Investigating Dihadron Fragmentation Functions in the NJL-Jet Model

Andrew Casey

CSSM, University of Adelaide

22 June 2011

Outline

- Motivation
- NJL-Jet Model
  - Field-Feynman formulation
- Single hadron Fragmentation Functions
- Dihadron Fragmentation Functions
- Results
- Conclusions

Motivation

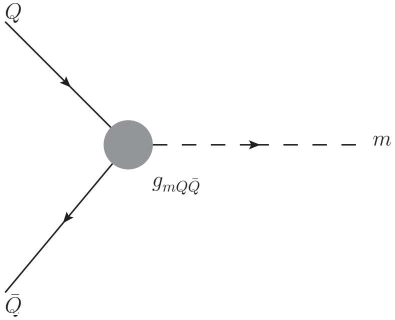
- Hadron structure is not fully understood, need as much information as possible
- Can be used to investigate the transversity of the nucleon

NJL-Jet Model

- Self-consistent framework for calculating quark distributions and fragmentation functions in an effective chiral quark theory *(T. Ito, W. Bentz, I. C. Cloët, A. W. Thomas, K. Yazaki, Phys.Rev.* **D***80 (2009) 074008)*
  - Lepage-Brodsky (LB) "invariant mass" cut-off regularization
- no ad-hoc parameters
- uses the quark jet model described by Field and Feynman *(R. D. Field and R. P. Feynman, Nucl.Phys.* **B***136 (1978) 1-76)*

Masses and Constants Used

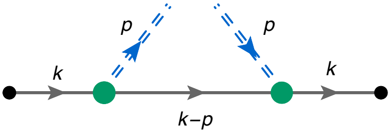
- Choose $M_u = 0.3$ GeV,
- Use decay constant $f_\pi = 0.093$, and masses $m_\pi = 0.14$ GeV and $m_K = 0.495$ GeV to obtain $M_s = 0.537$ GeV
- Coupling constants
  - $g_{\pi QQ} = 3.15$
  - $g_{KQQ} = 3.3876$



Single Hadron Fragmentation Functions

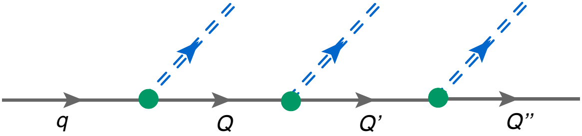
- $D_q^h(z) = \hat{d}_q^h(z) + \sum_Q \int_z^1 \frac{dy}{y} \hat{d}_q^Q(y) D_Q^m(\frac{z}{y})$
  - where $\hat{d}_q^Q(z) = \hat{d}_q^m(1-z)|_{m=q\bar{Q}}$

Single Hadron Fragmentation Functions



- $D_q^h(z) = \hat{d}_q^h(z) + \sum_Q \int_z^1 \frac{dy}{y} \hat{d}_q^Q(y) D_Q^m(\frac{z}{y})$
  - where $\hat{d}_q^Q(z) = \hat{d}_q^m(1 - z)|_{m=q\bar{Q}}$
- 1^{st} term: Driving function - corresponds to the probability of producing hadron h with momentum fraction z from the first step of the cascade.

Single Hadron Fragmentation Functions



- $D_q^h(z) = \hat{d}_q^h(z) + \sum_Q \int_z^1 \frac{dy}{y} \hat{d}_q^Q(y) D_Q^m(\frac{z}{y})$
  - where $\hat{d}_q^Q(z) = \hat{d}_q^m(1-z)|_{m=q\bar{Q}}$
- 1^{st} term: Driving function - corresponds to the probability of producing hadron h with momentum fraction z from the first step of the cascade.
- 2^{nd} term: Integral Term - corresponds to the probability of producing hadron h with momentum fraction z later in the cascade after a splitting to a quark Q with momentum fraction y.

Method

- Discretise the expression into values of z, writing $D_q^h(z)$ and $\hat{d}_q^h(z)$ into vectors and the integral term as a matrix multiplied by a vector.
- $D^m_{q\,i} = f_i + g_{ij} D^m_{q\,j} \rightarrow \vec{D}_q^m = \vec{f} + g.\vec{D}_q^m$
- Rearranging for D_q^m gives $\vec{D}_q^m = \dfrac{\vec{f}}{\mathcal{I} - g}$
- A similar method is used to solve for the Dihadron fragmentation functions.

Dihadron Fragmentation Functions

$$D_q^{h_1,h_2}(z_1,z_2) =$$
$$\hat{d}_q^{h_1}(z_1)\frac{D_{q_1}^{h_2}(\frac{z_2}{1-z_1})}{1-z_1}+\hat{d}_q^{h_2}(z_2)\frac{D_{q_2}^{h_1}(\frac{z_1}{1-z_2})}{1-z_2}+\sum_Q\int_{z_1+z_2}^1\frac{d\eta}{\eta^2}\hat{d}_q^Q(\eta)D_Q^{h_1,h_2}(\frac{z_1}{\eta},\frac{z_2}{\eta})$$

- z_1 and z_2 are the momentum fractions of hadrons h_1 and h_2, respectively.

Dihadron Fragmentation Functions

$$D_q^{h_1,h_2}(z_1,z_2) =$$

$$\hat{d}_q^{h_1}(z_1)\frac{D_{q_1}^{h_2}(\frac{z_2}{1-z_1})}{1-z_1}+\hat{d}_q^{h_2}(z_2)\frac{D_{q_2}^{h_1}(\frac{z_1}{1-z_2})}{1-z_2}+\sum_Q\int_{z_1+z_2}^1\frac{d\eta}{\eta^2}\hat{d}_q^Q(\eta)D_Q^{h_1,h_2}(\frac{z_1}{\eta},\frac{z_2}{\eta})$$

- z_1 and z_2 are the momentum fractions of hadrons h_1 and h_2, respectively.
- 1^{st} term: corresponds to the probability of producing hadron h_1 with momentum fraction z_1 from the first step of the cascade.

Dihadron Fragmentation Functions

$$D_q^{h_1,h_2}(z_1,z_2) =$$
$$\hat{d}_q^{h_1}(z_1)\frac{D_{q_1}^{h_2}(\frac{z_2}{1-z_1})}{1-z_1} + \hat{d}_q^{h_2}(z_2)\frac{D_{q_2}^{h_1}(\frac{z_1}{1-z_2})}{1-z_2} + \sum_Q \int_{z_1+z_2}^1 \frac{d\eta}{\eta^2}\hat{d}_q^Q(\eta) D_Q^{h_1,h_2}(\frac{z_1}{\eta},\frac{z_2}{\eta})$$

- z_1 and z_2 are the momentum fractions of hadrons h_1 and h_2, respectively.
- 1^{st} term: corresponds to the probability of producing hadron h_1 with momentum fraction z_1 from the first step of the cascade.
- 2^{nd} term: corresponds to the probability of producing hadron h_2 with momentum fraction z_2 from the first step of the cascade.

Dihadron Fragmentation Functions

$$D_q^{h_1,h_2}(z_1,z_2) =$$
$$\hat{d}_q^{h_1}(z_1)\frac{D_{q_1}^{h_2}(\frac{z_2}{1-z_1})}{1-z_1}+\hat{d}_q^{h_2}(z_2)\frac{D_{q_2}^{h_1}(\frac{z_1}{1-z_2})}{1-z_2}+\sum_Q\int_{z_1+z_2}^{1}\frac{d\eta}{\eta^2}\hat{d}_q^{Q}(\eta)D_Q^{h_1,h_2}(\frac{z_1}{\eta},\frac{z_2}{\eta})$$

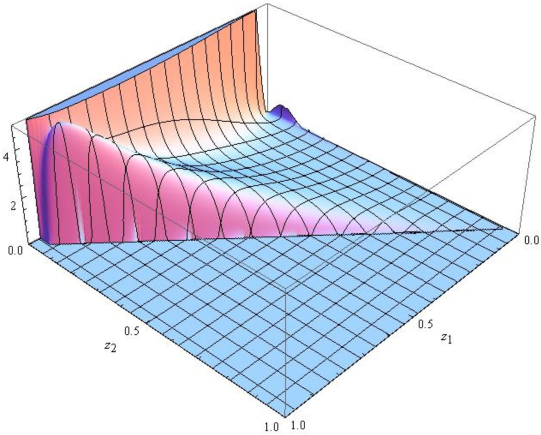
- z_1 and z_2 are the momentum fractions of hadrons h_1 and h_2, respectively.
- 1^{st} term: corresponds to the probability of producing hadron h_1 with momentum fraction z_1 from the first step of the cascade.
- 2^{nd} term: corresponds to the probability of producing hadron h_2 with momentum fraction z_2 from the first step of the cascade.
- 3^{rd} term: corresponds to the probability of producing hadron h_1 and h_2 later in the cascade after a splitting to a quark Q with momentum fraction η.

The Integral Term

- solving the integral part (3^{rd} term) numerically is difficult because of the division by η
- rearrange to remove this problem

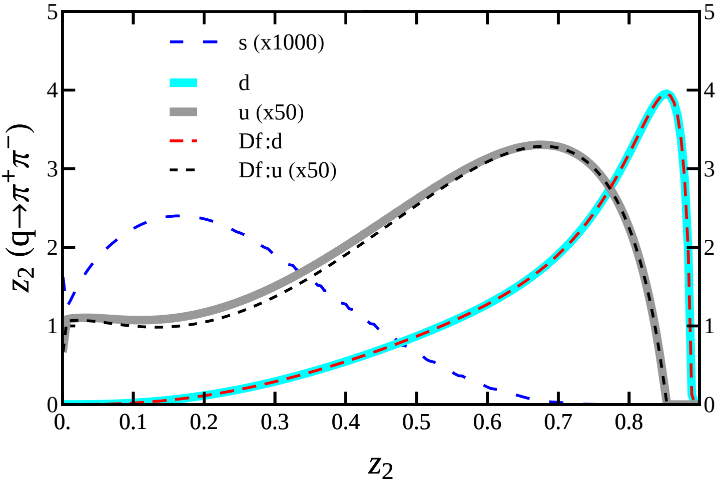
$$\begin{aligned}
& \int_{z_1+z_2}^{1} \frac{d\eta}{\eta^2} \hat{d}_q^Q(\eta) D_Q^{h_1,h_2}(\frac{z_1}{\eta}, \frac{z_2}{\eta}) \\
= & \int_{z_1}^{\frac{z_1}{z_1+z_2}} d\xi_1 \int_{z_2}^{\frac{z_2}{z_1+z_2}} d\xi_2 \int_{z_1+z_2}^{1} d\eta \delta(z_1 - \xi_1 \eta) \\
& \times \delta(z_2 - \xi_2 \eta) \hat{d}_q^Q(\eta) D_Q^{h_1,h_2}(\xi_1, \xi_2) \\
= & \int_{z_1}^{\frac{z_1}{z_1+z_2}} d\xi_1 \int_{z_2}^{\frac{z_2}{z_1+z_2}} d\xi_2 \delta(z_2 \xi_1 - z_1 \xi_2) \hat{d}_q^Q(z_1/\xi_1) D_Q^{h_1,h_2}(\xi_1, \xi_2)
\end{aligned}$$

Results by Casey, Matevosyan and Thomas, in preparation



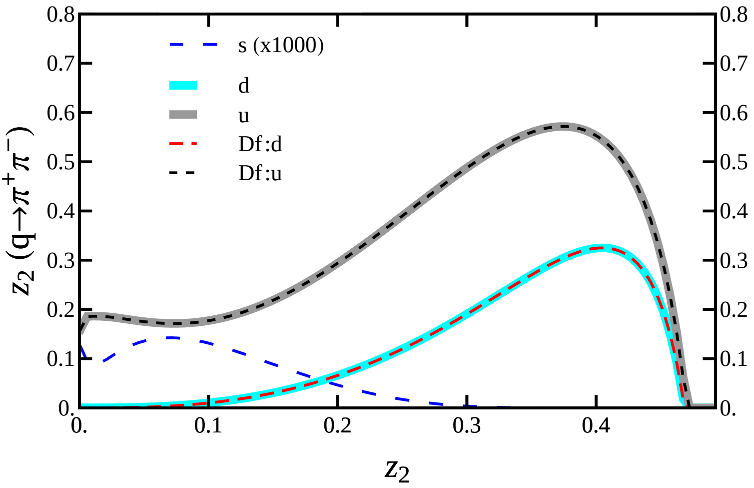
$D_u^{\pi^+\pi^-}(z_1, z_2) \qquad Q_0^2 = 0.2\mathrm{GeV}^2$

Driving Function vs. Full DFF



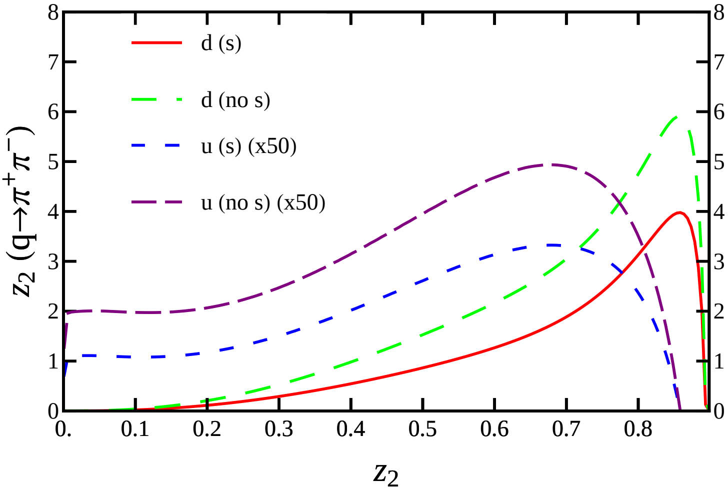
z_1=0.1 $\quad Q_0^2 = 0.2\,\text{GeV}^2$

Driving Function vs. Full DFF

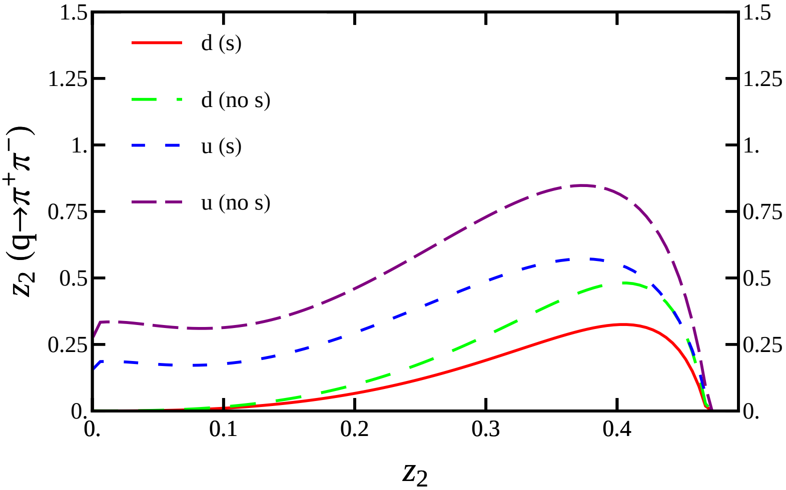


z_1=0.5 $\quad Q_0^2 = 0.2\,\text{GeV}^2$

Contribution of the Strange Quark



z_1=0.1 $\quad Q_0^2 = 0.2\,\mathrm{GeV}^2$

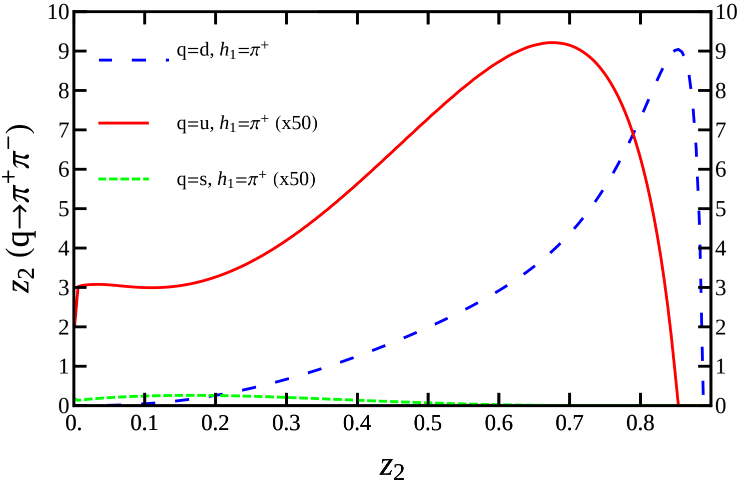


z_1=0.5 $\quad Q_0^2 = 0.2\,\mathrm{GeV}^2$

Evolution of DFF

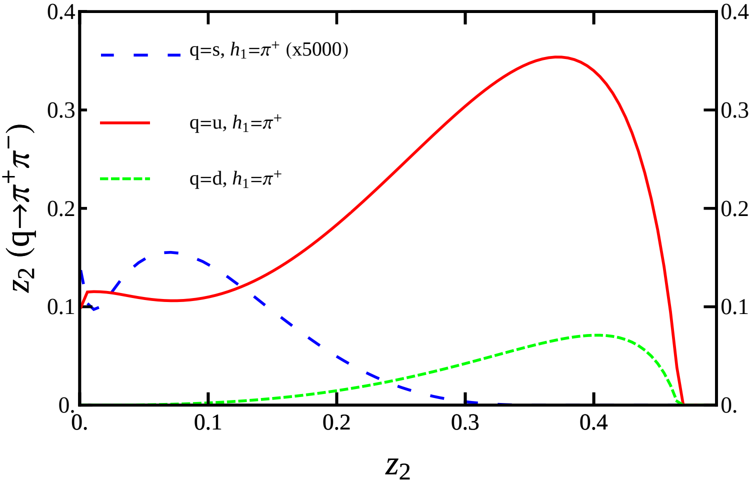
- $D_q^h(z, Q^2)$ is evolved using the QCDNUM program. *(M Botje, Comput.Phys.Commun.182:490-532,2011)*
- Ratio of $D_q^{h_1,h_2}(z_1, z_2, Q^2)/D_q^{h_1}(z_1, Q^2)$ stays relatively constant as Q^2 is increased. *(A. Majumder, Xin-Nian Wang, Phys.Rev* **D***70 (2004) 014007)*
- $$D_q^{h_1,h_2}(z_1, z_2, Q^2) \approx D_q^{h_1}(z_1, Q^2) \frac{D_q^{h_1,h_2}(z_1, z_2, Q_0^2)}{D_q^{h_1}(z_1, Q_0^2)} \tag{1}$$

Evolution of DFF



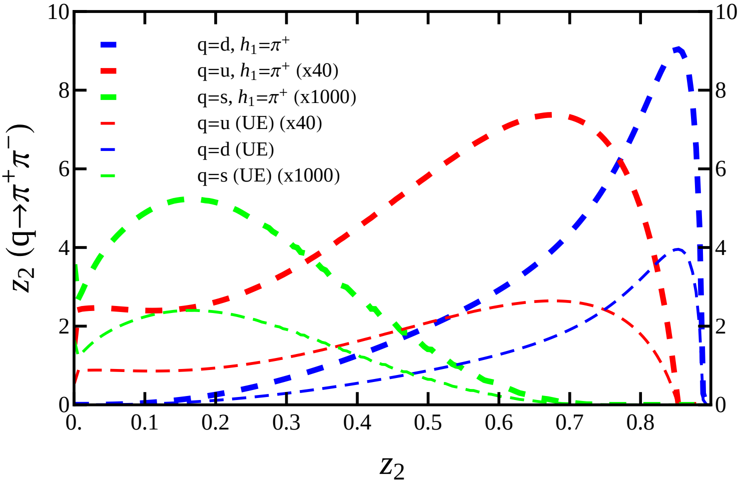
z_1=0.1 $\quad Q_0^2 = 0.2\,\text{GeV}^2 \quad Q^2 = 4\,\text{GeV}^2$

Evolution of DFF



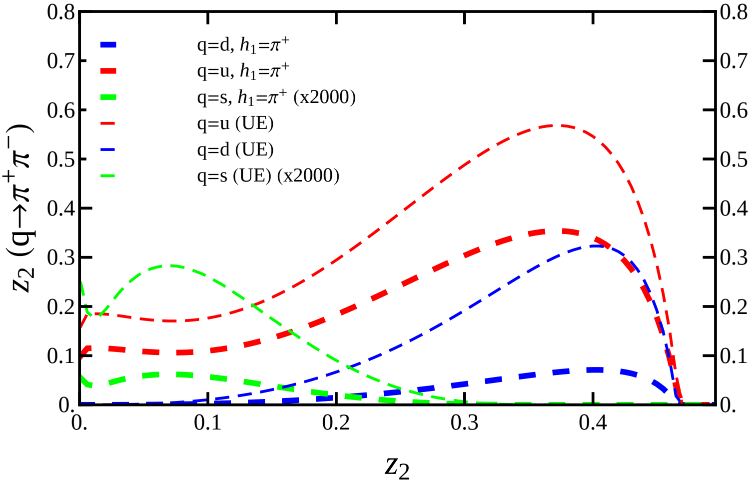
z_1=0.5 $\quad Q_0^2 = 0.2\,\text{GeV}^2 \quad Q^2 = 4\,\text{GeV}^2$

Evolved vs. Unevolved DFF's



z_1=0.1 $\quad Q_0^2 = 0.2\,\text{GeV}^2 \quad Q^2 = 4\,\text{GeV}^2$

Evolved vs. Unevolved DFF's



z_1=0.5 $\quad Q_0^2 = 0.2\,\text{GeV}^2 \quad Q^2 = 4\,\text{GeV}^2$

Conclusions

- Devised a process for obtaining Dihadron fragmentation functions.
- Obtained results for q $=$ u, d and s, for combinations of pions and kaons.
- Next Step: Compare the results obtained with those of other methods.
- Possible Future: Inclusion of transverse momentum.