SIDIS & Bessel Weighted Asymmetries

Pan Pacific SPIN 2011

Cairns, Australia

21 June 2011

Leonard Gamberg
Penn State University

D. Boer, B. Musch and A. Prokudin in preparation

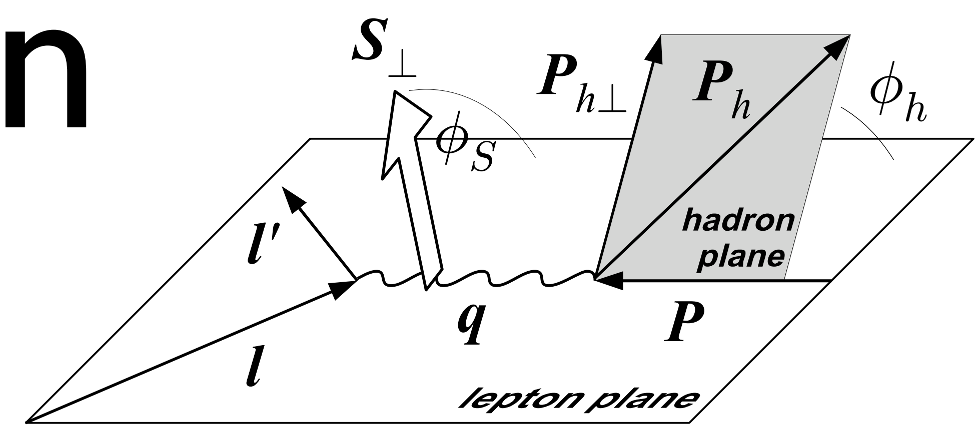
Outline

- **Heurstic Discussion Elements of factorization in hard QCD processes, "collinear" to "TMD"**
- Factorization and Transverse spin effects
  - Transverse Spin Effects-twist 3 & TMD
  - **Role of Gauge Links (hard processes)-**

    "process dependence", Soft Factor (in SIDIS)
- On the merit of Bessel Weighted asymmetries
- Predictions from QCD SIDIS as a tool for nucleon structure
- Fourier Transformed SIDIS cross section
- Cancellation of the Soft Factor from WA

(PRE-DIS wkshp http://conferences.jlab.org/QCDEvolution/index.html)

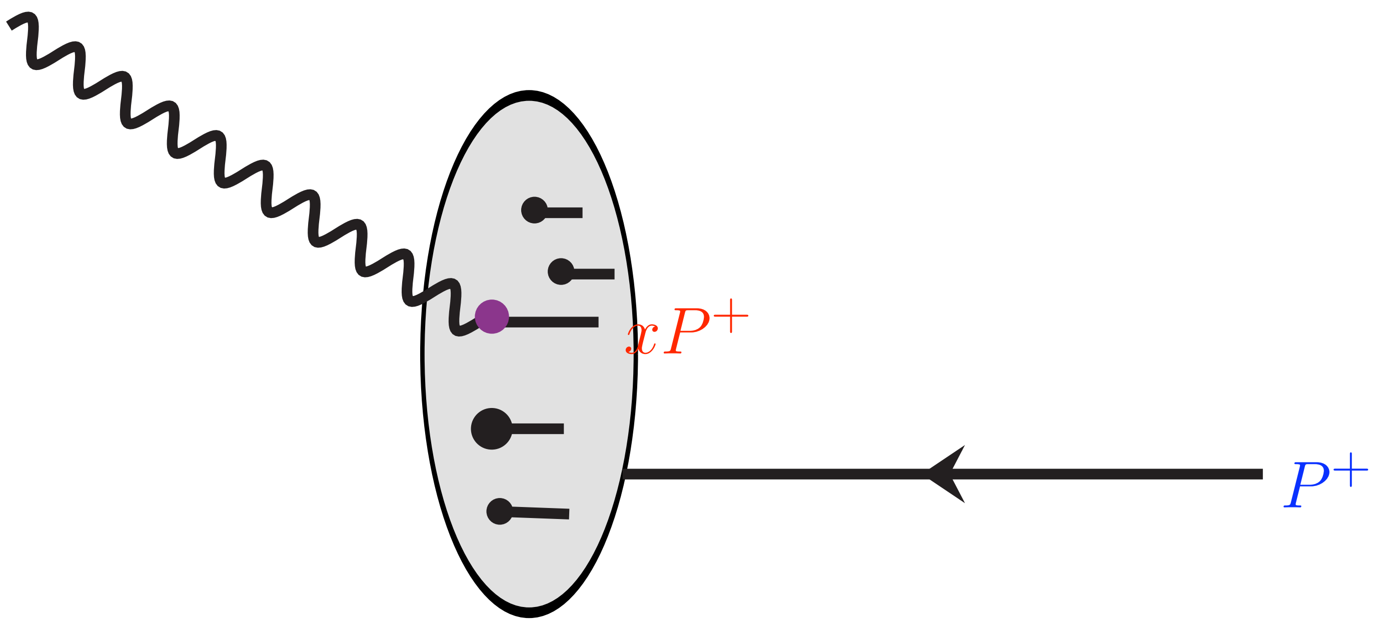
Factorization

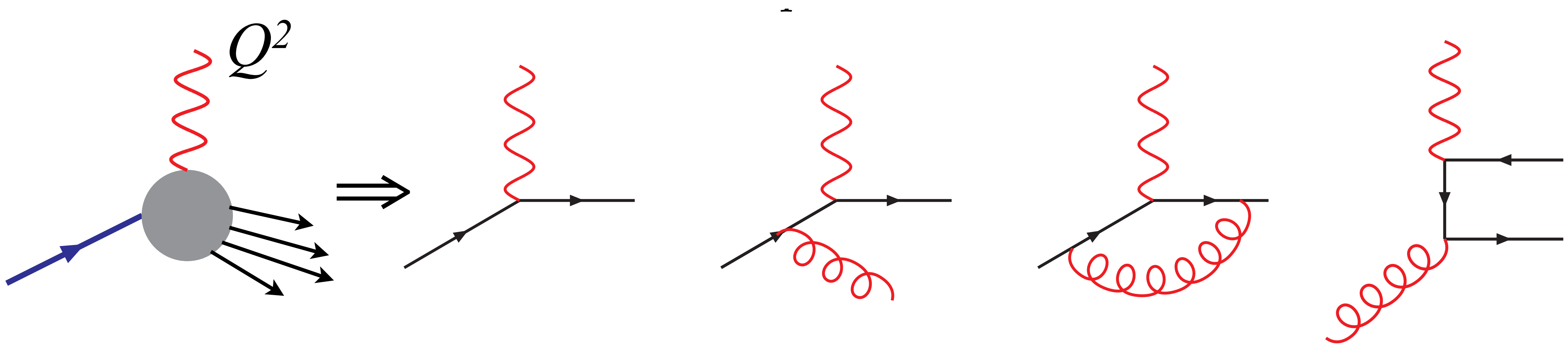


- Depends on momentum of probe $q^2 = -Q^2$ and momentum of produced hadron $\boldsymbol{P}_{h\perp}$ relative to hadronic scale $k_T^2 (\equiv k_\perp^2) \sim \Lambda_{\mathrm{QCD}}^2$
- Cases
  - $k_\perp^2 \sim P_{h\perp}^2 \ll Q^2$ sensitive hadronic scale-TMDs
  - $k_\perp^2 \ll P_{h\perp}^2 \ll Q^2$ twist 3 factorization-ETQSs
  - $k_\perp^2 \ll P_{h\perp}^2 \sim Q^2$ insensitive to hadronic scale

Collinear DIS picture of the nucleon
x & Q^2

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}Q^2} = \sum_q q(x, Q^2) \otimes \frac{\mathrm{d}\hat{\sigma}_q}{\mathrm{d}Q^2}$$

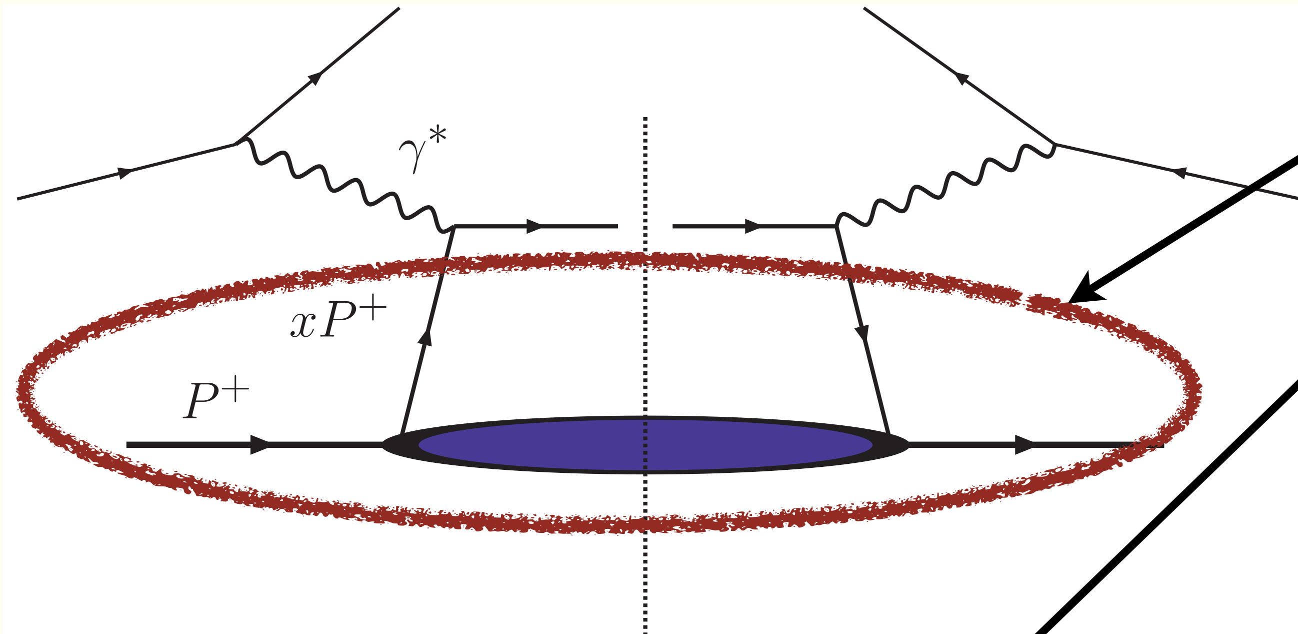




Note operator structure of DIS
Factorizes into hard and soft

$$\sigma = \underbrace{\hat{\sigma}_{\text{part.}}}_{\text{hard}} \otimes \underbrace{\text{PDF}}_{\text{soft}}$$

$$e^- + N \to e^- + X$$



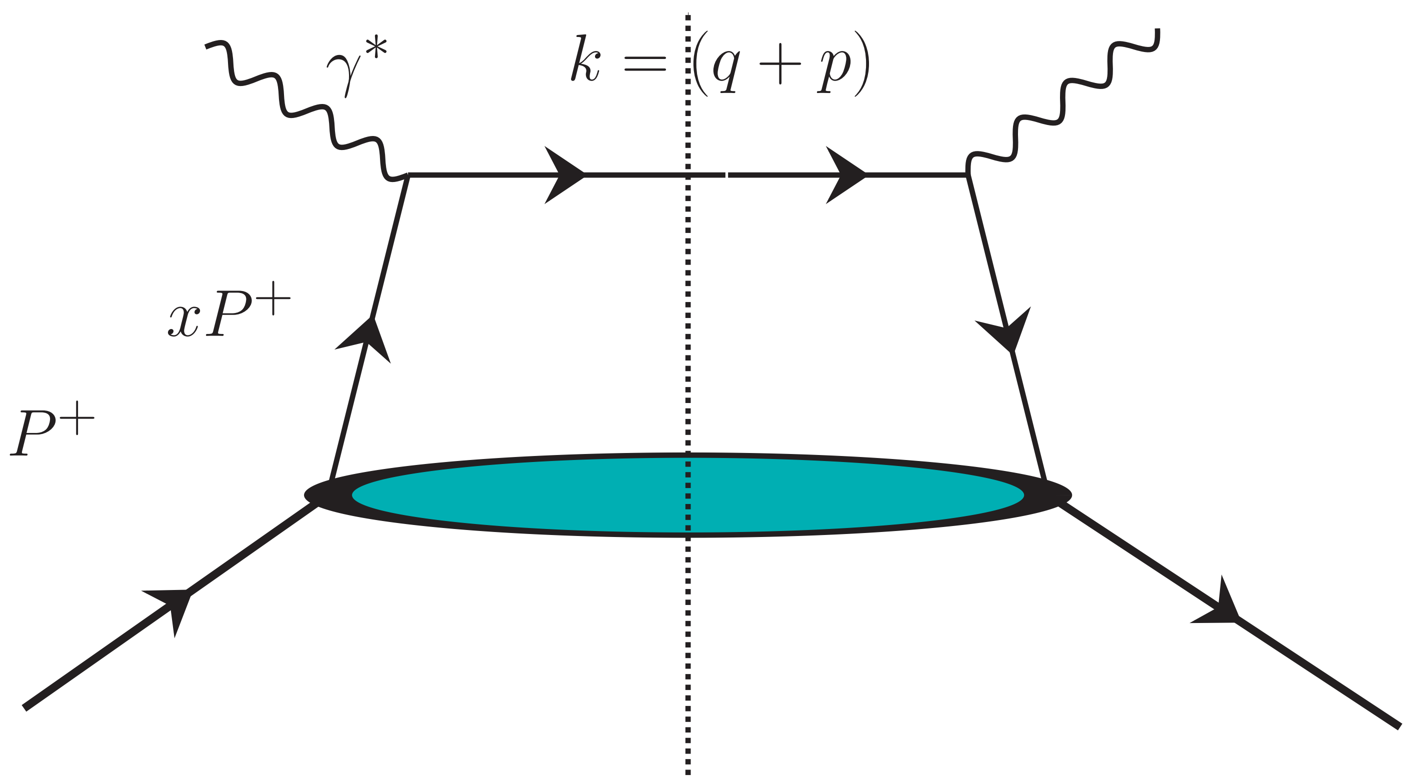
D. Amati, R. Petronzio, and G. Veneziano, Nucl. Phys. **B140** (1978) 54 and **B146** (1978) 29; R.K. Ellis, H. Georgi, M. Machacek, H.D. Politzer, and G.G. Ross, Nucl. Phys. **B152** (1979) 285; A.V. Efremov and A.V. Radyushkin, Teor. Mat. Fiz. **44** (1980) 17 [Eng. transl.: Theor. Math. Phys. **44** (1981) 573], Teor. Mat. Fiz. **44** (1980) 157 [Eng. transl.: Theor. Math. Phys. **44** (1981) 664], Teor. Mat. Fiz. **44** (1980) 327 [Eng. transl.: Theor. Math. Phys. **44** (1981) 774]; S. Libby and G. Sterman, Phys. Rev. **D18** (1978) 3252, 4737; A.H. Mueller, Phys. Rev. **D18** (1978) 3705.

$$f(x) = \int \frac{dz^-}{(4\pi)} e^{ixP^+z^-} \langle P|\bar{q}\left(\frac{-z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right)|P\rangle|_{z^+=\mathbf{z}_\perp=0}$$

Correlator

$$\Phi(x) = \tfrac{1}{2}\{f(x)\not{P} + \lambda_N \Delta f(x)\gamma_5\not{P} + \Delta_T f(x)\not{P}\gamma_5\not{S}_\perp\}.$$

Collinear DIS picture of the nucleon
x & Q^2

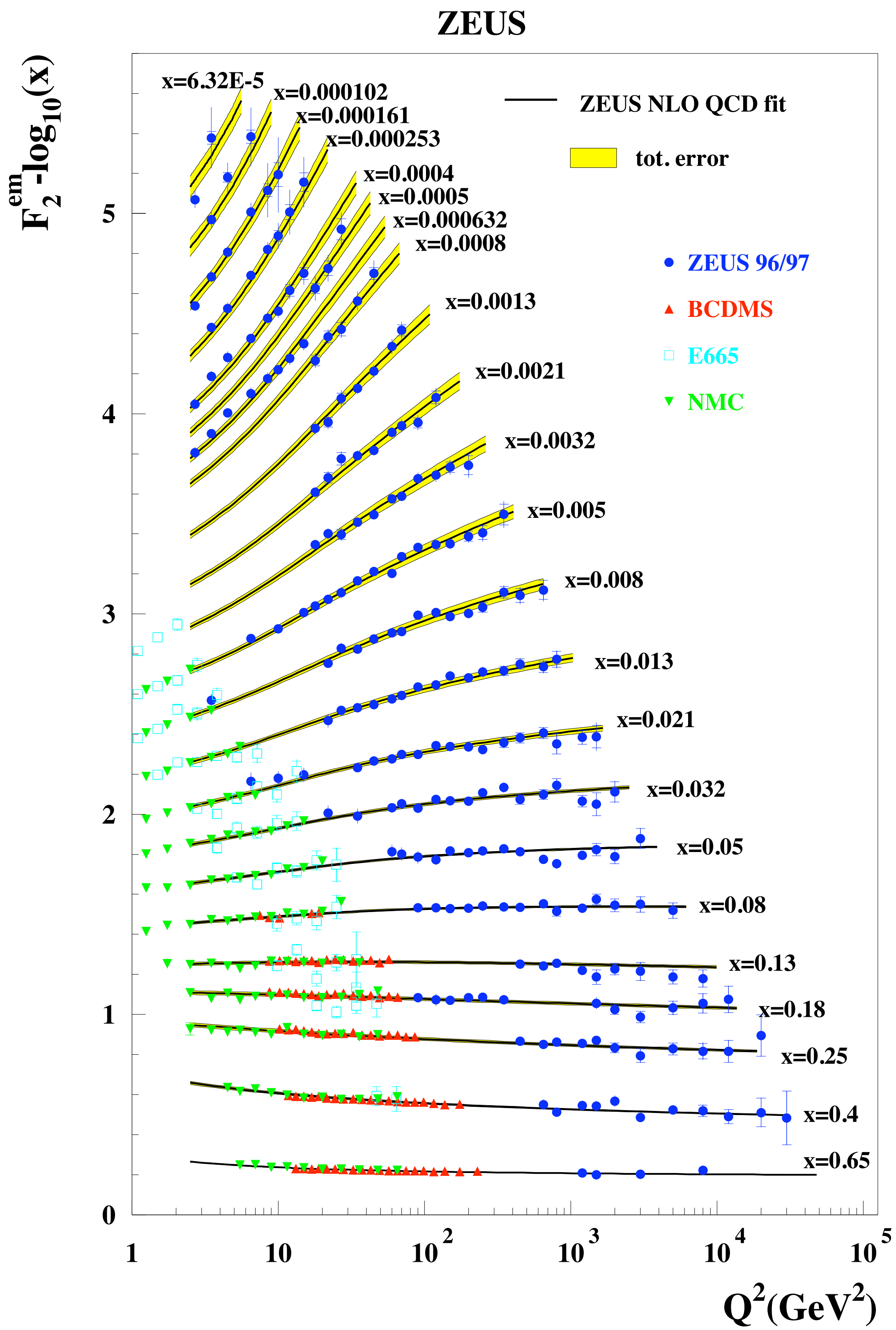


$\Delta_T q$

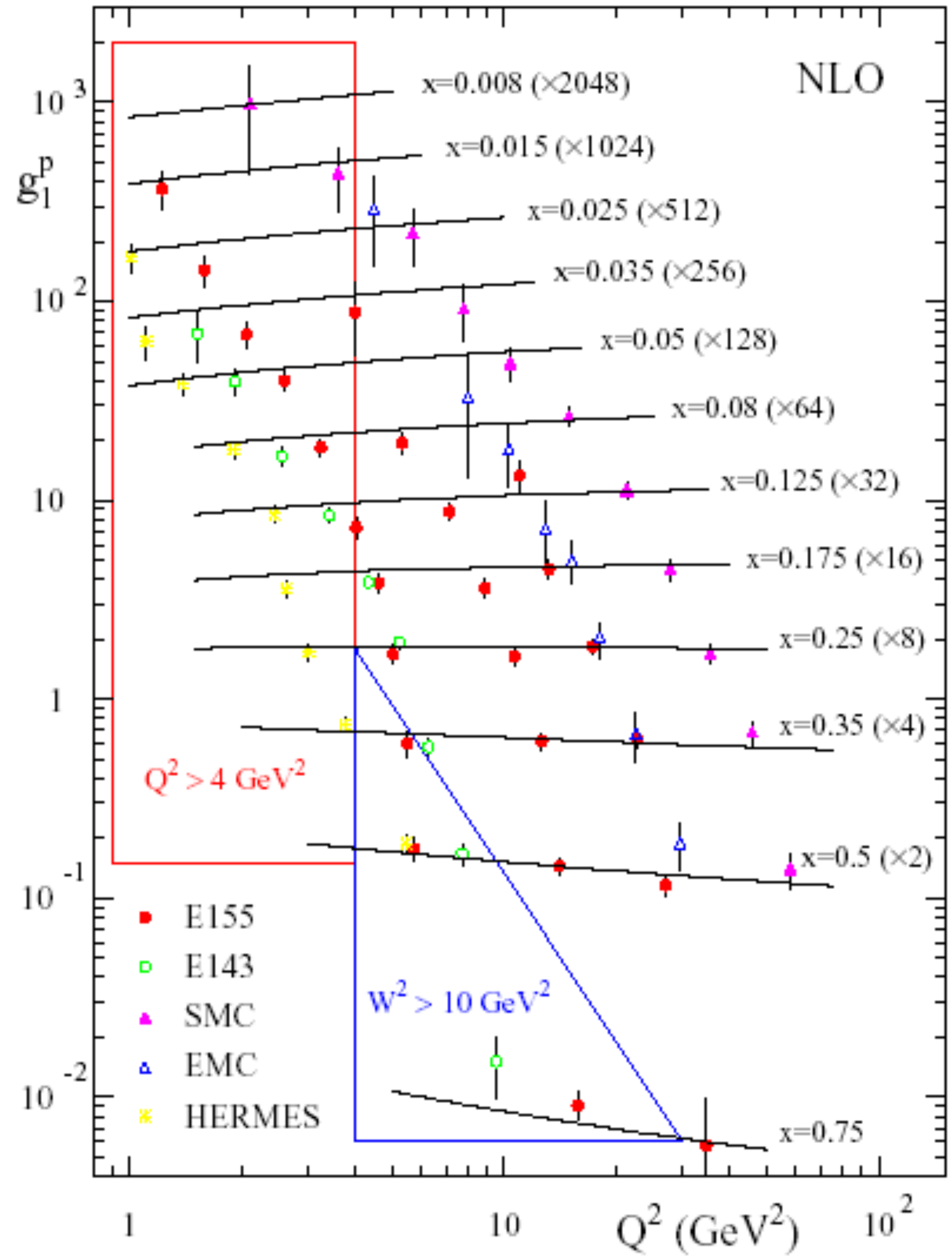
$$\Phi(x,S) = \frac{1}{2}\left[f_1(x)\,\not{n}_+ + S_L\, g_{1L}(x)\,\gamma^5\,\not{n}_+ \quad + h_1(x)\, i\sigma_{\mu\nu}\gamma^5 n_+^\mu S_T^\nu\right.$$

q $\qquad$ Δq $\qquad$?

The theoretical hint to go beyond coliner limit



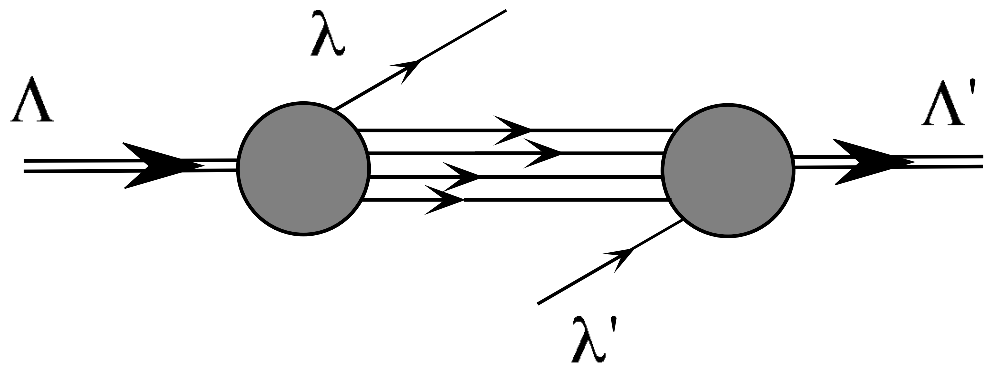
$$F_2 = \sum_q x\, q(x, Q^2)$$



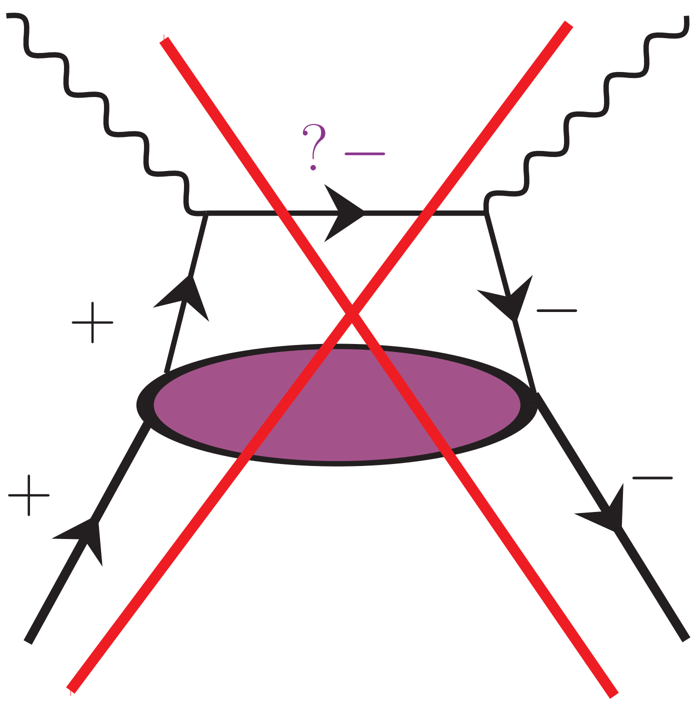
$$g_1 = \frac{1}{2} \sum_q e_q^2\, \Delta q(x, Q^2)$$

Either from helicity description or Dirac Structure $i\sigma_{\mu\nu}\gamma^5$
Transversity Decouples from DIS

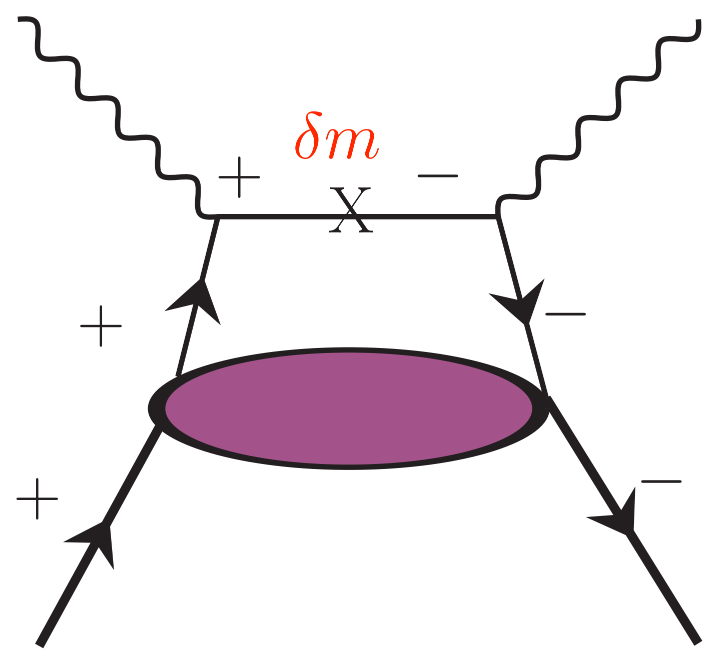
Jaffe Ji 1991 prl



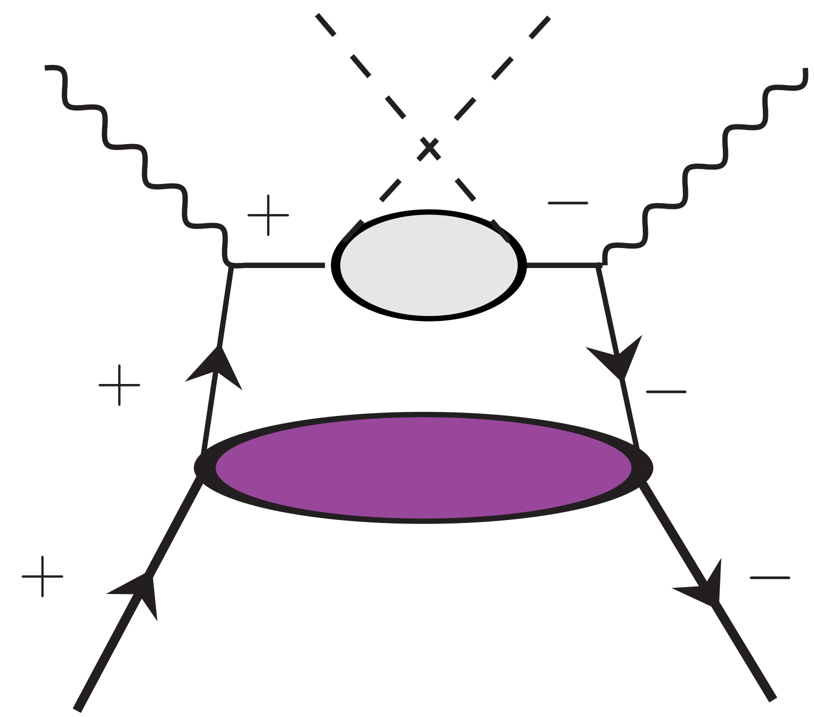
$QCD\,int.\,cannot\,flip\,helicity$



$helicity\,flip\,at\,twist-3$

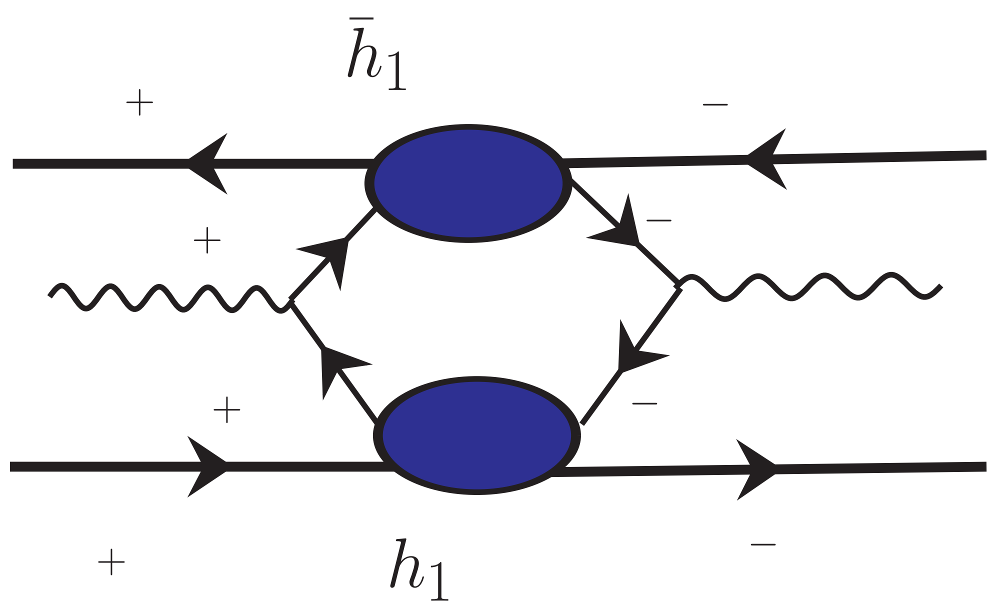


$helicity\,flip\,at\,twist-2$

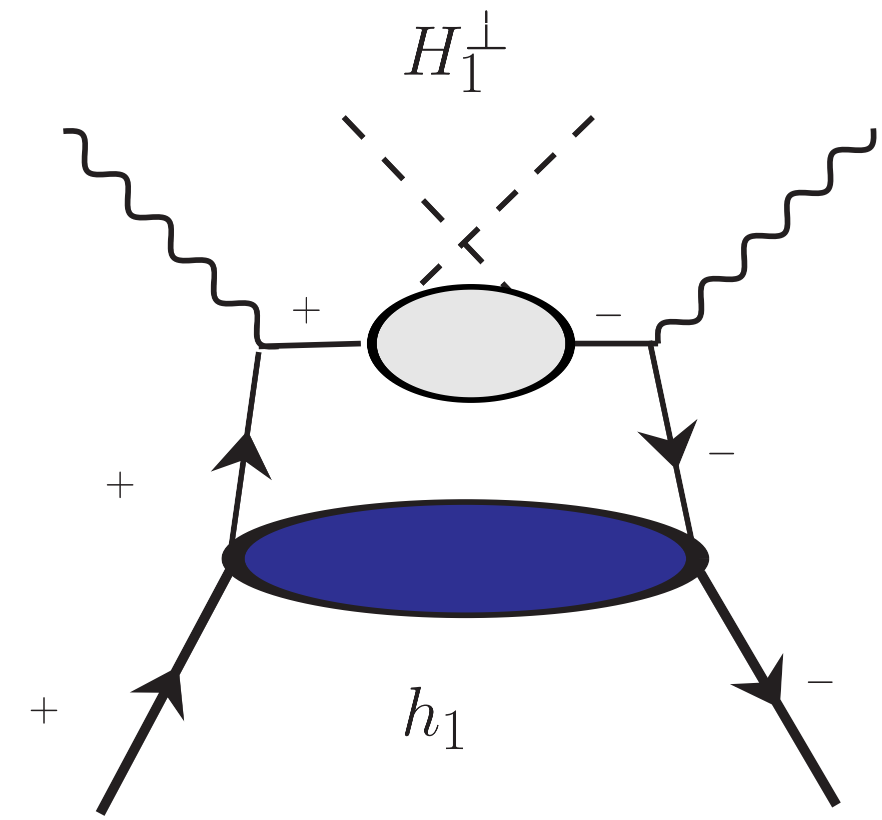


need second chiral odd structure

Need for theory of unintegrated PDFs
Transv Polarization and SIDIS



Drell-Yan Ralston Soper NPB 1979

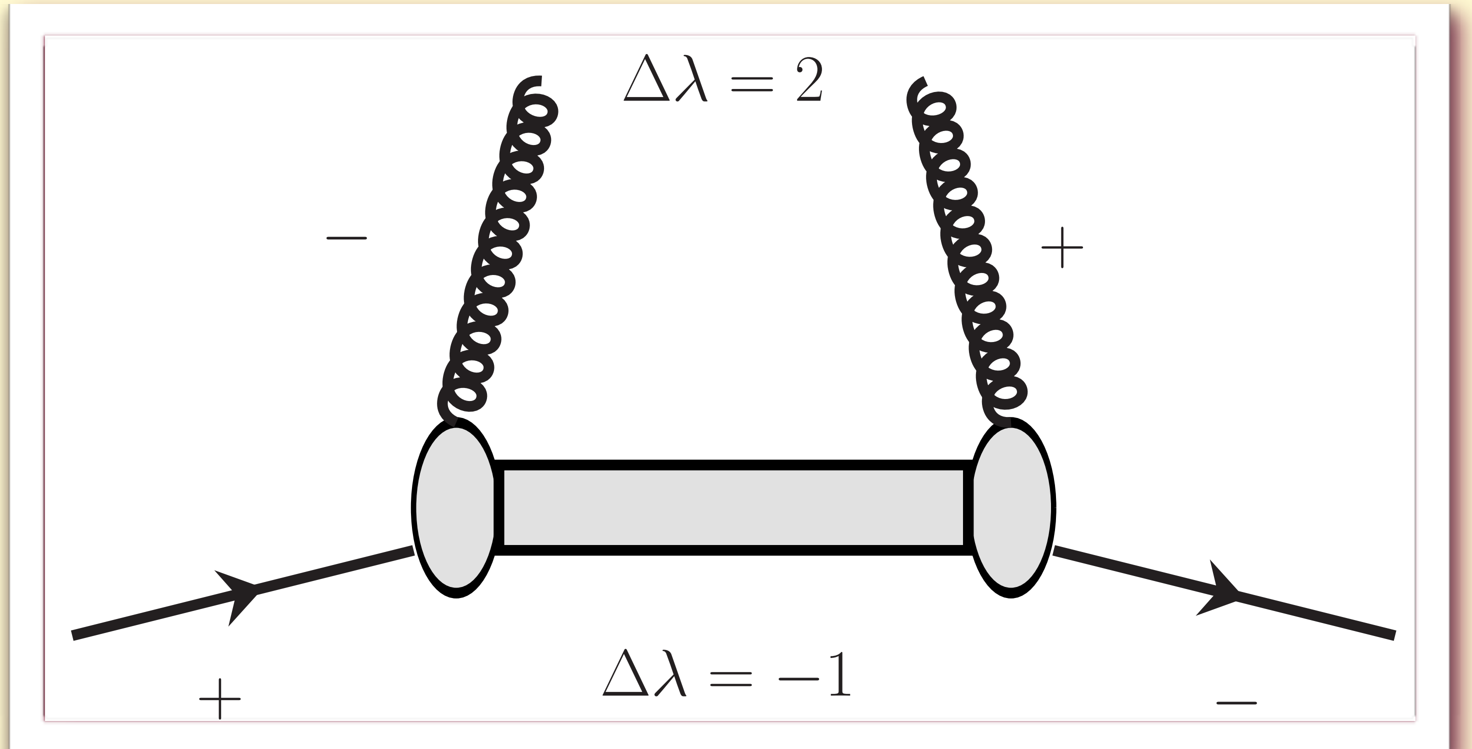


SIDIS Collins NPB 1993

No Gluon Transversity

- Evolves as a non-singlet!

  ie doesn't mix with glue
- Uniquely Valence-like
- A golden opportunity for JLAB 6 &12 GeV Hall A and CLAS programs and EIC

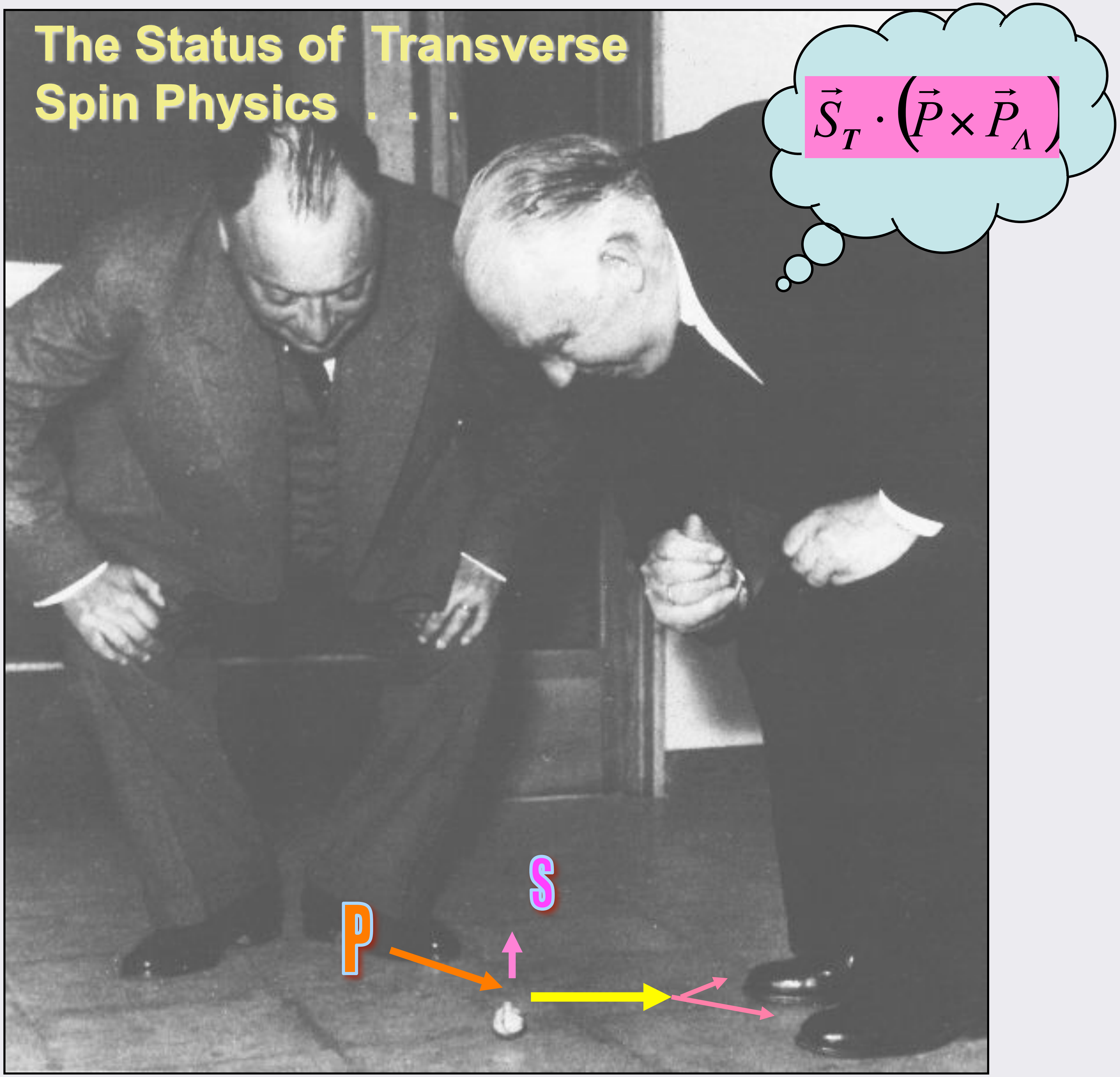


Eur. Phys. J. Plus (2011),
H. Gao, L. Gamberg, J.-P. Chen, X. Qian, Y. Qiang, M. Huang, A. Afanasev, M. Anselmino, H. Avakian, G. Cates, E. Chudakov, E. Cisbani, C. de Jager, F. Garibaldi, B.T. Hu, X. Jiang, K. S. Kumar, X.M. Li, H.J. Lu, Z.-E. Meziani, B.-Q. Ma, Y.J. Mao, J.-C. Peng, A. Prokudin, M. Schlegel, P. Souder, Z.G. Xiao, Y. Ye and L. Zhu

Study of Transverse Structure driven by Discrepancies between Exp results on TSSAs and Collinear QCD Pic

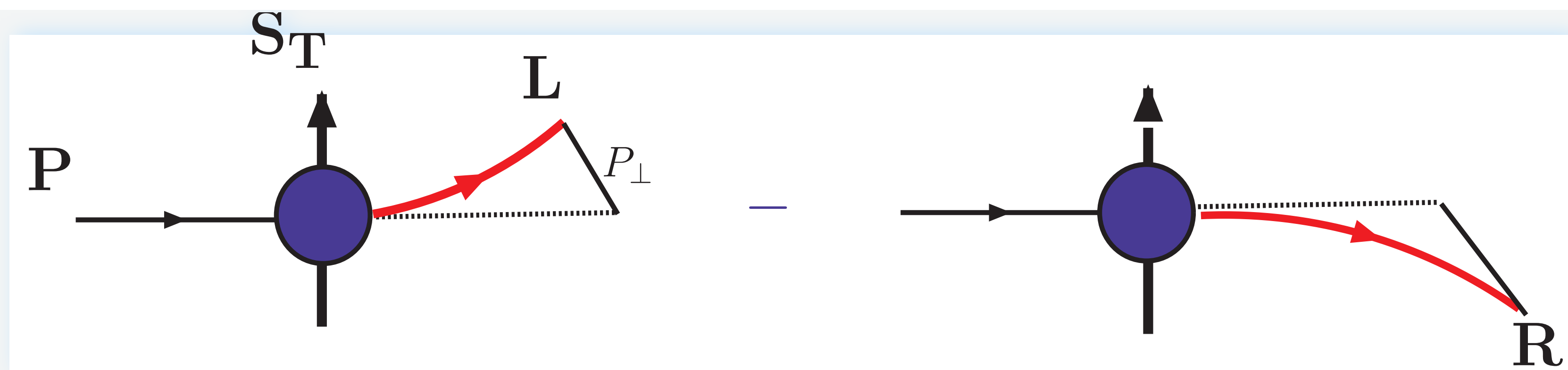
- Exp. Large TSSAs while collinear picture predicts triviality
- Quark transversity $\Delta_T\, q(x)$ distribution on the same footing as $q(x)$ and $\Delta q(x)$ yet inaccessible in DIS !
- While Collinear QCD can account for unpol. and long. pol. pheno, the pheno. of transverse structure demands the more general kinematic and dynamic structure of "unintegrated" pdfs---what Jaffe referred to as a "renaissance" in QCD spin physics (2001).
- 1993 Collins consider that SIDIS at low $P_{h\perp}$ provides a tool to measure quark transversity
- Physics potential reinforced by rediscovery of Sivers' effect by BHS

Transverse Spin Effects



Ingredients of factorization in Transverse SPIN Observables TSSA $P^{\uparrow}P \to \pi X$

- **Single Spin Asymmetry**



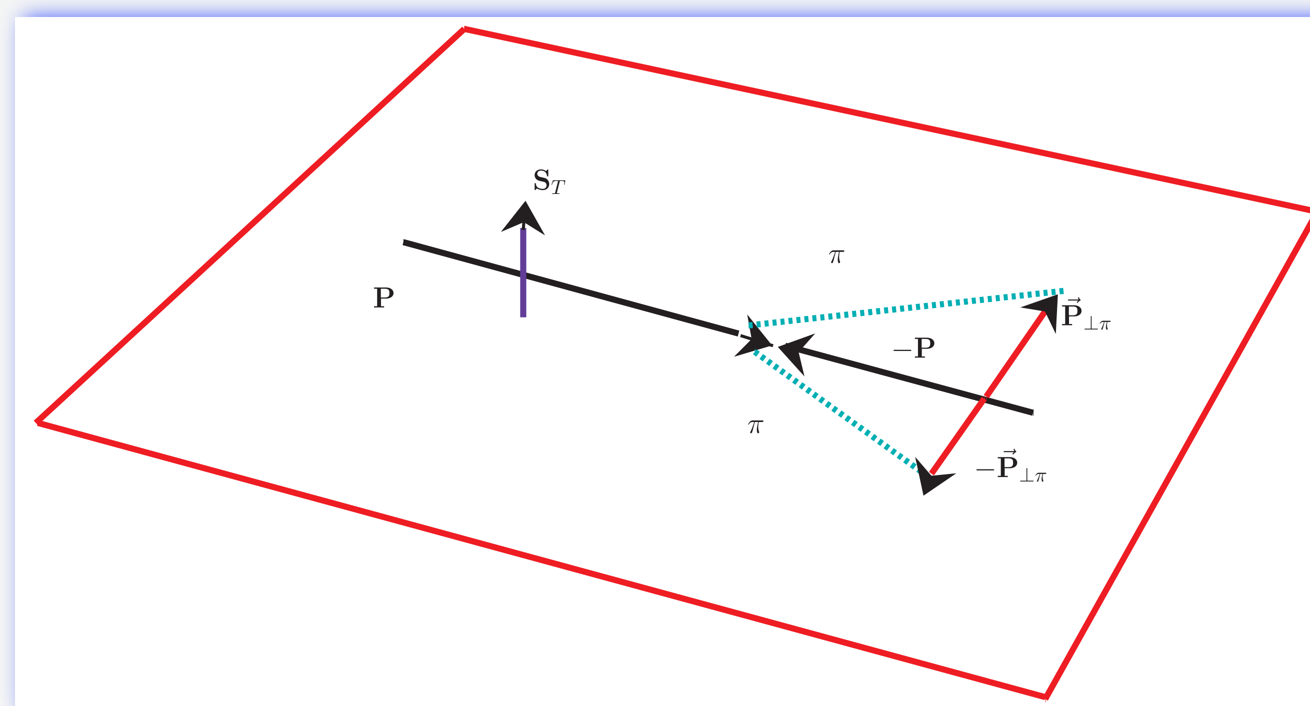
Parity Conserving interactions: SSAs Transverse Scattering plane

$$\Delta\sigma \sim iS_T \cdot (\mathbf{P} \times P_\perp^\pi)$$

- **Rotational invariance** $\sigma^{\downarrow}(x_F, \boldsymbol{p}_\perp) = \sigma^{\uparrow}(x_F, -\boldsymbol{p}_\perp)$

  $\Rightarrow$ ***Left-Right Asymmetry***

$$\boldsymbol{A}_N = \frac{\sigma^{\uparrow}(x_F,\boldsymbol{p}_\perp)-\sigma^{\uparrow}(x_F,-\boldsymbol{p}_\perp)}{\sigma^{\uparrow}(x_F,\boldsymbol{p}_\perp)+\sigma^{\uparrow}(x_F,-\boldsymbol{p}_\perp)} \equiv \Delta\sigma$$



Reaction Mechanism Partonic Description $P^{\uparrow} P \to \pi X$

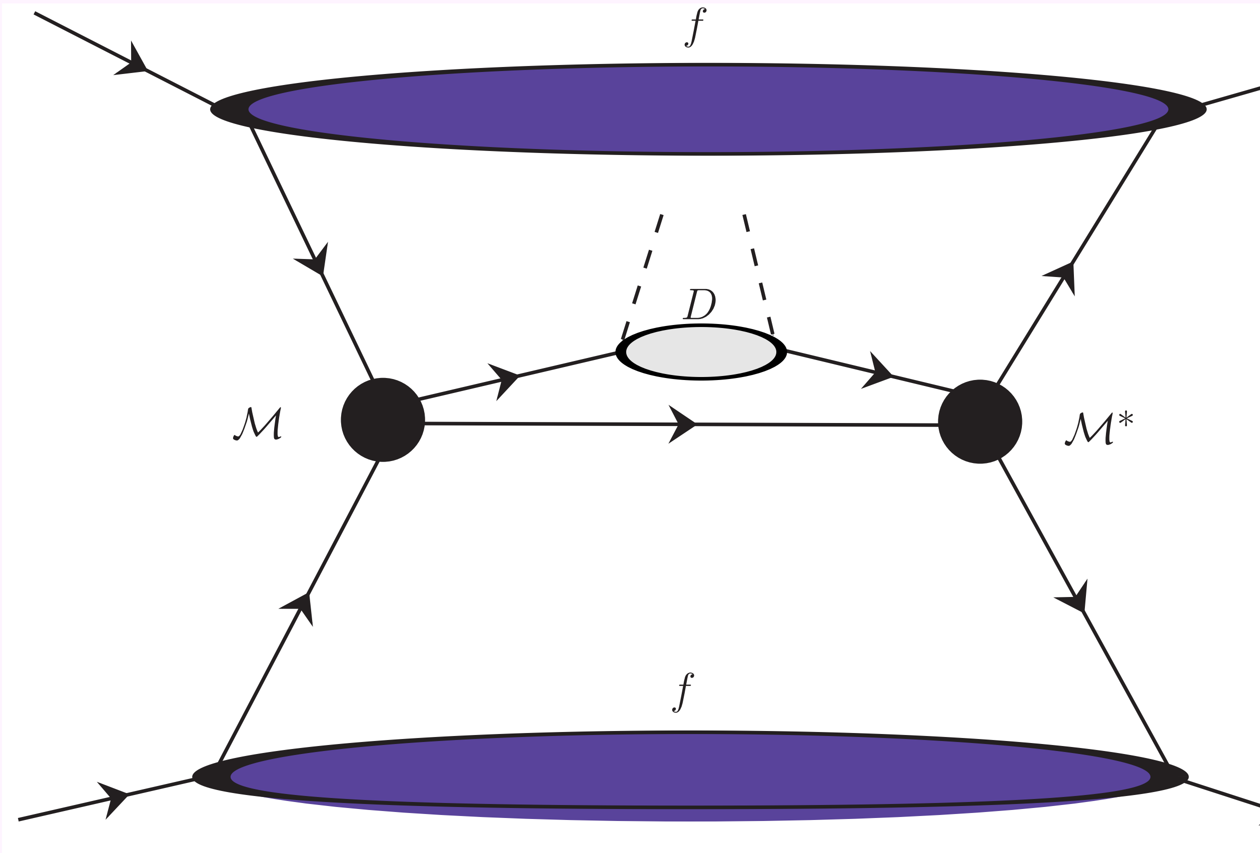
Collinear factorized QCD parton dynamics

$$\Delta\sigma^{pp^{\uparrow}\to\pi X} \sim f_a \otimes f_b \otimes \Delta\hat{\sigma} \otimes D^{q\to\pi}$$

$$\Delta\hat{\sigma} \equiv \hat{\sigma}^{\uparrow} - \hat{\sigma}^{\downarrow}$$

$$|\uparrow / \downarrow\rangle = (|+\rangle \pm i|-\rangle)$$

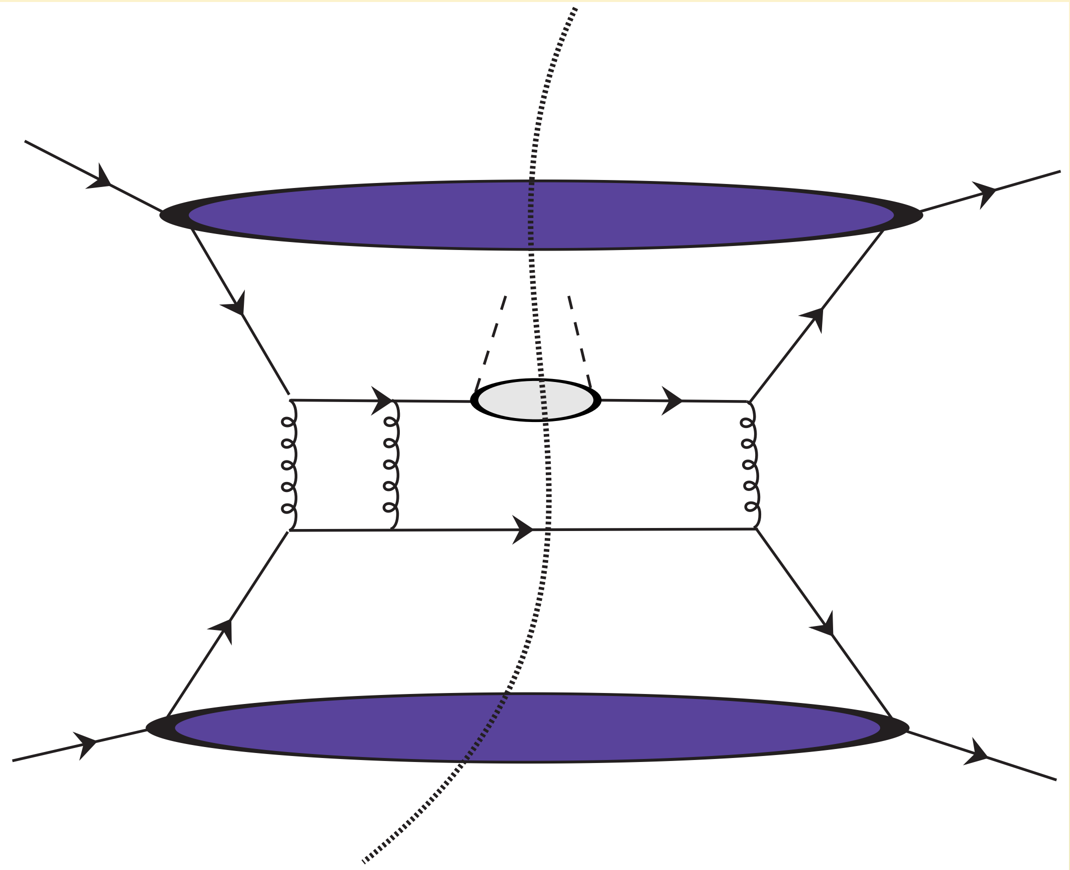
$$\hat{a}_N = \frac{\hat{\sigma}^{\uparrow} - \hat{\sigma}^{\downarrow}}{\hat{\sigma}^{\uparrow} + \hat{\sigma}^{\downarrow}} \sim \frac{\mathrm{Im}\left(\mathcal{M}^{+*}\mathcal{M}^{-}\right)}{|\mathcal{M}^{+}|^2 + |\mathcal{M}^{-}|^2}$$

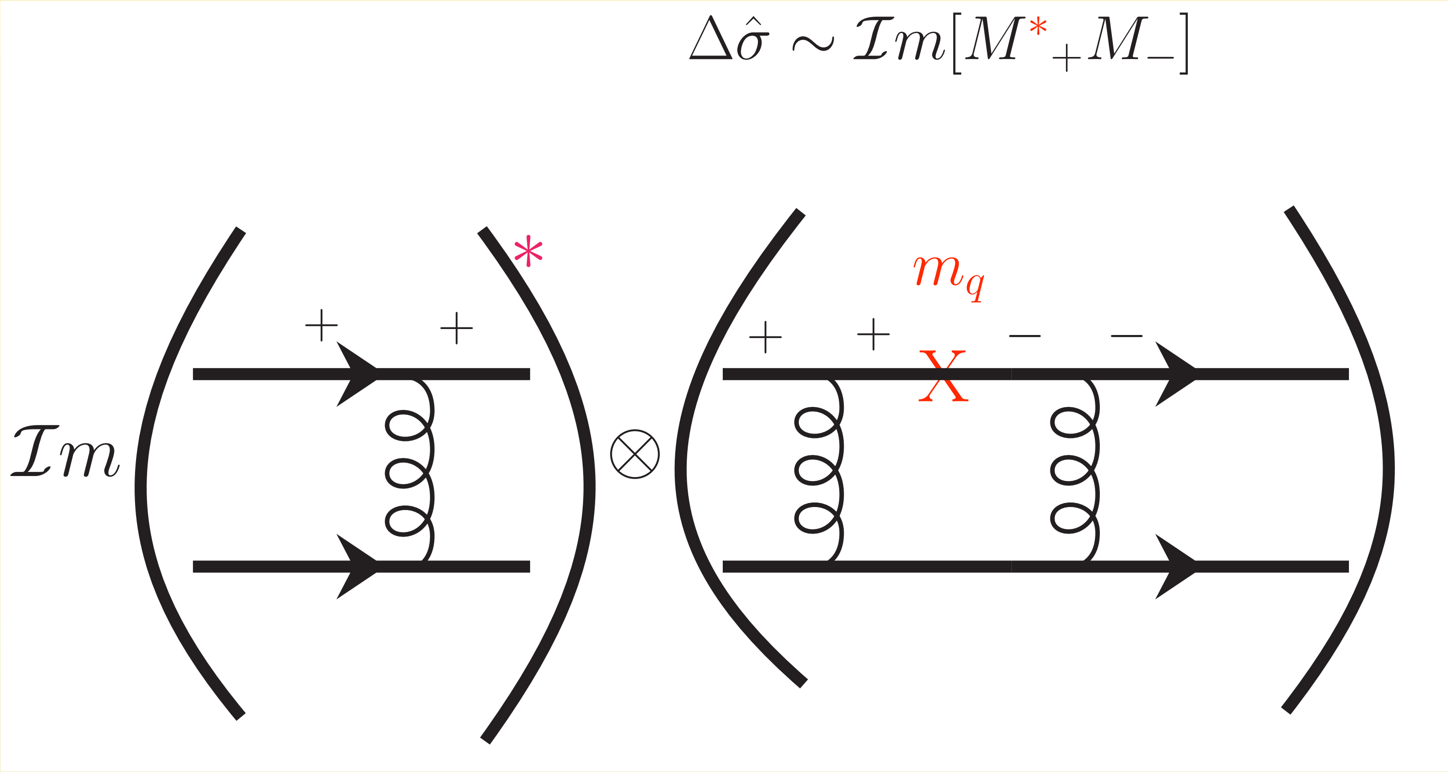


Transv. polarization cross section "interference" of helicity flip and non-flip amps.

Interference of helicity flip and non-flip amps
1) requires breaking of chiral symmetry m_q/E
2) relative phases require higher order correction

Factorization Theorem & SSAs at Partonic level





- **Born amps are real -- need "loops"----> phases**
- **QCD interactions conserve helicity up to corrections** $\mathcal{O}\left(\frac{m_q}{E_q}\right)$

Twist three and trivial in chiral limit

$A_N \propto \frac{m_q}{E}\alpha_s$ at the partonic level   Kane & Repko, PRL: 1978

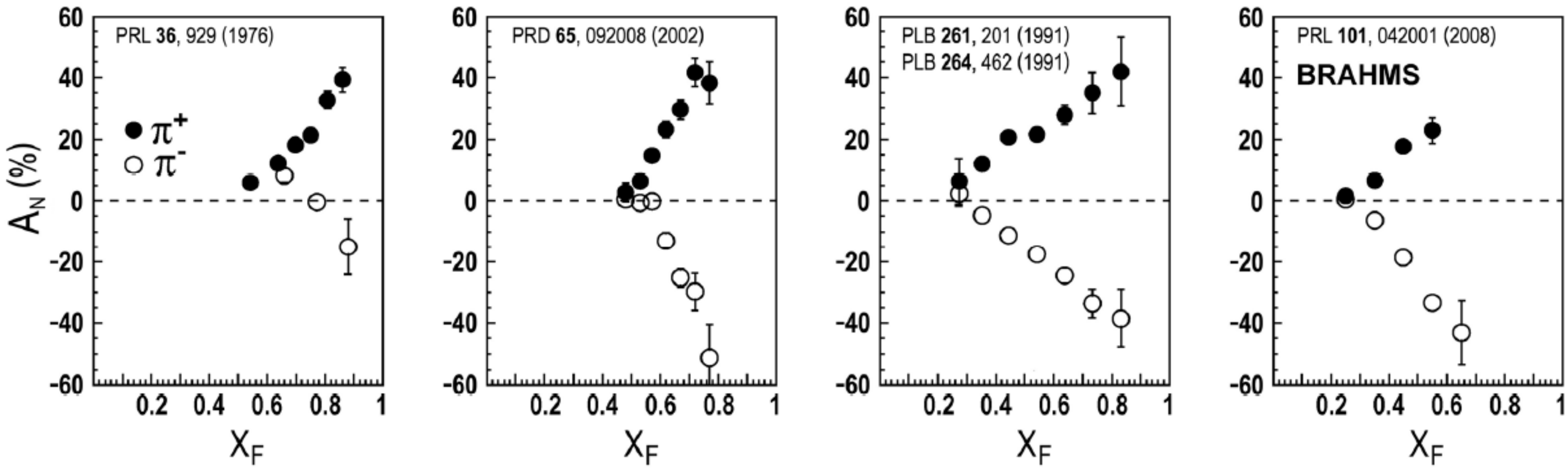
Large Transverse Polarization in Inclusive Reactions

Transverse Single-Spin Asymmetries: From Low to High Energies!

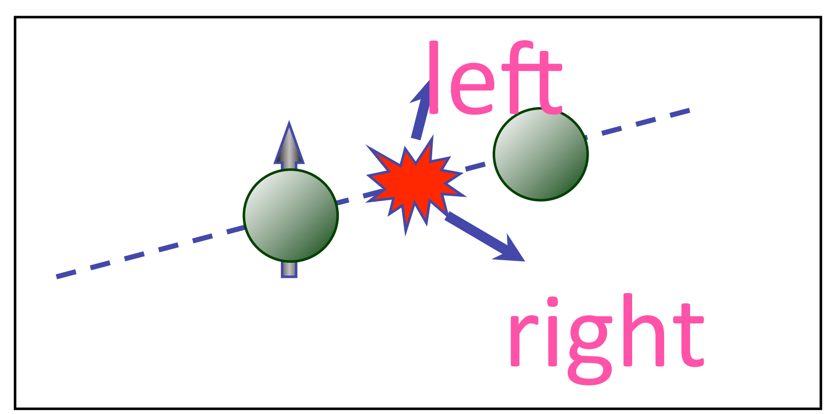
BRAHMS

Pbeam=12 Gev/c	Pbeam=22 Gev/c	Pbeam=200 Gev/c	
ANL	BNL	FNAL	RHIC
$\sqrt{s}$=4.9 GeV	$\sqrt{s}$=6.6 GeV	$\sqrt{s}$=19.4 GeV	$\sqrt{s}$=62.4 GeV
			Collider

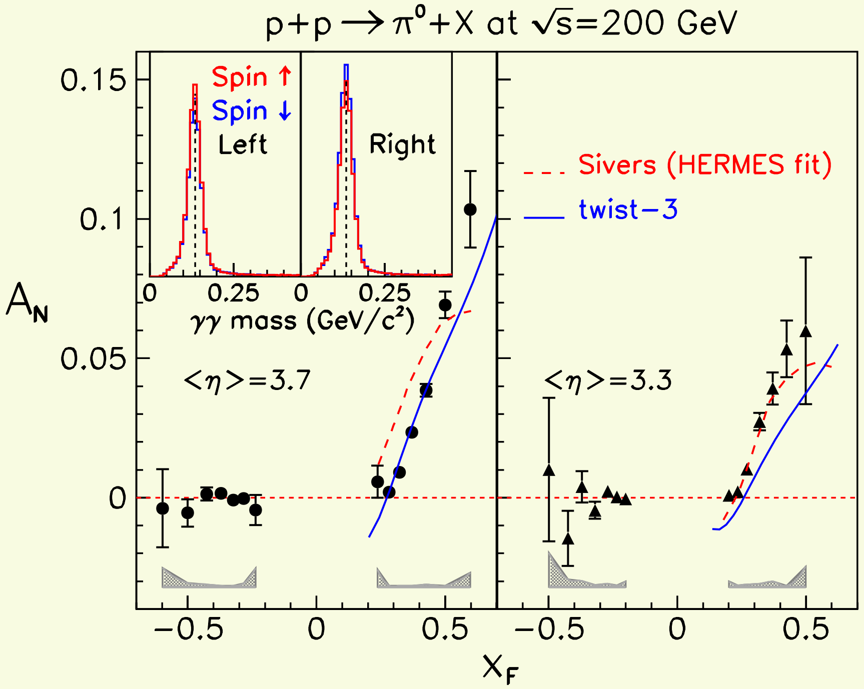
← Fixed target →



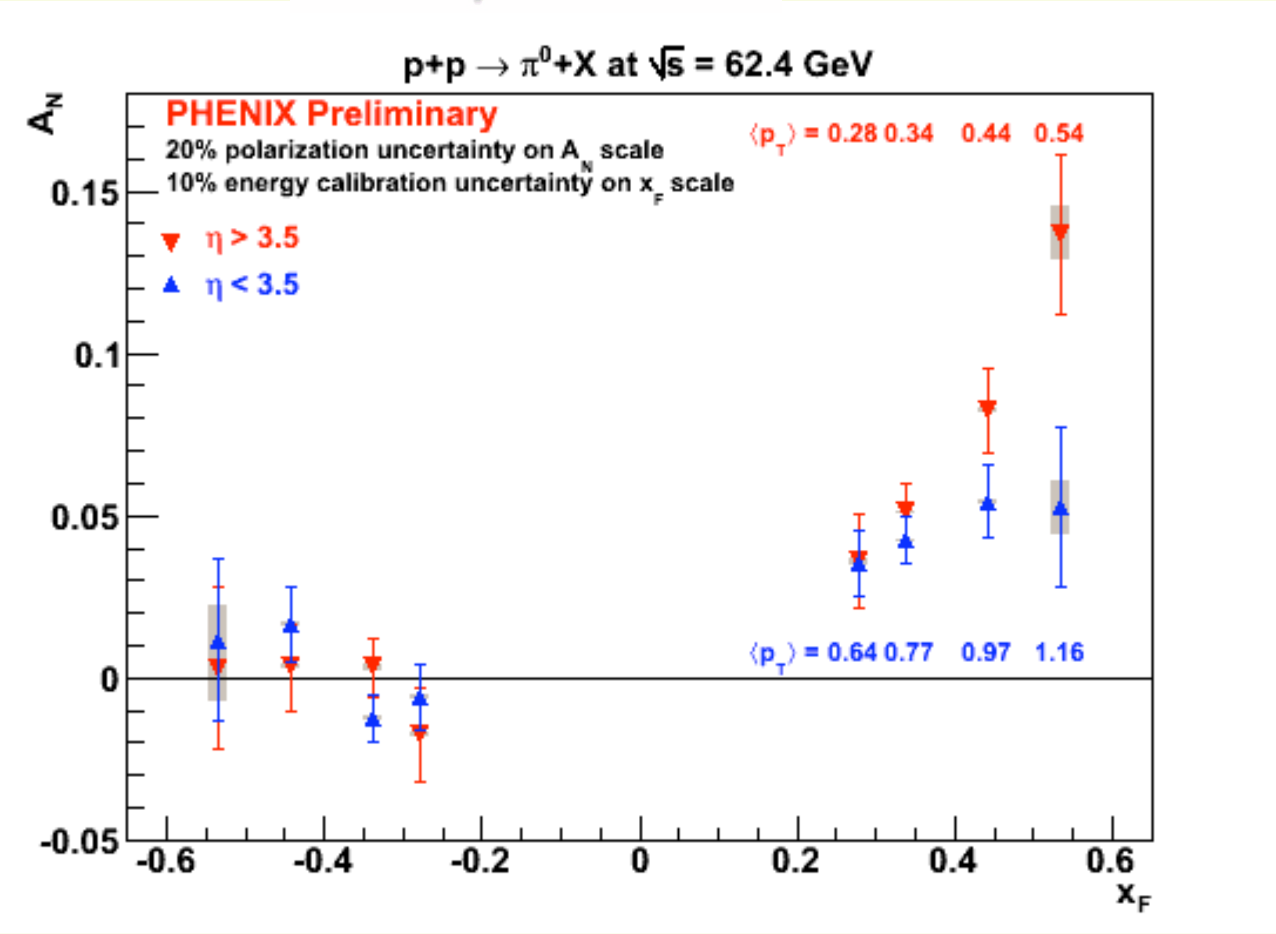
$$x_F = 2p_{long} / \sqrt{s}$$



Modern Era Transverse SSA's at $\sqrt{s}$ = 62.4 & 200 GeV at RHIC

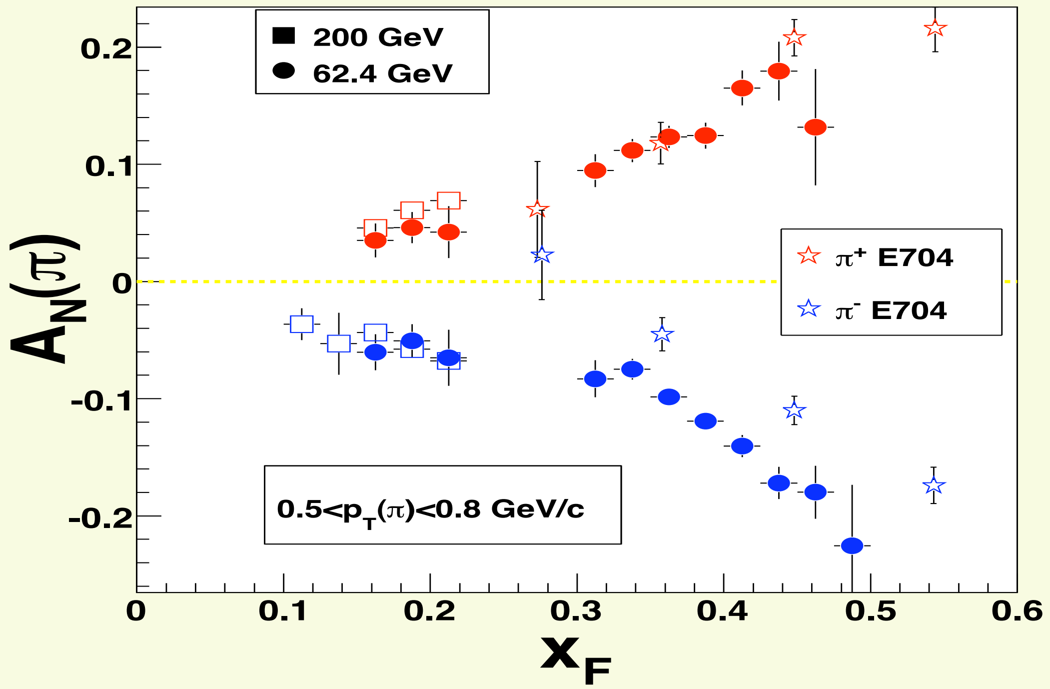


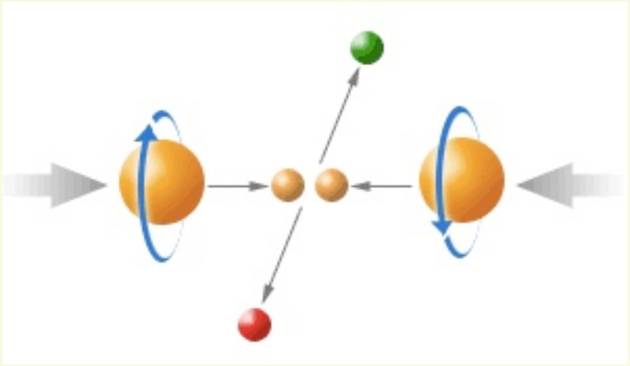
STAR



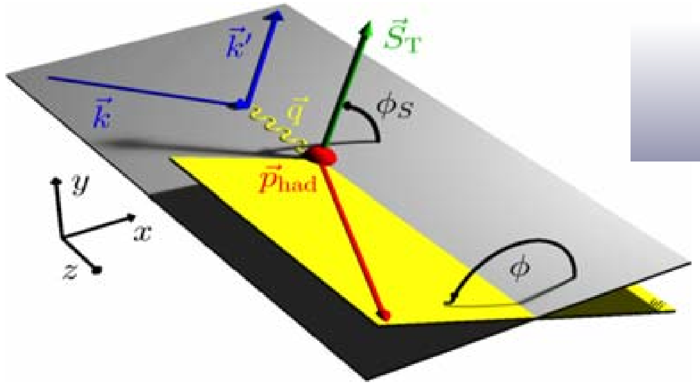
BRAHMS

PRL101, 042001 (2008)

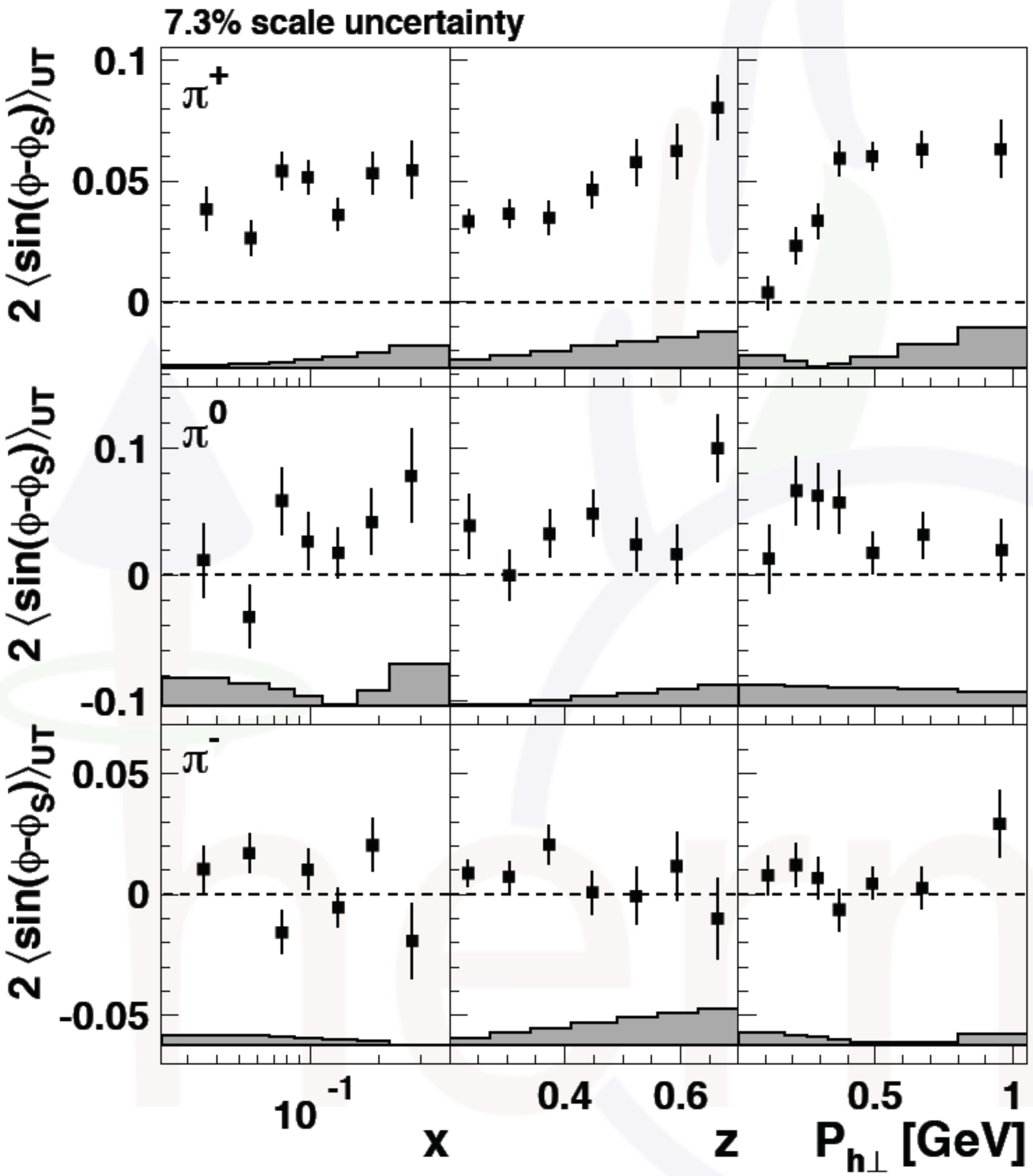


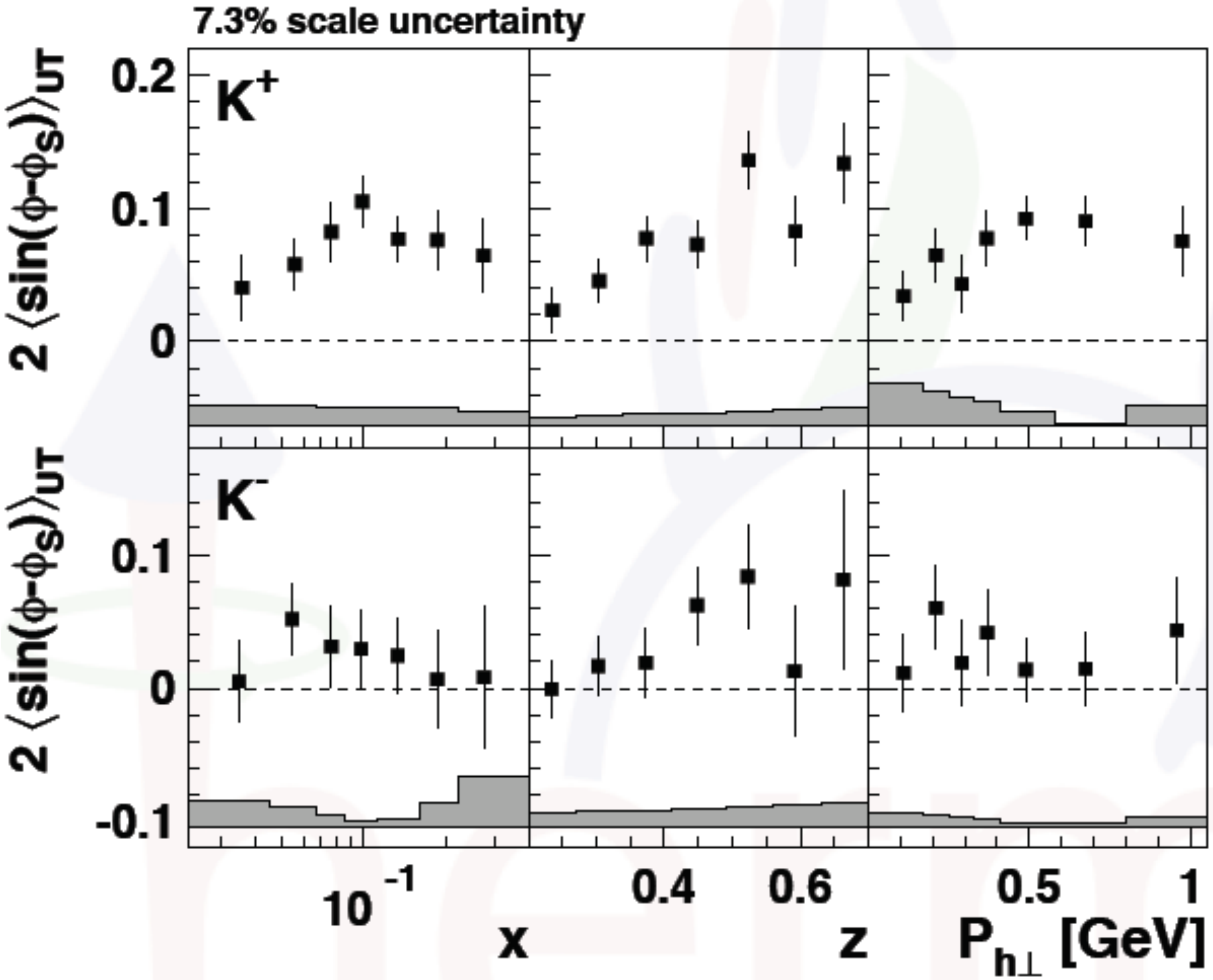


$$\ell p \to \ell' \pi X$$



Hermes PRL 2009





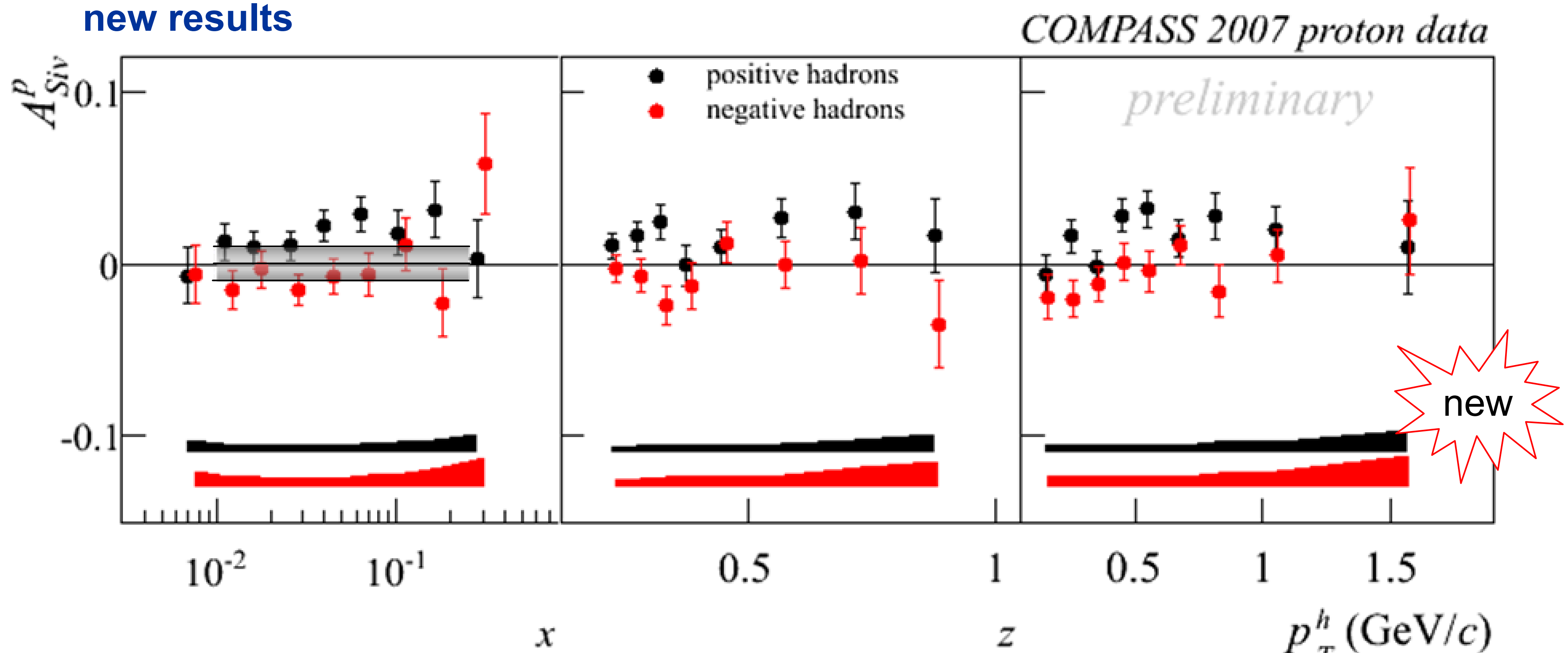
From Anna Martin DIS 2010

See talk of Krysz Kurek

Sivers asymmetry – proton data

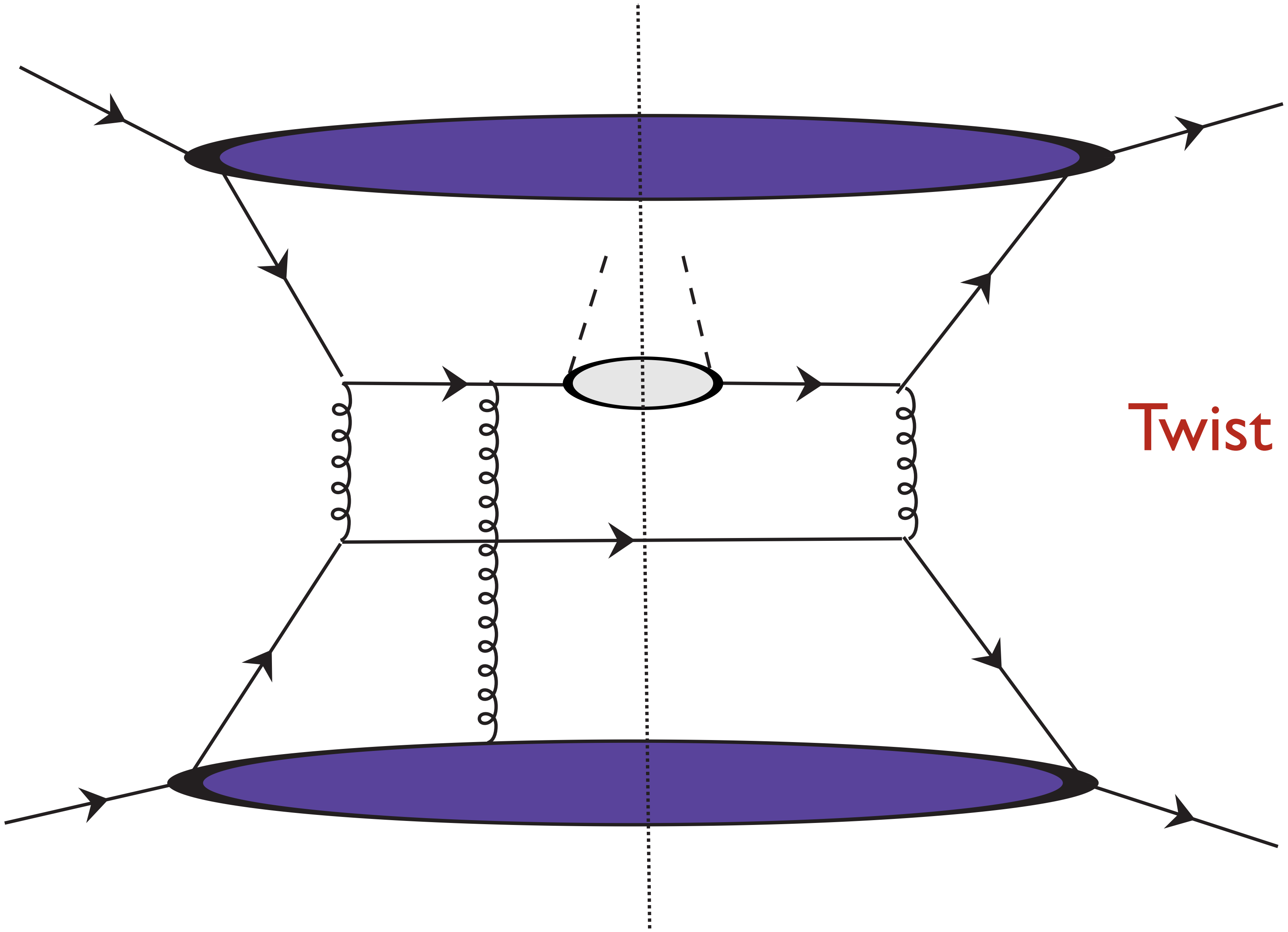
the analysis of the 2007 data is over

new results



evidence for a positive signal for h^+,
which extends to small x, in the region not measured before

Not the full story @ Twist 3 approach ETQS approach



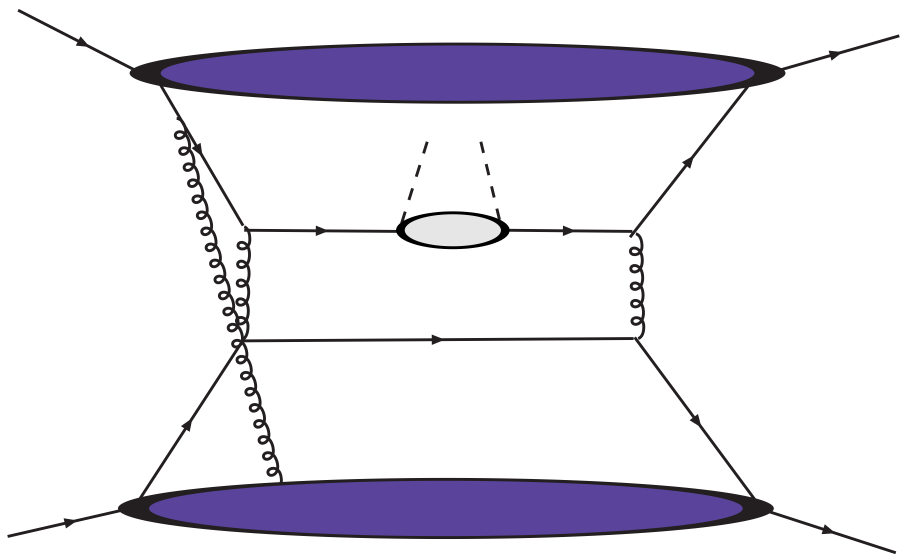
Twist three and non-trival?!

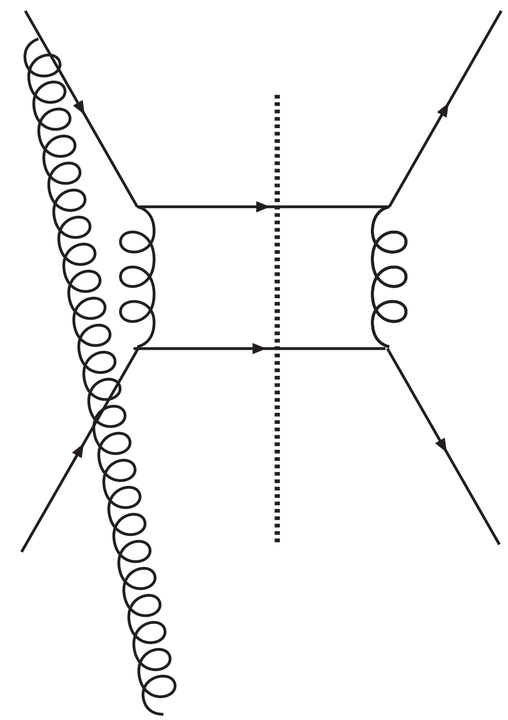
Phases in *soft* poles of propagator in hard subprocess Efremov & Teryaev :PLB 1982

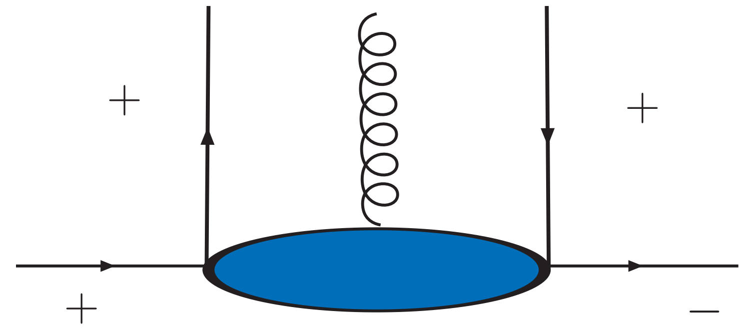
Factorization and Pheno: Qiu, Sterman 1991,1999..., Koike et al, 2000, ... 2010,
Ji, Qiu, Vogelsang, Yuan, 2005 ... 2008 ..???, Yuan, Zhou 2008, 2009, Kang, Qiu, 2008, 2009 ...
Kouvaris Ji, Qiu, Vogelsang! 2006, Vogelsang and Yuan 2007

$Q \sim P_T >> \Lambda_{\rm qcd}$ Co-linear Twist 3 Mechanism

Phases in *soft* poles of propagator in hard subprocess Efremov & Teryaev :PLB 1982







T_F

$\star$ **Get helicity flips and phases** $m_q \to\sim M_H$

- $\Delta\sigma \sim f_a \otimes T_F \otimes H_{ETQS} \otimes D^{q\to\pi}$

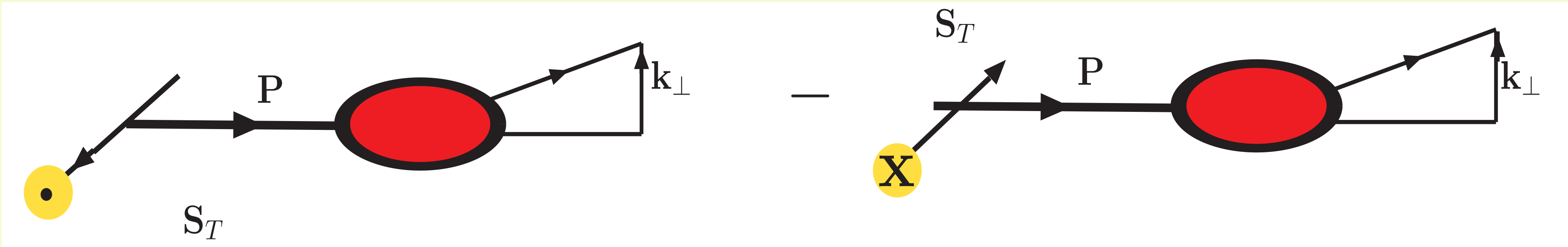
$$\frac{1}{xs + i\epsilon} = \mathcal{P}\left(\frac{1}{xs}\right) \pm i\pi\delta(xs)$$

quark-gluon-quark correlator

TSSAs thru "T-odd" non-pertb. **spin-orbit** *correlations....*

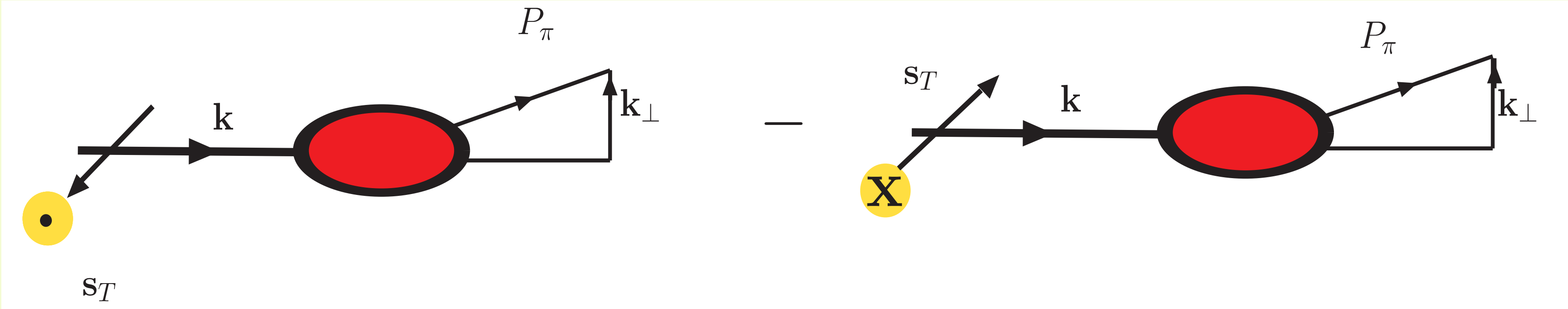
Sensitivity to $p_T \sim \mathbf{k}_T << \sqrt{Q^2}$

- **Sivers PRD: 1990** TSSA is associated w/ correlation *transverse* spin and momenta in initial state hadron



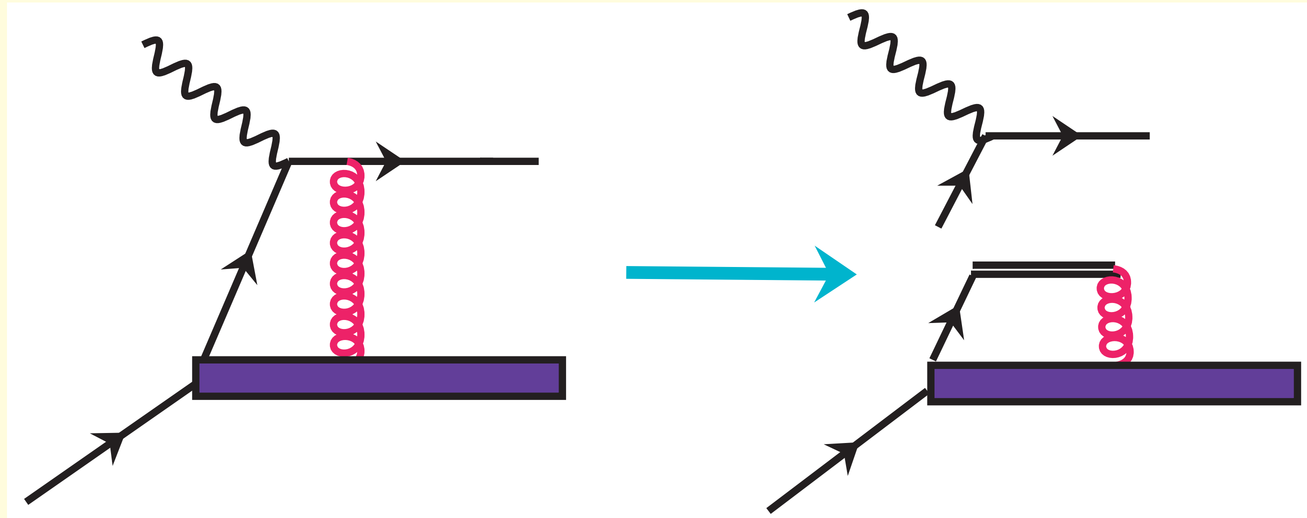
$$\Delta\sigma^{pp^\uparrow \to \pi X} \sim D \otimes f \otimes \Delta f^\perp \otimes \hat{\sigma}_{Born} \Longrightarrow \boldsymbol{\Delta f^\perp(x, k_\perp) = i S_T \cdot (P \times k_\perp)}\, f_{1T}^\perp(x, \boldsymbol{k}_\perp)$$

- **Collins NPB: 1993** TSSA is associated with *transverse* spin of fragmenting quark and transverse momentum of final state hadron



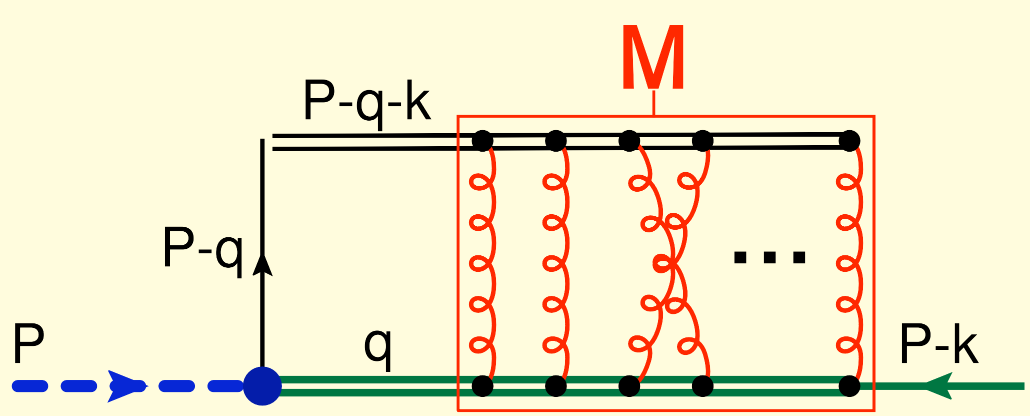
$$\Delta\sigma^{ep^\uparrow \to e\pi X} \sim \Delta D^\perp \otimes f \otimes \hat{\sigma}_{Born} \Longrightarrow \boldsymbol{\Delta D^\perp(x, p_\perp) = i s_T \cdot (P \times p_\perp)}\, H_1^\perp(x, \boldsymbol{p}_\perp)$$

FSI phases in TSSAs unsupressed

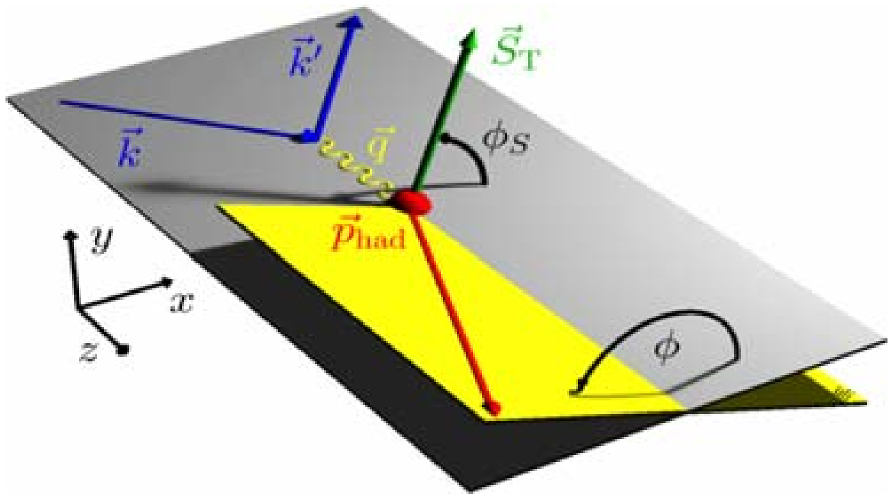


$$\Delta f^{\perp}(x,k_{\perp}) = iS_T \cdot (P \times k_{\perp})\ f_{1T}^{\perp}(x,\boldsymbol{k}_{\perp})$$

- **Unsurpressed reaction mech. Boer PRD 1999 context of DY process RHIC**
- ***Brodsky Hwang Schmidt PLB 2002- SIDIS w/ transverse polarized target***
- ***Collins PLB 2002- Gauge link Sivers function doesn't vanish***
- Ji, Yuan PLB: 2002 -Sivers fnct. FSI emerge from Color Gauge-links
- **LG, Goldstein, Oganessyan 2002, 2003** PRD Boer-Mulders Fnct, and Sivers -spectator model
- **LG, M. Schlegel, PLB 2010** Boer-Mulders Fnct, and Sivers beyond summing the FSIs through gauge link



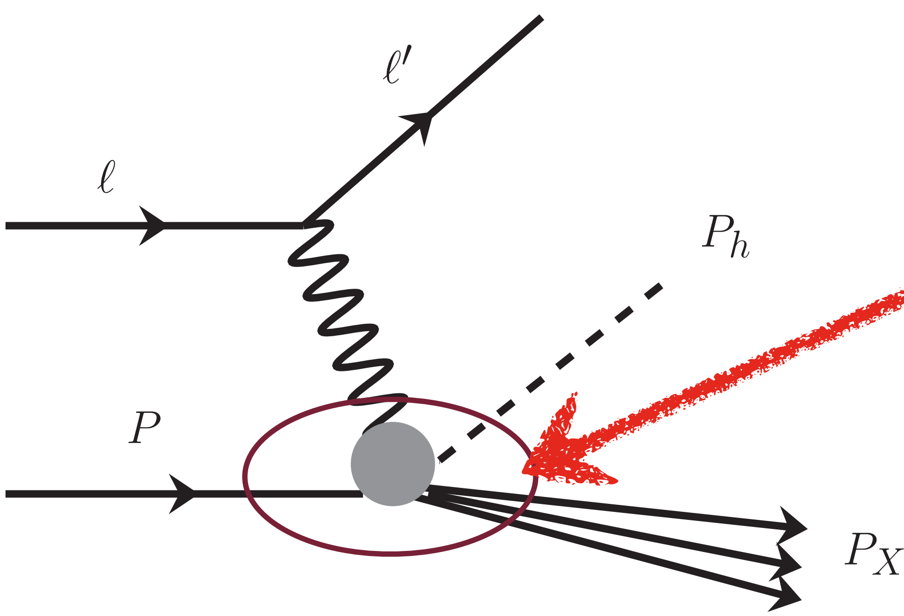
Factorization in SIDIS Paton Model



$$x_B = \frac{Q^2}{2P\cdot q}$$

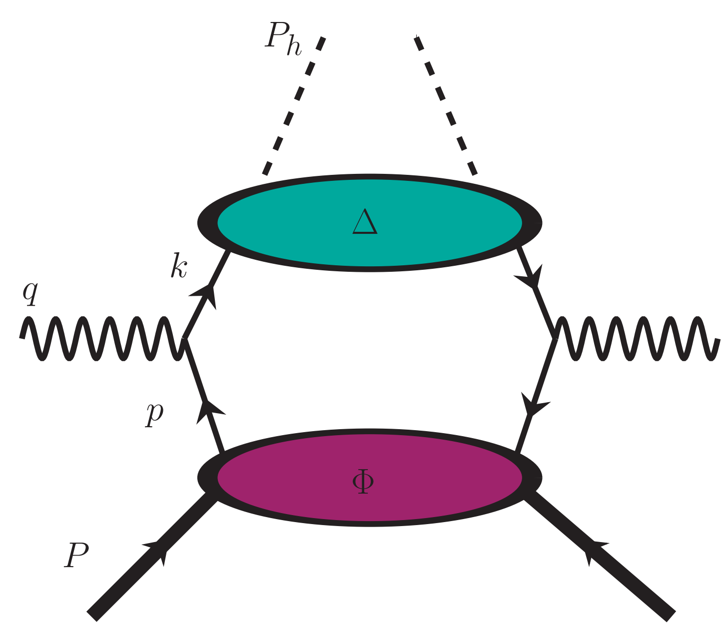
$$z_h = \frac{P\cdot P_h}{P\cdot q} \approx \frac{P_h^-}{q^-}$$

Parton model & DIS kinematics



"***structures***"

Factorize



Factorization parton model when P_T of the hadron small

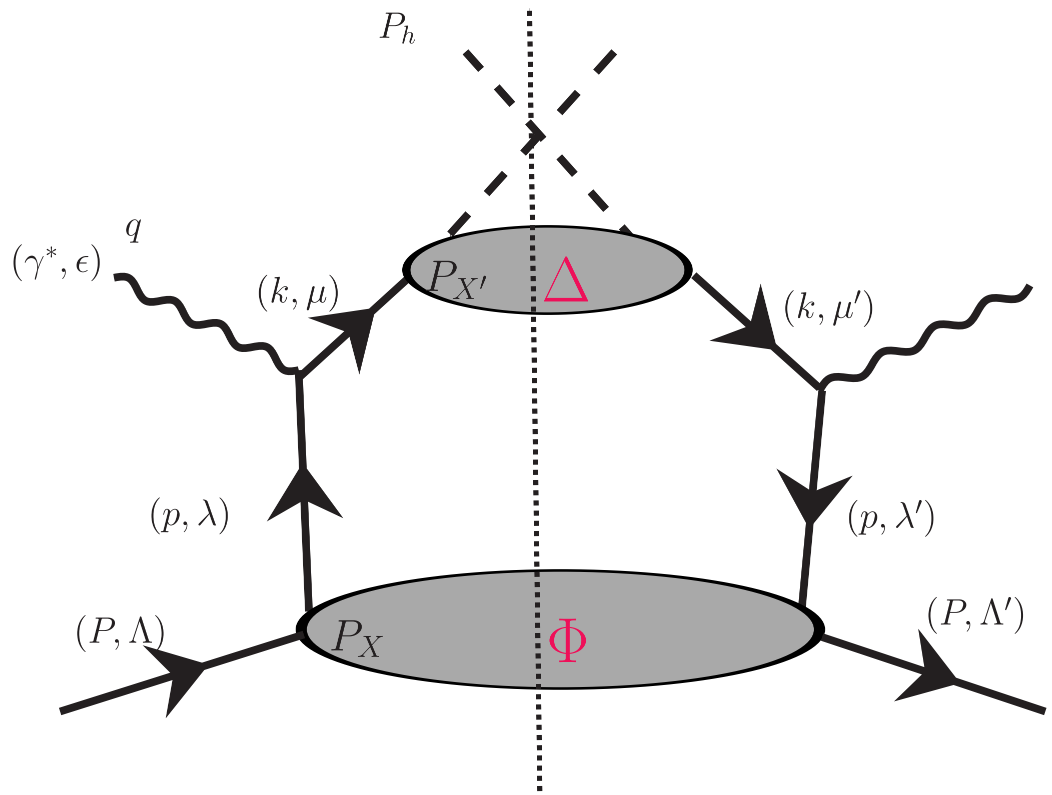
$$W^{\mu\nu}(q,P,S,P_h) \approx \sum_a e^2 \int \frac{d^2\mathbf{p}_T dp^- dp^+}{(2\pi)^4} \int \frac{d^2\mathbf{k}_T dk^- dk^+}{(2\pi)^4} \delta(p^+ - x_B P^+)\delta(k^- - \frac{P_h^-}{z})\delta^2(\mathbf{p}_T + \mathbf{q}_T - \mathbf{k}_T)$$

$$\times \mathrm{Tr}\left[\Phi(p,P,S)\gamma^\mu \Delta(k,P_h)\gamma^\nu\right]$$

$$W^{\mu\nu}(q,P,S,P_h) = \int \frac{d^2\mathbf{p}_T}{(2\pi)^4} \int \frac{d^2\mathbf{k}_T}{(2\pi)^4} \delta^2(\mathbf{p}_T - \frac{\mathbf{P}_{h\perp}}{z} - \mathbf{k}_T)\mathrm{Tr}\left[\left(\int dp^-\Phi\right)\gamma^\mu\left(\int dk^+\Delta\right)\gamma^\nu\right]$$

Small transverse momentum !!!

$$\Phi(x,\mathbf{p}_T,S) \equiv \int dp^- \Phi(p,P,S)\Big|_{p^+=x_B P^+}, \qquad \Delta(z,\mathbf{k}_T) \equiv \int dk^+ \Delta(k,P_h)\Big|_{k^-=\frac{P^-}{z_h}}$$

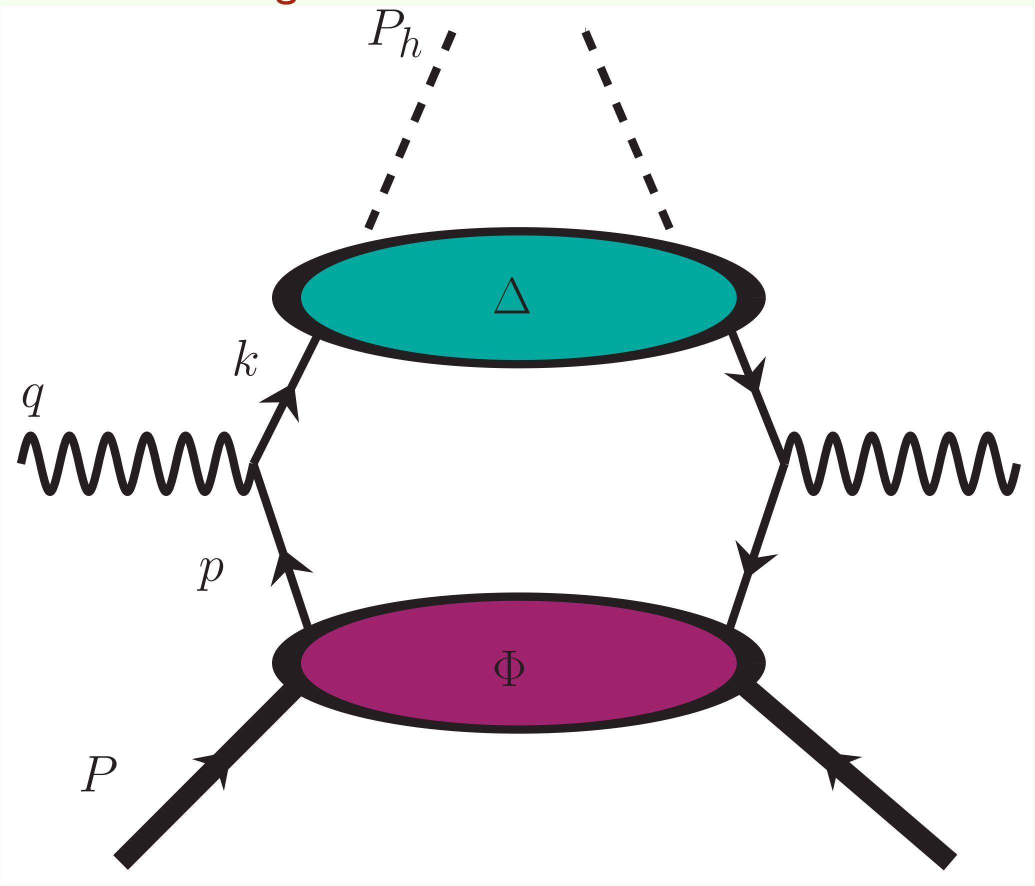


Factorization & Sensitivity to $P_T \sim k_\perp \longrightarrow$ TMDs

Based on QCD factorization at tree level

Ralson & Soper NPB 1979-Drell Yan, Mulders Tangerman NPB 1996-SIDIS

$$d^6\sigma = \hat{\sigma}_{\mathrm{hard}}\, \mathcal{C}[w f D]$$



Structure functions are convolutions in momentum-space

$$F_{XY,Z} \equiv \mathcal{C}\left[w f D\right]$$

$$\mathcal{C}\left[w\, f\, D\right] \equiv x_B \sum_a e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{K}_T\, \delta^{(2)}\left(z\boldsymbol{p}_T + \boldsymbol{K}_T - \boldsymbol{P}_{h\perp}\right) w\left(\boldsymbol{p}_T, -\frac{\boldsymbol{K}_T}{z}\right) f^a(x, p_T^2)\, D^a(z, K_T^2)$$

where $\boldsymbol{K}_T \equiv -\boldsymbol{k}_T z$ with $\hat{\boldsymbol{h}} \equiv \boldsymbol{P}_{h\perp}/|\boldsymbol{P}_{h\perp}|$

Lepton Hadron scatt expressed model indpen. thru structure functions

$$
\begin{aligned}
\frac{d\sigma}{dx_B\,dy\,d\psi\,dz_h\,d\phi_h\,dP_{h\perp}^2} &= \frac{\alpha^2}{x_B y Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x_B}\right)\Bigg\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} \\
&+ \quad \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} \\
&+ \quad S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&+ \quad S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&+ \quad |\boldsymbol{S}_\perp|\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
&+ \quad \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} \\
&+ \quad \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] \\
&+ \quad |\boldsymbol{S}_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} \\
&+ \quad \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\},
\end{aligned}
$$

Kotzinian NPB 95,
Mulders Tangermann NPB 96,
Bacchetta et al JHEP 08

More than postage stamp collecting!!!

DIS kinematics $d\psi \approx d\phi_S$ and variables are defined

Structure functs projected from cross section

$$
A_{XY}^{\mathcal{F}} \equiv 2\,\frac{\int d\phi_h\,d\phi_S\,\mathcal{F}(\phi_h,\phi_S)\,\left(d\sigma^\uparrow - d\sigma^\downarrow\right)}{\int d\phi_h d\phi_S\,\left(d\sigma^\uparrow + d\sigma^\downarrow\right)}\,,
$$

X Y-polarization

e.g. $\mathcal{F}(\phi_h,\phi_S)=\sin(\phi_h-\phi_S).$

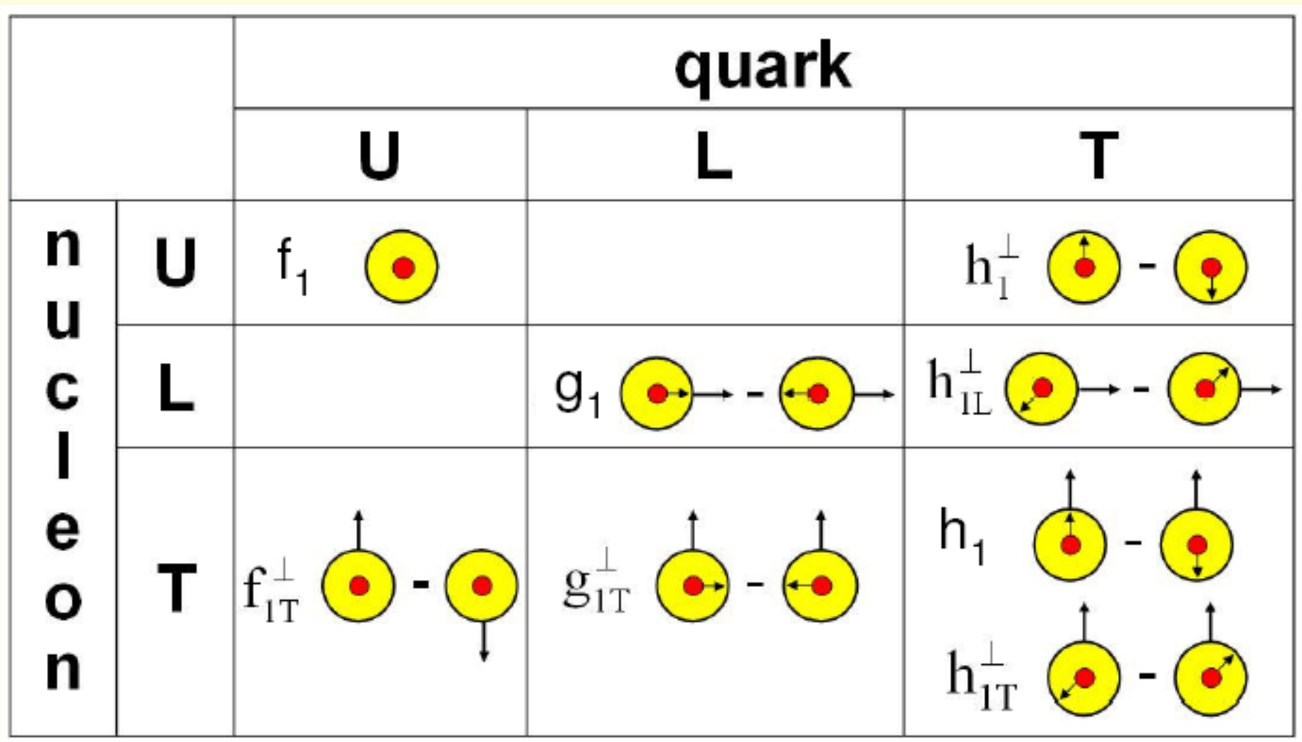
8 Leading Twist TMDs: Correlation Matrix Dirac space

$$\Phi^{[\gamma^+]}(x,\boldsymbol{p}_T) \equiv f_1(x,\boldsymbol{p}_T^2) + \frac{\epsilon_T^{ij} p_{Ti} S_{Tj}}{M}\, f_{1T}^{\perp}(x,\boldsymbol{p}_T^2)$$

$$\Phi^{[\gamma^+\gamma_5]}(x,\boldsymbol{p}_T) \equiv \lambda\, g_{1L}(x,\boldsymbol{p}_T^2) + \frac{\boldsymbol{p}_T\cdot\boldsymbol{S}_T}{M}\, g_{1T}(x,\boldsymbol{p}_T^2)$$

$$\Phi^{[i\sigma^{i+}\gamma_5]}(x,\boldsymbol{p}_T) \equiv S_T^i\, h_{1T}(x,\boldsymbol{p}_T^2) + \frac{p_T^i}{M}\left(\lambda\, h_{1L}^{\perp}(x,\boldsymbol{p}_T^2) + \frac{\boldsymbol{p}_T\cdot\boldsymbol{S}_T}{M}\, h_{1T}^{\perp}(x,\boldsymbol{p}_T^2)\right)$$

$$+\frac{\epsilon_T^{ij} p_T^j}{M}\, h_1^{\perp}(x,\boldsymbol{p}_T^2)$$

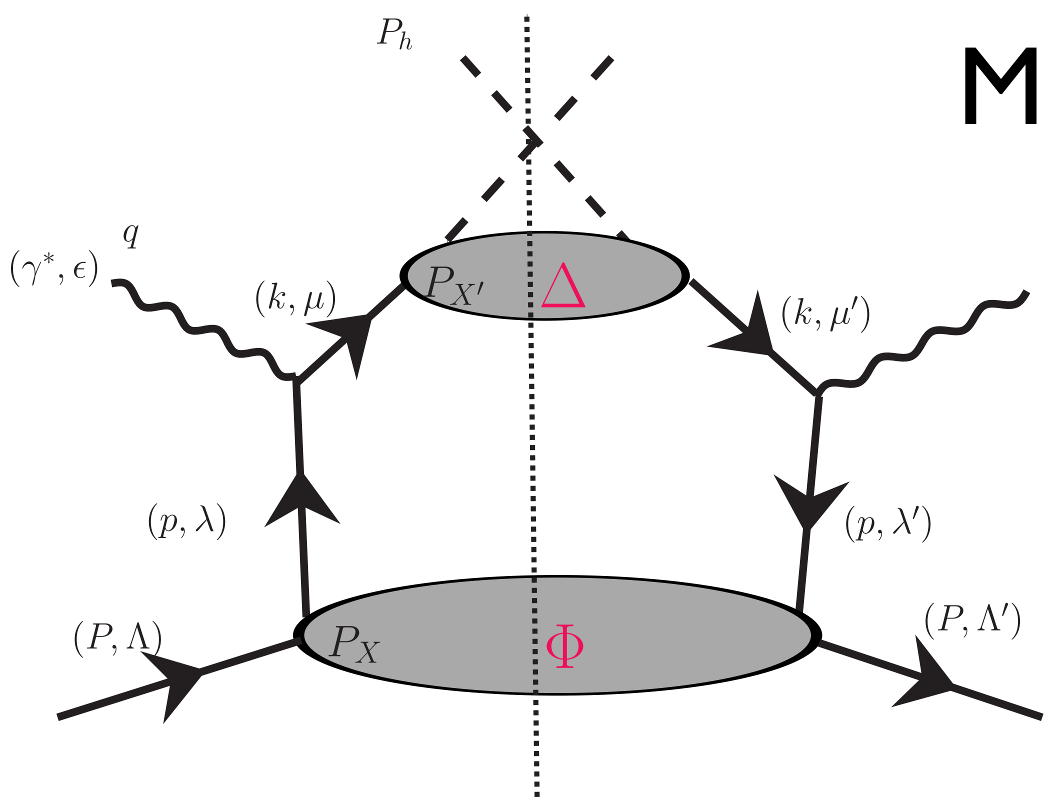


Factorization parton model when P_T of the hadron small

$$W^{\mu\nu}(q,P,S,P_h) = \int \frac{d^2\mathbf{p}_T}{(2\pi)^4} \int \frac{d^2\mathbf{k}_T}{(2\pi)^4} \delta^2\left(\mathbf{p}_T - \frac{\mathbf{P}_{h\perp}}{z} - \mathbf{k}_T\right) \mathrm{Tr}\left[\left(\int dp^- \Phi\right)\gamma^\mu \left(\int dk^+ \Delta\right)\gamma^\nu\right]$$

Small transverse momentum !!!

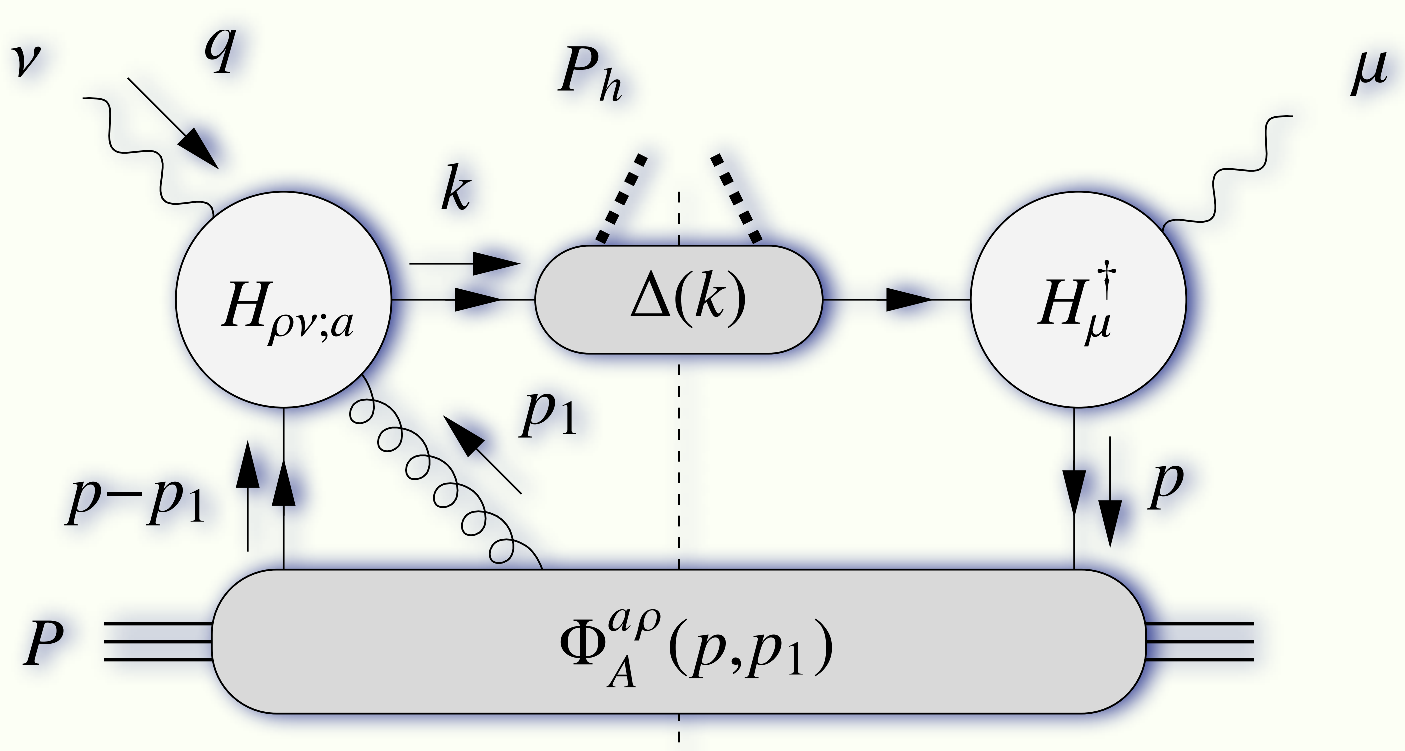
$$\Phi(x,\mathbf{p}_T,S) \equiv \int dp^- \Phi(p,P,S)\Big|_{p^+=x_B P^+}, \qquad \Delta(z,\mathbf{k}_T) \equiv \int dk^+ \Delta(k,P_h)\Big|_{k^-=\frac{P^-}{z_h}}$$



Minimal requirement satisfy color gauge invariance

FSIs and TSSAs in Extend Parton Model-Gauge Links

- Obtained by summing the "leading order" gluons that implement ***color gauge invariance***?
- How is the correlator modified?



$$H_{\rho,\nu} = \gamma^\nu$$

“T-Odd” Effects From Color Gauge Inv. Via Gauge links

Gauge link determined re-summing gluon interactions btwn soft and hard

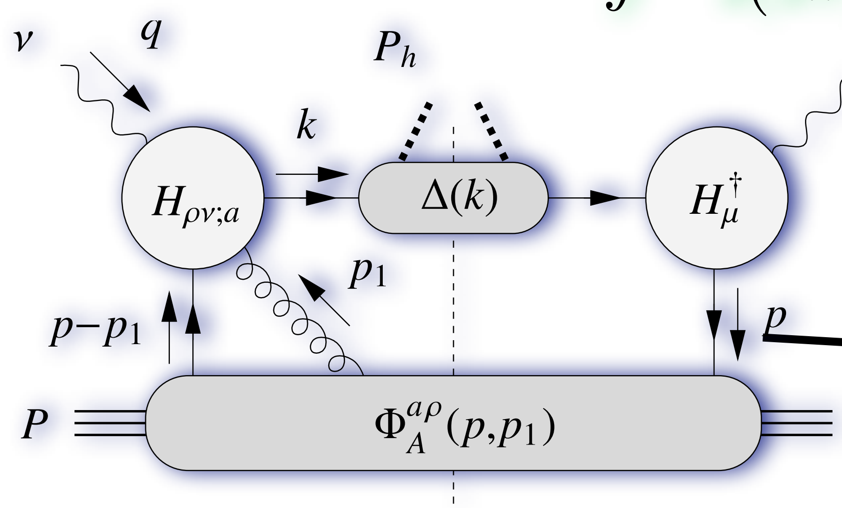
Efremov, Radyushkin *Theor. Math. Phys. 1981*

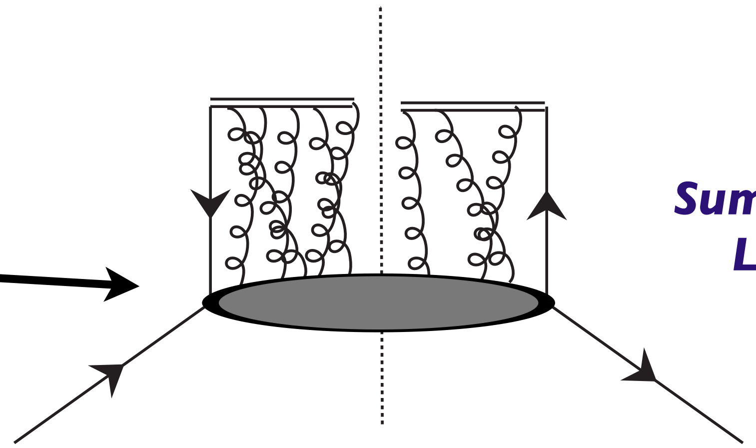
Belitsky, Ji, Yuan NPB 2003,

Boer, Bomhof, Mulders Pijlman, et al. 2003 - 2008- NPB, PLB, PRD

Vogelsang and Yuan PRD 2007

$$\Phi^{[\mathcal{U}[C]]}(x,p_T) = \int \frac{d\xi^- d^2\xi_T}{2(2\pi)^3} e^{ip\cdot\xi} \langle P|\overline{\psi}(0)\mathcal{U}^{[C]}_{[0,\xi]}\psi(\xi^-,\xi_T)|P\rangle|_{\xi^+=0}$$

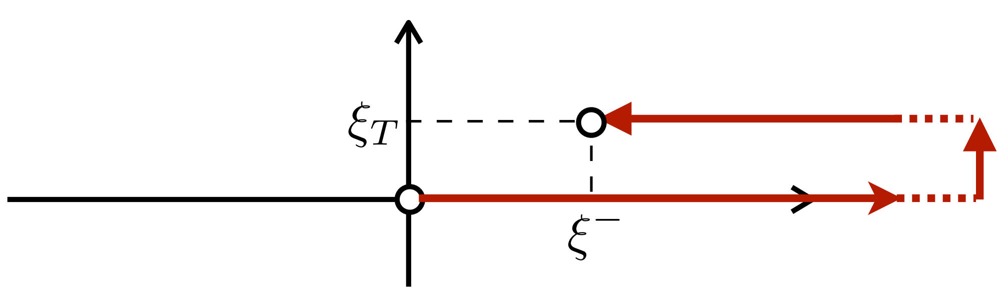




Summing gauge link with color
LG, M. Schlegel PLB 2010

- **The path $[C]$ is fixed by hard subprocess within hadronic process.**

$$\int d^4p d^4k \delta^4(p+q-k)\mathrm{Tr}\left[\Phi^{[U^{C}_{[\infty;\xi]}}(p) H^\dagger_\mu(p,k)\Delta(k)H_\nu(p,k)\right]$$

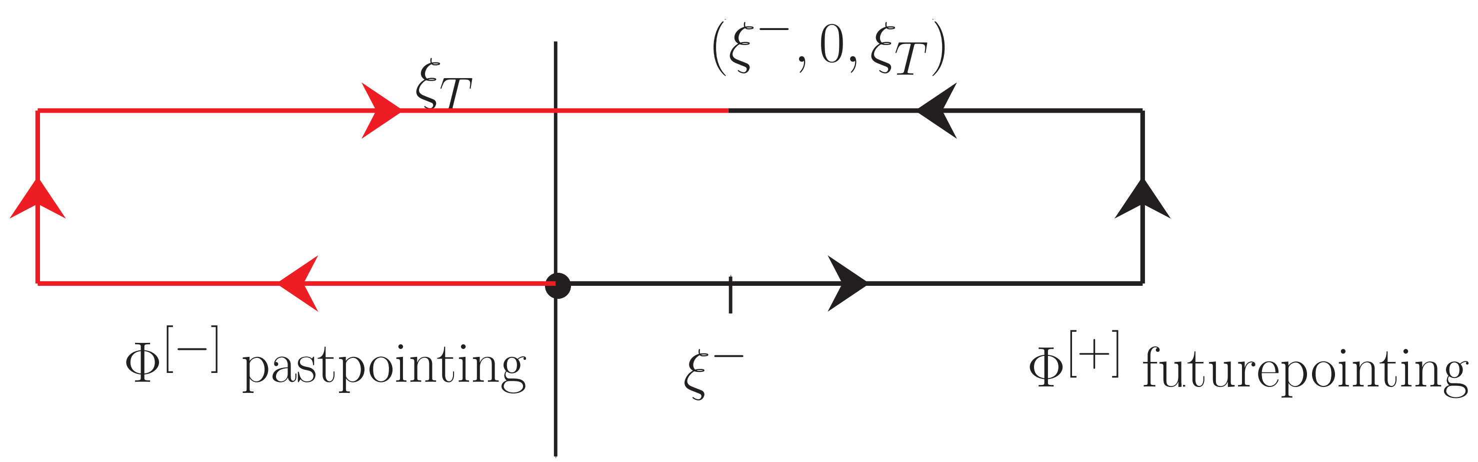


$U_{[+]}$

- Depends on the hard partonic subprocess. In particular it depends on the color-flow through the subprocess

$$W_{\mu\nu} = \int d^4p d^4k \delta^4(p+q-k) \mathrm{Tr}\left[\Phi^{\mathcal{U}^{[C]}_{[\infty;\xi]}}(p) H^\dagger_\mu(p,k) \Delta(k) H_\nu(p,k)\right]$$

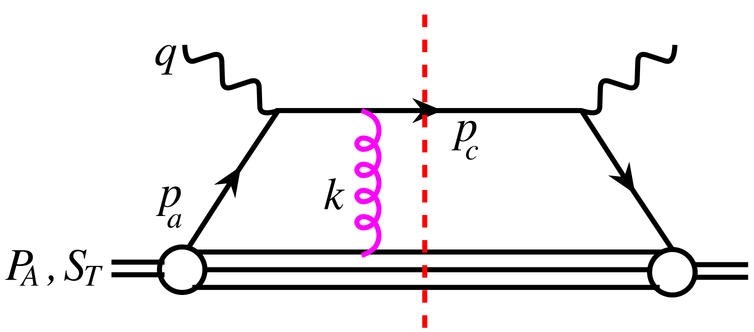
The path C is fixed by the hard subprocess within the full hadronic reaction

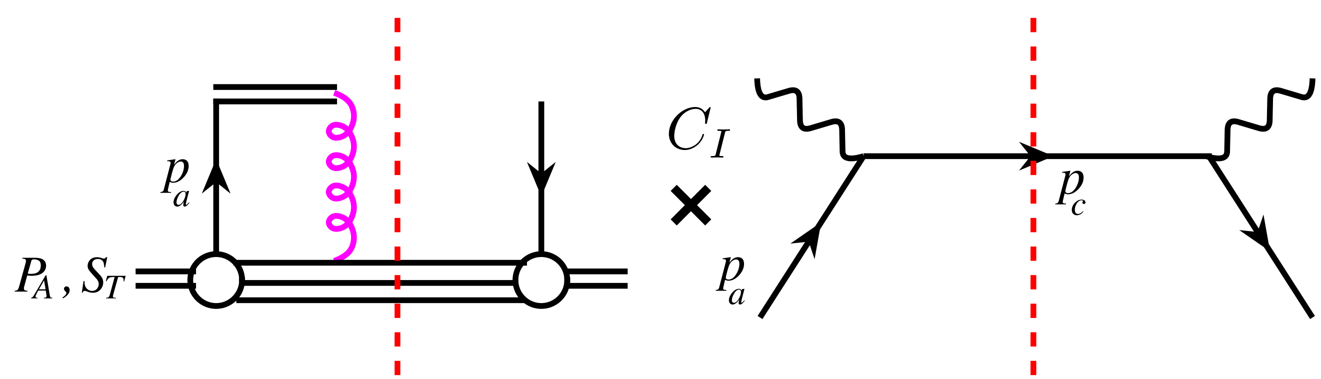


Process Dependence example SIDIS and DY

Factorize w/ leading 1 gluon exchange get GI & phase

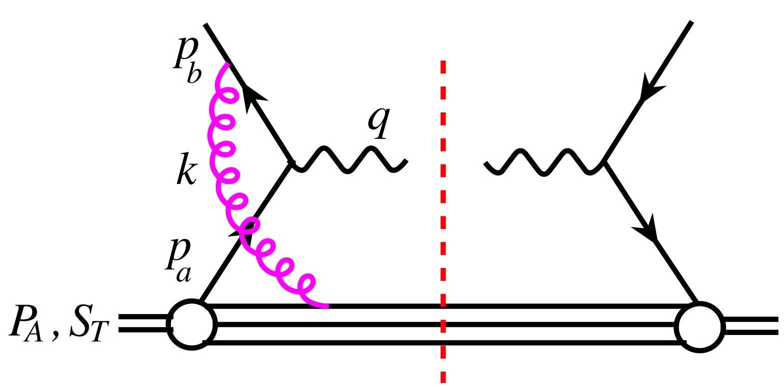
Final-state interaction in SIDIS

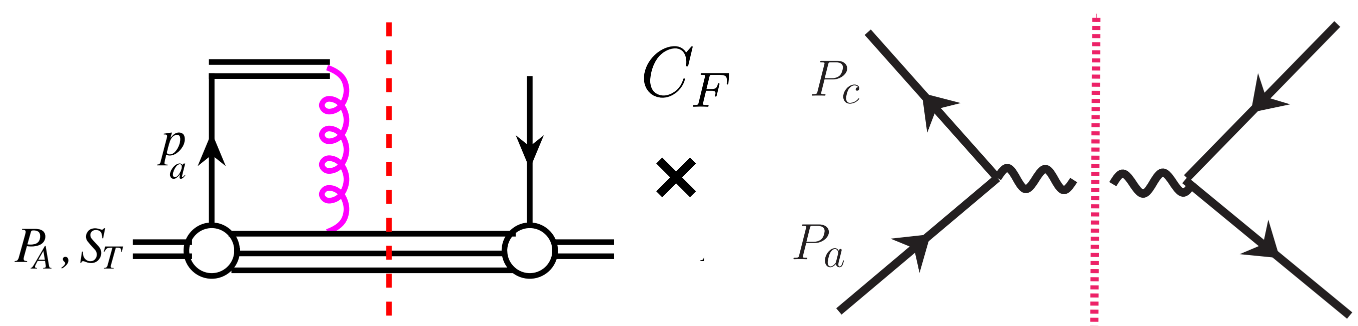




$$\bar{u}(p_c)(-ig)\gamma^- T^a \frac{i(\not{p}_c - \not{k})}{(p_c - k)^2 + i\epsilon} \approx \bar{u}(p_c)\left[\frac{g}{-k^+ + i\epsilon} T^a\right]$$

and initial-state interaction in DY





$$\bar{v}(p_b)(-ig)\gamma^- T^a \frac{-i(\not{p}_b + \not{k})}{(p_b + k)^2 + i\epsilon} \approx \bar{v}(p_b)\left[\frac{g}{-k^+ - i\epsilon} T^a\right],$$

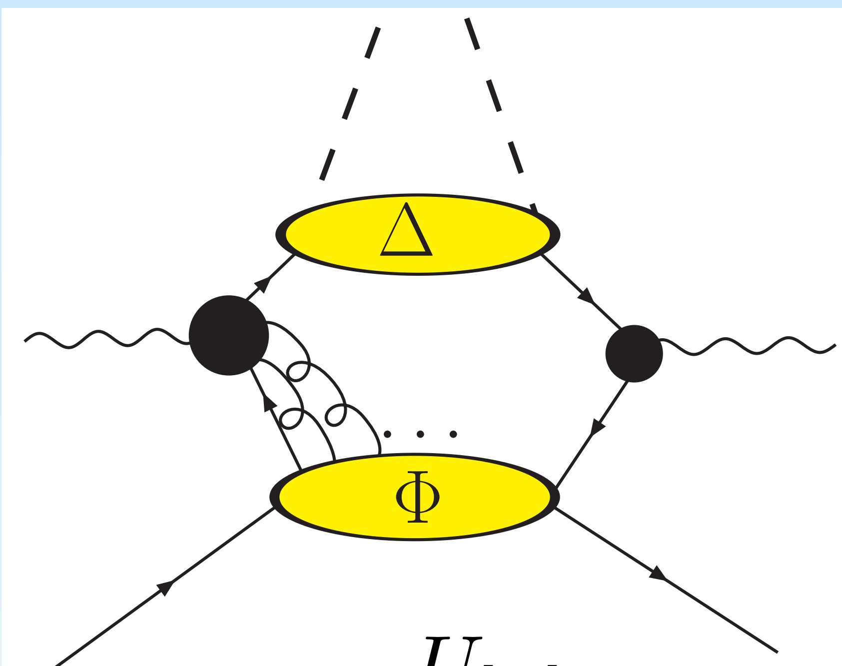
Collins PLB 02
BHW NPB 02

"Generalized Universality" Fund. Prediction of QCD Factorization

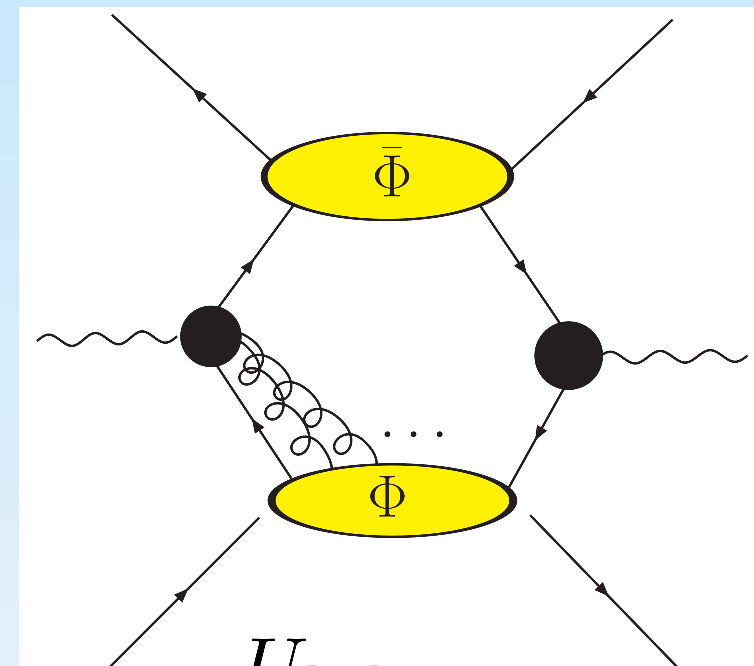
$$f_{1T_{sidis}}^{\perp}(x,k_T) = -f_{1T_{DY}}^{\perp}(x,k_T) \qquad p_T \sim \mathbf{k}_T << \sqrt{Q^2}$$

EIC conjunction with DY exp. E906-Fermi, RHIC II, Compass, JPARC

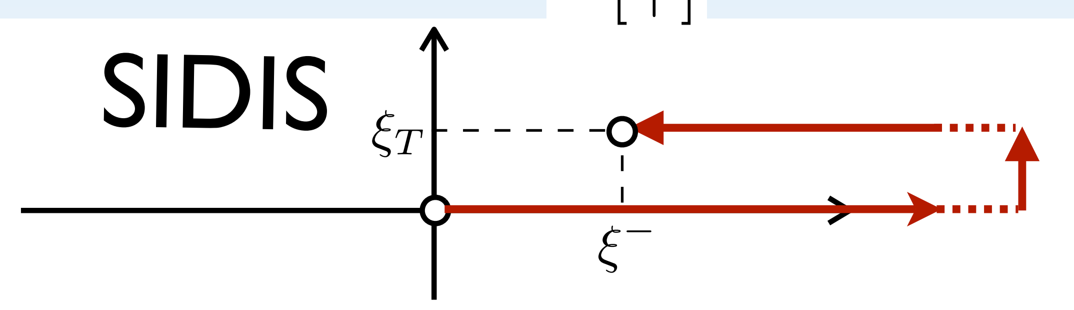
Process Dependence, Collins PLB 02, Brodsky et al. NPB 02, Boer Mulders Pijlman Bomhoff 03, 04 ...



$$d\sigma = L_{\mu\nu} W^{\mu\nu} \Rightarrow$$

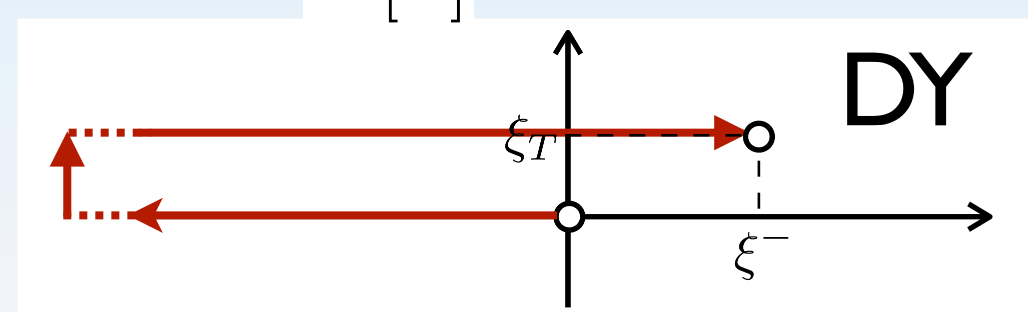


$U_{[+]}$



P&T

$U_{[-]}$

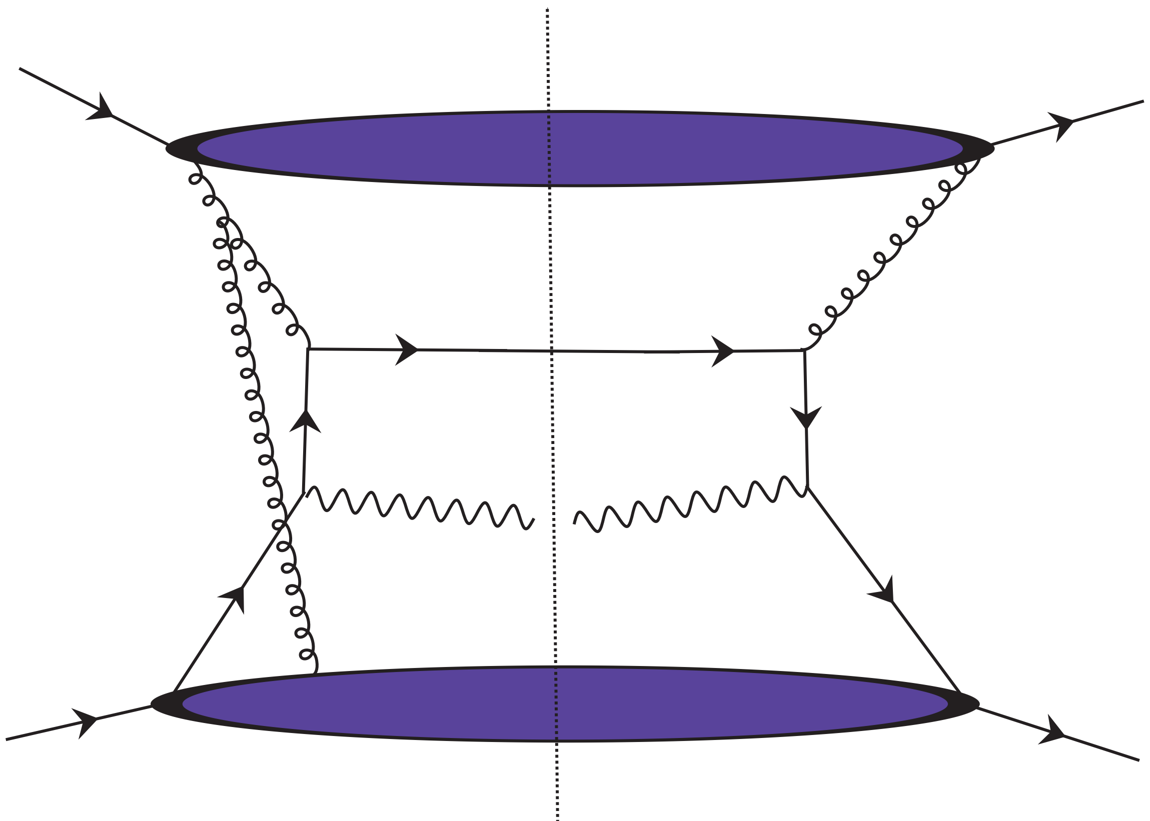


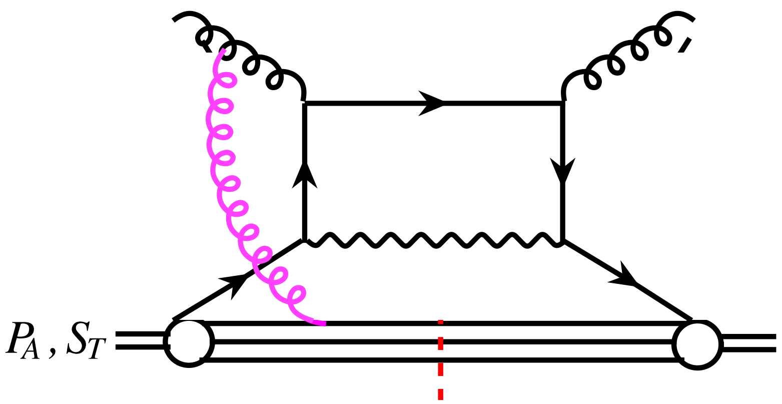
$$\Phi^{[+]*}(x,p_T) = i\gamma^1\gamma^3\Phi^{[-]}(x,p_T)i\gamma^1\gamma^3$$

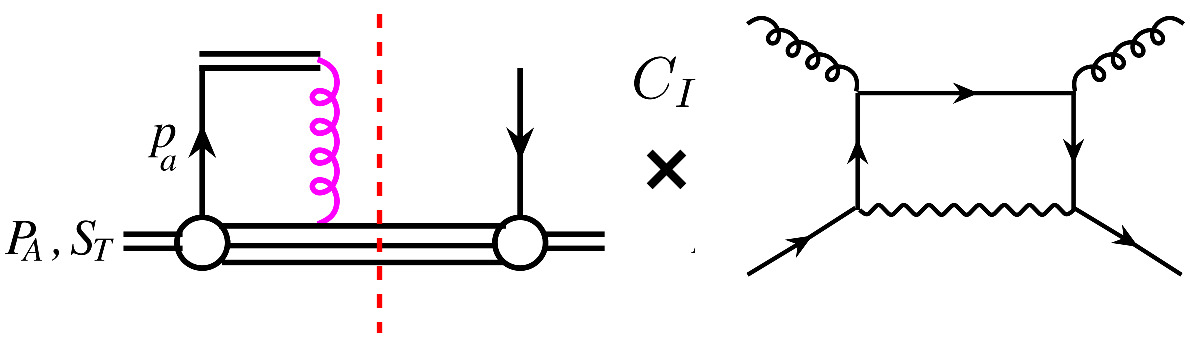
Consider direct Photon in GPM

GPM w/color
LG & Z. Kang
Phys.Lett. B696 2011

$$\Delta\sigma^{pp^{\uparrow}\to\gamma X} \sim \Delta f_a \otimes f_b \otimes \Delta\hat{\sigma}$$

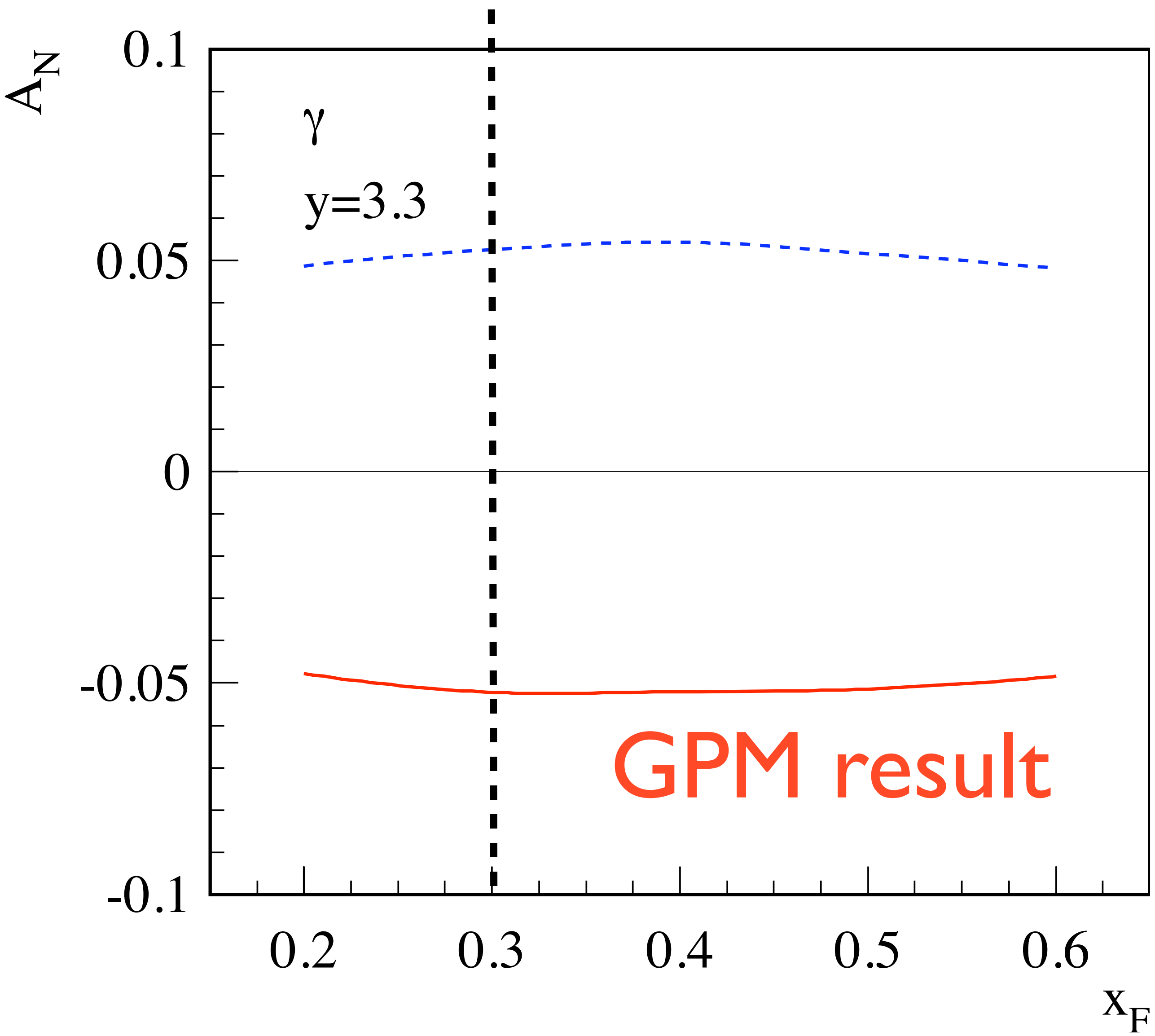






Get Sivers function for this process to use in GPM

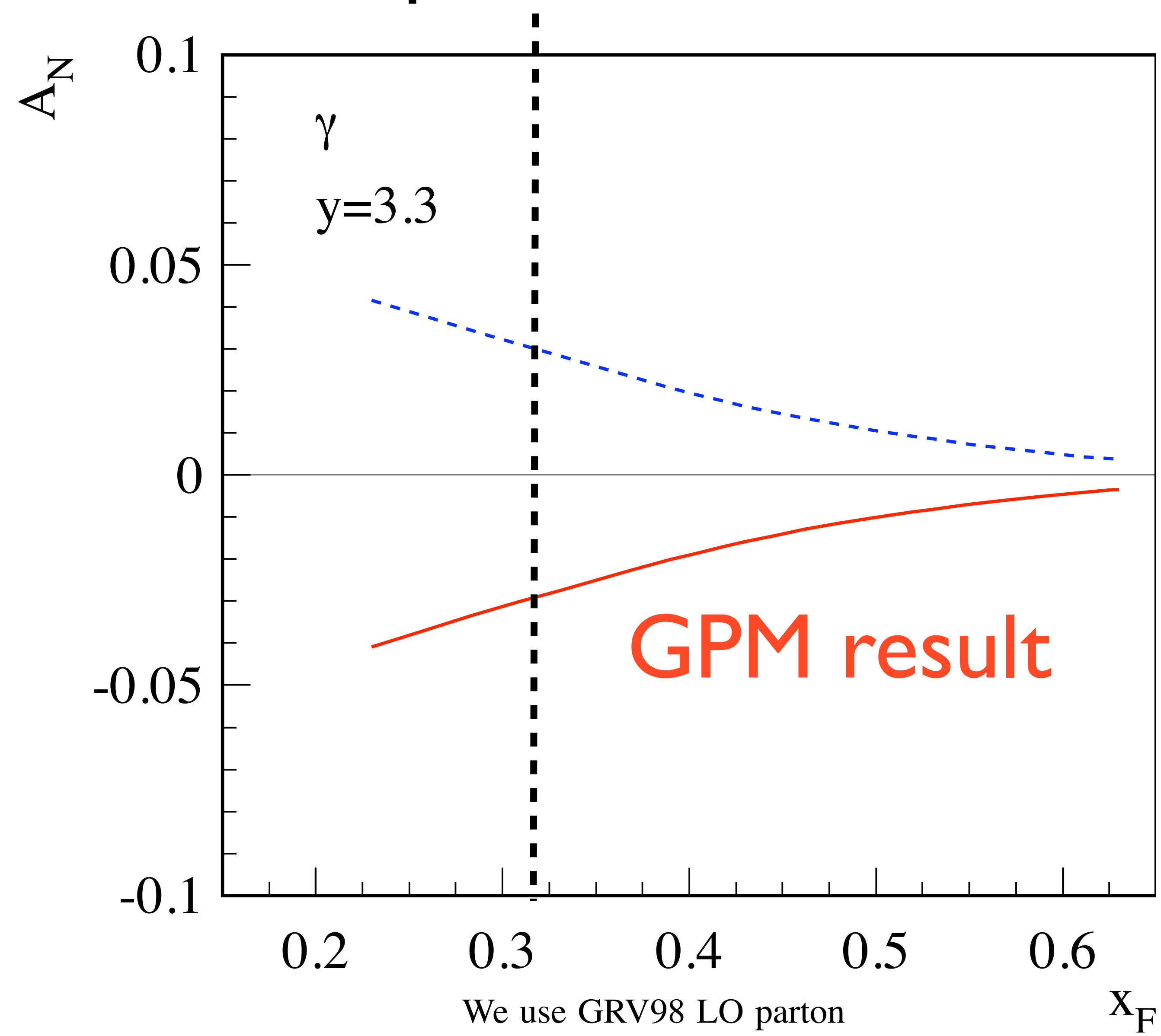
Old parameterization Torino



We use GRV98 LO parton

the old Sivers function from [4], and Kretzer fragmentation function [5].

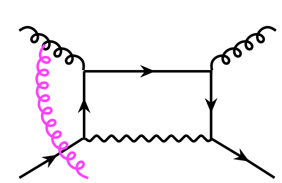
New parameterization ...



We use GRV98 LO parton

, the latest Sivers function from [2], and DSS fragmentation function [3].

$$H^{\rm Inc}_{qg\to\gamma q} \;=\; -\frac{N_c}{N_c^2-1}e_q^2\left[-\frac{\hat{t}}{\hat{s}}-\frac{\hat{s}}{\hat{t}}\right]$$

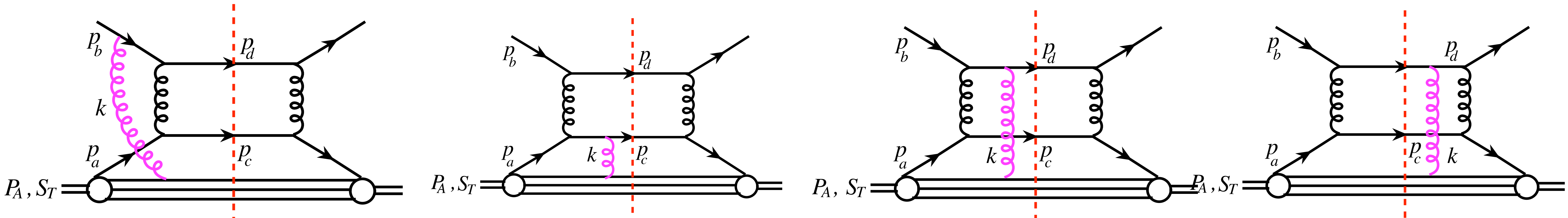


ISI drives result

Different color factors different processes ...

- Crucial point: Sivers function in inclusive single particle production contains both ISI and FSI
- consider channel $qq' \to qq'$

One gluon exchange approx for ISI and FSI



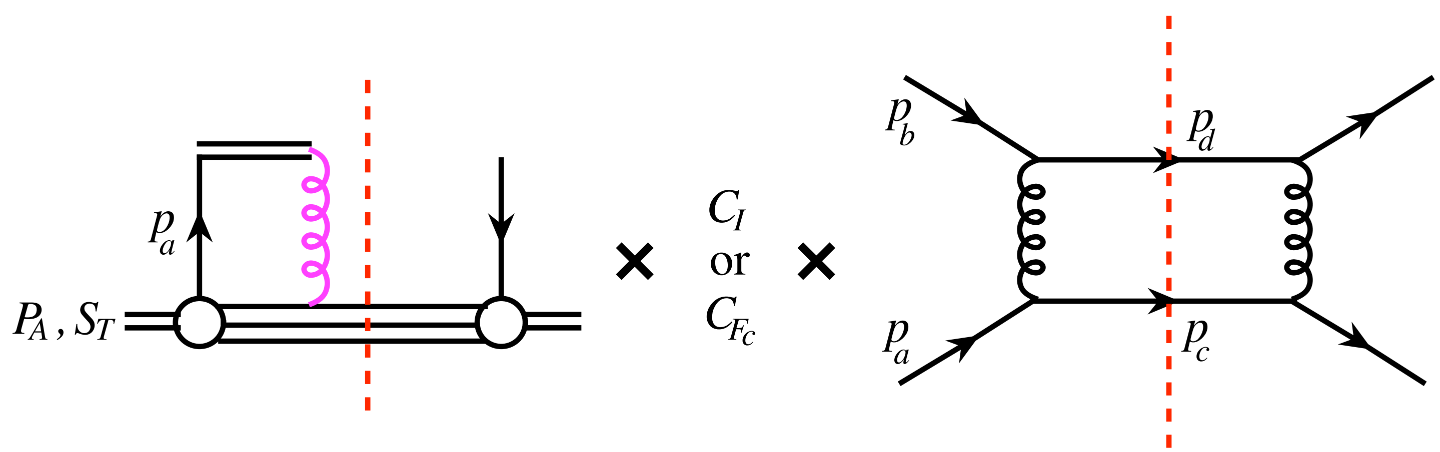
(a) (b) (c) (d)

$$\left[\frac{-g}{-k^+ - i\epsilon}T^a\right]$$

$$C_I = -\frac{1}{2N_c^2},$$

$$\left[\frac{g}{-k^+ + i\epsilon}T^a\right]$$

$$C_{F_c} = -\frac{1}{4N_c^2},$$

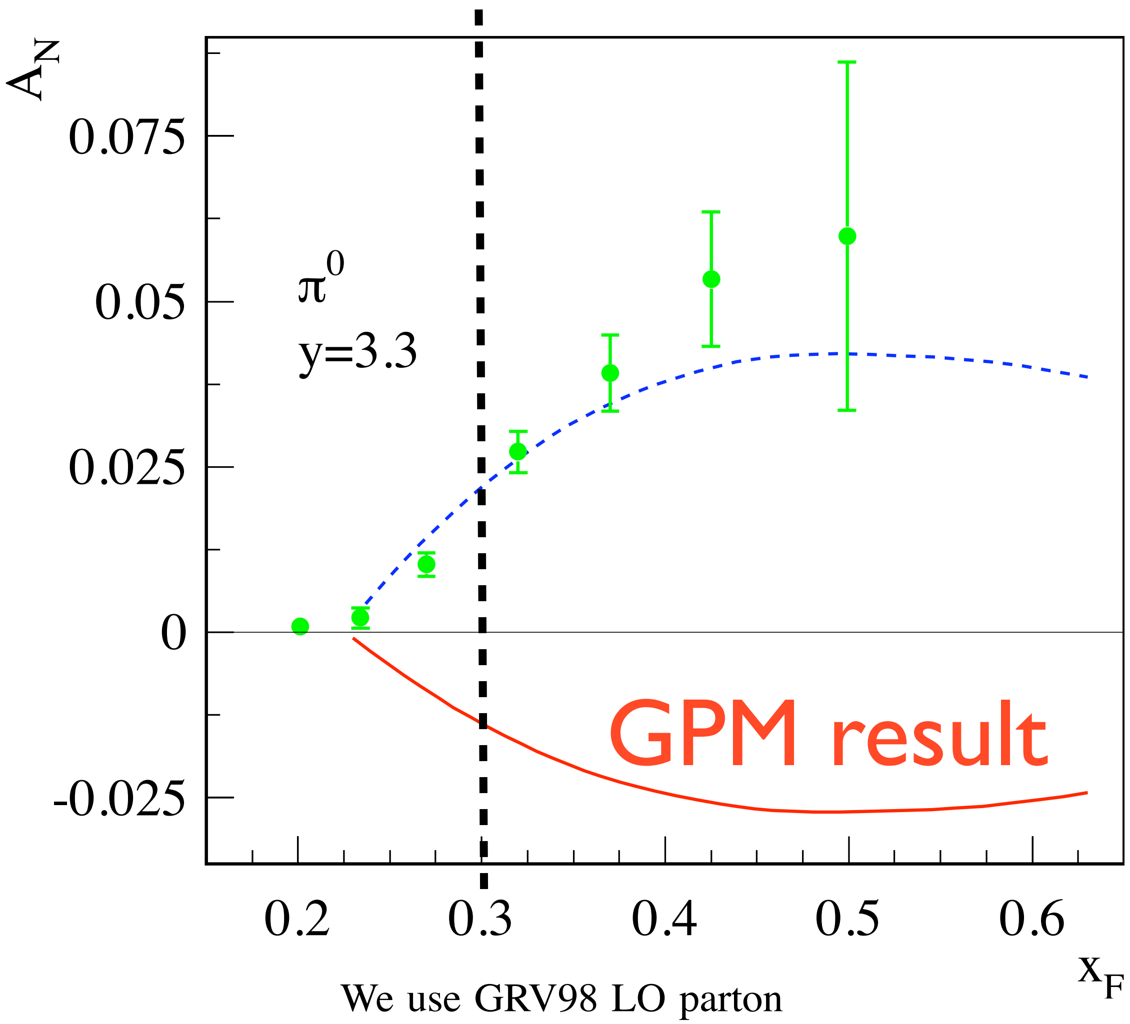


(a) (b)

Note unpolarized color factor

$$C_u = \frac{N_c^2 - 1}{4N_c^2}$$

Based on old parameterization

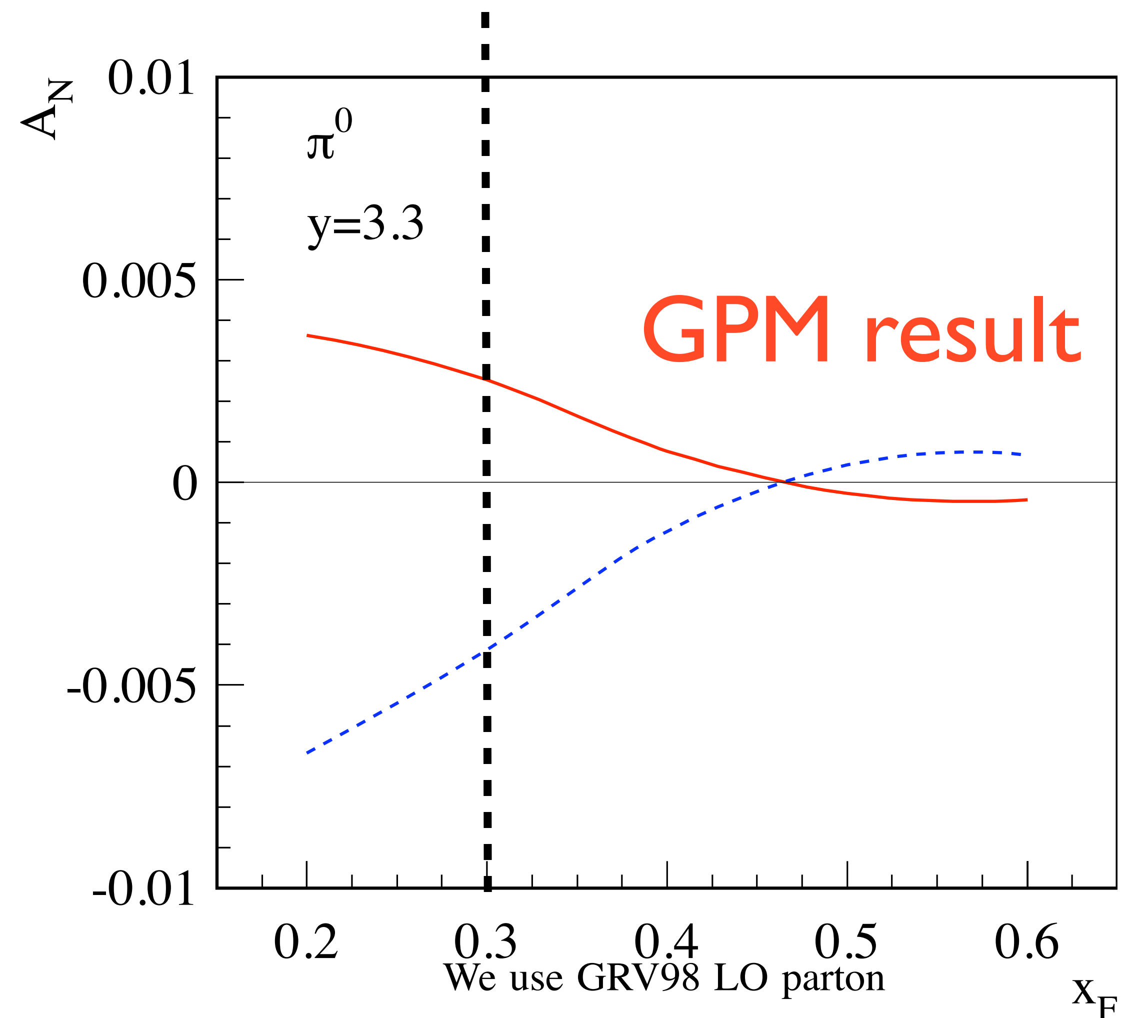


the old Sivers function from [4], and Kretzer fragmentation function [5].

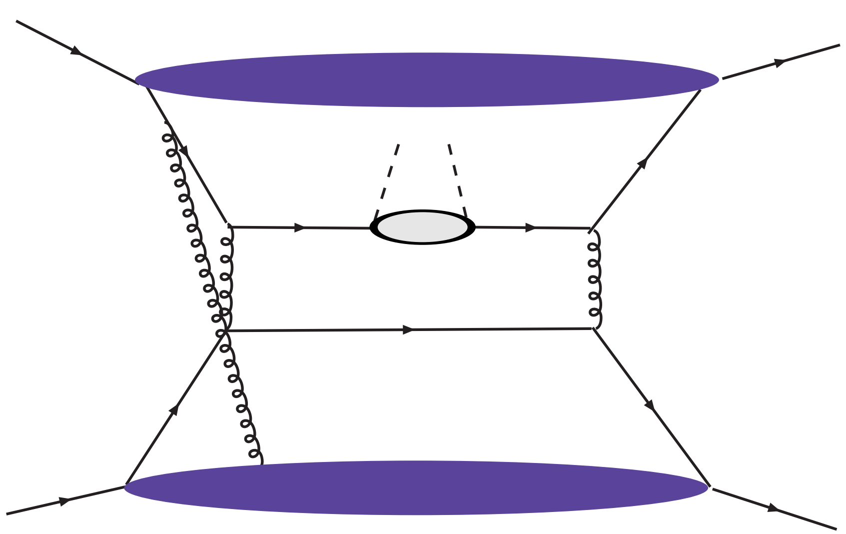
Based on new parameterization

LG & Z. Kang

Phys.Lett. B696 2011

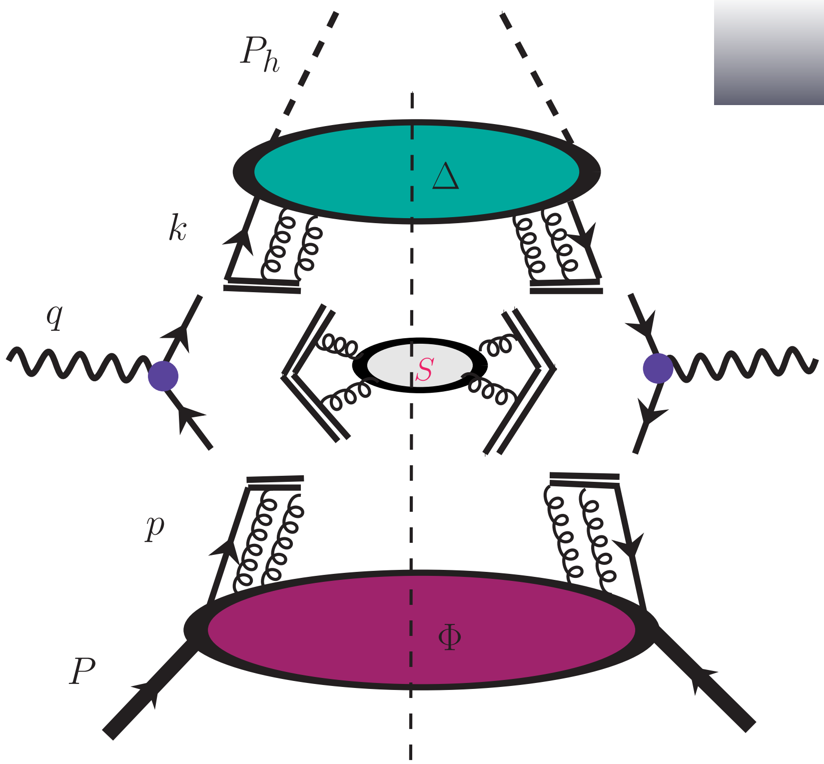


, the latest Sivers function from [2], and DSS fragmentation function [3].



$$\Delta\sigma^{pp^\uparrow \to \pi X} \sim \Delta f_a \otimes f_b \otimes \Delta\hat{\sigma} \otimes D^{q\to\pi}$$

Beyond "tree level" factorization



CS NPB 81,CSS NPB 1985 Collins Hautman PLB 00, Ji Ma Yuan PRD 05,
Cherednikov Karanikas Stefanis NPB 10, Collins Oxford Press 2011,
Abyat & Rogers arXiv: 2011

- Extra divergences at one loop and higher
- Various strategies to address them
- Extra variables needed to regulate divergences
- Modifies convolution integral by introduction soft factor
- Will show cancels in certain weighted asymmetries

Hard

$$\mathcal{C}\left[H; w f S D\right] \equiv x_B H(Q^2,\mu^2,\rho)\sum_a e_a^2 \int d^2p_T\, d^2K_T\, d^2\ell_T\, \delta^{(2)}\left(zp_T + K_T + \ell_T - P_{h\perp}\right) w\left(p_T, -\frac{K_T}{z}\right)$$
$$\times f^a(x, p_T^2, \mu^2, x\zeta, \rho)\, \underline{S(\ell_T^2, \mu^2, \rho)}\, D^a(z, K_T^2, \mu^2, \hat{\zeta}/z, \rho)$$

TMD

Soft

FF

Collins Soper NPB 1981, Collins Metz PRL 2004, Ji, Ma, Yuan PRD 2005, also Bacchetta Boer Diehl Mulders JHEP 2008

Comments on Soft factor

- Collective effect of soft gluons not associated with distribution or fragmentation function-factorizes
- Considered to be universal in hard processes
  (Collins & Metz PRL 04, Ji, Ma, Yuan, PRD 05)
- At tree level (zeroth order α_s) unity-parton model
- Absent tree level pheno analyses of experimental data
  (e.g. Anselmino et al PRD 05 & 07, Efremov et al PRD 07)
- Potentially, results of analyses can be difficult to compare at different energies **issue for EIC**
- Correct description of energy scale dependence of cross section and asymmetries in TMD picture, soft factor must be included (see Collins Oxford Press 2011, & Abyat & Rogers arXive: 1101.5057)
- However, possible to consider observables where it cancels e.g. weighted asymmetries

Structure Function

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\big[-(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T/M)\, f_{1T}^{\perp}\, D_1\big] \times (1+\mathcal{O}(\alpha_s))$$

Momentum conv. becomes

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\big[H_{UT,T}^{\sin(\phi_h-\phi_S)}; -(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T/M)\; f_{1T}^{\perp}\; S^{+}\, D_1\big]$$

or,

$$\begin{aligned} F_{UT,T}^{\sin(\phi_h-\phi_S)} &= x_B\, H_{UT,T}^{\sin(\phi_h-\phi_S)} \sum_a e_a^2 \\ &\times \int d^2p_T\, d^2K_T\, d^2l_T \delta^{(2)}\big(zp_T + K_T + l_T - P_{h\perp}\big) \\ &\times \frac{p_T \cos(\phi_h - \phi_p)}{M}\, f_{1T}^{\perp\, a}(x, p_T^2)\, S(l_T^2)\, D^a(z, K_T^2) \end{aligned}$$

Weighted asymmetries

Disentangle in model independent way cross section in terms of moments of TMDs

Kotzinian, Mulders PLB 97, Boer, Mulders PRD 98

$$A_{XY}^{\mathcal{W}} = 2\frac{\int d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|\,d\phi_h\,d\phi_S\,\mathcal{W}(|\boldsymbol{P}_{h\perp}|,\phi_h,\phi_S)\,\left(d\sigma^{\uparrow}-d\sigma^{\downarrow}\right)}{\int d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|\,d\phi_h\,d\phi_S\,(d\sigma^{\uparrow}+d\sigma^{\downarrow})}$$

e.g. $\mathcal{W}_{\text{Sivers}} = \frac{|\boldsymbol{P}_{h\perp}|}{M}\sin(\phi_h-\phi_S)$

$$A_{UT}^{\frac{|\boldsymbol{P}_{h\perp}|}{z_h M}\sin(\phi_h-\phi_s)} = -2\frac{\sum_a e_a^2\, f_{1T}^{\perp(1)}(x)\ D_1^{a(0)}(z)}{\sum_a e_a^2\, f_1^{a(0)}(x)\ D_1^{a(0)}(z)}$$

Undefined w/o regularization to subtract infinite contribution at large transverse momentum

Bacchetta et al. JHEP 08

Comments

- Propose generalize Bessel Weights-"BW"
- BW procedure has advantages
- Introduces a free parameter $\mathcal{B}_T\ [\mathrm{GeV}^{-1}]$ that is Fourier conjugate to $\boldsymbol{P}_{h\perp}$
- Provides a regularization of infinite contributions at lg. transverse momentum when $\mathcal{B}_T^2$ is non-zero for moments
- Addtnl. bonus soft factor eliminated from weighted asymmetries
- Possible to compare observables at different scales.... could be useful for an EIC

Advantages of Bessel Weighting

1. "Deconvolution"-SIDIS struct fnct simple products "$\mathcal{P}$"
2. Soft Factor Cancels
3. Circumvents the problem of ill-defined $\boldsymbol{p}_T$ moments
4. Bessel Weight asymmetris sensitive to low $\boldsymbol{P}_{h\perp}$ region

★ $w_1 = 2J_1(|\boldsymbol{P}_{h\perp}|\mathcal{B}_T)/zM\mathcal{B}_T$

$$A_{UT}^{\frac{2J_1(|\boldsymbol{P}_{h\perp}|\mathcal{B}_T)}{zM\mathcal{B}_T}\sin(\phi_h-\phi_s)} = -2\frac{\sum_a e_a^2\, \tilde{f}_{1T}^{\perp(1)a}(x, z^2\mathcal{B}_T^2)\, \tilde{D}_1^a(z, \mathcal{B}_T^2)}{\sum_a e_a^2\, \tilde{f}_1^a(x, z^2\mathcal{B}_T^2)\, \tilde{D}_1^a(z, \mathcal{B}_T^2)},$$

Where $\tilde{f}_1,\ \tilde{f}_{1T}^{\perp(1)}$, and $\tilde{D}_1$ are Fourier Transf. of TMDs/FFs and finite

1. “Deconvolution”-SIDIS structure functions simple products

1. "Deconvolution"-SIDIS structure functions simple products

a) F.T. SIDIS cross section w/ following defintions

$$\begin{aligned}
\tilde{f}(x, \boldsymbol{b}_T^2) &\equiv \int d^2\boldsymbol{p}_T \, e^{i\boldsymbol{b}_T \cdot \boldsymbol{p}_T} \; f(x, \boldsymbol{p}_T^2) \\
&= 2\pi \int d|\boldsymbol{p}_T||\boldsymbol{p}_T| \; J_0(|\boldsymbol{b}_T||\boldsymbol{p}_T|) \; f^a(x, \boldsymbol{p}_T^2) \; , \\
\tilde{f}^{(n)}(x, \boldsymbol{b}_T^2) &\equiv n! \left(-\frac{2}{M^2}\partial_{\boldsymbol{b}_T^2}\right)^n \tilde{f}(x, \boldsymbol{b}_T^2) \\
&= \frac{2\pi \; n!}{(M^2)^n} \int d|\boldsymbol{p}_T||\boldsymbol{p}_T| \left(\frac{|\boldsymbol{p}_T|}{|\boldsymbol{b}_T|}\right)^n J_n(|\boldsymbol{b}_T||\boldsymbol{p}_T|) \; f(x, \boldsymbol{p}_T^2) \; ,
\end{aligned}$$

b) n.b. connection to $\boldsymbol{p}_T$ moments

$$\tilde{f}^{(n)}(x, 0) = \int d^2\boldsymbol{p}_T \left(\frac{\boldsymbol{p}_T^2}{2M^2}\right)^n f(x, \boldsymbol{p}_T^2) \equiv f^{(n)}(x)$$

Structure functions are "products" $\mathcal{P}$ vs. "convolutions" $\mathcal{C}$

$$\frac{d\sigma}{dx_{_B}\,dy\,d\phi_S\,dz_h\,d\phi_h\,d|\boldsymbol{P}_{h\perp}|^2} \propto \frac{\alpha^2}{x_{_B}Q^2}\int\frac{d|\boldsymbol{b}_T|}{(2\pi)}|\boldsymbol{b}_T|\,\tilde{S}(\boldsymbol{b}_T^2)\Big\{\quad\ldots$$

$$+\,J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{P}[\tilde{f}_1\ \tilde{D}_1]$$

Soft factor is

- spin blind
- flavor blind
- factors in $\mathcal{P}$
- Universal

Ibildi,Ji,Ma,Yuan PRD 05

$$+\,|\boldsymbol{S}_\perp|\,\sin(\phi_h-\phi_S)\,J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{P}[\tilde{f}_{1T}^{\perp(1)}\ \tilde{D}_1]$$

$$+\varepsilon\cos(2\phi_h)\,J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{P}[\tilde{h}_1^{\perp(1)}\ \tilde{H}_1^{\perp(1)}]$$

$$+\quad\ldots 15\quad\text{more}\quad\text{structure}\quad\text{functions}$$

Products in terms of " $\boldsymbol{b}_T$ moments "

$$\mathcal{P}[\tilde{f}_{1T}^{\perp(1)}\,\tilde{D}^{(0)}] \equiv x_{_B}\,(zM|\boldsymbol{b}_T|)$$

$$\times\sum_a e_a^2\,\tilde{f}_{1T}^{\perp a(1)}(x,z^2\boldsymbol{b}_T^2)\,\tilde{D}^a(z,\boldsymbol{b}_T^2)$$

Full Cross Section expansion in Bessel functions

$$\frac{d\sigma}{dx_B\, dy\, d\phi_S\, dz_h\, d\phi_h\, |\boldsymbol{P}_{h\perp}| d|\boldsymbol{P}_{h\perp}|} = \frac{\alpha^2}{x_B y Q^2} \frac{y^2}{(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x_B}\right) \int \frac{d|\boldsymbol{b}_T|}{(2\pi)} |\boldsymbol{b}_T| \Big\{$$

$$+\ J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{UU,T} + \varepsilon\, J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \cos\phi_h\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{UU}^{\cos\phi_h}$$

$$+\ \varepsilon \cos(2\phi_h)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{UU}^{\cos(2\phi_h)} + \lambda_e \sqrt{2\,\varepsilon(1-\varepsilon)}\, \sin\phi_h\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{LU}^{\sin\phi_h}$$

$$+\ S_\parallel \left[\sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin\phi_h\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{UL}^{\sin 2\phi_h} \right]$$

$$+\ S_\parallel \lambda_e \left[\sqrt{1-\varepsilon^2}\, J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos\phi_h\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{LL}^{\cos\phi_h} \right]$$

$$+\ |\boldsymbol{S}_\perp| \Big[\sin(\phi_h - \phi_S)\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|) \left(\mathcal{P}_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\, \mathcal{P}_{UT,L}^{\sin(\phi_h-\phi_S)} \right)$$

$$+\ \varepsilon\, \sin(\phi_h + \phi_S)\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\, \sin(3\phi_h - \phi_S)\, J_3(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{UT}^{\sin(3\phi_h-\phi_S)}$$

truncates at J_3

$$+\ \sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin\phi_S\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{UT}^{\sin\phi_S} + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin(2\phi_h - \phi_S)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{UT}^{\sin(2\phi_h-\phi_S)} \Big]$$

$$+\ |\boldsymbol{S}_\perp| \lambda_e \Big[\sqrt{1-\varepsilon^2}\, \cos(\phi_h - \phi_S)\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos\phi_S\, J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{LT}^{\cos\phi_S}$$

$$+\ \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos(2\phi_h - \phi_S)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{LT}^{\cos(2\phi_h-\phi_S)} \Big] \Big\}$$

Projecting w/ basis functions

$$J_n(|\mathbf{P}_{h\perp}||\mathbf{b}_\perp|)\{\sin,\cos\}(m\phi \pm n\phi_s)$$

$$\begin{aligned}&\int_0^{2\pi}\frac{d\phi_S}{2\pi}\int_0^{2\pi}d\phi_h\int_0^{\infty}d|\boldsymbol{P}_{h\perp}|\;|\boldsymbol{P}_{h\perp}|\,J_0(|\boldsymbol{P}_{h\perp}||\boldsymbol{b}_T|)\left[\frac{d\sigma}{dx_{_B}\,dy\,d\phi_S\,dz_h\,d\phi_h\,|\boldsymbol{P}_{h\perp}|d|\boldsymbol{P}_{h\perp}|}\right]\\&\qquad=\frac{\alpha^2}{yQ^2}\,\frac{y^2}{(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x_{_B}}\right)\sum_a e_a^2\left\{\tilde{f}_1^{a(0)}(x,z^2\boldsymbol{b}_T^2)+S_\parallel\lambda_e\sqrt{1-\varepsilon^2}\;\tilde{g}_1^{(0)a}(x,z^2\boldsymbol{b}_T^2)\right\}\tilde{D}_1^{(0)a}(z,\boldsymbol{b}_T^2)\,,\end{aligned}$$

and .....

A similar integration allows us to project out information on the Sivers function $f_{1T}^{\perp}$:

$$\int_0^{2\pi}\frac{d\phi_S}{2\pi}\int_0^{2\pi}d\phi_h\;\sin(\phi_h-\phi_S)\int_0^{\infty}d|\boldsymbol{P}_{h\perp}|\;|\boldsymbol{P}_{h\perp}|\,\frac{-2J_1(|\boldsymbol{P}_{h\perp}||\boldsymbol{b}_T|)}{zM|\boldsymbol{b}_T|}\left[\frac{d\sigma}{dx_{\scriptscriptstyle B}\,dy\,d\phi_S\,dz_h\,d\phi_h\,|\boldsymbol{P}_{h\perp}|d|\boldsymbol{P}_{h\perp}|}\right]$$
$$=\frac{\alpha^2}{yQ^2}\,\frac{y^2}{(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x_{\scriptscriptstyle B}}\right)\;|\boldsymbol{S}_{\perp}|\;\sum_a e_a^2\;\tilde{f}_{1T}^{\perp(1)a}(x,z^2\boldsymbol{b}_T^2)\;\tilde{D}_1^{(0)a}(z,\boldsymbol{b}_T^2)\;.$$

A similar integration allows us to project out information on the Boer-Mulders function $h_1^{\perp}$:

$$\int_0^{2\pi}\frac{d\phi_S}{2\pi}\int_0^{2\pi}d\phi_h\;\cos(2\phi_h)\int_0^{\infty}d|\boldsymbol{P}_{h\perp}|\;|\boldsymbol{P}_{h\perp}|\,\frac{2J_2(|\boldsymbol{P}_{h\perp}||\boldsymbol{b}_T|)}{z^2MM_h\boldsymbol{b}_T^2}\left[\frac{d\sigma}{dx_{\scriptscriptstyle B}\,dy\,d\phi_S\,dz_h\,d\phi_h\,|\boldsymbol{P}_{h\perp}|d|\boldsymbol{P}_{h\perp}|}\right]$$
$$=\frac{\alpha^2}{yQ^2}\,\frac{y^2}{(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x_{\scriptscriptstyle B}}\right)\;\varepsilon\;\sum_a e_a^2\;\tilde{h}_1^{\perp(1)a}(x,z^2\boldsymbol{b}_T^2)\;\tilde{H}_1^{\perp(1)a}(z,\boldsymbol{b}_T^2)$$

Comments

- Old weights are asymptotic form of Bessel

$$|\boldsymbol{P}_{h\perp}|^n \to J_n(|\boldsymbol{P}_{h\perp}|\mathcal{B}_T)\, n! \left(\frac{2}{\mathcal{B}_T}\right)^n \equiv \ \mathcal{J}_n^{\mathcal{B}_T}(|\boldsymbol{P}_{h\perp}|)$$

- Spin orbit correlations???

$$\gamma \,+\, q \to 3/2 - (-3/2) = 3$$
$$\gamma \,+\, P \to 3/2 - (-3/2) = 3$$

- Twisted quarks

(? angular momentum w/ intrinsic helicity ?)

Ivanov 2011 on twisted photons spin orbit

2. Bessel Weighting & cancellation of soft factor

- Various strategies developed to take into account extra divergences that appear at 1 loop and beyond
- Requires introduction of variables that act as regularization scales--TMD evolution (PRE-DIS wkshp http://conferences.jlab.org/QCDEvolution/index.html
- Soft factor coming from gluon radiation can be absorbed in definition of TMDs or can appear in structure functions

Ji, Ma, Yuan PRD 05 , Collins 2011 Oxford Press, Abyet , Rogers

- With both definitions we show it cancels in weighted asymmetries Boer, LG, Musch, Prokudin (in preparation)

Hard

$$\mathcal{C}[H; w f S D] \equiv x_B H(Q^2, \mu^2, \rho) \sum_a e_a^2 \int d^2 p_T\, d^2 K_T\, d^2 \ell_T\, \delta^{(2)}(z p_T + K_T + \ell_T - P_{h\perp})\, w\left(p_T, -\frac{K_T}{z}\right)$$

$$\times f^a(x, p_T^2, \mu^2, x\zeta, \rho)\, \underbrace{S(\ell_T^2, \mu^2, \rho)}_{\text{Soft}}\, D^a(z, K_T^2, \mu^2, \hat{\zeta}/z, \rho)$$

TMD

FF

2. Bessel Weighting & cancellation of soft factor

Bessel weighting-projecting out Sivers using orthogonality of Bessel Fncts.

$$\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM} = \frac{2\,J_1(|\boldsymbol{P}_{hT}|\mathcal{B}_T)}{zM\mathcal{B}_T}$$

$$A_{UT}^{\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_S)}(\mathcal{B}_T) =$$

$$2\,\frac{\int d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|\,d\phi_h\,d\phi_S\,\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_S)\,(d\sigma^{\uparrow}-d\sigma^{\downarrow})}{\int d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|\,d\phi_h\,d\phi_S\;\mathcal{J}_0^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)\;(d\sigma^{\uparrow}+d\sigma^{\downarrow})}$$

$$A_{UT}^{\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_s)}(\mathcal{B}_T) =$$

$$-2\,\frac{\cancel{\tilde{S}(\mathcal{B}_T^2)}\,H_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2)\sum_a e_a^2\,\tilde{f}_{1T}^{\perp(1)a}(x,z^2\mathcal{B}_T^2)\,\tilde{D}_1^a(z,\mathcal{B}_T^2)}{\cancel{\tilde{S}(\mathcal{B}_T^2)}\,H_{UU,T}(Q^2)\sum_a e_a^2\;\tilde{f}_1^a(x,z^2\mathcal{B}_T^2)\,\tilde{D}_1^a(z,\mathcal{B}_T^2)}$$

Sivers asymmetry with full dependences

$$A_{UT}^{\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_s)}(\mathcal{B}_T) \quad =$$

$$-2\,\frac{\tilde{S}(\mathcal{B}_T^2,\mu^2,\rho^2)\,H_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2,\mu^2,\rho)\,\sum_a e_a^2\,\tilde{f}_{1T}^{\perp(1)a}(x,z^2\mathcal{B}_T^2;\mu^2,\zeta,\rho)\,\tilde{D}_1^a(z,\mathcal{B}_T^2;\mu^2,\hat{\zeta},\rho)}{\tilde{S}(\mathcal{B}_T^2,\mu^2,\rho^2)\,H_{UU,T}(Q^2,\mu^2,\rho)\,\sum_a e_a^2\,\tilde{f}_1^a(x,z^2\mathcal{B}_T^2;\mu^2,\zeta,\rho)\,\tilde{D}_1^a(z,\mathcal{B}_T^2;\mu^2,\hat{\zeta},\rho)}$$

3. Circumvents the problem of ill-defined $\boldsymbol{p}_T$ moments

$$A_{UT}^{\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_s)}(\mathcal{B}_T) \quad =$$

$$-2\frac{\tilde{S}(\mathcal{B}_T^2,\mu^2,\rho^2)H_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2,\mu^2,\rho)\sum_a e_a^2\,\tilde{f}_{1T}^{\perp(1)a}(x,z^2\mathcal{B}_T^2;\mu^2,\zeta,\rho)\,\tilde{D}_1^a(z,\mathcal{B}_T^2;\mu^2,\hat{\zeta},\rho)}{\tilde{S}(\mathcal{B}_T^2,\mu^2,\rho^2)H_{UU,T}(Q^2,\mu^2,\rho)\sum_a e_a^2\,\tilde{f}_1^a(x,z^2\mathcal{B}_T^2;\mu^2,\zeta,\rho)\,\tilde{D}_1^a(z,\mathcal{B}_T^2;\mu^2,\hat{\zeta},\rho)}$$

Traditional weighted asymmetry recovered but UV divergent

$$\lim_{\mathcal{B}_T\to 0} w_1 = 2J_1(|\boldsymbol{P}_{h\perp}|\mathcal{B}_T)/zM\mathcal{B}_T \longrightarrow |\boldsymbol{P}_{h\perp}|/zM$$

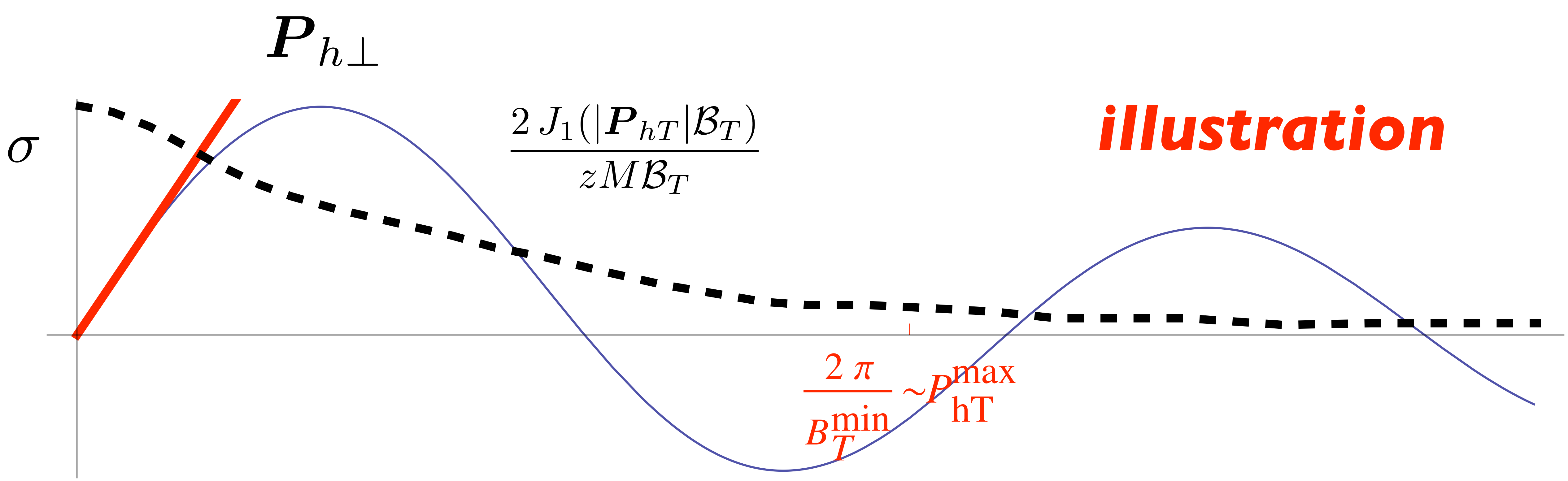
$$A_{UT}^{\frac{|\boldsymbol{P}_{h\perp}|}{z_h M}\sin(\phi_h-\phi_s)} = -2\frac{\sum_a e_a^2 f_{1T}^{\perp(1)}(x)\; D_1^{a(0)}(z)}{\sum_a e_a^2 f_1^{a(0)}(x)\; D_1^{a(0)}(z)}$$

Bacchetta et al. JHEP 08

undefined w/o regularization

4. More sensitive to low $\boldsymbol{P}_{h\perp}$ region

$\mathcal{B}_T$ can serve as a lever arm to enhance the low $\boldsymbol{P}_{h\perp}$ description and possibly dampen lg. momentum tail of cross section. We can use it to scan the cross section



Conclusions

- Propose generalize Bessel Weights
- New theoretical weighting procedure w/ advantages
- Introduces a free parameter $\mathcal{B}_T\,[\mathrm{GeV}^{-1}]$ that is Fourier conjugate to $\boldsymbol{P}_{h\perp}$
- Provides a regularization of infinite contributions at lg. transverse momentum when $\mathcal{B}_T^2$ is non-zero
- Addtnl. bonus soft factor eliminated from weighted asymmetries
- Possible to compare observables at different scales.... could be useful for an EIC

Generalized av. quark trans. momentum shift
Soft Factor cancels

From talk of Berni Musch Pre-DIS wkshp.

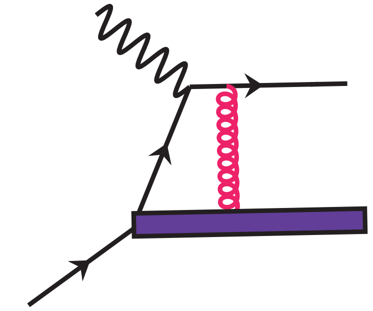
$\langle \boldsymbol{p}_y \rangle_{TU}$:= average quark momentum in transverse y-direction measured in a proton polarized in transverse x-direction.

"dipole moment", "shift"

attention divergences from high-$\boldsymbol{p}_T$-tails!

$$\langle \boldsymbol{p}_y \rangle_{TU}(\mathcal{B}_T) \equiv M \frac{\int dx\, \tilde{f}_{1T}^{\perp(1)}(x, \mathcal{B}_T^2)}{\int dx\, \tilde{f}_1^{(0)}(x, \mathcal{B}_T^2)} = \frac{\cancel{\tilde{S}(\mathcal{B}_T^2, \ldots)}\, \tilde{A}_{12B}(\mathcal{B}_T^2, 0, 0, \tilde{\zeta}, \mu)}{\cancel{\tilde{S}(\mathcal{B}_T^2, \ldots)}\, \tilde{A}_{2B}(\mathcal{B}_T^2, 0, 0, \tilde{\zeta}, \mu)}$$

Sivers



Anselmino et al. PRD 05, EPJA 08

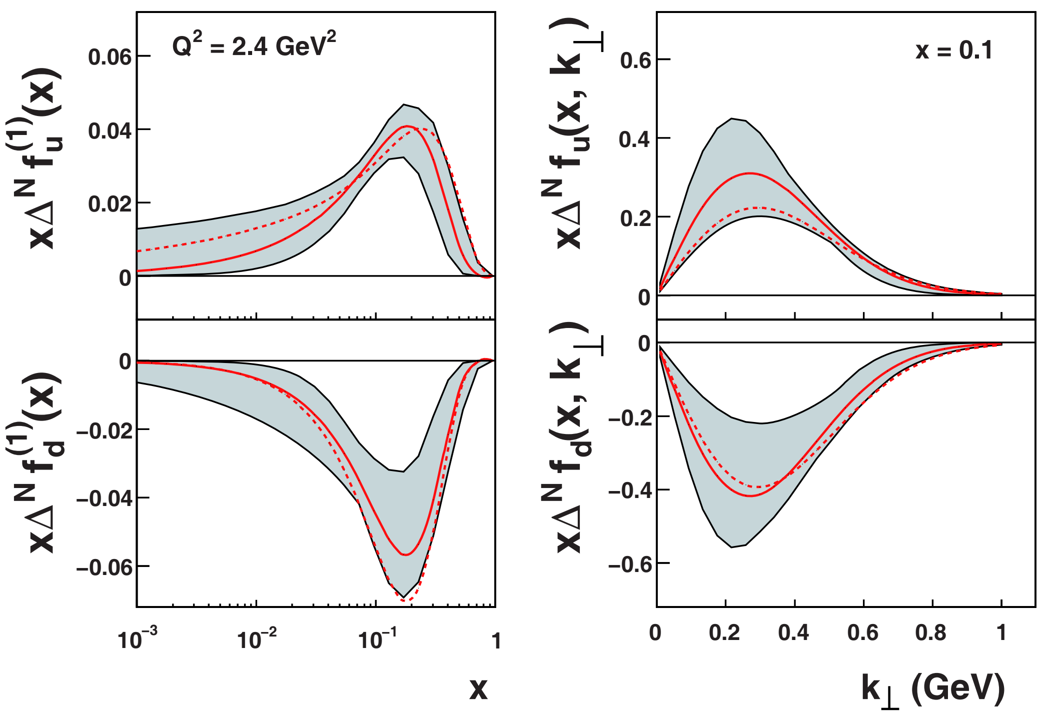
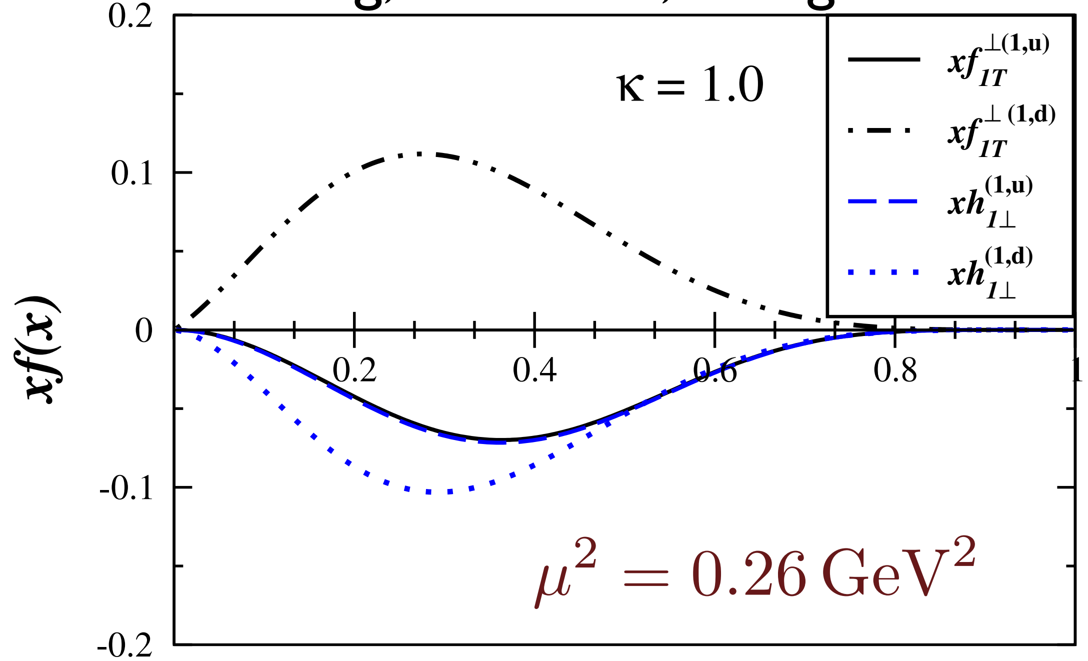
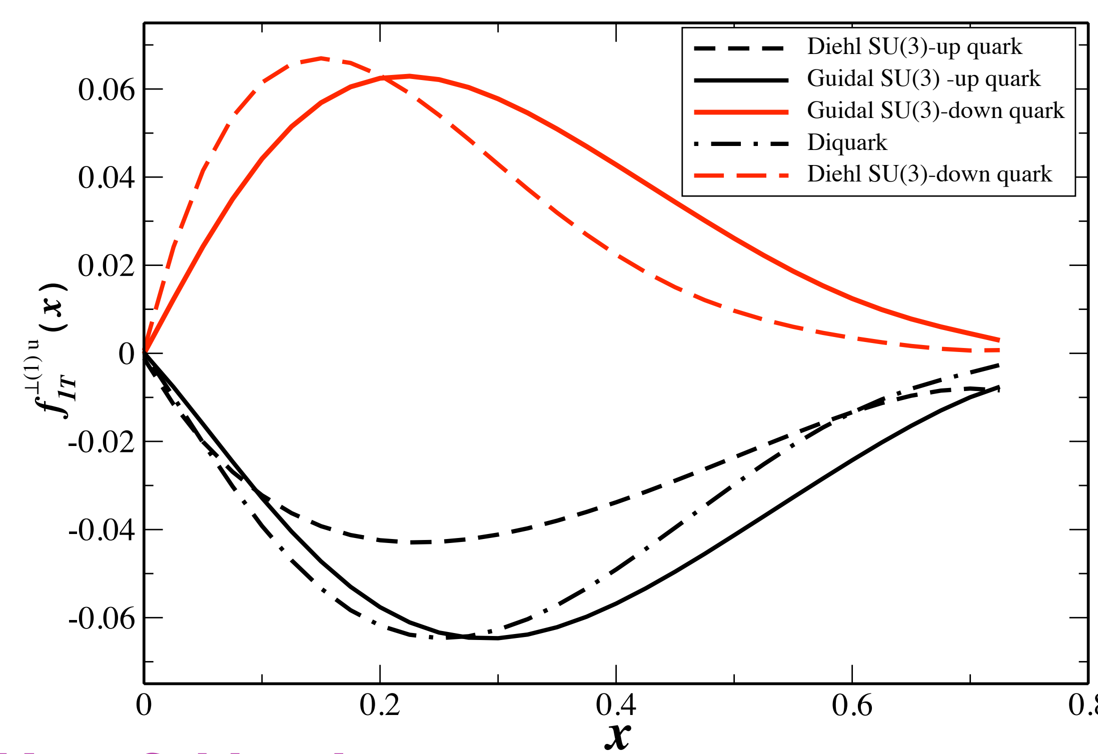
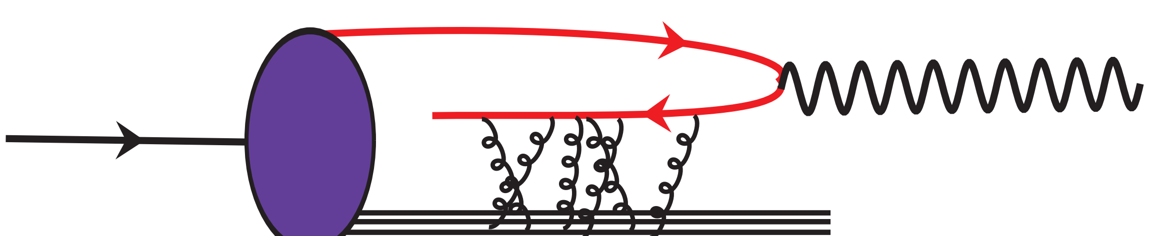


Fig. 7. The Sivers distribution functions for u and d flavours, at the scale $Q^2 = 2.4\,(\mathrm{GeV}/c)^2$, as determined by our present fit (solid lines), are compared with those of our previous fit [2] of SIDIS data (dashed lines), where π^0 and kaon productions were not considered and only valence quark contributions were taken into account. This plot clearly shows that the Sivers functions previously found are consistent, within the statistical uncertainty bands, with the Sivers functions presently obtained.

Gamberg, Goldstein,Schlegel PRD 77, 2008







L.G. & Marc Schlegel

Phys.Lett.B685:95-103,2010 & Mod.Phys.Lett.A24:2960-2972,2009.