SUM RULES FOR LIGHT-BY-LIGHT AND COMPTON SCATTERING

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OUTLINE

- Intro: causality, analyticity and GDH sum rule
- Sum rules for light-by-light scattering: derivation, implications
- Derivative of GDH in Yang-Mills theory
- Analyticity of chiral behavior

INTRODUCTION

causality:

$$B(t) = \int dt'\, G(t-t')A(t')$$

$$G(t-t') = 0, \quad t < t'$$

relativistic version:

$$B(x) = \int dx' G(x-x')A(x')$$

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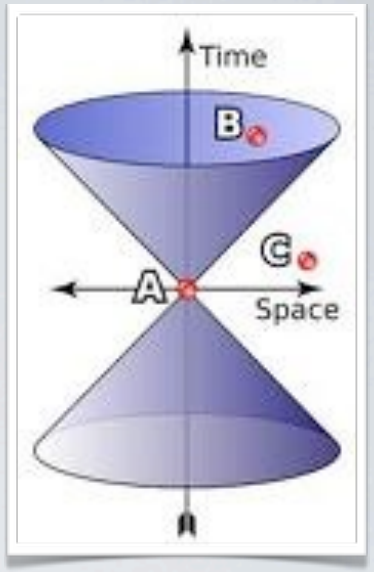

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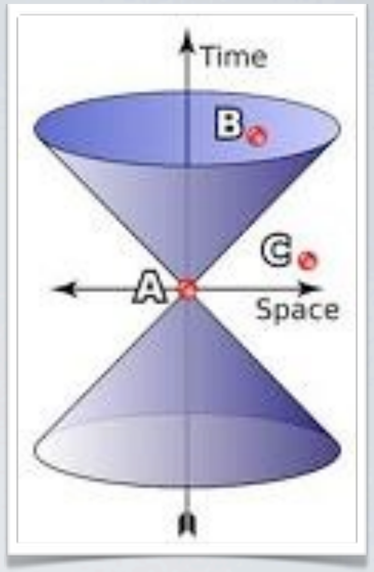

1. $a(t) = 0$ if $t \le 0$ and $a(t) \in \mathrm{L}^2$.
2. $a(\omega) = \mathrm{F}\left[a\left(t\right)\right] \in \mathrm{L}^2$ if $\omega \in \mathbb{R}$ and if

Statements 1 and 2 are equivalent (Titchmarsch theorem)

$$a(\omega) = \lim_{\omega' \to 0} a(\omega + \mathrm{i}\omega'),$$

then $a(\omega + \mathrm{i}\omega')$ is holomorphic if $\omega' > 0$.

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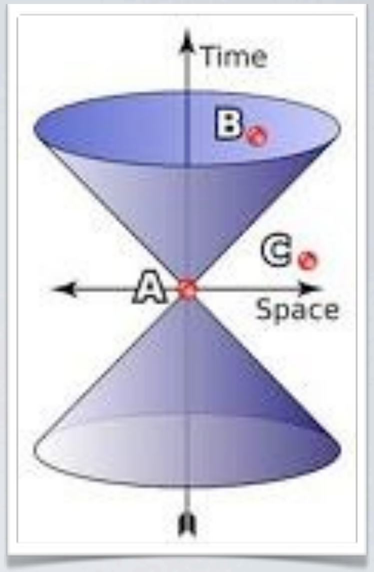

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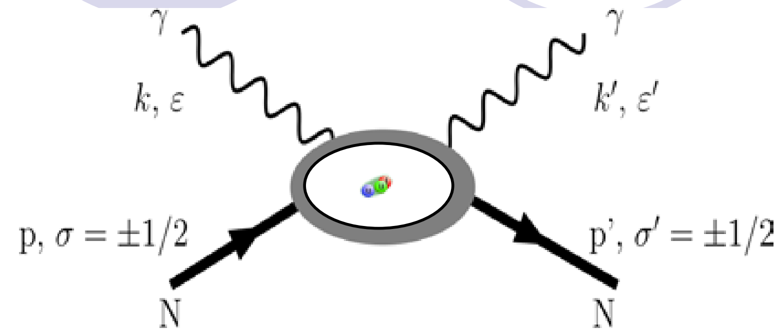
$$\mathrm{Re}\{a(\omega)\} = \frac{1}{\pi}\mathrm{P}\int_{-\infty}^{\infty} \frac{\mathrm{Im}\{a(\omega')\}}{\omega' - \omega}\mathrm{d}\omega',$$

$$\mathrm{Im}\{a(\omega)\} = -\frac{1}{\pi}\mathrm{P}\int_{-\infty}^{\infty} \frac{\mathrm{Re}\{a(\omega')\}}{\omega' - \omega}\mathrm{d}\omega'.$$

e.g., Kramers-Kronig relations

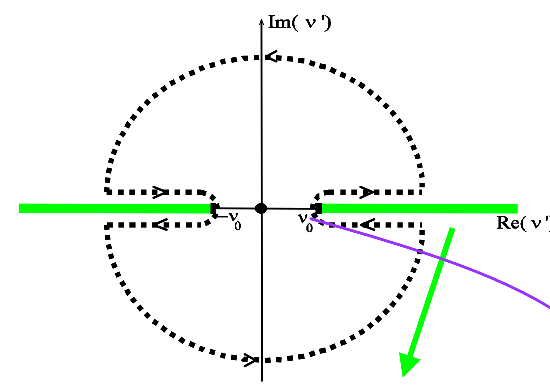
Derivation steps

Forward Compton scattering amplitude:



$$\mathrm{Amp}_{t=0} = \underbrace{f(\omega)\,\vec{\varepsilon}\,' \cdot \vec{\varepsilon} + g(\omega)\, i\vec{S} \cdot (\vec{\varepsilon}\,' \times \vec{\varepsilon}) + \ldots}_{2s+1 \text{ terms } \leftrightarrow \text{ \# of e.m. moments}}$$

Analyticity: $\mathrm{Re}\, g(\omega) = \frac{2\omega}{\pi} \int\limits_{\omega_{th}}^{\infty} d\omega' \, \frac{\mathrm{Im}\, g(\omega')}{\omega'^2 - \omega^2}$



Unitarity: $\mathrm{Im}\, g(\omega) = \frac{\omega}{2} \Delta\sigma(\omega)$

branch cuts :
πN, ππN, ...

Low-energy theorem: $g(\omega) = \frac{e^2 \kappa^2}{4 s M^2}\, \omega + O(\omega^3)$

$\omega_{th} \geq 0$

Stability condition

Gerasimov-Drell-Hearn (GDH) sum rule

$$\frac{e^2}{2M^2}\kappa^2 = \frac{1}{\pi}\int_0^\infty \frac{d\omega}{\omega}\Delta\sigma(\omega)$$

$\kappa = (g-2)\,s$ anomalous magnetic moment

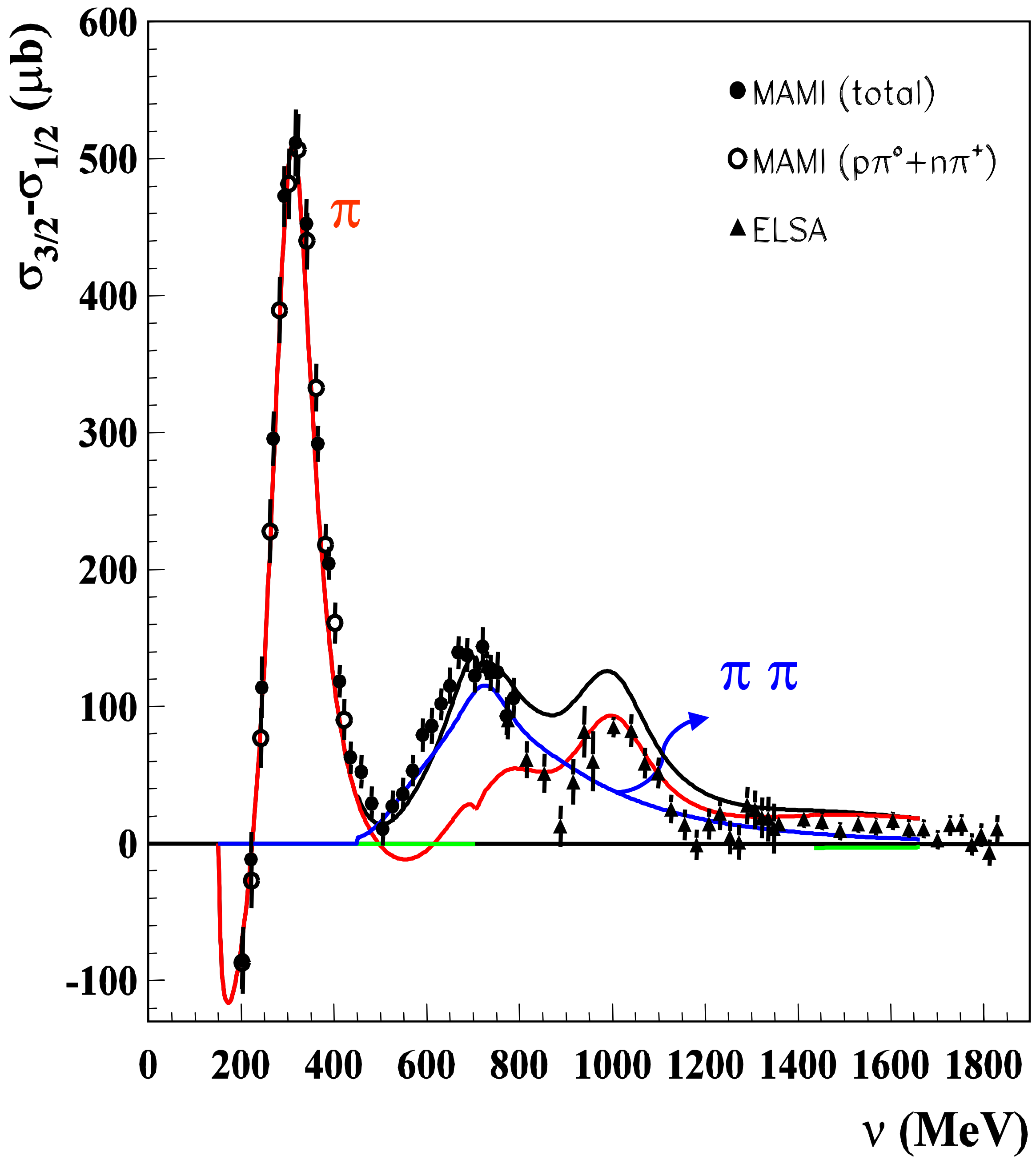
$\Delta\sigma = \sigma_{1+s} - \sigma_{1-s}$ doubly-polarized total photoabsorption cross section (photon circular polarized parallel or anti-parallel to the target's spin)

Principles/Assumptions:

- **Low-energy theorem for Compton scattering** (gauge-invariance, crossing symmetry,..
- **Analyticity** (forward Compton amplitude obeys disp. relations along the production cut)
- **Unitarity** (optical theorem: Im forward Compton amplitude = total phtotoabsorption)

verification of the GDH sum rule for the proton

$$\frac{e^2}{2M^2}\kappa^2 = \frac{1}{\pi}\int_0^\infty \frac{d\omega}{\omega}\Delta\sigma(\omega)$$



PROTON	Eγ [GeV]	GDH [μb]
MAID2000 estimate	< 0.2	-28.5 ± 2
MAMI experiment	0.2 - 0.8	226 ± 5 ± 12
ELSA experiment	0.8 - 2.9	27.5 ± 2.0 ± 1.2
Bianchi-Thomas Simula et al. : estimate	> 2.9	-14 ± 2
Total		211 ± 15
GDH sum rule		205

SUM RULES FOR LIGHT-BY-LIGHT

[V.P. & VANDERHAEGHEN, PRL (2010)]

$$M_{\lambda_1\lambda_2\lambda_3\lambda_4} = \varepsilon^{*\mu_4}_{\lambda_4}(\vec{q}_4)\,\varepsilon^{*\mu_3}_{\lambda_3}(\vec{q}_3)\,\varepsilon^{\mu_2}_{\lambda_2}(\vec{q}_2)\,\varepsilon^{\mu_1}_{\lambda_1}(\vec{q}_1)\,\mathcal{M}_{\mu_1\mu_2\mu_3\mu_4}$$

Helicity AMPl. **Feynman AMPl.**

In the forward direction ($t = 0, \quad s = 4\omega^2, \quad u = -s.$):

$$\mathcal{M}_{\mu_1\mu_2\mu_3\mu_4} = A(s)\,g_{\mu_4\mu_2}g_{\mu_3\mu_1} + B(s)\,g_{\mu_4\mu_1}g_{\mu_3\mu_2} + C(s)\,g_{\mu_4\mu_3}g_{\mu_2\mu_1}\,,$$

$$M_{++++}(s) = A(s) + C(s),$$
$$M_{+-+-}(s) = A(s) + B(s),$$
$$M_{++--}(s) = B(s) + C(s).$$

1) Crossing symmetry (1 <-> 3, 2 <-> 4):

$$M_{+-+-}(s) = M_{++++}(-s), \quad M_{++--}(s) = M_{++--}(-s)$$

SUM RULES FOR LIGHT-BY-LIGHT (DERIVATION CONTD)

Amplitudes with definite parity under Crossing:

$$f^{(\pm)}(s) = M_{++++}(s) \pm M_{+-+-}(s)$$

$$g(s) = M_{++--}(s)$$

2) Causality => Analyticity => dispersion relations:

$$\mathrm{Re} \left\{ \begin{array}{l} f^{(\pm)}(s) \\ g(s) \end{array} \right\} = \frac{1}{\pi} \fint_{-\infty}^{\infty} \frac{ds'}{s'-s} \, \mathrm{Im} \left\{ \begin{array}{l} f^{(\pm)}(s') \\ g(s') \end{array} \right\},$$

3) Optical theorem (unitarity):

$$\mathrm{Im}\, f^{(\pm)}(s) = -\frac{s}{8} \left[\sigma_0(s) \pm \sigma_2(s) \right],$$

$$\mathrm{Im}\, g(s) = -\frac{s}{8} \left[\sigma_{||}(s) - \sigma_{\perp}(s) \right].$$

$\sigma_{0,2}(\sigma_{||,\perp})$ **Are circularly (linearly) polarized Photon-Photon Fusion cross-sections**

SUM RULES FOR LIGHT-BY-LIGHT (DERIVATION CONTD)

Sum rules:

$$\mathrm{Re}\, f^{(+)}(s) = -\frac{1}{2\pi} \fint_0^\infty ds'\, s'^2\, \frac{\sigma(s')}{s'^2 - s^2}\,, \qquad \sigma = (\sigma_0 + \sigma_2)/2 = (\sigma_{||} + \sigma_\perp)/2$$

$$\mathrm{Re}\, f^{(-)}(s) = -\frac{s}{4\pi} \fint_0^\infty ds'\, \frac{s'\,\Delta\sigma(s')}{s'^2 - s^2}\,, \qquad \Delta\sigma = \sigma_2 - \sigma_0$$

$$\mathrm{Re}\, g(s) = -\frac{1}{4\pi} \fint_0^\infty ds'\, s'^2\, \frac{\sigma_{||}(s') - \sigma_\perp(s')}{s'^2 - s^2}\,,$$

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4) "Low-energy Theorem": $\mathcal{L}_{\mathrm{EH}} = c_1 (F_{\mu\nu} F^{\mu\nu})^2 + c_2 (F_{\mu\nu} \tilde{F}^{\mu\nu})^2,$

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Low-energy expansion

$$f^{(+)}(s) = -2(c_1 + c_2)s^2 + O(s^4)$$

$$f^{(-)}(s) = O(s^5)$$

$$g(s) = -2(c_1 - c_2)s^2 + O(s^4)$$

SUM RULES FOR LIGHT-BY-LIGHT

$O(s^0)$:

$$0 = \int_0^\infty \mathrm{d}s \left[\sigma_{||}(s) \pm \sigma_\perp(s)\right]$$

$O(s^1)$:

$$0 = \int_0^\infty \frac{\mathrm{d}s}{s} \left[\sigma_2(s) - \sigma_0(s)\right]$$

$O(s^2)$:

$$c_1 = \frac{1}{8\pi} \int_0^\infty \frac{\mathrm{d}s}{s^2}\, \sigma_{||}(s)\,,$$

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SUM RULES FOR LIGHT-BY-LIGHT

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Diverges!
Gerasimov & Moulin (1976)

$O(s^1):$

$$0 = \int_0^\infty \frac{\mathrm{d}s}{s} \left[\sigma_2(s) - \sigma_0(s)\right]$$

P. Roy (1974)
Brodsky & SCHMIDT (1995)

$O(s^2):$

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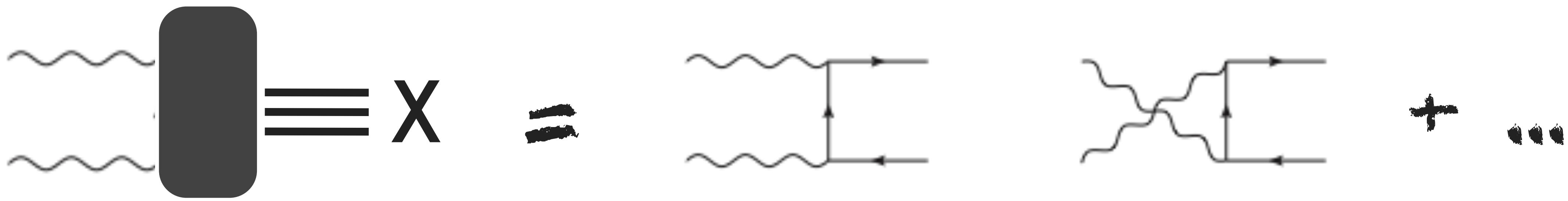
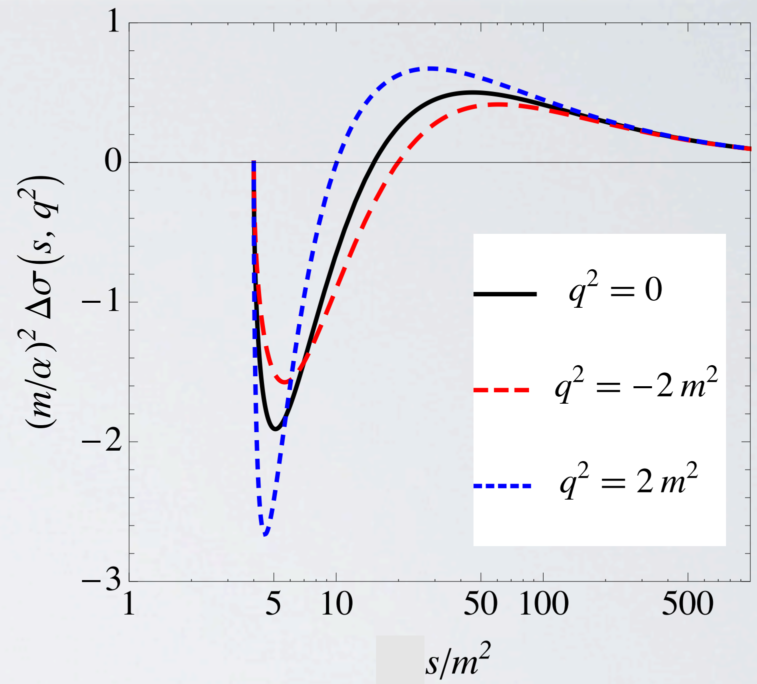
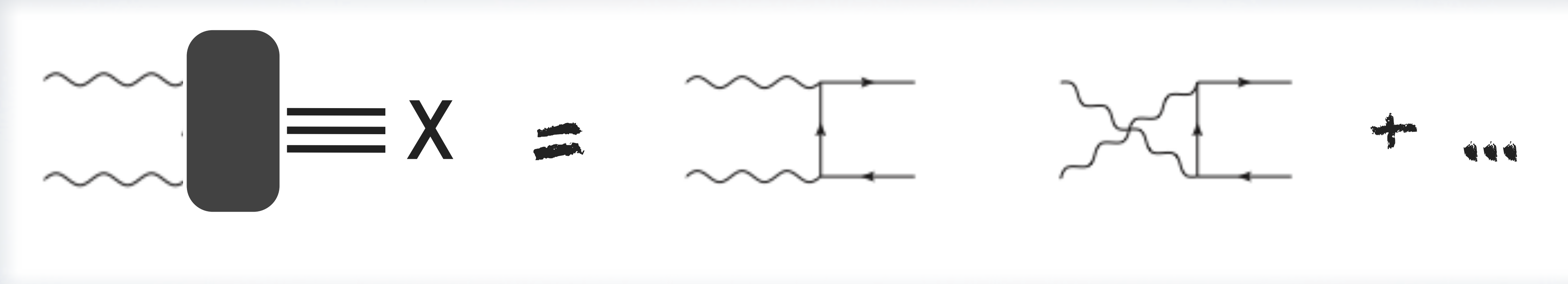


$$0 \;=\; \int_0^\infty \mathrm{d}s\, \frac{\sigma_2(s) - \sigma_0(s)}{s}\,, \qquad \xRightarrow{\text{PT}}$$





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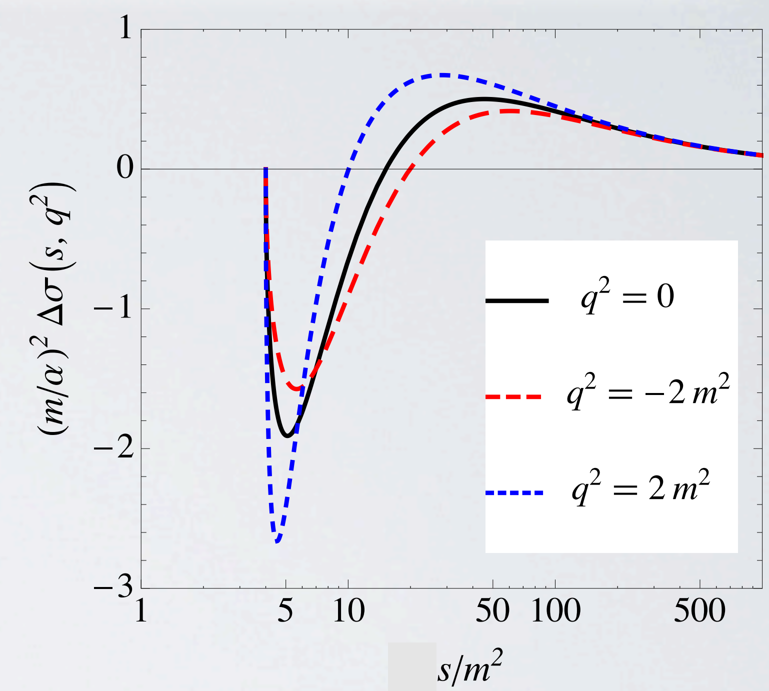
PT



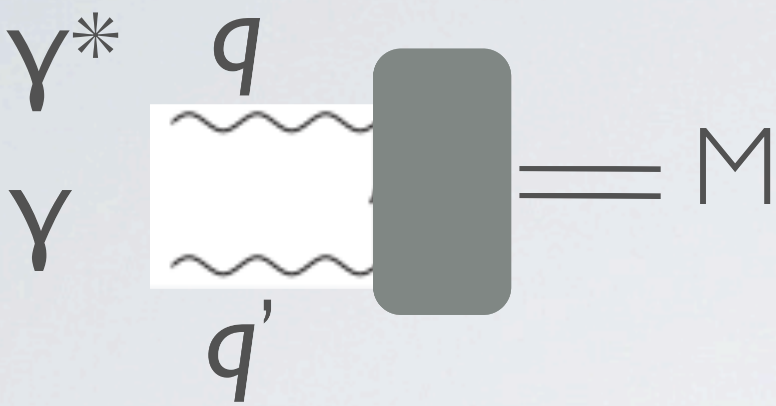
non-PT

cancellation of (pseudo)scalar and tensor meson contributions

	m_M [MeV]	$\Gamma_{\gamma\gamma}$ [keV]	$\int ds\ \Delta\sigma/s$ [nb]
π^0	134.98	$(7.8 \pm 0.6) \times 10^{-3}$	-195.0 ± 15.0
η	547.85	0.51 ± 0.03	-190.7 ± 11.2
η'	957.66	4.30 ± 0.15	-301.0 ± 10.5
Sum η, η'			-492 ± 22

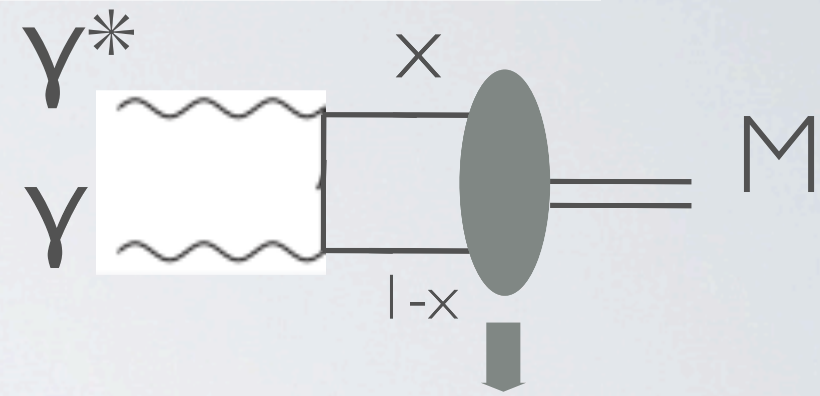
	m_M [MeV]	$\Gamma_{\gamma\gamma}$ [keV]	$\int ds\ \Delta\sigma/s$ narrow res. [nb]	$\int ds\ \Delta\sigma/s$ Breit-Wigner [nb]
$a_2(1320)$	1318.3	1.00 ± 0.06	134 ± 8	137 ± 8
$f_2(1270)$	1275.1	3.03 ± 0.35	448 ± 52	479 ± 56
$f_2'(1525)$	1525	0.081 ± 0.009	7 ± 1	7 ± 1
Sum f_2, f_2'			455 ± 53	486 ± 57

γ* γ -> M transition form factors



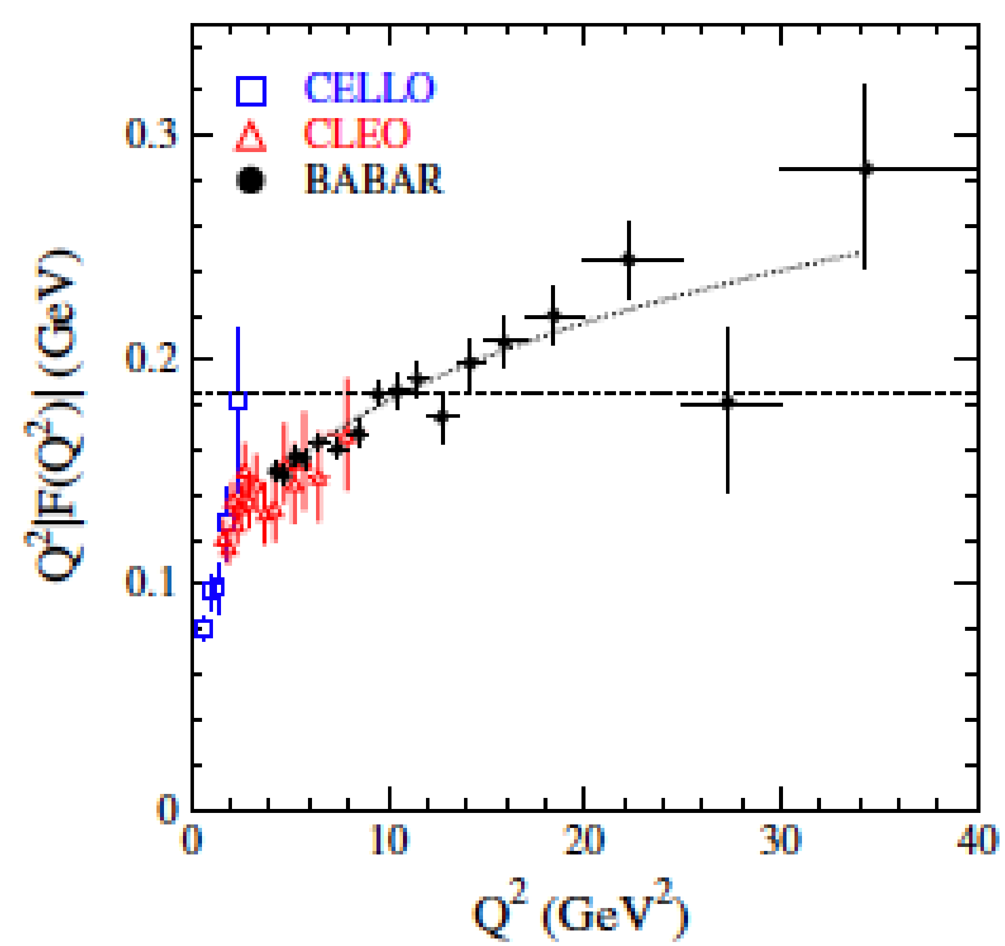
$$T_{\mu\nu} = ie^2\, \varepsilon_{\mu\nu\alpha\beta}\, q^\alpha\, q'^\beta\, F_{\pi^0\gamma^*\gamma}(Q^2)$$

Q^2 = - q^2 > 0



meson distribution amplitude (DA) φ(x)

asymptotic DA:
φ(x) = 6 x (1 - x)



$$F_{\pi^0\gamma^*\gamma} \xrightarrow{Q^2\to 0} \frac{1}{4\pi^2 f_\pi}$$

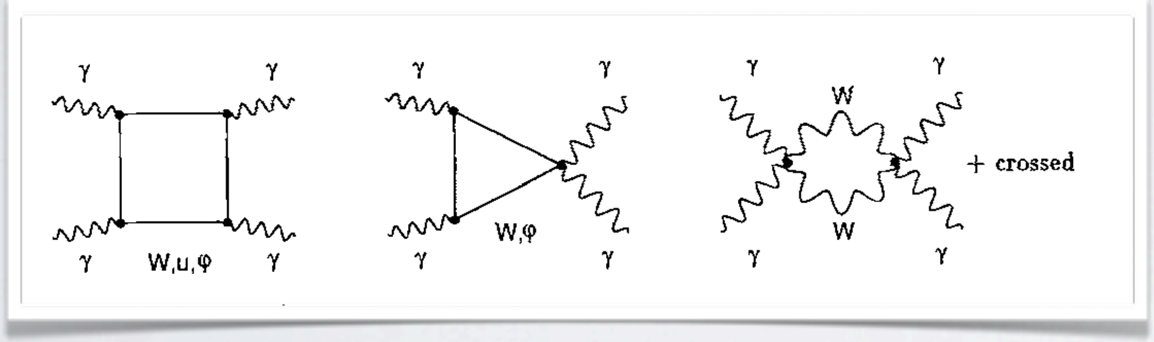
chiral anomaly
f_π = 92.4 MeV

$$F_{\pi^0\gamma^*\gamma} \xrightarrow{Q^2\to\infty} \frac{2\, f_\pi}{Q^2}$$

PERTURBATIVE VERIFICATION OF SUM RULES FOR LIGHT-BY-LIGHT

$$c_1 = \frac{1}{8\pi} \int_0^\infty \frac{\mathrm{d}s}{s^2}\, \sigma_{||}(s)\,,$$

$$c_2 = \frac{1}{8\pi} \int_0^\infty \frac{\mathrm{d}s}{s^2}\, \sigma_{\perp}(s)$$



Böhm & Schuster (1994)

Low-energy Expansion

$$c_1 = \frac{29\alpha^2}{160 M^4}$$

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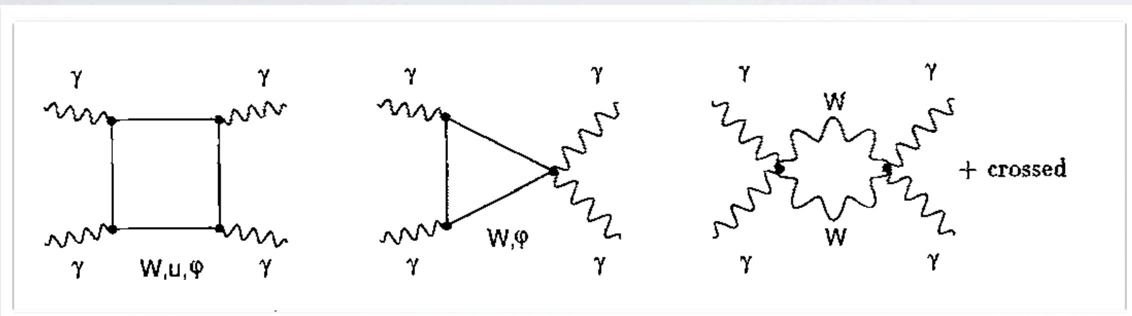

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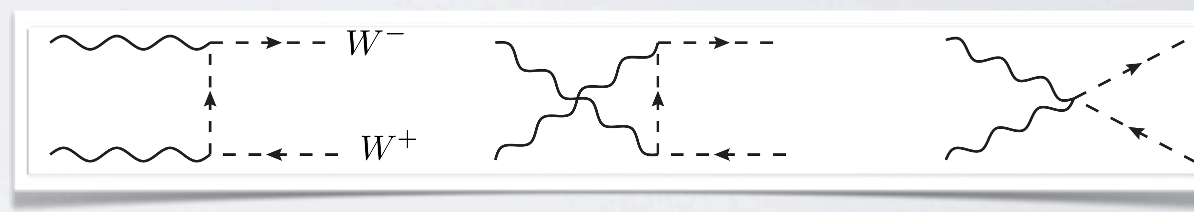

V. Pauk (2011)

$$\sigma_{||} = \frac{2\pi\alpha^2}{M^2 s^3}\left(s(18M^4+3M^2s+4s^2)\sqrt{1-\frac{4M^2}{s}} - 24M^4(s-3M^2)\mathrm{ArcTanh}\left(\sqrt{1-\frac{4M^2}{s}}\right)\right)$$

$$\sigma_{\perp} = \frac{2\pi\alpha^2}{M^2 s^3}\left(s(6M^4+3M^2s+4s^2)\sqrt{1-\frac{4M^2}{s}} - 24M^4(s-M^2)\mathrm{ArcTanh}\left(\sqrt{1-\frac{4M^2}{s}}\right)\right)$$

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Integrate

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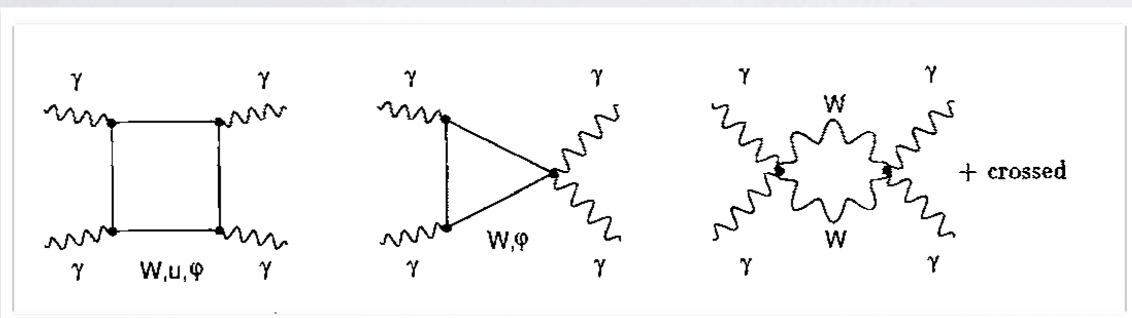

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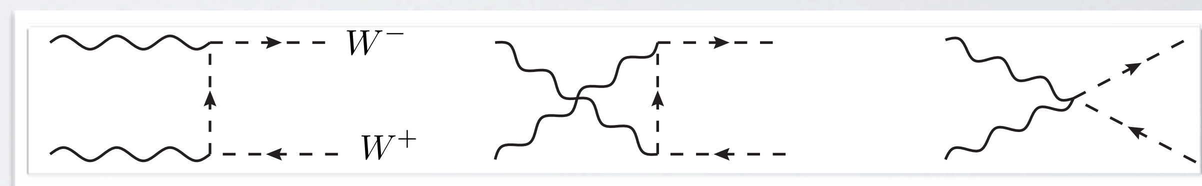

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Ghosts, Higgs Sector

Unitarity, Causality

SUM RULES FOR E.M. MOMENTS OF MASSIVE VECTOR BOSONS

E.M. coupling:

$$\begin{aligned}\mathcal{L} = & \quad -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{2}W^*_{\mu\nu}W^{\mu\nu} - M^2 W^*_\mu W^\mu \\ + & \quad ie\, W^*_{\mu\nu}A^\mu W^\nu - ie\, A_\mu W^*_\nu W^{\mu\nu} + e^2 A^2\, W^*_\mu\, W^\mu + ie\,\ell_1 W^*_\mu W_\nu\, F^{\mu\nu} \\ + & \quad e\,\ell_2\, \left[(D^*_\mu W^*_\nu)W^\alpha \partial_\alpha F^{\mu\nu} + W^*_\alpha (D_\mu W_\nu)\partial^\alpha F^{\mu\nu}\right]/(2M^2)\end{aligned}$$

where anomalous magnetic-dipole and electric-Quadrupole moments Are:

$$\kappa = \tfrac{1}{2}(\ell_1 - 1),$$
$$\lambda = \tfrac{1}{2}(\ell_2 - \ell_1 + 1).$$

Sum Rules ($\omega = (s - M^2)/2$):

$$O(\omega^1): \qquad \frac{e^2}{2M^2}\kappa^2 = \frac{1}{\pi}\int_0^\infty \frac{\mathrm{d}s}{s}\left[\sigma_2(s) - \sigma_0(s)\right]$$

$$O(\omega^2): \qquad 3\kappa(\kappa+\lambda) + \gamma_1 - \gamma_2 = \frac{1}{8\pi}\int_0^\infty \frac{\mathrm{d}s}{s^2}\left[\sigma_{||}(s) - \sigma_\perp(s)\right],$$

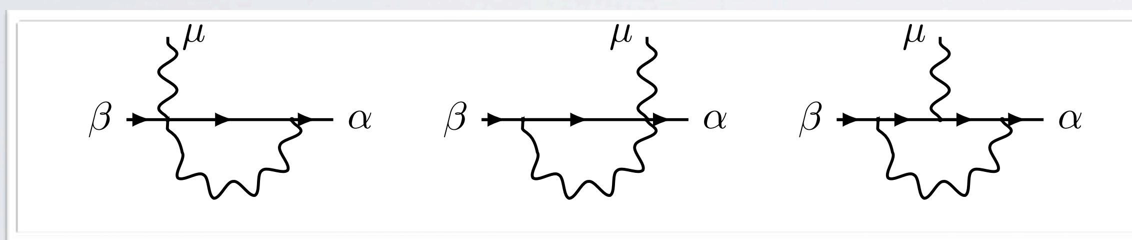
$\gamma_{1,2}$ **-- transversal spin polarizabilities** *etc.*

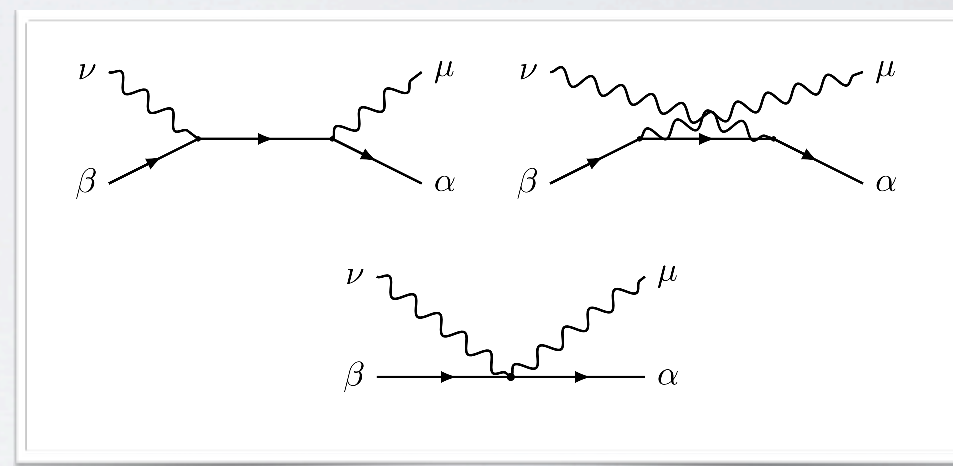
GDH SUM RULE AT ONE LOOP

Consider

$$\frac{e^2}{2M^2}\kappa^2 = \frac{1}{\pi}\int_0^\infty \frac{\mathrm{d}s}{s}\left[\sigma_2(s) - \sigma_0(s)\right]$$

Take a *derivative* of the sum rule with respect to a.m.m.
[V.P., Holstein, Vanderhaeghen PLB (2004), PRD (2005)]





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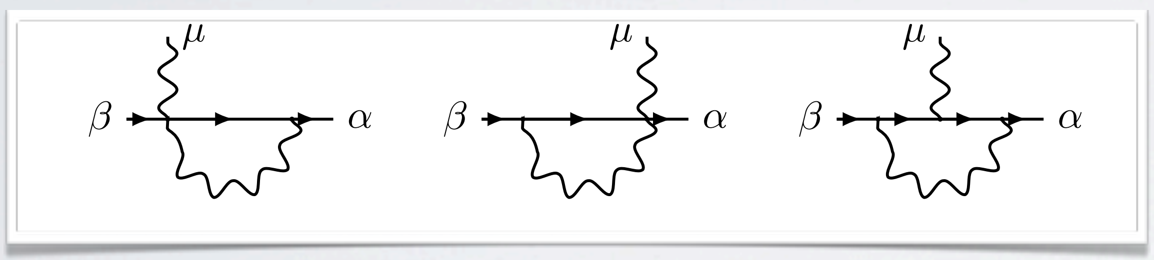

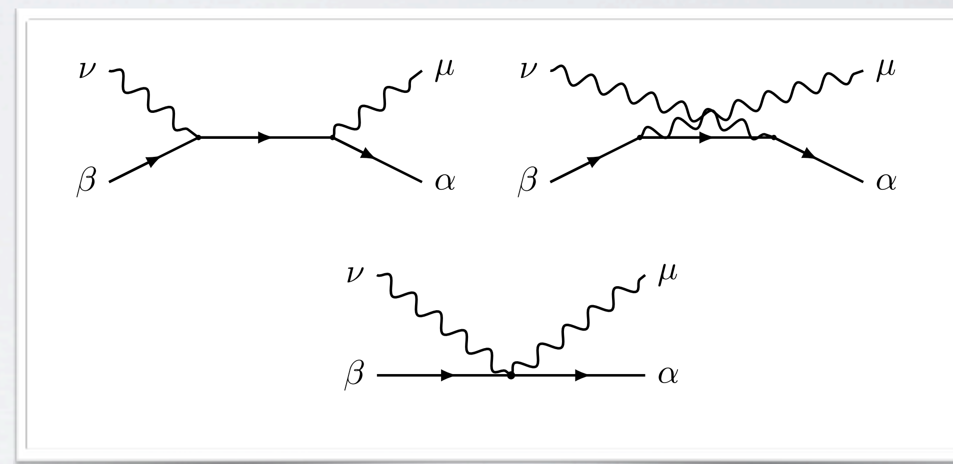

Dispersion IntegraL

$$\kappa = \frac{5\alpha}{3\pi}$$

GDH SUM RULE AT ONE LOOP

Consider

$$\frac{e^2}{2M^2}\kappa^2 = \frac{1}{\pi}\int_0^\infty \frac{\mathrm{d}s}{s}\left[\sigma_2(s) - \sigma_0(s)\right]$$

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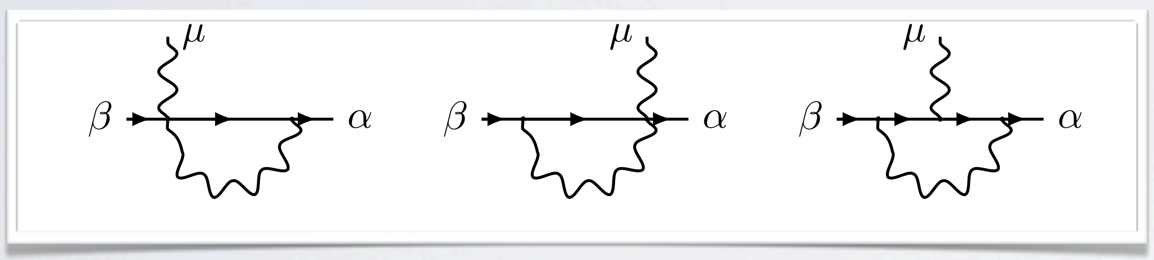
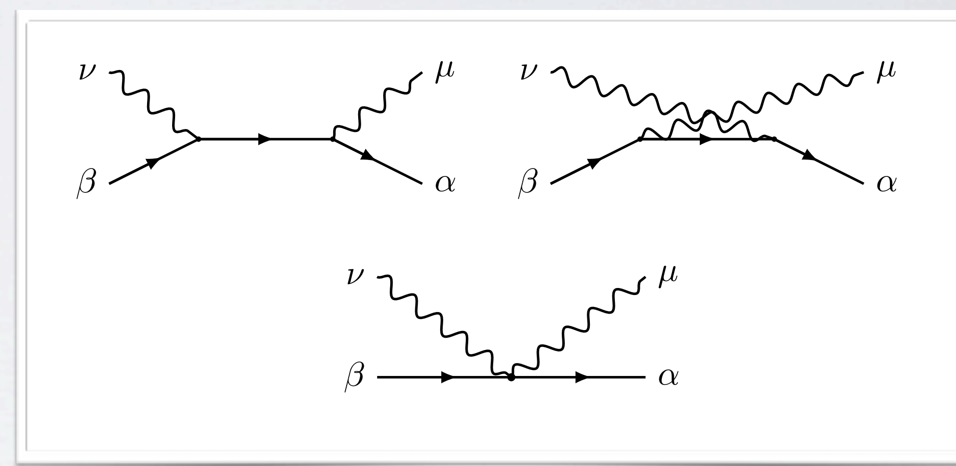


UV-divergent κ



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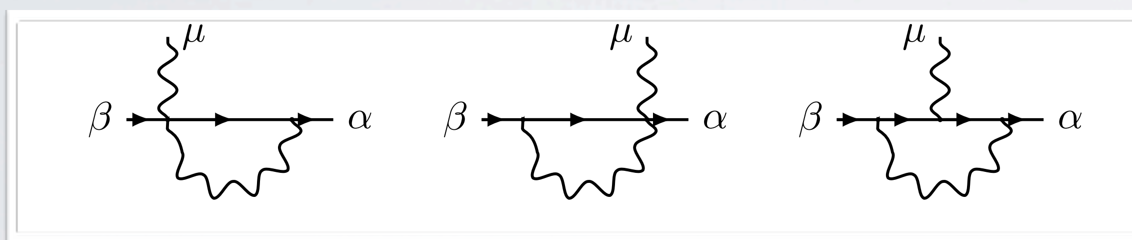

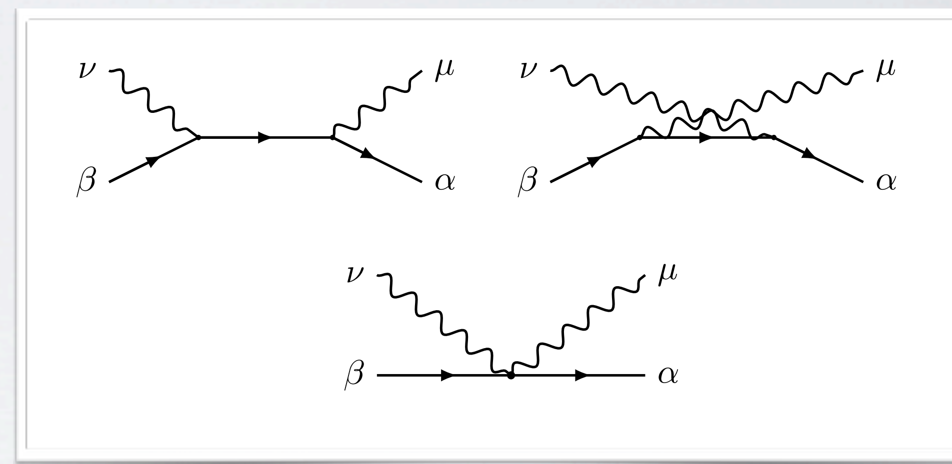

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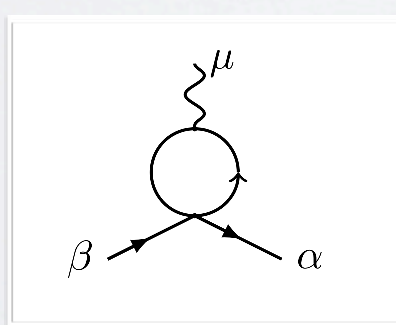
add



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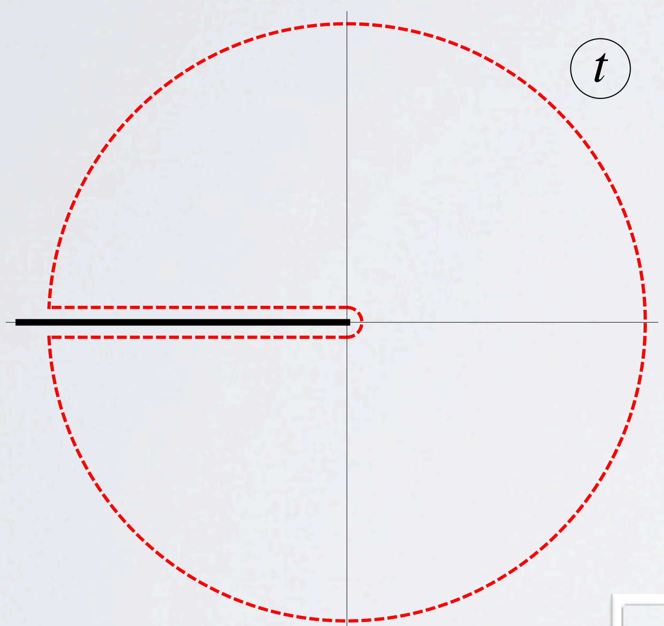
ANALYTICITY IN PION-MASS SQUARED

[Ledwig, V.P. & Vanderhaeghen, PLB (2010)]

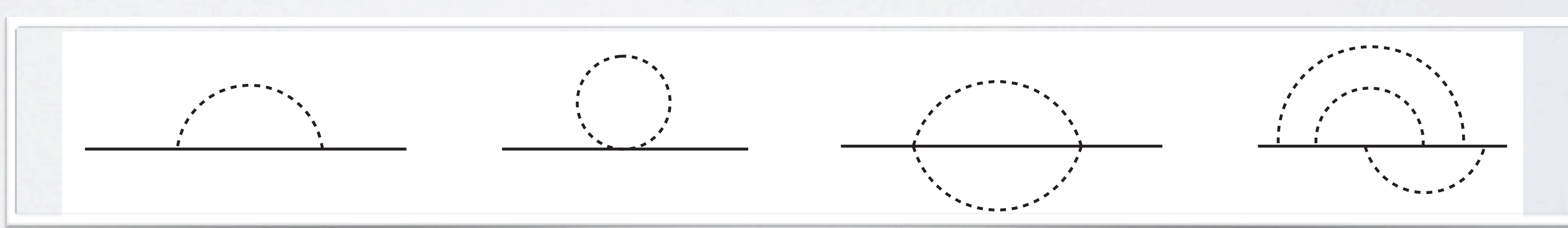
Motivation: chiral perturbation theory and lattice QCD, which compute the pion-mass dependence of hadron properties

$t = m_\pi^2$

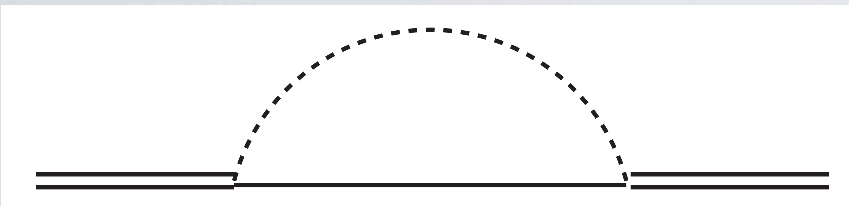


$$f(m_\pi^2) = -\frac{1}{\pi}\int_{-\infty}^{0} \mathrm{d}t \, \frac{\mathrm{Im}\, f(t)}{t - m_\pi^2 + i0^+}$$

Verified for nucleon mass, a.m.m., polarizabilities at order p^3.



DELTA(1232) RESONANCE



$$f(m_\pi^2) = -\frac{1}{\pi}\int_{-\infty}^{\Delta^2} \mathrm{d}t' \, \frac{\mathrm{Im}\, f(t')}{t' - m_\pi^2 + i0^+} \, .$$

with $\Delta = M_\Delta - M_N$, the Delta-nucleon mass difference.

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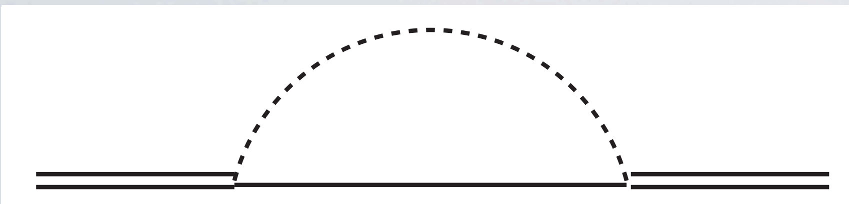

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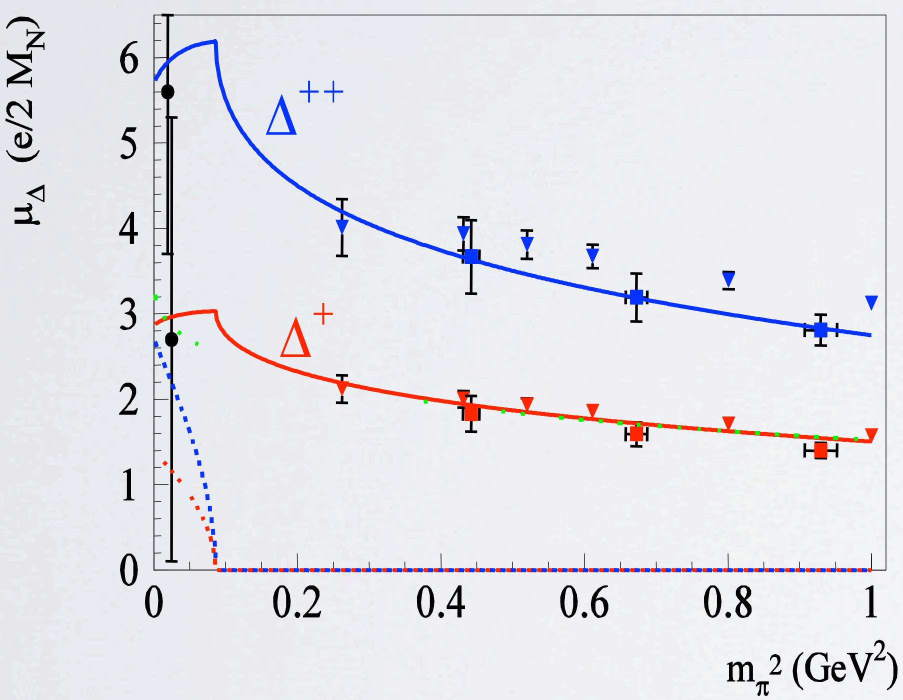
Magnetic moment:



Real parts

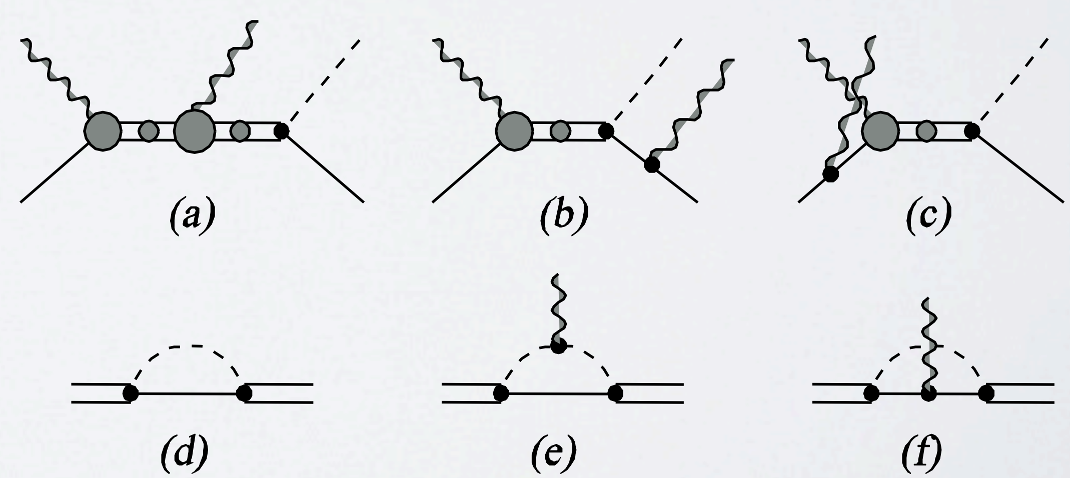
Imag. parts

quenched lattice points :

Leinweber, PRD (1992)

Cloet,Leinweber,Thomas, PLB (2003)

Lee *et.al.*, PLB (2004) – revised (2006)



Curves - chiral EFT calculation: V.P, Vanderhaeghen, PRL (2005)

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