$

- ☆ little direct data on interference structure functions
  (neutral currents at HERA at very small x)

- ☆ in parton model $F_2^{\gamma Z} = 2x\sum_q e_q\, g_V^q\,(q+\bar{q}) = 2xF_1^{\gamma Z}$

  → $F_2^{\gamma Z} \approx F_2^{\gamma}$ good approximation at *low x*

  → provides upper limit at *large x* $(F_2^{\gamma Z} \lesssim F_2^{\gamma})$

Vector h correction

- in resonance region use phenomenological input for F_2 , empirical SLAC fit for $R = \sigma_L/\sigma_T = (1 + 4M^2x^2/Q^2)F_2/(2xF_1) - 1$

→ for transitions to $I = 3/2$ states ($e.g.$ Δ), CVC and isospin symmetry give $F_i^{\gamma Z} = (1 + Q_W^p)\,F_i^\gamma$

→ for transitions to $I = 1/2$ states, SU(6) wave functions predict Z & γ transition couplings equal to few percent

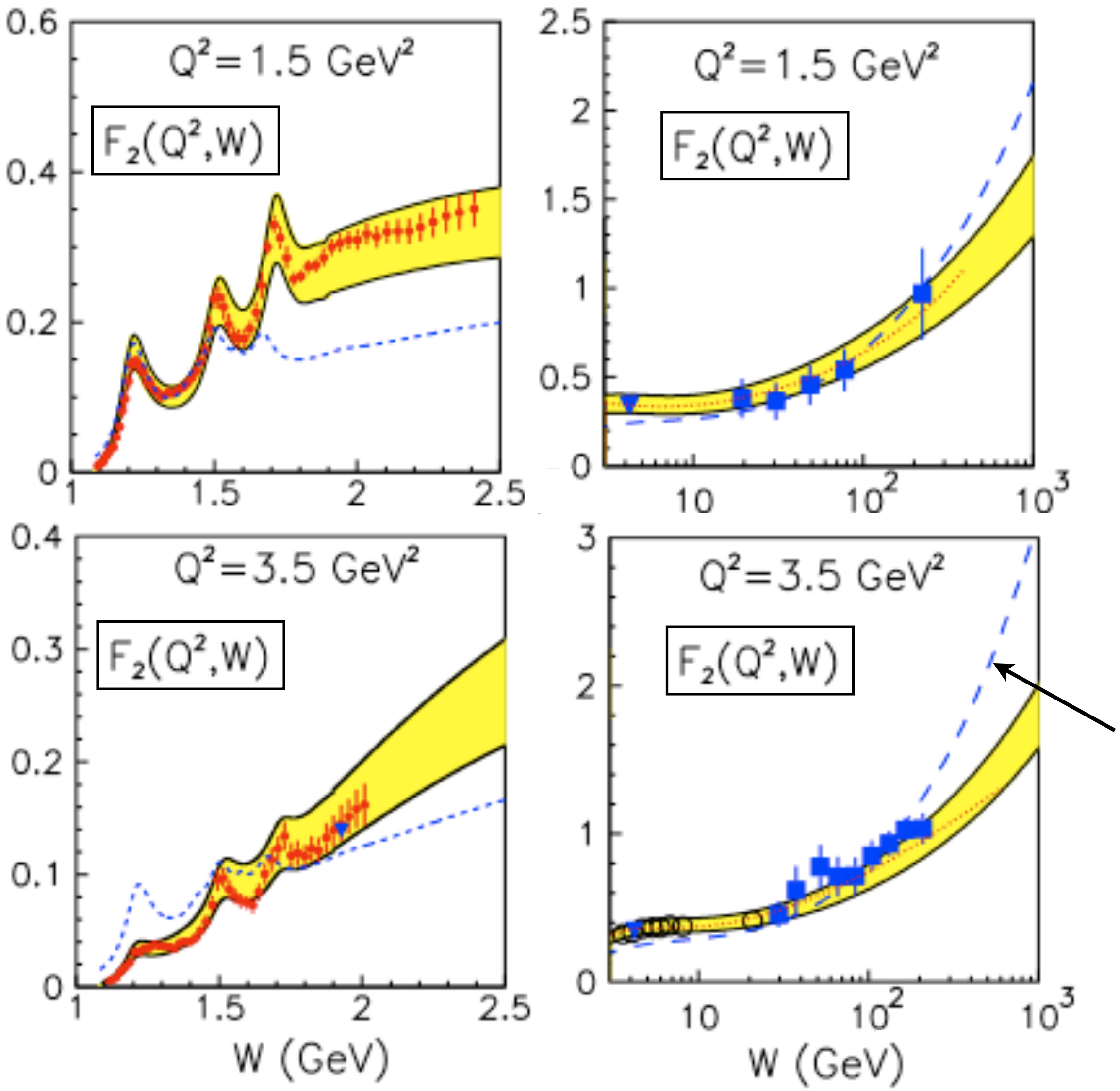
→ include contributions from prominent resonances:

$$P_{33}(1232),\ D_{13}(1520),\ F_{15}(1680),\ F_{37}(1950)$$

Vector h correction

- in resonance region use phenomenological input for F_2 , empirical SLAC fit for $R = \sigma_L/\sigma_T = (1 + 4M^2x^2/Q^2)F_2/(2xF_1) - 1$

low W



high W

GVMD model
(used as input by
Gorchtein & Horowitz)

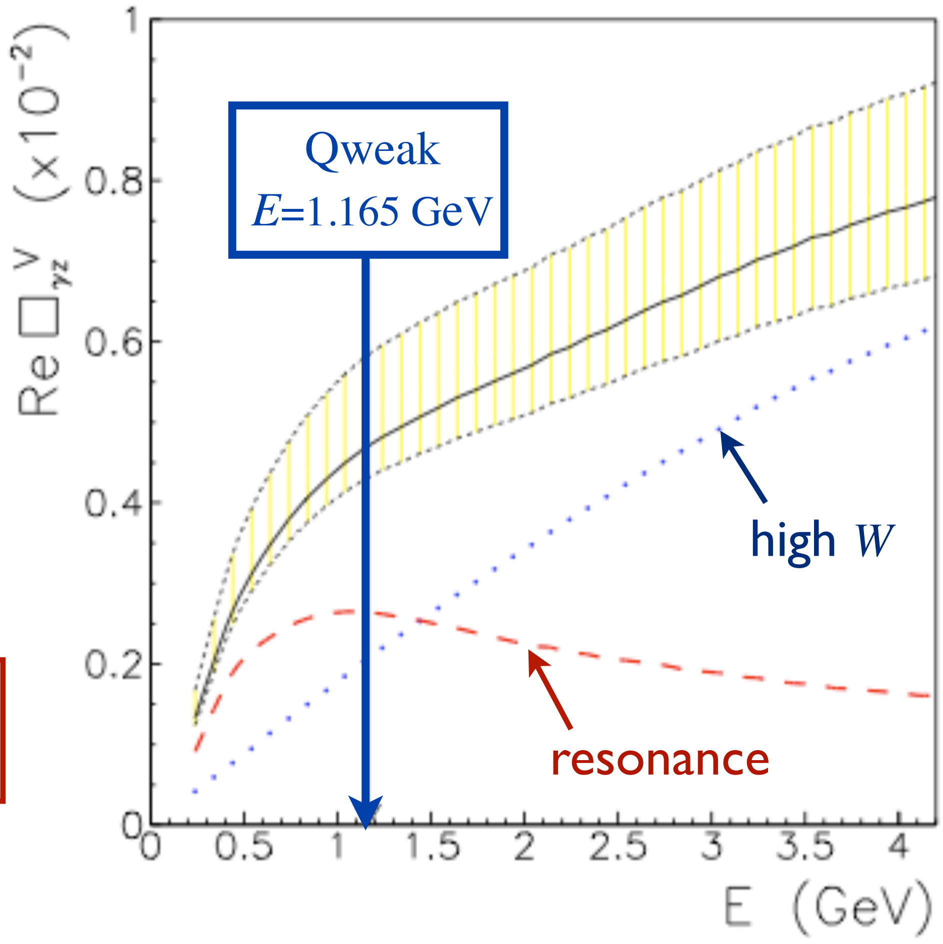
Vector h correction

- total $\Box^{V}_{\gamma Z}$ correction:

$$\Re e\, \Box^{V}_{\gamma Z} = 0.0047^{+0.0011}_{-0.0004}$$

or $6.6^{+1.5}_{-0.6}\,\%$ of uncorrected Q^p_W

$$\boxed{Q^p_W = 0.0713(8) \to 0.0760^{+0.0014}_{-0.0009}}$$



→ significant shift in central value, errors within projected experimental uncertainty $\Delta Q^p_W = \pm 0.003$

Axial h correction

- axial h correction $\Box_{\gamma Z}^{A}$ dominant γZ correction in atomic parity violation at very low (zero) energy

  → computed by Marciano & Sirlin as sum of two parts:

  - ★ low-energy part approximated by *Born* contribution (elastic intermediate state)
  - ★ high-energy part (above scale $\Lambda \sim 1\ \mathrm{GeV}$) computed in terms of scattering from *free quarks*

$$\Box_{\gamma Z}^{A} = \frac{5\alpha}{2\pi}(1 - 4\sin^2\theta_W)\left[\underbrace{\ln\frac{M_Z^2}{\Lambda^2}}_{\boxed{\text{short−distance}}} + \underbrace{C_{\gamma Z}(\Lambda)}_{\boxed{\text{long−distance}}}\right]$$

$$\approx\ 0.0028$$

Marciano, Sirlin, PRD ***29****, 75 (1984)*
Erler et al., PRD ***68****, 016006 (2003)*

Axial h correction

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- ★ repeat calculation for realistic (structure function) input

Axial h correction

→ imaginary part given by interference $F_3^{\gamma Z}$ structure function

$$\Im m \, \Box^A_{\gamma Z}(E) = \frac{\alpha}{(s-M^2)^2} \int_{W_\pi^2}^{s} dW^2 \int_0^{Q^2_{\max}} \frac{dQ^2}{1+Q^2/M_Z^2}$$

$$\times \frac{g_V^e}{2g_A^e} \left(\frac{4ME}{W^2-M^2+Q^2} - 1 \right) F_3^{\gamma Z}$$

with $g_A^e = -\frac{1}{2}, \quad g_V^e = -\frac{1}{2}(1-4\sin^2\theta_W)$

- ★ axial h contribution *anti*symmetric under $E' \leftrightarrow -E'$:

$$\Re e \, \Box^A_{\gamma Z}(E) = \frac{2}{\pi} \int_0^\infty dE' \frac{E'}{E'^2-E^2} \, \Im m \, \Box^A_{\gamma Z}(E')$$

- ★ imaginary part can only grow as $\log E' / E'$

Axial h correction

- $F_3^{\gamma Z}$ structure function
  - ☆ elastic part given by $G_M^p \, G_A^Z$
  - ☆ resonance part from parametrization of ν scattering data

    (Lalakulich-Paschos)
  - ☆ DIS part dominated by leading twist PDFs at small x

    (MSTW, CTEQ, Alekhin)

Axial h correction

- $F_3^{\gamma Z}$ structure function
  - elastic part given by $G_M^p\, G_A^Z$
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  - DIS part dominated by leading twist PDFs at small x

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- real part of $\Box_{\gamma Z}^{A}$ from dispersion relation

$$\Re e\, \Box_{\gamma Z}^{A}(0) \;=\; \underset{\underset{\text{elastic}}{\uparrow}}{0.0006} \;+\; \underset{\underset{\text{resonance}}{\uparrow}}{0.0002} \;+\; \underset{\underset{\text{DIS}}{\uparrow}}{0.0025} \;=\; 0.0033$$

→ additional $+\,0.7\%$ correction $\boxed{Q_W^p = 0.0760 \rightarrow 0.0765}$

Blunden et al. (2010)

Summary

- Two-photon exchange corrections resolve most of Rosenbluth / polarization transfer G_E^p/G_M^p discrepancy

  → striking demonstration of limitation of one-photon exchange approximation in ep scattering

- Dramatic effect of $\gamma(Z\gamma)$ corrections at forward angles on proton weak charge, $\Delta Q_W^p \sim 6–7\%$

  → would shift extracted weak angle by $\underline{\Delta \sin^2\theta_W \approx 0.0013}$

  → will be better constrained by direct measurement of $F_{1,2,3}^{\gamma Z}$ ($e.g.$ in PVDIS at JLab)

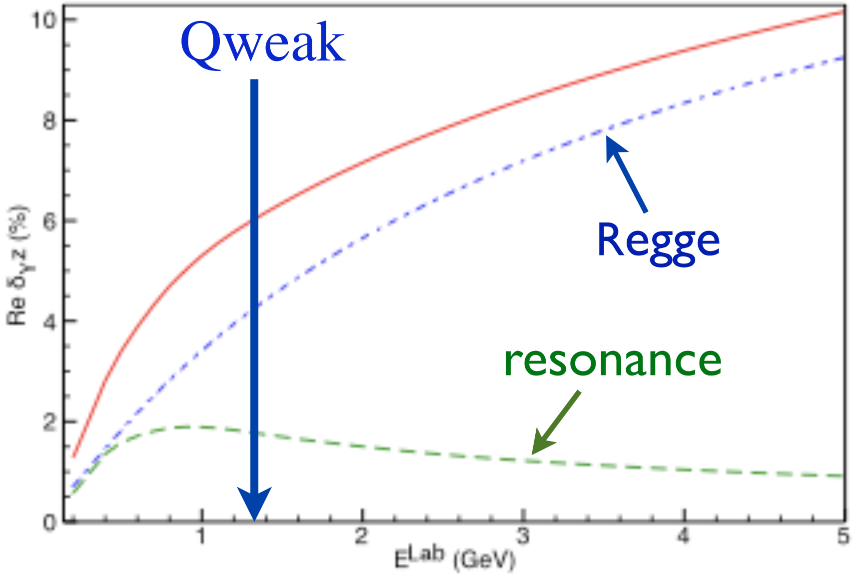
The End

- Mo-Tsai: soft γ approximation
  $\longrightarrow$ integrand most singular when $k = 0$ and $k = q$
  $\longrightarrow$ replace γ propagator which is not at pole by $1/q^2$
  $\longrightarrow$ approximate numerator $N(k) \approx N(0)$
  $\longrightarrow$ neglect all structure effects

- Maximon-Tjon: improved loop calculation
  $\longrightarrow$ exact treatment of propagators
  $\longrightarrow$ still evaluate $N(k)$ at $k = 0$
  $\longrightarrow$ first study of form factor effects
  $\longrightarrow$ additional ε dependence

- Blunden-WM-Tjon: exact loop calculation
  $\longrightarrow$ no approximation in $N(k)$ or $D(k)$
  $\longrightarrow$ include form factors

Gorchtein, Horowitz, PRL ***102****, 091806 (2009)*



$$\Re e\, \delta_{\gamma Z} = \Re e\, \Box^V_{\gamma Z}/Q^p_W \approx 6\%$$

mostly from high-W ("Regge") contribution

→ our formula for $\Im m\, \Box^V_{\gamma Z}$ factor 2 larger
(incorrect definition of parton model structure functions: "nuclear physics" *vs.* "particle physics" weak charges!)

→ GH omit factor $(1-x)$ in definition of $F_{1,2}$
(spurious ~30% enhancement)

→ GH use $Q^p_W \sim 0.05$ $cf. \sim 0.07$
(spurious ~40% enhancement)

→ numerical agreement purely coincidental!