Unpolarized and polarized parton distributions in the nucleon and nuclei

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Contents

Tensor polarization of the deuteron
  New spin structure of spin-1 hadrons

Unusual nuclear modification of $^9\text{Be}$ by Jlab
  Clustering effects in a nucleus?

$Q^2$ evolution code for fragmentation functions
  Useful code for theorists and experimentalists

Comments on

CDF anomaly on $W+2\text{jets}$

J-PARC theory activities (delayed by earthquake)
Tensor-polarized parton distribution functions in the deuteron

**Situation**

• **Spin structure of the spin-1/2 nucleon**

  **Nucleon spin puzzle:** This issue is not solved yet, but it is rather well studied theoretically and experimentally.

• **Spin-1 hadrons (e.g. deuteron)**

  There are some theoretical studies especially on tensor structure in electron-deuteron deep inelastic scattering.

    → HERMES experimental results

    → Letter of intent to JLab PAC-37

No investigation has been done for hadron \((p, \pi, \ldots)\) - polarized deuteron processes.

    → hadron facility \((J-PARC, RHIC, COMPASS, GSI, \ldots)\) experiment?
Almost none of nucleon spin is carried by quarks!

Nucleon spin crisis!?  

Naïve Quark Model

Sea-quarks and gluons?  
Orbital angular momenta?

Tensor structure $b_1$ (e.g. deuteron)

“old” standard model

only S wave $b_1 = 0$

S + D waves

standard model $b_1 \neq 0$

$\neq b_1$ “standard model”

Tensor-structure crisis!?
Structure Functions

\[ F_1 \propto \langle d\sigma \rangle \]

\[ g_1 \propto d\sigma(\uparrow,+1) - d\sigma(\uparrow,-1) \]

\[ b_1 \propto d\sigma(0) - \frac{d\sigma(+1) + d\sigma(-1)}{2} \]

\[ \text{note: } \sigma(0) - \frac{\sigma(+1) + \sigma(-1)}{2} = 3\langle\sigma\rangle - \frac{3}{2} [\sigma(+1) + \sigma(-1)] \]

Parton Model

\[ F_1 = \frac{1}{2} \sum_i e_i^2 (q_i + \bar{q}_i) \]

\[ q_i = \frac{1}{3} (q_i^{+1} + q_i^0 + q_i^{-1}) \]

\[ g_1 = \frac{1}{2} \sum_i e_i^2 \left( \Delta q_i + \Delta \bar{q}_i \right) \]

\[ \Delta q_i = q_{i\uparrow}^{+1} - q_{i\downarrow}^{+1} \]

\[ b_1 = \frac{1}{2} \sum_i e_i^2 \left( \delta_T q_i + \delta_T \bar{q}_i \right) \]

\[ \delta_T q_i = q_i^0 - \frac{q_i^{+1} + q_i^{-1}}{2} \]

\[ \left[ q^H(x,Q^2) \right] \]

\[ \left[ q^H(x,Q^2) \right] = \left[ \sum_i e_i^2 (\delta_T q_i + \delta_T \bar{q}_i) \right] \]
**Personal studies**

- **Sum rule for $b_1$**
  

- **Polarized proton-deuteron Drell-Yan: General formalism**
  

- **Polarized proton-deuteron Drell-Yan: Parton model**
  

- **Extraction of $\Delta \bar{u}/\Delta \bar{d}$ and $\Delta_1 \bar{u}/\Delta_1 \bar{d}$ from polarized pd Drell-Yan**
  

- **Projections to $b_1$, ..., $b_4$ from $W_{\mu\nu}$**
  

- **Tensor-polarized distributions from HERMES data**
  

*This talk*

Motivated by the following works.

Hoodbhoy-Jaffe-Manohar (1989)


HERMES measurement on $b_1$ (2005)

Future possibilities at JLab, J-PARC, RHIC, ...

JLab, letter of intent
Analysis of HERMES data to obtain tensor-polarized quark distributions


Purposes

• Understanding of current situation on tensor-polarized distributions
• Useful for future proposals at JLab, J-PARC, ...
• Test of theoretical model estimates
• Description of tensor structure in terms of quark-gluon degrees of freedom
• Understanding of hadron spins with orbital angular momenta

...
**HERMES measurements on $b_1$**

A. Airapetian *et al.* (HERMES), PRL 95 (2005) 242001.

- **27.6 GeV/c** $\leftrightarrow$, 0
- **positron** $\rightarrow$ **deuteron**

$b_1$ measurements in the kinematical region

$0.01 < x < 0.45$, $0.5 \text{ GeV}^2 < Q^2 < 5 \text{ GeV}^2$
Constraint on valence-tensor polarization (sum rule)

\[
\int dx \begin{vmatrix}
\begin{array}{c}
\delta_T u_v \\
\delta_T d_v
\end{array}
\end{vmatrix}
\begin{vmatrix}
\begin{array}{c}
\delta_T \bar{u}^D \\
\delta_T \bar{d}^D
\end{array}
\end{vmatrix}
\begin{vmatrix}
\begin{array}{c}
\delta_T \bar{s}^D
\end{array}
\end{vmatrix}
\leftrightarrow
q \rightarrow 0
\]

\[
\int dx b_i^D(x) = \frac{5}{18} \int dx \left[ \delta_T u_v + \delta_T d_v \right] + \frac{1}{18} \int dx \left[ 8 \delta_T \bar{u}^D + 2 \delta_T \bar{d}^D + \delta_T \bar{s}^D \right]
\]

Elastic amplitude in a parton model

\[
\Gamma_{H,H} = \langle p, H | J_0(0) | p, H \rangle = \sum_\xi e_\xi \int dx \left[ q_{i \uparrow}^H + q_{i \downarrow}^H - \bar{q}_{i \uparrow}^H - \bar{q}_{i \downarrow}^H \right]
\]

\[
\frac{1}{2} \left[ \Gamma_{0,0} - \frac{1}{2} \left( \Gamma_{1,1} + \Gamma_{-1,-1} \right) \right] = \frac{1}{3} \int dx \left[ \delta_T u_v(x) + \delta_T d_v(x) \right]
\]

Macroscopically

\[
\Gamma_{0,0} = \lim_{t \to 0} \left[ F_c(t) - \frac{t}{3} F_Q(t) \right], \quad \Gamma_{1,1} = \Gamma_{-1,-1} = \lim_{t \to 0} \left[ F_c(t) + \frac{t}{6} F_Q(t) \right]
\]

\[
\frac{1}{2} \left[ \Gamma_{0,0} - \frac{1}{2} \left( \Gamma_{1,1} + \Gamma_{-1,-1} \right) \right] = -\lim_{t \to 0} \frac{t}{2} F_Q(t)
\]

\[
\int dx b_i^D(x) = \frac{5}{18} \left[ \Gamma_{0,0} - \frac{1}{2} \left( \Gamma_{1,1} + \Gamma_{-1,-1} \right) \right] + \frac{1}{18} \int dx \left[ 8 \delta_T \bar{u}^D + 2 \delta_T \bar{d}^D + \delta_T \bar{s}^D \right]
\]

\[
= -\frac{5}{6} \lim_{t \to 0} t F_Q(t) + \frac{1}{18} \int dx \left[ 8 \delta_T \bar{u}^D + 2 \delta_T \bar{d}^D + \delta_T \bar{s}^D \right]
\]

\[
= 0 \text{ (valence)} + \frac{1}{18} \int dx \left[ 8 \delta_T \bar{u}^D + 2 \delta_T \bar{d}^D + \delta_T \bar{s}^D \right]
\]

Constraint on tensor-polarized valence quarks: \( \int dx \delta_T q_v(x) = 0 \)

Intuitive derivation without calculation:

\[
\int dx b_i(x) = \text{dimensionless quantity} = (\text{mass})^2 \cdot (\text{quadrupole moment})
\]

**Functional form of parametrization**

Assume flavor-symmetric antiquark distributions: \( \delta q^D = \delta u^D = \delta d^D = \delta s^D = \delta \bar{s}^D \)

\[
b_1^D(x)_{\text{LO}} = \frac{1}{18} \left[ 4\delta_T u^D_v(x) + \delta_T d^D_v(x) + 12 \delta_T \bar{q}^D(x) \right]
\]

At \( Q_0^2 = 2.5 \text{ GeV}^2 \), \( \delta_T q^D_v(x, Q_0^2) = \delta_T w(x) q^D_v(x, Q_0^2), \quad \delta_T \bar{q}^D(x, Q_0^2) = \alpha_q \delta_T w(x) \bar{q}^D(x, Q_0^2) \)

Certain fractions of quark and antiquark distributions are tensor polarized and such probabilities are given by the function \( \delta_T w(x) \) and an additional constant \( \alpha_{\bar{q}} \) for antiquarks in comparison with the quark polarization.

\[
b_1^D(x, Q_0^2)_{\text{LO}} = \frac{1}{18} \left[ 4\delta_T u^D_v(x, Q_0^2) + \delta_T d^D_v(x, Q_0^2) + 12 \delta_T \bar{q}^D(x, Q_0^2) \right]
\]

\[
= \frac{1}{36} \delta_T w(x) \left[ 5 \{ u_v(x, Q_0^2) + d_v(x, Q_0^2) \} + 4\alpha_{\bar{q}} \{ 2\bar{u}(x, Q_0^2) + 2\bar{d}(x, Q_0^2) + s(x, Q_0^2) + \bar{s}(x, Q_0^2) \} \right]
\]

\( \delta_T w(x) = ax^b (1 - x)^c (x_0 - x) \)

Two types of analyses

- **Set 1**: \( \delta_T \bar{q}^D(x) = 0 \)  Tensor-polarized antiquark distributions are terminated \( (\alpha_{\bar{q}} = 0) \),

- **Set 2**: \( \delta_T \bar{q}^D(x) \neq 0 \)  Finite tensor-polarized antiquark distributions are allowed \( (\alpha_{\bar{q}} \neq 0) \).
Theoretical background for the parametrization

(1) Tensor-polarized valence quarks: $\int dx \delta_r q_v(x) = 0$

(2) Standard convolution approach

Convolution model: $A_{H,hH}(x) = \int dy \sum_s f_s^H(y) \hat{A}_{hs,hs} (x/y) \equiv \sum_s f_s^H(y) \otimes \hat{A}_{hs,hs}(y)$

$A_{H,hH'} = \mathcal{E}_{h'}^\mu W_{\mu \nu}^H \mathcal{E}_h^\nu$, $b_1 = A_{+0,+0} - \frac{A_{++} + A_{+-}}{2}$

$\hat{A}_{+\uparrow,\uparrow} = F_1 - g_1$, $\hat{A}_{+\downarrow,\downarrow} = F_1 + g_1$

$b_1 = A_{+0,+0} - \frac{A_{++} + A_{+-}}{2} = \int \frac{dy}{y} \sum_s \left[ f_s^0(y) - \frac{f_s^+(y) + f_s^-(y)}{2} \right] F_1(x/y)$

Momentum distribution of a nucleon: $f^H(y) = \int d^3 p |\phi^H(\vec{p})|^2 \delta \left( y - \frac{E + p_z}{M} \right)$

D-state admixture: $\phi^H(\vec{p}) = \phi^H(\vec{p})^{\ell=0} \cos \alpha + \phi^H(\vec{p})^{\ell=2} \sin \alpha$

$= \cos \alpha \psi_0(p)Y_{00}(\hat{p})\chi_H + \sin \alpha \sum_{m_L} \langle 2m_L : 1m_s \mid 1H \rangle \psi_2(p)Y_{2m_L}(\hat{p})\chi_{m_s}$

Numerical estimates indicate the oscillatory function with $\int dx b_1(x) = 0$. 
Results

Two-types of fit results:

- set-1: $\chi^2 / \text{d.o.f.} = 2.83$
  Without $\delta_T q$, the fit is not good enough.

- set-2: $\chi^2 / \text{d.o.f.} = 1.57$
  With finite $\delta_T q$, the fit is reasonably good.

Obtained tensor-polarized distributions $\delta_T q(x), \delta_T \bar{q}(x)$ from the HERMES data.

$\rightarrow$ They could be used for
  - experimental proposals,
  - comparison with theoretical models.

Finite tensor polarization for antiquarks:

$$\int_0^1 dx b_1(x) = 0.058$$

$$= \frac{1}{9} \int_0^1 dx \left[ 4\delta_T \bar{u}(x) + \delta_T \bar{d}(x) + \delta_T \bar{s}(x) \right]$$
Summary

(1) The tensor-polarized distributions: $\delta_T q(x)$, $\delta_T \bar{q}(x)$ were obtained from the HERMES data on $b_1$.

(2) Finite tensor polarization was obtained for antiquarks: $\int dx \delta_T \bar{q}(x) \neq 0$.

Physics mechanism of $\delta_T \bar{q}(x)$ is missing.

Prospects

Future experimental possibilities
at JLab, EIC, J-PARC, RHIC, COMPASS, GSI-FAIR, ...

Experimental proposal is considered at JLab (Letter of intent, PAC-37).

Unpolarized proton + polarized deuteron

Spin asymmetry in $p + \bar{d} \rightarrow \mu^+ \mu^- + X$

$$A_{UQ_0} = \frac{\sum_a e_a^2 \left[ q_a(x_A) \delta_T \bar{q}_a(x_B) + \bar{q}_a(x_A) \delta_T q_a(x_B) \right]}{\sum_a e_a^2 \left[ q_a(x_A) \bar{q}_a(x_B) + \bar{q}_a(x_A) q_a(x_B) \right]}$$

Unique advantage of Drell-Yan ($\delta \bar{q}$ measurement)

$$A_{UQ_0} (\text{large } x_F) \approx \frac{\sum_a e_a^2 q_a(x_A) \delta \bar{q}_a(x_B)}{\sum_a e_a^2 q_a(x_A) \bar{q}_a(x_B)}$$

Gottfried: $\int \frac{dx}{x} \left[ F_2^p(x) - F_2^n(x) \right] = \frac{1}{3} + \frac{2}{3} \int dx \left[ \bar{u} - \bar{d} \right]$

Polarized proton-deuteron Drell-Yan
(Theory) S. Hino and SK,
PR D 59 (1999) 094026,
D 60 (1999) 054018.
JLab anomaly on $^9$Be
(A clustering aspect in DIS?)

M. Hirai, S. Kumano, K. Saito, and T. Watanabe
Purpose

A theoretical-model density with cluster structure for $^9$Be

To find a signature of nuclear clustering in high-energy processes, particularly in structure functions of deep inelastic scattering.
Nuclear modifications of structure function $F_2$

$F_2(x,Q^2)_{LO} = \sum_i e_i^2 [q_i(x,Q^2) + \bar{q}_i(x,Q^2)]$

- Anti-shadowing
- Fermi motion of the nucleon
- Shadowing
- Nuclear binding (+ Nucleon modification)

J. Seely \textit{et al.},


\begin{equation}
R_{EMC} = \frac{\sigma_A}{\sigma_D}
\end{equation}

\textbf{Slope:} \quad \frac{dR_{EMC}}{dx}, \quad R_{EMC} = \frac{\sigma_A}{\sigma_D}

\textit{9}Be anomaly = EMC slope is too large to be estimated from its nuclear density.
Convolution formalism

Charged-lepton deep inelastic scattering from a nucleus

\[ d\sigma \sim L^{\mu\nu} W^A_{\mu\nu}, \quad L^{\mu\nu} = \text{Lepton tensor}, \]

Hadron tensor: \[ W_{\mu\nu} = \frac{1}{4\pi} \int d^4 \xi \ e^{iq\cdot\xi} \langle p| J^{em}_\mu (\xi), J^{em}_\nu (0) \rangle |p\rangle \]

Convolution: \[ W^A_{\mu\nu}(p_A, q) = \int d^4 p \ S(p) \ W^N_{\mu\nu}(p_N, q) \]

\( S(p) = \text{Spectral function} = \text{nucleon momentum distribution in a nucleus} \)

In a simple model: \[ S(p_N) = \left| \phi(\vec{p}_N) \right|^2 \delta \left( p^0_N - M_A + \sqrt{M^2_A + \vec{p}_N^2} \right) \]

\( F_2 \) needs to be projected out from \( W_{\mu\nu} \) by the projection operator \[ \hat{P}^{\mu\nu} = -\frac{M^2_N v}{2\vec{p}^2} \left( g^{\mu\nu} - \frac{3\vec{p}^\mu \vec{p}^\nu}{\vec{p}^2} \right) \):

\[ W_{\mu\nu} = -F_1 \frac{1}{M_N} \left( g_{\mu\nu} - \frac{q_{\mu} q_{\nu}}{q^2} + \frac{\vec{p}_\mu \vec{p}_\nu}{M^2_N v} \right) + F_2 \frac{\vec{p}_\mu \vec{p}_\nu}{M^2_N v}, \quad \vec{p}_\mu = p_\mu - \frac{p \cdot q}{q^2} q_\mu; \quad \hat{P}^{\mu\nu} W_{\mu\nu} = F_2 \]

\[ F_2^A(x, Q^2) = \hat{P}^{\mu\nu}(A) W^A_{\mu\nu}(p_A, q) = \int d^4 p S(p) \hat{P}^{\mu\nu}(A) W^N_{\mu\nu}(p_N, q) \]

We obtain \[ F_2^A(x, Q^2) = \int dy f(y) F_2^N(x/y, Q^2), \quad f(y) = \int d^3 p_N y \delta \left( y - \frac{p_N \cdot q}{M_N v} \right) \left| \phi(\vec{p}_N) \right|^2 \]

\[ f(y) = \text{lightcone momentum distribution for a nucleon} \]

\[ y = \frac{p_N \cdot q}{M_N v} = \frac{p^0_N v - \vec{p}_N \cdot \vec{q}}{M_N v} = \frac{p_N \cdot q}{p_A \cdot q / A} = \frac{p^+_N}{p^+_A / A} = \text{lightcone momentum fraction}, \quad p^+ = \frac{p^0 \pm p^3}{\sqrt{2}} \]

Two theoretical models

\[ F_2^A(x, Q^2) = \int dy f(y) F_2^N(x / y, Q^2), \quad f(y) = \int d^3p_N y \delta \left( y - \frac{p_N \cdot q}{M_N \nu} \right) \rho(p_N) \]

Nuclear density \( \rho(p_N) \) is calculated by

1. Simple shell model
2. Anti-symmetrized molecular dynamics (AMD)

Simple shell model

\[
\psi_{n\ell m}(r, \theta, \phi) = R_{n\ell}(r)Y_{\ell m}(\theta, \phi)
\]

\[
R_{n\ell}(r) = \frac{2\kappa^{2\ell+3}(n-1)!}{\sqrt{[\Gamma(n+\ell+1/2)]^3}} r^\ell e^{-\frac{1}{2}\kappa^2 r^2} L_{n-\ell}^{\ell+1/2}(\kappa^2 r^2)
\]

\[ \kappa^2 \equiv M_N \omega, \quad V = \frac{1}{2} M_N \omega^2 r^2 \]

AMD: variational method with effective NN potentials

Slater determinant: \( \Phi(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_A) = \frac{1}{\sqrt{A!}} \)

\[
\begin{vmatrix}
\varphi_1(\vec{r}_1) & \varphi_1(\vec{r}_2) & \cdots & \varphi_1(\vec{r}_A) \\
\varphi_2(\vec{r}_1) & \varphi_2(\vec{r}_2) & \cdots & \varphi_2(\vec{r}_A) \\
\vdots & \vdots & \ddots & \vdots \\
\varphi_A(\vec{r}_1) & \varphi_A(\vec{r}_2) & \cdots & \varphi_A(\vec{r}_A)
\end{vmatrix}
\]

Single-particle wave function: \( \varphi_j(\vec{r}_j) = \left( \frac{2\nu}{\pi} \right)^{3/4} \exp \left[ -\nu \left( \vec{r}_j - \frac{\vec{Z}_j}{\sqrt{\nu}} \right)^2 \right] \)

Parameters are determined by a variational method with effective NN potentials.
Cluster structure in $^9$Be

Density distributions in $^4$He and $^9$Be by AMD

Two models:
(1) Shell model
(2) AMD (antisymmetrized molecular dynamics) to describe clustering structure

However, if the densities are averaged over the polar and azimuthal angles, differences from shell structure are not so obvious although there are some differences in $^9$Be in comparison with $^4$He.
EMC effect

Momentum (p) distributions

4He

9Be

Convolution model

Cluster effects

F^A_2(x, Q^2) = \int_x^A dy f(y) F^N_2(x/y, Q^2)
EMC slopes plotted by maximum local densities

The $^9$Be anomaly can be explained by the high-densities, which are created by clustering in the $^9$Be nucleus.
Our results indicate

\[ F_2^A = (\text{mean part}) + (\text{part created by large densities due to cluster formation}) \]

Convolution model indicates clustering effects are small in this term. JLab data could be related to this effect due to the nuclear cluster.

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Prospects

JLab proposal to measure structure functions of other light nuclei.

Jefferson Lab PAC-35 proposal, PR12-10-008 (2009)

Jefferson Lab Experiment E1210008

Detailed studies of the nuclear dependence of F2 in light nuclei.

Spokespersons:

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Summary

1. We developed a convolution formalism with clustering structure.

2. We showed density differences between shell and AMD models in nuclei (\(^4\)He, \(^9\)Be, \(^{12}\)C).

   *Nuclear clustering produce high-momentum components.*

3. Clustering effects on \(F_2^A\) by comparing shell and AMD model calculations; however, the effects are not large.

4. The JLab \(^9\)Be anomaly can be “explained” if nuclear modifications are shown by maximum local densities of the AMD not by the ones of the shell model.

   *a clear signature of clustering effects in high-energy processes*

5. *More investigations at JLab after 12-GeV upgrade (~2014)*
$Q^2$ evolution code for fragmentation functions

Purpose: to provide useful $Q^2$ evolution codes

S. Kumano and J. T. Londergan,

For general users, the following codes are available at
http://research.kek.jp/people/kumanos/program.html.

- **Unpolarized PDFs**

- **Longitudinally-polarized PDFs**

- **Transversity distributions**

- **Fragmentation functions**

This talk
**Fragmentation Functions**

Fragmentation function is defined by

$$ F^h(z,Q^2) = \frac{1}{\sigma_{\text{tot}}} \frac{d\sigma(e^+e^- \rightarrow hX)}{dz} $$

$$ \sigma_{\text{tot}} = \text{total hadronic cross section} $$

A fragmentation process occurs from quarks, antiquarks, and gluons, so that $F^h$ is expressed by their individual contributions:

$$ F^h(z,Q^2) = \sum_i \int \frac{dy}{y} C_i \left( \frac{z}{y}, Q^2 \right) D_i^h(y,Q^2) $$

Calculated in perturbative QCD

$$ C_i(z,Q^2) = \text{coefficient function} $$

$$ D_i^h(z,Q^2) = \text{fragmentation function of hadron } h \text{ from a parton } i $$

**Variable $z$**

- Hadron energy / Beam energy
- Hadron energy / Primary quark energy

Variable $z$ is defined as:

$$ z \equiv \frac{E_h}{\sqrt{s}/2} = \frac{2E_h}{Q} = \frac{E_h}{E_q}, \quad s = Q^2 $$

**Non-perturbative**

(determined from experiments)

**Fragmentation:** hadron production from a quark, antiquark, or gluon

Fragmentation process occurs from quarks, antiquarks, and gluons, so that $F^h$ is expressed by their individual contributions:

$$ F^h(z,Q^2) = \sum_i \int \frac{dy}{y} C_i \left( \frac{z}{y}, Q^2 \right) D_i^h(y,Q^2) $$

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**Non-perturbative**

(determined from experiments)
nonsinglet \[ \frac{\partial}{\partial \ln(Q^2)} D_{NS}(x,Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} P_{NS}\left(\frac{x}{y}\right) D_{NS}(y,Q^2) \quad D_s = \sum_i (D_{q_i} + D_{\bar{q}_i}) \]

singlet \[ \frac{\partial}{\partial \ln Q^2}\left( \begin{array}{c} D_s(x,Q^2) \\ D_g(x,Q^2) \end{array} \right) = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} \left( \begin{array}{ccc} P_{qq}(x/y) & P_{gq}(x/y) \\ P_{qs}(x/y) & P_{gq}(x/y) \end{array} \right) \left( \begin{array}{c} D_s(y,Q^2) \\ D_g(y,Q^2) \end{array} \right) \]

**Purposes of studying numerical solution**

1. **No analytical solution.** Complicated integrodifferential equations especially for those including higher-order effects.
2. **Frequently used in theoretical and experimental analyses.**
3. **In the PDF fit,** the evolution is calculated 1,000,000 times!

**Numerical solution methods**

- "Brute-force" → Gauss-Legendre quadrature (this work)
- Orthogonal polynomials (e.g. Laguerre)
- Mellin transformation
Gauss-Legendre quadrature

DGLAP: \[ \frac{\partial}{\partial t} D(x,t) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} P(y) D(x/y,t) \]

- \( t \rightarrow \) divided into \( N_t \) steps \( (t = \ln Q^2) \).
  \[ \frac{\partial}{\partial t} D(x,t) \Rightarrow \frac{D(x_i,t_{\ell+1}) - D(x_i,t_\ell)}{\Delta t} \]

- \( x \rightarrow \) divided into \( N_x \) steps.

Then, there are \( N_{GL} \) Gauss-Legendre points in each step.

\[ \int_x^1 \frac{dy}{y} P\left(\frac{x}{y}\right) D(y,t) \Rightarrow \frac{1-x_m}{2} \sum_{j=m}^{N_x} \sum_{k=1}^{N_{GL}} \frac{w_k}{x_k} P(x_k) D\left(\frac{x_m}{x_k},t_\ell\right) \]

DGLAP \( \Rightarrow \) \[ D(x_m,t_{\ell+1}) = D(x_m,t_\ell) + \Delta t \frac{\alpha_s(t_\ell)}{2\pi} \frac{1-x_m}{2} \sum_{j=m}^{N_x} \sum_{k=1}^{N_{GL}} \frac{w_k}{x_k} P(x_k) D\left(\frac{x_m}{x_k},t_\ell\right) \]

Repeating this step \( N_t \) times, we obtain the evolved function at \( Q^2 \).
Evolution results

Initial fragmentation functions
= HKNS07

The initial functions are evolved to $Q^2=10, 100, 1000 \text{ GeV}^2$. 
How to obtain the code

(1) Please look at our report,

(2) Please read the web page
    http://research.kek.jp/people/kumanos/program.html.

(3) Then, the evolution code can be obtained by email request
    to Masanori Hirai [mhirai(AT)ph.noda.tus.ac.jp]
    and/or Shunzo Kumano [shunzo.kumano(AT)kek.jp].

Distributed files
    FF_DFLAP.f: Evolution subroutine for fragmentation functions
    fort.10: Input file for calculating evolution
    sample.f: Test file for running FF_DGLAP.f
CDF anomaly
on W+2jet production
New discovery or?

One of subprocesses

(Figures from V. Cavaliere on April 6, 2011 at Fermilab)

T. Aaltonen et al. (CDF),
Phys. Rev. Lett. 106 (2011) 171801,
W+2jets

+ many processes ...
Recent D0

June 9, 2011

V. M. Abazov et al. (D0 collaboration), arXiv: 1106.1921 [hep-ex].
J-PARC theory activities

(delayed by earthquake)
Recovering from earthquake

KEK theory: no damage

My office is almost the worst in our KEK theory center.

Activities are normal now in theory.

J-PARC: some damage

Recovery is in progress.

Plan to restart experiments by the end of this year!
J-PARC theory group (expected to start on April, 1, 2011 but delayed)

Theory in the J-PARC center
- 4 KEK staff members in hadron-nuclear theory
- 5 KEK visiting staff members on strangeness, exotics, neutrino)
Start in hadron-nuclear theory (particle?) in the near future.
Staff and visitor rooms are ready

Theory activities will start soon!

You are welcome to join the activities on J-PARC theory.

For inquiry, please send email to S. Kumano.