Physical Degrees of Freedom for Gauge Fields and the Question of Spin

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Outline

I. Spin, boosts and angular momentum
II. Lorentz irreps and angular momentum
III. Aside: Gauge non-Invariance of Hydrogen Eigenenergies
IV. Gauge invariance and canonical angular momentum in QED
V. Gauge Invariance and canonical commutation relation in QCD (for nucleon spin operators)
VI. Conclusion

I. Spin, boosts and angular momentum

Binding moves spin from static non-relativistic view to include orbital contributions

e.g. Electron in hydrogenic atom: (where $\gamma = \sqrt{1 - Z^2\alpha^2}$)

$$\psi \propto \begin{bmatrix} 1 \\ 0 \\ -\imath\frac{(1-\gamma)}{Z\alpha}\cos\theta \\ \imath\frac{(1-\gamma)}{Z\alpha}\sin\theta e^{\imath\phi} \end{bmatrix}$$

but norm

$$\psi^\dagger\psi \propto \{1 + \left[\frac{(1-\gamma)}{Z\alpha}\right]^2 [(\cos\theta)^2 + (\sin\theta)^2]\}$$

{1st correction at $\mathcal{O}(Z^4\alpha^4)$ }

and so "spin"

$$\psi^\dagger\Sigma_3\psi \propto \frac{1}{1 + \left[\frac{(1-\gamma)}{Z\alpha}\right]^2}\{1 + \left[\frac{(1-\gamma)}{Z\alpha}\right]^2 [(\cos\theta)^2 - (\sin\theta)^2]\}$$

which integrates to

$$\frac{1}{1 + \left[\frac{(1-\gamma)}{Z\alpha}\right]^2} < 1$$

Difference must be made up by orbital

Axial current or generator of rotations?

$$\frac{1}{2}\bar{\psi}\gamma^3\gamma^5\psi = \frac{1}{2}\psi^\dagger\Sigma_3\psi \qquad \Sigma_3 = \begin{bmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{bmatrix}$$

from generator of 3-axis rotations:

$$\frac{1}{2}\bar{\psi}\sigma_{12}\psi = \frac{1}{2}\psi^\dagger \begin{bmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{bmatrix} \psi$$

Same effect for bound state wavefunctions,
but --

$\int(c^2 - s^2) = \int(s^2 - c^2) = 0$

Or recall Melosh: $\vec{S}_\perp$ and $\vec{S}_\parallel$ boost differently + Wigner rotation

Basic Boosts:

Accelerating a polarized fermion from rest distributes angular momentum from spin to spin plus orbital angular momentum

Rest Frame solution of Dirac equation for spin up fermion:

$$\psi(x,t) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} e^{-\imath(mt=p^{\mu}x_{\mu})}$$

Boost FRAME - along spin direction

$$\Psi(p^\mu x_\mu) \rightarrow e^{-i\sigma_{03}\omega/2}\,\Psi(p^\mu x_\mu)$$

$$= e^{\left\{-\begin{pmatrix}0 & \sigma^3\\ \sigma^3 & 0\end{pmatrix}\omega/2\right\}}\,\Psi(p^\mu x_\mu)$$

$$= \left\{\cosh(\omega/2)\mathbf{1} - \begin{pmatrix}0 & \sigma^3\\ \sigma^3 & 0\end{pmatrix}\sinh(\omega/2)\right\}\begin{pmatrix}1\\0\\0\\0\end{pmatrix} e^{-i(Et-pz)}$$

$$= \begin{pmatrix}\cosh(\omega/2)\\0\\ \sinh(\omega/2)\\0\end{pmatrix} e^{-i(Et-px)}$$

$$\cosh(\omega) = E/m$$

$$\sinh(\omega) = p/m$$

$$\cosh(\omega/2) = \sqrt{\{[1+\cosh(\omega)]/2\}}$$

$$\sinh(\omega/2) = \sqrt{\{[\cosh(\omega)-1]/2\}}$$

or in terms of energy and momentum

$$\Psi_L(p^\mu x_\mu) = \sqrt{\frac{E+m}{2m}} \begin{pmatrix} 1 \\ 0 \\ p/(E+m) \\ 0 \end{pmatrix} e^{-i(Et-px)}$$

Cf. $\sigma \cdot p/(E+m) \rightarrow$ Spin-flip + Orbital L=1

¡ *lim* p → E → ∞ ! c.f. p = 0

$$\psi^\dagger \Sigma_3 \psi = \frac{(E+m)^2 + p^2}{2m(E+m)} = \frac{E}{m}$$

$$\bar{\psi} \sigma_{12} \psi = \frac{(E+m)^2 - p^2}{2m(E+m)} = 1$$

Boost FRAME- transverse to spin direction

$$\Psi(p^\mu x_\mu) \rightarrow e^{-i\sigma_{01}\omega/2}\,\Psi(p^\mu x_\mu)$$

$$= e^{-\left\{\begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix}\omega/2\right\}}\,\Psi(p^\mu x_\mu)$$

$$= \left\{ \cosh(\omega/2)\mathbf{1} - \begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix}\sinh(\omega/2)\right\}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^{-i(Et-pz)}$$

$$= \begin{pmatrix} \cosh(\omega/2) \\ 0 \\ 0 \\ -\sinh(\omega/2) \end{pmatrix} e^{-i(Et-px)}$$

$$\cosh(\omega) = E/m$$

$$\sinh(\omega) = p/m$$

$$\cosh(\omega/2) = \sqrt{\{[1+\cosh(\omega)]/2\}}$$

$$\sinh(\omega/2) = \sqrt{\{[\cosh(\omega)-1]/2\}}$$

or in terms of energy and momentum

$$\Psi_T(p^\mu x_\mu) = \sqrt{\frac{E+m}{2m}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ -p/(E+m) \end{pmatrix} e^{-i(Et-px)}$$

$$\psi^\dagger \Sigma_3 \psi = \frac{(E+m)^2 - p^2}{2m(E+m)} = 1$$

$$\bar{\psi}\sigma_{12}\psi = \frac{(E+m)^2 + p^2}{2m(E+m)} = \frac{E}{m}$$

II. Lorentz irreps and angular momentum

Rest Frame Spin-j Weyl Spinors

$$\phi^R_j(0^\mu) = \frac{m^j}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad \phi^R_{j-1}(0^\mu) = \frac{m^j}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}$$

$$\cdots \qquad \phi^R_{-j}(0^\mu) = \frac{m^j}{\sqrt{2}} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

Weinberg-Soper front-form formalism

$$0^\mu \equiv (p^+ = m, p^1 = p^2 = 0, p^- = m)$$

Column Index = $\dot{\zeta}$

Wigner Conjugation Operators

$$\Theta_{[1/2]} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

transforms R = (1/2, 0) Weyl spinor into L = (0,1/2) spinor

$$\psi^{R}_{\frac{1}{2}\dot{\zeta}} \rightarrow \psi^{L}_{\frac{1}{2}\zeta}$$

$$\Theta_{[1]} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

transforms R = (1,0) Weyl-like spinor into L = (0,1) spinor

$$\phi^{R}_{1\dot{\zeta}} \rightarrow \phi^{L}_{1\zeta}$$

Two independent R's, one transformed to L make (1/2, 0) ⊕ (0,1/2) Dirac bispinor

One R ⊕ same transformed to L makes self (or anti-self)-conjugate Majorana bispinor

Similarly for spin-1: self-conjugate bi"spinor" has no charge

After boosting along the 3-axis (quantization axis):

$$\phi_1^R \rightarrow \frac{m}{\sqrt{2}} \begin{bmatrix} \frac{p^+}{m} \\ 1 \\ \frac{m}{p^+} \end{bmatrix}$$

In limit $p^+ \rightarrow \infty$, bispinor $\rightarrow$ $\phi_1 \propto \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ Wigner-Weyl representation

ignoring the plane wave factor.

Note that only helicity = ±1 survive in the massless limit -- and only +1 or -1, violating parity, for each part {(1,0) or (0,1)} separately.

2nd state $\phi_1 \sim \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

Structure of spin-1 interaction with spin-½ field using ϕ^1_ξ

where $\xi = \begin{bmatrix} \dot{\zeta} \\ \zeta \end{bmatrix}$

$$\bar{\Psi}\Gamma^\xi \phi^1_\xi \Psi \quad : \quad \Gamma^\xi \leftrightarrow \gamma^\mu \tilde{\Gamma}^\xi_\mu \quad ?$$

Need a Lorentz group Clebsch-Gordan coefficient $\tilde{\Gamma}^\xi_\mu$

to form a relation to the conventional photon field:

$$\tilde{\Gamma}^\xi_\mu \phi^1_\xi \equiv A_\mu$$

RHS in $(\frac{1}{2}, \frac{1}{2})$ irrep

(J = 0 + 1)

III. Gauge non-Invariance of Pauli Hamiltonian Hydrogen Eigenenergies

$$H_D\psi = \imath\frac{\partial}{\partial t}\psi$$

$$= E\psi$$

$$A_0 = \frac{Z}{r}$$

$$H_D = \vec{\gamma}\cdot(\vec{p} - e\vec{A}) + m - eA_0$$

$$\vec{J} = \vec{L} + \vec{S}$$

$$= \vec{r}\times\vec{p} + \frac{1}{2}\vec{\Sigma}$$

$$\vec{\Sigma} = \begin{bmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{bmatrix}$$

Bjorken & Drell, Vol.1 (1964) p.52

T. Goldman, *Phys. Rev*. D15 (1977) 1063.

See also: Wei-min Sun: Time Evolution Op $\neq \int T_{00}$

$$i\frac{\partial\psi'}{\partial t} = \left(UHU^{-1} - iU\frac{\partial U^{-1}}{\partial t}\right)\psi' \equiv H'\psi'$$

$$U = \exp[-iHf(t)]$$

$$H_P = UH_DU^{-1} - iU\frac{\partial}{\partial t}U^{-1}$$

$$\int d^3x\, \psi'^{\dagger}H'\psi' = (1+\dot{f})\sum_n |c_n|^2 E_n$$

$$\neq \sum_n |c_n|^2 E_n = \int d^3x\, \psi^{\dagger}H\psi$$

Pauli and Dirac Hamiltonians are not unitarily equivalent

$$U = \exp[-iHf(t)] \rightarrow U = e^{[\beta\vec{\alpha}\cdot(\vec{p}-e\vec{A})/2m]}$$

Foldy-Wouthuysen

$$H_P = UH_DU^{-1} - iU\frac{\partial}{\partial t}U^{-1}$$

$$\vec{E} = -\vec{\nabla}A_0 - \frac{\partial\vec{A}}{\partial t}$$

No problem only if:

$$\vec{\nabla}\cdot\vec{A} = 0$$

$$\vec{\sigma}\cdot\frac{\partial\vec{A}}{\partial t}\times\vec{p} = 0$$

$$\vec{\sigma}\cdot\vec{\nabla}\times\frac{\partial\vec{A}}{\partial t} = 0$$

$$\simeq \beta\left[m + \frac{(\vec{p}-e\vec{A})^2}{2m}\right] - eA_0 - \frac{e}{2m}\beta\vec{\sigma}\cdot\vec{B}$$

$$-\frac{ie}{8m^2}\vec{\sigma}\cdot\vec{\nabla}\times\vec{E} - \frac{e}{4m^2}\vec{\sigma}\cdot\vec{E}\times\vec{p} - \frac{e}{2m^2}\vec{\nabla}\cdot\vec{E}$$

i.e., OK only in Coulomb gauge

Hydrogen atom (and em multipole radiation) has the same problem

We use canonical momentum and orbital angular momentum, even though not gauge invariant, in the Hydrogen atom. The Hamiltonian itself of the hydrogen atom used in the Schroedinger equation is not gauge invariant.

IV. Gauge invariance and canonical angular momentum

Straightforward angular momentum decomposition not gauge invariant:

$$\vec{J}_{QED} = \vec{S}_e + \vec{L}_e + \vec{S}_\gamma + \vec{L}_\gamma$$

$$\vec{S}_e = \int d^3x \, \psi^\dagger \frac{\vec{\Sigma}}{2} \psi$$

$$\vec{L}_e = \int d^3x \, \psi^\dagger \vec{x} \times \frac{1}{i} \vec{\nabla} \psi$$

$$\vec{S}_\gamma = \int d^3x \, \vec{E} \times \vec{A}$$

$$\vec{L}_\gamma = \int d^3x \, \vec{x} \times E^i \vec{\nabla} A^i$$

BUT gauge invariant form does not obey canonical commutation relations:

$$\vec{J}_{QED} = \vec{S}_e + \vec{L}'_e + \vec{J}'_\gamma$$

$$\vec{S}_e = \int d^3x \, \psi^\dagger \frac{\vec{\Sigma}}{2} \psi$$

$$\vec{L}'_e = \int d^3x \, \psi^\dagger \vec{x} \times \frac{1}{i} \vec{D} \psi$$

$$\vec{J}'_\gamma = \int d^3x \, \vec{x} \times (\vec{E} \times \vec{B})$$

Therefore, despite the labels, $\vec{L}'_e$ and $\vec{J}'_\gamma$ are NOT angular momenta!

QM example: $[(\vec{x} \times \frac{1}{i}\vec{\nabla})_j, (\vec{x} \times \frac{1}{i}\vec{\nabla})_k] = \imath\epsilon_{jkl}[\vec{x} \times \frac{1}{i}\vec{\nabla}]_l$

Using the gauge invariant “mechanical” momentum generates an extra term

$$[(\vec{x} \times \frac{1}{i}(\vec{\nabla} - ie\vec{A}))_j, (\vec{x} \times \frac{1}{i}(\vec{\nabla} - ie\vec{A}))_k]$$
$$= \imath\epsilon_{jkl}\{[\vec{x} \times \frac{1}{i}(\vec{\nabla} - ie\vec{A})]_l + ex_l\vec{x} \cdot (\vec{\nabla} \times \vec{A})\}$$

But OK if we define a part of the vector field as $\vec{A} = \vec{A}_{pur}$

such that $\vec{\nabla} \times \vec{A}_{pur} = 0$

See, e.g.: D. Singleton and V. Dzhunushaliev, *Found. Phys.* **30** (2000) 1093.

Both requirements can be satisfied by identifying physical and pure gauge parts of the gauge field:

$$\vec{A} \equiv \vec{A}_{fys} + \vec{A}_{pur}\,,\; \vec{D}_{pur} \equiv \vec{\nabla} - \imath e \vec{A}_{pur}$$

$$\vec{\nabla} \cdot \vec{A}_{fys} = 0\,,\; \vec{\nabla} \times \vec{A}_{pur} = 0$$

$$\begin{aligned}
\vec{J}_{QED} &= \vec{S}_e + \vec{L}_e'' + \vec{S}_\gamma'' + \vec{L}_\gamma'' \\
\vec{S}_e &= \int d^3x\, \psi^\dagger \frac{\vec{\Sigma}}{2} \psi \\
\vec{L}_e'' &= \int d^3x\, \psi^\dagger \vec{x} \times \frac{1}{\imath} \vec{D}_{pur} \psi \\
\vec{S}_\gamma'' &= \int d^3x\, \vec{E} \times \vec{A}_{fys} \\
\vec{L}_\gamma'' &= \int d^3x\, \vec{x} \times E^i \vec{\nabla} A^i_{fys}
\end{aligned}$$

NOT Coulomb gauge: $\vec{\nabla} \cdot \vec{A} \neq 0$

$$\vec{A} \equiv \vec{A}_{fys} + \vec{A}_{pur}$$ only $$\vec{\nabla} \cdot \vec{A}_{fys} = 0$$

This defines $\vec{A}_{pur}$ piece

$$\begin{aligned} -\vec{E}_{pur} &= F^{i0}_{pur} \\ &= \partial^i A^0_{pur} - \partial^0 A^i_{pur} \\ &= 0 \end{aligned}$$

Full constraint:
$F^{\mu\nu}_{pur} = 0$

$$-(\vec{\nabla})^2 A^0_{pur} - \partial_t \vec{\nabla} \cdot \vec{A}_{pur} = 0$$

So $\vec{A}_{pur}$ does not contribute to charge either: $\vec{\nabla} \cdot \vec{E}_{pur} = 0$

Cf.: Momentum operator in quantum mechanics

$$\vec{p} = m\dot{\vec{r}} + q\vec{A} = m\dot{\vec{r}} + q\vec{A}_{\perp} + q\vec{A}_{||}$$

$$\vec{p} - q\vec{A}_{||} = m\dot{\vec{r}} + q\vec{A}_{\perp}$$

$$\vec{\nabla} \cdot \vec{A}_{\perp} = 0 \quad \vec{\nabla} \times \vec{A}_{||} = 0$$

Generalized momentum for a charged particle moving in EM field:

➥1st form is not gauge invariant, but satisfies the canonical momentum commutation relation.

➥2nd form is both gauge invariant and the canonical momentum commutation relation is satisfied.

We recognize

$$\vec{D}_{pur} = \vec{p} - q\vec{A}_{||} = \frac{1}{i}\vec{\nabla} - q\vec{A}_{||}$$

as the physical momentum.

It is neither the canonical momentum:

$$\vec{p} = m\dot{\vec{r}} + q\vec{A} = \frac{1}{i}\vec{\nabla}$$

nor the mechanical momentum:

$$\vec{p} - q\vec{A} = m\dot{\vec{r}} = \frac{1}{i}\vec{D}$$

Gauge transformation

$$\psi' = e^{\imath q\omega(x)}\psi\,, \qquad A'_\mu = A_\mu + \partial_\mu\omega(x),$$

only affects the longitudinal part of the vector potential:

$$A'_{||} = A_{||} + \nabla\omega(x),$$

and the time component:

$$\phi' = \phi - \partial_t\omega(x).$$

It does not affect the transverse part:

$$A'_\perp = A_\perp,$$

so $A_\perp$ is physical.

Hamiltonian of hydrogen atom

Coulomb gauge:

$$\vec{A}_{//}^{c} = 0, \qquad \vec{A}_{\perp}^{c} \neq 0, \qquad A_0^c = \varphi^c \neq 0.$$

Hamiltonian of a nonrelativistic particle:

$$H_c = \frac{(\vec{p} - q\vec{A}_{\perp}^{c})^2}{2m} + q\varphi^c.$$

Gauge transformed becomes:

$$\vec{A}_{//} = \vec{A}_{//}^{c} + \vec{\nabla}\omega(x) = \vec{\nabla}\omega(x), \vec{A}_{\perp} = \vec{A}_{\perp}^{c}, \varphi = \varphi^c - \partial_t \omega(x)$$

$$H = \frac{(\vec{p} - q\vec{A})^2}{2m} + q\varphi = \frac{(\vec{p} - q\vec{\nabla}\omega - q\vec{A}_{\perp}^{c})^2}{2m} + q\varphi^c - q\partial_t \omega.$$

Following this recipe, we introduce a new Hamiltonian:

$$H_{fys} = H + q\partial_t\omega(x) = \frac{(\vec{p} - q\vec{\nabla}\omega - q\vec{A}_\perp^c)^2}{2m} + q\varphi^c$$

The matrix elements are gauge invariant, i.e.,

$$\langle\psi \mid H_{fys} \mid\psi\rangle = \langle\psi^c \mid H_c \mid\psi^c\rangle$$

i.e., the hydrogen energy states calculated in Coulomb gauge are both gauge invariant and physical.

See also Wei-min Sun.

Coulomb gauge Lorentz invariant: $\partial_k[A^k, J^{ab}] = 0$

-- E. B. Manoukian, *J. Phys. G: Nucl.Phys*. **13** (1987) 1013.

QED:

$$\vec{A}_{pur} = \vec{A} - \vec{A}_{fys}$$

$$F^{\mu\nu}_{pur} = 0 \ ; \ F^{\mu\nu}_{fys} = F^{\mu\nu}$$

$$\vec{\nabla} \times \vec{A}_{fys} = \vec{\nabla} \times \vec{A}$$

$$\vec{\nabla} \cdot \vec{A}_{fys} = 0 \ ; \ \vec{A}_{fys}(|x| \to \infty) = 0$$

$$\vec{A}_{fys}(x) = \vec{\nabla} \times \frac{1}{4\pi} \int d^3y \frac{\vec{\nabla} \times \vec{A}(y)}{|\vec{x} - \vec{y}|}$$

$$\vec{A}'_{fys} = \vec{A}_{fys} \ ; \ \vec{A}'_{pur} = \vec{A}'_{pur} - \vec{\nabla}\omega$$

$$A^0_{fys}(x) = \int_{-\infty}^{x} dx^i (\partial_i A^0 + \partial_t A^i - \partial_t A^i_{fys})$$

$$\phi(x) = -\frac{1}{4\pi} \int d^3y \frac{\vec{\nabla} \cdot \vec{A}(y)}{|\vec{x} - \vec{y}|} + \phi_0(x)$$

$$\vec{A}_{pur} = -\vec{\nabla}\phi(x) \ ; \ A^0_{pur} = \partial_t \phi(x) \ ; \ \nabla^2 \phi_0(x) = 0$$

Multipole Radiation

Multipole radiation analysis is based on the decomposition of EM vector potential in Coulomb gauge. The results are physical and gauge invariant, i.e., gauge transformed to other gauges one obtains the same results.

$2P_{3/2} \rightarrow 2P_{1/2} \leftrightarrow$ spin-flip

$2P_{1/2} \rightarrow 1S_{1/2} \leftrightarrow \Delta L$ of 1

Similarly in Dalitz plot analysis to determine particle spin.

V.Gauge Invariance and canonical commutation relation in QCD (for nucleon spin operators)

From the QCD Lagrangian, one can obtain the total angular momentum by a Noether theorem:

$$\vec{J} = \vec{S}_q + \vec{L}_q + \vec{S}_g + \vec{L}_g$$

$$\vec{S}_q = \int d^3x\, \psi^\dagger \frac{1}{2}\vec{\Sigma}\,\psi$$

$$\vec{L}_q = \int d^3x\, \psi^\dagger\, (\vec{x} \times \frac{1}{i}\vec{\nabla})\psi$$

$$\vec{S}_g = 2\int d^3x\, \mathrm{Tr}\,\{\vec{E} \times \vec{A}\}$$

$$\vec{L}_g = 2\int d^3x\, \mathrm{Tr}\,\{\vec{x} \times E^i\, \vec{\nabla} A^i\}$$

- Each term in this decomposition satisfies the canonical angular momentum algebra, so they may properly be called, respectively, quark spin, quark orbital angular momentum, gluon spin and gluon orbital angular momentum operators.
- However they are not individually gauge invariant, except for the quark spin.

→ physical meaning obscure

A Gauge Invariant Decomposition:

$$\vec{J} = \vec{S}_q + \vec{L}'_q + \vec{J}'_G$$

$$\vec{S}_q = \frac{1}{2}\int d^3x\, \psi^\dagger \vec{\Sigma} \psi$$

$$\vec{L}'_q = \int d^3x\, \psi^\dagger \vec{x} \times \frac{\vec{D}}{i} \psi$$

$$\vec{J}'_G = 2\int d^3x\, \{\vec{x} \times (\vec{E}^a \times \vec{B}^a)\}$$

- These terms do not separately satisfy the canonical angular momentum algebra (except the quark spin). In this sense the second and third terms are not quark orbital and gluon angular momentum operators.
- The physical meaning of these operators is obscure also.
- Gluon spin and orbital angular momentum operators are not separately gauge invariant; only the total angular momentum of the gluon is gauge invariant.

(Similarly for the photon, but we do have polarized photon beams!)

Our Solution - A different decomposition: Gauge invariance and angular momentum algebra both satisfied for individual terms. Key point is to separate out the transverse and longitudinal parts of the gauge field.

Essential task: to separate properly

the pure gauge field: $\vec{A}_{pur}$

from the physical one: $\vec{A}_{fys}$

$$\vec{A} = \vec{A}_{pur} + \vec{A}_{fys} \qquad \vec{A}_{\Box} = T^a \vec{A}^a_{\Box}$$

Fundamental: $\vec{D}_{pur} = \vec{\nabla} - ig\vec{A}_{pur}$

$$\vec{D}_{pur} \times \vec{A}_{pur} = \vec{\nabla} \times \vec{A}_{pur} - ig\vec{A}_{pur} \times \vec{A}_{pur} = 0$$

Adjoint: $\vec{\mathcal{D}}_{pur} = \vec{\nabla} - ig[\vec{A}_{pur}, \ ]$

$$\vec{\mathcal{D}}_{pur} \cdot \vec{A}_{fys} = \vec{\nabla} \cdot \vec{A}_{fys} - ig[A^i_{pur}, A^i_{fys}] = 0$$

QCD:

$$\vec{\nabla}\cdot\vec{A}_{fys} = \imath g[A^i - A^i_{fys}, A^i_{fys}] = \imath g[A^i, A^i_{fys}]$$

$$\vec{\nabla}\times\vec{A}_{fys} = \vec{\nabla}\times\vec{A} = \imath g(A^i - A^i_{fys})\times(A^i - A^i_{fys})$$

$$\partial_t A^0_{fys} = \partial_i A^0 + \partial_t(A^i - A^i_{fys}) - \imath g[A^i - A^i_{fys}, A^0 - A^0_{fys}]$$

Solve perturbatively:

$$\vec{\nabla}\times\vec{A}_{pur} = ig\vec{A}_{pur}\times\vec{A}_{pur}$$

$$\vec{\nabla}\cdot\vec{A}_{pur} = \vec{\nabla}\cdot\vec{A} - ig[A^i_{pur}, A^i]$$

$$\partial_i A^0_{pur} = -\partial_t A^i_{pur} + ig[A^i_{pur}, A^0_{pur}]$$

Gauge transformation:

$$\vec{A}'_{fys} = U\vec{A}_{fys}U^\dagger$$

$$\vec{A}'_{pur} = U\vec{A}_{pur}U^\dagger - \frac{i}{g}U\vec{\nabla}U^\dagger$$

New decomposition

$$\vec{J}_{QCD} = \vec{S}_q + \vec{L}''_q + \vec{S}''_g + \vec{L}''_g$$

$$\vec{S}_q = \int d^3x\,\psi^\dagger \frac{\vec{\Sigma}}{2}\psi$$

$$\vec{L}''_q = \int d^3x\,\psi^\dagger\,\vec{x}\times\frac{1}{i}\vec{D}_{pur}\psi$$

$$\vec{S}''_g = \int d^3x\,\vec{E}\times\vec{A}_{fys}$$

$$\vec{L}''_g = \int d^3x\,\vec{x}\times E^i\,\vec{\mathcal{D}}_{pur}A^i_{fys}$$

We have chosen a separation between physical and gauge pieces of the gauge vector potential and consistently separated the gauge boson and fermion degrees of freedom in the interacting case.

Alternative Proposals

- Jaffe-Manohar -- light cone; helicity only[†]
- Ji -- gauge invariant but violates commutation algebra
- Wakamatsu -- different apportionment of decomposition violates commutation algebra; frame independence conflicts with Lorentz (J is covariant; boost shifts rest frame spin to orbital angular momentum.)
- Cho-Ge-Zhang -- valence/binding decomposition violates commutation algebra
- Leader -- gauge invariant but violates commutation algebra

< Experiment does not prefer other sources: $\Delta S \sim 0 \ , \ \Delta G \sim 0$ >

[†]Cf.: Sivers effect: QM includes P-wave even in IMF (as required by Lorentz)

[Recent: Zhou-Huang -- $\vec{E} \longrightarrow \vec{E}_\perp$ in gauge orbital.]

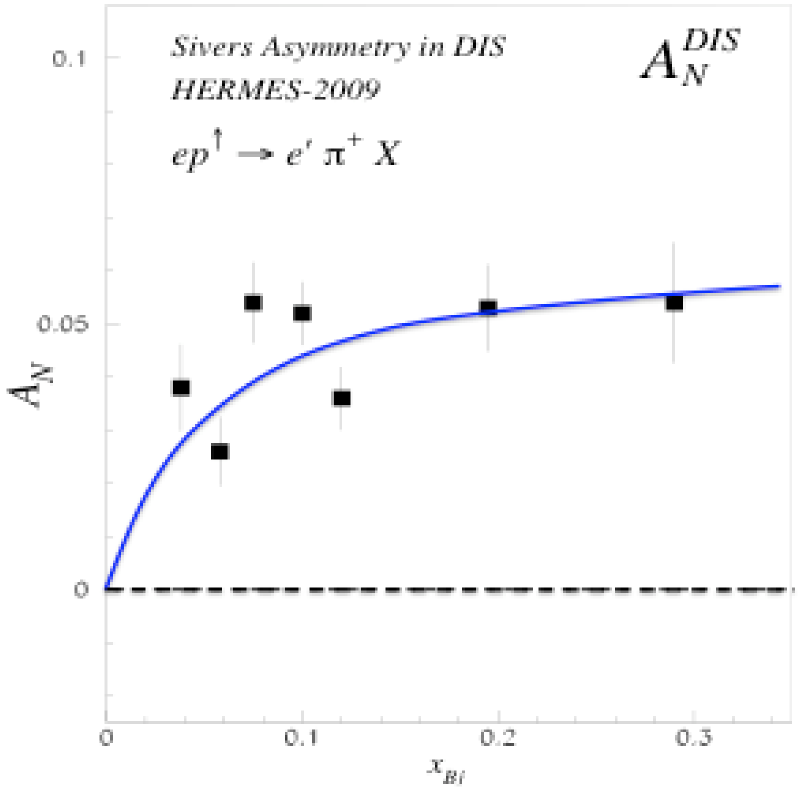
{See also: Wong, Wang, Sun, Lü}

VI. Conclusion

- The physical component of a vector gauge field can be identified in a gauge covariant fashion.
- The gauge covariant derivatives needed to extract orbital angular momentum (and mechanical momentum) of fermions coupled to the gauge field must include only the non-physical, pure gauge part of the vector gauge field so that:
- Both gauge invariance and canonical commutation relations are satisfied in order to allow physical interpretation of the matrix elements of these operators.

Additional Pages

How can there be transverse orbital motion
in the Infinite Momentum Frame? (Large, Finite)



Sivers Effect

Quantum Mechanics:

P-wave without
classical motion

DVCS

Experimentally, there is!

from Bass review: arXiv:hep-ph/0411005v2 10 Jun 2005

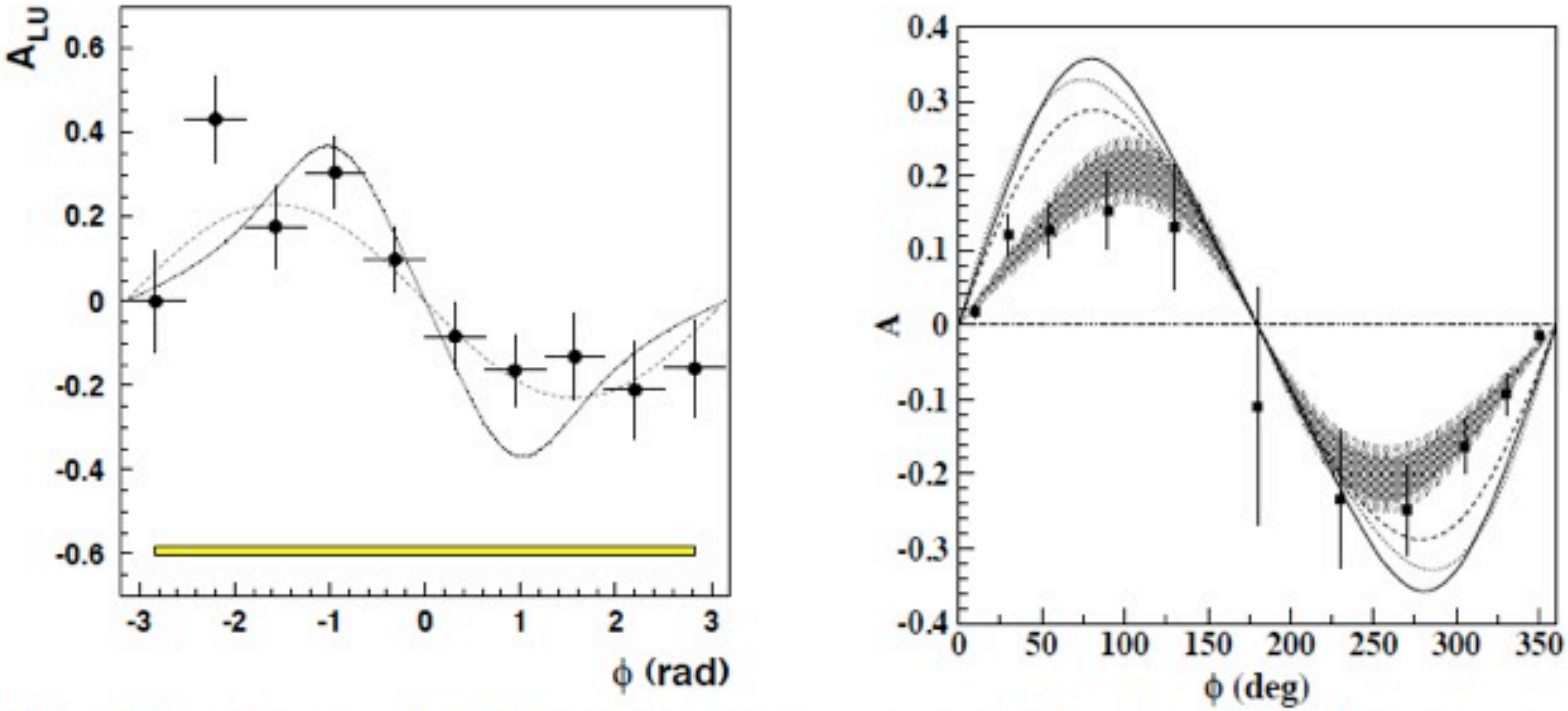


FIG. 21 Recent data from HERMES (left) and the CLAS experiment at Jefferson Laboratory (right) in the realm of DVCS Bethe-Heitler interference. The sin ϕ azimuthal dependence of the single spin asymmetry is clearly visible in the data (Airapetian *et al.*, 2001; Stepanyan *et al.*, 2001).

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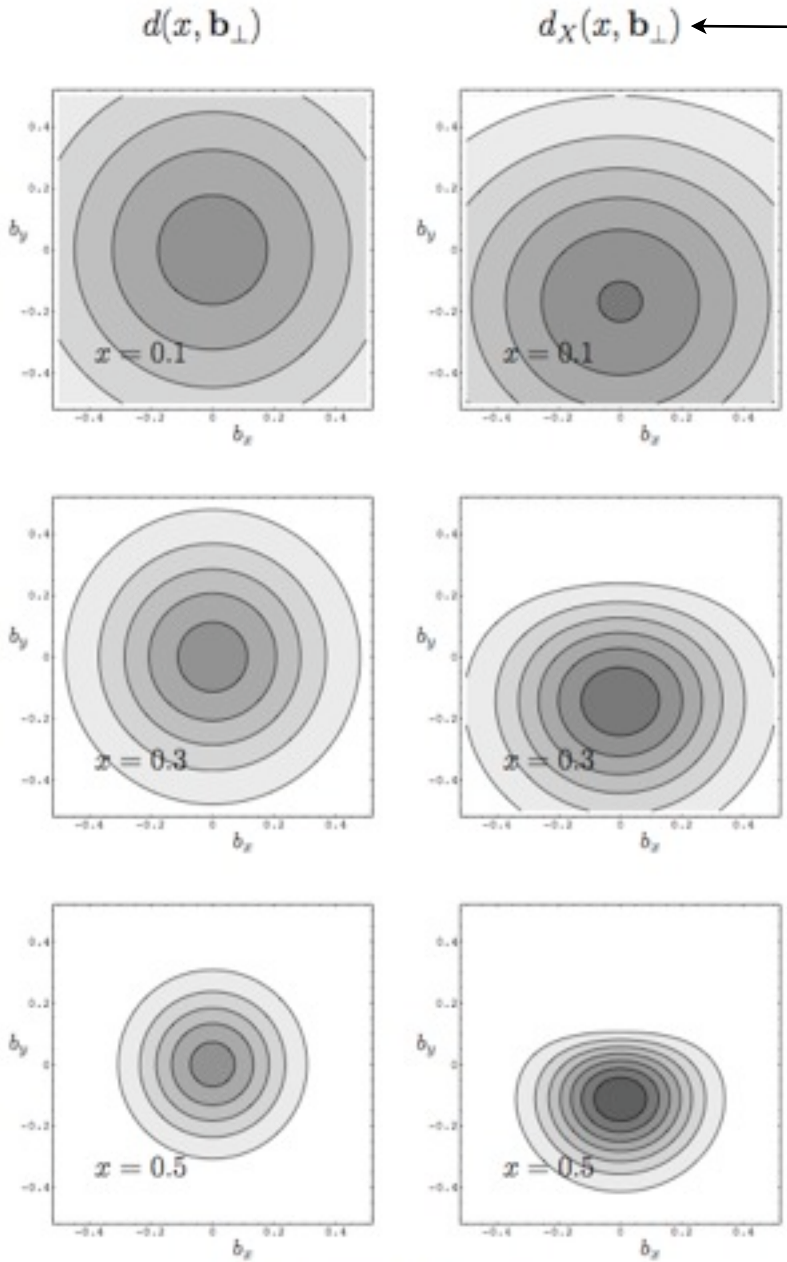

Fig. 4. Same as Fig. 3, but for d quarks.

for transversely polarized proton

M. Burkhardt, IJMPA 18 (2003) 173

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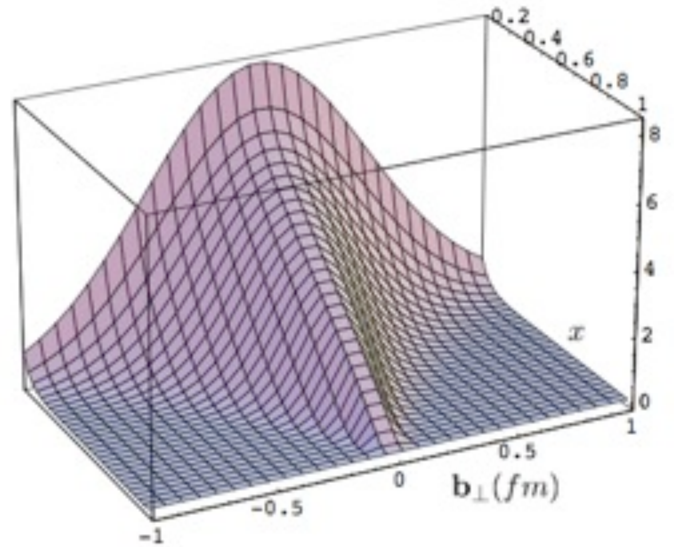
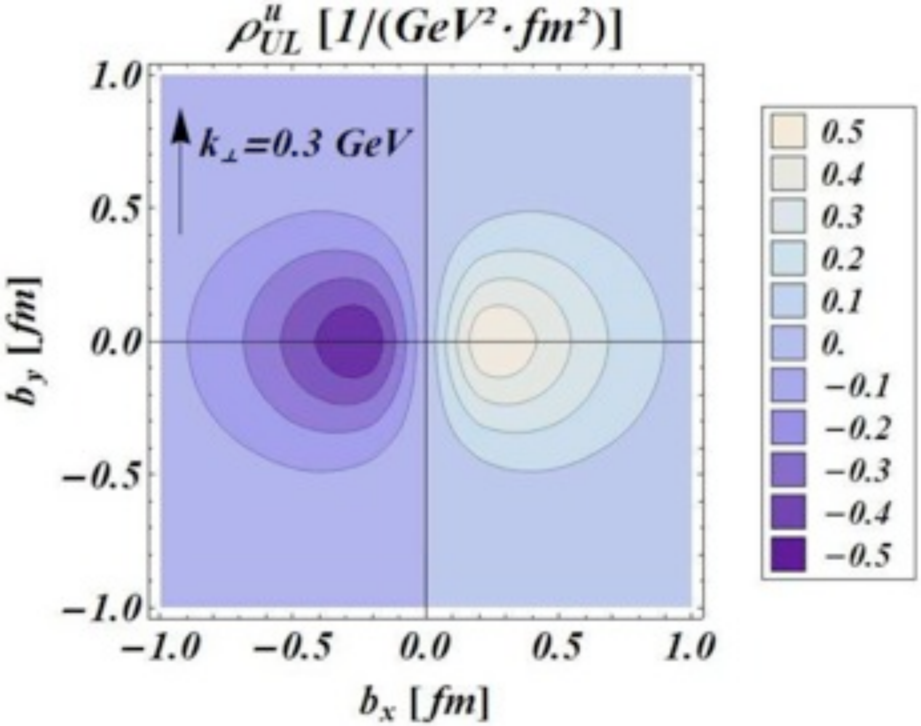


Fig. 1. Impact parameter dependent parton distribution $u(x, \mathbf{b}_\perp)$ for the simple model (31).

Lorce, Pasquini arXiv:1106.0139

Hagler et al., arXiv:0908.1283



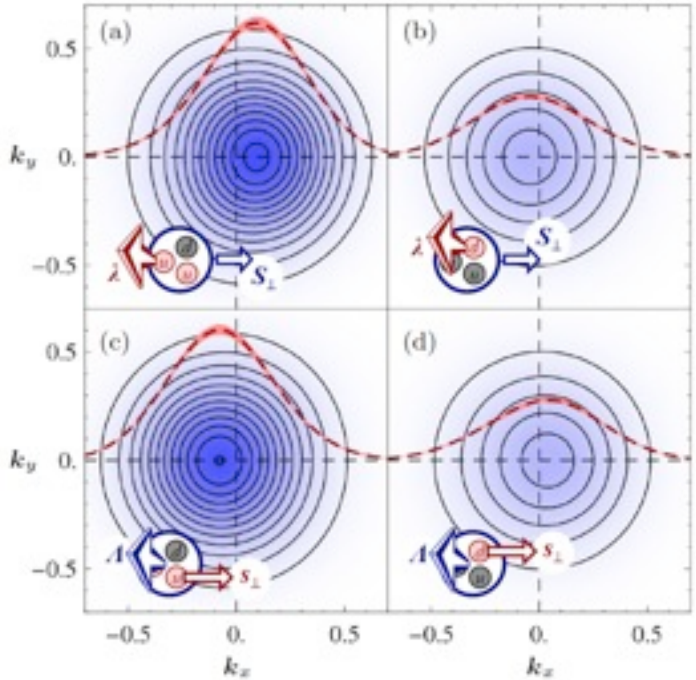


FIG. 3: Quark densities in the $k_\perp$-plane, for $m_\pi \approx 500$ MeV. (a) ρ_L for u-quarks and $\lambda = 1$, $S_\perp = (1, 0)$, (b) the same for d-quarks, (c) ρ_T for u-quarks and $\Lambda = 1$, $s_\perp = (1, 0)$, (d) the same for d-quarks. The error bands show the density profile at $k_y = 0$ as a function of k_x (scale not shown).

see also: G.A. Miller, arXiv:0802.3731v1

Quark and Gluon momentum contributions are also affected by these considerations:

PRL 103, 062001 (2009) PHYSICAL REVIEW LETTERS week ending 7 AUGUST 2009

Do Gluons Carry Half of the Nucleon Momentum?

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We examine the conventional picture that gluons carry about half of the nucleon momentum in the asymptotic limit. We show that this large fraction is due to an unsuitable definition of the gluon momentum in an interacting theory. If defined in a gauge-invariant and consistent way, the asymptotic gluon momentum fraction is computed to be only about one-fifth. This result suggests that the asymptotic limit of the nucleon spin structure should also be reexamined. A possible experimental test of our finding is discussed in terms of novel parton distribution functions.

$$\mathcal{P}_{q/h}(\xi) = \frac{1}{2}\int_{-\infty}^{\infty}\frac{dx^-}{2\pi}e^{-\imath\xi P^+ x^-} < \bar{\psi}(0,x^-,0_\perp)\gamma^+ \mathcal{P}e^{\imath g\int_0^{x^-} dy^- A^+(0,y^-,0_\perp)}\psi(0) >_h$$

$$\int_{-\infty}^{\infty} d\xi \xi \mathcal{P}_{q/h}(\xi) = \frac{1}{2(P^+)^2} < \bar{\psi}\gamma^+ \imath D^+ \psi >_h$$

$$\mathcal{P}_{q/h}(\xi) = \frac{1}{2}\int_{-\infty}^{\infty}\frac{dx^-}{2\pi}e^{-\imath\xi P^+ x^-} < \bar{\psi}(0,x^-,0_\perp)\gamma^+ \mathcal{P}e^{\imath g\int_0^{x^-} dy^- A^+_{pur}(0,y^-,0_\perp)}\psi(0) >_h$$

$$\int_{-\infty}^{\infty} d\xi \xi \mathcal{P}_{q/h}(\xi) = \frac{1}{2(P^+)^2} < \bar{\psi}\gamma^+ \imath D^+_{pur} \psi >_h$$

$$\mathcal{P}_{g/h}(\xi) = \frac{1}{\xi P^+}\int_{-\infty}^{\infty}\frac{dx^-}{2\pi}e^{-\imath\xi P^+ x^-} < F^{+\nu}(0,x^-,0_\perp)\mathcal{P}e^{\imath g\int_0^{x^-} dy^- A^+(0,y^-,0_\perp)}F_\nu{}^+(0) >_h$$

$$\mathcal{P}_{g/h}(\xi) = \frac{1}{\xi P^+}\int_{-\infty}^{\infty}\frac{dx^-}{2\pi}e^{-\imath\xi P^+ x^-} < F^{+i}(0,x^-,0_\perp)\mathcal{P}e^{\imath g\int_0^{x^-} dy^- A^+_{pur}(0,y^-,0_\perp)}A^i_{fys}(0) >_h$$

Polarized glue:

$$\mathcal{P}_{\Delta g/h}(\xi) = \frac{1}{\xi P^+}\int_{-\infty}^{\infty}\frac{dx^-}{2\pi}e^{-\imath\xi P^+ x^-} < F^{+i}(0,x^-,0_\perp)\mathcal{P}e^{\imath g\int_0^{x^-} dy^- A^+_{pur}(0,y^-,0_\perp)}\epsilon_{ij+}A^j_{fys}(0) >_h$$

Conventional gluon momentum defintion:

$$\int d^3x\, \vec{E} \times \vec{B}$$

$$\gamma^{\mathscr{P}} = -\frac{\alpha_s}{4\pi} \begin{pmatrix} -\frac{8}{9}n_g & \frac{4}{3}n_f \\ \frac{8}{9}n_g & -\frac{4}{3}n_f \end{pmatrix}$$

$$\hookrightarrow \quad \vec{\mathscr{P}}_g^R = \frac{2n_g}{2n_g + 3n_f} \vec{P}_{\text{total}}$$

becomes

$$\int d^3x\, E^i \vec{\mathcal{D}}_{pur} A^i_{fys}$$

$$\gamma^P = -\frac{\alpha_s}{4\pi} \begin{pmatrix} -\frac{2}{9}n_g & \frac{4}{3}n_f \\ \frac{2}{9}n_g & -\frac{4}{3}n_f \end{pmatrix}$$

$$\hookrightarrow \quad \vec{P}_g^R = \frac{\frac{1}{2}n_g}{\frac{1}{2}n_g + 3n_f} \vec{P}_{\text{total}}.$$

for $n_f = 5$:
gluon momentum fraction

1/2 ➡ 1/5

There is no proton spin crisis but only quark spin-axial charge confusion

The quark spin contributions measured in DIS are:

$$\Delta u + \Delta d + \Delta s$$

$$= \begin{cases} 0.82(6) - 0.44(6) - 0.10(7) & = & 0.29(19) \\ 0.80(2) - 0.46(2) - 0.12(2) & = & 0.23(6) \\ 0.82(4) - 0.44(4) - 0.11(4) & = & 0.27(12) \end{cases} \quad Q^2 = \begin{cases} 10 \\ 5 \quad GeV^2. \\ 3 \end{cases}$$

while the pure valence q^3 S-wave quark model calculated values are:

$$\Delta u = \frac{4}{3}, \Delta d = -\frac{1}{3}, \Delta s = 0$$

More recent values for sum:

$$\Sigma = 0.330 \pm 0.011(\text{thry}) \pm 0.025(\text{exp}) \pm 0.028(\text{evol}) \quad \text{Hermes}$$

$$\Sigma = 0.33 \pm 0.03(\text{stat}) \pm 0.05(\text{syst}) \qquad \text{COMPASS}.$$

There appear to be two contradictions between these two results:

1. The total quark spin contribution to nucleon spin measured by DIS is ~ 1/3 while the quark model value is 1;
2. The strange quark contribution measured in DIS is nonzero while the quark model value is zero. (A new measurement gives a smaller strange contribution.)

- To clarify, first recognize that the value measured in DIS is the matrix element of the quark axial-vector current operator in a nucleon state:

$$2a_0 S^\mu =< ps| \int d^3x \, \bar{\psi} \, \gamma^\mu \gamma^5 \, \psi \, |ps >$$

Here, $a_0 = \Delta u + \Delta d + \Delta s$ which is not the quark spin contribution calculated in the CQM. The value calculated in the CQM is the matrix element of the Pauli spin part only.

The axial-vector current operator can be expanded as:

$$\int d^3x \bar{\psi}\vec{\gamma}\gamma^5\psi = \sum_{i\lambda\lambda'} \int d^3k \chi_\lambda^\dagger \vec{\sigma}\chi_{\lambda'}(a^\dagger_{i\vec{k}\lambda}a_{i\vec{k}\lambda'} - b^\dagger_{i\vec{k}\lambda'}b_{i\vec{k}\lambda})$$

$$- \sum_{i\lambda\lambda'} \int d^3k \chi_\lambda^\dagger \frac{\vec{\sigma}\cdot\vec{k}}{k_0(k_0+m_i)} i\vec{\sigma}\vec{k}\chi_{\lambda'}$$

$$\times (a^\dagger_{i\vec{k}\lambda}a_{i\vec{k}\lambda'} - b^\dagger_{i\vec{k}\lambda'}b_{i\vec{k}\lambda})$$

$$+ \sum_{i\lambda\lambda'} \int d^3k \chi_\lambda^\dagger \frac{i\vec{\sigma}\times\vec{k}}{k_0} \chi_{\lambda'} a^\dagger_{i\vec{k}\lambda} b^\dagger_{i-\vec{k}\lambda'} + \text{H.c.}$$

Spin is 1/2 of this.

- Only the first term of the axial-vector current operator, which is the Pauli spin part, has been calculated in non-relativistic quark models.
- The second term, the relativistic correction, has not been included in non-relativistic quark model calculations. The relativistic quark model does include this correction and it reduces the quark spin contribution by about 25%.
- The third term, $q\bar{q}$ creation and annihilation, does not contribute in a model with only valence quark configurations and so it has not been calculated in any quark model to our knowledge.

An Extended CQM with Sea Quark Components

- To understand nucleon spin structure quantitatively within the CQM and to clarify the quark spin-axial vector confusion further a CQM was developed with sea quark components:

$$|N >= c_0|q^3 > +\Sigma C_{\alpha\beta}|(q^3)_\alpha(q\bar{q})_\beta >$$

PHYSICAL REVIEW D, VOLUME 58, 114032

Is nucleon spin structure inconsistent with the constituent quark model?

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(Received 23 February 1998; published 9 November 1998)

TABLE III. The spin contents of the proton.

	q^3	$q^3-q^4\bar{q}$	$q^4\bar{q}-q^4\bar{q}$	sum	exp.	lattice [9]	lattice [9,15]
Δu	0.773	−0.125	0.100	0.75	0.80	0.79(11)	0.638(54)
Δd	−0.193	−0.249	−0.041	−0.48	−0.46	−0.42(11)	−0.347(46)
Δs	0	−0.064	−0.002	−0.07	−0.12	−0.12(1)	−0.109(30)

TABLE I. Proton model wave function.

q^3	$N\eta$	$N\pi$	$\Delta\pi$	$N\eta'$	ΛK	ΣK	$\Sigma^* K$
−0.923	0.044	0.232	−0.252	0.065	0.109	−0.036	−0.106

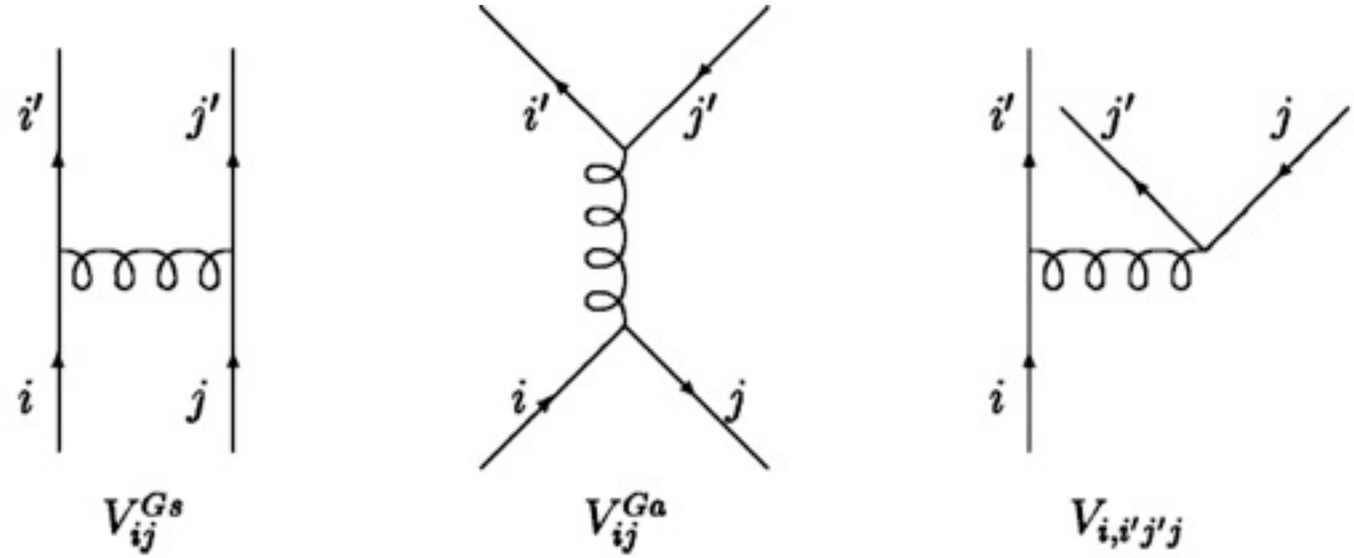


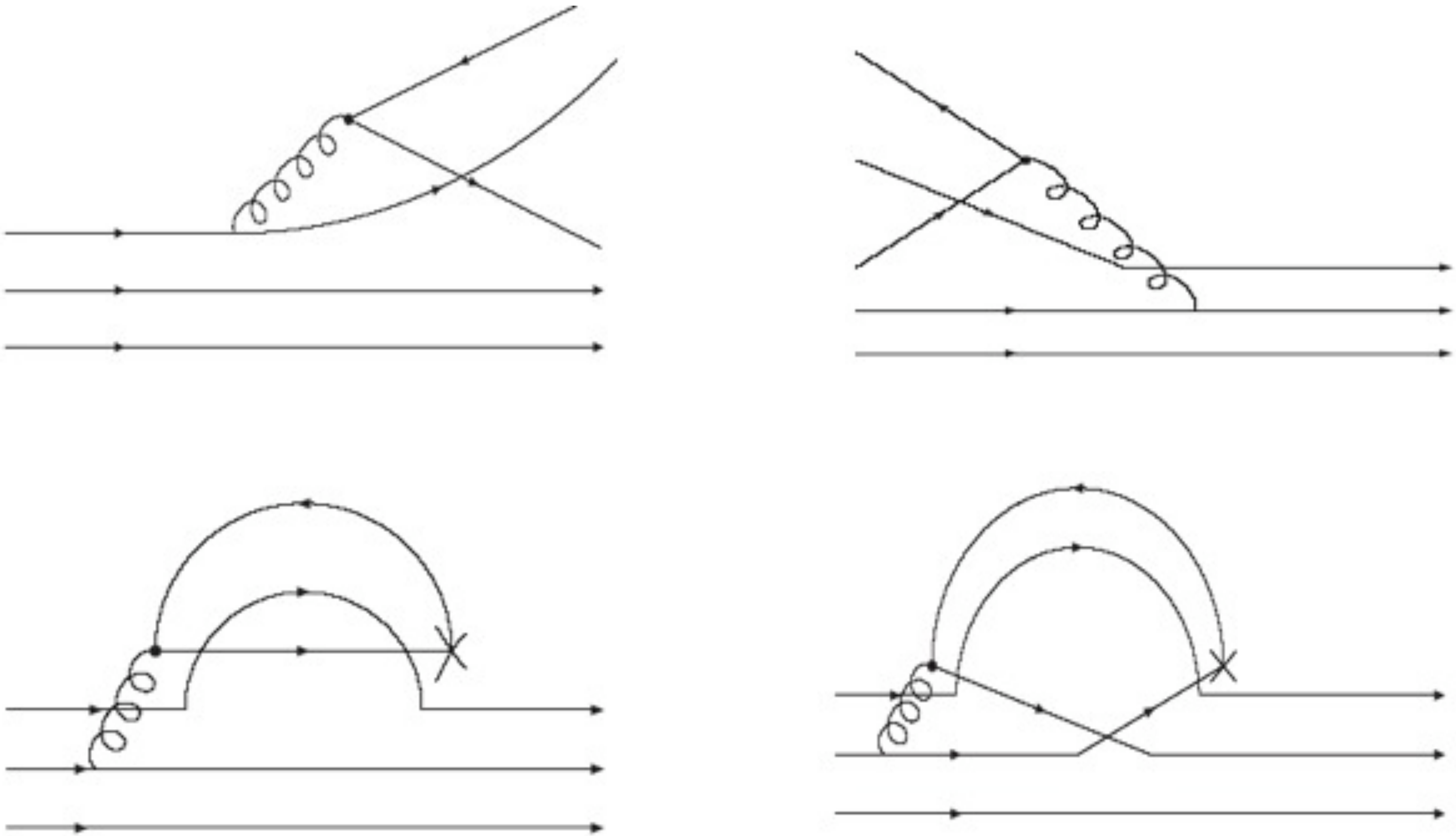
FIG. 2. Quark interaction diagrams.

TABLE II. Masses and magnetic moments of the baryon octet and decuplet. $m=330$ (MeV), $m_s=564$ (MeV), $b=0.61$ (fm), $\alpha_s=1.46$, $a_c=48.2$ (MeV fm^{-2}).

		p	n	Λ	Σ^+	Σ^-	Ξ^0	Ξ^-	Δ	Σ^*	Ξ^*	Ω
	M(Mev)	939		1116	1193		1346		1232	1370	1523	1659
Theor.	E1(MeV)	2203		2323	2306		2409		2288	2306	2450	2638
	$\mu(\mu_N)$	2.780	−1.818	−0.522	2.652	−1.072	−1.300	−0.412				
	$\sqrt{\langle r^2\rangle}$(fm)	0.802	0.124	← Improved over Isgur-Karl (all other results almost identical)								
	M(MeV)	939		1116	1189		1315		1232	1385	1530	1672
Exp.	$\mu(\mu_N)$	2.793	−1.913	−0.613	2.458	−1.160	−1.250	−0.651				
	$\sqrt{\langle r^2\rangle}$(fm)	0.836	0.34									

NOTE: $^3S_1$ NOT $^3P_0$ -- Vector Gluons, not 0^+ pairs

Coupling between 3-quark and 5-quark sectors



$$H=\sum_i \left(m_i+\frac{p_i^2}{2m_i}\right)+\sum_{i<j}\left(V_{ij}^c+V_{ij}^G\right)$$

$$+\sum_{i<j}\left(V_{i,i'j'j}+V_{i,i'j'j}^{\dagger}\right),$$

$$V_{ij}^c=-a_c\vec{\lambda}_i\cdot\vec{\lambda}_j r_{ij}^2,$$

$$V_{ij}^G=V_{ij}^{Gs}+V_{ij}^{Ga},$$

$$V_{ij}^{Gs}=\alpha_s\frac{\vec{\lambda}_i\cdot\vec{\lambda}_j}{4}$$

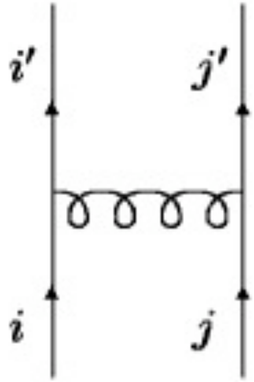
$$\times\left[\frac{1}{r_{ij}}-\frac{\pi}{2}\left(\frac{1}{m_i^2}+\frac{1}{m_j^2}+\frac{4\vec{\sigma}_i\cdot\vec{\sigma}_j}{3m_im_j}\right)\delta(\vec{r}_{ij})+\cdots\right],$$

$$V_{ij}^{Ga}=\pi\alpha_s\left(\frac{\vec{\lambda}_i+\vec{\lambda}_j}{2}\right)^2\left(\frac{1}{3}-\frac{\vec{f}_i\cdot\vec{f}_j}{2}\right)$$

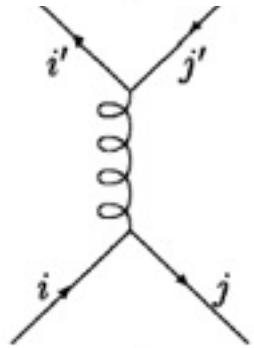
$$\times\left(\frac{\vec{\sigma}_i+\vec{\sigma}_j}{2}\right)^2\frac{2}{3}\frac{1}{(m_i+m_j)^2}\delta(\vec{r}_{ij}),$$

$$V_{i,i'j'j}=i\alpha_s\frac{\vec{\lambda}_i\cdot\vec{\lambda}_j}{4}\frac{1}{2r_{ij}}$$

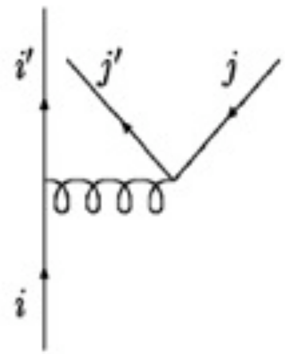
$$\times\left\{\left[\left(\frac{1}{m_i}+\frac{1}{m_j}\right)\vec{\sigma}_j+\frac{i\vec{\sigma}_j\times\vec{\sigma}_i}{m_i}\right]\cdot\frac{\vec{r}_{ij}}{r_{ij}^2}-\frac{2\vec{\sigma}_j\cdot\vec{\nabla}_i}{m_i}\right\}$$



V_{ij}^{Gs}



V_{ij}^{Ga}



$V_{i,i'j'j}$

If one allows sea quark Fock component mixing as shown in Eq. (6) used in our model, then the third term of Eq. (11), the quark-antiquark pair creation and annihilation term, will contribute to the matrix element of QAVCO. Table III shows our model results of the quark spin contents Δq of proton, in fact the matrix element of the QAVCO (axial charge). The experimental value and lattice QCD results are listed for comparison. In Table III, the second column is the q^3 valence quark contribution, where

$$\Delta u = \frac{4}{3}(1-0.32)(-0.923)^2,$$

$$\Delta d = -\frac{1}{3}(1-0.32)(-0.923)^2,$$

$$\Delta s = 0, \qquad (12)$$

Motion Fock

the first factors $\frac{4}{3}$, $-\frac{1}{3}$, 0 are the well known proton spin contents of the nonrelativistic quark model. $-0.32 = -1/3m^2b^2$ is the relativistic reduction and -0.923 is the amplitude of the q^3 component of our model. The third column is the contribution of the quark-antiquark pair creation (annihilation) term. It is another important reduction of the quark spin contribution and Δs is mainly due to this term. The fourth column lists the contribution of $q^3 q\bar{q}$ Fock components; due to quark antisymmetrization it cannot be separated into the valence and sea quark part. However, the antiquark contribution is very small (the largest one is $\Delta\bar{d}=0.004$), and has not been listed in Table III. The fifth column lists the sum. Our model quark spin contents Δu, Δd, and Δs are quite close to the experiment ones in Eq. (3) and column 6, even though we have not made any model parameter adjustments aimed at fitting the proton spin content.

PHYSICAL REVIEW D, VOLUME 58, 114032

Is nucleon spin structure inconsistent with the constituent quark model?

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−0.923	0.044	0.232	−0.252	0.065	0.109	−0.036	−0.106

Where does the nucleon get its spin?

- The spin of the nucleon has four contributions:

$$
\begin{aligned}
\vec{J} &= \vec{S}_q + \vec{L}_q + \vec{S}_G + \vec{L}_G \\
\vec{S}_q &= \frac{1}{2}\int d^3x\ \psi^\dagger \vec{\Sigma}\psi \\
\vec{L}_q &= \int d^3x\ \psi^\dagger \times \frac{\vec{\nabla}}{\imath}\psi \\
\vec{S}_G &= 2\int d^3x\ \mathrm{Tr}\{\vec{E}\times\vec{A})\} \\
\vec{L}_G &= 2\int d^3x\ \mathrm{Tr}\{E^i\vec{r}\times\vec{\nabla}A^i)\}
\end{aligned}
$$

- In the CQM, the gluon field is assumed to be frozen in the ground state and does not contribute to the nucleon spin.
- The only other contribution is the quark orbital angular momentum $\vec{L}_q$.
- One may well wonder how quark orbital angular momentum can contribute for a pure S-wave configuration.

- The quark orbital angular momentum operator can be expanded as:

$$\vec{L}_q = \sum_{i\lambda} \int d^3k (a^{\dagger}_{i\vec{k}\lambda} i\vec{\partial}_k \times \vec{k} a_{i\vec{k}\lambda} + b^{\dagger}_{i\vec{k}\lambda} i\vec{\partial}_k \times \vec{k} b_{i\vec{k}\lambda})$$

$$+ \frac{1}{2} \sum_{\lambda\lambda'} \int d^3k \chi^{\dagger}_{\lambda} \frac{\vec{\sigma}\cdot\vec{k}}{k_0(k_0+m)} i\vec{\sigma}\vec{k}\chi_{\lambda'}$$

$$\times (a^{\dagger}_{i\vec{k}\lambda} a_{i\vec{k}\lambda'} - b^{\dagger}_{i\vec{k}\lambda'} b_{i\vec{k}\lambda})$$

$$- \sum_{i\lambda\lambda'} \int d^3k \chi^{\dagger}_{\lambda} \frac{i\vec{\sigma}\times\vec{k}}{2k_0} \chi_{\lambda'} a^{\dagger}_{i\vec{k}\lambda} b^{\dagger}_{i-\vec{k}\lambda'} + \text{H.c.}$$

- The first term is the nonrelativistic quark orbital angular momentum operator used in the CQM, which does not contribute to nucleon spin in a pure valence S-wave configuration.
- The second term is a relativistic correction, which undoes the relativistic spin reduction.
- The third term is the $q\bar{q}$ creation and annihilation contribution, which also replaces missing spin.

- The quark orbital angular momentum operator can be expanded as:

$$\vec{L}_q = \sum_{i\lambda} \int d^3k (a^\dagger_{i\vec{k}\lambda} i\vec{\partial}_k \times \vec{k} a_{i\vec{k}\lambda} + b^\dagger_{i\vec{k}\lambda} i\vec{\partial}_k \times \vec{k} b_{i\vec{k}\lambda})$$

$$+ \frac{1}{2} \sum_{\lambda\lambda'} \int d^3k \chi^\dagger_\lambda \frac{\vec{\sigma}\cdot\vec{k}}{k_0(k_0+m)} i\vec{\sigma}\vec{k}\chi_{\lambda'}$$

$$\times (a^\dagger_{i\vec{k}\lambda} a_{i\vec{k}\lambda'} - b^\dagger_{i\vec{k}\lambda'} b_{i\vec{k}\lambda})$$

$$- \sum_{i\lambda\lambda'} \int d^3k \chi^\dagger_\lambda \frac{i\vec{\sigma}\times\vec{k}}{2k_0} \chi_{\lambda'} a^\dagger_{i\vec{k}\lambda} b^\dagger_{i-\vec{k}\lambda'} + \text{H.c.}$$

Add to half of (see next page) cancels 2nd & 3rd terms.

RECALL:: axial-vector current operator can be expanded as:

$$\int d^3x \bar{\psi}\vec{\gamma}\gamma^5\psi = \sum_{i\lambda\lambda'} \int d^3k \chi_\lambda^\dagger \vec{\sigma} \chi_{\lambda'} (a^\dagger_{i\vec{k}\lambda} a_{i\vec{k}\lambda'} - b^\dagger_{i\vec{k}\lambda'} b_{i\vec{k}\lambda})$$

$$- \sum_{i\lambda\lambda'} \int d^3k \chi_\lambda^\dagger \frac{\vec{\sigma}\cdot\vec{k}}{k_0(k_0+m_i)} i\vec{\sigma}\vec{k}\chi_{\lambda'}$$

$$\times (a^\dagger_{i\vec{k}\lambda} a_{i\vec{k}\lambda'} - b^\dagger_{i\vec{k}\lambda'} b_{i\vec{k}\lambda})$$

$$+ \sum_{i\lambda\lambda'} \int d^3k \chi_\lambda^\dagger \frac{i\vec{\sigma}\times\vec{k}}{k_0} \chi_{\lambda'} a^\dagger_{i\vec{k}\lambda} b^\dagger_{i-\vec{k}\lambda'} + \text{H.c.}$$

Spin is 1/2 of this.

Note that the relativistic correction and the $q\overline{q}$ creation and annihilation terms of the quark spin and the orbital angular momentum operator are exactly the same but with opposite sign. Adding them together produces:

$$\vec{S}_q + \vec{L}_q = \vec{S}_q^{NR} + \vec{L}_q^{NR}$$

where $\vec{S}_q^{NR}$, $\vec{L}_q^{NR}$ are the non-relativistic parts of the quark spin and angular momentum operators.

$$\vec{S}_q + \vec{L}_q = \vec{S}_q^{NR} + \vec{L}_q^{NR}$$

- This shows that the nucleon spin can be either solely attributed to the quark Pauli spin, as has long been done in the CQM, with no contribution from the non-relativistic quark orbital angular momentum to the nucleon spin; or
- part of the nucleon spin can be attributed to the relativistic quark spin, as measured in DIS (and more appropriately called axial-charge to distinguish it from the Pauli spin), and part of the nucleon spin can be attributed to the relativistic quark orbital angular momentum, that provides the exact compensation missing in the relativistic "quark spin" no matter what quark model is used.
- The right combination must be used; otherwise the nucleon spin structure will be misunderstood.

3. We suggest using the physical momentum, angular momentum, etc. in hadron physics in the same manner as is done in atomic physics, which is both gauge invariant and satisfies canonical commutation relations, and has been measured in atomic physics with established and well-defined physical meaning.

PRL 100, 232002 (2008) PHYSICAL REVIEW LETTERS week ending 13 JUNE 2008

Spin and Orbital Angular Momentum in Gauge Theories: Nucleon Spin Structure and Multipole Radiation Revisited

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$$\vec{A}_{pur} = \vec{A} - \vec{A}_{fys}$$

$$F^{\mu\nu}_{pur} = 0 \ ; \ F^{\mu\nu}_{fys} = F^{\mu\nu}$$

$$\vec{\nabla} \times \vec{A}_{fys} = \vec{\nabla} \times \vec{A}$$

$$\vec{\nabla} \cdot \vec{A}_{fys} = 0 \ ; \ \vec{A}_{fys}(|x| \rightarrow \infty \ ) = 0$$

$$\vec{A}_{fys}(x) = \vec{\nabla} \times \frac{1}{4\pi} \int d^3y \frac{\vec{\nabla}_{(y)} \times \vec{A}(y)}{|\vec{x} - \vec{y}|}$$

$$\vec{A}'_{fys} = \vec{A}_{fys} \ ; \ \vec{A}'_{pur} = \vec{A}'_{pur} - \vec{\nabla}\omega$$

$$A^0_{fys}(x) = \int_{-\infty}^{x} dx^i (\partial_i A^0 + \partial_t A^i - \partial_t A^i_{fys})$$

$$\phi(x) = -\frac{1}{4\pi} \int d^3y \frac{\vec{\nabla}_{(y)} \cdot \vec{A}(y)}{|\vec{x} - \vec{y}|} + \phi_0(x)$$

$$\vec{A}_{pur} = -\vec{\nabla}\phi(x) \ ; \ A^0_{pur} = \partial_t \phi(x) \ ; \ \nabla^2 \phi_0(x) = 0$$

First Argument

- A matrix element of a gauge non-invariant operator taken in a gauge invariant state is gauge invariant (Elliott Leader). Nucleon is a color singlet so QCD gauge invariance of ME is guaranteed.
- Atomic analog: QED gauge invariance of spin and angular momentum of neutral atom, but not of (nucleus) electrons in it.

Can the two fundamental requirements:

1. Gauge Invariance
2. Canonical Commutation Relation for S, L

(i.e., angular momentum algebra for the individual components of the nucleon spin),

both be satisfied or must only one be kept, while the other is violated?

Only $\vec{J} = \vec{L} + \vec{S}$ is conserved

Lorentz covariant

We argue from what is known in atomic quantum physics.

At least one of those must correspond to net spin less than 1/2 -- rest must be in angular momentum since J is conserved.

SemiClassically: Think of lower components of amplitude
rotating about CM line of momentum
QM: L ≠ 0 without "physical" rotation

Solution?

A decomposition of the angular momentum operator for atom (QED) and nucleon (QCD), such that both the gauge invariance and angular momentum algebra are satisfied for individual components.

Energy and momentum of hydrogen atom also gauge invariant, as expect.

Key point is to separate the transverse and longitudinal components of the gauge field.