Applications of symmetry breaking in determining PDFs of the nucleon

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1. Symmetries in the PDFs

    flavour symmetry, quark-antiquark symmetry

    charge symmetry

2. Symmetry breaking in the meson cloud model
3. Strange distributions of the nucleon
4. Spin dependent structure functions
5. Summary

1. Introduction

Parton Distribution Functions (PDFs) of the Nucleon

$$q(x,Q^2) = q^{\downarrow}(x,Q^2) + q^{\downarrow}(x,Q^2) \quad \text{unpolarized PDFs}$$

$$\Delta q(x,Q^2) = q^{\downarrow}(x,Q^2) - q^{\downarrow}(x,Q^2) \quad \text{ploarized PDFs}$$

$q^{\uparrow}(x), q^{\downarrow}(x)$: number densities of quarks whose spin orientation is parallel (antiparallel) to the longitudinal spin direction of the proton

x : the fractional parton momentum

- Non-perturbative inputs in higher-energy hardon processes
- The LHC will measure the PDFs in as yet unexplored kinematic regions of small x and high Q^2.
- Understanding the PDFs are important to the discovery of New Physics at the LHC.

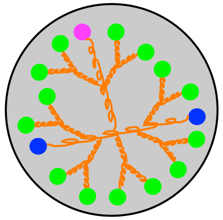
Parton Distribution Functions of the Nucleon

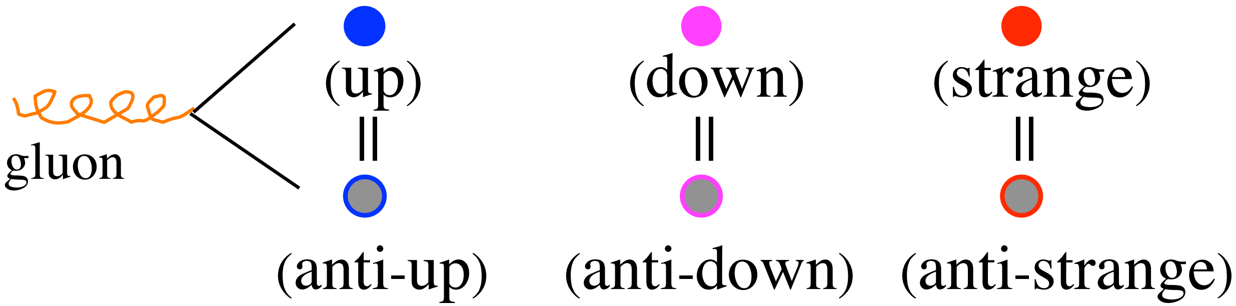
- Determined via a global fit of experimental data
  - Certain function forms are assumed at an initial scale (but not in neutral network methods)
  - QCD evolution equations give PDFs at different scales
  - Comparing theoretical calculations with experimental measurements for various processes
- Can be calculated using various quark models
- Distributions for the sea quarks are not well determined

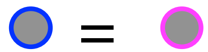
Sea quarks: $\bar{u}, \bar{d}, s, \bar{s}, c, \bar{c}, b, \bar{b}$

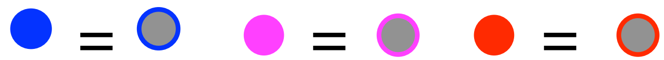
Generation of the nucleon sea

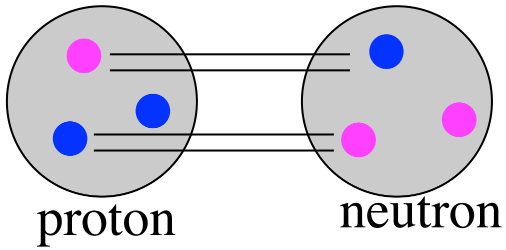
Perturbative mechanism for the nucleon sea





Flavour symmetry  $(\bar{u} = \bar{d})$

Quark-antiquark symmetry  $(q = \bar{q})$

Charge symmetry 

$$\begin{pmatrix} d^p = u^n \\ u^p = d^n \end{pmatrix}$$

Non-perturbative mechanism for the nucleon sea

→ Meson cloud model $\left|\pi^+ n\right\rangle > \left|\pi^- \Delta^{++}\right\rangle$

→ Pauli blocking

→ Chiral perturbation theory

$$u \to d\,\pi^+, \quad d \to u\pi^-$$

→ Chiral quark-soliton model

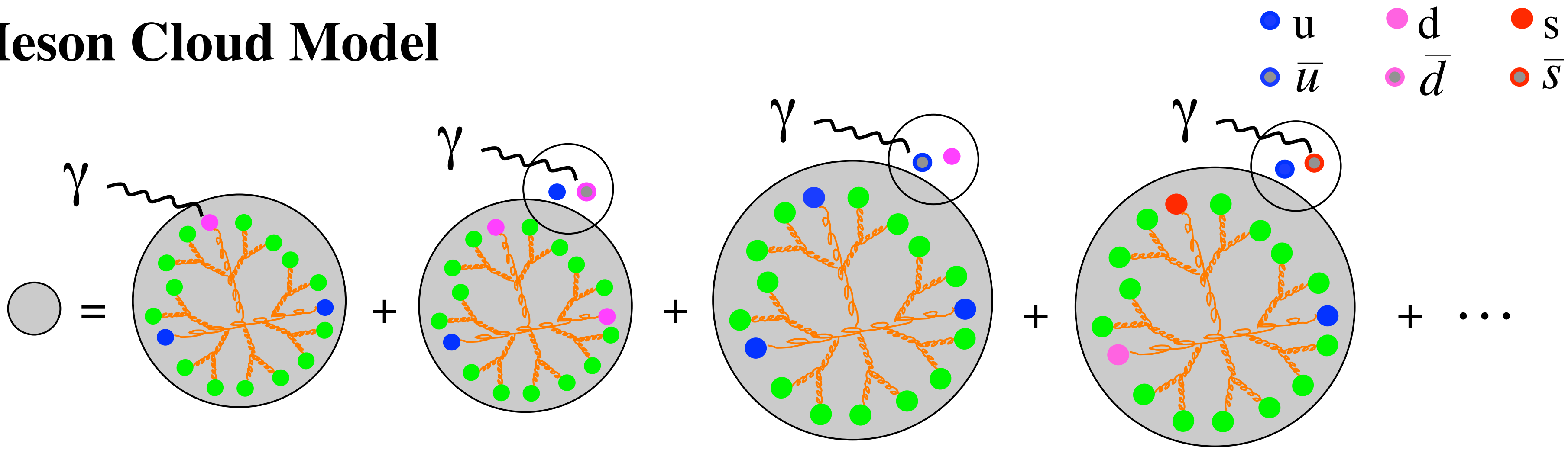
$$\bar{d} - \bar{u} = N_c f(xN_c)$$

→ Instanton model

→ Isospin breaking

2. Symmetry breaking in the meson cloud model

Meson Cloud Model



$$|p\rangle_{phys} = |p\rangle_{bare} + |n\pi^{+}\rangle + |\Delta^{++}\pi^{-}\rangle + |\Lambda K^{+}\rangle + \cdots$$

-- Fock state expansion of proton's wave function

- The photons may 'see' the anti-quarks in the mesons.
- Observed PDFs:

$$q_{phys} = q_{bare} + \delta q \quad \text{with } \delta q = \int_x^1 \frac{dy}{y} f_{BM}(y)\, q^{B(M)}\left(\frac{x}{y}\right)$$

- Each NBM vertex is described by an effective Lagrangian

$$e.g. \quad L = i\, g_{NN\pi} \overline{N} \gamma_5\, \pi\, N \quad \text{for the } NN\pi \text{ vertex}$$

- f is calculated using time-order perturbative theory (TOPT) in the infinite momentum frame

$$f_{BM}(y) = \sum_{\lambda\lambda'} \int_0^\infty dk_\perp^2 \left| \phi_{BM}^{\lambda\lambda'}(y, k_\perp^2) \right|^2, \quad \phi_{BM}^{\lambda\lambda'}(y, k_\perp^2) \propto V_{IMF}(y, k_\perp^2)\, G(y, k_\perp^2)$$

↑ Phenomenological form factor

- Prescriptions for $q^{B(M)}$

→ Bag model calculations

→ Ansatz based on lattice calculations $\int_0^1 \Delta V_\rho(x) dx = 0.6 \int_0^1 V_\rho(x) dx$

$\rightarrow \Delta V_\rho = 0.6\, V_\rho = 0.6\, V_\pi$

→ SU(3) symmetry $s^\Lambda = s^\Sigma = \frac{1}{2} u^N$

Mechanism for symmetry breaking:

Probabilities are different;

PDFs of meson and baryon are different

$$q_{phys} = q_{bare} + \delta q \quad \text{with } \delta q = \int_x^1 \frac{dy}{y} f_{BM}(y)\, q^{B(M)}\left(\frac{x}{y}\right)$$

Possible symmetry breaking:

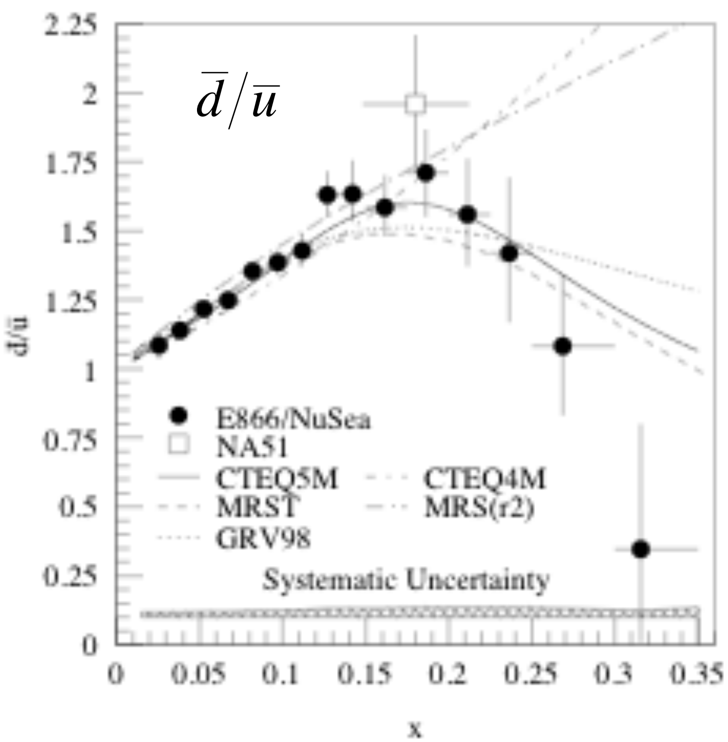
1. Flavor symmetry breaking
2. Quark-antiquark symmetry breaking
3. Charge symmetry breaking

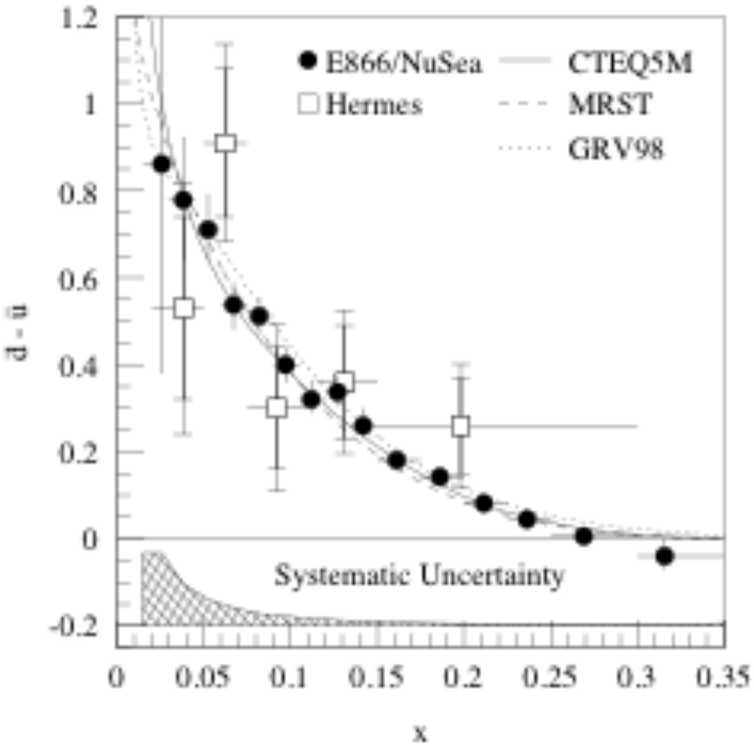
Flavor symmetry breaking

- SU(2) flavour asymmetry in the unpolarized nucleon sea is well established

$$\mathrm{Prob}(p \to n\,\pi^{+}(u\bar{d})) > \mathrm{Prob}(p \to \Delta\,\pi^{0}(\bar{u}\,d)) \Rightarrow \bar{d} > \bar{u}$$

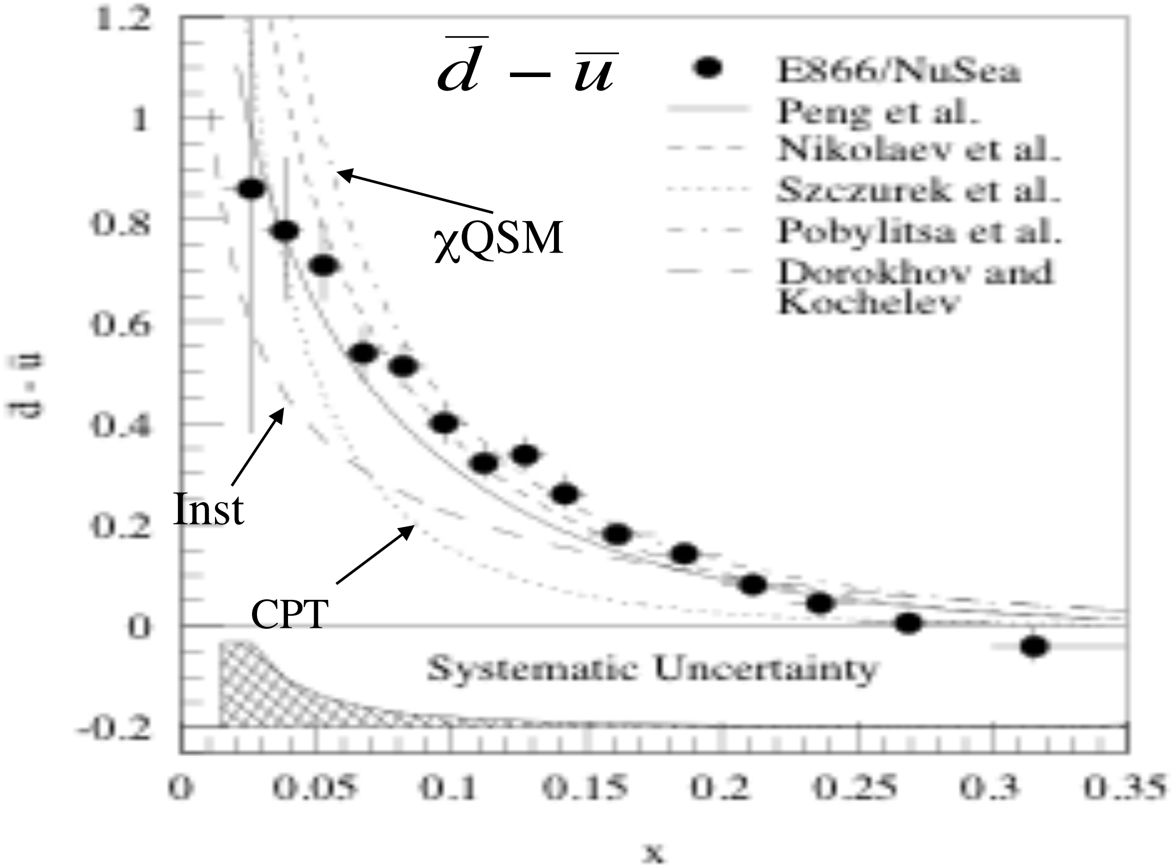
Flavour asymmetry in the unpolarized nucleon sea





Garvey&Peng, Prog.Part.Nucl.Phys. 47 (2001) 203-243

Flavour asymmetry in the unpolarized nucleon sea



Data from Phys. Rev. D64 (2001) 052002

Flavor symmetry breaking

- SU(2) flavour asymmetry in the unpolarized nucleon sea is well established

$$\mathrm{Prob}(p \to n\,\pi^{+}(u\bar{d})) > \mathrm{Prob}(p \to \Delta\,\pi^{0}(\bar{u}d)) \Rightarrow \bar{d} > \bar{u}$$

- Possible SU(2) flavour asymmetry in the polarized nucleon sea?
- The extent of SU(3) flavour symmetry breaking

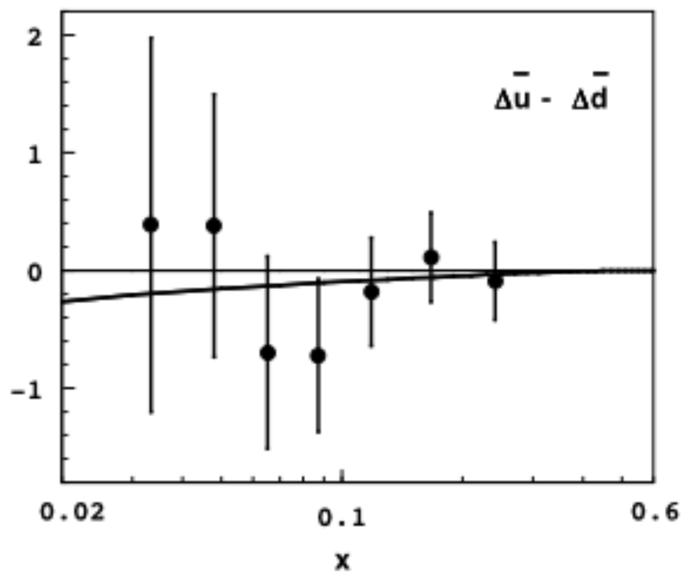
Common practice in most global QCD analyses of PDFs is

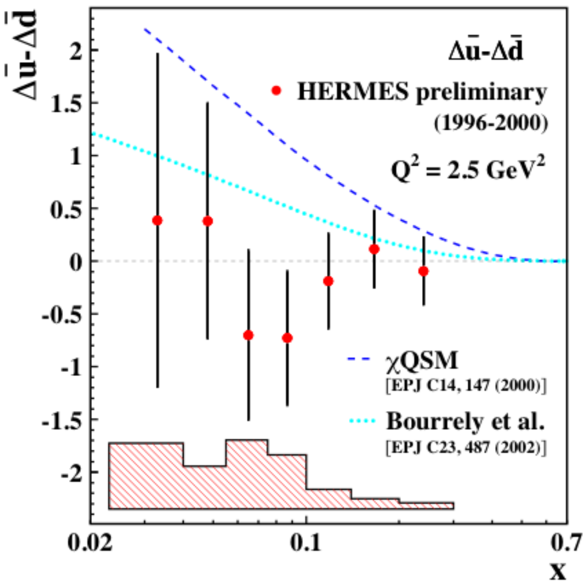
$$s(x) + \bar{s}(x) = r\left[\bar{u}(x) + \bar{d}(x)\right] \text{ with } r = 0.50 \text{ (CTEQ6.5M)}$$

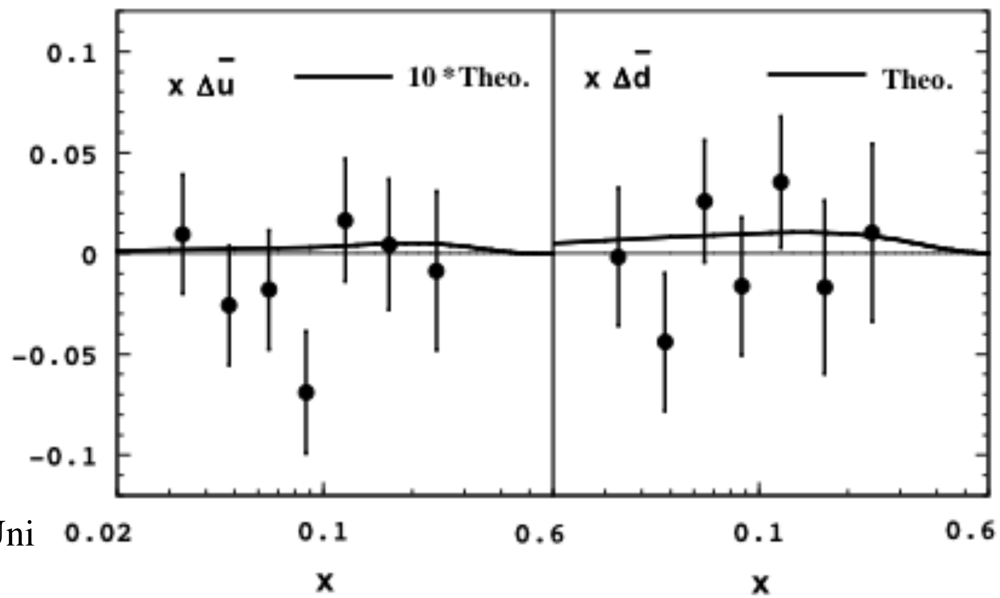
while $r = 1.0$ under SU(3) symmetry and $q - \bar{q}$ symmetry.

Direct experimental evidence for the value of r is very weak.

SU(2) flavour asymmetry in the polarized nucleon sea

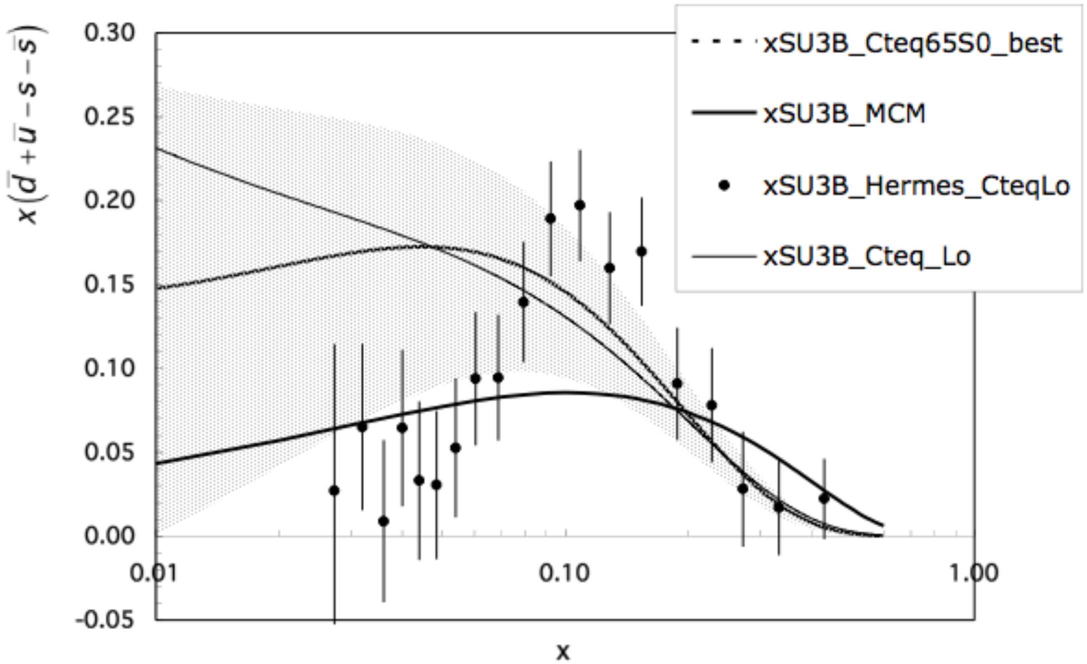






SU(3) flavour asymmetry in the unpolarized nucleon sea

$$x\Delta(x) = x\left[\bar{d}(x) + \bar{u}(x) - s(x) - \bar{s}(x)\right]$$



H. Chen, F.-G., A.I., Signal, JPG37(2010) 105006

Early refs. e.g., S. Kumano, PRD43(1991)59

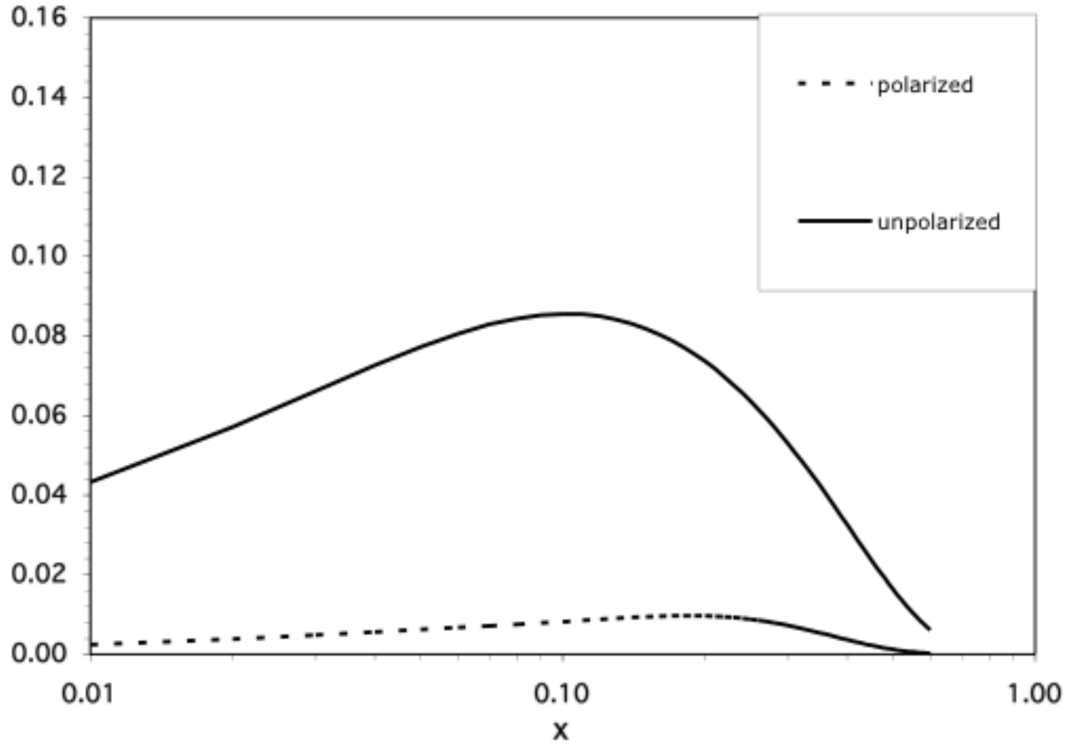
CTEQ65S [JHEP 0704 : 089,2007]:

$s(x) + \bar{s}(x)$ has different shape from $\bar{d}(x) + \bar{u}(x)$

HERMES[PLB666(2008)446 also arXiv : 0803.2993]:

a measurement of $s(x) + \bar{s}(x)$ and $\Delta s(x) + \Delta\bar{s}(x)$

SU(3) flavour asymmetry in the polarized nucleon sea



Quark-antiquark symmetry breaking

Strange-antistrange asymmetry and the measurement of $\sin^2\theta_W$

NuTeV (2002): 0.2277 ± 0.0013(stat) ± 0.0009(syst)
World Average: 0.2227 ± 0.0004

2% difference ➔ 3σ discrepancy →

The probability that it is consistent with the expected result is only about 1 in 400

NuTeV anomaly: Tony Thomas' talk on Thursday

Quark-antiquark symmetry breaking

Strange-antistrange asymmetry and the measurement of $\sin^2\theta_W$

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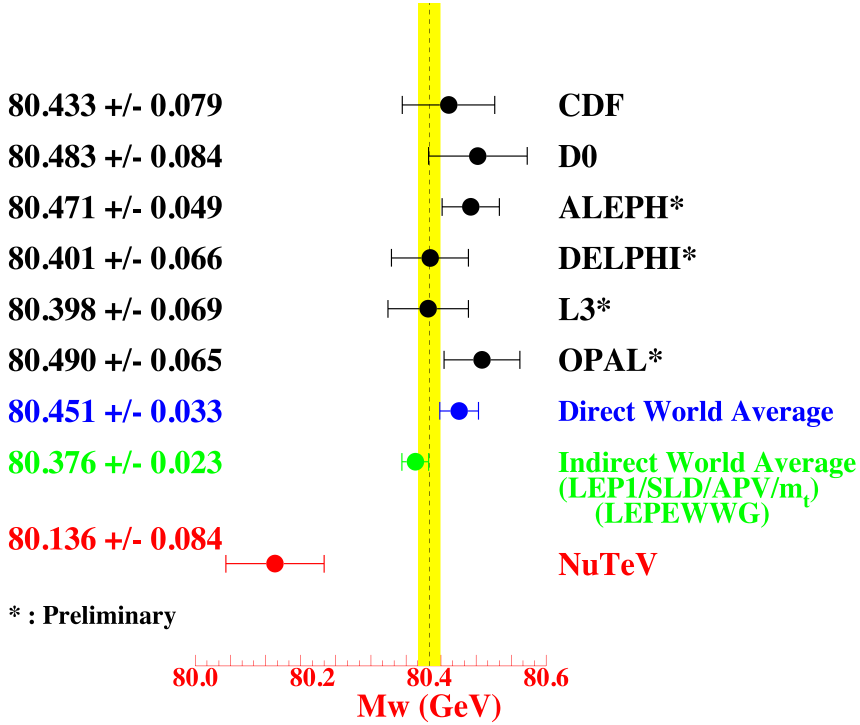


$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}$$

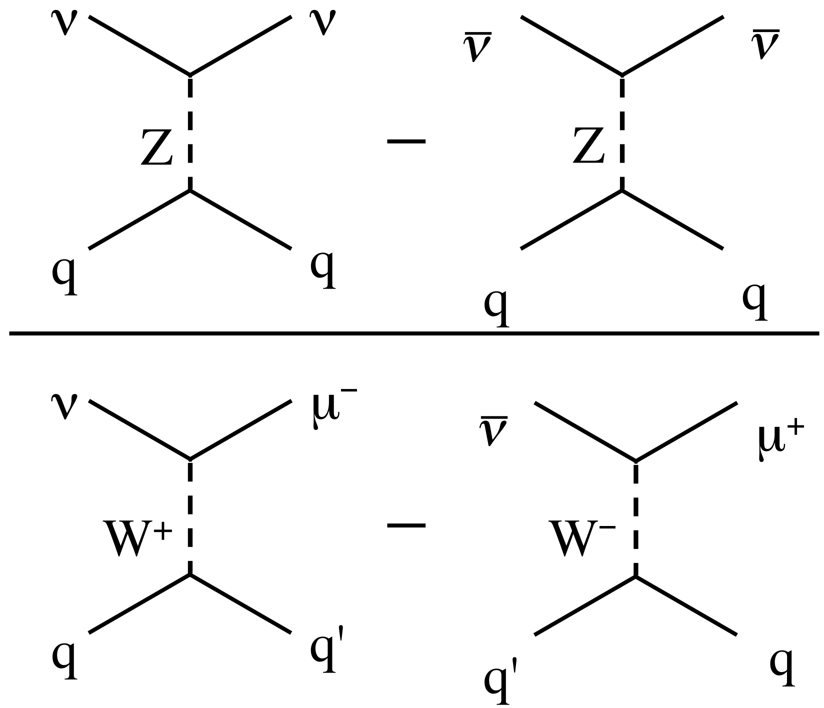
$$M_Z = 91.1882 \pm 0.0022$$

← $M_W = 80.451 \pm 0.033$

← $M_W = 80.136 \pm 0.084$

- How to extract? Paschos-Wolfenstein ratio

$$R^- = \frac{\sigma_{NC}^{\nu} - \sigma_{NC}^{\bar{\nu}}}{\sigma_{CC}^{\nu} - \sigma_{CC}^{\bar{\nu}}} = \frac{1}{2} - \sin^2\theta_w \equiv$$



- QCD corrections to the Paschos-Wolfenstein ratio

$$R^{-} = \frac{\sigma_{NC}^{\nu} - \sigma_{NC}^{\bar{\nu}}}{\sigma_{CC}^{\nu} - \sigma_{CC}^{\bar{\nu}}} = \frac{1}{2} - \sin^2\theta_W + 1.3\left[\frac{1}{2}\left(\langle \delta u \rangle - \langle \delta d \rangle\right) - \left(\langle s \rangle - \langle \bar{s} \rangle\right)\right]$$

$\delta u = u^p - d^n; \quad \delta d = d^p - u^n$ Charge symmetry breaking

$\langle s \rangle = \int_0^1 dx\, x\, s(x); \quad \langle \bar{s} \rangle = \int_0^1 dx\, x\, \bar{s}(x)$ s-sbar asymmetry

[…]=−0.0038 is needed to explain the NuTeV anomaly

- No well established experimental evidence for these symmetry breakings
- Models to break these symmetries are known, e.g. the Meson Cloud Model

Strange-antistrange asymmetry: unpolarized nucleon sea

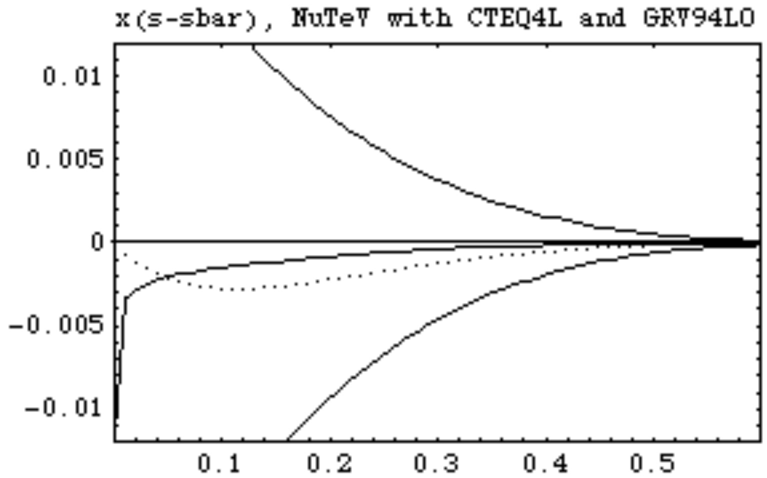
- No net strangeness: $\int_0^1 dx\, s(x) = \int_0^1 dx\, \bar{s}(x)$

- CCFR (Z. Phys. C65 (1995) 189)

$$x\, s = \kappa\, \frac{\bar{u} + \bar{d}}{2} (1 - x)^{\alpha}, \quad x\, \bar{s} = \bar{\kappa}\, \frac{\bar{u} + \bar{d}}{2} (1 - x)^{\bar{\alpha}}$$

$\Rightarrow$ No evidence for $s(x) \neq \bar{s}(x)$

- NuTeV (PRD 64 (2001) 112006)

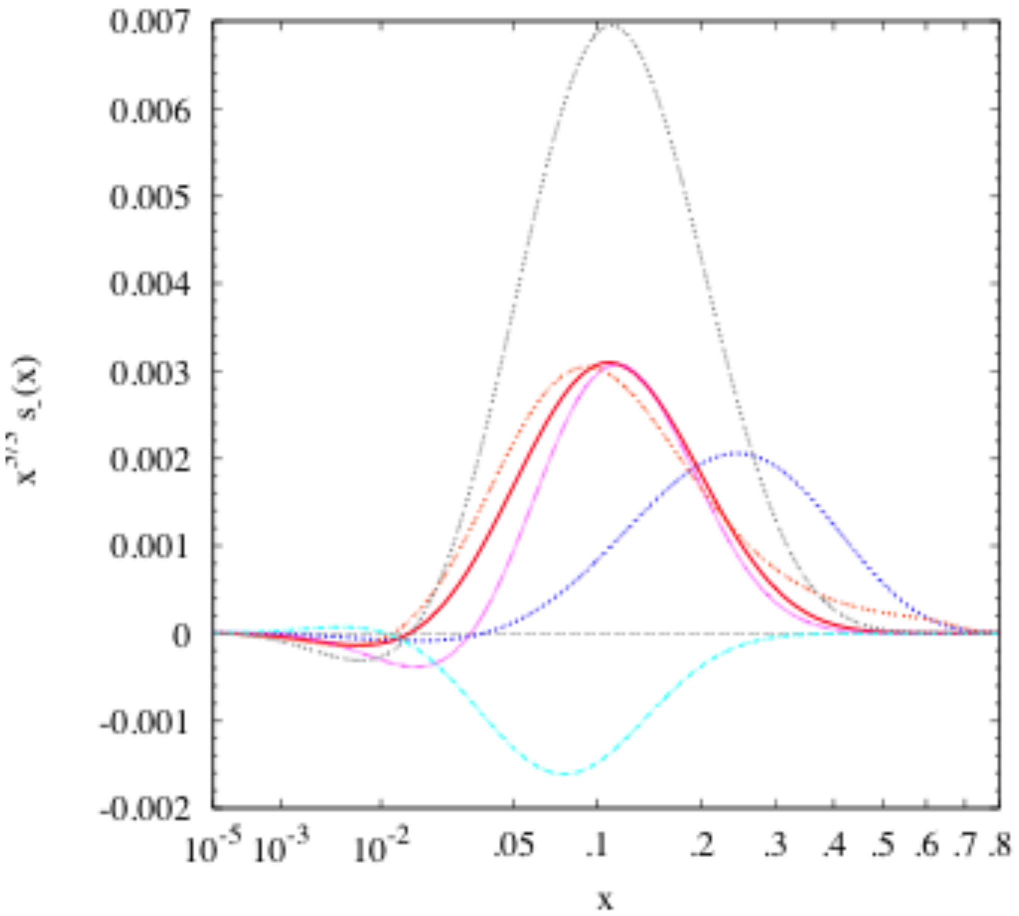


o CTEQ4L: $\langle s \rangle - \langle \bar{s} \rangle = -0.0004$

o GRV94LO: $\langle s \rangle - \langle \bar{s} \rangle = -0.0008$

- H. L. Lai et. al (CTEQ6.5S), JHEP 0704:089 (2007)

$$s_{-}(x,Q_0) = s_{+}(x,Q_0)\frac{2}{\pi}\tan^{-1}\left[cx^a(1-\tfrac{x}{b})e^{dx+ex^2}\right]$$



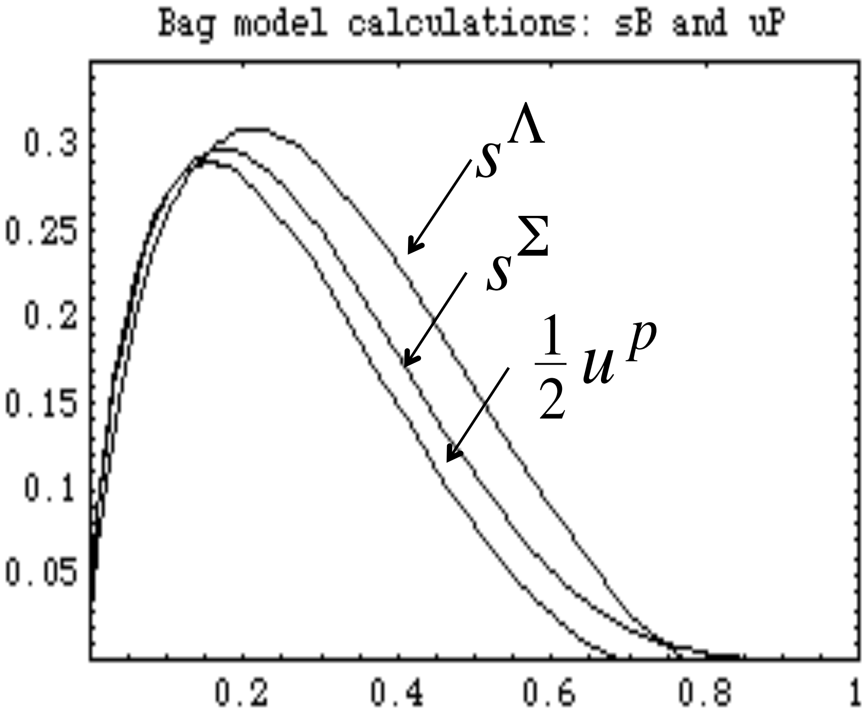
$$-0.001 < \langle x \rangle_{s_-} < 0.005$$

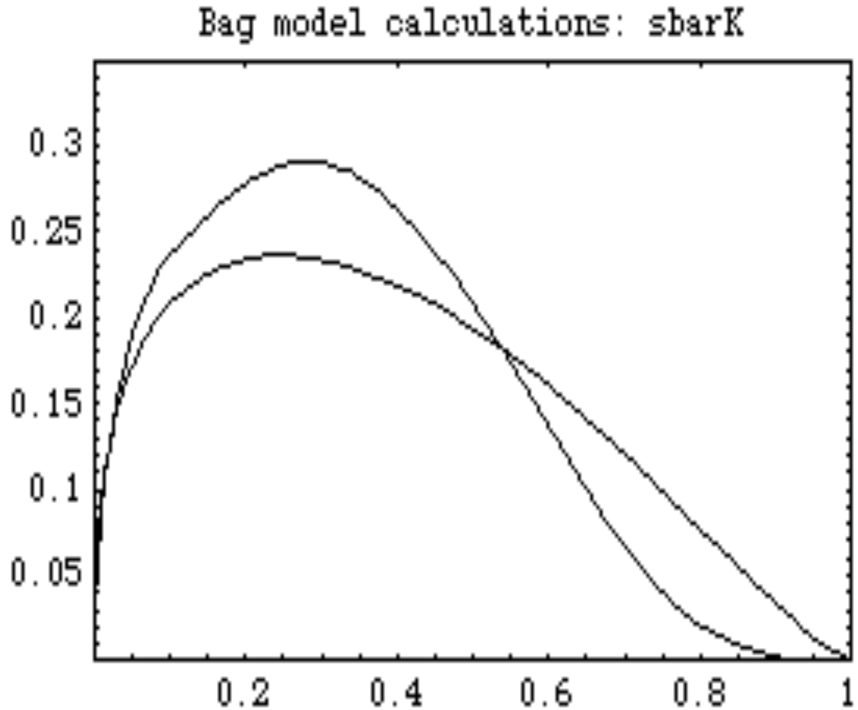
Large experimental uncertainties

Strange-antistrange asymmetry: unpolarized nucleon sea

- MCM calculation: $p \to \Lambda K; \Sigma K; \Lambda K^*; \Sigma K^*$

  Prescription for the PDFs: $s^\Lambda, s^\Sigma, \bar{s}^K$





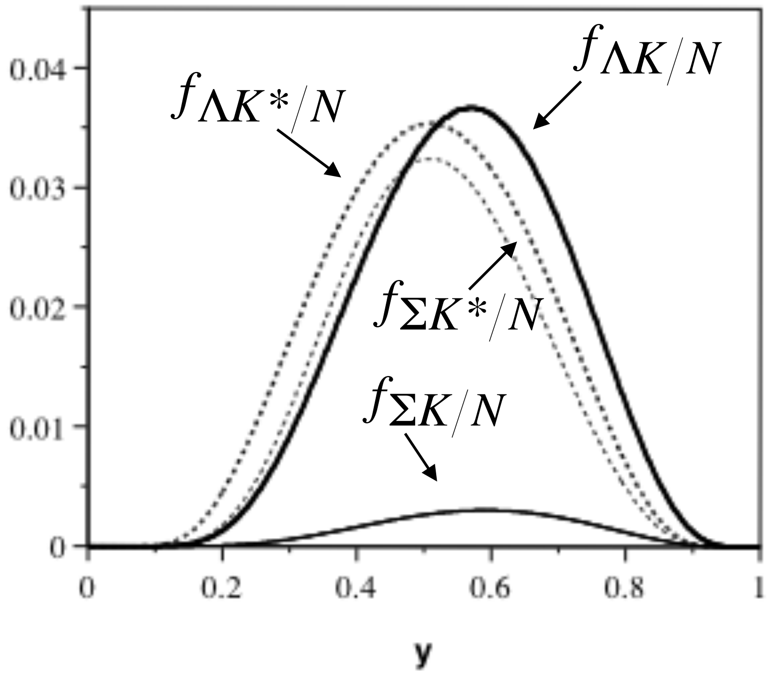
$\rightarrow$ Bag model calculations suggest $s^\Lambda \neq s^\Sigma \neq \frac{1}{2} u^N$

Strange-antistrange asymmetry: unpolarized nucleon sea

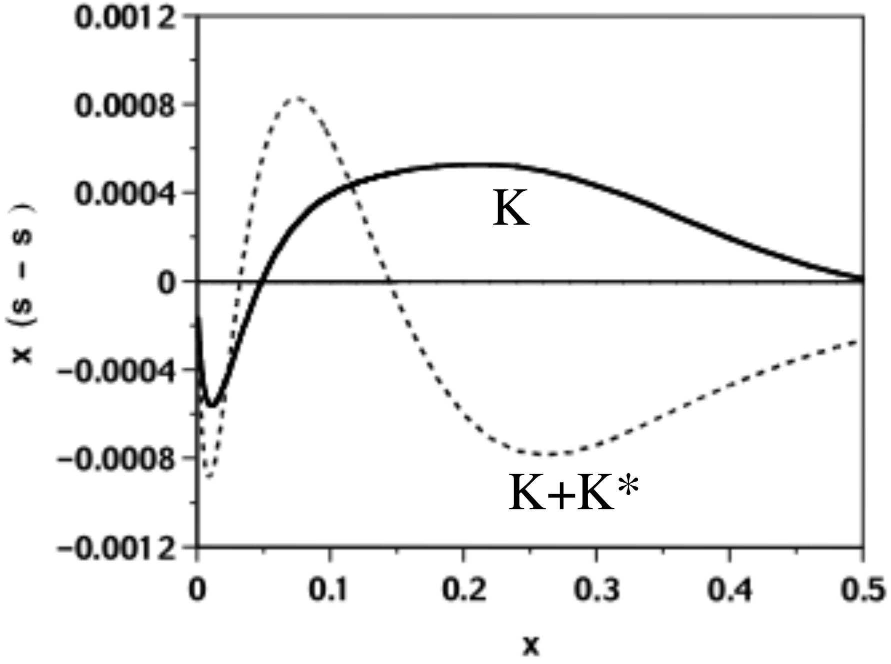
- Contributions from $p \to \Lambda K^*; \Sigma K^*$

  Expected to be suppressed due to higher mass of K*

- Fluctuation functions



Strange-antistrange asymmetry: unpolarized nucleon sea



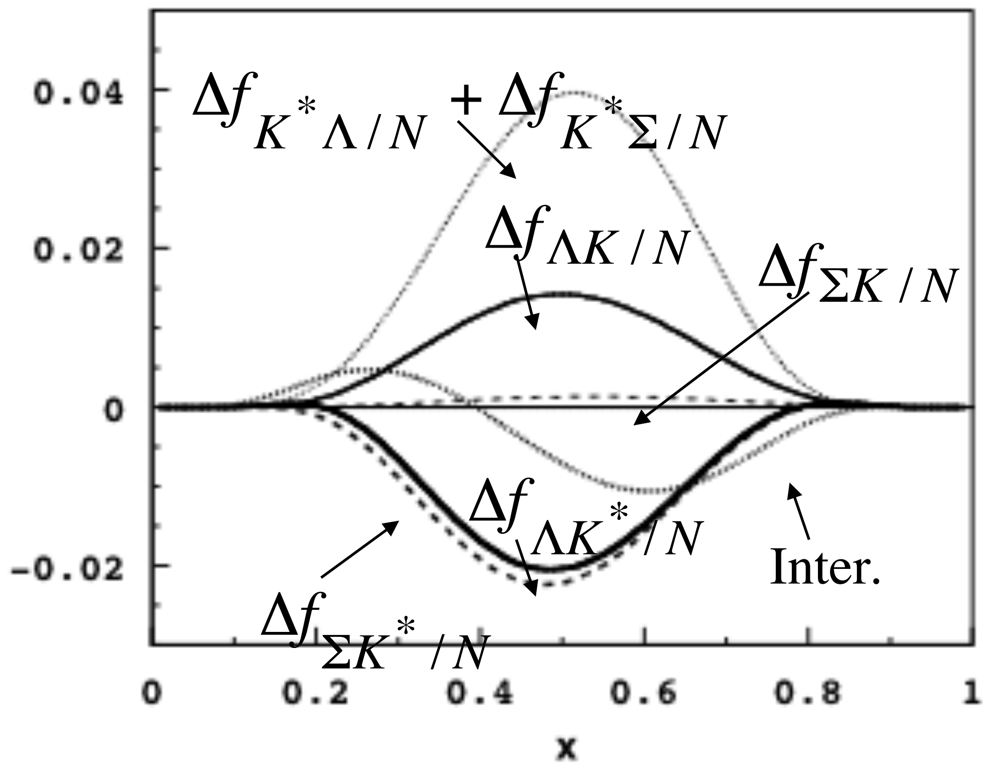
$\langle s \rangle - \langle \bar{s} \rangle = 0.00014$ including only K

$\langle s \rangle - \langle \bar{s} \rangle = -0.00014$ including K+K*

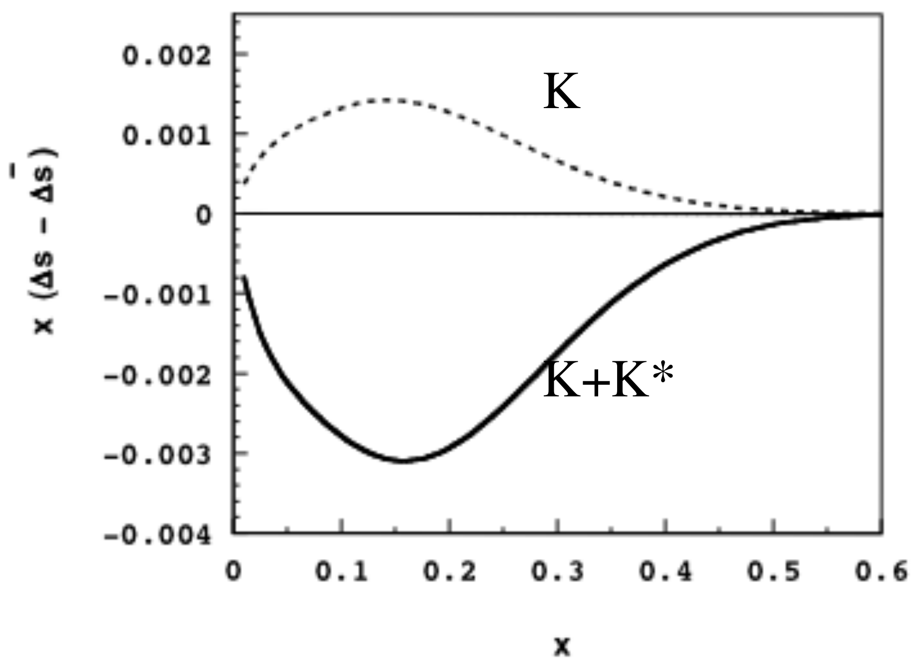
- Contributions from fluctuations involving K* are important

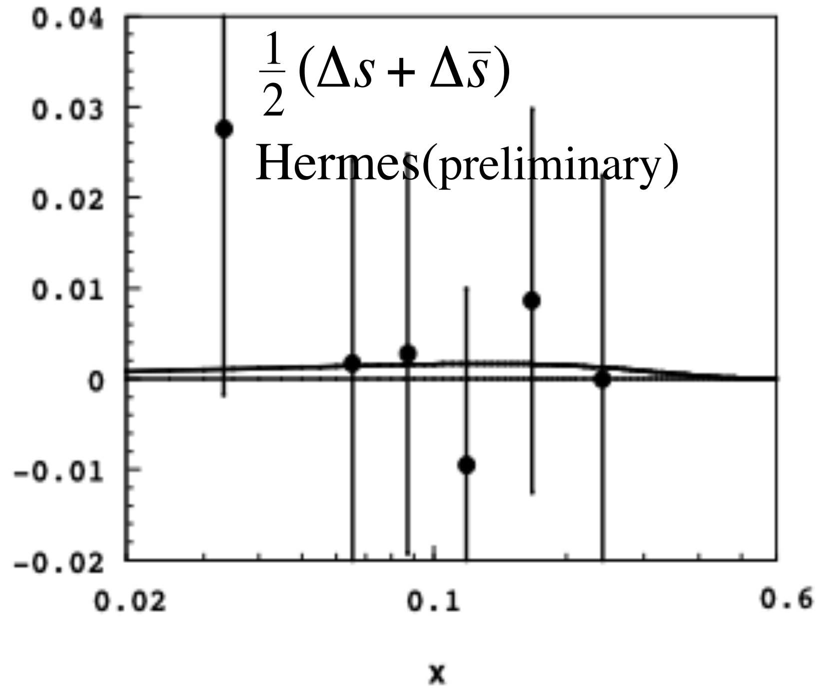
Strange-antistrange asymmetry: polarized nucleon sea

- Fluctuation functions



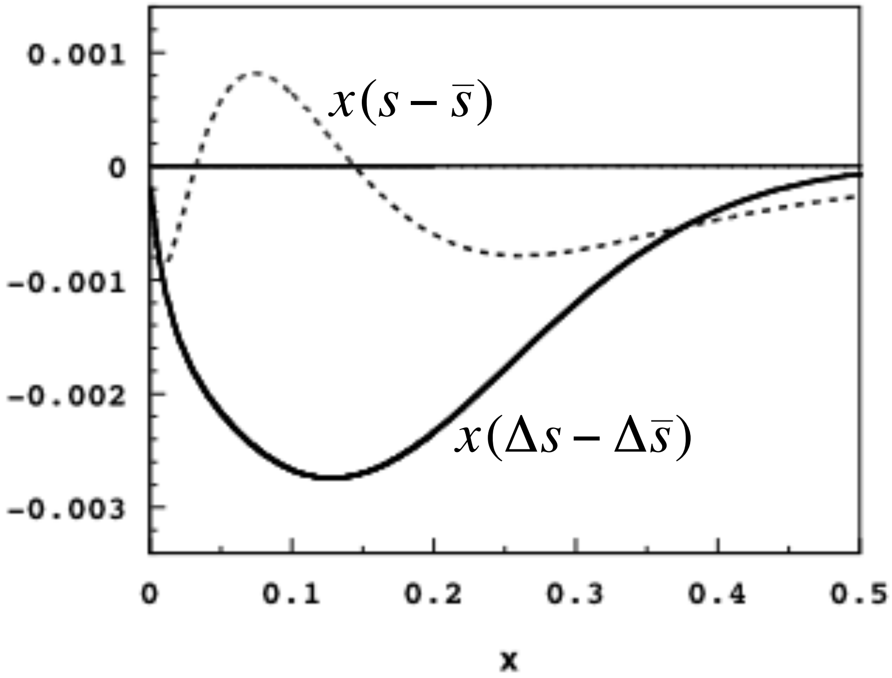
Strange-antistrange asymmetry: polarized nucleon sea





- Strange-antistrange symmetry is broken in the polarized nucleon sea.

Strange-antistrange asymmetry: polarized nucleon sea



- Strange-antistrange asymmetry is more significant in the polarized nucleon sea than that in the unpolarized nucleon sea.

Charge symmetry breaking in the PDFs

- Definitions

$$\delta u_V = u_V^p - d_V^n; \quad \delta d_V = d_V^p - u_V^n;$$

$$\delta \bar{u} = \bar{u}_V - \bar{d}_V; \ \delta \bar{d} = \bar{d}_V - \bar{u}_V; \ \delta s = s^p - s^n; \ \delta \bar{s} = \bar{s}^p - \bar{s}^n$$

Tim Londergan (next talk):
CSB in spin distributions

Charge symmetry breaking in the PDFs

- Definitions

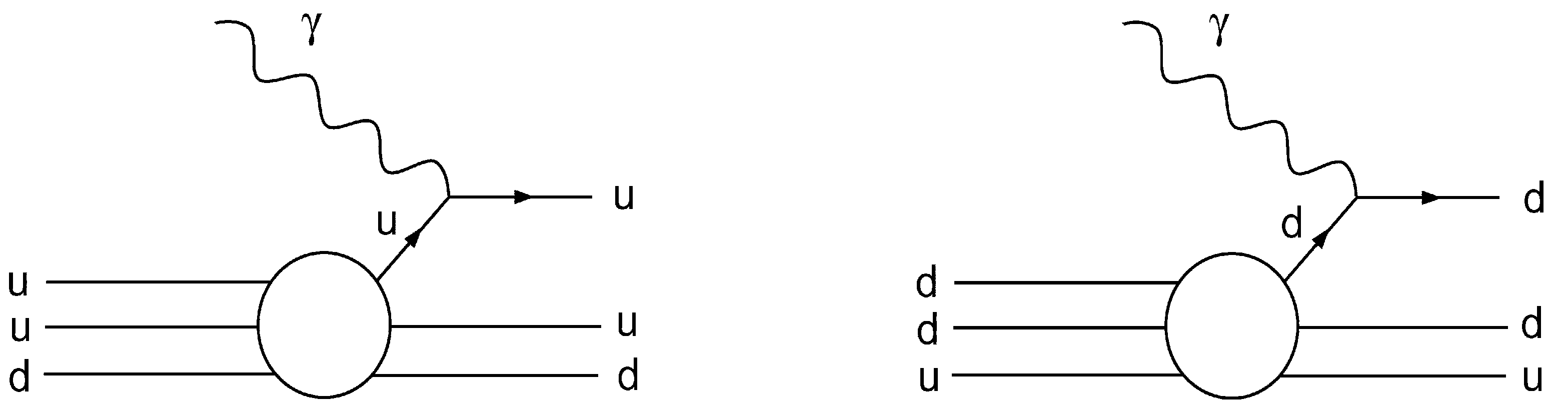
$$\delta u_V = u_V^p - d_V^n; \quad \delta d_V = d_V^p - u_V^n;$$

$$\delta \bar{u} = \bar{u}_V - \bar{d}_V; \; \delta \bar{d} = \bar{d}_V - \bar{u}_V; \; \delta s = s^p - s^n; \; \delta \bar{s} = \bar{s}^p - \bar{s}^n$$

- CS is universally assumed in the quark phenomenology
- Nuclear physics: 1%
- EW interaction: $\frac{m_d - m_u}{M} = \frac{3 \sim 5\,MeV}{0.5 \sim 1GeV} < 1\%$

- Quark model calculations:

  EW interaction; mass differences of the struck quark; mass differences of the di-quark; quark wavefunction



(Ref: Prog. Par. Nucl 41 (1998) 41; Londegran and Thomas)

- MCM calculation for the charge symmetry breaking

$$d^p = d^p_{bare} + d^p_{per.} + d^p_{non}; \quad u^n = u^n_{bare} + u^n_{per.} + u^n_{non};$$

$$\overline{d}^p = \overline{d}^p_{per.} + \overline{d}^p_{non}; \quad \overline{u}^n = \overline{u}^n_{per.} + \overline{u}^n_{non};$$

$$\Downarrow d^p_{per.} = \overline{d}^p_{per.};$$

$$d^p_V = d^p_{bare} + d^p_{non} - \overline{d}^p_{non}; \quad u^n_V = u^n_{bare} + u^n_{non} - \overline{u}^n_{non};$$

$$\delta d_V = \left[d^p_{bare} - u^n_{bare} \right] + \left[\left(d^p_{non} - \overline{d}^p_{non} \right) - \left(u^n_{non} - \overline{u}^n_{non} \right) \right];$$

$\Uparrow$ Calculated with quark models

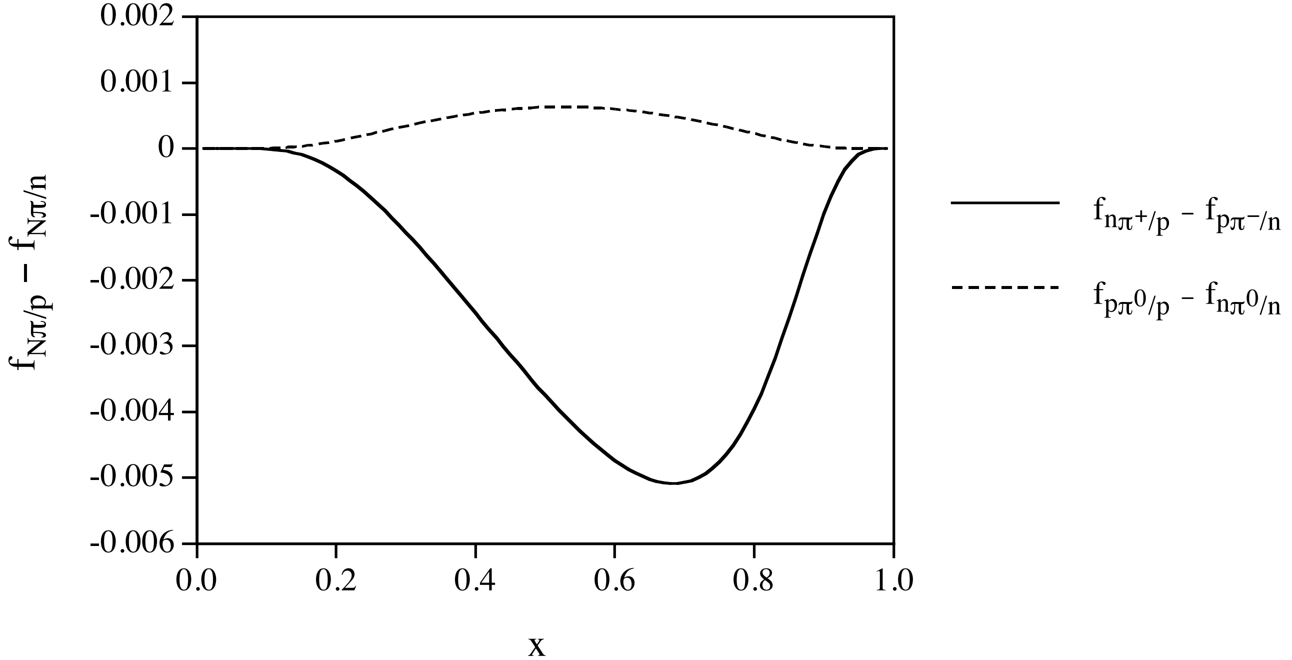
$\Uparrow$ Calculated with the Meson cloud model

- MCM calculation for the charge symmetry breaking

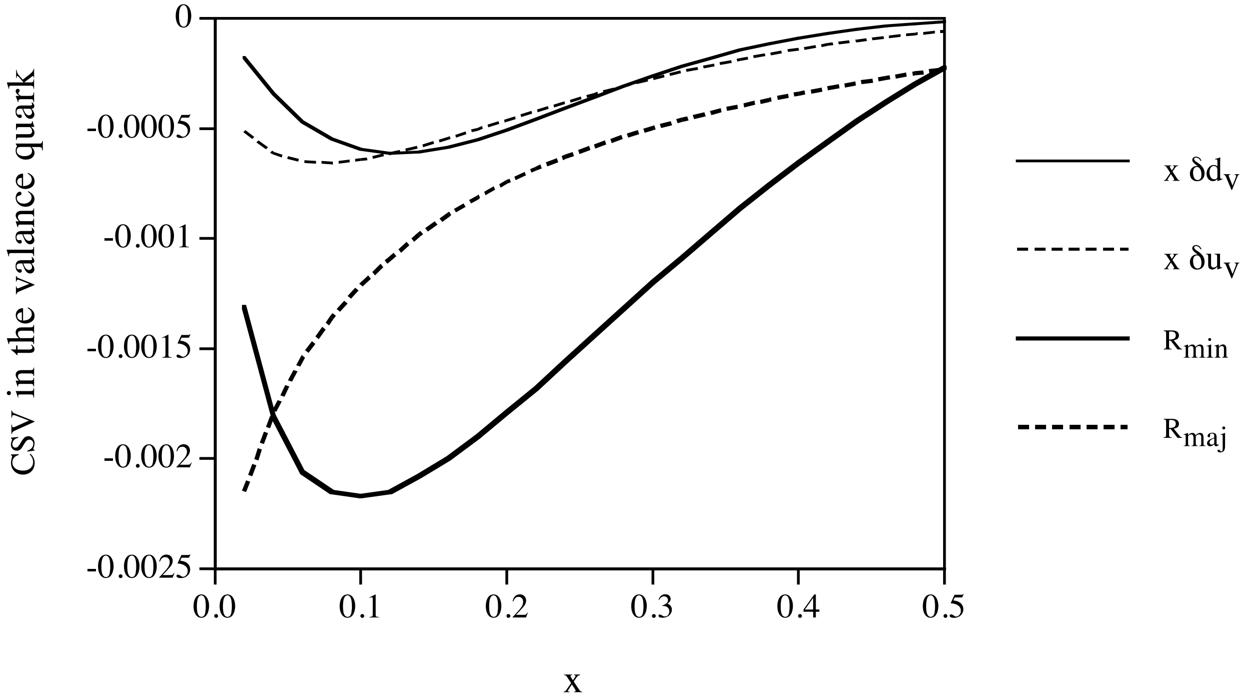
  Fluctuations considered include:

$$p \to n\,\pi^+; \qquad n \to p\,\pi^-;$$
$$p \to \Delta^0\,\pi^+; \qquad n \to \Delta^+\,\pi^-;$$
$$p \to \Lambda\,K^+; \qquad n \to \Lambda\,K^o$$

$$m_p - m_n = -1.3\,MeV,$$
$$m_{\pi^\pm} - m_{\pi^0} = 4.6\,MeV,\ etc$$



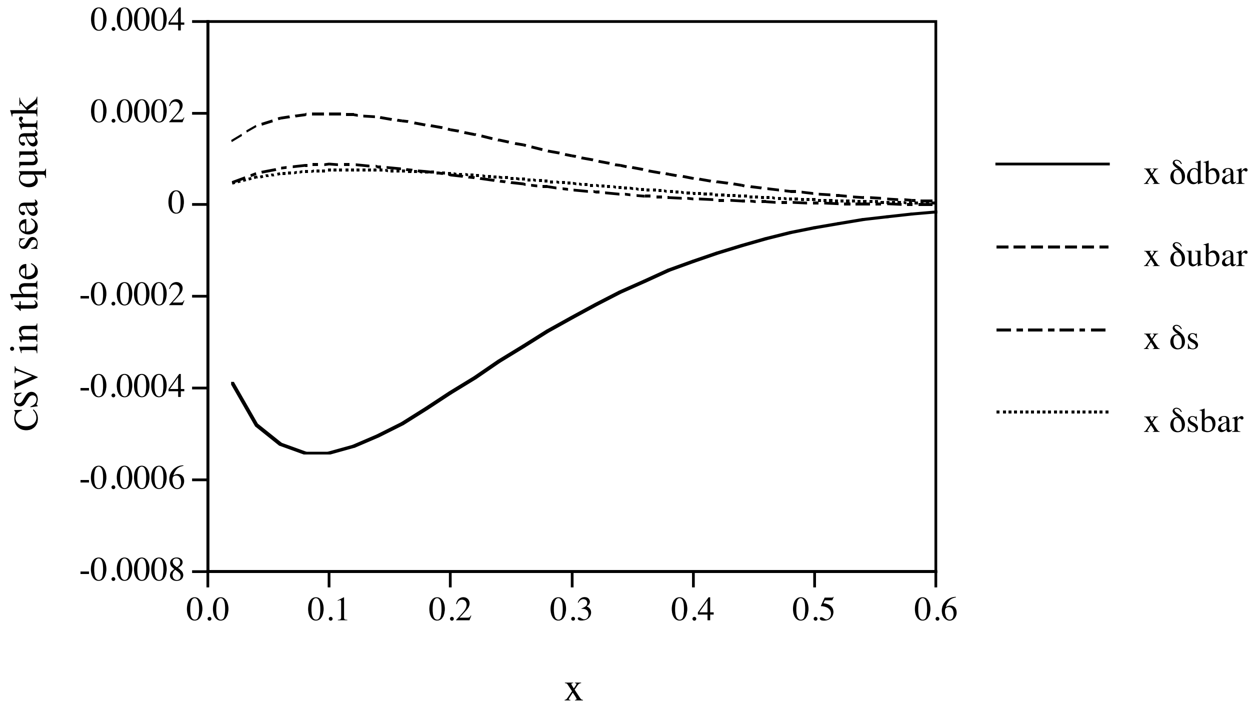
Charge symmetry breaking in the valence quarks



$$\delta u_V = u_V^p - d_V^n, \quad \delta d_V = d_V^p - u_V^n;$$

$$R_{\min} = \delta d_V / d_V, \quad R_{\mathrm{maj}} = \delta u_V / u_V$$

Charge symmetry breaking in the sea quarks



$$\delta\bar{u} = \bar{u}_V - \bar{d}_V;\ \delta\bar{d} = \bar{d}_V - \bar{u}_V;\ \delta s = s^p - s^n;\ \delta\bar{s} = \bar{s}^p - \bar{s}^n$$

Meson cloud contributions to the CSV are much smaller than that calculated with quark models.

3. Strange sea distributions

Strange sea distributions are not well determined compared with the valence distributions and light quark sea.

CTEQ6.5S [H. L. Lai et. al, JHEP 0704:089 (2007)]

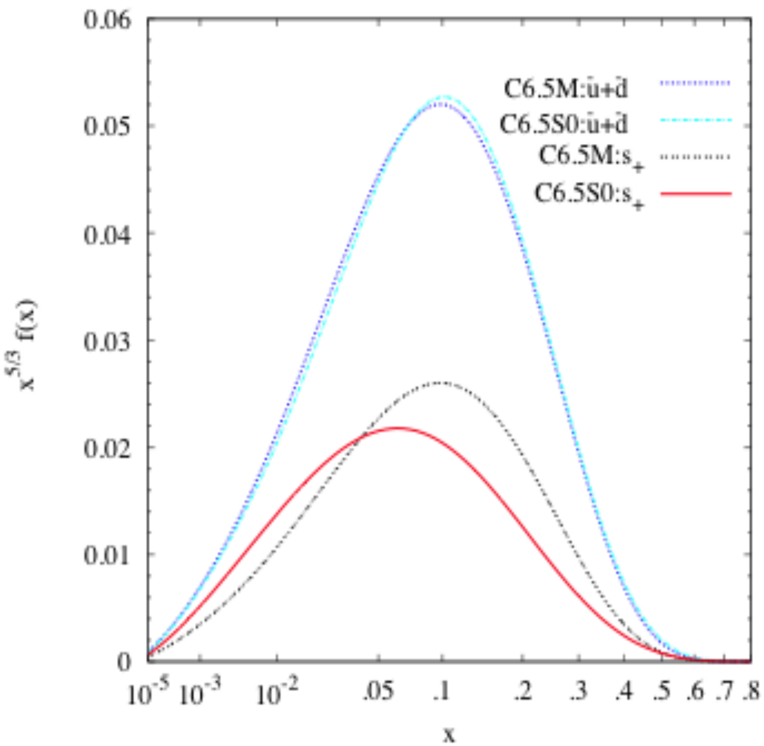
$$s_{\pm}(x,Q_0) = s(x,Q_0) \pm \bar{s}(x,Q_0)$$

$$s_{+}(x,Q_0) = A_0 x^{A_1}(1-x)^{A_2}$$

$A_1^{s_+} = A_1^{(\bar{u}+\bar{d})_+}$ is assumed

A_0 is related to suppression factor

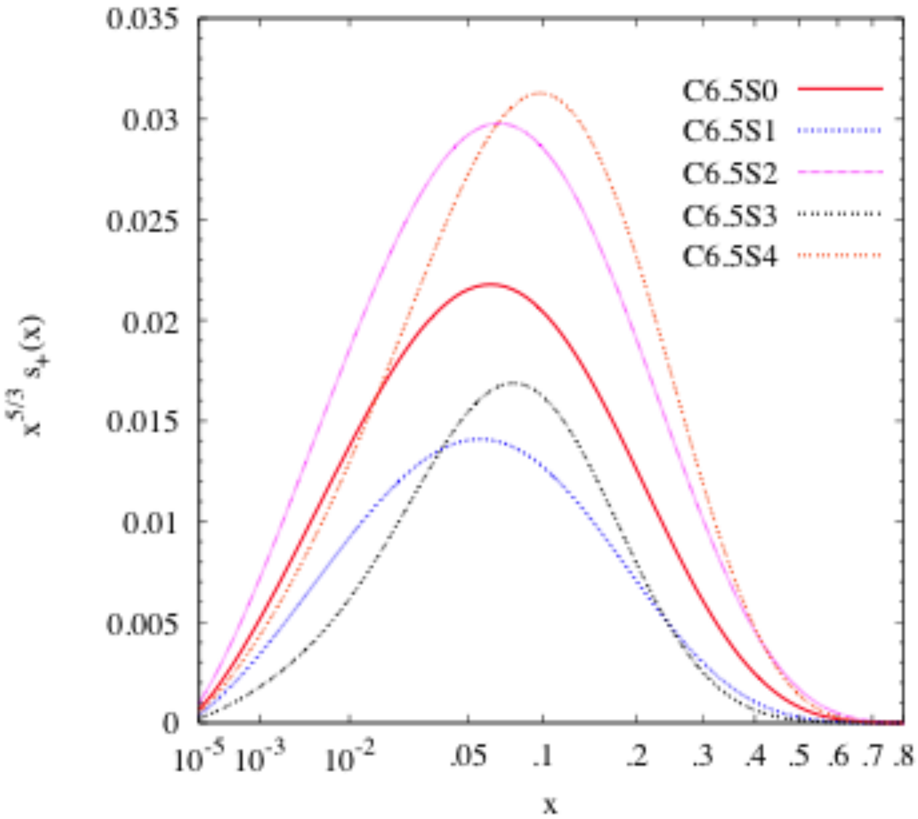
$$r = \frac{\langle x \rangle_{s+}}{\langle x \rangle_{\bar{u}(x)+\bar{d}(x)}}$$



Light sea is almost unchanged while $s_+(x)$ becomes smaller and softer compared to CTEQ6.5M.

$r = 0.44$ (CTEQ6.5S$_0$) vs. 0.50 (CTEQ6.5M)

- Allowed range for $s_+(x)$

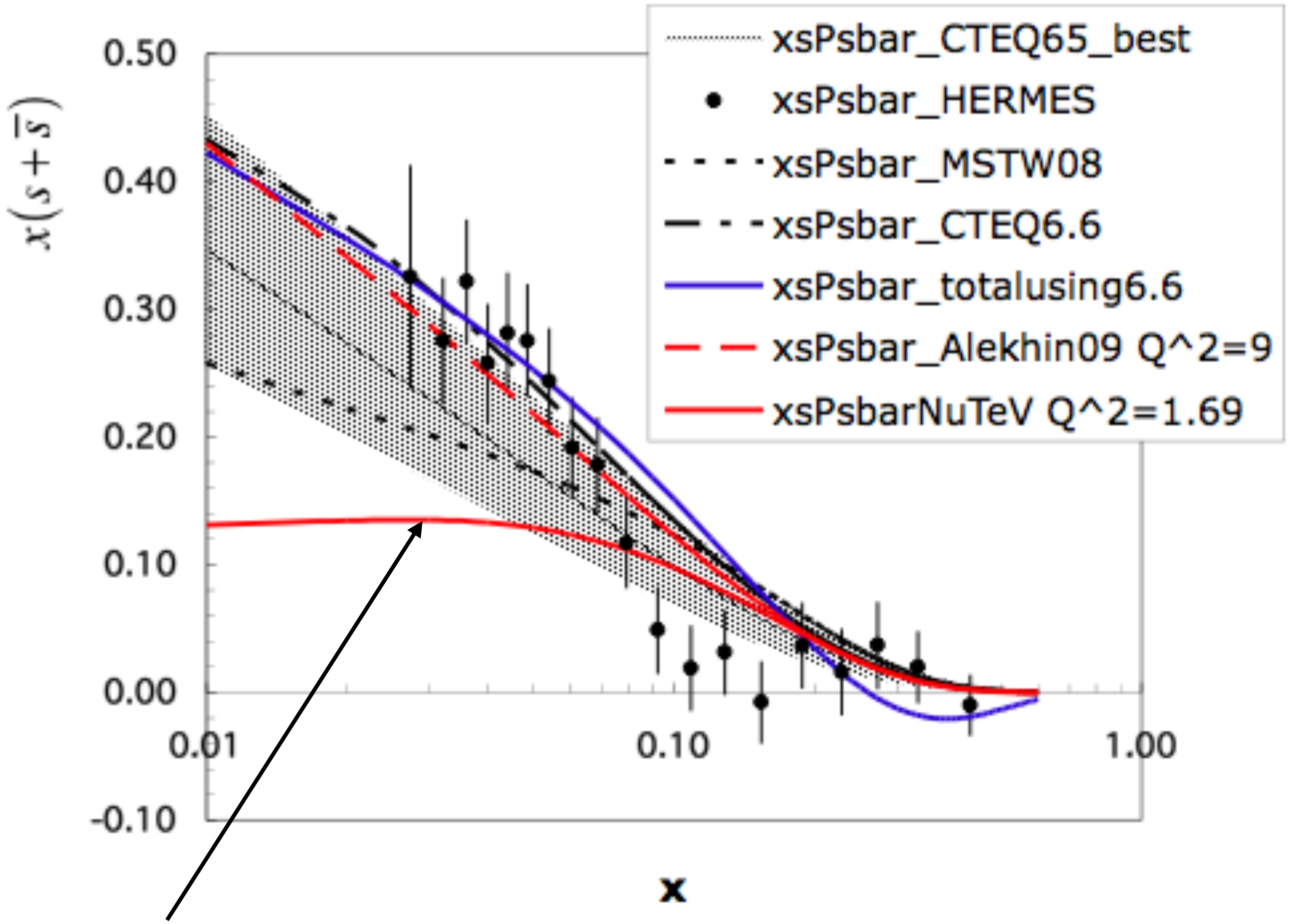


Monentum fraction

$0.018 < \langle x \rangle < 0.040;$

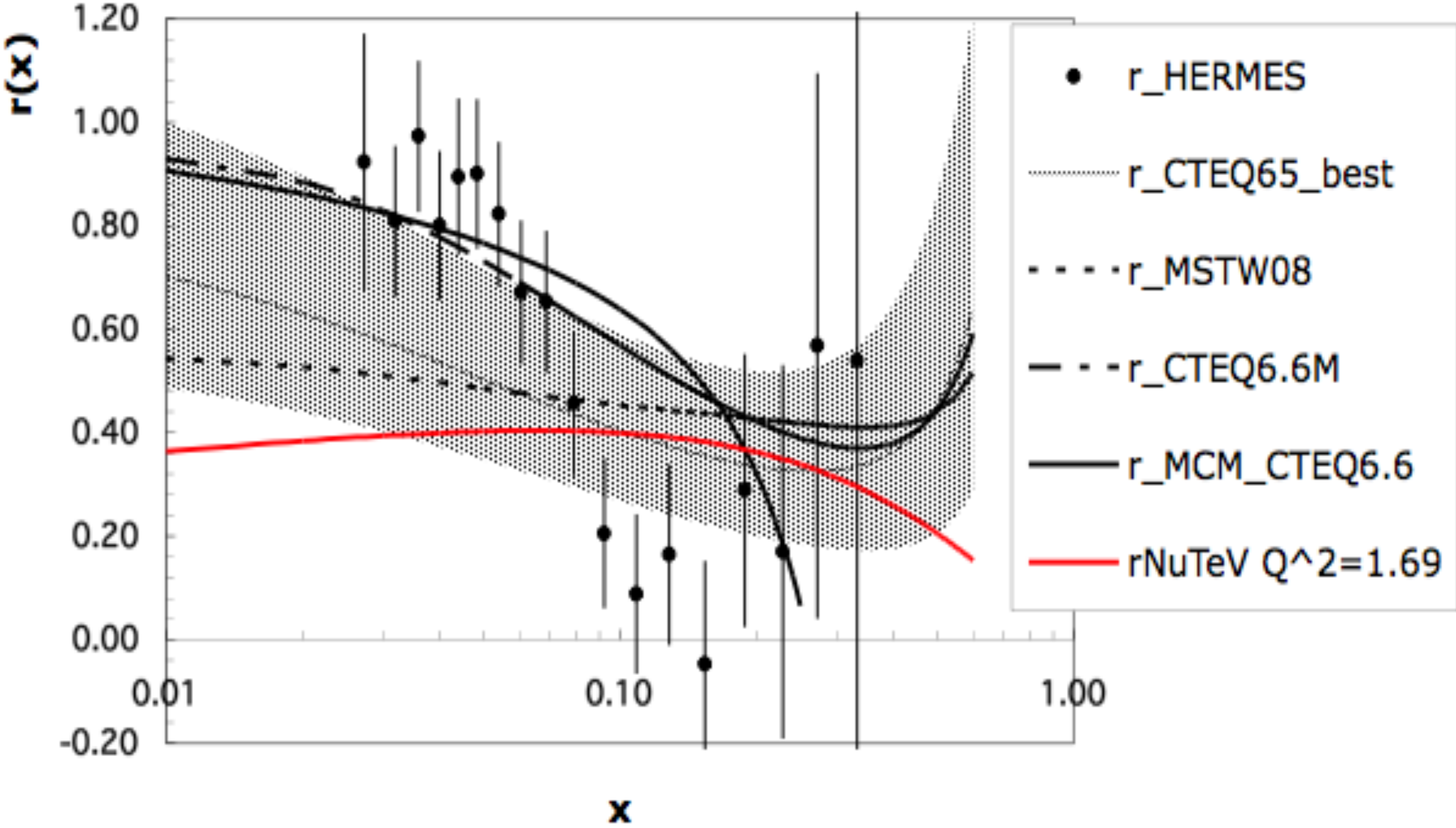
Different parameterizations

$$xs_{+}(x)=\left[\bar{d}(x)+\bar{u}(x)\right]_{\text{Fit}}-x\Delta(x)_{\text{MCM}}$$



NLO analysis of NuTeV data PRL99(2007)192001

The suppression factor $r(x) = \dfrac{s(x) + \overline{s}(x)}{\overline{d}(x) + \overline{u}(x)}$



4. Spin Dependent Structure Functions

$$g_1(x,Q^2) = \sum_q e_q^2 \left[\Delta q(x,Q^2) + \Delta\bar{q}(x,Q^2)\right]$$

- Dominated by valence distributions

  $N \rightarrow N\pi$, $N \rightarrow \Delta\pi$ most important fluctuations

$$\mathcal{L}_{int} = i g_{NN\pi} \bar{\psi}\gamma_5 \pi \psi, \quad f_{N\Delta\pi} \bar{\psi}\pi\partial_\mu \chi^\mu + \text{h.c.}$$

- At finite Q^2 spin of cloud hadrons are not parallel with initial nucleon spin

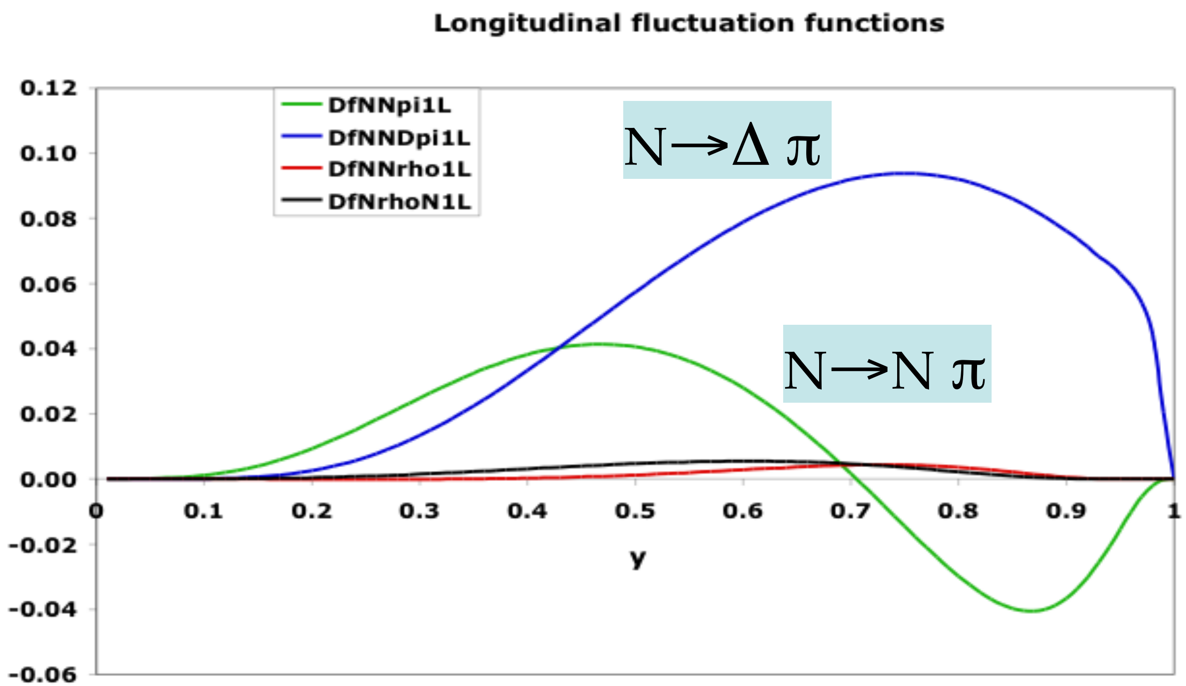
- Both longitudinal and transverse spin components of cloud contribute to observed structure functions

Spin Dependent Structure Functions

$$\begin{aligned}\delta g_1(x,Q^2) &= \frac{1}{1+\gamma^2}\int_x^1 \frac{dy}{y}\Big([\Delta f_{1L}(y)+\Delta f_{1T}(y)]g_1^B(\frac{x}{y},Q^2) \\ &\quad +[\Delta f_{2L}(y)+\Delta f_{2T}(y)]g_2^B(\frac{x}{y},Q^2)\Big) \\ \delta g_2(x,Q^2) &= -\frac{1}{1+\gamma^2}\int_x^1 \frac{dy}{y}\Big([\Delta f_{1L}(y)-\Delta f_{1T}(y)/\gamma^2]g_1^B(\frac{x}{y},Q^2) \\ &\quad -[\Delta f_{2L}(y)-\Delta f_{2T}(y)/\gamma^2]g_2^B(\frac{x}{y},Q^2)\Big)\end{aligned}$$

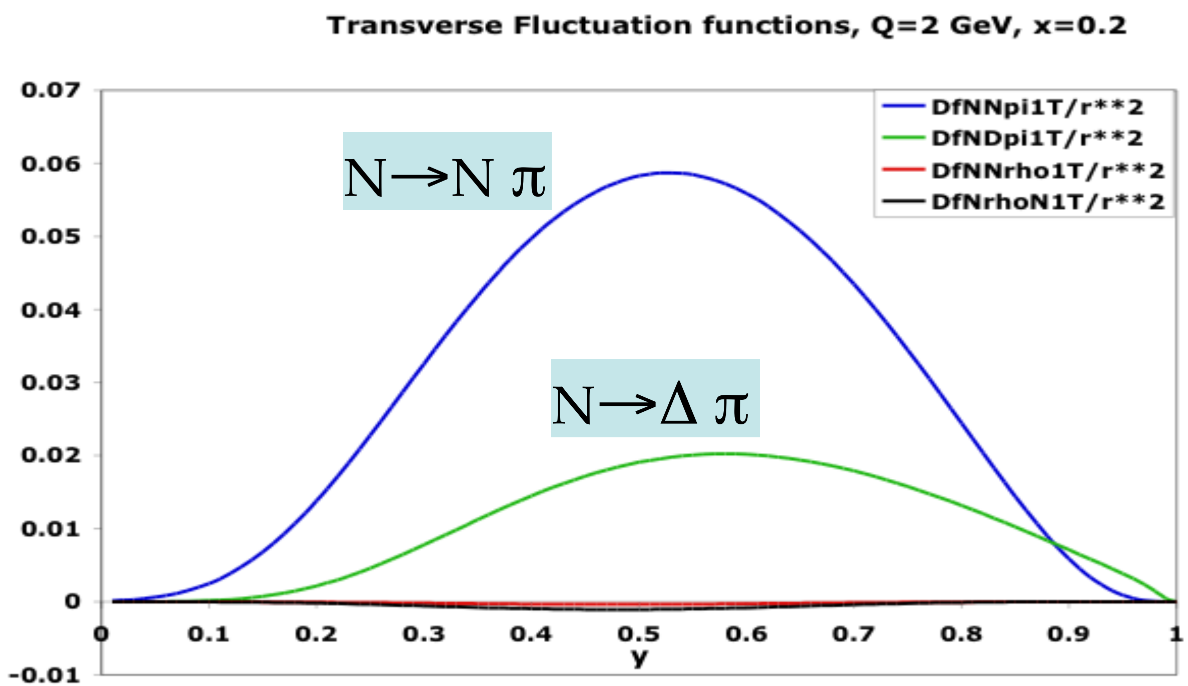
$$\gamma^2 = \frac{4x^2 m_N^2}{Q^2}$$

Spin Dependent Fluctuations



- Long. fluctuations require both N and Δ
- s = 3/2 state important

Spin Dependent Fluctuations



- N is more important for transverse fluct.
- n.b $\gamma \approx 0.19$

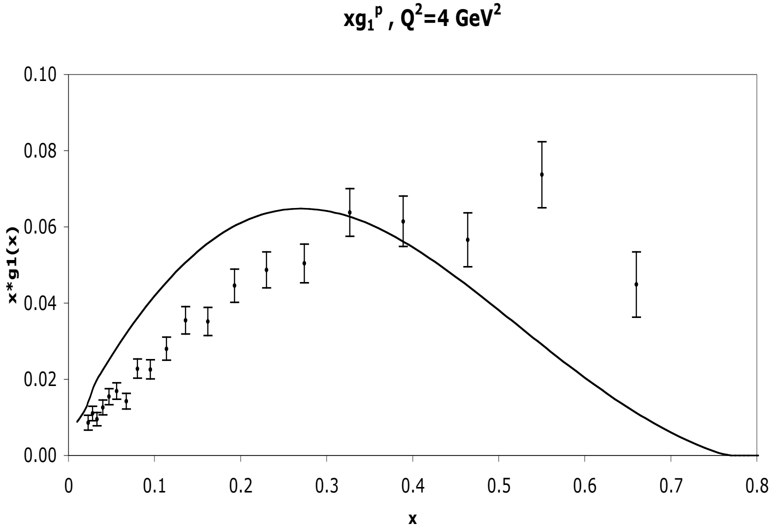
'Bare' Hadron SFs

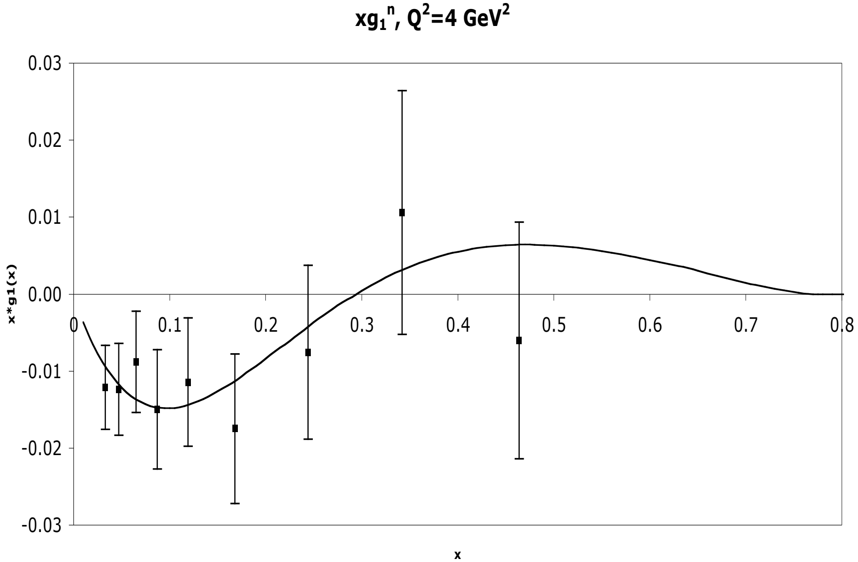
- Use bag model for N, Δ parton distributions
  - Add $\Delta g(x)$ 'by hand'
  - Hyperfine splitting between N and Δ
  - Use NLO evolution
  - Unpol. dists agree with DIS data
- $g_2(x)$ from Wandzura-Wilczek

$$g_2^{WW}(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$$

- No higher twist component

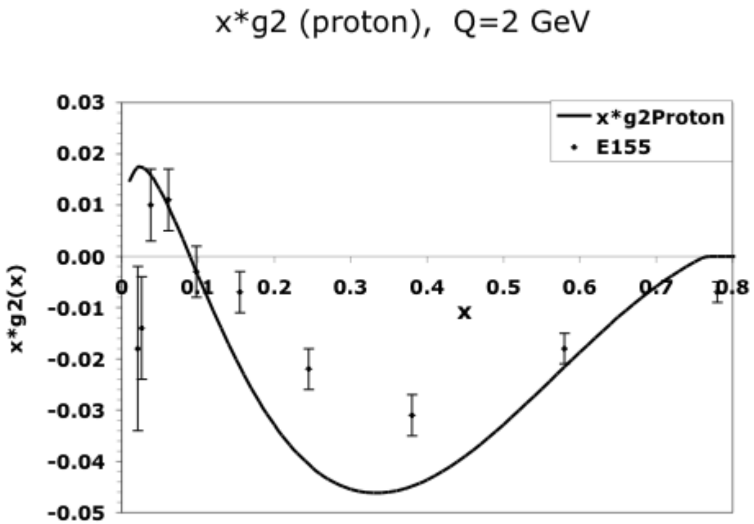
'Bare' Nucleon $g_1(x)$

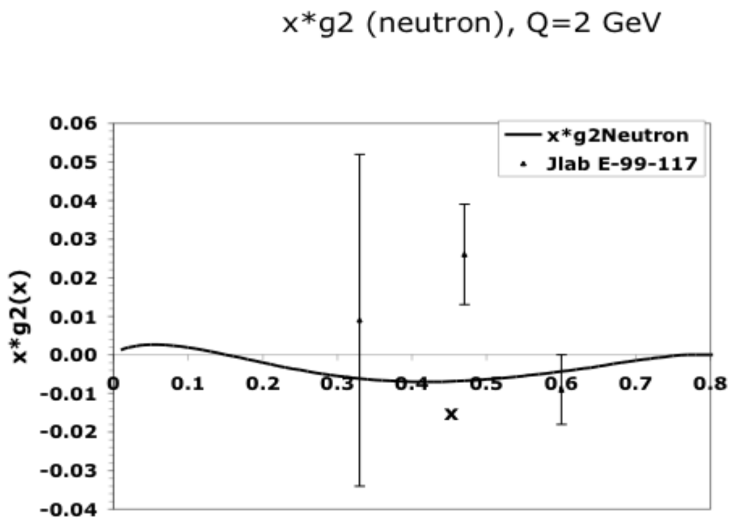




- Data from Hermes

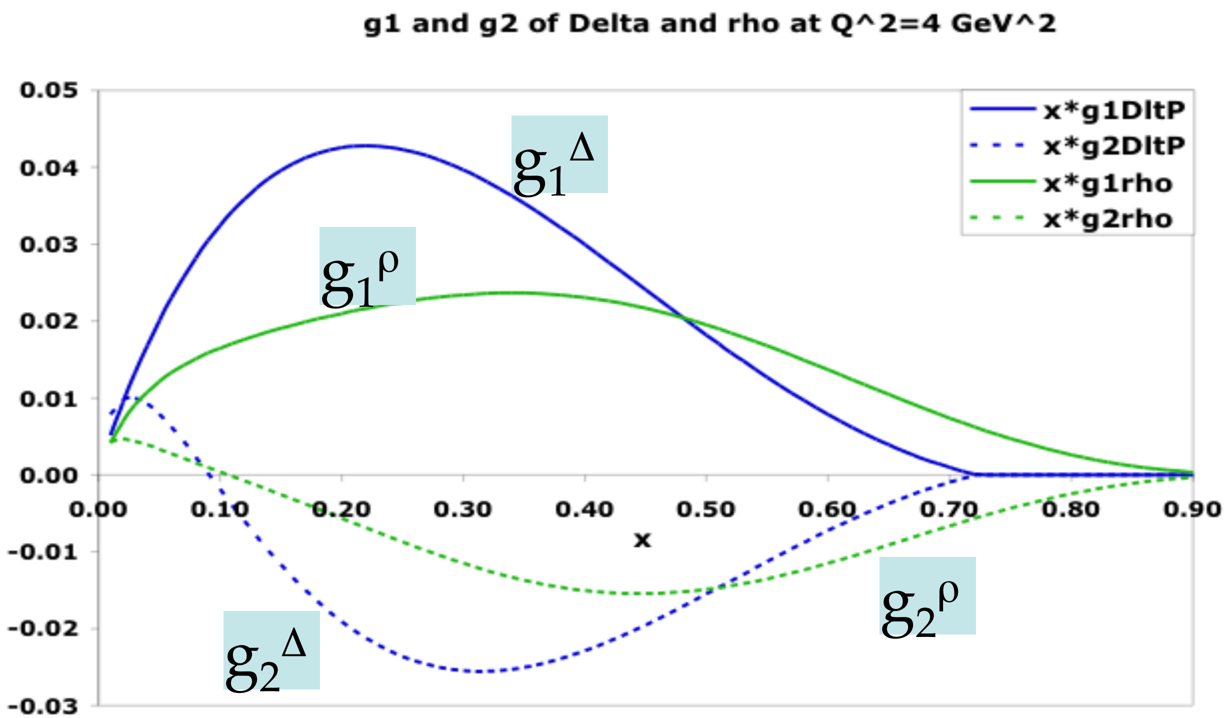
‘Bare’ Nucleon $g_2(x)$



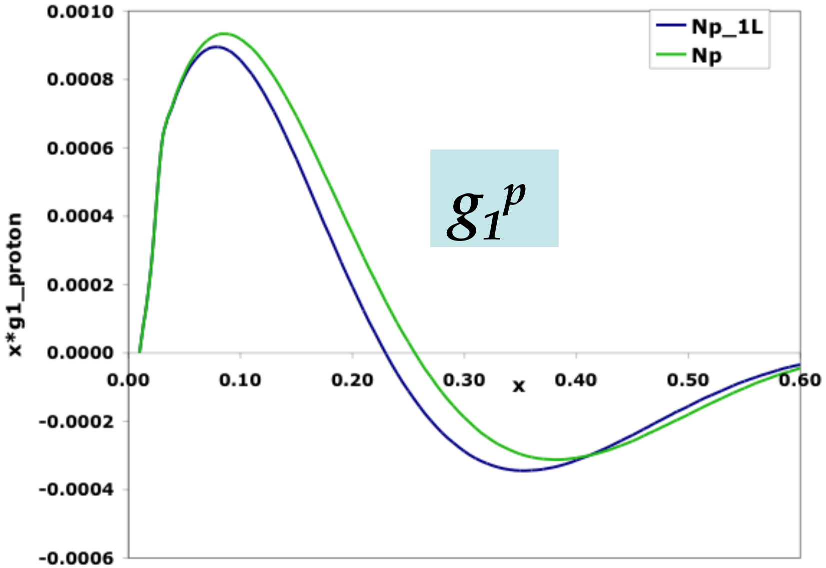


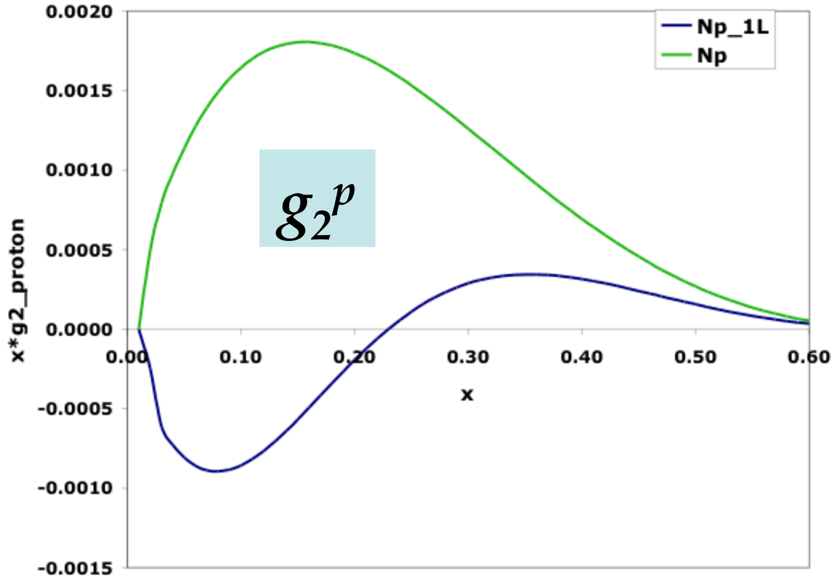
- Data from E155, Jlab E-99-117

‘Bare’ Hadrons Δ and ρ, $g_1(x)$ and $g_2(x)$



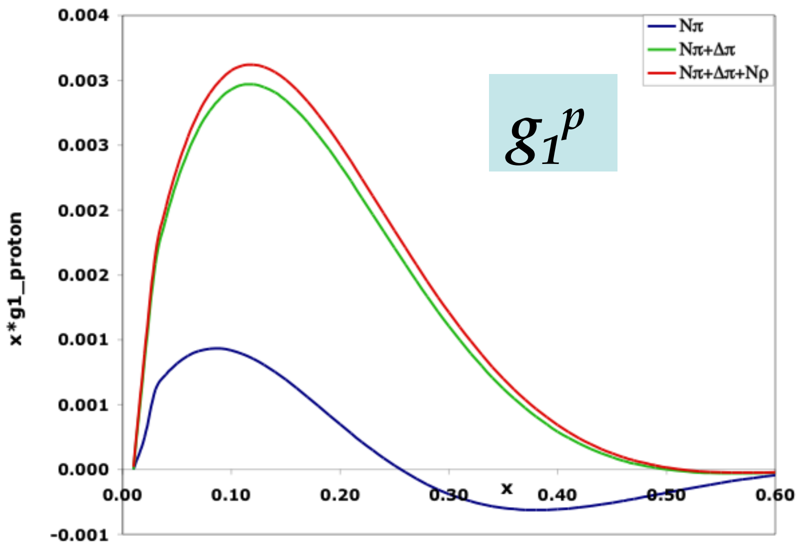
MC Contributions to g_1^p and g_2^p

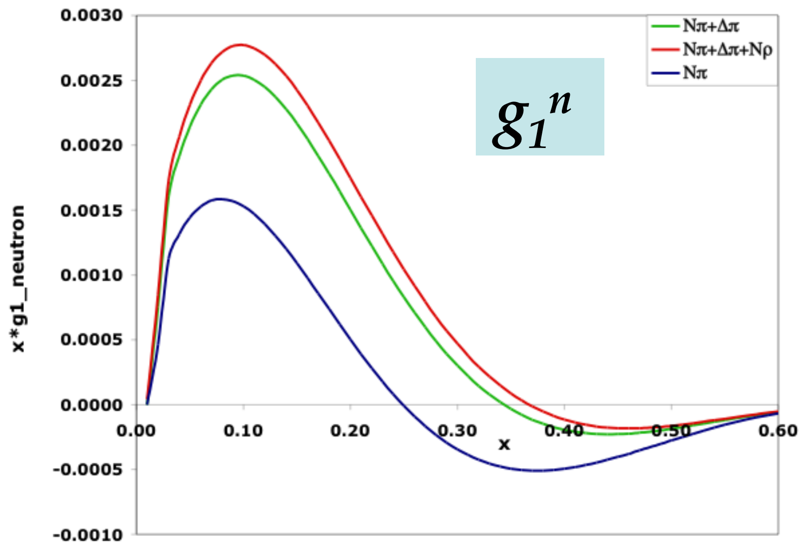




- The results for the neutron are very similar
- Δf_{1T}, Δf_{2L}, Δf_{2T} are important to g_2^p and g_2^n

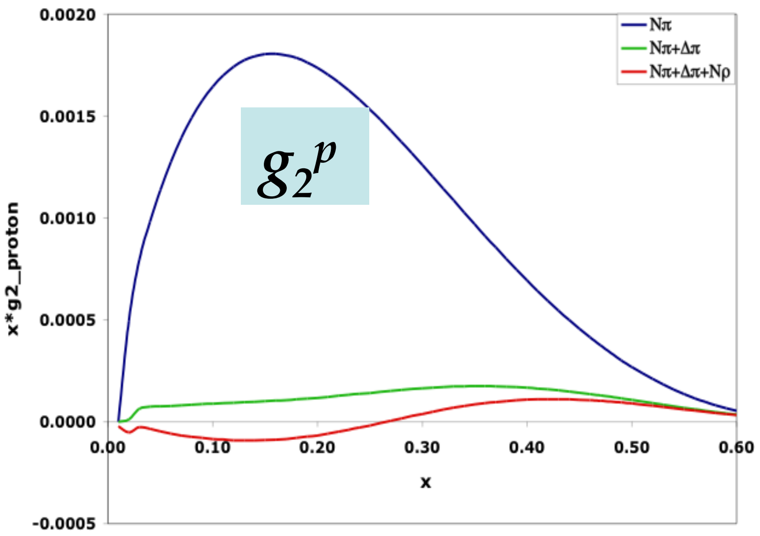
MC Contributions to g_1^p and g_1^n

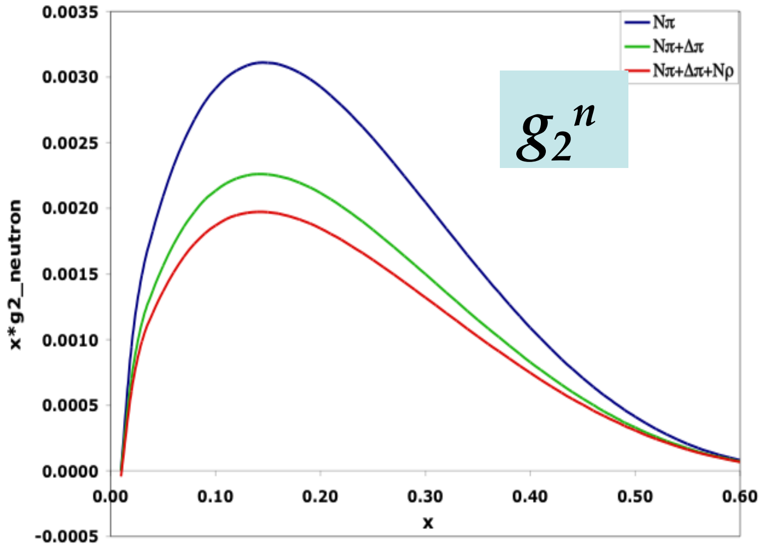




- Δπ is important while Nρ is not
- Δπ increases g_1^p more than that for g_1^n

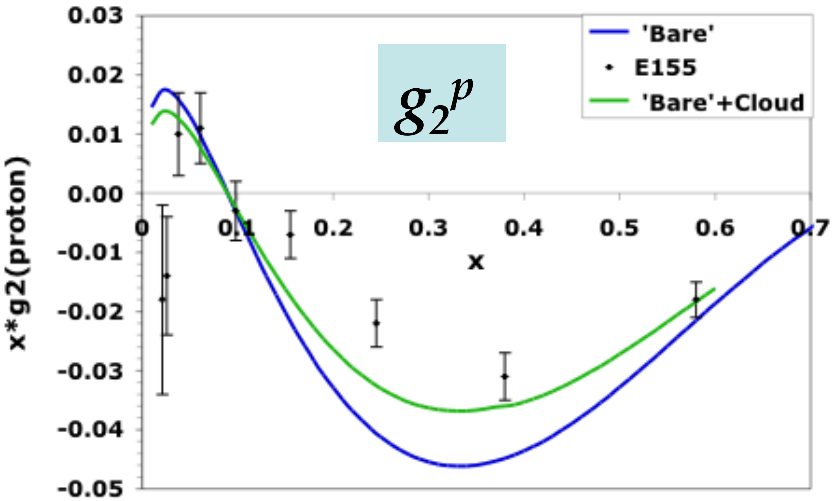
MC Contributions to g_2^p and g_2^n

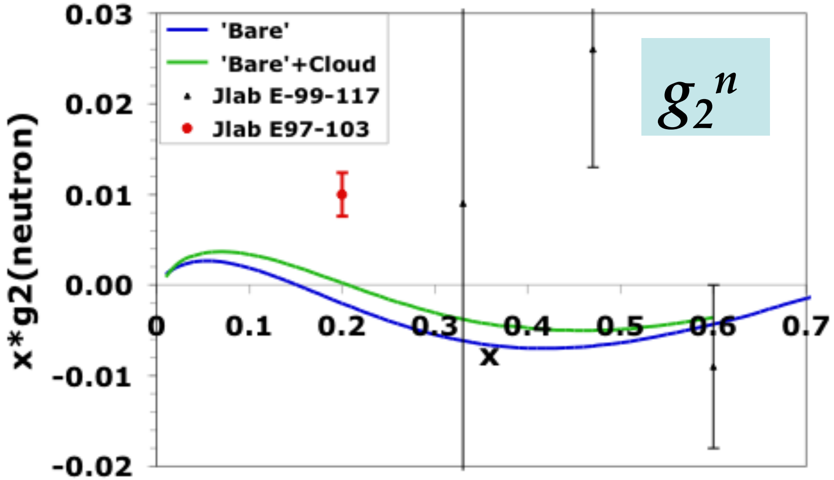




- Δπ is important
- Δπ affects g_2^p more than that for g_2^n

Comparison with data: g_2^p and g_2^n





- 20~30% corrections from MC
- Improve the agreement with the experiment

Nucleon spin

- Sea quarks also contribute to the nucleon spin

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

$$\Delta\Sigma = \Delta U + \Delta D + \Delta\bar{U} + \Delta\bar{D} + \Delta S + \Delta\bar{S}$$

$$\Delta\Sigma \approx 0.3 \text{ from DIS}$$

Bjorken sum rule

$$S_B = \int_0^1 dx\left[g_1^p(x) - g_1^n(x)\right]$$

$$= \frac{1}{6}\int_0^1 dx[\Delta u(x) - \Delta d(x)] + \frac{1}{6}\int_0^1 dx[\Delta\bar{u}(x) - \Delta\bar{d}(x)]$$

Ellis-Jaffe sum rule

$$S_{EJ} = \int_0^1 dx\, g_1^p(x) = \frac{1}{2}\sum_q e_q^2 \int_0^1 dx[\Delta q + \Delta\bar{q}(x)]$$

Nucleon spin

- Nucleon spin decomposition

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L'_q + L'_g$$

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L''_q + L''_g$$

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L'''_q + L'''_g$$

Certainly not trivial and/or purely academic

What processes can be used to measure the OAMs of the quark and gluon?

5. Summary

1. Non-perturbative QCD models for the nucleon structure can make reliable predictions for the symmetry breaking effects.
2. Strange sea distributions are not well constrained.
3. Combining the MCM calculations for the $SU(3)_f$ breaking effect with global analysis results for the light quark sea, we estimated the total strange sea distributions. The calculations agree with HERMES results, but not with the NLO analysis of NuTeV dimuon data.
4. Possible strange-antistrange asymmetry is of great interest.
5. MC contributions to g_1 are small and to g_2 are 20%.