A covariant description of nucleon [and nuclear] spin structure and TMDs

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8th Circum-Pan-Pacific Symposium On High Energy Spin Physics

Cairns, June 2011

DIS on Nuclear Targets

- Why nuclear targets?
  - only targets with $J > \frac{1}{2}$ are nuclei
  - study QCD and nucleon structure at finite density

- Hadronic Tensor: in Bjorken limit & Callen-Gross ($F_2 = 2x\, F_1$)
  - For $J = \frac{1}{2}$ target

$$W_{\mu\nu} = \left(g_{\mu\nu} \frac{p\cdot q}{q^2} + \frac{p_\mu p_\nu}{p\cdot q} \right) F_2(x, Q^2) + \frac{i\varepsilon_{\mu\nu\lambda\sigma} q^\lambda p^\sigma}{p\cdot q}\, g_1(x, Q^2)$$

  - For arbitrary J: $-J \leqslant H \leqslant J$ [$2J + 1$ EM structure functions]

$$W^H_{\mu\nu} = \left(g_{\mu\nu} \frac{p\cdot q}{q^2} + \frac{p_\mu p_\nu}{p\cdot q} \right) F^H_{2A}(x_A, Q^2) + \frac{i\varepsilon_{\mu\nu\lambda\sigma} q^\lambda p^\sigma}{p\cdot q}\, g^H_{1A}(x_A, Q^2)$$

- Parton model expressions [$2J + 1$ quark distributions]

$$g^H_{1A}(x_A) = \frac{1}{2} \sum_q e_q^2 \left[\Delta q^H_A(x_A) + \Delta \bar{q}^H_A(x_A) \right]; \quad \text{parity} \implies g^H_{1A} = -g^{-H}_{1A}$$

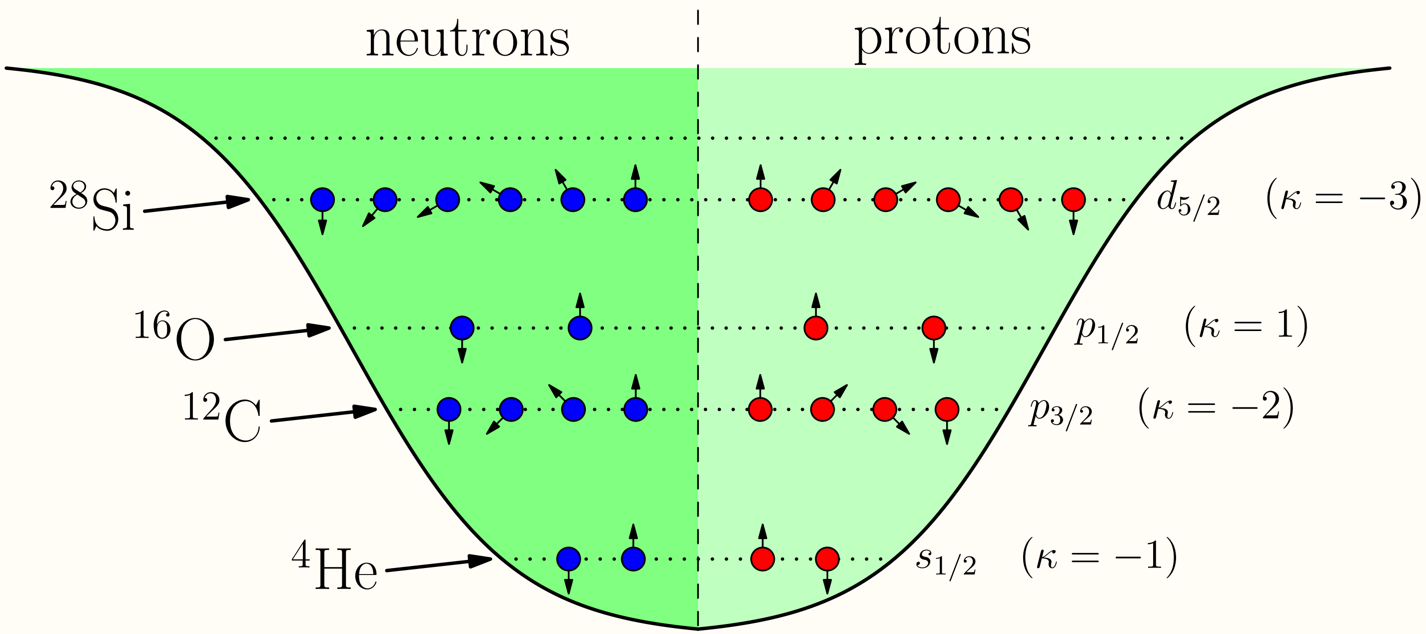
Finite nuclei quark distributions

- Definition of finite nuclei quark distributions

$$\Delta q_A^H(x_A) = \frac{P^+}{A}\int \frac{d\xi^-}{2\pi} e^{iP^+ x_A \xi^-/A}\langle A, P, H|\overline{\psi}_q(0)\,\gamma^+\gamma_5\,\psi_q(\xi^-)|A, P, H\rangle$$

- Approximate using a modified convolution formalism

$$\Delta q_A^H(x_A) = \sum_{\alpha,\kappa,m}\int dy_A \int dx\; \delta(x_A - y_A\, x)\, \Delta f^{(H)}_{\alpha,\kappa,m}(y_A)\; \Delta q_{\alpha,\kappa}(x)$$



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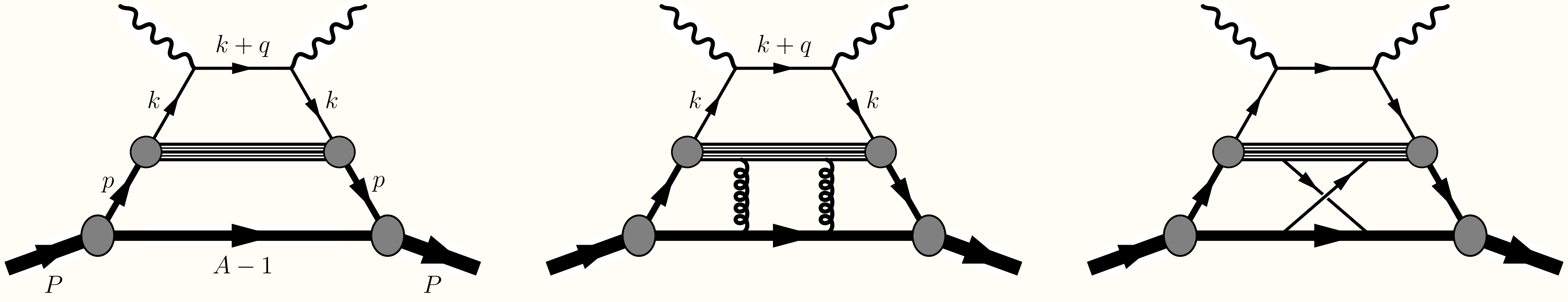
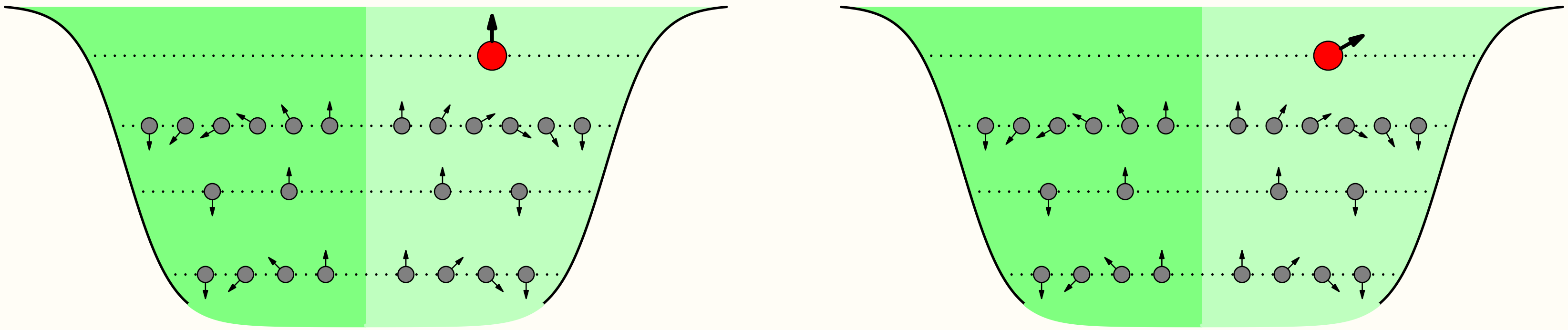
- Convolution formalism diagrammatically:



Convolution Formalism: implications

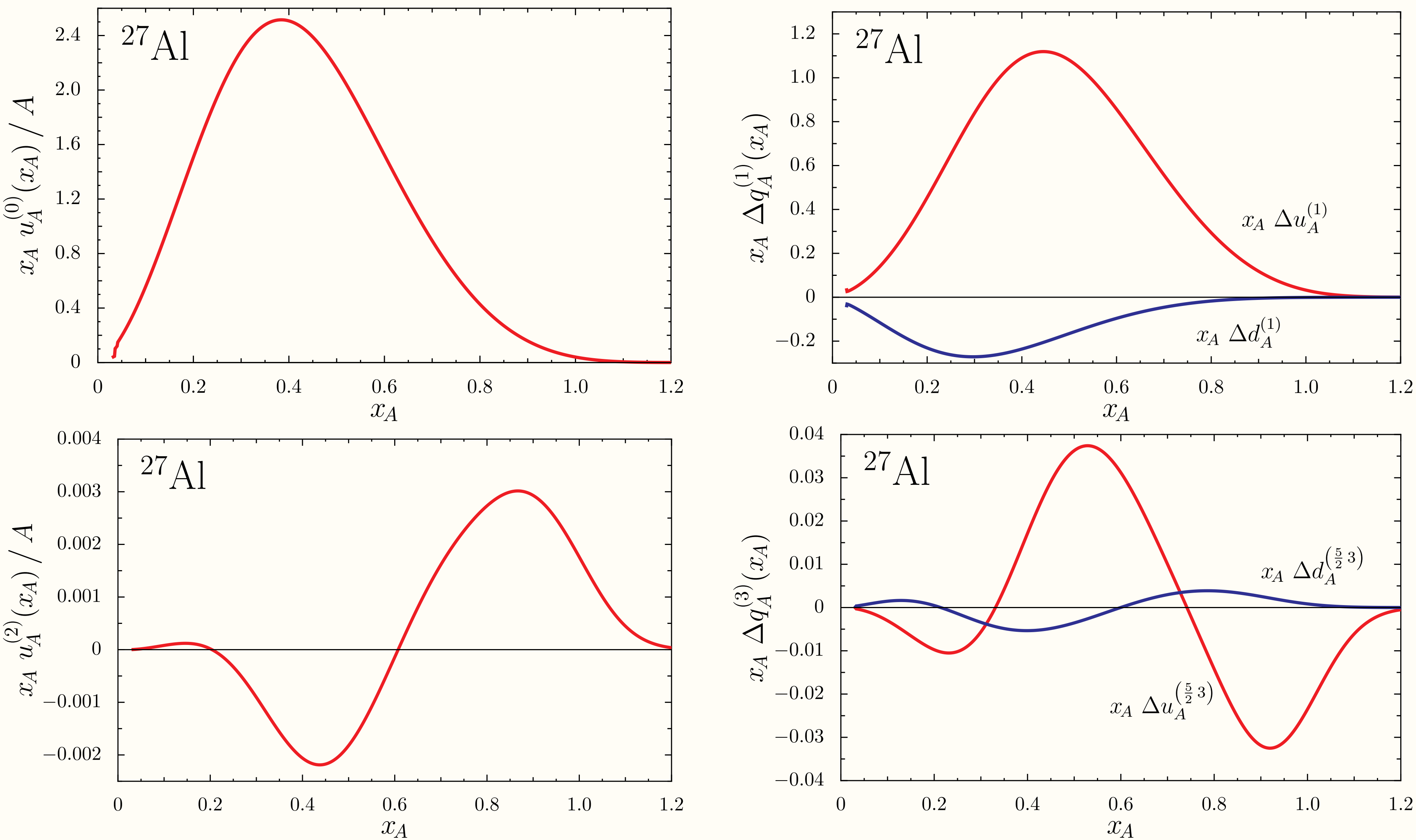


- Assume all spin is carried by the valence nucleons
  - If $A \gtrsim 8$ and for example if: $J = \frac{3}{2} \implies F_{2A}^{3/2} \simeq F_{2A}^{1/2}$
- Basically a model independent result within the convolution formalism
- Introduce multipole quark distributions

$$\Delta q^{(K)}(x) \equiv \sum_H (-1)^{J-H} \sqrt{2K+1} \begin{pmatrix} J & J & K \\ H & -H & 0 \end{pmatrix} \Delta q^H(x), \quad K = 1, 3, \ldots, 2J$$

- $J = \frac{3}{2} \longrightarrow \Delta q^{(0)} = \frac{1}{\sqrt{5}} [3\, \Delta q^{\frac{3}{2}} + \Delta q^{\frac{1}{2}}] \quad \Delta q^{(2)} = \frac{1}{\sqrt{5}} [\Delta q^{\frac{3}{2}} - 3\, \Delta q^{\frac{1}{2}}]$
- Higher multipoles encapsulate difference between helicity distributions

Some multipole quark distributions results



- Large $K > 1$ multipole PDFs would be very surprising
  - $\Longrightarrow$ large off-shell effects &/or non-nucleon components, etc

New Sum Rules

- Sum rules for multipole quark distributions

$$\int dx\, x^{n-1}\ q^{(K)}(x) \ = 0, \quad K, n \text{ even}, \quad 2 \leqslant n < K,$$
$$\int dx\, x^{n-1}\, \Delta q^{(K)}(x) = 0, \quad K, n \text{ odd}, \quad 1 \leqslant n < K.$$

- Examples:

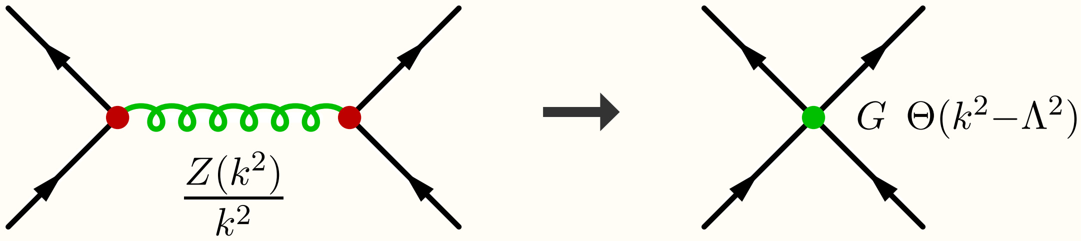
$$J = \tfrac{3}{2} \implies \left\langle \Delta q^{(3)}(x) \right\rangle = 0$$
$$J = 2 \implies \left\langle \Delta q^{(3)}(x) \right\rangle = \left\langle q^{(4)}(x) \right\rangle = 0$$
$$J = \tfrac{5}{2} \implies \left\langle \Delta q^{(3)}(x) \right\rangle = \left\langle q^{(4)}(x) \right\rangle = \left\langle \Delta q^{(5)}(x) \right\rangle = \left\langle x^2\, \Delta q^{(5)}(x) \right\rangle = 0$$

- Sum rules place tight constraints on multipole PDFs

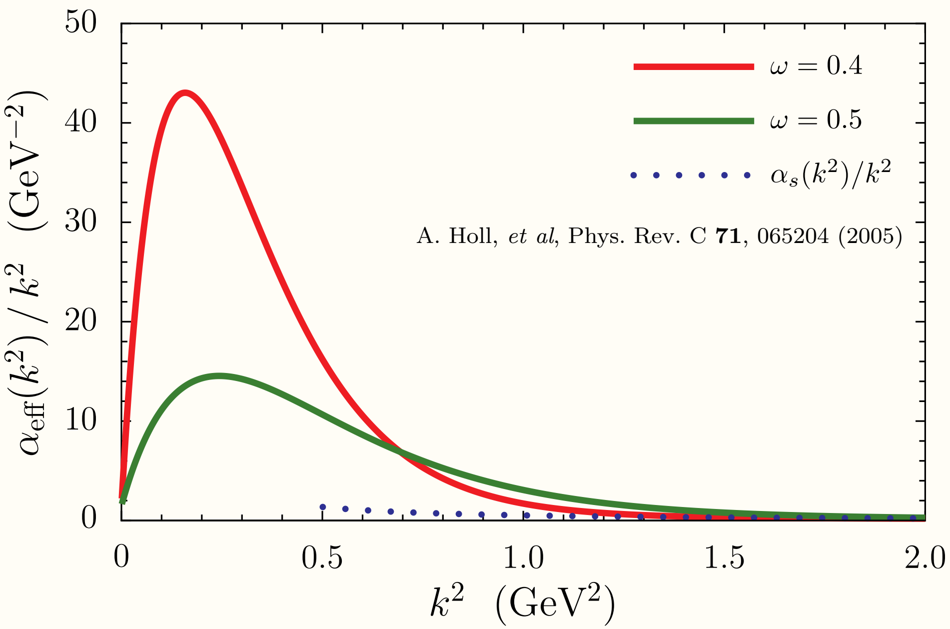
- Jaffe and Manohar, *DIS from arbitrary spin targets*, Nucl. Phys. B **321**, 343 (1989).

Nambu–Jona-Lasinio Model

- Interpreted as low energy chiral effective theory of QCD

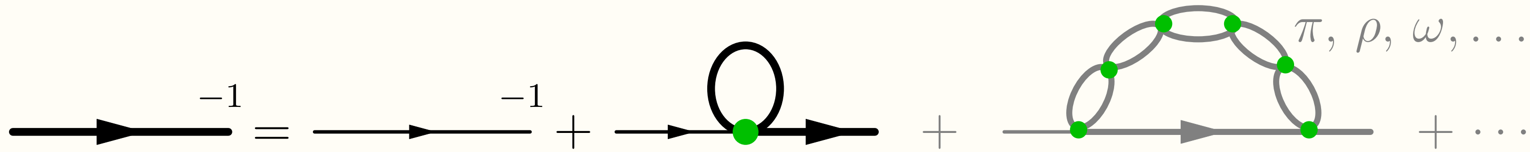


- Can be motivated by infrared enhancement of quark–gluon interaction
  e.g. DSEs and Lattice QCD

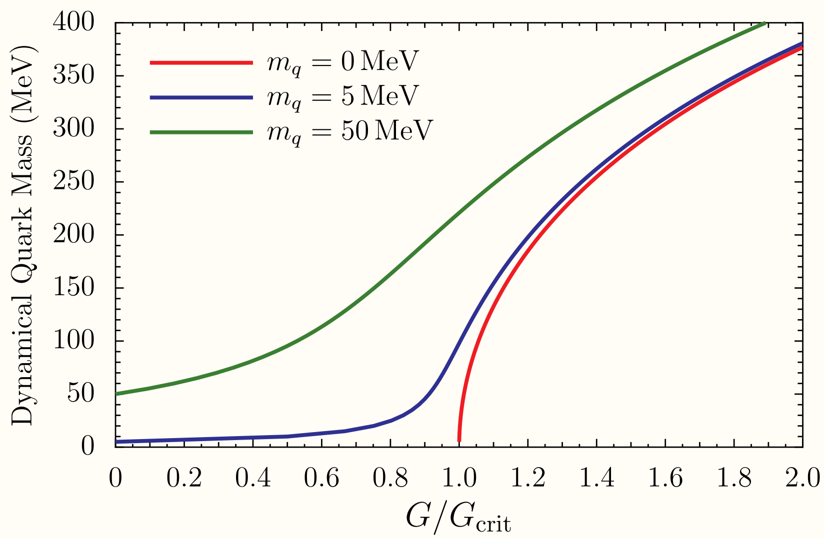


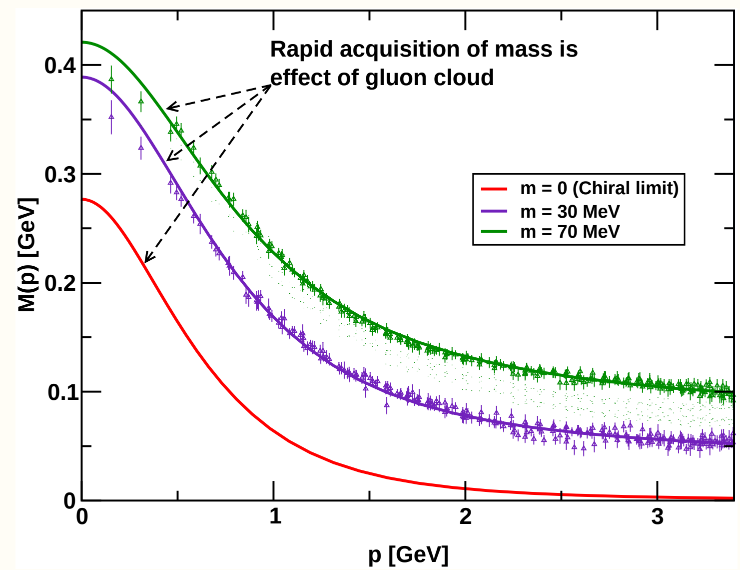
- Investigate the role of quark degrees of freedom
- NJL has same flavour symmetries as QCD
- Lagrangian: $\mathcal{L}_{NJL} = \overline{\psi}_q\left(i\partial\!\!\!/ - m_q\right)\psi_q + G\left(\overline{\psi}_q\,\Gamma\,\psi_q\right)^2$

Gap Equation & Mass Generation



- Quark propagator: $$\frac{1}{\not p - m + i\varepsilon} \rightarrow \frac{1}{\not p - M + i\varepsilon}$$
- Mass is generated via interaction with vacuum



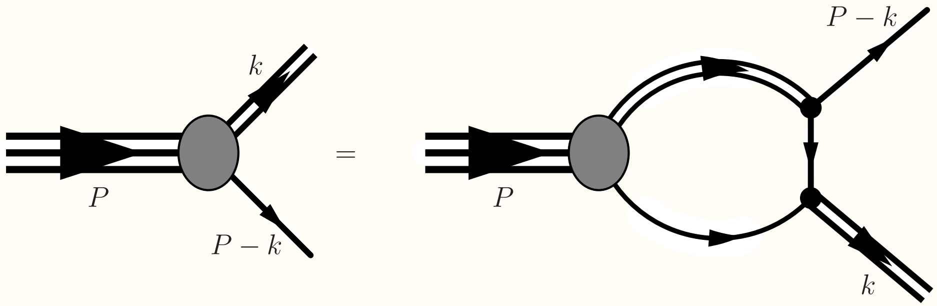


- Dynamically generated quark masses $\Longleftrightarrow$ $\langle \bar\psi \psi \rangle \neq 0$
- Proper-time regularization: Λ_{IR} and Λ_{UV}

→ $\boxed{Z(p^2 = M^2) = 0}$ $\Longrightarrow$ No free quarks $\Longrightarrow$ Confinement

Nucleon in the NJL model

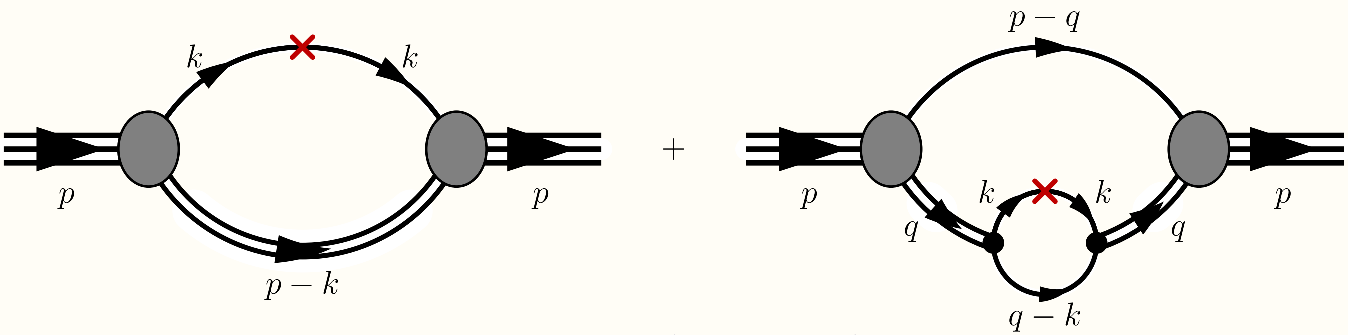
- Nucleon approximated as quark-diquark bound state
- Use relativistic Faddeev approach:



- Nucleon quark distributions

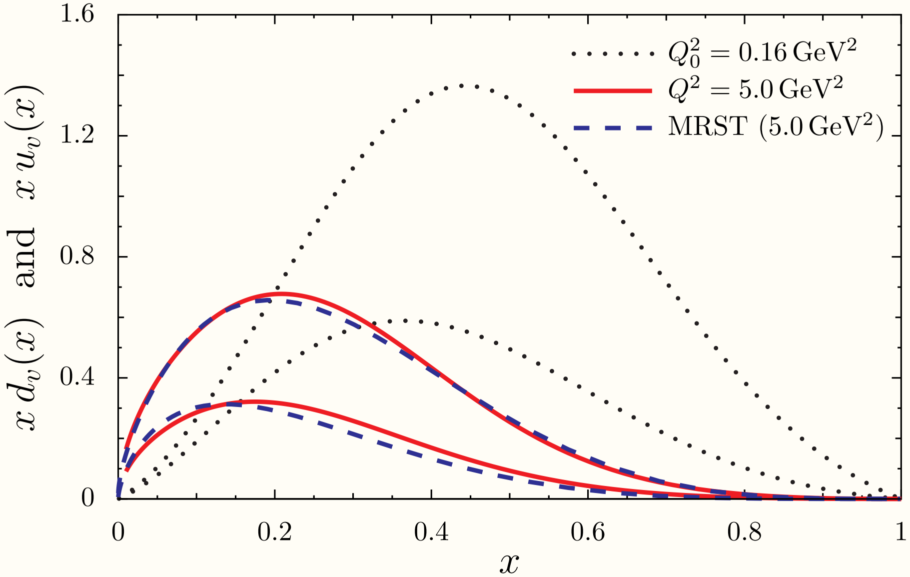
$$q(x) = p^+ \int \frac{d\xi^-}{2\pi} e^{i\,x\,p^+\xi^-} \langle p, s|\overline{\psi}_q(0)\gamma^+\psi_q(\xi^-)|p, s\rangle_c, \quad \Delta q(x) \quad = \langle\gamma^+\gamma_5\rangle$$

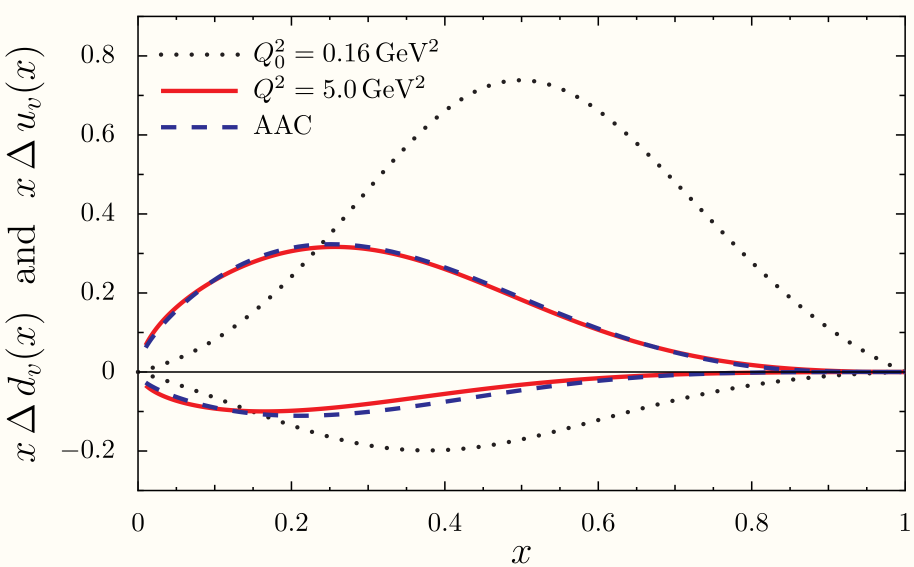
- Associated with a Feynman diagram calculation



❖ $[q(x), \Delta q(x), \Delta_T q(x)]$ ➞ $\text{X} = \delta\left(x - \frac{k^+}{p^+}\right)[\gamma^+, \gamma^+\gamma_5, \gamma^+\gamma^1\gamma_5]$

Results: proton quark distributions





- Covariant, correct support, satisfies baryon and momentum sum rules

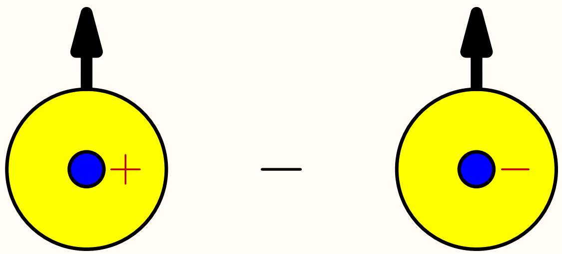
$$\int dx\,[q(x) - \bar{q}(x)] = N_q, \qquad \int dx\,x\,[u(x) + d(x) + \ldots] = 1$$

- Satisfies positivity constraints and Soffer bound

$$|\Delta q(x)|\,, \;\; |\Delta_T q(x)| \leqslant q(x), \qquad q(x) + \Delta q(x) \geqslant 2\,|\Delta_T q(x)|$$

- Martin, Roberts, Stirling and Thorne, Phys. Lett. B **531**, 216 (2002).
- M. Hirai, S. Kumano and N. Saito, Phys. Rev. D **69**, 054021 (2004).

Why is Transversity Interesting?

$\Delta_T q(x) =$ 

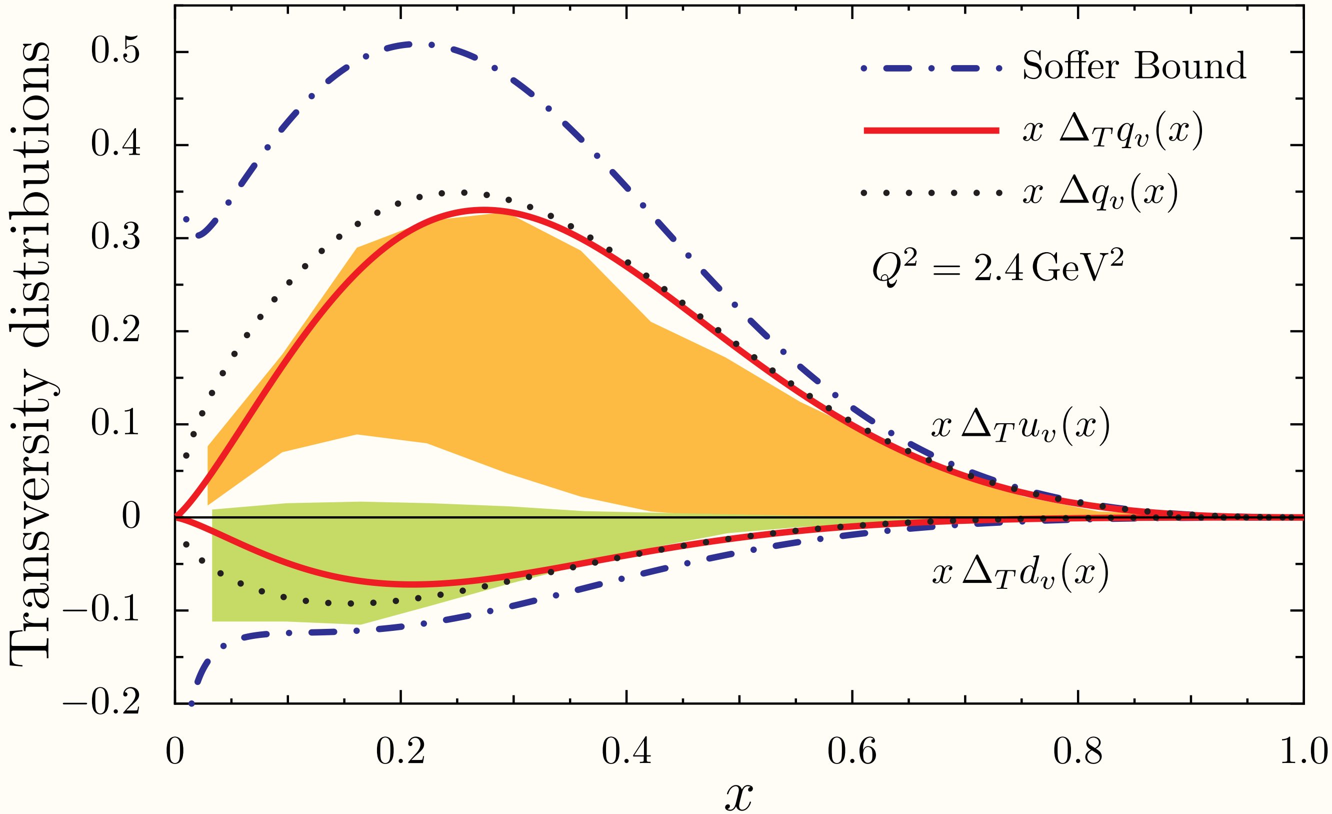
- Quarks in eigenstates of $\gamma^{\perp}\gamma_5$

- Tensor charge [c.f. Bjorken sum rule for g_A]

$$g_T = \int dx\,[\Delta_T u(x) - \Delta_T d(x)]$$

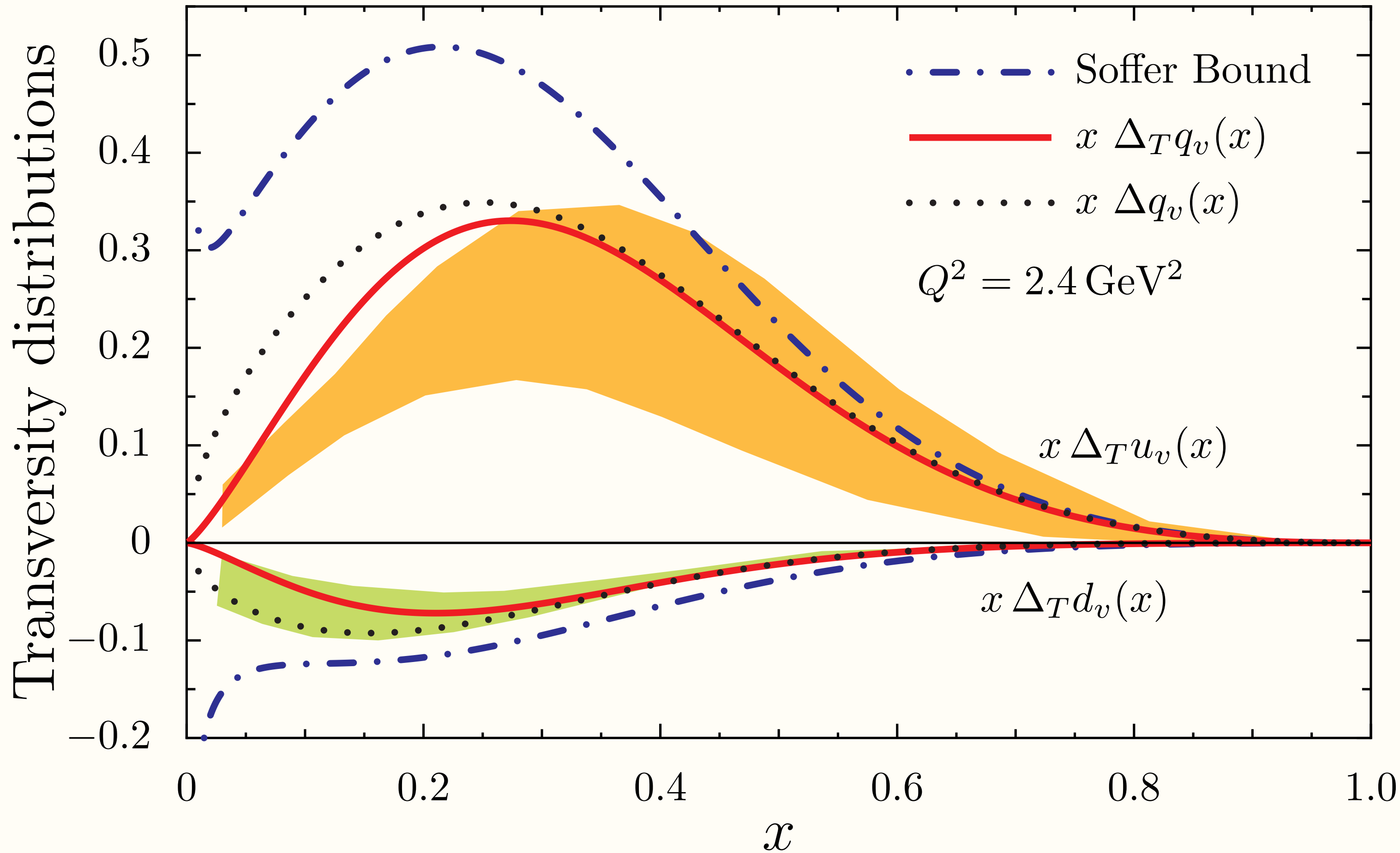
- In non-relativistic limit: $\Delta_T q(x) = \Delta q(x)$
  - therefore $\Delta_T q(x)$ is a measure of relativistic effects
- Helicity conservation $\Longrightarrow$ no mixing between $\Delta_T q$ & $\Delta_T g$
- For $J \leqslant \frac{1}{2}$ we have $\Delta_T g(x) = 0$
- Therefore for the nucleon $\Delta_T q(x)$ is valence quark dominated
- Transversity moment $\neq$ spin quarks in transverse direction [c.f. $g_T(x)$]

$\Delta_T u_v(x)$ and $\Delta_T d_v(x)$ distributions



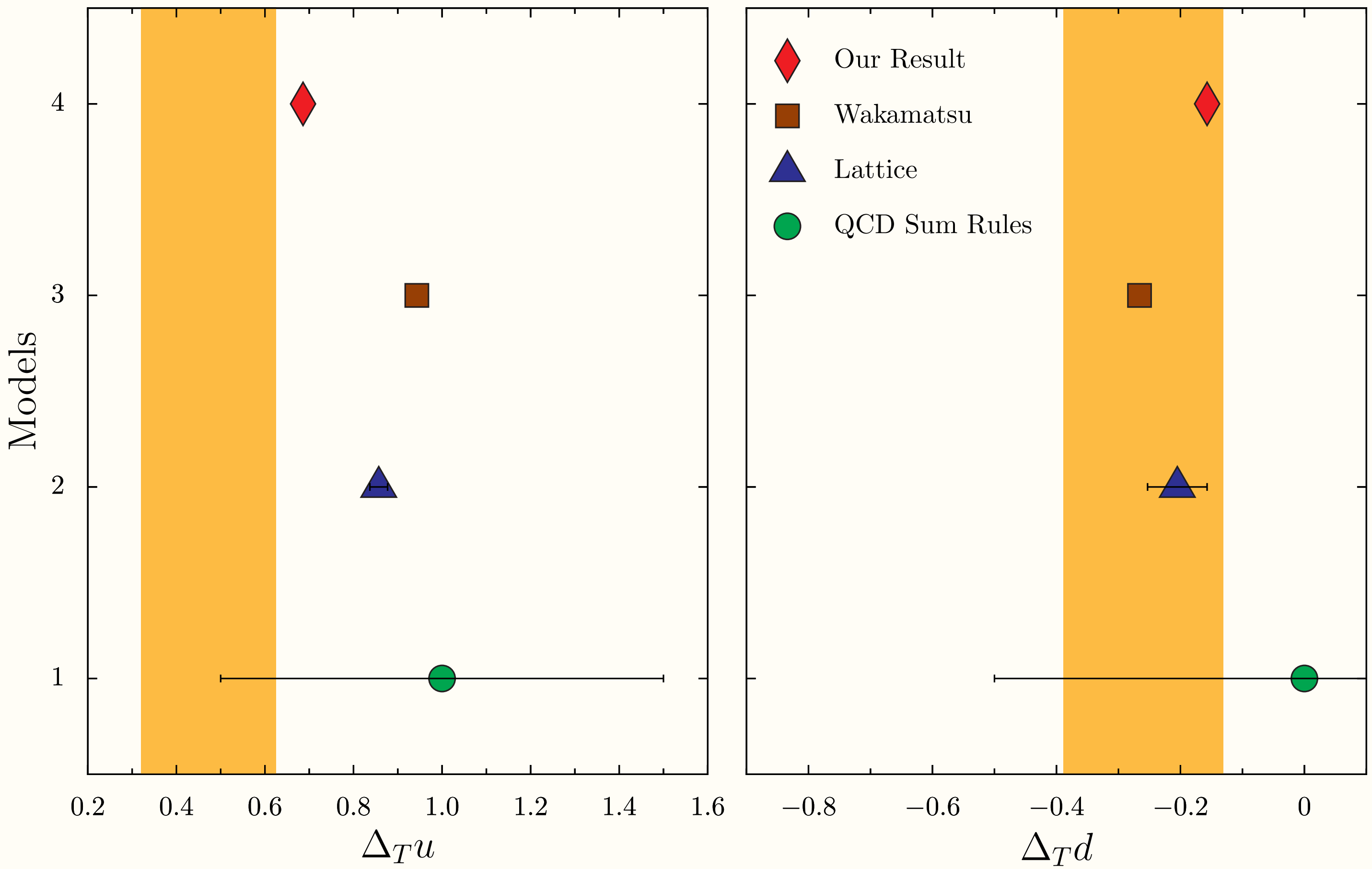
- Predict small relativistic corrections
- Empirical analysis *potentially* found large relativistic corrections
  - M. Anselmino *et. al.*,Phys. Rev. D **75**, 054032 (2007).
- Large effects difficult to support with quark mass ~ 0.4 GeV
  - maybe running quark mass is needed

Transversity: Reanalysis



- M. Anselmino *et al*, Nucl. Phys. Proc. Suppl. **191**, 98 (2009)
- Our results are now in better agreement updated distributions
- Concept of constituent quark models safe . . . for now

Transversity Moments

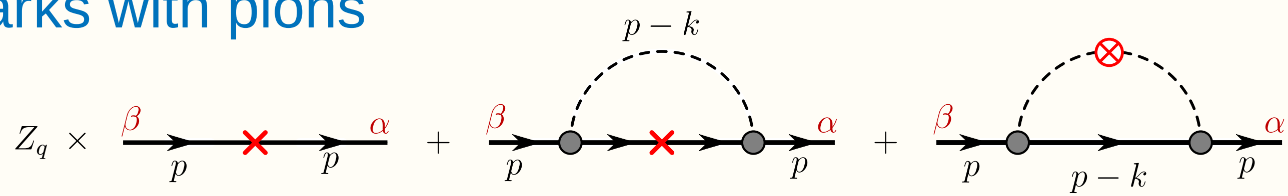


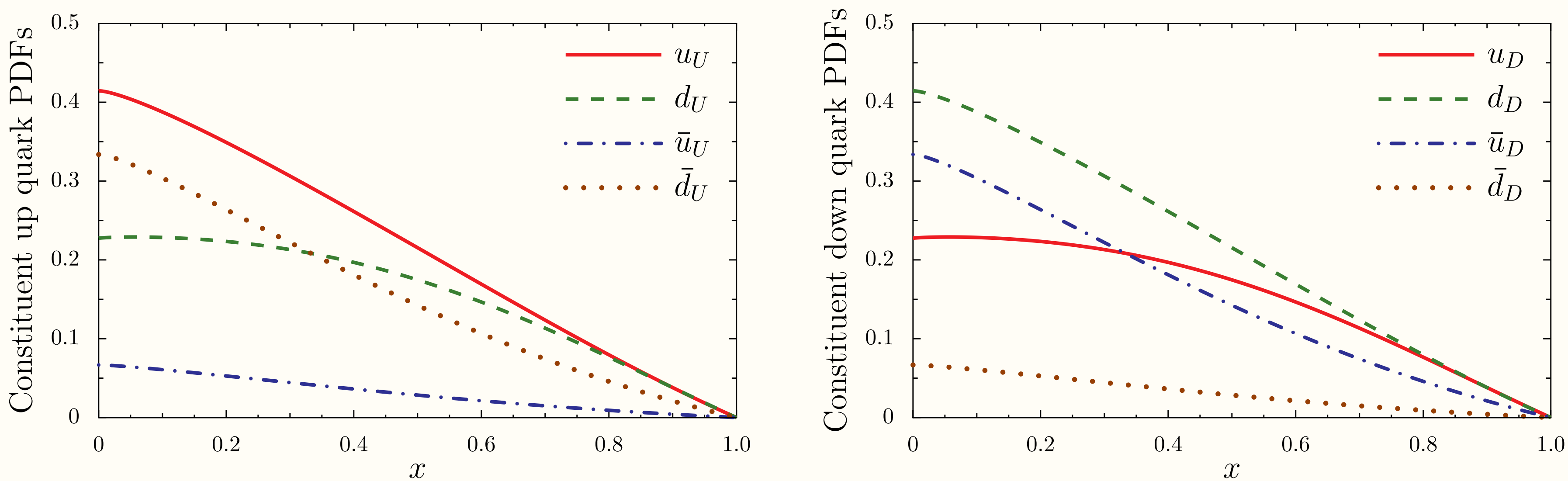
- M. Anselmino *et al*, Nucl. Phys. Proc. Suppl. **191**, 98 (2009)
- At model scale we find tensor charge

$$g_T = 1.28 \qquad \text{compared with} \qquad g_A = 1.267$$

Including Anti-quarks

- Dress quarks with pions





- Gottfried Sum Rule: NMC 1994: $S_G = 0.258 \pm 0.017 \; [Q^2 = 4\,\text{GeV}^2]$

$$S_G = \int_0^1 \frac{dx}{x} \left[F_{2p}(x) - F_{2n}(x)\right] = \frac{1}{3} - \frac{2}{3} \int_0^1 dx \, \left[\bar{d}(x) - \bar{u}(x)\right]$$

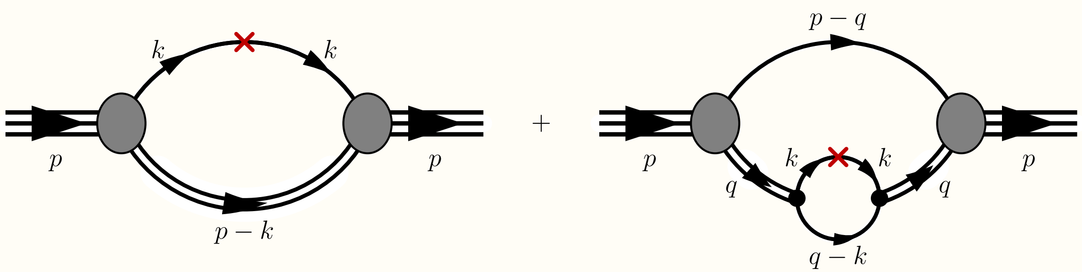
- We find: $S_G = \frac{1}{3} - \frac{4}{9}\,(1 - Z_q) = 0.252 \qquad [Z_q = 0.817]$

Spin Sum in NJL Model

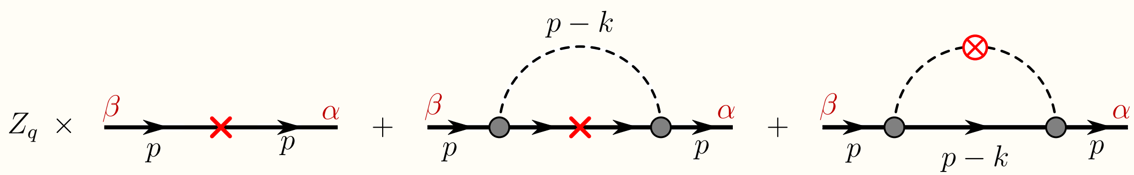
- Nucleon angular momentum must satisfy: $J = \frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$

$\Delta\Sigma = 0.33 \pm 0.03(stat.) \pm 0.05(syst.)$ [COMPASS & HERMES]

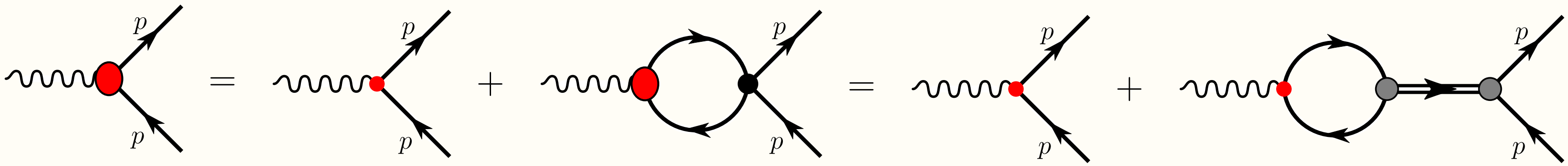
- Result from Faddeev calculation: $\Delta\Sigma = 0.66$



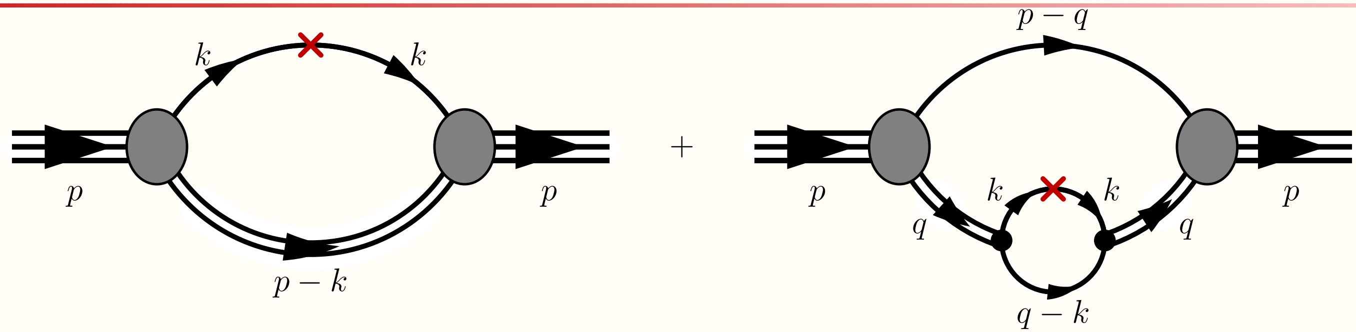
- Correction from pion cloud: $\Delta\Sigma = 0.79 \times 0.66 = 0.52$



- Bare operator $\gamma^\mu\gamma_5$ gets renormalized: $\Delta\Sigma = 0.91 \times 0.52 = 0.47$

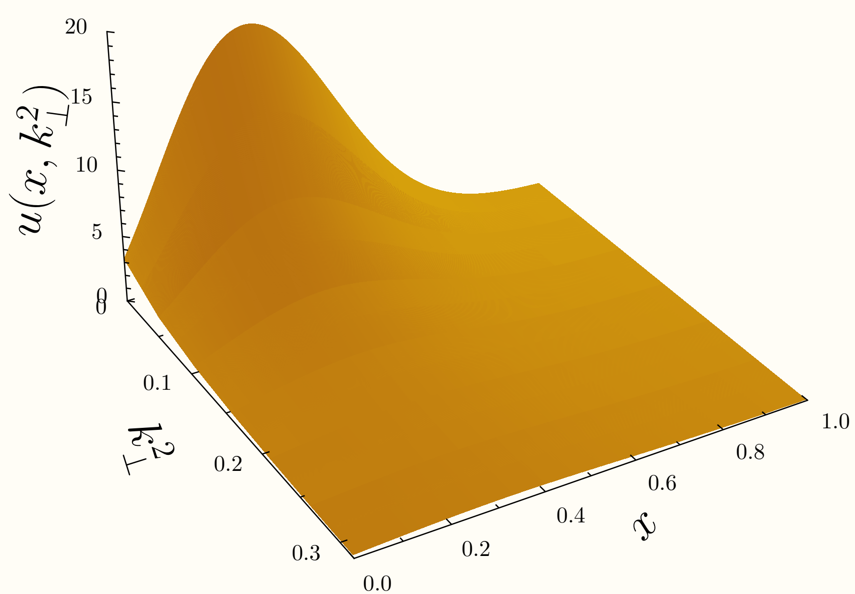


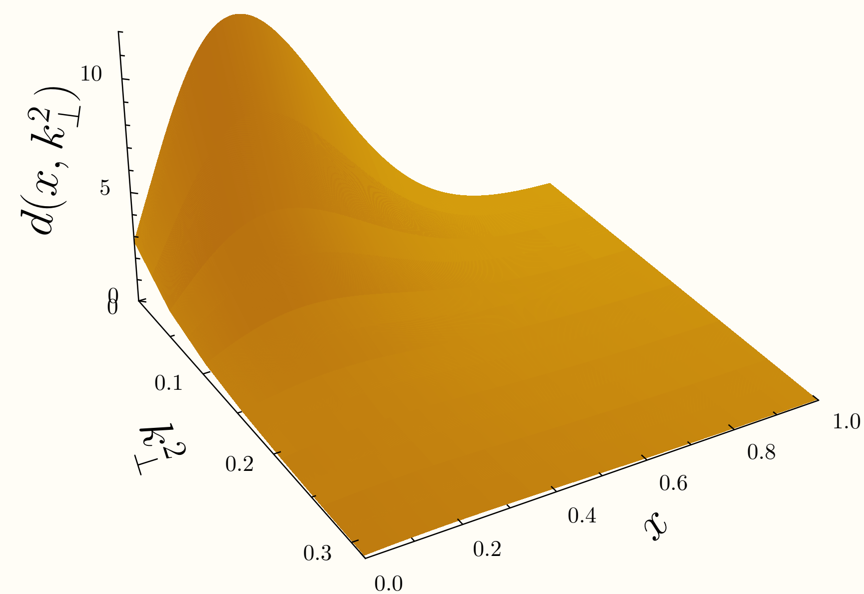
TMDs in the NJL model



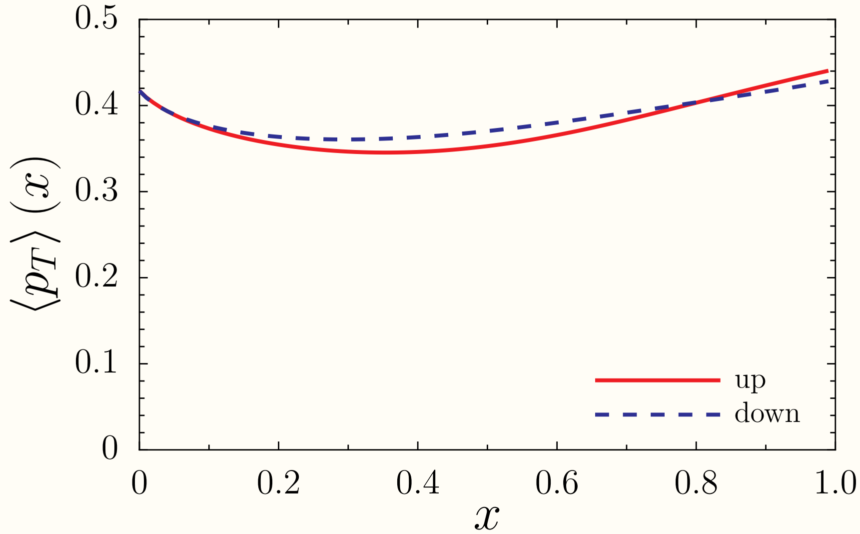
$$\Delta f_{sq}(x) = \overline{\Gamma}_N(p) \int \frac{d^4k}{(2\pi)^4}\, \delta\!\left(x - \frac{k^+}{p^+}\right) S(k)\, \gamma^+\gamma_5\, S(k)\, \tau_s(p-k)\, \Gamma_N(p)$$

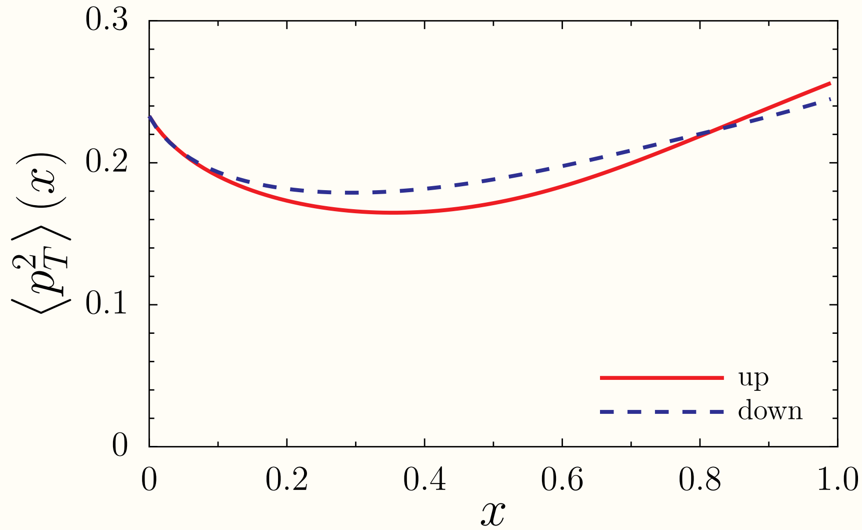
- For TMDs simply do not integrate over $\vec{k}_\perp$ – have so far $q(x, \vec{k}_\perp^{\,2})$





p_T dependence





$$\langle p_T \rangle (x) = \frac{\int d\vec{k}_\perp \, k_\perp \, q(x,k_\perp^2)}{\int d\vec{k}_\perp \, q(x,k_\perp^2)} \qquad \langle p_T^2 \rangle (x) = \frac{\int d\vec{k}_\perp \, k_\perp^2 \, q(x,k_\perp^2)}{\int d\vec{k}_\perp \, q(x,k_\perp^2)}$$

- For the average p_T we find

$$\langle p_T \rangle_u = 0.36\,\text{GeV} \qquad \langle p_T \rangle_d = 0.37\,\text{GeV}$$

- This compares with values derived from data

$$\langle p_T \rangle_{\text{Gauss}} (x) = 0.64\,\text{GeV [EMC]} \qquad \langle p_T \rangle_{\text{Gauss}} (x) = 0.56\,\text{GeV [HERMES]}$$

H. Avakian, *et al.*, Phy. Rev. D81, 074035 (2010)

The N^* (Roper) Resonance

- N^* manifests as second pole in Faddeev equation kernel
  - $M_N = 0.940$ GeV and $M_{N^*} = 1.8$ GeV
  - Agrees very well with EBAC value for quark core mass
- Vertex function is given by eigenvector at pole: $p^2 = m_i^2$
- For our NJL model N, N^* vertex function has the simple form

$$\Gamma(p) = \begin{bmatrix} \alpha_1 \\ \alpha_2 \, \frac{p^\mu}{M}\gamma_5 + \alpha_3 \, \gamma^\mu\gamma_5 \end{bmatrix} u(p)$$

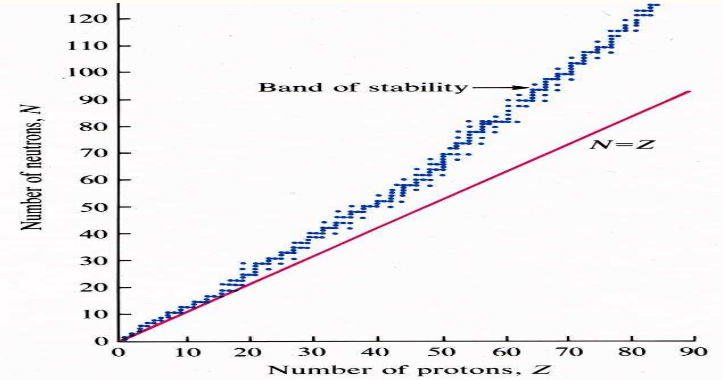
- For the nucleon: $\alpha_1 = 0.43, \quad \alpha_2 = 0.024, \quad \alpha_3 = -0.45$
- For the Roper: $\alpha_1 = 0.0011, \quad \alpha_2 = 0.94, \quad \alpha_3 = -0.051$
- N^* is completely dominated by the axial–vector diquark
- $\Delta\Sigma_N = 0.68 - 0.21 = 0.47, \qquad \Delta\Sigma_{N^*} = -0.02 + 0.01 \simeq 0.0$

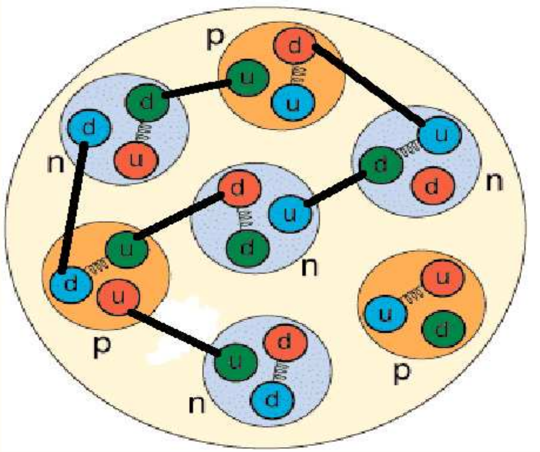
Nuclear Matter

- Finite density Lagrangian: add $\bar{q}q$ interaction in $\sigma,\ \omega,\ \rho$ channels

$$\mathcal{L} = \overline{\psi}_q \left(i\,\not{\partial} - M^* - \not{V}_q\right)\psi_q + \mathcal{L}'_I$$

- Fundamental physics: mean fields couple to the quarks in nucleons





- Finite density quark propagator

$$S(k)^{-1} = \not{k} - M - i\varepsilon \quad \rightarrow \quad S_q(k)^{-1} = \not{k} - M^* - \not{V}_q - i\varepsilon$$

- Hadronization + mean–field $\Longrightarrow$ effective potential that provides

$$V_{u(d)} = \omega_0 \pm \rho_0, \qquad \omega_0 = 6\,G_\omega\,(\rho_p + \rho_n)\,, \qquad \rho_0 = 2\,G_\rho\,(\rho_p - \rho_n)$$

  - $G_\omega \Leftrightarrow Z = N$ saturation & $G_\rho \Leftrightarrow$ symmetry energy

Finite nuclei EMC effects

- EMC ratio

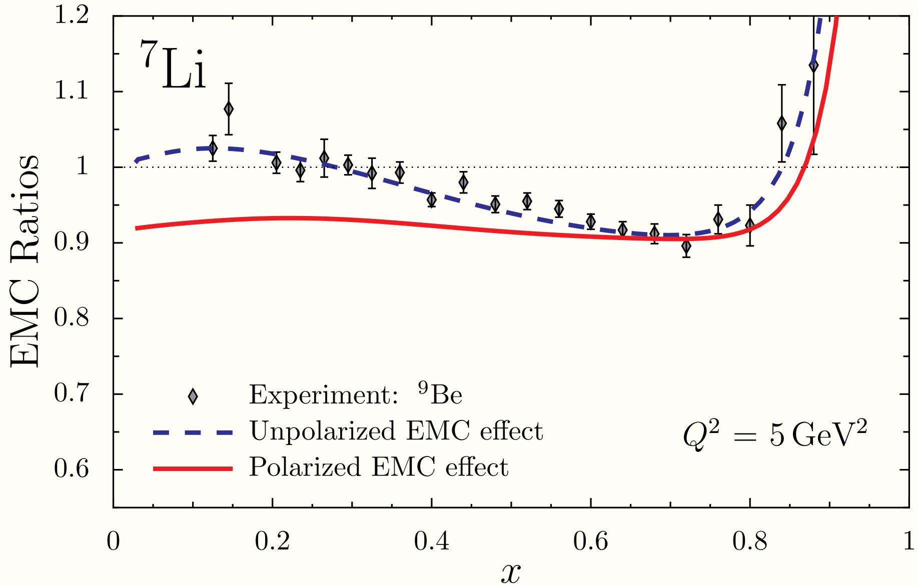
$$R = \frac{F_{2A}}{F_{2A}^{\text{naive}}} = \frac{F_{2A}}{Z\,F_{2p} + N\,F_{2n}}$$

- Polarized EMC ratio

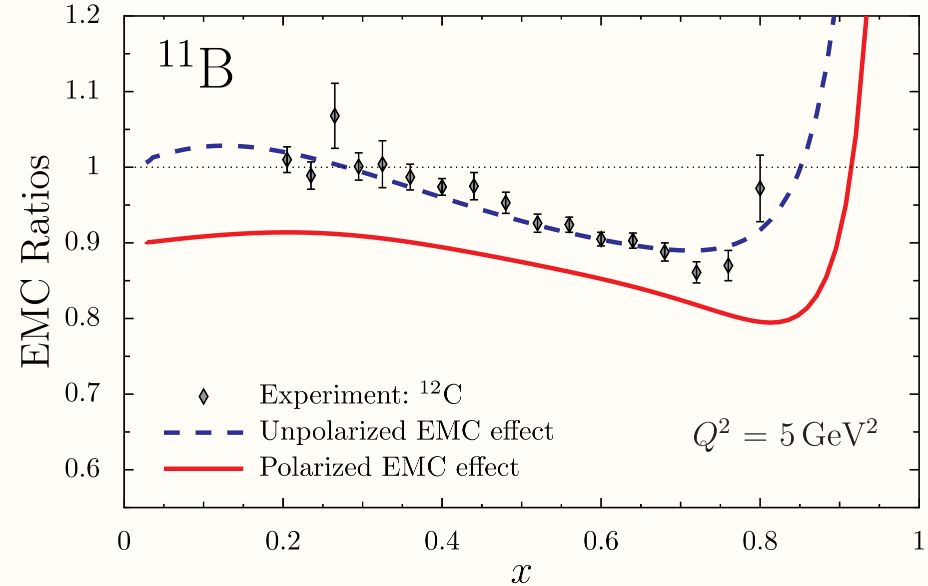
$$R_s^H = \frac{g_{1A}^H}{g_{1A}^{H,\text{naive}}} = \frac{g_{1A}^H}{P_p^H\,g_{1p} + P_n^H\,g_{1n}}$$

- Spin-dependent cross-section is suppressed by $1/A$
  - Must choose nuclei with $A \lesssim 27$
  - protons should carry most of the spin e.g. $\Longrightarrow$ ^{7}Li, ^{11}B, $\ldots$
- Ideal nucleus is probably ^{7}Li
  - From Quantum Monte–Carlo: $P_p^J = 0.86$ & $P_n^J = 0.04$
- Ratios equal 1 in non-relativistic and no-medium modification limit

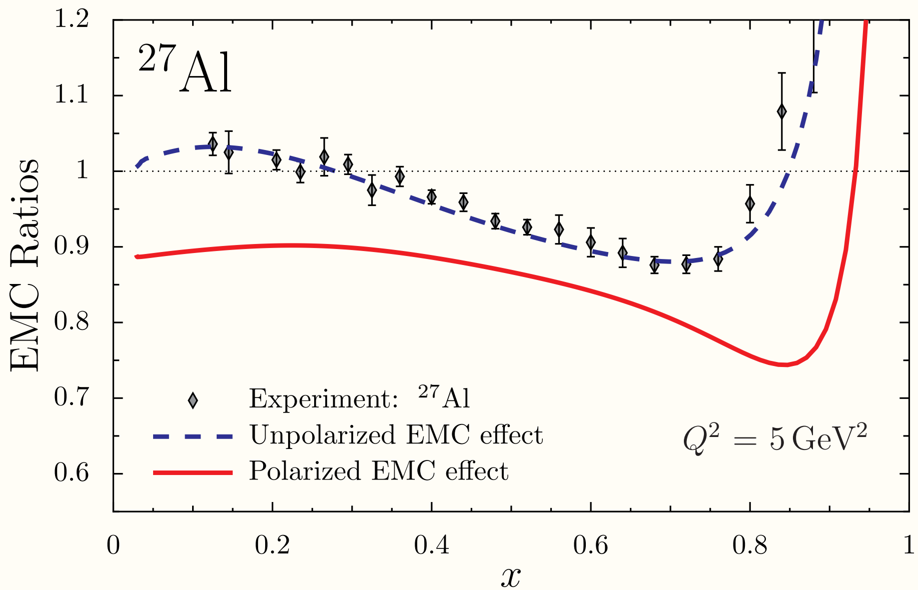
EMC ratios



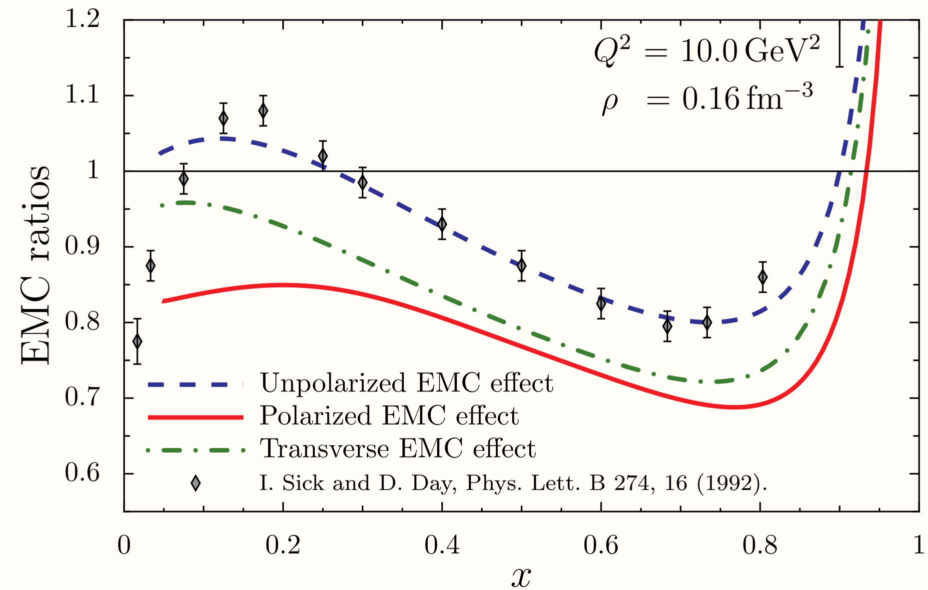
^{7}Li
EMC Ratios
Experiment: ^{9}Be
Unpolarized EMC effect
Polarized EMC effect
$Q^2 = 5\,\mathrm{GeV}^2$
x



^{11}B
EMC Ratios
Experiment: ^{12}C
Unpolarized EMC effect
Polarized EMC effect
$Q^2 = 5\,\mathrm{GeV}^2$
x

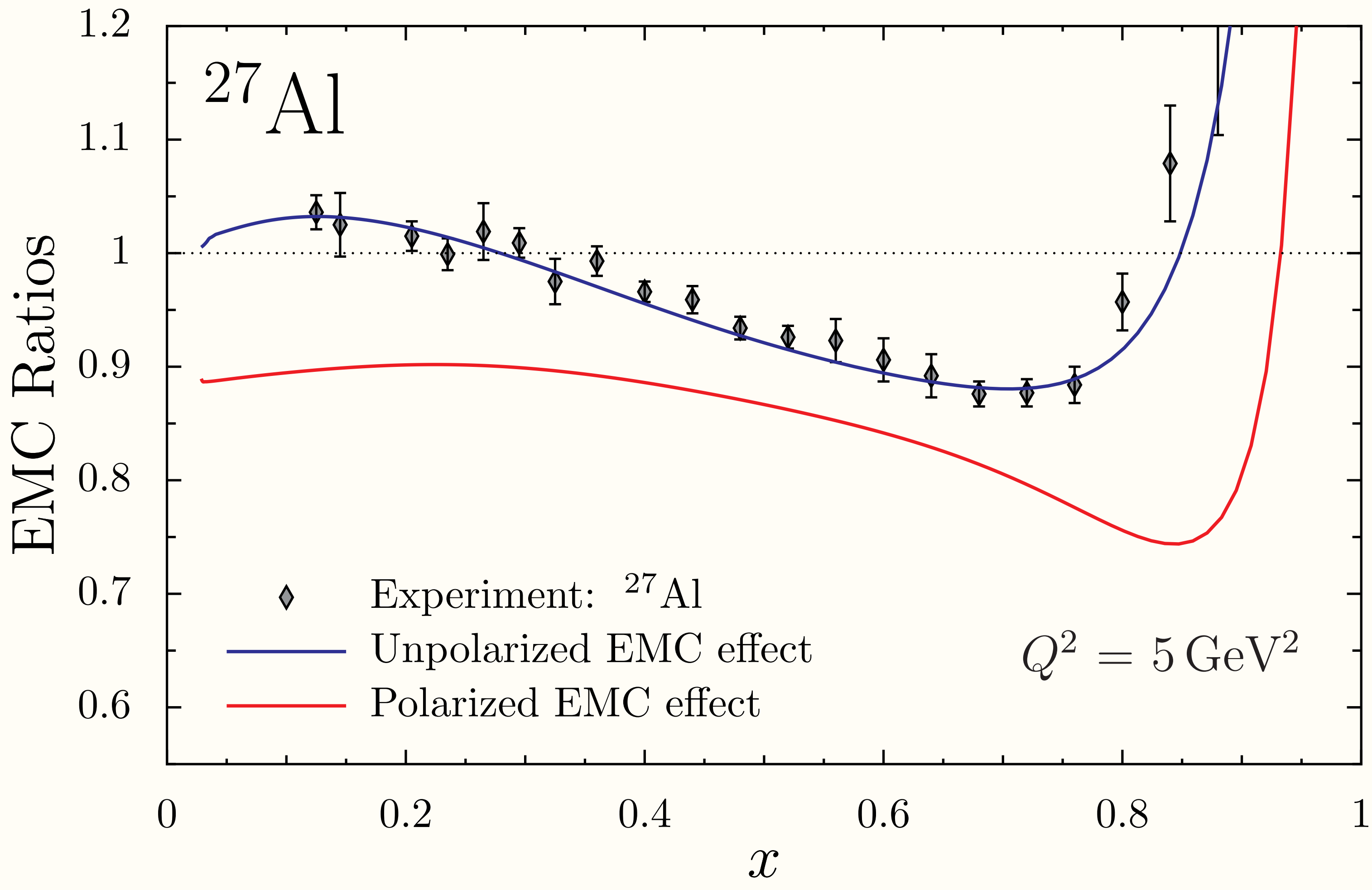


^{27}Al
EMC Ratios
Experiment: ^{27}Al
Unpolarized EMC effect
Polarized EMC effect
$Q^2 = 5\,\mathrm{GeV}^2$
x

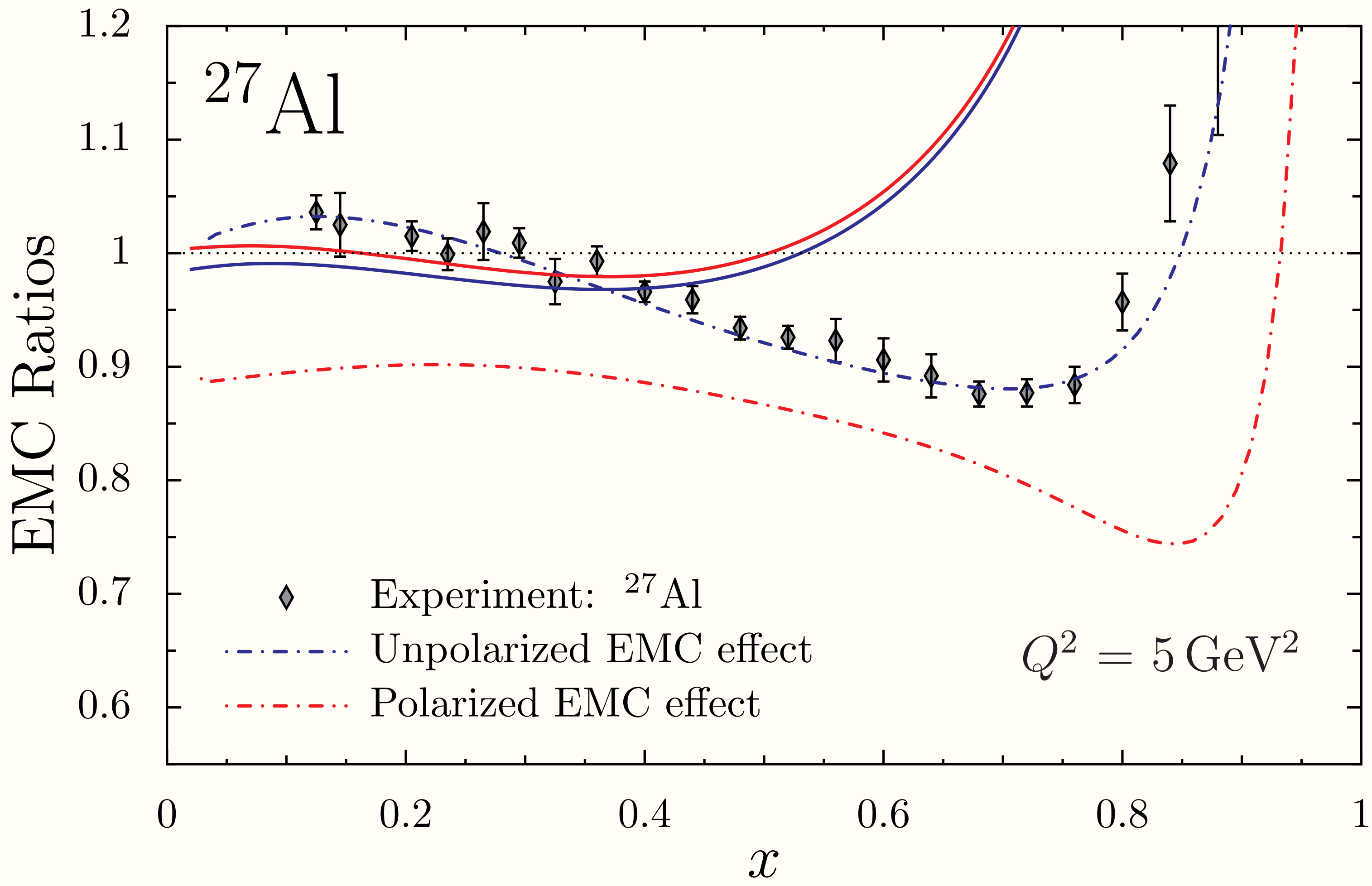


$Q^2 = 10.0\,\mathrm{GeV}^2$
$\rho = 0.16\,\mathrm{fm}^{-3}$
EMC ratios
Unpolarized EMC effect
Polarized EMC effect
Transverse EMC effect
I. Sick and D. Day, Phys. Lett. B 274, 16 (1992).
x

Is there medium modification



Is there medium modification



- Medium modification of nucleon has been switched off
- Relativistic effects remain
- Large splitting would be strong evidence for medium modification

Nuclear Spin Sum

Proton spin states	Δu	Δd	Σ	g_A
p	0.97	-0.30	0.67	1.267
^{7}Li	0.91	-0.29	0.62	1.19
^{11}B	0.88	-0.28	0.60	1.16
^{15}N	0.87	-0.28	0.59	1.15
^{27}Al	0.87	-0.28	0.59	1.15
Nuclear Matter	0.79	-0.26	0.53	1.05

- Angular momentum of nucleon: $J = \frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$
  - in medium $M^* < M$ and therefore quarks are more relativistic
  - lower components of quark wavefunctions are enhanced
  - quark lower components usually have larger angular momentum
  - $\Delta q(x)$ very sensitive to lower components
- Conclusion: quark spin → orbital angular momentum in-medium

Conclusion

- Illustrated the inclusion of quarks into a traditional description of nuclei
  - complementary approach to traditional nuclear physics
- NJL model is a useful tool to study nucleon and nuclear structure
  - covariant, confining, dynamical chiral symmetry breaking
- NJL gives a good description of Twist-2 PDFs
  - soon have results for twist 3 and 4 PDFs and TMDs
- EMC effect is interpreted as evidence for the medium modification of the bound nucleon wavefunction
  - will be tested in forthcoming experiments – PV DIS, Drell-Yan
  - NuTeV anomaly
- Polarized structure functions of nuclei are potentially interesting
  - polarized EMC effect [quark spin converted $\rightarrow L_q$ in nuclei]

Model Parameters

- Free Parameters:

  $\Lambda_{IR},\ \Lambda_{UV},\ M_0,\ G_\pi,\ G_s,\ G_a,\ G_\omega$ and G_ρ

- Constraints:
  - $f_\pi = 93$ MeV, $m_\pi = 140$ MeV & $M_N = 940$ MeV
  - $\int_0^1 dx\ (\Delta u_v(x) - \Delta d_v(x)) = g_A = 1.267$
  - $(\rho, E_B/A) = (0.16\,\text{fm}^{-3}, -15.7\ \text{MeV})$
  - $a_4 = 32$ MeV
  - $\Lambda_{IR} = 240$ MeV
- We obtain [MeV]:
  - $\Lambda_{UV} = 644$
  - $M_0 = 400,\quad M_s = 690,\quad M_a = 990,\quad \ldots$
- Can now study a very large array of observables:
  - e.g. meson and baryon: quark distributions, form factors, GPDs, finite temp. and density, neutron stars

Regularization

- Proper-time regularization

$$\frac{1}{X^n} = \frac{1}{(n-1)!} \int_0^\infty d\tau\, \tau^{n-1}\, e^{-\tau X}$$
$$\longrightarrow \quad \frac{1}{(n-1)!} \int_{1/(\Lambda_{UV})^2}^{1/(\Lambda_{IR})^2} d\tau\, \tau^{n-1}\, e^{-\tau X}$$

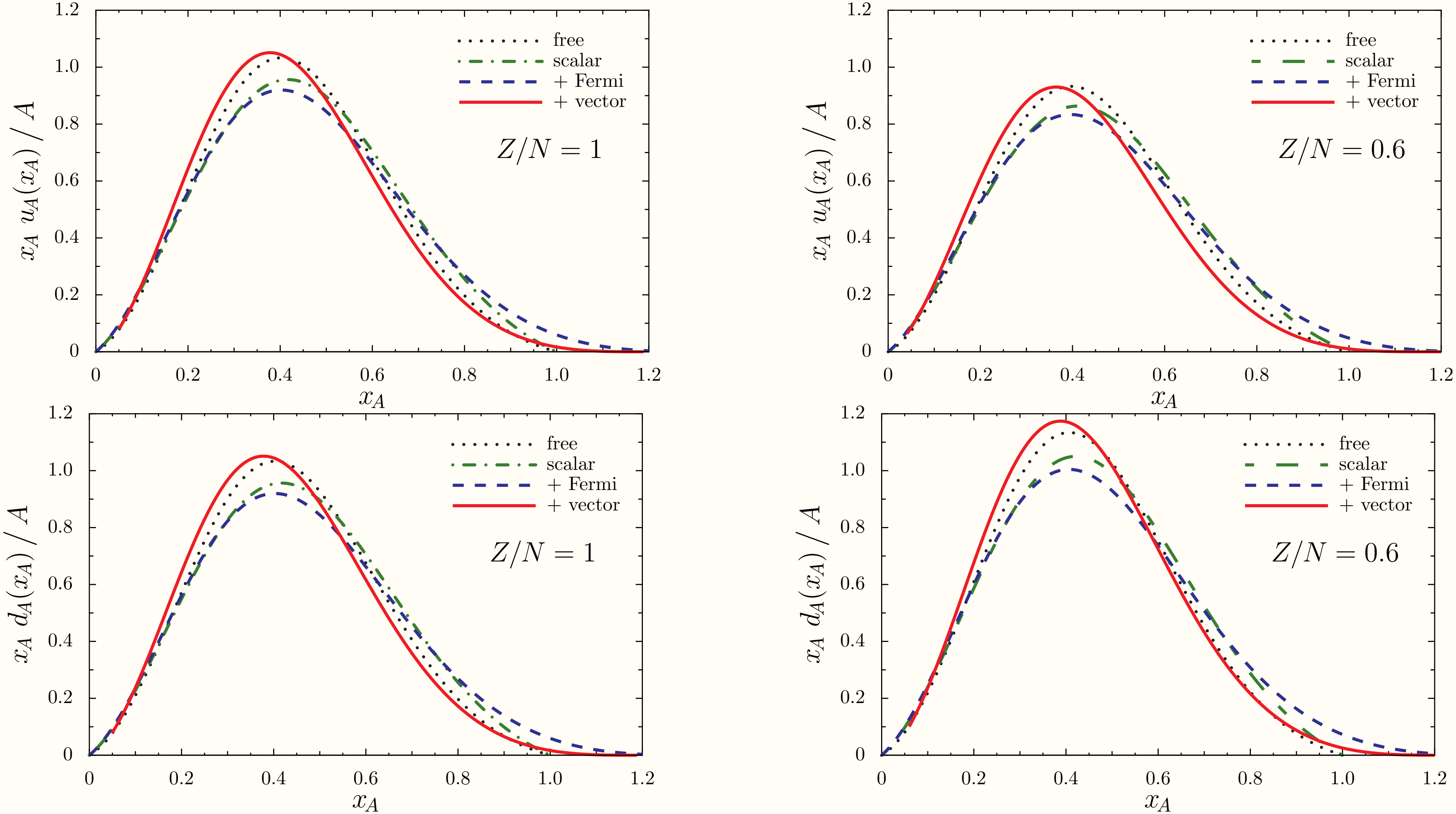
- Λ_{IR} eliminates unphysical thresholds for the nucleon to decay into quarks: $\rightarrow$ simulates confinement
  - D. Ebert, T. Feldmann and H. Reinhardt, Phys. Lett. B **388**, 154 (1996).

- E.g.: Quark wave function renormalization
  - $Z(k^2) = e^{-\Lambda_{UV}(k^2-M^2)} - e^{-\Lambda_{IR}(k^2-M^2)}$

  → $Z(k^2 = M^2) = 0 \quad \Longrightarrow \quad$ no free quarks

- Needed for: nuclear matter saturation, Δ baryon, etc
  - W. Bentz, A.W. Thomas, Nucl. Phys. A **696**, 138 (2001)

Results: Nuclear Matter



- $\rho_p + \rho_n =$ fixed – Differences arise from:
  - naive: different number protons and neutrons
  - medium: p & n Fermi motion and $V_{u(d)}$ differ $\rightarrow$ $u_p(x) \neq d_n(x), \ldots$

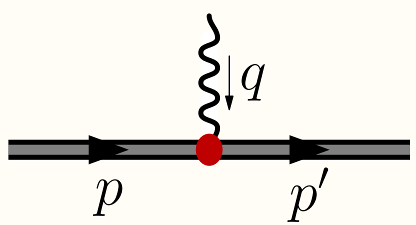
Off-Shell Effects

- For an off-shell nucleon, photon–nucleon vertex given by

$$\Gamma^\mu_N(p',p) = \sum_{\alpha,\beta=+,-} \Lambda^\alpha(p') \left[\gamma^\mu\, f_1^{\alpha\beta} + \frac{i\sigma^{\mu\nu}q_\nu}{2M}\, f_2^{\alpha\beta} + q^\mu\, f_3^{\alpha\beta}\right] \Lambda^\alpha(p)$$

- In-medium nucleon is off-shell, extremely difficult to quantify effects
  - However must understand to fully describe in-medium nucleon

- Simpler system: off-shell pion form factors
  - relax on-shell constraint $p'^2 = p^2 = m_\pi^2$
  - Very difficult to calculate in many approaches, e.g. Lattice QCD



$$= (p'+p)^\mu\, F_{\pi,1}(p'^2,p^2,Q^2) + (p'-p)^\mu\, F_{\pi,2}(p'^2,p^2,Q^2)$$

- For $p'^2 = p^2 = m_\pi^2$ we have $F_{\pi,1} \to F_\pi$ and $F_{\pi,2} = 0$