Studies of heavy hadron physics

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1. Introduction

Heavy flavor physics is a good area to

- Find CP violation in heavy hadron decays
- Measure parameters in SM accurately
- Look for new physics beyond SM
- Extract non-perturbative QCD information

Many heavy mesons and baryons have been observed. Comparing with heavy mesons, heavy baryons have been less studied both experimentally and theoretically.

Most heavy baryons with more than one heavy quarks have not been observed.

Some new heavy hadron states were measured by BaBar (SLAC), Belle (KEKB), CDF, D0, SELEX, FOCUS (Fermilab), CLEO (Cornell) etc. For example:

- **excited states of B mesons:** $B_2^*(5747)(2^+)$, $B_{s2}^*(5840)(2^+)$
- **excited states of heavy baryons containing one c quark:** $\Lambda_c(2880)(5/2^+)$, $\Lambda_c(2940)$ **(**J^P **undetermined),** $\Xi_c(2980, 3080)$ **(**J^P **undetermined),** $\Xi_c(3055, 3123)$ **(**J^P **undetermined)**
- **heavy baryons containing one b quark:** Σ_b, Σ_b^*, Ξ_b, Ω_b
- **heavy baryons containing two c quarks:** $\Xi_{cc}(3520)$ **(reported by SELEX, not observed by BaBar, Belle, FOCUS,** J^P **undetermined)**

In recent years, some experimental results for decays:

- Λ_b**'s lifetime by CDF, D0**
- $\Lambda_b \to \Lambda_c$ **semileptonic decay by DELPHI**
- $\Lambda_b \to \Lambda_c \pi$ **by CDF**
- $\Gamma(\Sigma_c^* \to \Lambda_c \pi)$ **and** $\Gamma(\Sigma_c \to \Lambda_c \pi)$ **by CLEO and FOCUS**
- $\Gamma(\Sigma_b^* \to \Lambda_b \pi)$ **and** $\Gamma(\Sigma_b \to \Lambda_b \pi)$ **by CDF in 2010**

In the future we expect more and more data. Experiments need predictions.

In QCD, hadrons with structures like glueball (gg,ggg), hybrid (qqg), multiple quark states (quark numbers are bigger than 3), molecular states of hadrons may exist.

In recent years some possible exotic states were observed, for instance, $D^*_{s0}(2317)^{\pm}$, $D_{s1}(2460)^{\pm}$, $X(3872)$. They may have structures which are not those of traditional mesons and baryons. There are many theoretical works to study their structures with different models (like potential model, QCD sum rules, chiral perturbation theory).

Heavy quark symmetry $SU(2)_f \times SU(2)_s$ simplifies heavy hadron processes like $B \to D(D^*)$, $\Lambda_b \to \Lambda_c$. In the heavy quark limit only one form factor (Isgur-Wise function) is needed. This helps to extract nonperturbative QCD information.

QCD factorization improves theoretical treatment of two-body nonleptonic decays of B-meson, $B \to M_1 M_2$. It provides a way to calculate QCD corrections to conventional factorization. It is shown that when $m_b \gg \Lambda_{QCD}$, QCD corrections to conventional factorization can be calculated systematically in terms of short-distance coefficients and light-cone distribution amplitudes of mesons.

For heavy baryon decays, QCD factorization has been studied much less.

2. Bethe-Salpeter approach for heavy baryons and exotic states

Consider a heavy baryon with one heavy quark. When $m_Q \to \infty$, due to $SU(2)_f \times SU(2)_s$, the light degrees of freedom is blind to the quantum numbers of the heavy quark, hence have good quantum numbers, including angular momentum and isospin.

Therefore, we assume that a heavy baryon is composed of a heavy quark and a light diquark. This is our underlying assumption. In this picture: Three body system $\to$ two body system

$J = \frac{1}{2} + j$ (J: total spin of heavy baryon; j: spin of the brown muck)

Λ_Q: $j = 0$. Isospin of light diquark = 0 in order to guarantee that the total wave function of the hadron is antisymmetric.

$\omega_Q^{(*)}$ ($\omega = \Sigma$, Ξ, Ω): $j = 1$. Isospin of light diquark = 1.
$J = \frac{1}{2}$ (Σ_Q) or $\frac{3}{2}$ (Σ_Q^*)

The light degrees of freedom of $\omega_Q^{(*)}$ belong to a 6 representation of flavor SU(3). Taking $Q = b$ as an example, $\omega_b^{(*)}$ includes $\Sigma_b^{(*)+,0,-}$, $\Xi_b^{(*)0,-}$ and $\Omega_b^{(*)-}$.

In a baryon with two heavy quarks, these two heavy quarks are reasonably bound into a color-antitriplet heavy diquark which radius is much smaller than the scale $1/\Lambda_{QCD}$. The light quark moves in the color field induced by the heavy diquark. Then the three body system is also reduced to a two body system.

Two heavy quarks with the same flavor (bb or cc) can only constitute an axial-vector diquark with spin 1. Two quarks with different flavors (bc) can constitute both a scalar diquark and an axial-vector diquark.

Bethe-Salpeter (BS) equation is a formally exact equation to describe the relativistic bound state. In the heavy quark limit, we established BS equations for the heavy baryons with either one heavy quark or two heavy quarks in the diquark picture.

The kernel of the BS equation describes the interaction between the constituents of the bound state. Due to its nonperturbative nature, usually phenomenological models have to be applied for the kernel. In our series of work we assume the kernel containing scalar confinement and one gluon exchange terms motivated by potential model.

In a baryon containing one heavy quark the light diquark is not point-like, we introduce a few form factors to describe the effects of the diquark's structure.

In a baryon containing two heavy quarks, the heavy diquark is not really a point object and its radius is enhanced by $\ln m_Q$ with respect to $1/m_Q$ (Coulomb potential), so in the kernel we also introduce a few form factors for the effective vertex of the heavy diquark coupling to gluon to reflect the inner structure of the heavy diquark.

In order to study the structure of the diquarks, we also established the BS equations for the diquarks, with the kernel also including scalar confinement and one gluon exchange terms.

Take Λ_Q as an example. The BS wave function of Λ_Q ($QS_{[ud]}$)

$$\chi(x_1, x_2, P) = \langle 0|T\psi(x_1)\varphi(x_2)|\Lambda_Q(P)\rangle,$$

obeys the BS equation

$$\chi_P(p) = S_F(\lambda_1 P + p)\int \frac{\mathrm{d}^4 q}{(2\pi)^4} K(P, p, q)\chi_P(q)S_D(-\lambda_2 P + p).$$

In the limit $m_Q \to \infty$, $\chi_P(p) = \phi_P(p)u_{\Lambda_Q}(v, s)$ ($\phi_P(p)$ is a scalar function). Motivated by potential model and heavy quark spin symmetry we assume the kernel

$$-iK = I \otimes IV_1 + v_\mu \otimes (p_2 + p_2')^\mu V_2,$$

V_1: scalar confinement, V_2: one gluon exchange diagram.

In the convariant instantaneous approximation

$$\begin{aligned}\tilde{V}_1 &= \frac{8\pi\kappa}{[(p_t-q_t)^2+\mu^2]^2}\\ &\quad -(2\pi)^3\delta^3(p_t-q_t)\int\frac{\mathrm{d}^3k}{(2\pi)^3}\frac{8\pi\kappa}{(k^2+\mu^2)^2},\\ \tilde{V}_2 &= -\frac{16\pi}{3}\frac{\alpha_{seff}^2 Q_0^2}{[(p_t-q_t)^2+\mu^2][(p_t-q_t)^2+Q_0^2]}.\end{aligned}$$

The BS equation was solved numerically by discretizing the integration region $(0, \infty)$ into n pieces (n is chosen to be sufficiently large) and solving the eigenvalue equation.

Numerical results for BS wave function:

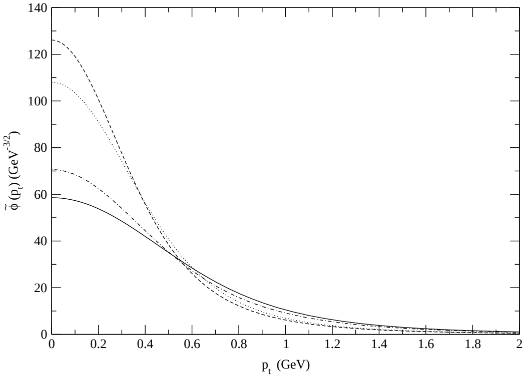
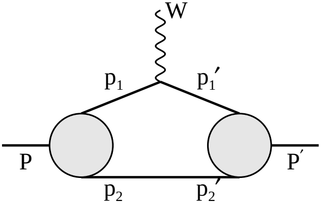


Figure: Numerical results for $\tilde{\phi}_P(p_t) \equiv \int \frac{dp_l}{2\pi}\phi_P(p)$. The solid (dashed) line corresponds to $m_D = 0.7$GeV and $\kappa = 0.02(0.1)$GeV^3, respectively. The dotted (dot-dashed) line corresponds to $\kappa = 0.04$GeV^3 and $m_D = 0.65(0.8)$GeV, respectively.

- $1/m_Q$ **corrections for** Λ_Q **were also studied.**
- $\omega_Q^{(*)}$ **can be studied in a similar way. It is more complicated. The BS wave function includes three scalar functions.**

$\Lambda_b \to \Lambda_c$ transition matrix is related to the BS wave functions of Λ_b and Λ_c:

$$\langle \Lambda_c(v') | J_\mu | \Lambda_b(v) \rangle = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \bar{\chi}_{P'}(p') \gamma_\mu (1-\gamma_5) \chi_P(p) S_D^{-1}(p_2),$$



- **Semileptonic decays and nonleptonic decays $\Lambda_b \to \Lambda_c$ (to order $O(1/m_Q)$)**

Table: Predictions for the decay rates for $\Lambda_b \to \Lambda_c l \bar{\nu}$, in units $10^{10}\mathrm{s}^{-1}$ for $\kappa = 0.02\mathrm{GeV}^3$ ($0.10\mathrm{GeV}^3$), $V_{cb} = 0.045$

m_D**(GeV)**	Γ_0	Γ_{1/m_Q}	$\Gamma_{1/m_Q+\mathrm{QCD}}$
0.65	**4.77 (7.20)**	**4.26 (6.62)**	**3.10 (4.76)**
0.70	**5.12 (7.12)**	**4.60 (6.56)**	**3.34 (4.72)**
0.75	**5.40 (7.02)**	**4.89 (6.50)**	**3.54 (4.67)**

XHG and T. Muta, Phys. Rev. D54 (1996) 4629
XHG, A.W. Thomas, and A.G. Williams, Phys. Rev. D61 (2000) 116015

Experimental data (PDG, by Delphi in 2004):
$\Gamma = 2.5 \sim 5.1 \times 10^{10} s^{-1}$. **Consistent.**

▶ Nonleptonic decays $\Lambda_b \to \Lambda_c \ P \ (V)$

Table: Decay rates (in units $10^{10}\mathrm{s}^{-1} a_1^2$), and up-down asymmetry parameters for $\Lambda_b \to \Lambda_c P(V)$ for $m_D = 0.7$GeV with $\kappa = 0.02\mathrm{GeV}^3$ ($0.10\mathrm{GeV}^3$)

Product	Γ_0	Γ_{1/m_Q}	$\Gamma_{1/m_Q+\mathrm{QCD}}$	α_{total}
$\Lambda_c^+\pi^-$	**0.30 (0.56)**	**0.36 (0.67)**	**0.29 (0.55)**	**-1.00**
$\Lambda_c^+\rho^-$	**0.44 (0.78)**	**0.51 (0.94)**	**0.42 (0.77)**	**-0.89**
$\Lambda_c^+ D_s^-$	**1.03 (1.57)**	**1.16 (1.81)**	**1.02 (1.59)**	**-0.98**
$\Lambda_c^+ D_s^{*-}$	**0.78 (1.17)**	**0.89 (1.35)**	**0.76 (1.15)**	**-0.38**
$\Lambda_c^+ K^-$	**0.022 (0.039)**	**0.026 (0.048)**	**0.021 (0.039)**	**-1.00**
$\Lambda_c^+ K^{*-}$	**0.023 (0.041)**	**0.027 (0.049)**	**0.022 (0.040)**	**-0.85**
$\Lambda_c^+ D^-$	**0.037 (0.057)**	**0.042 (0.066)**	**0.036 (0.057)**	**-0.98**
$\Lambda_c^+ D^{*-}$	**0.027 (0.041)**	**0.031 (0.048)**	**0.026 (0.040)**	**-0.42**

XHG, A.W. Thomas, and A.G. Williams, Phys. Rev. D61 (2000) 116015

Experimental data (PDG, by CDF in 2007):
$\Gamma(\Lambda_c^+\pi^-) = (0.39 \sim 0.89) \times 10^{10} s^{-1}$.
Consistent (taking $a_1 (= c_1 + 1/N_c^{\mathrm{eff}} c_2) \sim 1$).

▶ **Nonleptonic decays $\Omega_b \to \Omega_c^{(*)}\ P\ (V)$**

Table: Predictions for decay widths and asymmetry parameters for $\Omega_b \to \Omega_c^{(*)}\ P\ (V)$ for $m_D = 1.20\text{GeV}$ with $\kappa = 0.02\text{GeV}^3$ (0.10GeV^3).

Process	$\Gamma(10^{10}s^{-1})$	α
$\Omega_b^- \to \Omega_c^0 \pi^-$	$0.052a_1^2$ $(0.154a_1^2)$	-0.67 (-0.70)
$\Omega_b^- \to \Omega_c^0 D_s^-$	$0.261a_1^2$ $(0.592a_1^2)$	-0.56 (-0.58)
$\Omega_b^- \to \Omega_c^0 \rho^-$	$0.073a_1^2$ $(0.207a_1^2)$	-0.68 (-0.71)
$\Omega_b^- \to \Omega_c^0 D_s^{*-}$	$0.115a_1^2$ $(0.245a_1^2)$	-0.73 (-0.74)
$\Omega_b^- \to \Omega_c^{*0} \pi^-$	$0.046a_1^2$ $(0.133a_1^2)$	-0.61 (-0.58)
$\Omega_b^- \to \Omega_c^{*0} D_s^-$	$0.165a_1^2$ $(0.370a_1^2)$	-0.54 (-0.52)
$\Omega_b^- \to \Omega_c^{*0} \rho^-$	$0.134a_1^2$ $(0.354a_1^2)$	0.59 (0.59)
$\Omega_b^- \to \Omega_c^{*0} D_s^{*-}$	$0.462a_1^2$ $(0.960a_1^2)$	0.31 (0.31)

XHG, A.W. Thomas, and A.G. Williams, Phys. Rev. D59 (1999) 116007

- **Average kinetic energy of heavy quark in Λ_b (μ_π^2)**

Interesting: contribute to inclusive semileptonic decays of heavy hadrons ($\Lambda_b \to X l \bar{\nu}$) when contributions from higher order terms in $1/m_b$ are considered. $1/m_b^2$ corrections are characterized by μ_π^2.

$\to$ influence determination of CKM matrix elements V_{ub} and V_{cb}. No direct measurement of μ_π^2 yet.

$$\mu_\pi^2 = \frac{\langle \Lambda_Q | \bar{h}_v (iD_\perp)^2 h_v | \Lambda_Q \rangle}{2M},$$

Result: $\mu_\pi^2 : 0.25 \sim 0.95 GeV^2$ **in variation ranges of model parameters.**

XHG and H.-K. Wu, Phys. Lett. B654 (2007) 97

$\mu_\pi^2(B) = 0.401 \pm 0.040 GeV^2$ **(obtained by fitting data). No direct experimental measurement of μ_π^2 for Λ_b.**

Expanding heavy hadron masses to $1/m_Q \rightarrow$

$$\mu_\pi^2(\Lambda_b) - \mu_\pi^2(B)$$

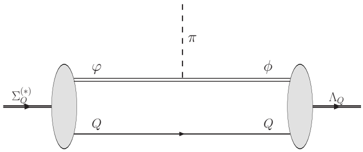
$$= \frac{2M(B)M(D)}{M(B) - M(D)}\{[M(\Lambda_c) - M(D)_{\rm avg}] - [M(\Lambda_b) - M(B)_{\rm avg}]\}.$$

Assuming that $1/m_Q^2$ terms in $[M(\Lambda_c) - M(D)_{\rm avg}] - [M(\Lambda_b) - M(B)_{\rm avg}]$ is of order $\bar{\Lambda}^3/m_Q^2$ to make a conservative estimate, one may expect $\mu_\pi^2(\Lambda_b)$ to be in the range $0.27 GeV^2 \sim 0.58 GeV^2$. Consistent with our result, $0.25 \sim 0.95 GeV^2$.

Conversely, one may give a rough constraint on the ranges of the parameters in the BS model from the range of $\mu_\pi^2(\Lambda_b)$. κ : $0.02 GeV^3 \sim 0.08 GeV^3$ ($m_D = 0.65 GeV$); $0.02 GeV^3 \sim 0.06 GeV^3$ ($m_D = 0.7 GeV$); $0.02 GeV^3 \sim 0.04 GeV^3$ ($m_D = 0.8 GeV$).

- $\Sigma_Q^{(*)} \rightarrow \Lambda_Q + \pi$ **(BS equation for light diquark)**

$\Gamma^{\rm exp}(\Sigma_c^*) \approx (13\text{-}18)$ **MeV (PDG, CLEO and FOCUS in 97, 01, 05)**
$\Gamma^{\rm exp}(\Sigma_c) \approx (1.8\text{-}2.6)$ **MeV (PDG, CLEO in 01, 05)**
The light diquark in $\Sigma_Q^{(*)}$ emits a very soft pion, then combines with the spectator heavy quark to form Λ_Q.



The vertex of the pion, the scalar diquark, and axial-vector diquark is calculated with the obtained BS wave functions for the diquarks and with the aid of PCAC, the reduction formula, and the low energy effective Lagrangian for the diquarks and the soft pion.

Results: in the range of model parameters, the decay widths for $\Sigma_Q^{(*)} \to \Lambda_Q + \pi$:

$$\Gamma(\Sigma_c) \approx (2.77\text{-}6.61)\,\textbf{MeV}\,, \qquad \Gamma(\Sigma_c^*) \approx (11.69\text{-}18.88)\,\textbf{MeV}\,,$$
$$\Gamma(\Sigma_b) \approx (6.73\text{-}13.45)\,\textbf{MeV}\,, \qquad \Gamma(\Sigma_b^*) \approx (10.00\text{-}17.74)\,\textbf{MeV}\,.$$

XHG, X.-H. Wu, and K.-W. Wei, Phys. Rev. D77 (2008) 036003

Nearly consistent with data for $\Gamma(\Sigma_c^{(*)})$. ($1/m_c$ corrections are larger than $1/m_b$ corrections.)

New results from CDF in 2010: the widths of Σ_b and Σ_b^* were measure for the first time. $\Gamma(\Sigma_b^+) = 9.2^{+3.8+1.0}_{-2.9-1.1}$, $\Gamma(\Sigma_b^{*+}) = 10.4^{+2.7+0.8}_{-2.2-1.2}$.

Consistent with our predictions.

▶ Heavy quark distribution functions in Λ_Q and $\omega_Q^{(*)}$

Heavy baryon $|B\rangle$ with momentum $P^+ = \frac{1}{\sqrt{2}}(P^0 + P^3)$ and $\mathbf{P}_\perp = 0$. The twist-2 heavy quark distribution function in $A^+ = 0$ gauge is defined as:

$$Q(\alpha) = \sqrt{2}P^+ \int \frac{\mathrm{d}x^-}{2\pi} e^{-i\alpha P^+ x^-} \langle B| T \bar{\psi}_Q(x^-) \gamma^+ \psi_Q(0) |B\rangle, \quad (1)$$

where $\gamma^+ = \frac{1}{\sqrt{2}}(\gamma^0 + \gamma^3)$, $\psi_Q(x^-)$ denotes $\psi_Q(x)$ at $x^+ = \mathbf{x}_\perp = 0$.

$Q(\alpha)$ measures probability to find Q with “plus” momentum fraction α.

With the BS wave functions the heavy quark distribution functions were obtained numerically at low energy scale ν_0 (a few hundred MeV).

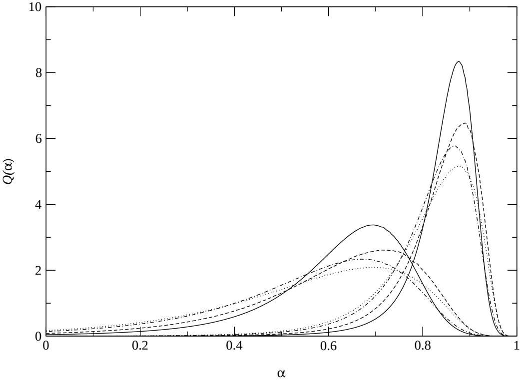


Figure: Λ_Q, $m_Q \to \infty$, ν_0. Solid (dot) lines: $m_D = 0.70$GeV and $\kappa = 0.02\text{GeV}^3$ ($\kappa = 0.10\text{GeV}^3$). Dashed (dot dashed) lines: $\kappa = 0.06\text{GeV}^3$ and $m_D = 0.65$GeV ($m_D = 0.75$GeV).

XHG, A.W. Thomas, and A.G. Williams, Phys. Rev. D64 (2001) 096004

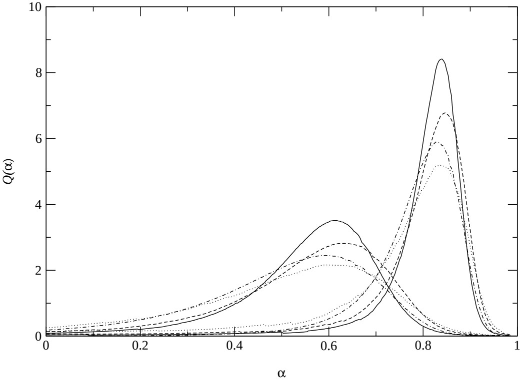


Figure: Σ_Q, $m_Q \to \infty$, ν_0. Solid (dot) lines: $m_D = 0.95$GeV and $\kappa = 0.02\text{GeV}^3$ ($\kappa = 0.10\text{GeV}^3$). Dashed (dot dashed) lines: $\kappa = 0.06\text{GeV}^3$ and $m_D = 0.90$GeV ($m_D = 1.00$GeV).

XHG, A.W. Thomas, and A.G. Williams, Phys. Rev. D64 (2001) 096004

From these figures we can see that:

- **For different heavy baryons with the same heavy quark flavor the shapes of the heavy quark distribution functions are rather similar.**
- **There is an obvious peak at some "plus" momentum fraction carried by the heavy quark, α_0, and this peak is much sharper for b-baryons than c-baryons.**
- α_0 **is much closer to 1 for b-baryons than c-baryons.**
- α_0 **is a little closer to 1 for Λ_Q than Σ_Q.**

QCD running: ν_0 is evolved to higher scale, ν, by Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations. The distinction between b-quark and c-quark distribution functions is still obvious at high ν^2.

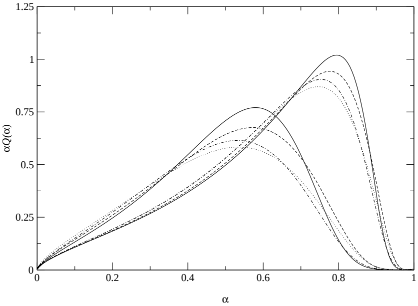
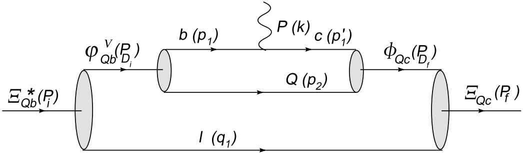


Figure: Λ_Q, $m_Q \to \infty$, $\nu^2 = 10\mathrm{GeV}^2$, $\nu_0^2 = 0.25\mathrm{GeV}^2$.

- **Non-leptonic decays of doubly heavy baryons emitting a pseudo-scalar meson**



The BS equations for the heavy diquarks and the heavy baryons are established respectively. With the kernel containing one gluon exchange and linear confinement terms the BS wave functions for the heavy diquarks and the heavy baryons are solved out numerically. Then the non-leptonic decay widths of the doubly heavy baryons are predicted.

M.-H. Weng, X.-H. Guo, and A.W. Thomas, Phys. Rev. D83 (2011) 056006

Table: Predictions for the non-leptonic decay widths for the doubly heavy baryons emitting π and K mesons (in units of $10^{-14} a_1^2 GeV$)

$\Gamma(\Xi^{*\frac{1}{2}}_{bb} \to \Xi^{\frac{1}{2}}_{bc}\pi)$	**0.343∼0.362**	$\Gamma(\Omega^{*\frac{1}{2}}_{bb} \to \Omega^{\frac{1}{2}}_{bc}\pi)$	**0.591∼0.607**
$\Gamma(\Xi^{*\frac{1}{2}}_{bb} \to \Xi^{*\frac{1}{2}}_{bc}\pi)$	**0.205∼0.211**	$\Gamma(\Omega^{*\frac{1}{2}}_{bb} \to \Omega^{*\frac{1}{2}}_{bc}\pi)$	**0.380∼0.381**
$\Gamma(\Xi^{*\frac{3}{2}}_{bb} \to \Xi^{*\frac{3}{2}}_{bc}\pi)$	**4.110∼4.234**	$\Gamma(\Omega^{*\frac{3}{2}}_{bb} \to \Omega^{*\frac{3}{2}}_{bc}\pi)$	**7.606∼7.643**
$\Gamma(\Xi^{\frac{1}{2}}_{bc} \to \Xi^{*\frac{1}{2}}_{cc}\pi)$	**0.848∼1.101**	$\Gamma(\Omega^{\frac{1}{2}}_{bc} \to \Omega^{*\frac{1}{2}}_{cc}\pi)$	**1.708∼1.876**
$\Gamma(\Xi^{*\frac{1}{2}}_{bc} \to \Xi^{*\frac{1}{2}}_{cc}\pi)$	**0.415∼0.587**	$\Gamma(\Omega^{*\frac{1}{2}}_{bc} \to \Omega^{*\frac{1}{2}}_{cc}\pi)$	**0.965∼1.019**
$\Gamma(\Xi^{*\frac{3}{2}}_{bc} \to \Xi^{*\frac{3}{2}}_{cc}\pi)$	**8.626∼12.110**	$\Gamma(\Omega^{*\frac{3}{2}}_{bc} \to \Omega^{*\frac{3}{2}}_{cc}\pi)$	**19.435∼20.529**
$\Gamma(\Xi^{*\frac{1}{2}}_{bb} \to \Xi^{\frac{1}{2}}_{bc}K)$	**0.265∼0.267**	$\Gamma(\Omega^{*\frac{1}{2}}_{bb} \to \Omega^{\frac{1}{2}}_{bc}K)$	**0.469∼0.482**
$\Gamma(\Xi^{*\frac{1}{2}}_{bb} \to \Xi^{*\frac{1}{2}}_{bc}K)$	**0.165∼0.171**	$\Gamma(\Omega^{*\frac{1}{2}}_{bb} \to \Omega^{*\frac{1}{2}}_{bc}K)$	**0.307∼0.308**
$\Gamma(\Xi^{*\frac{3}{2}}_{bb} \to \Xi^{*\frac{3}{2}}_{bc}K)$	**3.317∼3.425**	$\Gamma(\Omega^{*\frac{3}{2}}_{bb} \to \Omega^{*\frac{3}{2}}_{bc}K)$	**6.150∼6.170**
$\Gamma(\Xi^{\frac{1}{2}}_{bc} \to \Xi^{*\frac{1}{2}}_{cc}K)$	**0.649∼0.845**	$\Gamma(\Omega^{\frac{1}{2}}_{bc} \to \Omega^{*\frac{1}{2}}_{cc}K)$	**1.313∼1.557**
$\Gamma(\Xi^{*\frac{1}{2}}_{bc} \to \Xi^{*\frac{1}{2}}_{cc}K)$	**0.328∼0.466**	$\Gamma(\Omega^{*\frac{1}{2}}_{bc} \to \Omega^{*\frac{1}{2}}_{cc}K)$	**0.767∼0.806**
$\Gamma(\Xi^{*\frac{3}{2}}_{bc} \to \Xi^{*\frac{3}{2}}_{cc}K)$	**6.810∼9.612**	$\Gamma(\Omega^{*\frac{3}{2}}_{bc} \to \Omega^{*\frac{3}{2}}_{cc}K)$	**15.436∼16.237**

Table: Predictions for the non-leptonic decay widths for the doubly heavy baryons emitting D and D_s mesons (in units of $10^{-14} a_1^2 GeV$)

$\Gamma(\Xi_{bb}^{*\frac{1}{2}} \to \Xi_{bc}^{\frac{1}{2}} D)$	**0.818∼0.832**	$\Gamma(\Omega_{bb}^{*\frac{1}{2}} \to \Omega_{bc}^{\frac{1}{2}} D)$	**1.404∼1.466**
$\Gamma(\Xi_{bb}^{*\frac{1}{2}} \to \Xi_{bc}^{*\frac{1}{2}} D)$	**0.693∼0.741**	$\Gamma(\Omega_{bb}^{*\frac{1}{2}} \to \Omega_{bc}^{*\frac{1}{2}} D)$	**1.292∼1.319**
$\Gamma(\Xi_{bb}^{*\frac{3}{2}} \to \Xi_{bc}^{*\frac{3}{2}} D)$	**13.885∼14.834**	$\Gamma(\Omega_{bb}^{*\frac{3}{2}} \to \Omega_{bc}^{*\frac{3}{2}} D)$	**25.873∼26.419**
$\Gamma(\Xi_{bc}^{\frac{1}{2}} \to \Xi_{cc}^{*\frac{1}{2}} D)$	**1.136∼1.528**	$\Gamma(\Omega_{bc}^{\frac{1}{2}} \to \Omega_{cc}^{*\frac{1}{2}} D)$	**2.434∼2.552**
$\Gamma(\Xi_{bc}^{*\frac{1}{2}} \to \Xi_{cc}^{*\frac{1}{2}} D)$	**0.945∼1.464**	$\Gamma(\Omega_{bc}^{*\frac{1}{2}} \to \Omega_{cc}^{*\frac{1}{2}} D)$	**2.383∼2.426**
$\Gamma(\Xi_{bc}^{*\frac{3}{2}} \to \Xi_{cc}^{*\frac{3}{2}} D)$	**19.525∼30.028**	$\Gamma(\Omega_{bc}^{*\frac{3}{2}} \to \Omega_{cc}^{*\frac{3}{2}} D)$	**47.895∼48.741**
$\Gamma(\Xi_{bb}^{*\frac{1}{2}} \to \Xi_{bc}^{\frac{1}{2}} D_s)$	**0.285∼0.290**	$\Gamma(\Omega_{bb}^{*\frac{1}{2}} \to \Omega_{bc}^{\frac{1}{2}} D_s)$	**0.511∼0.515**
$\Gamma(\Xi_{bb}^{*\frac{1}{2}} \to \Xi_{bc}^{*\frac{1}{2}} D_s)$	**0.253∼0.271**	$\Gamma(\Omega_{bb}^{*\frac{1}{2}} \to \Omega_{bc}^{*\frac{1}{2}} D_s)$	**0.471∼0.483**
$\Gamma(\Xi_{bb}^{*\frac{3}{2}} \to \Xi_{bc}^{*\frac{3}{2}} D_s)$	**5.065∼5.435**	$\Gamma(\Omega_{bb}^{*\frac{3}{2}} \to \Omega_{bc}^{*\frac{3}{2}} D_s)$	**9.437∼9.668**
$\Gamma(\Xi_{bc}^{\frac{1}{2}} \to \Xi_{cc}^{*\frac{1}{2}} D_s)$	**0.368∼0.497**	$\Gamma(\Omega_{bc}^{\frac{1}{2}} \to \Omega_{cc}^{*\frac{1}{2}} D_s)$	**0.794∼0.828**
$\Gamma(\Xi_{bc}^{*\frac{1}{2}} \to \Xi_{cc}^{*\frac{1}{2}} D_s)$	**0.328∼0.513**	$\Gamma(\Omega_{bc}^{*\frac{1}{2}} \to \Omega_{cc}^{*\frac{1}{2}} D_s)$	**0.828∼0.851**
$\Gamma(\Xi_{bc}^{*\frac{3}{2}} \to \Xi_{cc}^{*\frac{3}{2}} D_s)$	**6.763∼10.520**	$\Gamma(\Omega_{bc}^{*\frac{3}{2}} \to \Omega_{cc}^{*\frac{3}{2}} D_s)$	**16.650∼17.099**

- **Possible molecular state $D_{s0}^*(2317)^+$**

$D_{s0}^*(2317)^+$ **was observed by BaBar, Belle and CLEO in 2003 and 2004. The isospin and spin-parity quantum numbers are** $I(J^P) = 0(0^+)$ **(PDG). The decay width is narrow,** $\Gamma \leq$ **3.8 MeV.**

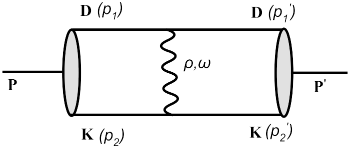
Various models have been proposed:

a traditional $c\bar{s}$ state

an exotic meson state such as
a four-quark state
a $D_s\pi$ bound state
a DK bound state
a state of $c\bar{s}$ mixed with DK or with a four-quark state

In the BS equation approach, we studied the possibility that $D_{s0}^*(2317)^+$ is an S-wave DK molecular bound state.

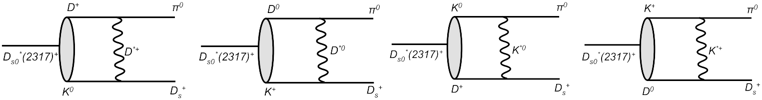
The BS equation of the two pseudo-scalar meson system can be easily established. The kernel is generated by exchanging ρ and ω mesons between D and K:



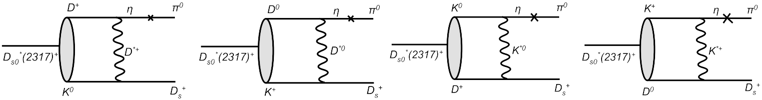
In order to reflect the effects of the non-point interaction of the hadrons, a form factor is introduced in each interaction vertex (a cutoff is involved), $F(\mathbf{k}) = \frac{2\Lambda^2 - M_V^2}{2\Lambda^2 + \mathbf{k}^2}$.

It is found that the DK bound system can exist. Therefore, the bound state of DK system does contribute to the state $D_{s0}^*(2317)^+$.

The bound state $D_{s0}^{*}(2317)^{+}$ can decay to $D_s^{+}\pi^0$ through exchanging vector mesons D^* or K^*.



The decay is isospin-violating and thus we also include $\eta-\pi$ mixing which gives important contribution.



The decay width is $29 \sim 38$ KeV, consistent with $\Gamma^{\rm exp} < 3.8$MeV.

Z.-X. Xie, G.-Q. Feng, XHG, Phys. Rev. D81(2010)036014

3. Proof of QCD factorization for $\Lambda_b \to \Lambda_c \pi$

QCD factorization was proven to be applicable to b-meson decays by Beneke, Buchalla, Neubert, and Sachrajda. They showed that in the case of heavy-light final states ($B \to D\pi$) factorization holds at $\mathcal{O}(\alpha_s)$ when $m_b \to \infty$.

$\Lambda_b^0 \to \Lambda_c^+ \pi^-$ is much more complex: the additional light quark in Λ_b (and Λ_c) generates many more Feynman diagrams.

In the framework of QCD factorization, we proved that most of these diagrams are $1/m_b$ suppressed, leaving only the vertex corrections at $\mathcal{O}(\alpha_s)$. $\to$ Factorization still holds at $\mathcal{O}(\alpha_s)$ for $\Lambda_b^0 \to \Lambda_c^+ \pi^-$ when $m_b \to \infty$.

Furthermore, the decay amplitude is renormalization-scale- and scheme-independent at $\mathcal{O}(\alpha_s)$.

Z.-H. Zhang, XHG, and G. Lu, Phys. Rev. D83 (2011) 031501(R)

- **Effective Hamiltonian for $\Lambda_b^0 \to \Lambda_c^+ \pi^-$**

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{cb} \left[c_1(\mu) Q_1 + c_2(\mu) Q_2\right] + H.c.,$$

c_1 **and** c_2 **are Wilson coefficients at the scale** μ **(** $\sim \mathcal{O}(m_b)$**),**

$$Q_1 = \bar{d}\gamma_\mu(1-\gamma_5)u\bar{c}\gamma^\mu(1-\gamma_5)b,$$
$$Q_2 = \bar{d}^i\gamma_\mu(1-\gamma_5)u^j\bar{c}^j\gamma^\mu(1-\gamma_5)b^i.$$

▶ **Wave functions of hadron H**

$$|H(q)\rangle = \sum_n |H(q)\rangle_n,$$

$$|H(q)\rangle_n = \int [d\xi dq] a_1^\dagger \cdots a_n^\dagger |0\rangle \Psi_H^{(n)}(\xi_*, q_{*\perp}),$$

$\Psi_H^{(n)}$: **BS wave function for the n-parton Fock state,**

$$[d\xi dq] = \left[\prod_l^n \frac{d\xi_l d^2 q_{l\perp}}{2(2\pi)^3\sqrt{\xi_l}}\right] 2(2\pi)^3 \delta\left(1 - \sum_l \xi_l\right) \delta^{(2)}\left(\sum_l \vec{q}_{l\perp}\right).$$

Normalization condition:

$$\langle H(q)|H(q')\rangle = (2\pi)^3 E_{\vec{q}} \delta^{(3)}(\vec{q} - \vec{q}\,').$$

$${}_n\langle H(q)|H(q')\rangle_m = \lambda_H^{(n)} (2\pi)^3 E_{\vec{q}} \delta^{(3)}(\vec{q} - \vec{q}\,')\delta_{nm},$$

where

$$\lambda_H^{(n)} = \int \{d\xi dq\} |\Psi_H^{(n)}(\xi_*, q_{*\perp})|^2, \quad \sum_n \lambda_H^{(n)} = 1.$$

Power-counting rules for valence Fock state wave functions

Light cone distribution amplitude Φ_H and decay constant f_H:

$$\int \{dq_\perp\}\, \Psi_H(\xi_*, q_{*\perp}) \sim f_H \Phi_H(\xi_*), \quad \int \mathcal{D}\xi \Phi_H(\xi_*) = 1.$$

For π,

$$\Psi_\pi(\xi_*, q_{*\perp}) \sim \begin{cases} -if_\pi \Phi_\pi(\xi_*)/\Lambda_{QCD}^2, & \text{for } |q_{*\perp}| \sim \Lambda_{QCD}, \\ 0, & \text{for } |q_{*\perp}| \gg \Lambda_{QCD}; \end{cases}$$

$$\Phi_\pi(\xi_*) \sim \begin{cases} \mathcal{O}(1), & \text{for } \xi_1 \& \xi_2 \sim \mathcal{O}(1), \\ \xi_i, & \text{for } \xi_i \sim 0; \end{cases}$$

For Λ_Q,

$$\Psi_{\Lambda_Q}(\xi_*, q_{*\perp}) \sim \begin{cases} -f_{\Lambda_Q} \Phi_{\Lambda_Q}(\xi_*)/\Lambda_{QCD}^4, & \text{for } |q_{*\perp}| \sim \Lambda_{QCD}, \\ 0, & \text{for } |q_{*\perp}| \gg \Lambda_{QCD}; \end{cases}$$

$$\Phi_{\Lambda_Q}(\xi_*) \sim \begin{cases} (m_Q/\Lambda_{QCD})^2\,, & \text{for } \xi_1 \& \xi_2(\text{light}) \sim \mathcal{O}(\Lambda_{QCD}/m_Q), \\ 0, & \text{else.} \end{cases}$$

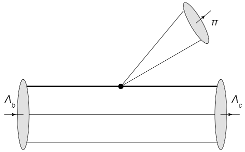
- **Infinite-momentum frame**

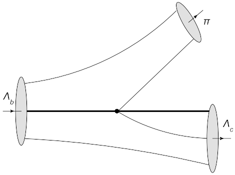
The decay amplitude is Lorentz-invariant. For convenience, we work in the infinite-momentum frame of Λ_b. We start with the rest frame of Λ_b, in which π moves along the z axis; then, go to the infinite-momentum frame by making a Lorentz boost along the z axis.

In this frame, the transverse momenta of three particles are zero and $p_{\Lambda_b} : p_{\Lambda_c} : p_\pi = 1 : z^2 : (1 - z^2)$ ($z = m_c/m_b$).

- **Valence Fock state contributions at tree level**

Factorizable (left, T_A) and non-factoriazable (right, T_B) diagrams at tree level:





Using the power counting rules for the wave functions:

$$T_A \sim \bar{u}(p_{\Lambda_c}) \not{p}_\pi (1-\gamma_5) u(p_{\Lambda_c}) \Lambda_{QCD},$$

$$T_B \sim T_A \left(\frac{\Lambda_{QCD}}{m_b}\right)^2 .$$

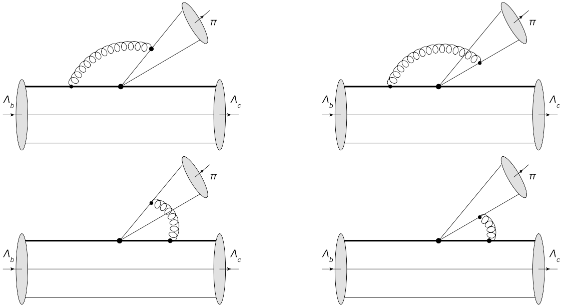
Therefore, the only diagram that contributes at the leading order of α_s is T_A in the heavy-quark limit, which is factorizable.

▶ **Valence Fock state contributions at $\mathcal{O}(\alpha_s)$**

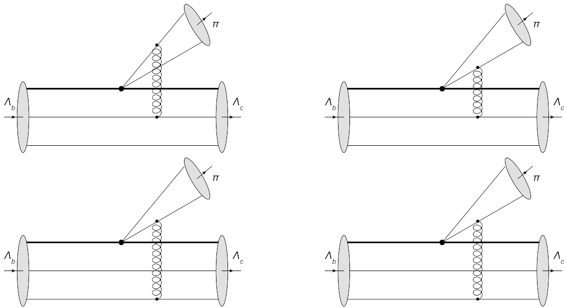
At $\mathcal{O}(\alpha_s)$, the diagrams can be classified into three categories: (a) corrections to T_A, (b) corrections to T_B, (c) annihilation diagrams.

(a) Corrections to T_A: There are two kinds of diagrams.

The first one: vertex corrections, which are $\mathcal{O}(1)$ in $1/m_b$ expansion.



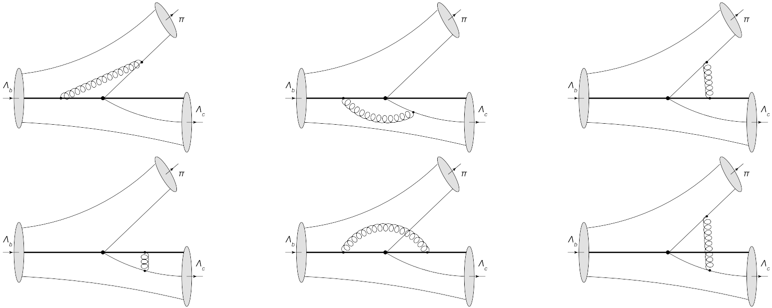
The second one: spectator-scattering diagrams, which are nonfactorizable but $1/m_b$ suppressed.

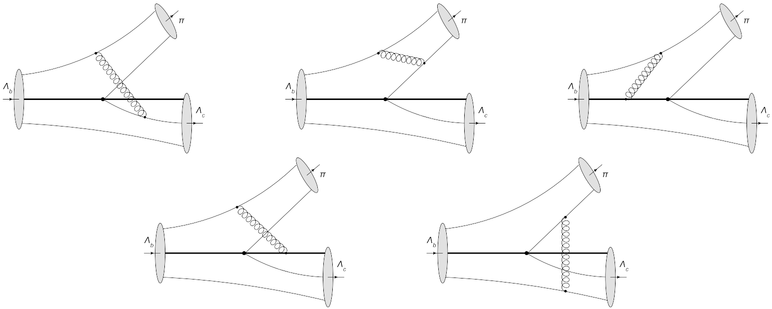


Unlike $B \to D\pi$, there are four such diagrams for $\Lambda_b^0 \to \Lambda_c^+ \pi^-$ due to an additional light quark in Λ_b (and Λ_c). One can find that each of them are of the order $\alpha_s T_A$, which is not suppressed. However, the leading terms cancel when one sums up all of the four diagrams, leading to $1/m_b$ suppression.

(b) Corrections to T_B: There are 11 diagrams.

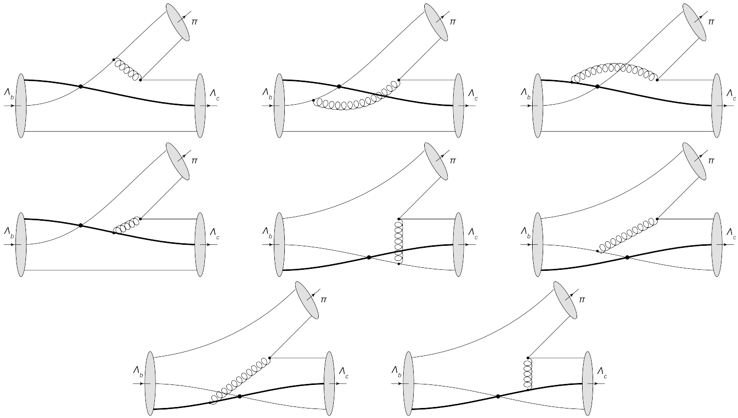
There are two kinds of diagrams: the first one is vertex corrections to T_B that contain one loop, while the second one is tree diagrams.





We prove that all the diagrams are $1/m_b$ suppressed. The first kind of diagrams contain infrared and ultraviolet divergences because of the loops, but since this kind of diagrams are of the order $\alpha_s T_B$, they are still $1/m_b$ suppressed.

(c) Annihilation diagrams:



We prove that all the diagrams are $1/m_b$ suppressed. For instance, the first diagram $\sim \alpha_s T_A \frac{\Lambda^2_{\rm QCD}}{m_b^2}$.

- **Nonvalence Fock state contributions**

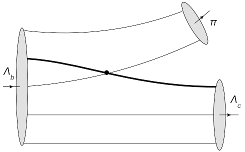
The proof for the suppression of higher-Fock-state contributions of the π meson is very similar to the decay $B \to D\pi$ given by Beneke, Buchalla, Neubert, and Sachrajda (NPB, 2000, hard collinear gluon and soft gluon are considered respectively).

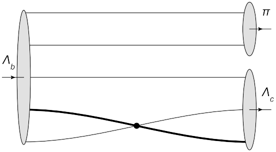
We need to consider the situation when Λ_b and/or Λ_c are at their nonvalence Fock states while π is at its valence Fock state.

Most of this kind of diagrams are factorizable and can be absorbed into the form factor of the transition $\Lambda_b \to \Lambda_c$.

However, when (at least) one of the valence quarks of π comes form the sea quarks of Λ_b, the diagram is nonfactorizable and cannot be absorbed into the form factor. We have to consider this kind of diagrams explicitly.

Two examples:





Power-counting rules for the nonvalence Fock states of Λ_Q:

The probability that Λ_Q is in its valence Fock state is $\mathcal{O}(1)$, $\rightarrow$ the power of the wave function of the nonvalence Fock state is, at most, the order obtained, if we assume that the probability that Λ_Q is in the nonvalence Fock state is a constant (not suppressed by $1/m_Q$).

Then we can adopt similar power-counting rules to that of the valence Fock state.

n-parton Fock state:

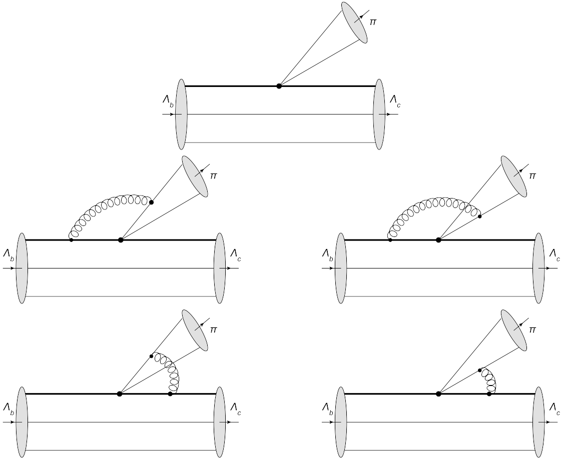
$$\Psi^{(n)}_{\Lambda_Q}(\xi_*, q_{*\perp}) \sim \begin{cases} -f^{(n)}_{\Lambda_Q}\Phi^{(n)}_{\Lambda_Q}(\xi_*)/\Lambda^{(2n-2)}_{QCD}, & \text{for } |q_{*\perp}| \sim \Lambda_{QCD}, \\ 0, & \text{for } |q_{*\perp}| \gg \Lambda_{QCD}; \end{cases}$$

$$\Phi^{(n)}_{\Lambda_Q}(\xi_*) \sim \begin{cases} \left(\frac{m_Q}{\Lambda_{QCD}}\right)^{(n-1)}, & \text{for } \xi_1 ... \& \xi_{n-1}(\text{light}) \sim \mathcal{O}(\frac{\Lambda_{QCD}}{m_Q}), \\ 0, & \text{else.} \end{cases}$$

With these power-counting rules, the two example diagrams are $1/m_b$ suppressed.

There may be other diagrams with even more partons in Λ_b and Λ_c. But with the restriction that π should be at its valence Fock state, the extra partons should go directly from Λ_b to Λ_c. This situation is similar to either the first or the second example diagrams.

Conclusion: when $m_b \to \infty$, up to $\mathcal{O}(\alpha_s)$, only the following diagrams contribute to $\Lambda_b^0 \to \Lambda_c^+ \pi^-$.



After removing ultraviolet and infrared divergences, we have the following factorizable form:

$$\mathcal{A}_{\Lambda_b^0 \to \Lambda_c^+ \pi^-} = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{cb} \langle \pi^- | \bar{d} \gamma_\mu (1-\gamma_5) u | 0 \rangle \langle \Lambda_c | \bar{c} \gamma^\mu (a_{1V} - a_{1A} \gamma_5) b | \Lambda_b \rangle$$

$$a_{1j} = \bar{c}_1(m_b) \quad + \frac{\bar{c}_2(m_b)}{N_c} \left[1 + \frac{\alpha_s(m_b)}{4\pi} C_F \int_0^1 dx \Phi_\pi(x) F_j(x,z) \right],$$

$j = V, A$, $F_j(x,z)$ **are defined as**

$$F_j(x,z) = \left(3 + 2 \ln \frac{x}{\bar{x}} \right) \ln z^2 - 7 + f(x, \epsilon^j z) + f(\bar{x}, \epsilon^j / z),$$

$\bar{x} = 1 - x$, $\epsilon^j = 1(-1)$ **for** $j = V(A)$, **and** $f(x,z)$ **(complicated).**

a_{1V} **and** a_{1A} **are independent of renormalization scale and scheme at** $\mathcal{O}(\alpha_s)$**.**

Branching ratio for $\Lambda_b^0 \to \Lambda_c^+ \pi^-$:

$$\mathrm{BR}(\Lambda_b^0 \to \Lambda_c^+ \pi^-) = |\xi(\omega)|^2 \times 1.74 \times 10^{-2} \times (1 \pm 5.4\%),$$

where the uncertainty (5.4%) is mainly from the CKM matrix element $|V_{cb}|$.

The Isgur-Wise function ξ is model-dependent.

Branching ratio:
$(5.7 \pm 0.3) \times 10^{-3}$ **for soliton model (E. Jenkins, A. Manohar, and M. Wise, NPB, 1993);**
$(3.2 \pm 0.2) \times 10^{-3}$ **for MIT bag model (M. Sadzikowski and K. Zalewski, ZPC, 1993);**
$(4.5 \pm 0.9) \times 10^{-3}$ **for Bethe-Salpeter-equation model (XHG and T. Muta, PRD, 1996).**

Experimental data: $(8.8 \pm 3.2) \times 10^{-3}$. It seems that the soliton model and the Bethe-Salpeter equation model agree with the experimental data better. ($1/m_Q$ corrections not considered yet.)

4. Summary

- When $m_Q \to \infty$, in the diquark picture, we established BS equations for ground states of heavy baryons containing one heavy quark and two heavy quarks, respectively. The BS equations for both heavy and light diquarks were also established.
- Assuming kernel to consist of a scalar confinement term and a one-gluon-exchange term we solved BS equations numerically in the covariant instantaneous approximation.
- Applications: semileptonic and nonleptonic decay widths; average kinetic energy of heavy quark in Λ_Q; $\Sigma_Q^{(*)} \to \Lambda_Q + \pi$ decay widths; heavy quark distribution functions
- Possible molecular heavy bound state.
- Some of our predictions have been confirmed by experiments.
- Proof of QCD factorization for $\Lambda_b \to \Lambda_c \pi$.