The Λ(1405) Resonance in Full-QCD

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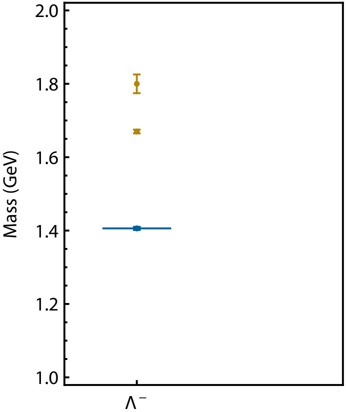
8th Circum-Pan-Pacific Symposium
on High Energy Spin Physics

The Λ(1405)

- The negative-parity ground state of the Lambda has a mass of 1406 ± 4 MeV.



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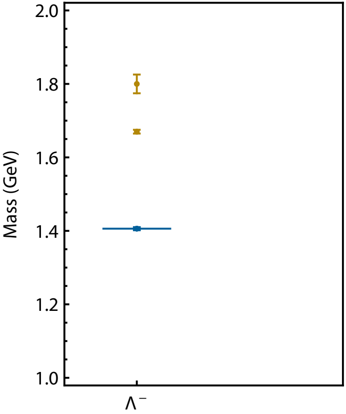

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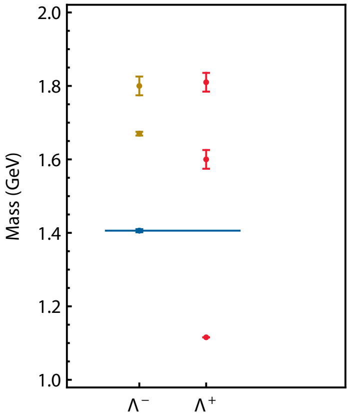

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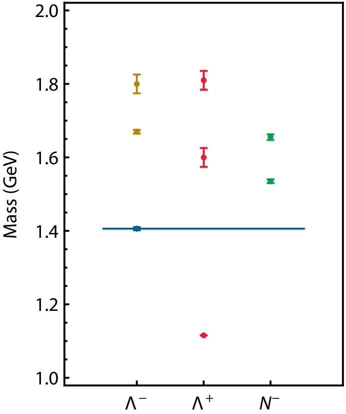
- Such a low mass is puzzling:
    - lies lower than the $\Lambda(1600)$, but has negative parity.
    - lies lower than the $N(1535)$, but has a valence strange quark.



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- The Roper resonance of the nucleon is also abnormally low-lying, and Lattice QCD has had similar trouble in isolating it.
- The CSSM Lattice Collaboration has developed a technique that has successfully isolated the Roper.

M. Selim Mahbub, et al., arXiv:1011.5724
M. S. Mahbub, et al., PoS Lattice 2010, 112, arXiv:1011.0480

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- We apply the same techniques to the Lambda in an attempt to isolate the $\Lambda(1405)$.
- Last year we showed that such an analysis is necessary to isolate this state.

BM, et al., AIP Conf. Proc., **1354**, 213 (2011), `arXiv:1102.3492`

Outline

1. **Technique**
   - Variational Analysis
   - Lattice Details
2. **Results**
   - Common Interpolator Results
   - Octet & Singlet Flavour-Symmetry Results
3. **Conclusions**

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2. Use eigenanalysis to project out correlation functions for individual baryon states.
3. Analyse these projected correlation functions using normal techniques.

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- Calculate the $N \times N$ matrix of zero-momentum, parity-projected cross correlation functions from these operators:

$$
\begin{aligned}
G_{ij}^{\pm}(t) &= \sum_{\mathbf{x}} \mathrm{tr}\left(\Gamma_{\pm} \langle \Omega | \chi_i(\mathbf{x}, t) \overline{\chi}_j(\mathbf{x}, 0) | \Omega \rangle \right) \\
&= \sum_{\alpha=1}^{N} \lambda_i^{\alpha} \overline{\lambda}_j^{\alpha} \mathrm{e}^{-m_\alpha t}.
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$$= \sum_{\alpha=1}^{N} \lambda_i^{\alpha} \overline{\lambda}_j^{\alpha} \mathrm{e}^{-m_\alpha t}.$$

- Construct a set of N "perfect" operators $\varphi_\alpha(\mathbf{x}, t)$ that completely isolate the N lowest states from linear combinations of our original operators:

$$\chi_i = \sum_{\alpha} v_i^{\alpha} \varphi_\alpha \quad \text{and} \quad \overline{\chi}_j = \sum_{\alpha} u_j^{\alpha} \varphi_\alpha.$$

Correlation Matrices

- With a bit of algebraic manipulation, can show that u^α and v^α are the right- and left-eigenvectors of $G^\pm(t)^{-1}G^\pm(t+\Delta t)$, with eigenvalue $\mathrm{e}^{-m_\alpha \Delta t}$.

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- Moreover, the quadratic form $v^\alpha G^\pm(t) u^\beta \propto \delta^{\alpha\beta} e^{-m_\alpha t}$ has t-dependence only in a single exponential term.
- Hence, defining $G^\pm_\alpha(t) := v^\alpha G^\pm(t) u^\alpha$, can extract the masses m_α using the usual

$$m_\alpha = \ln\left(\frac{G^\pm_\alpha(t)}{G^\pm_\alpha(t+1)}\right).$$

Available Operators

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- Flavour symmetry gives three operators:
  - Octet:

$$\chi_i^8 = \frac{1}{\sqrt{6}}\varepsilon_{abc}(2(u_a^{\mathrm{T}}A_i d_b)B_i s_c + (u_a^{\mathrm{T}}A_i s_b)B_i d_c - (d_a^{\mathrm{T}}A_i s_b)B_i u_c),$$

  - Singlet:

$$\chi^1 = -2\varepsilon_{abc}(-(u_a^{\mathrm{T}}C\gamma_5 d_b)s_c + (u_a^{\mathrm{T}}C\gamma_5 s_b)d_c - (d_a^{\mathrm{T}}C\gamma_5 s_b)u_c),$$

  - Common:

$$\chi_i^c = \frac{1}{\sqrt{2}}\varepsilon_{abc}((u_a^{\mathrm{T}}A_i s_b)B_i d_c - (d_a^{\mathrm{T}}A_i s_b)B_i u_c,$$

- The i-index indicates the spin-structure of the operator:

$$(A_1, B_1) = (C\gamma_5, I), \quad (A_2, B_2) = (C, \gamma_5), \text{ and}$$
$$(A_4, B_4) = (C\gamma_5\gamma_4, I).$$

Available Operators

- Similarly to the CSSM Collaboration's investigation of the Roper, we use gauge-invariant Gaussian smearing of the source and sink to further increase our operator basis.
  - We use 16, 35, 100, and 200 sweeps of smearing.

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- Similarly to the CSSM Collaboration's investigation of the Roper, we use gauge-invariant Gaussian smearing of the source and sink to further increase our operator basis.
  - We use 16, 35, 100, and 200 sweeps of smearing.
- This gives us a total of 28 available operators.
  - There will not be enough signal to extract 28 states, but smaller subsets should give useful isolation of the lowest states.

Lattice Details

- We use the PACS-CS $(2+1)$-flavour full-QCD lattices, available from the ILDG.

  PACS-CS Collaboration, Phys. Rev. D, **79**, 034503 (2009), `arXiv:0807.1661`

  - Lattice size is $32^3 \times 64$, with a lattice spacing of $0.0907(13)$ fm.
  - There are 5 light quark masses, with the strange quark mass held fixed.
  - These correspond to pion masses of 623.40 ± 0.75 MeV down to 170.7 ± 2.1 MeV.

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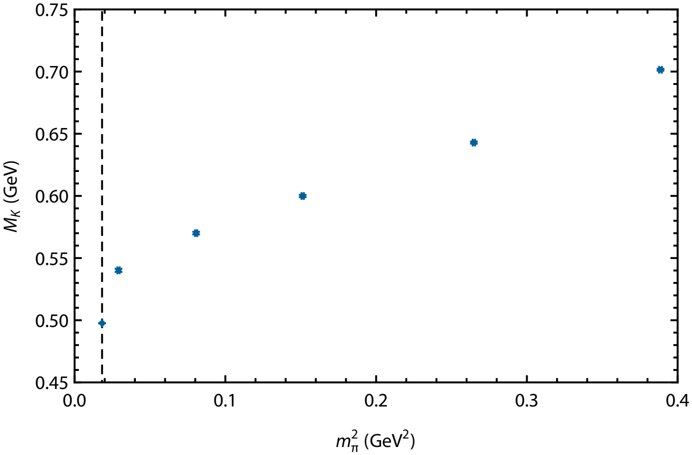
- **PROBLEM**: The strange quark is too heavy!

Kaon Mass



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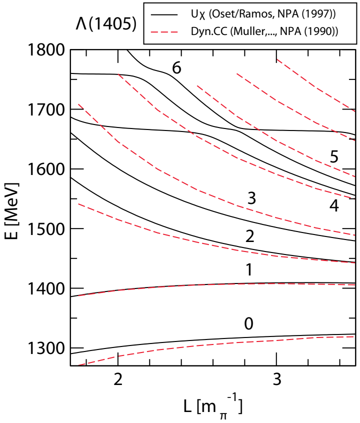

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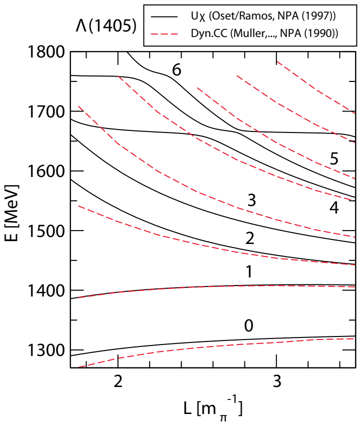
- However, don't have the avoided level crossings that are present in, e.g., the Roper resonance.
- We instead look for the correct arrangement of the states.



Results from Common Interpolating Fields

BM, Waseem Kamleh, Derek B. Leinweber, M. Selim Mahbub, in preparation

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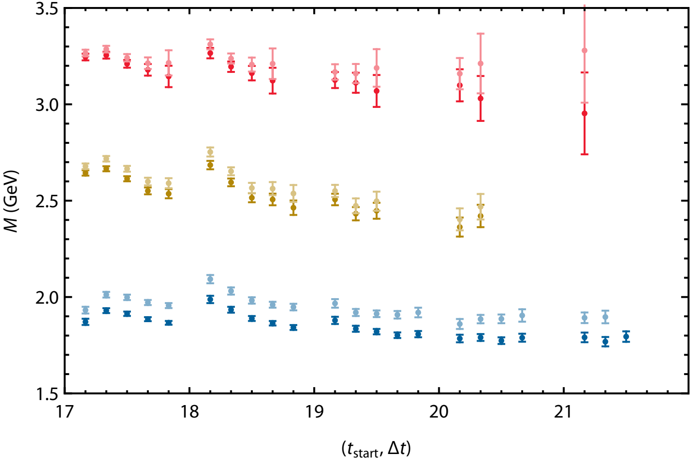
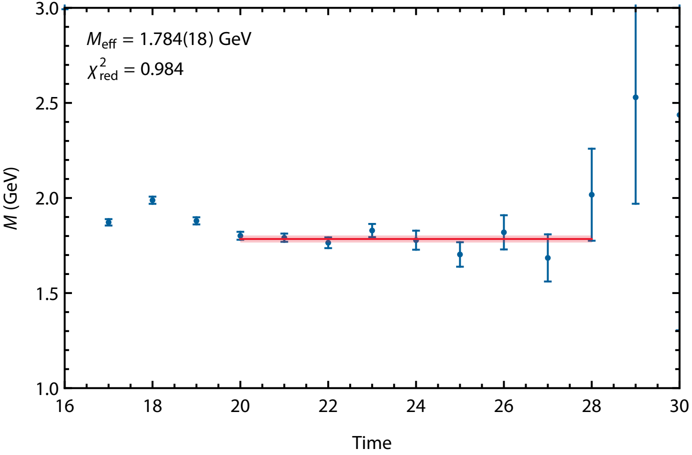
- Need to determine which t_{start} and Δt, and which matrix basis to use.
  - Pick $\kappa_{\text{u,d}} = 0.13727$ ($m_\pi = 514.6 \pm 0.7$ MeV).
  - Use 6×6 basis from χ_1^c and χ_2^c with 16, 100, and 200 sweeps of smearing to investigate t_{start} and Δt.

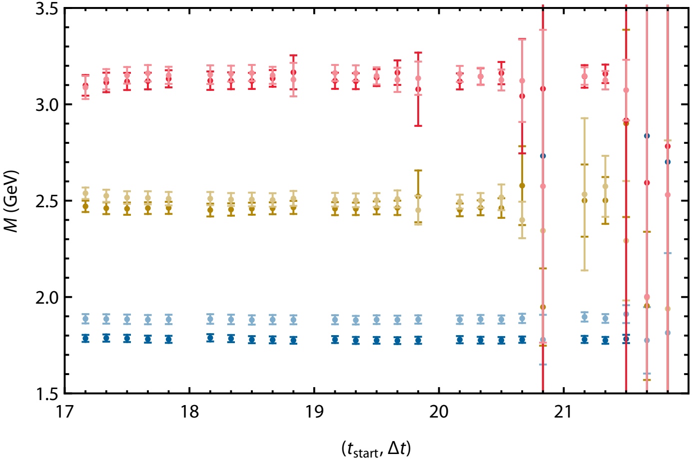
Eigenvalues



Ground State Effective Mass



Fits



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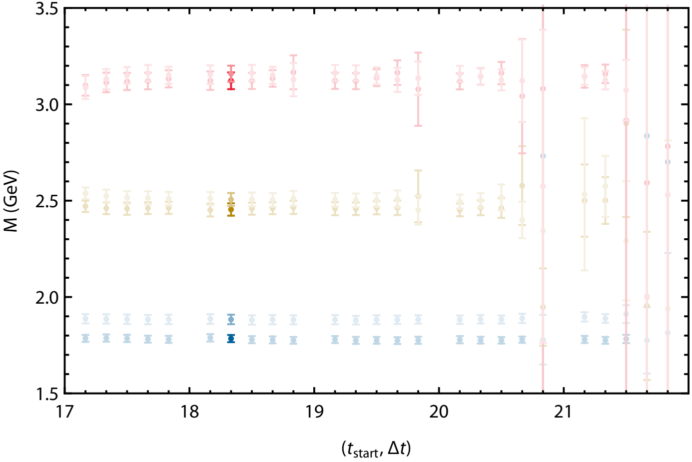
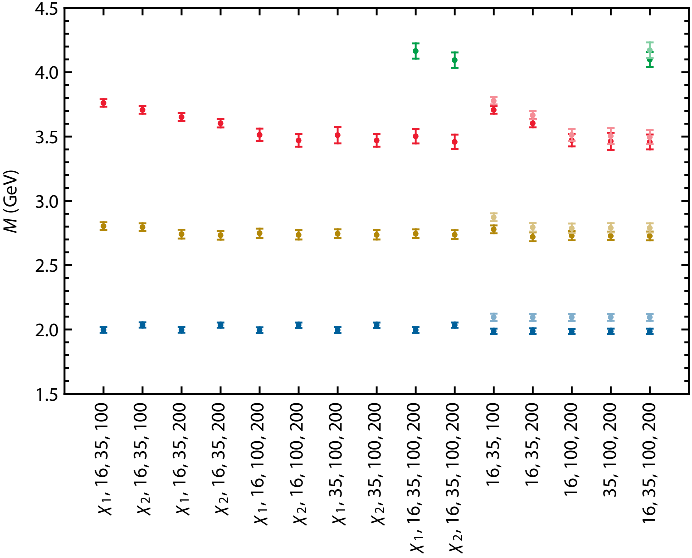


Comparison of Bases



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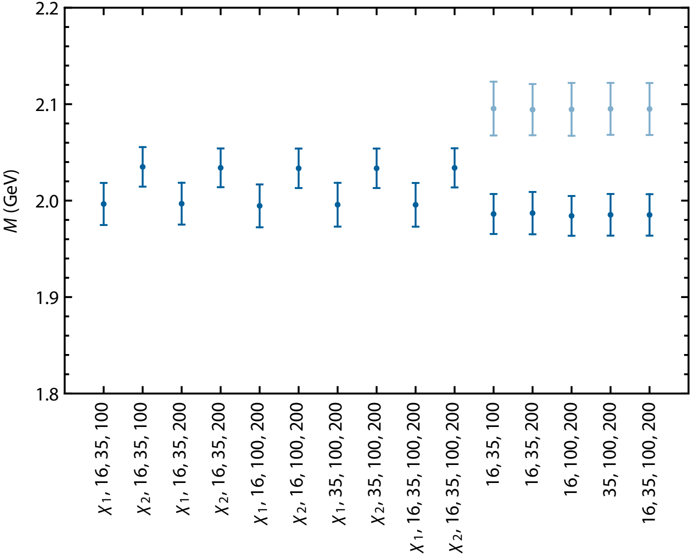

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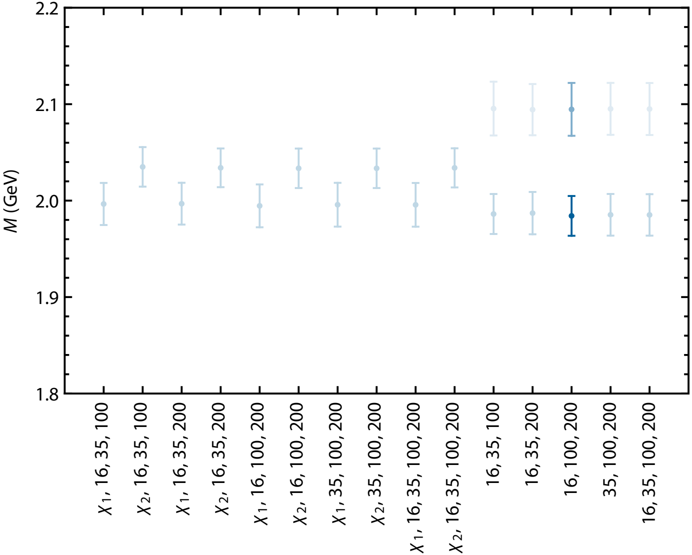

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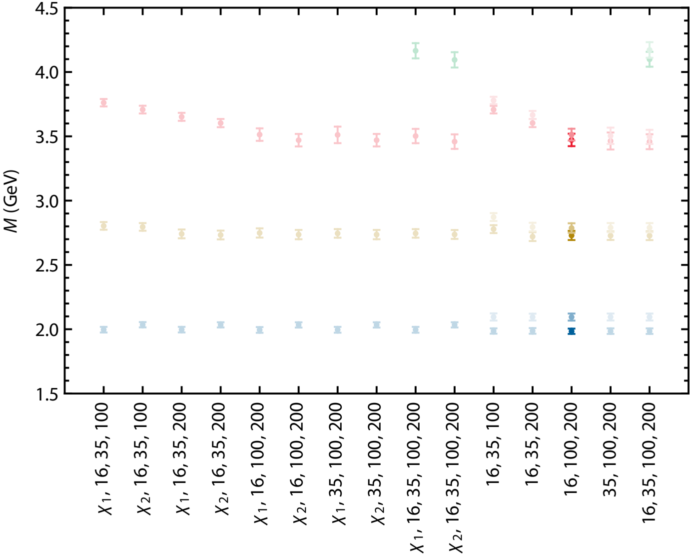

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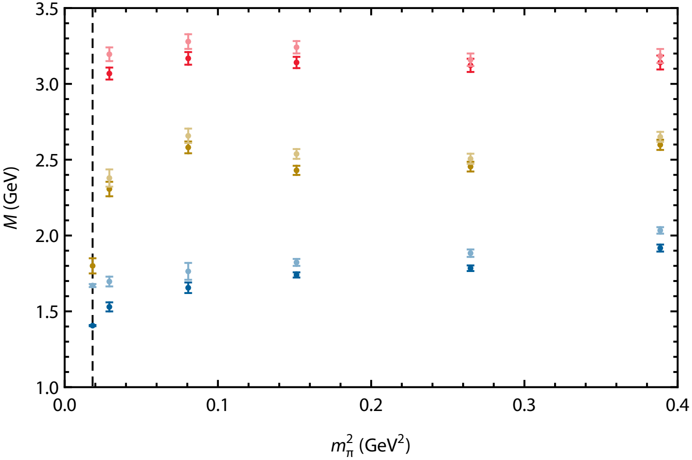

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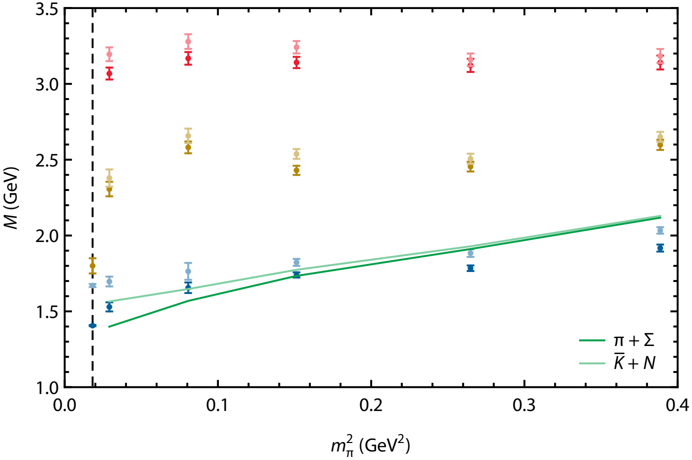
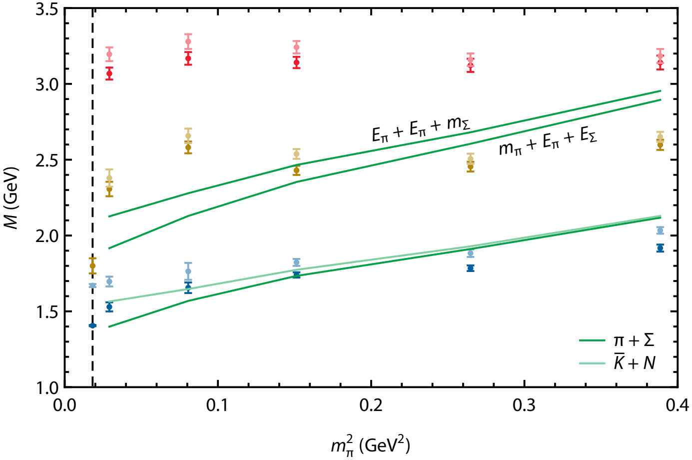

3.5
3.0
2.5
2.0
1.5
1.0
0.0
0.1
0.2
0.3
0.4
M (GeV)
m_π^2 (GeV^2)
$\pi + \Sigma$
$\overline{K} + N$

All States


$E_\pi + E_\pi + m_\Sigma$
$m_\pi + E_\pi + E_\Sigma$
$\pi + \Sigma$
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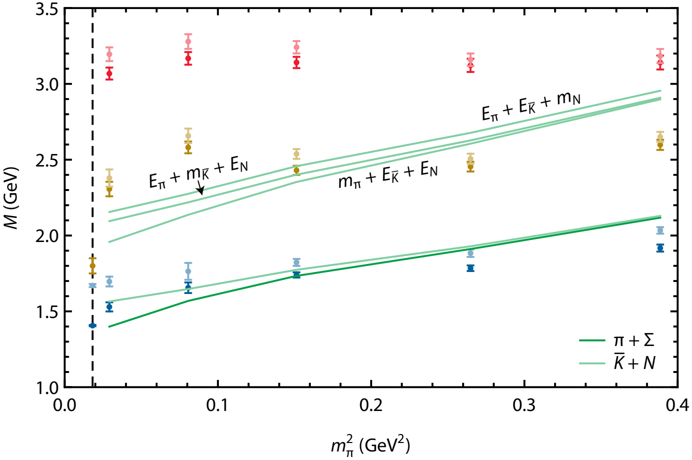

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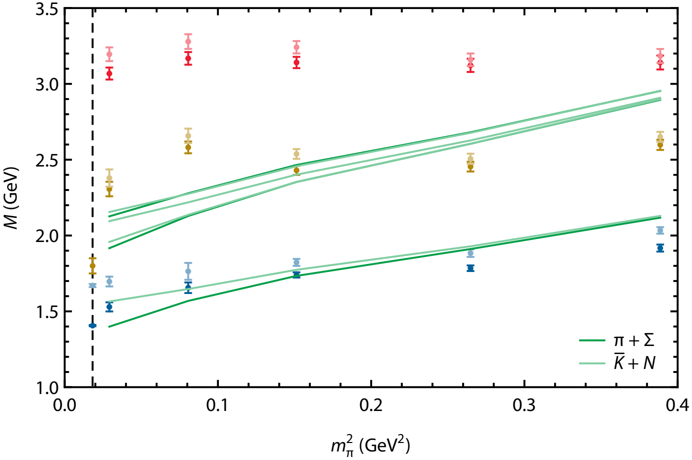
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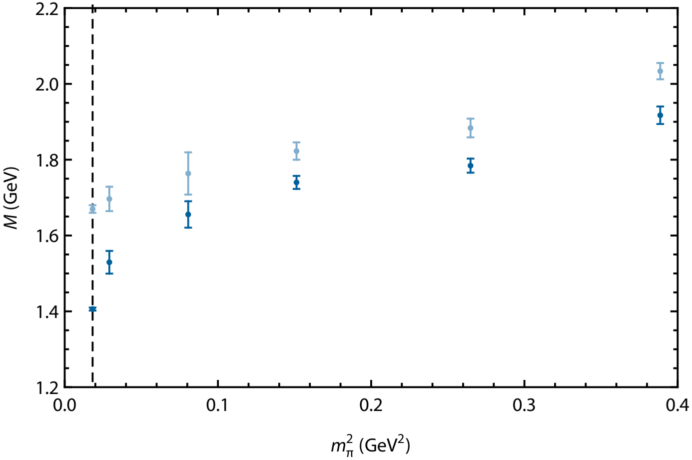
Lowest Two States



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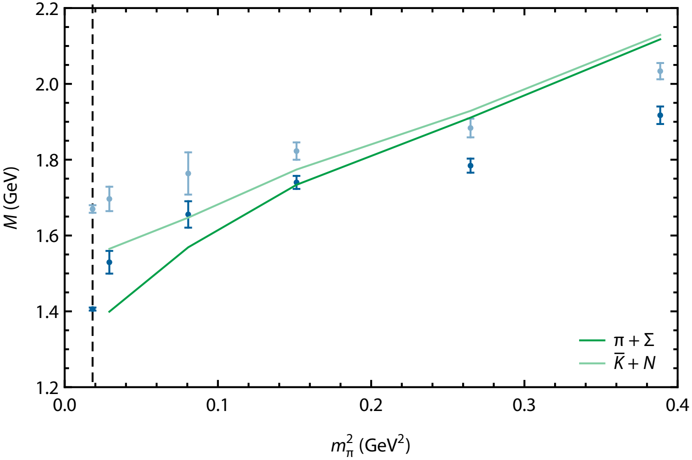

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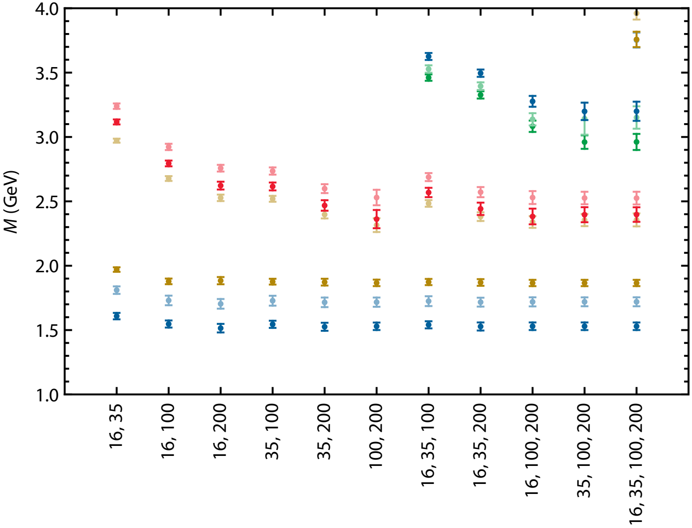
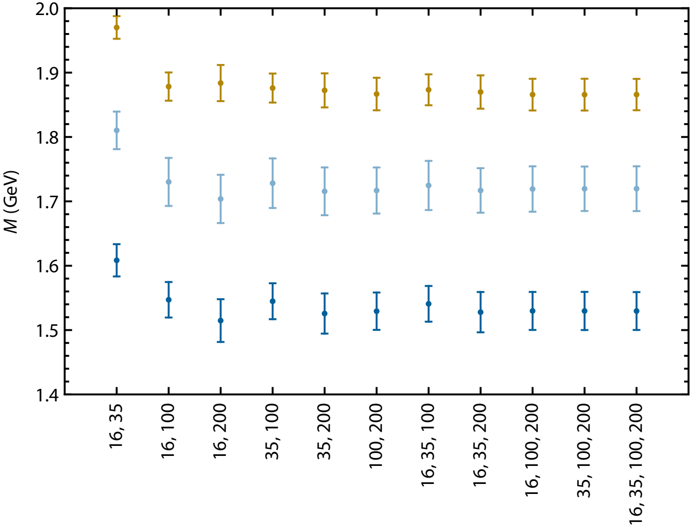
- To begin with, we can extend the analysis to include the octet and singlet flavour-symmetric operators.
- So far, looked at lightest quark mass ($m_\pi = 170.7 \pm 2.1$ MeV).
  - Use $\chi^8_{1,2}$ and χ^1 with various sets of smearing.

Comparison of Bases – All States



Comparison of Bases – Lowest Three States



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- Using this, we can investigate the properties of this unusual resonance.
    - Can attempt to identity the flavour-symmetry associated with each state.
    - Have also begun a form factor study of the $\Lambda(1405)$.

Acknowledgements

This research was undertaken on the NCI National Facility in Canberra, Australia, which is supported by the Australian Commonwealth Government.