The first moments of nucleon GPDs

Ping Wang, A. W. Thomas

1. Introduction

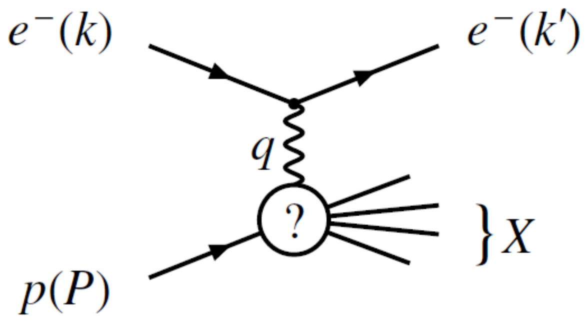
2. Conventional extrapolation method

3. Two quark models

4. Finite-Range-Regularization

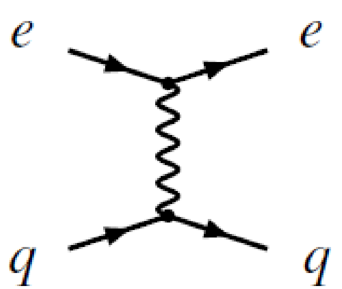
5. Summary

Introduction

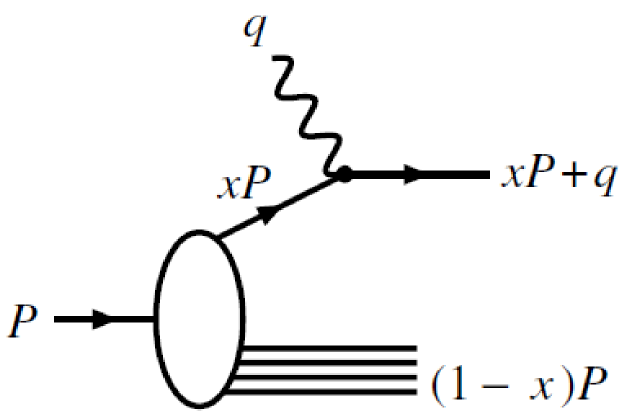


$$\frac{d^2\sigma^{\text{em}}}{dx_B\,dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[y^2\,F_1(x_B,Q^2) + \left(\frac{1-y}{x_B} - \frac{x_B y^2 M^2}{Q^2}\right)\,F_2(x_B,Q^2)\right]$$

$$x_B = Q^2/(2\,P\cdot q) \qquad y = (P\cdot q)/(P\cdot k) = 1 - E'/E$$



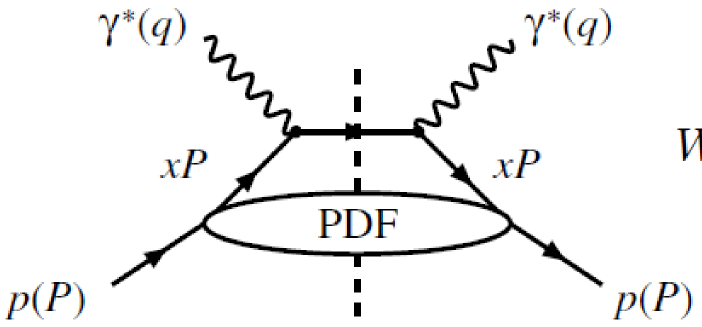
$$\frac{d^2\widehat{\sigma}^q}{dx\,dQ^2} = \frac{4\pi\,\alpha^2}{Q^4}\,\left[1+(1-y)^2\right]\,\frac{1}{2}\,e_q^2\,\delta(x-x_B)$$



$q(x)$

$$\frac{d^2\sigma^{\text{em}}}{dx_B dQ^2} = \int_0^1 dx \sum_{q,\bar{q}} \frac{d^2\widehat{\sigma}^q}{dx dQ^2}\,q(x)$$

$$F_2^{\text{em}}(x_B) = 2x_B\,F_1^{\text{em}}(x_B) = \sum_{q,\bar{q}} e_q^2\,x_B\,q(x_B)$$



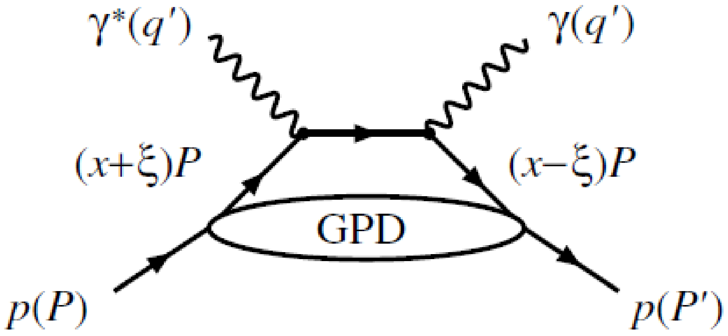
$$W_{\mu\nu}(P,q) \propto \mathrm{Im}\left[i \int d^4z\, e^{iqz} \langle p(P)|\mathrm{T}\left[J_\mu^\dagger(z) J_\nu(0)\right]|p(P)\rangle \right]$$

$$J_\mu(x) = \sum_q e_q\, \bar{\psi}_q(x)\gamma_\mu\psi_q(x)$$

The product of currents can be expanded around values of *z* which lie on the light-cone orthogonal to *P* .

$$q(x) = \frac{1}{2}\sum_{\text{spin}} \int \frac{d\lambda}{2\pi}\, e^{-i\,\lambda\, x}\, \langle p(P)|\bar{\psi}_q(z)\, \not{n}\, \psi_q(0)|p(P)\rangle$$

$$z^\mu = \lambda\, n^\mu \qquad \text{with } n\cdot P \equiv 1 \quad \text{and} \quad n^2 = 0$$



$$F_q(x,\xi,t) = \frac{1}{2}\int \frac{\mathrm{d}\lambda}{2\pi} e^{\mathrm{i}\lambda x} \left\langle P' \left| \overline{\psi}_q\left(-\frac{\lambda}{2}n\right) \not{n} \mathcal{P} e^{-\mathrm{i}g\int_{\lambda/2}^{-\lambda/2} \mathrm{d}\alpha\, n\cdot A(\alpha n)} \psi_q\left(\frac{\lambda}{2}n\right) \right| P \right\rangle$$
$$= H_q(x,\xi,t)\tfrac{1}{2}\overline{U}(P')\not{n}U(P) + E_q(x,\xi,t)\tfrac{1}{2}\overline{U}(P')\frac{\mathrm{i}\sigma^{\mu\nu}n_\mu\Delta_\nu}{2M}U(P)$$

$$n\cdot p = 1-\xi, \qquad n\cdot p' = 1+\xi$$

Theoretical research:

Parameterization method:
M. Guidal, M.V. Polyakov, A.V. Radyushkin, M. Vanderhaegen, Phys. Rev. D72(2005)054013

Quark models:
Bag model: X. Ji, W. Melnitchouk, X. Song, Phys. Rev. D56(1997)5511
Cloudy bag mode: B. Pasquini, S.Boffi, Nucl.Phys.A782(2007)86
Constituent quark model: S. Scopetta, V. Vento, Phys. Rev. D69(2004)094004
Light-front bag model: H. Choi, C.R. Ji, L.S. Kisslinger, Phys. Rev. D64(2001)093006
Betha-Salpeter approach: B.C. Tiburzi, G.A. Miller, Phys. Rev. D65(2002)074009
NJL model: H. Mineo, S.N. Yang, C.Y. Cheung, W. Bentz, Phys. Rev. C72(2005)025202
Color glass condensate model: K. Goeke, V. Guzey, M. Siddidov, Eur. Phys. J. C56(2008)203

Experiments:

ZEUS and H1: $10^{(-4)} < x < 0.02$
EIC: Up to $x = 0.3$
HERMES: $0.02 < x < 0.3$
JLab 12 GeV: $0.1 < x < 0.7$
COMPASS: $0.006 < x < 0.3$

Zero-th order moments:

$$\int_{-1}^{1} dx x^0 H^q(x,\xi,t) = F_1^q(t)$$

$$\int_{-1}^{1} dx x^0 E^q(x,\xi,t) = F_2^q(t)$$

First Moments:

$$\int_{-1}^{1} dx x H^q(x,\xi,t) = A_{2,0}^q(t) + (-2\xi)^2 C_{2,0}^q(t)$$

$$\int_{-1}^{1} dx x E^q(x,\xi,t) = B_{2,0}^q(t) - (-2\xi)^2 C_{2,0}^q(t)$$

$$i\langle p^{'}|\bar{q}\gamma_{\{\mu}\overleftrightarrow{D}_{\nu\}}|p\rangle = u(p^{'})\left[A_{2,0}^q(\Delta^2)\gamma_{\{\mu}\bar{p}_{\nu\}} - \frac{B_{2,0}^q(\Delta^2)}{2M_N}\Delta^\alpha i\sigma_{\alpha\{\mu}\bar{p}_{\nu\}} + \frac{C_{2,0}^q(\Delta^2)}{M_N}\Delta_{\{\mu}\Delta_{\nu\}}\right]u(p)$$

the bracket $\{...\}$ denote the symmetrized and traceless combination

Lattice simulation:

LHPC Collaborations (Ph. Haegler *et al*.), Phys. Rev. D77 (2008) 094502
QCDSF Collaboration (M. Gockeler *et al.*), Phys. Rev. Lett. 92 (2004) 042002
QCDSF Collaboration and UKQCD Collaboration (M. Gockeler *et al.*), Phys. Lett. B627 (2005) 113
QCDSF Collaboration and UKQCD Collaboration (D. Brommel *et al.*), Phys. Rev. Lett. 101 (2008) 122001.

Chiral perturbation theory:

M. Dorati, T. A. Gail, T. R. Hemmert, Nucl. Phys. A798 (2008) 96
M. Diehl, A. Manashov, A. Schafer, Eur.Phys.J.A31 (2007) 335

The isospin scalar and vector form factors X (X=A, B or C) are defined as

$$X_{2,0}^{u+d} = X_{2,0}^{u} + X_{2,0}^{d}$$

$$X_{2,0}^{u-d} = X_{2,0}^{u} - X_{2,0}^{d}$$

The lowest order Lagrangian is

$$\begin{aligned}\mathcal{L}^{(0)} &= \frac{1}{2}\bar{\psi}_N \left\{ i \left[\frac{a_{2,0}^v}{2} u^\dagger V_{\mu\nu}^3 \tau^3 u + \frac{a_{2,0}^v}{2} u V_{\mu\nu}^3 \tau^3 u^\dagger + \frac{\Delta a_{2,0}^v}{2} u^\dagger V_{\mu\nu}^3 \tau^3 u \gamma_5 \right.\right. \\ &- \left.\left. \frac{\Delta a_{2,0}^v}{2} u V_{\mu\nu}^3 \tau^3 u^\dagger \gamma_5 + a_{2,0}^s V_{\mu\nu}^0 \right] \gamma^{\{\mu} \overleftrightarrow{D}^{\nu\}} \right\} \psi_N,\end{aligned}$$

The $\mathcal{O}(p^1)$ part of the interaction Lagrangian is expressed as

$$\begin{aligned}\mathcal{L}^{(1)} &= \bar{\psi}_N \left\{ i\gamma^\mu D_\mu - M_0 + \frac{g_A}{2}\gamma^\mu \gamma_5 u_\mu + \left(\frac{i b_{2,0}^v}{8M_0} \left[D_\alpha, u^\dagger V_{\mu\nu}^3 \tau^3 u + u V_{\mu\nu}^3 \tau^3 u^\dagger \right] \sigma^{\alpha\{\mu} \overleftrightarrow{D}^{\nu\}} + h.c. \right) \right. \\ &+ \left. \left(\frac{i b_{2,0}^s}{4M_0} \left[\nabla_\alpha, V_{\mu\nu}^0 \right] \sigma^{\alpha\{\mu} \overleftrightarrow{D}^{\nu\}} + h.c. \right) \right\} \psi_N\end{aligned}$$

The $\mathcal{O}(p^2)$ part of the interaction can be written as

$$\begin{aligned}\mathcal{L}^{(2)} &= F_0^2 Tr \left[\nabla^{\{\mu} U^\dagger \nabla^{\nu\}} U x_\pi^0 V_{\mu\nu}^0 \right] - \frac{c_{2,0}^v}{2M_0} \bar{\psi}_N \left[D^{\{\mu}, \left[D^{\nu\}}, u^\dagger V_{\mu\nu}^3 \tau^3 u + u V_{\mu\nu}^3 \tau^3 u^\dagger \right] \right] \psi_N \\ &- \frac{c_{2,0}^s}{M_0} \bar{\psi}_N \left[D^{\{\mu}, \left[D^{\nu\}}, V_{\mu\nu}^0 \right] \right] \psi_N.\end{aligned}$$

$$J_{00}^q \equiv i\langle p'|\bar{q}\gamma_{\{0}\overleftrightarrow{D}_{0\}}|p\rangle = \frac{3\bar{P}_0}{2}\mathcal{A}_{2,0}^q(\Delta^2) + \frac{\Delta^2}{2M_N}\mathcal{C}_{2,0}^q(\Delta^2)$$

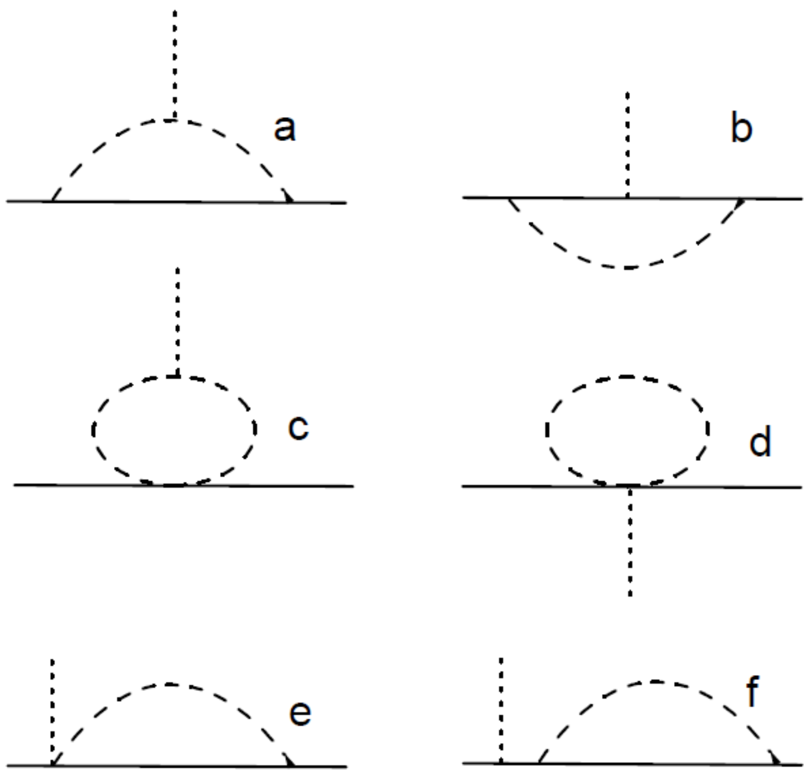
$$J_{33}^q \equiv i\langle p'|\bar{q}\gamma_{\{3}\overleftrightarrow{D}_{3\}}|p\rangle = \frac{\bar{P}_0}{2}\mathcal{A}_{2,0}^q(\Delta^2) + \frac{3\Delta^2}{2M_N}\mathcal{C}_{2,0}^q(\Delta^2)$$

$$J_{03}^q \equiv i\langle p'|\bar{q}\gamma_{\{0}\overleftrightarrow{D}_{3\}}|p\rangle = \frac{i\bar{P}_0}{2M_N}\mathcal{B}_{2,0}^q(\Delta^2)(\vec{\sigma}\times\vec{\Delta})_3$$

$$\mathcal{A}_{2,0}^q(\Delta^2) = A_{2,0}^q(\Delta^2) - \frac{\Delta^2}{8M(E+M_N)}A_{2,0}^q(\Delta^2) - \frac{\Delta^2}{4M_N^2}B_{2,0}^q(\Delta^2)$$

$$\mathcal{B}_{2,0}^q(\Delta^2) = B_{2,0}^q(\Delta^2) + A_{2,0}^q(\Delta^2) - \frac{\Delta^2}{8M(E+M_N)}B_{2,0}^q(\Delta^2)$$

$$\mathcal{C}_{2,0}^q(\Delta^2) = C_{2,0}^q(\Delta^2) + \frac{\Delta^2}{8M(E+M_N)}C_{2,0}^q(\Delta^2)$$

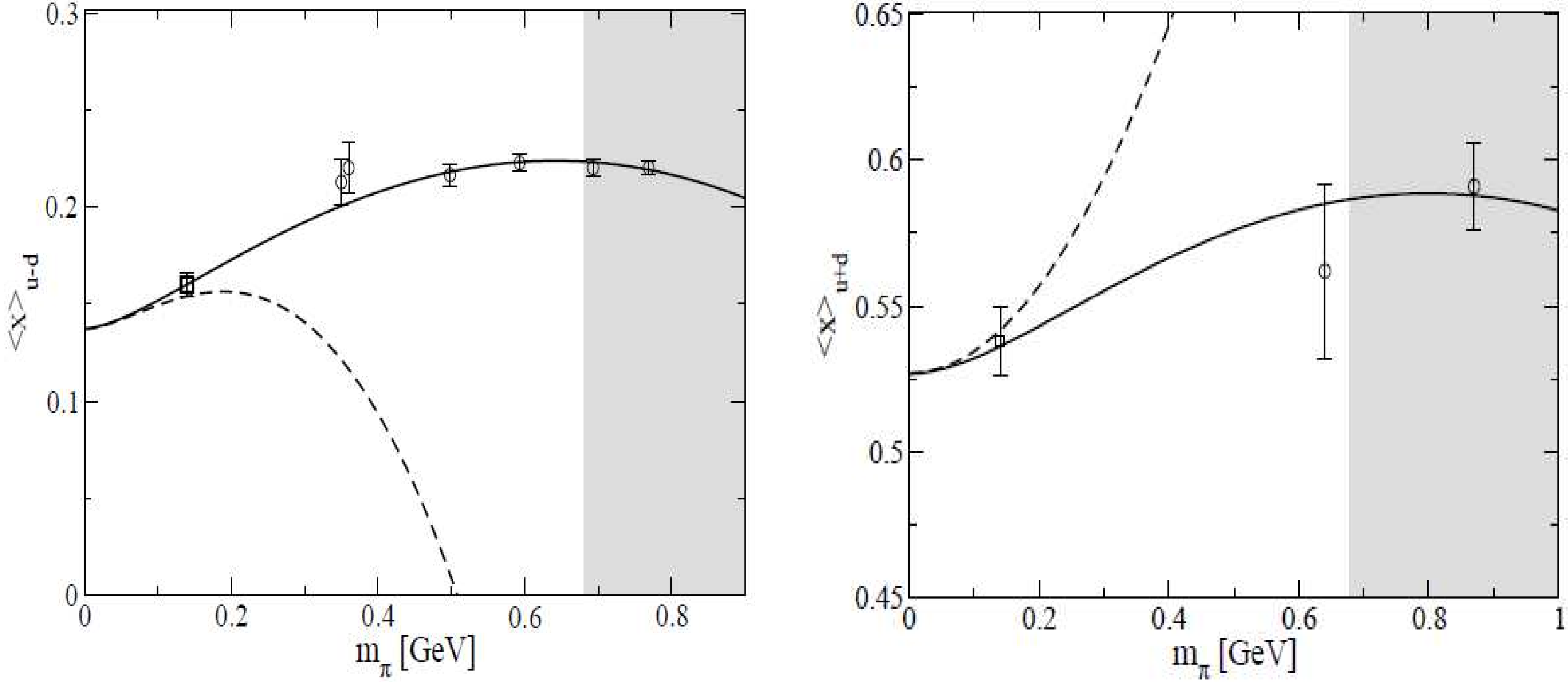

a
b
c
d
e
f

Conventional extrapolation method

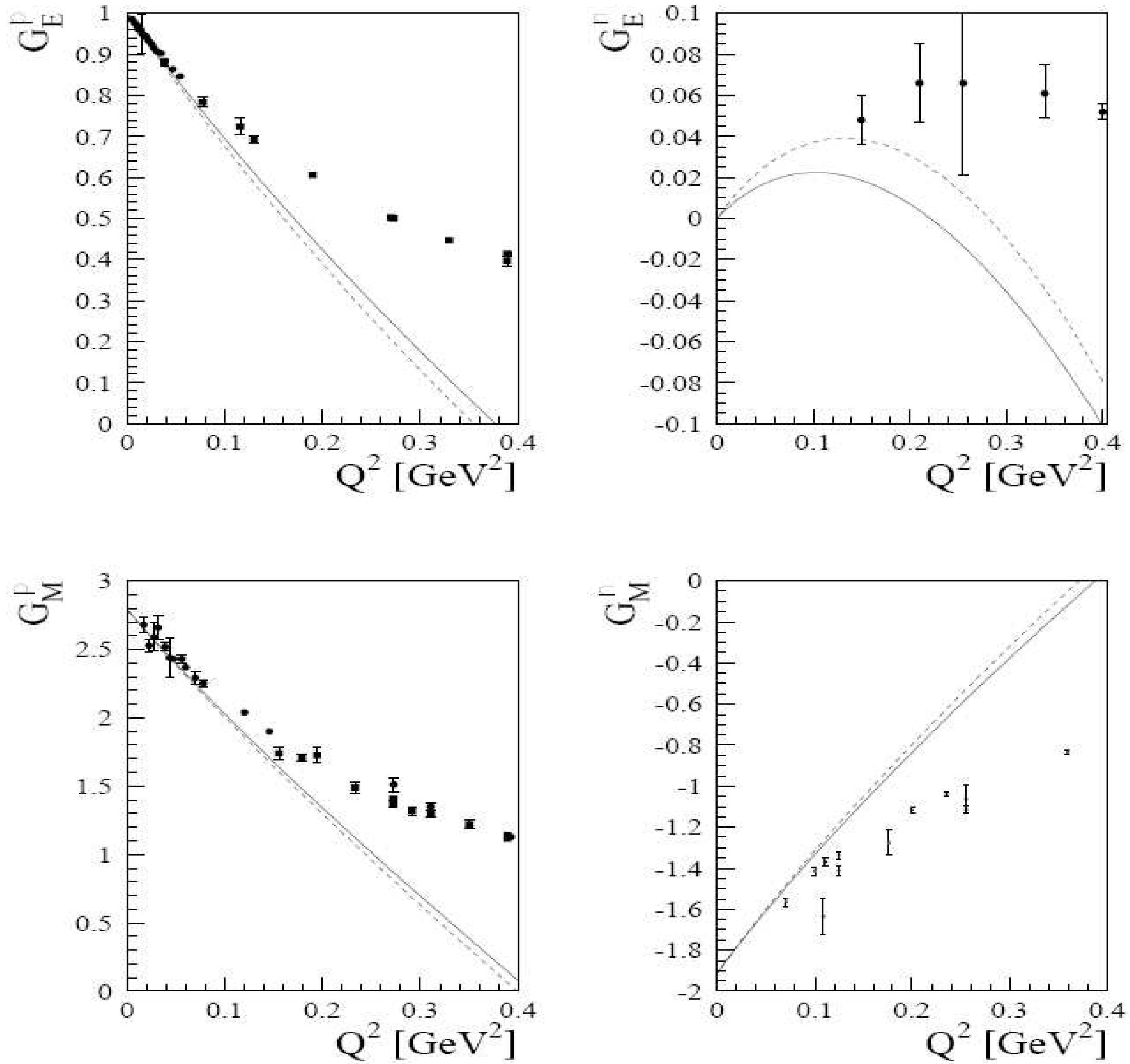
$$\begin{aligned}
A^v_{2,0}(0) &\equiv \langle x \rangle_{u-d} \\
&= a^v_{2,0} + \frac{a^v_{2,0} m_\pi^2}{(4\pi F_\pi)^2} \Bigg\{ -(3g_A^2+1)\log\frac{m_\pi^2}{\lambda^2} - 2g_A^2 + g_A^2 \frac{m_\pi^2}{M_0^2}\left(1 + 3\log\frac{m_\pi^2}{M_0^2}\right) \\
&\quad -\frac{1}{2} g_A^2 \frac{m_\pi^4}{M_0^4} \log\frac{m_\pi^2}{M_0^2} + g_A^2 \frac{m_\pi}{\sqrt{4M_0^2 - m_\pi^2}} \left(14 - 8\frac{m_\pi^2}{M_0^2} + \frac{m_\pi^4}{M_0^4}\right) \arccos\left(\frac{m_\pi}{2M_0}\right) \Bigg\} \\
&\quad + \frac{\Delta a^v_{2,0} g_A m_\pi^2}{3(4\pi F_\pi)^2} \Bigg\{ 2\frac{m_\pi^2}{M_0^2}\left(1 + 3\log\frac{m_\pi^2}{M_0^2}\right) - \frac{m_\pi^4}{M_0^4}\log\frac{m_\pi^2}{M_0^2} + \frac{2m_\pi(4M_0^2 - m_\pi^2)^{\frac{3}{2}}}{M_0^4} \arccos\left(\frac{m_\pi}{2M_0}\right) \Bigg\} \\
&\quad + 4m_\pi^2 \frac{c_8^{(r)}(\lambda)}{M_0^2} + \mathcal{O}(p^3). \qquad (28)
\end{aligned}$$

$$\begin{aligned}
A^v_{2,0}(0)\big|^{p^2}_{HBChPT} &= a^v_{2,0}\left\{1 - \frac{m_\pi^2}{(4\pi F_\pi)^2}\left(2g_A^2 + (3g_A^2+1)\log\frac{m_\pi^2}{\lambda^2}\right)\right\} + 4m_\pi^2 \frac{c_8^{(r)}(\lambda)}{M_0^2} \\
&\quad + \mathcal{O}\left(\frac{1}{16\pi^2 F_\pi^2 M_0}\right).
\end{aligned}$$

M. Dorati, T. A. Gail, T. R. Hemmert, Nucl. Phys. A798 (2008) 96



M. Dorati, T. A. Gail, T. R. Hemmert, Nucl. Phys. A798 (2008) 96



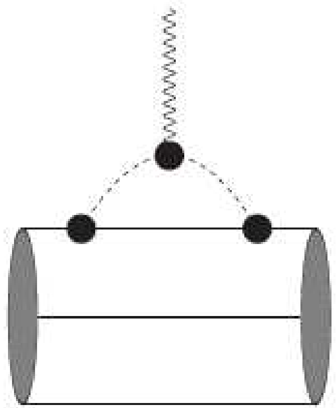
T. Fuchs, J. Gegelia, S. Scherer, J. Phys. G30 (2004) 1407

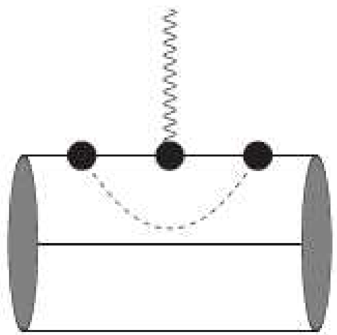
Perterbative chiral quark model

$$\mathcal{L}_{\rm inv}(x) = \bar\psi(x)[i\not\partial - \gamma^0 V(r)]\psi(x) + \frac{1}{2}[D_\mu \Phi_i(x)]^2 - S(r)\bar\psi(x)\exp\left[i\gamma^5\frac{\hat\Phi(x)}{F}\right]\psi(x),$$

$$\mathcal{L}_{\chi SB}(x) = -\bar\psi(x)\mathcal{M}\psi(x) - \frac{B}{2}{\rm Tr}\left[\hat\Phi^2(x)\,\mathcal{M}\right],$$

$$iG_\psi(x,y) \longrightarrow iG_0(x,y) \doteq u_0(\vec x)\,\bar u_0(\vec y)\,e^{-i\mathcal{E}_\alpha(x_0-y_0)}\,\theta(x_0-y_0),$$





$$G_E^N(Q^2)\Big|_{MC} = \frac{9}{400}\left(\frac{g_A}{\pi F}\right)^2 \int_0^\infty dp\, p^2 \int_{-1}^{1} dx\, (p^2 + p\sqrt{Q^2}x)$$

$$\times \mathcal{F}_{\pi NN}(p^2, Q^2, x)\, t_E^N(p^2, Q^2, x)\Big|_{MC},$$

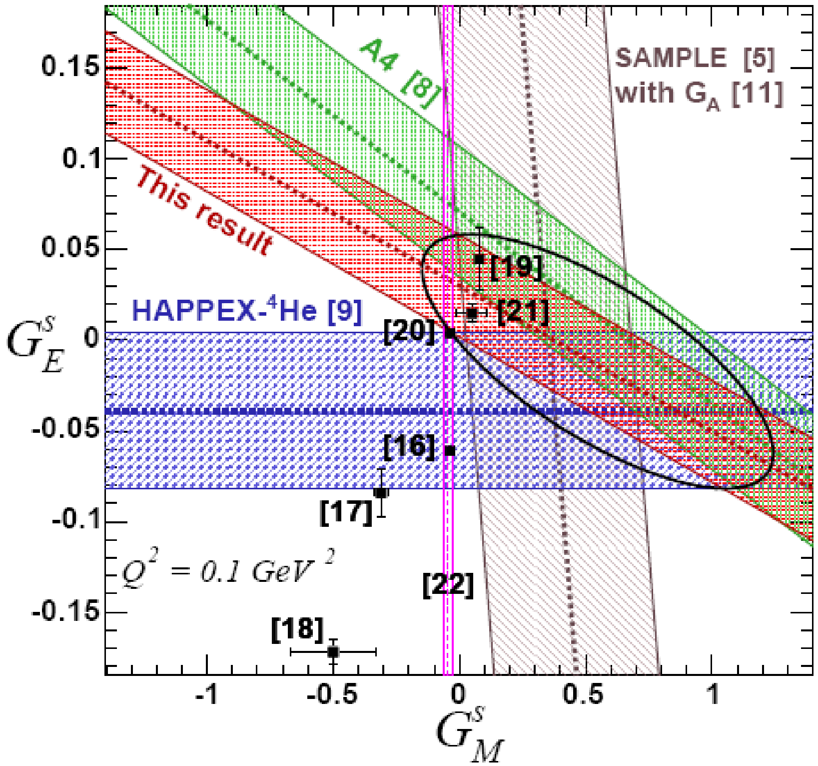
$$t_E^p(p^2)|_{VC} = \frac{1}{2} W_\pi(p^2) - W_K(p^2) + \frac{1}{6} W_\eta(p^2), \qquad W_\Phi(p^2) = \frac{1}{w_\Phi^3(p^2)}$$

$$t_E^n(p^2)|_{VC} = W_\pi(p^2) - W_K(p^2),$$

$$\mathcal{F}_{\pi NN}(p^2, Q^2, x) = F_{\pi NN}(p^2) F_{\pi NN}(p^2 + Q^2 + 2p\sqrt{Q^2}x),$$

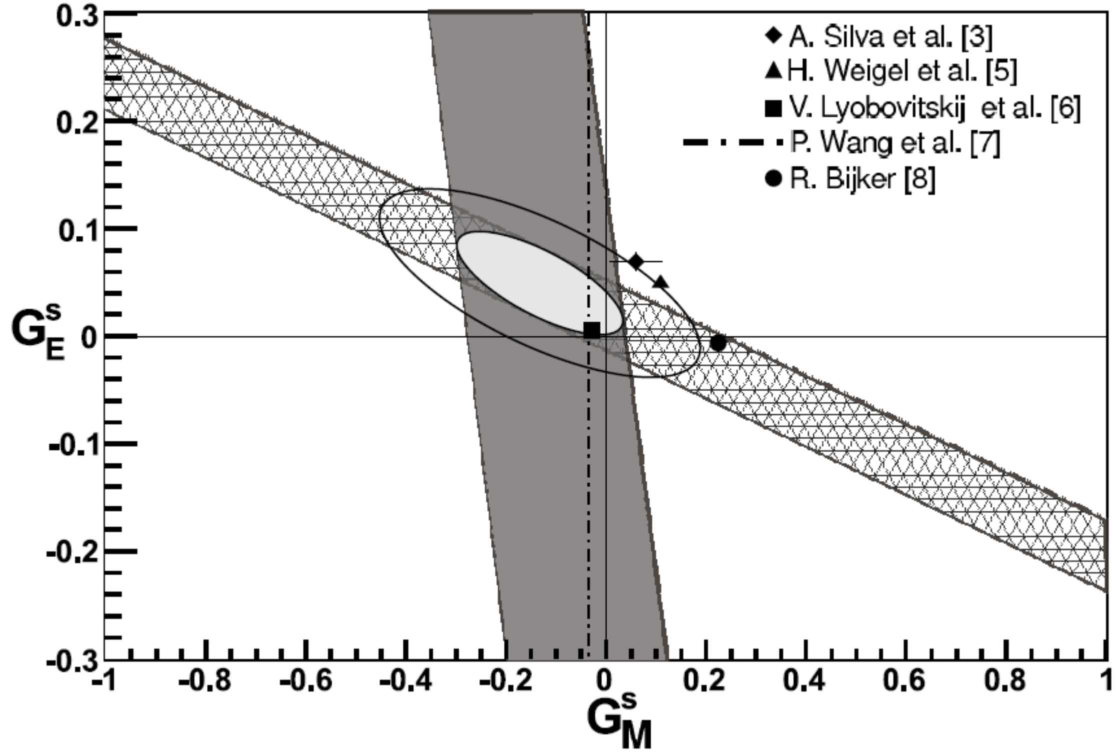
$$F_{\pi NN}(p^2) = \exp\left(-\frac{p^2R^2}{4}\right)\left\{1 + \frac{p^2R^2}{8}\left(1 - \frac{5}{3g_A}\right)\right\}$$

HAPPEX Collaboration
Phys. Lett. B635 (2006) 275



[5] D. T. Spayder *et al*, Phy. lett. B583 (2004) 79
[8] F. E. Mass *et al*, Phys. Rev. Lett. 94 (2005) 152001
[9] D. S. Armstrong *et al*, Phys. Rev. Lett. 95 (2005) 092001
[11] S. Escoffier *et al*, Nucl. Instrum. Meth. A551 (2005) 563
[20] V. Lyubovitskij, P. Wang, T. Gutsche, A. Faessler, Phys. Rev. C 66 (2002) 055204

S. Baunack *et al.*,
Phys.Rev.Lett.102(2009)151803



[3] A. Silva *et al.*, Phys. Rev. D74 (2006) 054011
[5] H.Weigel *et al.,* Phys. Lett. B353 (1995) 20
[6] V. Lyubovitskij, P. Wang, T. Gutsche, A. Faessler, Phys.Rev.C66 (2002) 055204
[7] P. Wang, D. Leinweber, A. Thomas, R. Young, Phys. Rev. C79 (2009) 065202
[8] R. Bijker, J. Phys. G32 (2006) L49

Non-local quark-meson coupling model

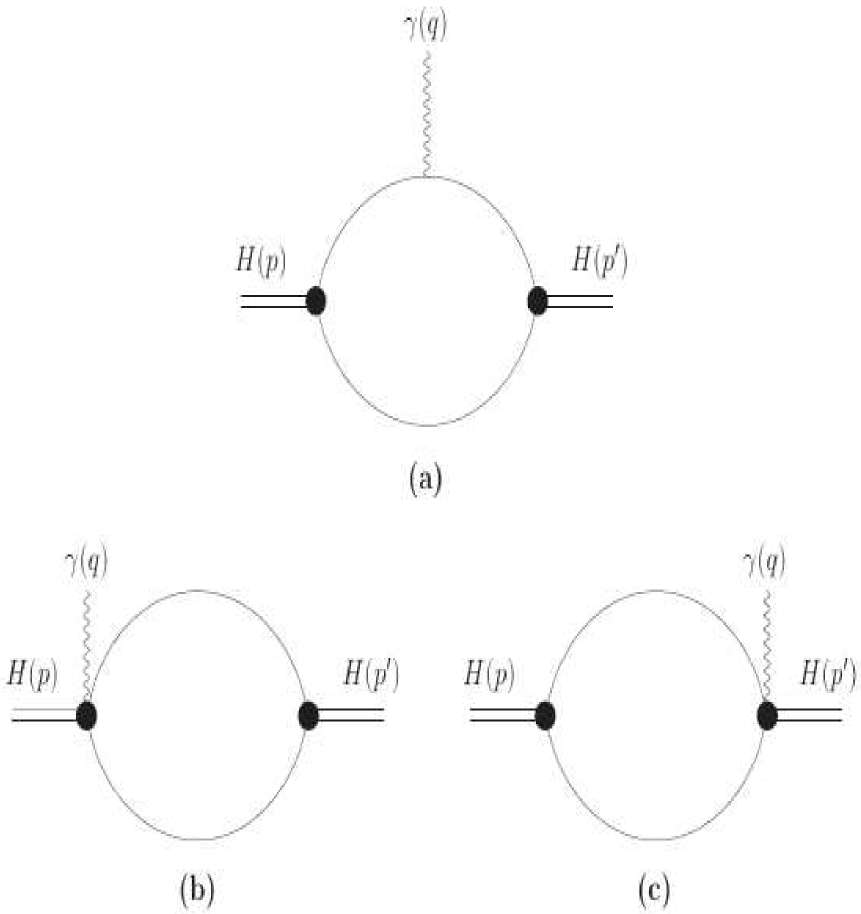
$$\mathcal{L}_{\rm int}^{\rm str}(x) = g_H H(x) \int dx_1 \int dx_2 F_H(x, x_1, x_2) \bar{q}_2(x_2) \Gamma_H \lambda_H q_1(x_1)$$

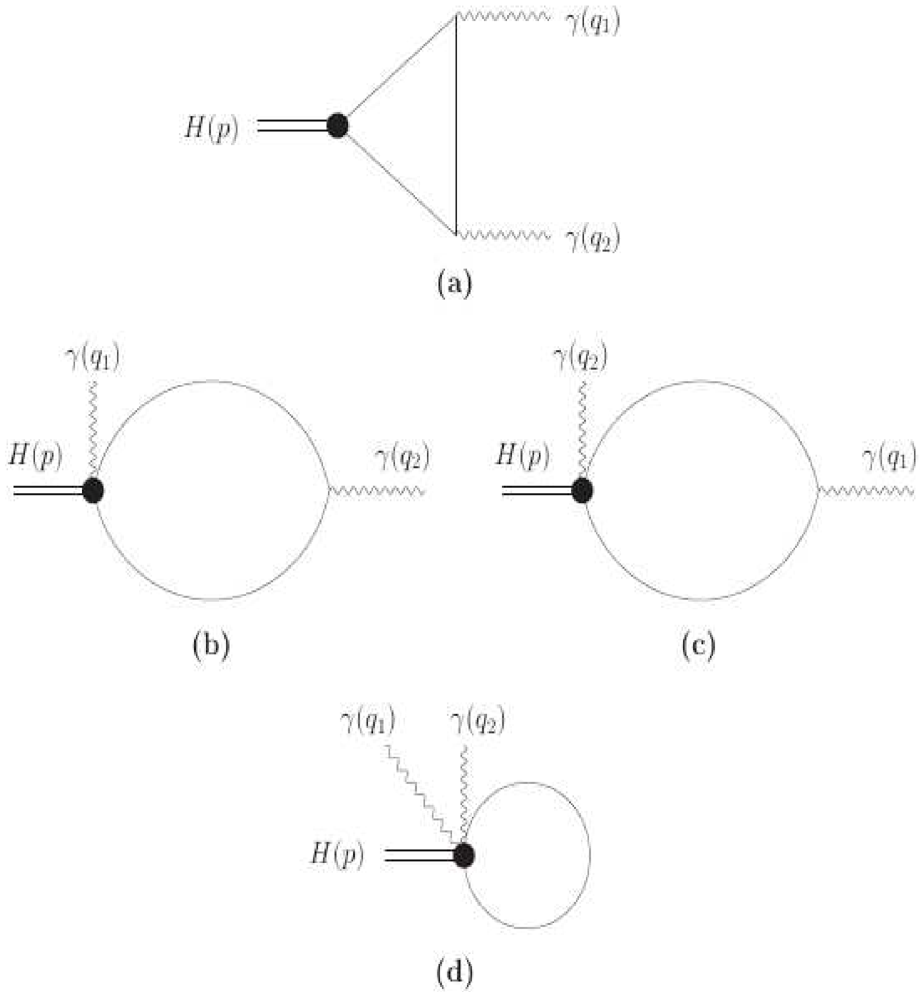
$$F_H(x, x_1, x_2) = \delta(x - w_{21} x_1 - w_{12} x_2) \Phi_H((x_1 - x_2)^2)$$

$$\begin{aligned}\mathcal{L}_{\rm int}^{\rm em(1)}(x) &= e\bar{q}(x) \not{A} Q\, q(x) \\ &+ ieA_\mu(x) \Big(H^-(x) \partial^\mu H^+(x) - H^+(x) \partial^\mu H^-(x) \Big) + e^2 A_\mu^2(x) H^-(x) H^+(x)\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{\rm int}^{\rm str+em(2)}(x) &= g_H H(x) \int dx_1 \int dx_2 F_H(x, x_1, x_2) \bar{q}_2(x_2)\, e^{ie_{q_2} I(x_2, x, P)} \\ &\times \Gamma_H \lambda_H\, e^{-ie_{q_1} I(x_1, x, P)}\, q_1(x_1),\end{aligned}$$

$$I(x_i, x, P) = \int_x^{x_i} dz_\mu A^\mu(z)$$


$\gamma(q)$
$H(p)$
$H(p')$
(a)
$\gamma(q)$
$H(p)$
$H(p')$
(b)
$\gamma(q)$
$H(p)$
$H(p')$
(c)


$\gamma(q_1)$
$H(p)$
$\gamma(q_2)$
(a)
$\gamma(q_1)$
$H(p)$
$\gamma(q_2)$
(b)
$\gamma(q_2)$
$H(p)$
$\gamma(q_1)$
(c)
$\gamma(q_1)$
$\gamma(q_2)$
$H(p)$
(d)

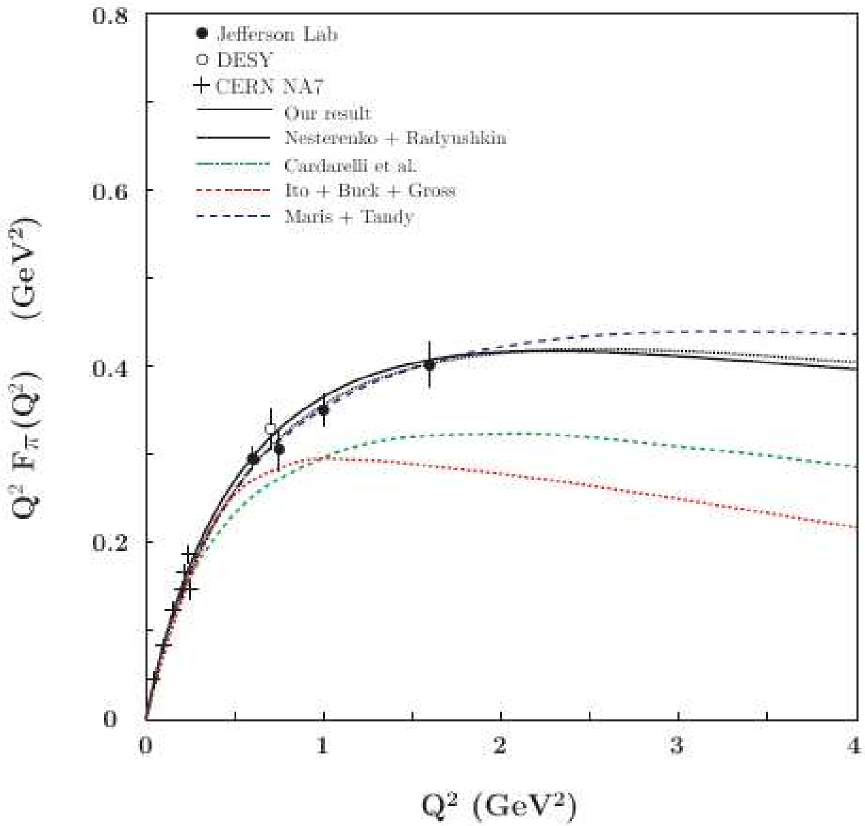
$$\Lambda^{\mu}(p,p') = \frac{q^{\mu}}{q^2}[\tilde{\Sigma}_{\pi}(p^2) - \tilde{\Sigma}_{\pi}(p'^{\,2})] \; + \; \Lambda^{\mu}_{\perp}(p,p')$$

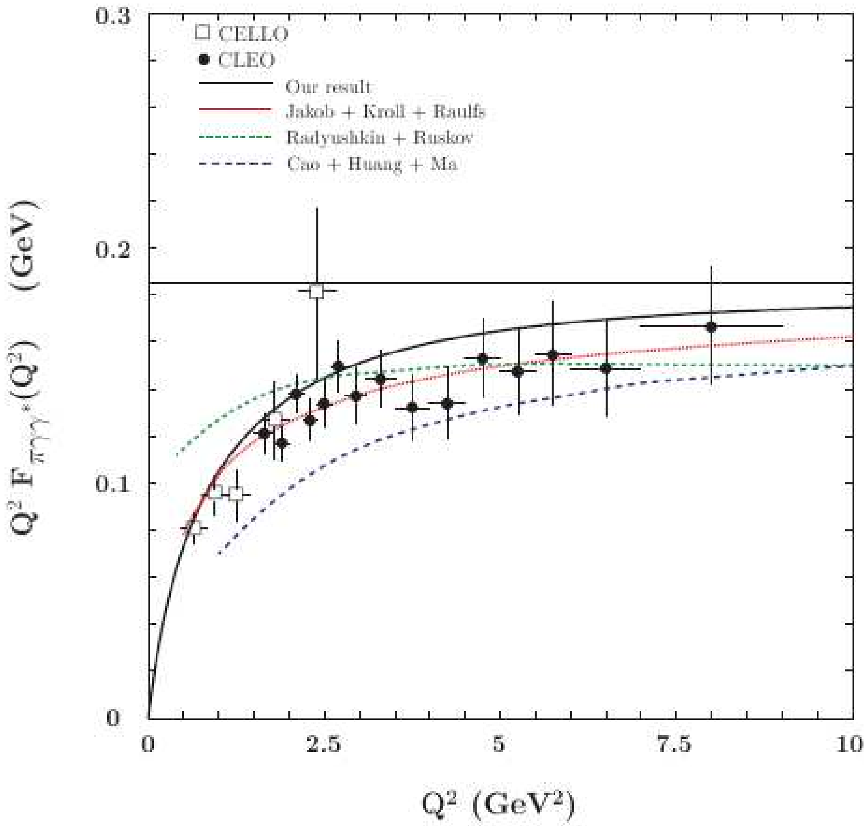
$$\Lambda^{\mu}(p,p')\Big|_{p^2=p'^{\,2}=M_{\pi}^2} = P^{\mu}\,F_{\pi}(Q^2)$$

$$\Lambda^{\mu}_{\perp}(p,p') \; = \; \Lambda^{\mu}_{\triangle_{\perp}}(p,p') \; + \; \Lambda^{\mu}_{\mathrm{bub}_{\perp}}(p,p')$$

$$\Lambda^{\mu}_{\triangle_{\perp}(\mathrm{bub}_{\perp})}(p,p') \; = \; \frac{3g^2}{4\pi^2}\, I^{\mu}_{\triangle_{\perp}(\mathrm{bub}_{\perp})}(p,p')$$

$$I^{\mu}_{\triangle_{\perp}}(p,p') = \int \frac{d^4k}{4\pi^2 i}\,\tilde{\Phi}\left(-\left[k+\frac{p}{2}\right]^2\right)\tilde{\Phi}\left(-\left[k+\frac{p'}{2}\right]^2\right)\mathrm{tr}[\gamma^5 S(k+p')\gamma^{\mu}_{\perp;q} S(k+p)\gamma^5 S(k)]$$





A. Faessler, T. Gutsche, M. Ivanov, V. Lyubovitskij, P. Wang, Phys. Rev. D 68 (2003) 014011

Finite-Range-Regularization

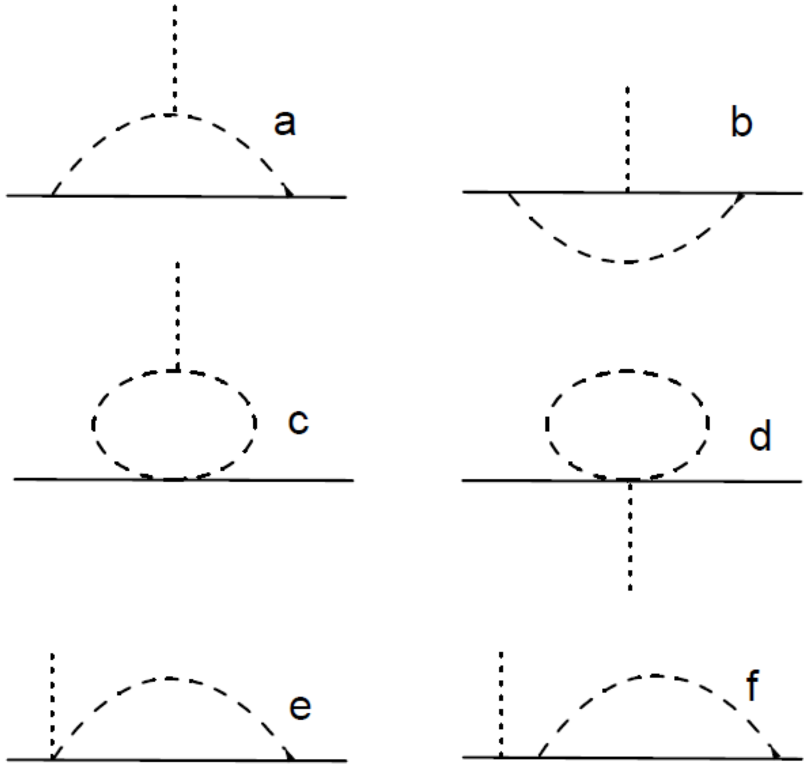
$$\mathcal{A}_{2,0}^{v,b+f} = Z a_{2,0}^v - \frac{g_A^2 a_{2,0}^v}{64\pi^3 F_\pi^2} \int d\vec{k} \frac{\vec{k}^2 u^2(\vec{k})}{\omega^3(\vec{k})} - \frac{5\mathcal{C}^2 a_{2,0}^v}{72\pi^3 F_\pi^2} \int d\vec{k} \frac{\vec{k}^2 u^2(\vec{k})}{\omega(\vec{k})(\omega(\vec{k}) + \Delta)^2}$$

$$\begin{aligned}\mathcal{B}_{2,0}^{v,b+f} &= Z b_{2,0}^v + \frac{g_A^2 b_{2,0}^v}{192\pi^3 F_\pi^2} \int d\vec{k} \frac{\vec{k}^2 u^2(\vec{k})}{\omega^3(\vec{k})} + \frac{5\mathcal{C}^2 b_{2,0}^v}{216\pi^3 F_\pi^2} \int d\vec{k} \frac{\vec{k}^2 u^2(\vec{k})}{\omega(\vec{k})(\omega(\vec{k}) + \Delta)^2} \\ &\quad + \frac{g_A \mathcal{C} b_{2,0}^v}{90\pi^3 F_\pi^2} \int d\vec{k} \frac{\vec{k}^2 u^2(\vec{k})}{\omega(\vec{k})^2(\omega(\vec{k}) + \Delta)},\end{aligned}$$

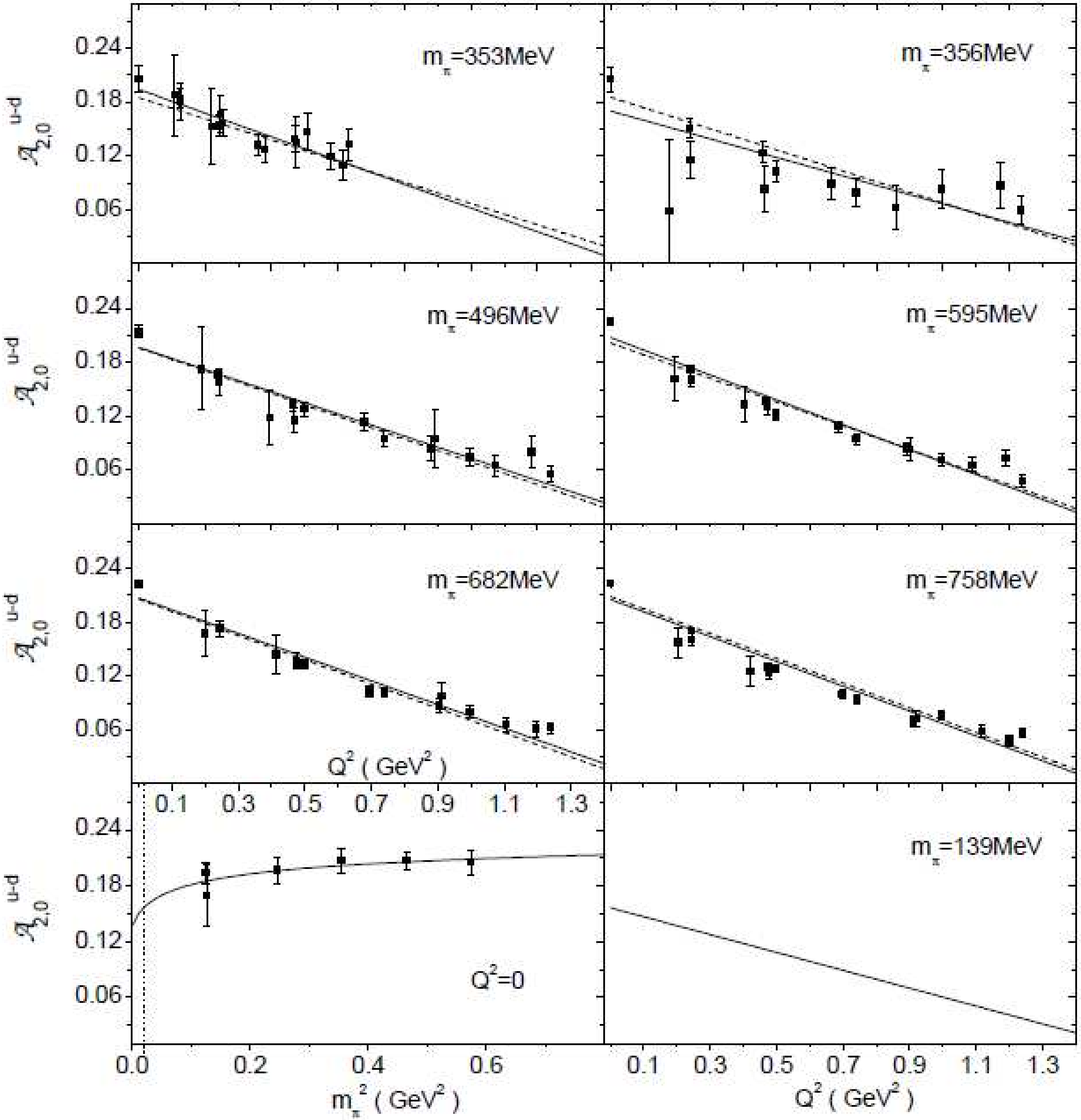
$$\mathcal{C}_{2,0}^{v,b+f} = Z c_{2,0}^v - \frac{g_A^2 c_{2,0}^v}{64\pi^3 F_\pi^2} \int d\vec{k} \frac{\vec{k}^2 u^2(\vec{k})}{\omega^3(\vec{k})} - \frac{5\mathcal{C}^2 c_{2,0}^v}{72\pi^3 F_\pi^2} \int d\vec{k} \frac{\vec{k}^2 u^2(\vec{k})}{\omega(\vec{k})(\omega(\vec{k}) + \Delta)^2},$$

$$Z = 1 - \frac{3 g_A^2}{64\pi^3 F_\pi^2} \int d\vec{k} \frac{\vec{k}^2 \vec{k}^2 u^2(\vec{k})}{\omega^3(\vec{k})} - \frac{g_A^2}{24\pi^3 F_\pi^2} \int d\vec{k} \frac{\vec{k}^2 \vec{k}^2 u^2(\vec{k})}{\omega^3(\vec{k})}$$

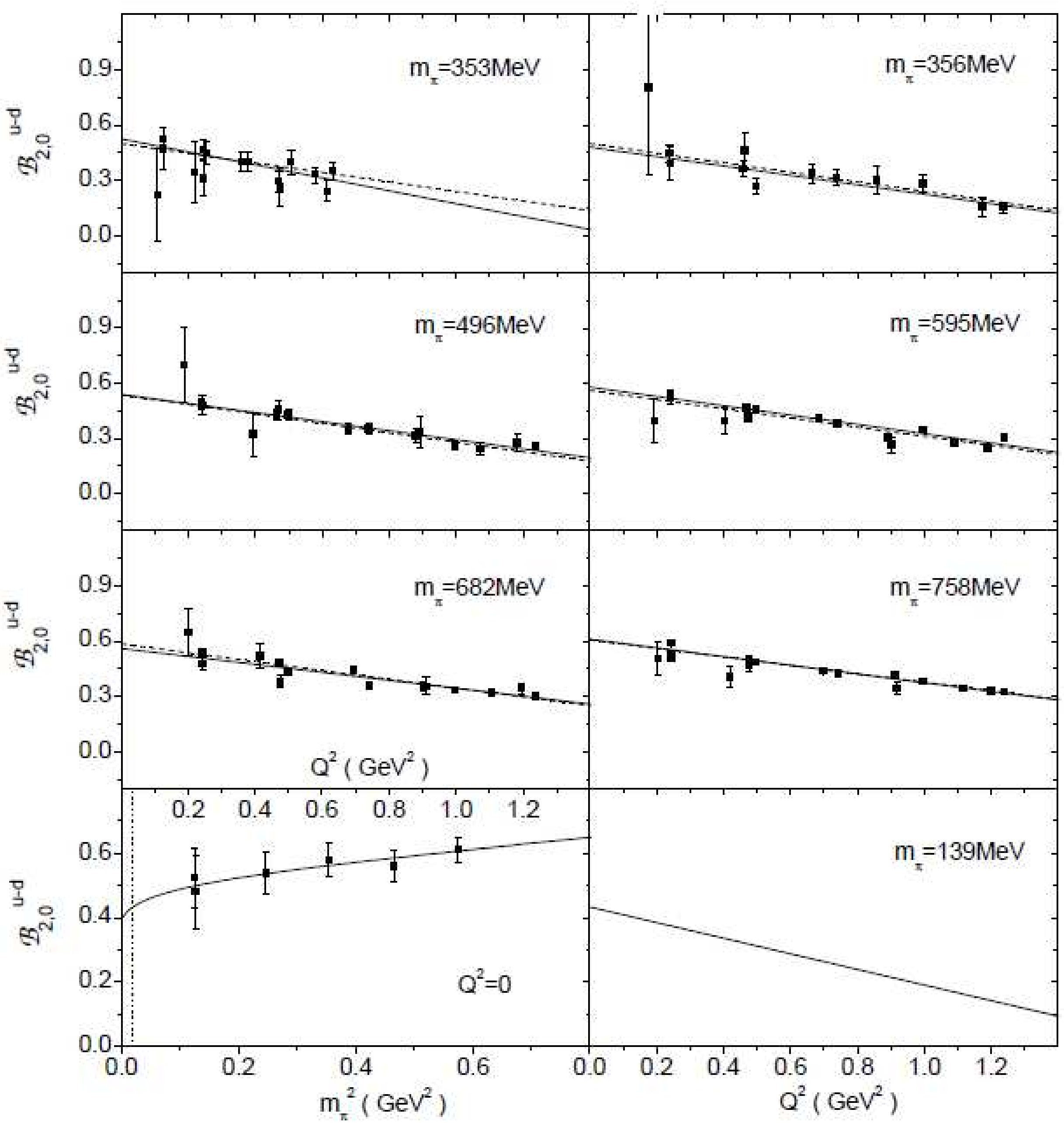
$$u(k) = \frac{1}{(1 + k^2/\Lambda^2)^2} \qquad \Lambda = 0.8 \text{ GeV}$$



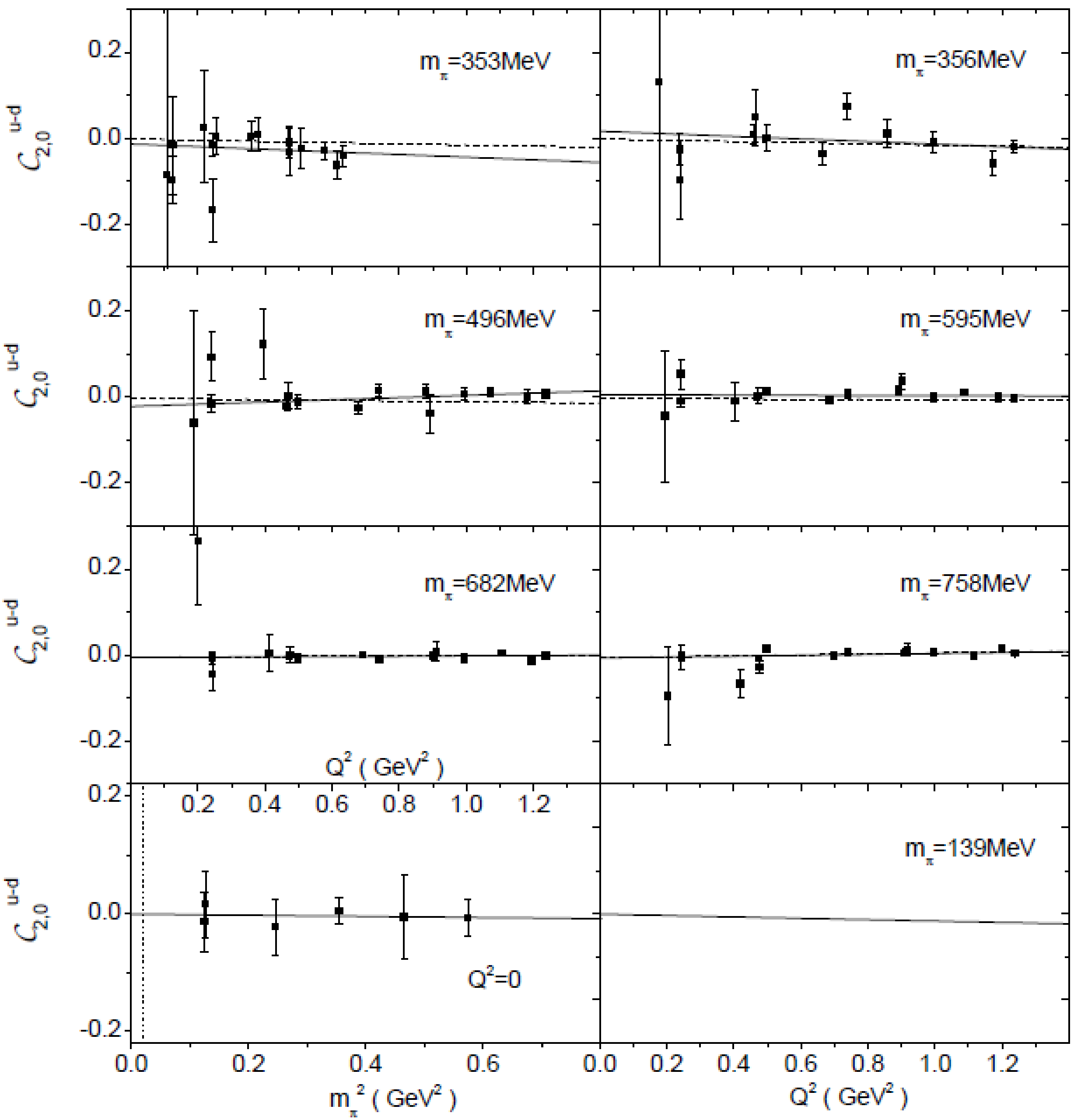
$$\mathcal{G}^v_{2,0}(Q^2, m_\pi^2) = Z(g^v_{2,0} + g^v_\pi m_\pi^2 + (g^v_t + g^v_{\pi,t} m_\pi^2)Q^2) + \sum_{i=a}^{f} \mathcal{G}^{v,i}_{2,0}$$



The lattice data is from LHPC Collaborations (Ph.Haeler *et al.*), Phys.Rev.D77 (2008) 094502



m_π=353MeV
m_π=356MeV
m_π=496MeV
m_π=595MeV
m_π=682MeV
m_π=758MeV
m_π=139MeV
Q²=0
B_{2,0}^{u-d}
Q² (GeV²)
m_π² (GeV²)


m_π=353MeV
m_π=356MeV
m_π=496MeV
m_π=595MeV
m_π=682MeV
m_π=758MeV
m_π=139MeV
Q²=0
$C_{2,0}^{u-d}$
Q² (GeV²)
m_π² (GeV²)

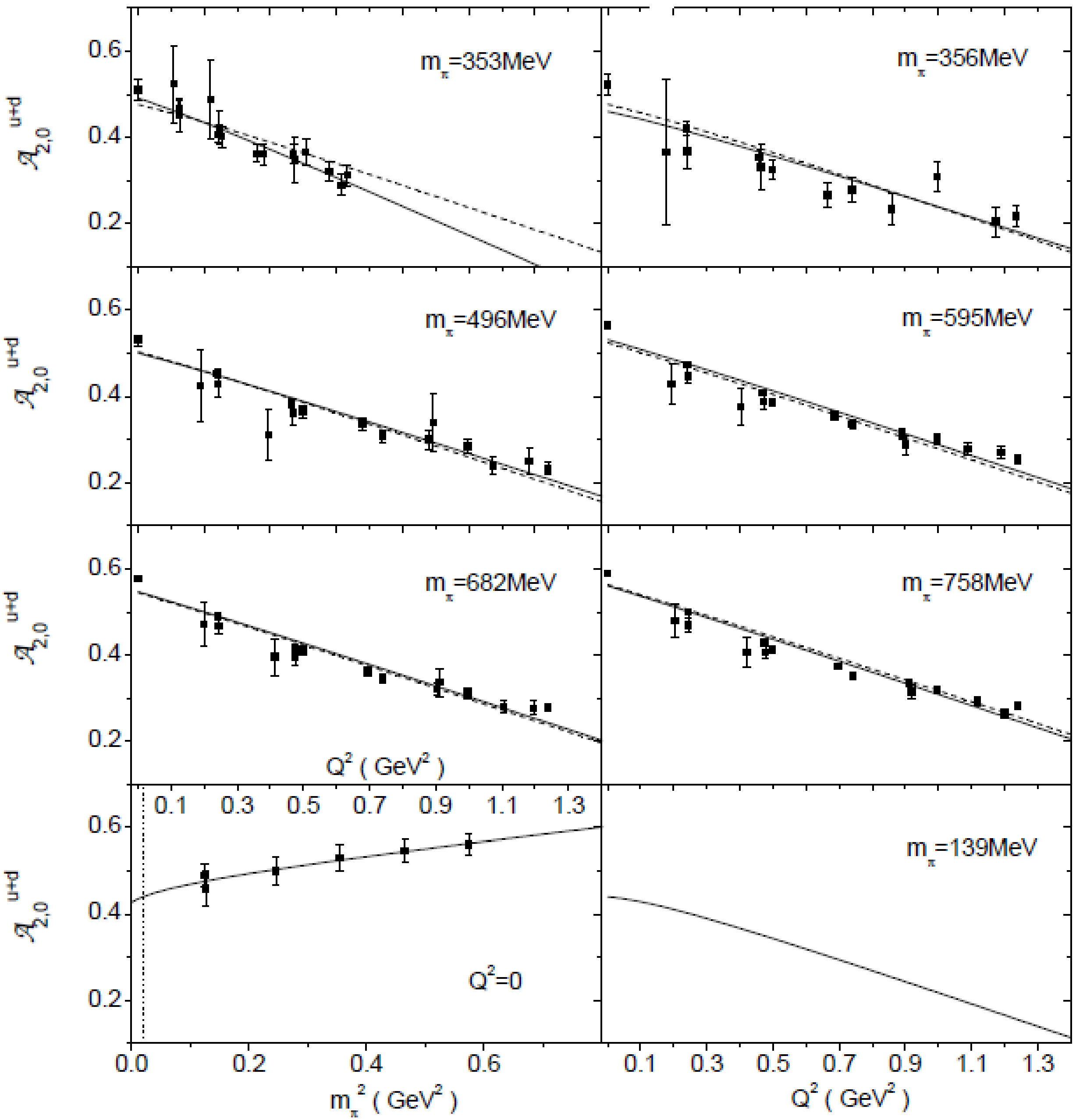
$$J_{00}^s = -\frac{3g_A^2 x_\pi^0}{128\pi^3 F_\pi^2}\int d\vec{k}\frac{u(\vec{k})u(\vec{k}-\vec{q})[\vec{k}\cdot(\vec{k}-\vec{q})]^2}{\omega^2(\vec{k})\omega^2(\vec{k}-\vec{q})} - \frac{\mathcal{C}^2 x_\pi^0}{48\pi^3 F_\pi^2}\int d\vec{k}u(\vec{k})u(\vec{k}-\vec{q})[\vec{k}\cdot(\vec{k}-\vec{q})]^2 f(\omega), \tag{34}$$

$$\begin{aligned} J_{33}^s &= -\frac{3g_A^2 x_\pi^0}{128\pi^3 F_\pi^2}\int d\vec{k}\frac{u(\vec{k})u(\vec{k}-\vec{q})[4k_z(k_z-q_z)-\vec{k}\cdot(\vec{k}-\vec{q})]\vec{k}\cdot(\vec{k}-\vec{q})}{\omega^2(\vec{k})\omega^2(\vec{k}-\vec{q})} \\ &\quad -\frac{\mathcal{C}^2 x_\pi^0}{48\pi^3 F_\pi^2}\int d\vec{k}u(\vec{k})u(\vec{k}-\vec{q})[4k_z(k_z-q_z)-\vec{k}\cdot(\vec{k}-\vec{q})]\vec{k}\cdot(\vec{k}-\vec{q})f(\omega), \end{aligned} \tag{35}$$

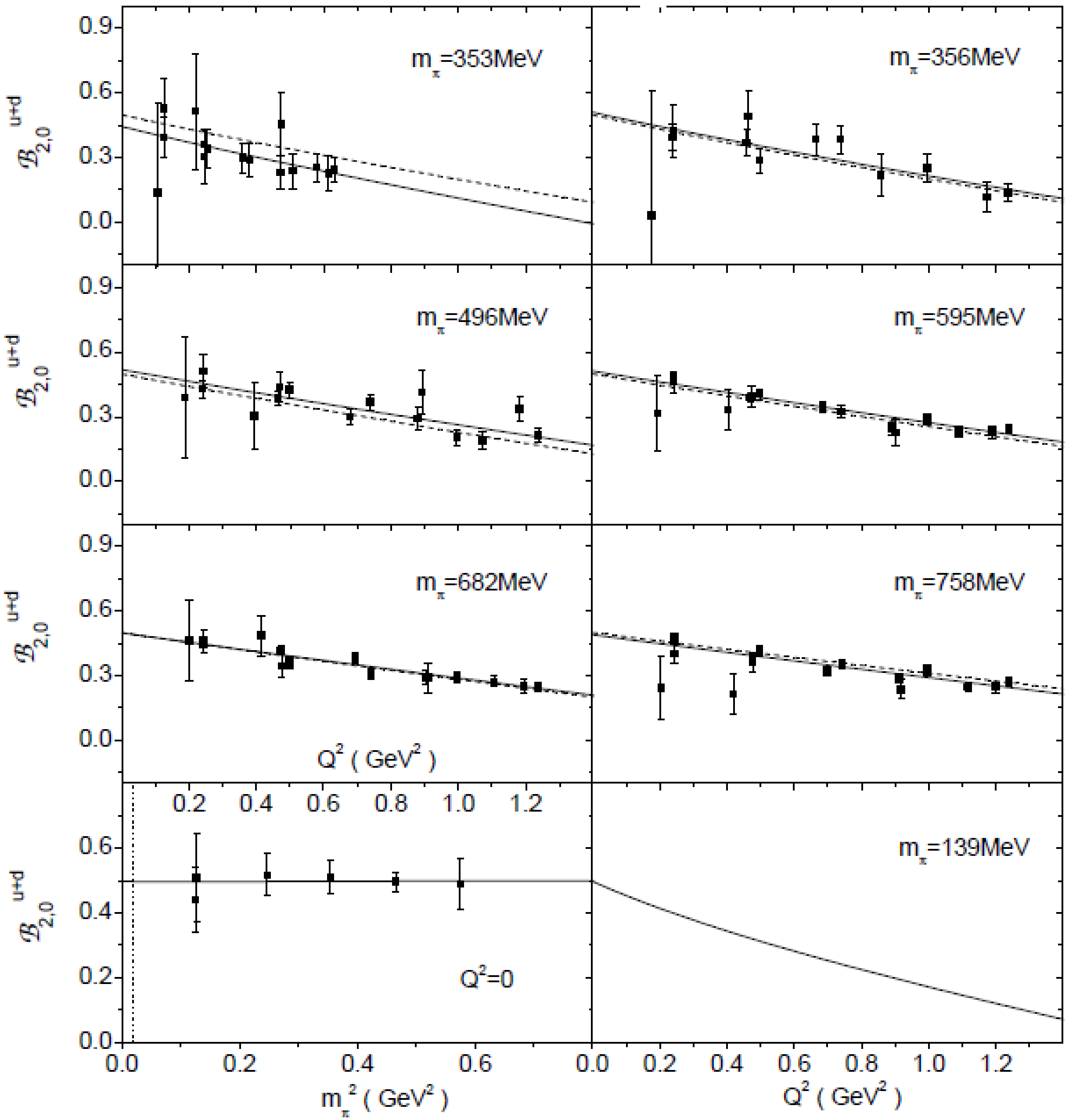
$$\begin{aligned} J_{03}^s &= \frac{i3g_A^2 x_\pi^0}{32\pi^3 F_\pi^2}\int d\vec{k}\frac{u(\vec{k})u(\vec{k}-\vec{q})k_z^2(\vec{\sigma}\times\vec{q})_z}{\omega(\vec{k})\omega(\vec{k}-\vec{q})[\omega(\vec{k})+\omega(\vec{k}-\vec{q})]} \\ &\quad -\frac{i\mathcal{C}^2 x_\pi^0}{24\pi^3 F_\pi^2}\int d\vec{k}\frac{u(\vec{k})u(\vec{k}-\vec{q})k_z^2(\vec{\sigma}\times\vec{q})_z}{[\omega(\vec{k})+\Delta][\omega(\vec{k}-\vec{q})+\Delta][\omega(\vec{k})+\omega(\vec{k}-\vec{q})]}, \end{aligned} \tag{36}$$

where $f(\omega)$ is expressed as

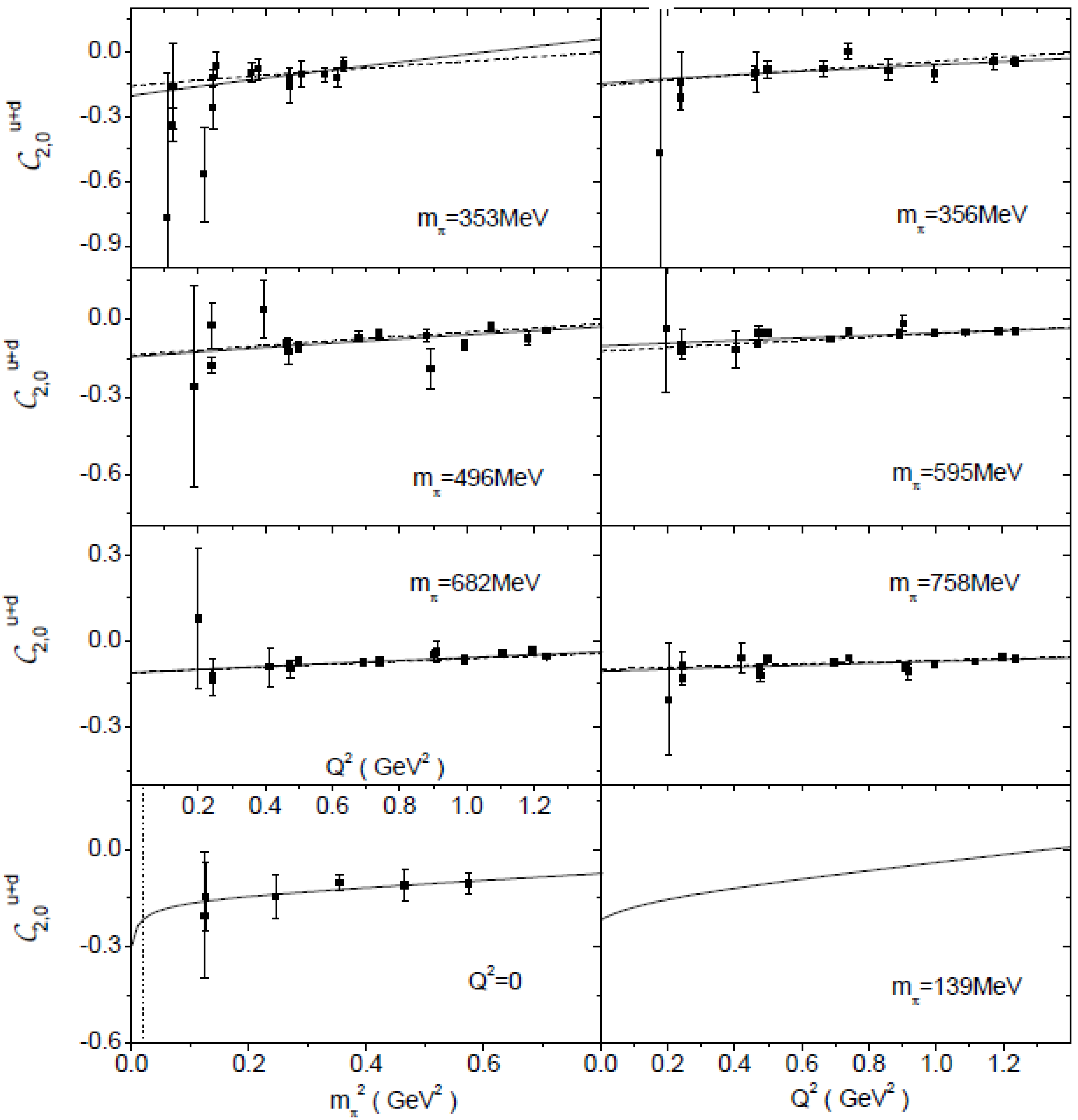
$$f(\omega) = \frac{\omega(\vec{k})+\omega(\vec{k}-\vec{q})+\Delta}{\omega(\vec{k})[\omega(\vec{k})+\Delta]\omega(\vec{k}-\vec{q})[\omega(\vec{k}-\vec{q})+\Delta][\omega(\vec{k})+\omega(\vec{k}-\vec{q})]}. \tag{37}$$



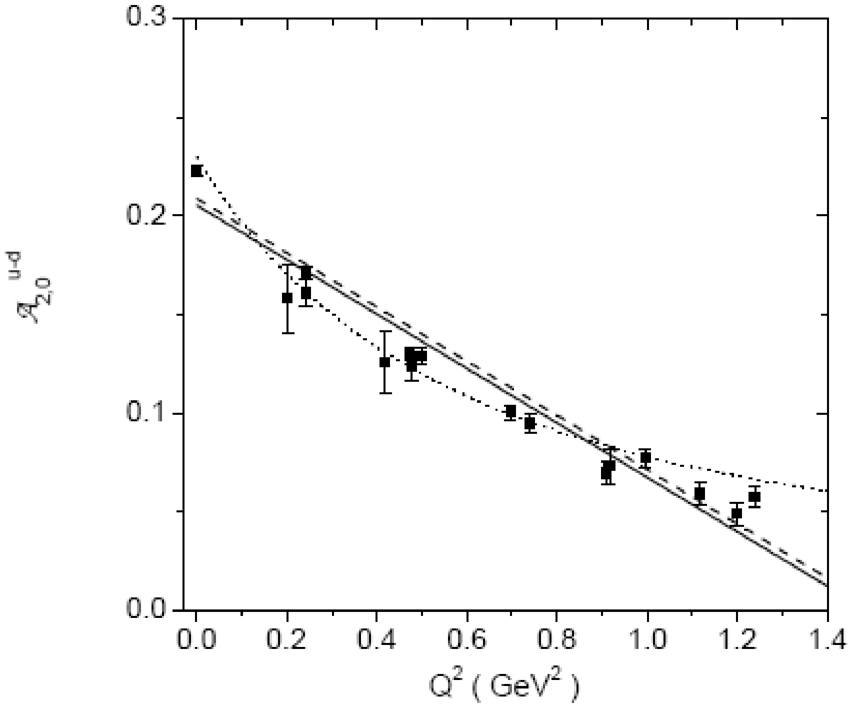
m_π=353MeV
m_π=356MeV
m_π=496MeV
m_π=595MeV
m_π=682MeV
m_π=758MeV
m_π=139MeV
Q^2=0
$\mathcal{A}_{2,0}^{u+d}$
Q^2 (GeV^2)
m_π^2 (GeV^2)
0.2 0.4 0.6
0.0 0.2 0.4 0.6
0.1 0.3 0.5 0.7 0.9 1.1 1.3



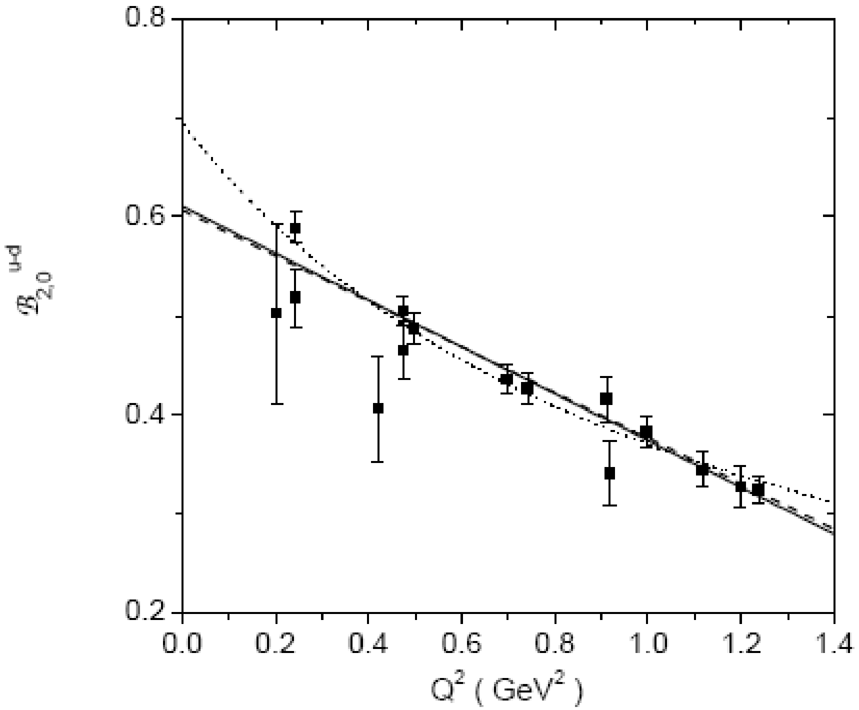
m_π=353MeV
m_π=356MeV
m_π=496MeV
m_π=595MeV
m_π=682MeV
m_π=758MeV
m_π=139MeV
Q²=0
B_{2,0}^{u+d}
Q² (GeV²)
m_π² (GeV²)



m_π=353MeV
m_π=356MeV
m_π=496MeV
m_π=595MeV
m_π=682MeV
m_π=758MeV
Q²=0
m_π=139MeV
C_{2,0}^{u+d}
Q² (GeV²)
m_π² (GeV²)



0.3
0.2
0.1
0.0
$\mathcal{A}_{2,0}^{u-d}$
0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.4
Q^2 (GeV^2)



0.8
0.6
0.4
0.2
$\mathcal{B}_{2,0}^{u-d}$
0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.4
Q^2 (GeV^2)

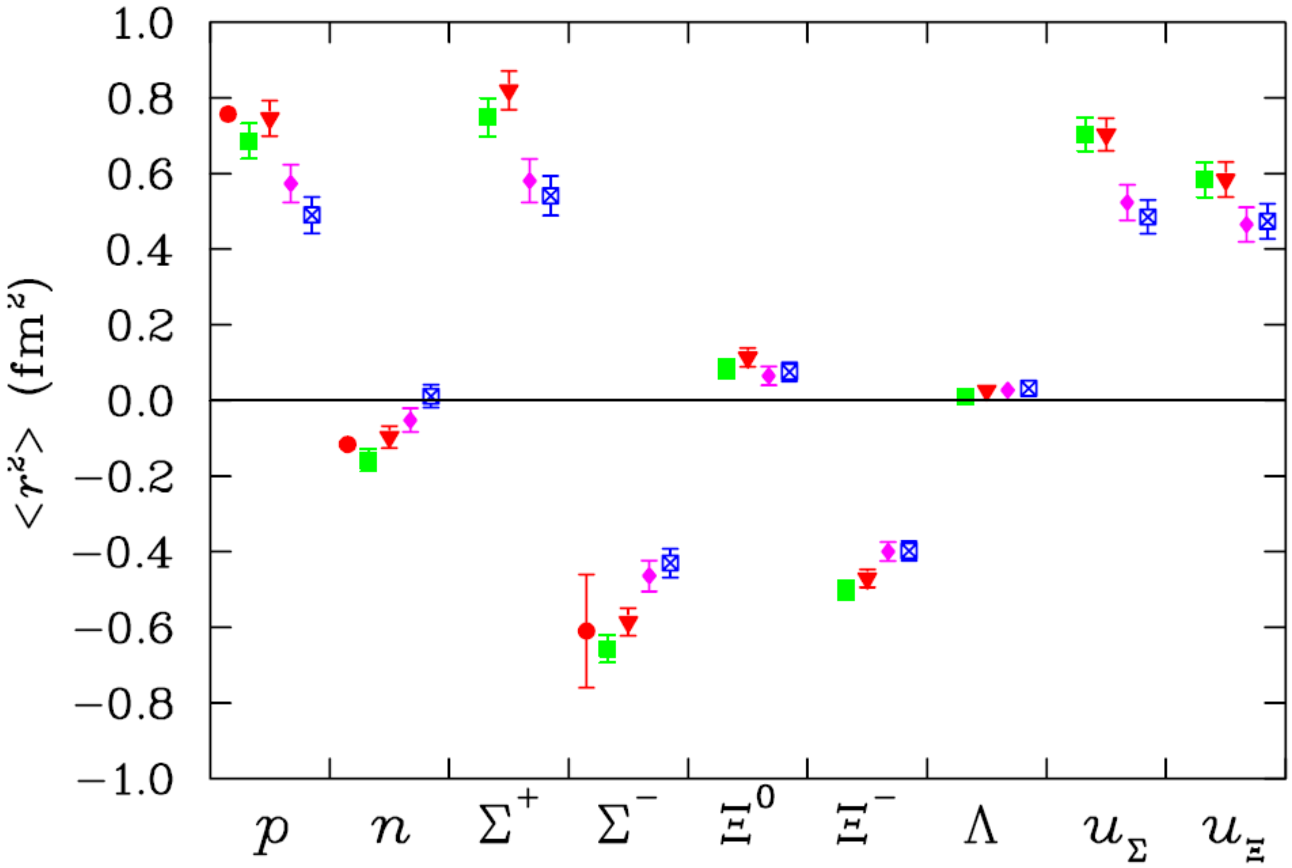
$$\mathcal{G}^v_{2,0}(Q^2, m_\pi^2) = Z(g^v_{2,0} + g^v_\pi m_\pi^2 + (g^v_t + g^v_{\pi,t} m_\pi^2)Q^2) + \sum_{i=a}^{f} \mathcal{G}^{v,i}_{2,0}$$

TABLE I: Low energy constants and moments at physical pion mass. The results in the table are for the linear fit where the momentum dependence of the tree level term is up to Q^2. For the dipole fit, the first moments at $Q^2 = 0$ are about $10\% - 20\%$ larger. The results of LHPC are also listed in the last column.

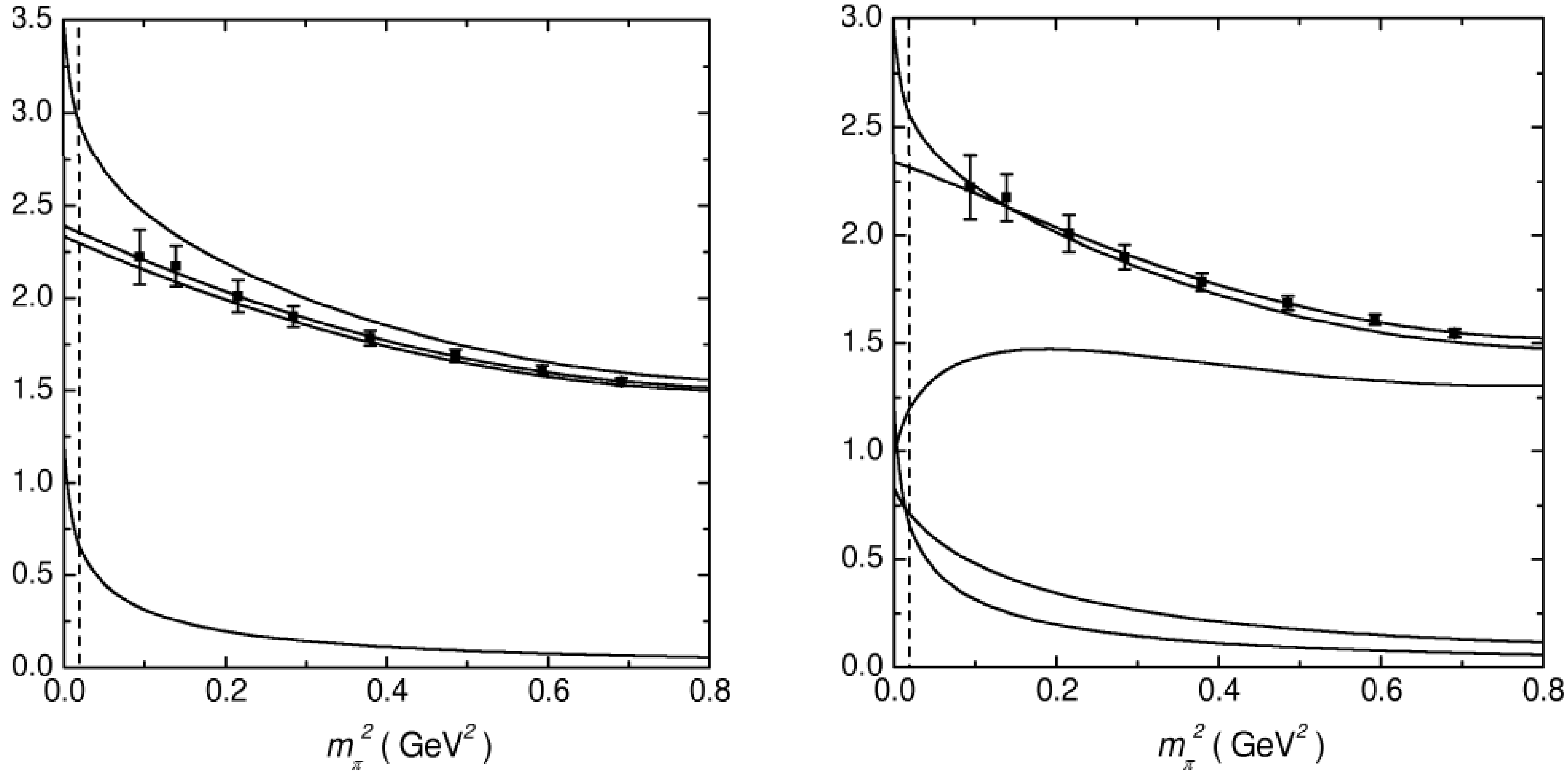
	$g_{2,0}$	$g_\pi(\mathrm{GeV}^{-2})$	$g_t(\mathrm{GeV}^{-2})$	$g_{\pi,t}(\mathrm{GeV}^{-4})$	$\mathcal{G}_{2,0}(0)$	$\mathcal{G}^{\mathrm{LHPC}}_{2,0}(0)$[29]
$\mathcal{A}^{u+d}_{2,0}$	0.489	0.152	−0.266	0.019	0.440 ± 0.033	0.520 ± 0.024
$\mathcal{B}^{u+d}_{2,0}$	0.496	0.005	−0.339	0.268	0.497 ± 0.089	0.425 ± 0.086
$\mathcal{C}^{u+d}_{2,0}$	−0.152	0.102	0.119	−0.159	-0.217 ± 0.103	-0.267 ± 0.062
$\mathcal{A}^{u-d}_{2,0}$	0.207	0.010	−0.139	−0.009	0.156 ± 0.020	0.157 ± 0.010
$\mathcal{B}^{u-d}_{2,0}$	0.526	0.158	−0.298	0.111	0.433 ± 0.071	0.430 ± 0.063
$\mathcal{C}^{u-d}_{2,0}$	−0.001	−0.009	−0.025	0.062	-0.001 ± 0.050	-0.017 ± 0.041

$J_q = \mathcal{B}_{2,0}/2$

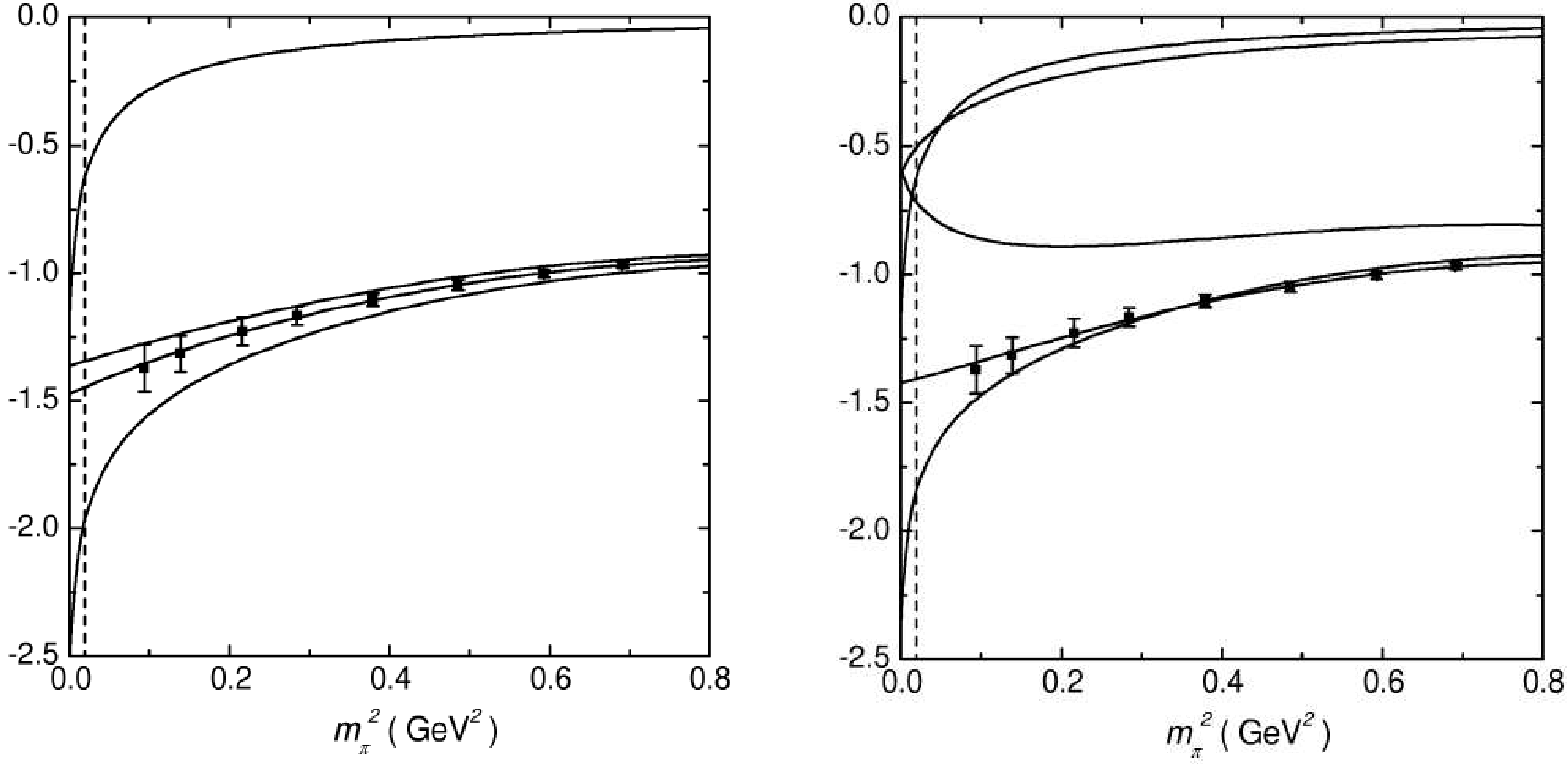
P. Wang, A. W. Thomas, Phys. Rev. D81 (2010) **114015**



P. Wang, D. Leinweber, A. Thomas, R. Young, Phys. Rev. D79 (2009) 094001



3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
m_π^2 (GeV^2)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
m_π^2 (GeV^2)


0.0
-0.5
-1.0
-1.5
-2.0
-2.5
0.0
0.2
0.4
0.6
0.8
m_π^2 (GeV^2)
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
0.0
0.2
0.4
0.6
0.8
m_π^2 (GeV^2)

Point Quantization

$$[\phi(\vec{x},t),\phi(\vec{y},t)] \;=\; [\pi(\vec{x},t),\pi(\vec{y},t)]=0,$$
$$[\phi(\vec{x},t),\pi(\vec{y},t)] \;=\; i\delta^{(3)}(\vec{x}-\vec{y})\,.$$

$$\phi(\vec{x},t)=\int \widetilde{dp}\left[a(\vec{p})e^{i\vec{p}\cdot\vec{x}-i\omega_p t}+a^\dagger(\vec{p})e^{-i\vec{p}\cdot\vec{x}+i\omega_p t}\right]$$

$$[a(\vec{p}),a(\vec{q})] \;=\; \left[a^\dagger(\vec{p}),a^\dagger(\vec{q})\right]=0,$$
$$\left[a(\vec{p}),a^\dagger(\vec{q})\right] \;=\; (2\pi)^3 2\omega_p\delta^{(3)}(\vec{p}-\vec{q})$$

$$Z_0(J)=\int \mathcal{D}\phi e^{i\int d^4x[\mathcal{L}_0+J\phi]}$$

$$\mathcal{L}_0=-\frac{1}{2}\partial^\mu\phi\partial_\mu\phi-\frac{1}{2}m^2\phi^2$$

Solid Quantization

$$[\phi(\vec{x},t),\phi(\vec{y},t)] \;=\; [\pi(\vec{x},t),\pi(\vec{y},t)]=0$$
$$[\phi(\vec{x},t),\pi(\vec{y},t)] \;=\; i\Phi(\vec{x}-\vec{y})\,.$$

$$[A(\vec{p}),A(\vec{q})] \;=\; \left[A^\dagger(\vec{p}),A^\dagger(\vec{q})\right]=0,$$
$$\left[A(\vec{p}),A^\dagger(\vec{q})\right] \;=\; (2\pi)^3\delta^{(3)}(\vec{p}-\vec{q})\Psi(\vec{p})$$

$$\Phi(\vec{x})=\int \frac{d^3p}{(2\pi)^3}\frac{\Psi(\vec{p})}{2}(e^{i\vec{p}\cdot\vec{x}}+e^{-i\vec{p}\cdot\vec{x}})$$

$$\Delta_F(x'-x)=\int \frac{d^4k}{(2\pi)^4}\frac{i\Psi(\vec{k})e^{-ik\cdot(x'-x)}}{k^2-m^2+i\epsilon}$$

$$\mathcal{L}_0(x)=\int d^3a\bar{\psi}(t,\vec{x}+\frac{\vec{a}}{2})\gamma^\mu i\partial_\mu\psi(t,\vec{x}-\frac{\vec{a}}{2})F(\vec{a})$$

$$\mathcal{L}_0(x)=\bar{\psi}(x)\gamma^\mu i\partial_\mu\tilde{F}(i\vec{\partial})\psi(x)$$

$$\tilde{F}(i\vec{\partial})=\int d^3a e^{i\vec{a}\cdot i\vec{\partial}}F(\vec{a})$$

P. Wang, Chin. Phys. C (to be published)
arXiv:1006.0842

Summary

The first moments of the nucleon GPDs are extrapolated from the lattice data using FRR with chiral perturbation theory.

The lattice data of the first moments with different Q^2 and pion mass can be fitted very well. The obtained values at physical pion mass are reasonable.

High order terms in the chiral expansion are important which can be built in the one loop contribution in FRR. The residual analytic terms have a good convergent behavior.

The Q^2 and pion mass dependence of the first moments of nucleon GPDs Is quite different from that of electromagnetic form factors.