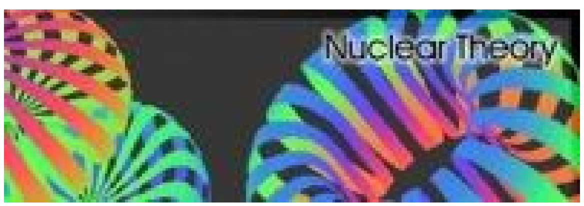
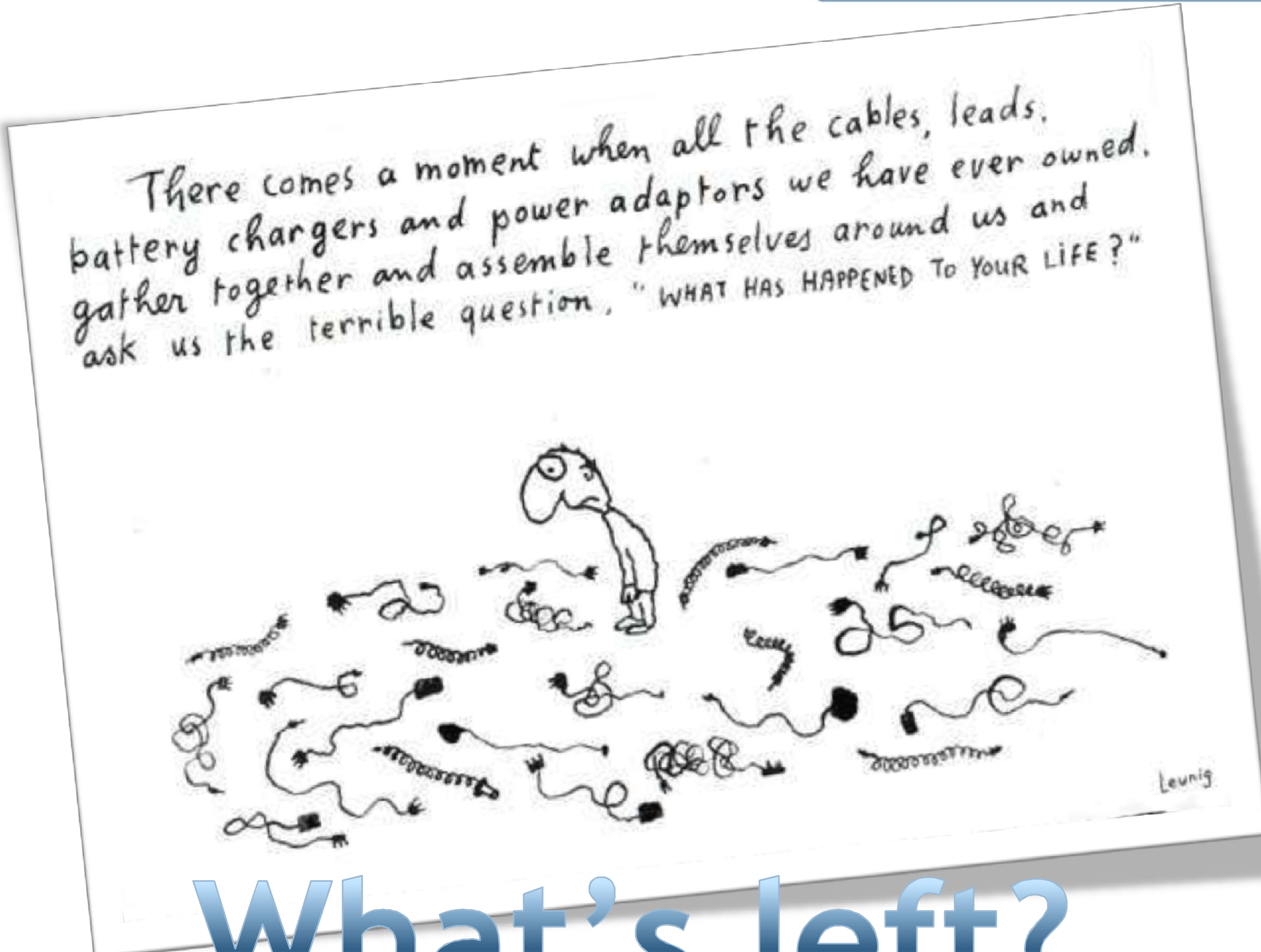
Continuum strong QCD

Craig Roberts



Physics Division


There comes a moment when all the cables, leads, battery chargers and power adaptors we have ever owned, gather together and assemble themselves around us and ask us the terrible question, "WHAT HAS HAPPENED TO YOUR LIFE?"
Leunig

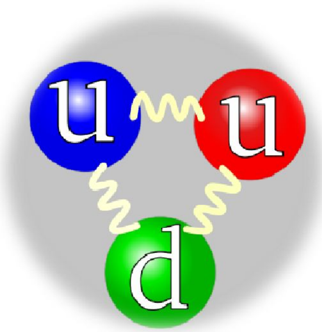
What's left?

New Challenges

- Computation of spectrum of hybrid and exotic mesons

  exotic mesons: quantum numbers not possible for quantum mechanical quark-antiquark systems
  hybrid mesons: normal quantum numbers but non-quark-model decay pattern
  BOTH suspected of having "constituent gluon" content

- Equally pressing, some might say more so, is the three-body problem; viz., baryons in QCD



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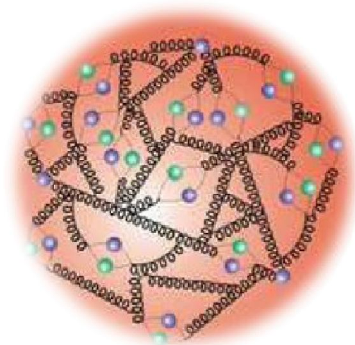


Grand Unification

Unification of Meson & Baryon Properties

- Correlate the properties of meson and baryon ground- and excited-states within a *single, symmetry-preserving framework*
  - Symmetry-preserving means:
    Poincaré-covariant & satisfy relevant Ward-Takahashi identities
- Constituent-quark model has hitherto been the most widely applied spectroscopic tool; whilst its weaknesses are emphasized by critics and acknowledged by proponents, it is of continuing value because there is nothing better that is yet providing a bigger picture.
- Nevertheless,
  - no connection with quantum field theory & therefore not with QCD
  - not symmetry-preserving & therefore cannot veraciously connect meson and baryon properties

Fully-covariant computation of nucleon form factors

- First such calculations:
  - G. Hellstern *et al*., Nucl.Phys. A627 (1997) 679-709 , restricted to $Q^2 < 2\,\mathrm{GeV}^2$
  - J.C.R. Bloch *et al*., Phys.Rev. C60 (1999) 062201(R) , restricted to $Q^2 < 3\,\mathrm{GeV}^2$

  Exploratory:
  - Included some correlations within the nucleon, but far from the most generally allowed
  - Used very simple photon-nucleon interaction current
- Did not isolate and study $G_E^p(Q^2)/G_M^p(Q^2)$
- How does one study baryons in QCD?

R.T. Cahill *et al.*,
Austral. J. Phys. 42 (1989) 129-145

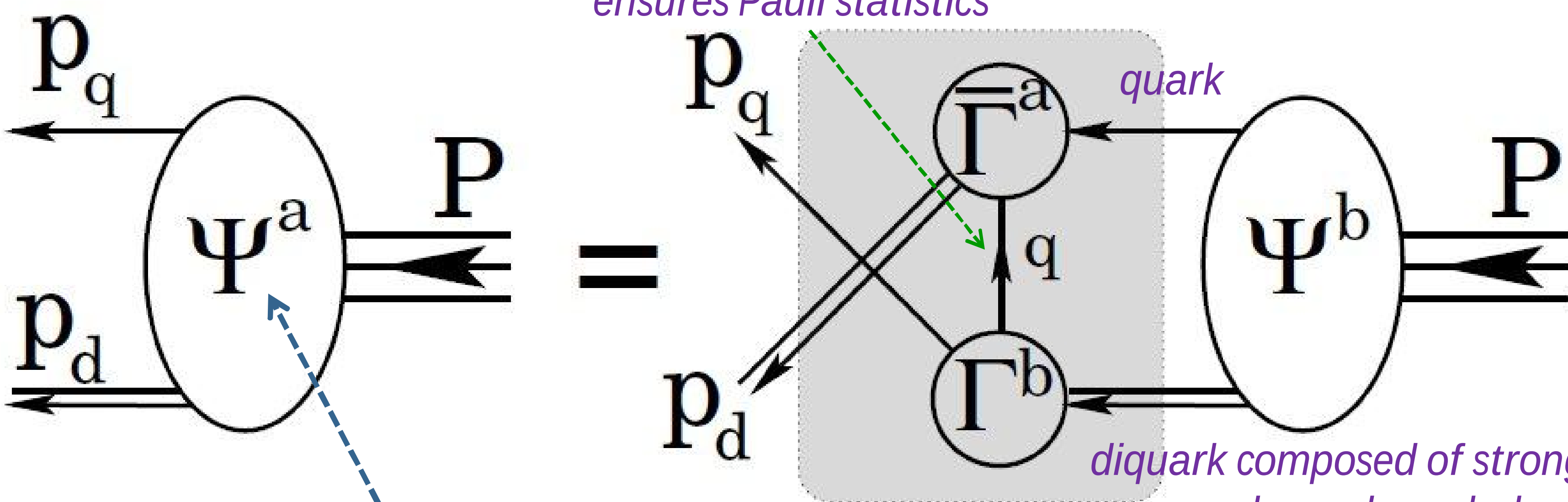
DSEs & Baryons

- *Dynamical chiral symmetry breaking* (DCSB)
  – has enormous impact on meson properties.
  - *Must be included in description and prediction of baryon properties*.
- *DCSB* is essentially a quantum field theoretical effect.
  In quantum field theory
  - Meson appears as pole in four-point quark-antiquark Green function → Bethe-Salpeter Equation
  - *Nucleon appears as a pole in a six-point quark Green function → Faddeev Equation*.
- Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks
- Tractable equation is based on the observation that an interaction which describes colour-singlet mesons also generates *nonpointlike* quark-quark (diquark) correlations in the colour-antitriplet channel

$SU_c(3)$: $3 \otimes 3 = \bar{3} \oplus 6$

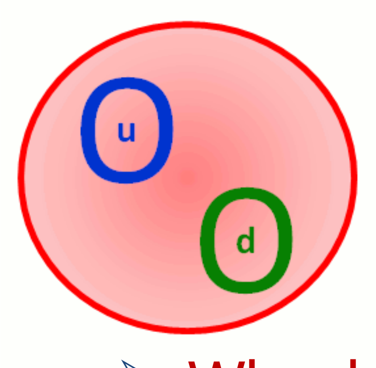
R.T. Cahill *et al.*,
[Austral. J. Phys. 42 (1989) 129-145]

Faddeev Equation



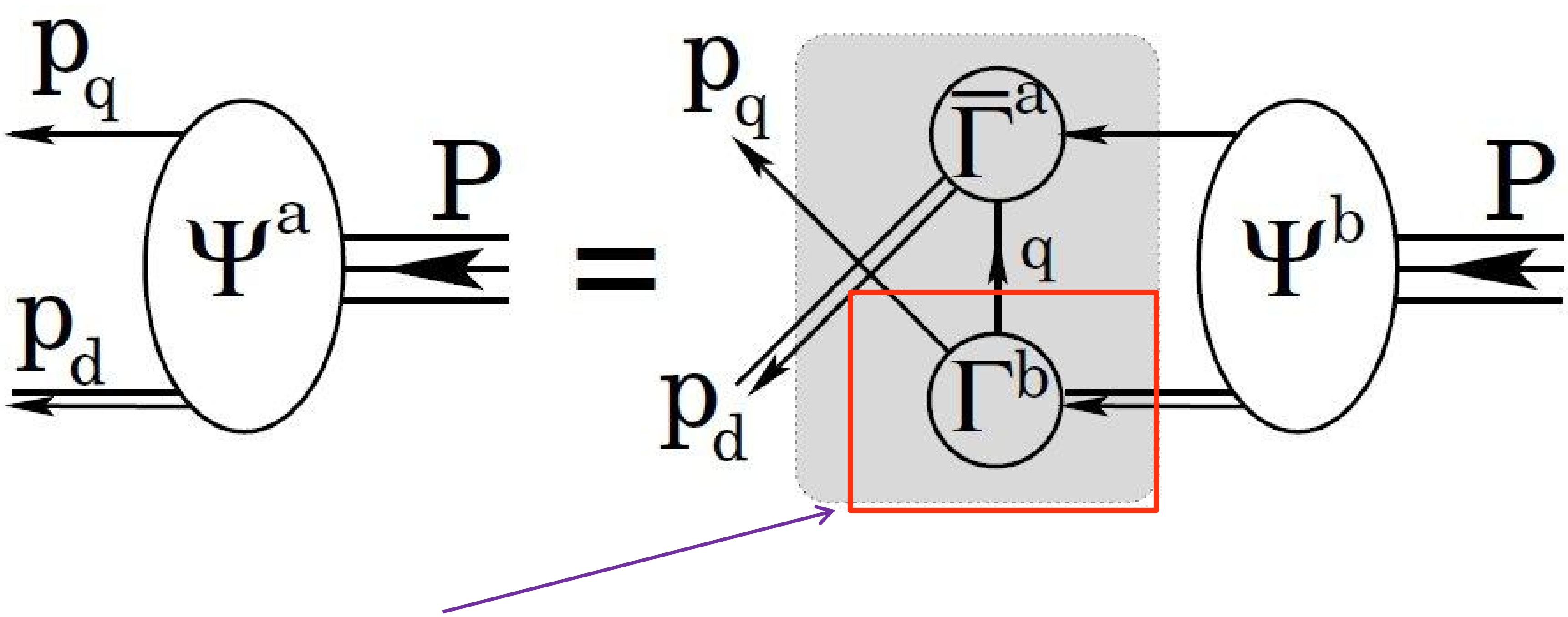
- Linear, Homogeneous Matrix equation
  - Yields *wave function (Poincaré Covariant Faddeev* Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks . . .
  - Both have "*correct*" parity and "*right*" masses
  - In Nucleon's Rest Frame Amplitude has

    s−, p− & d−wave correlations

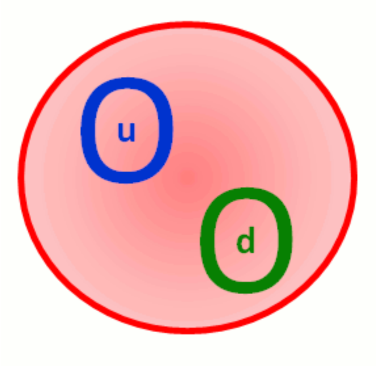


Faddeev Equation

- Why should a pole approximation produce reliable results?



quark-quark scattering matrix
- a pole approximation is used to arrive at the Faddeev-equation



Calculation of diquark masses in QCD
R.T. Cahill, C.D. Roberts and J. Praschifka
Phys.Rev. D**36** (1987) 2804

Diquarks

Consider the rainbow-gap and ladder-Bethe-Salpeter equations

$$S(p)^{-1} = i\gamma \cdot p + m + \int \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2}\gamma_\mu S(q) \frac{\lambda^a}{2}\gamma_\nu(q,p),$$

$$\Gamma(k;P) = -\int \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2}\gamma_\mu S(q+P)\Gamma(q;P)S(q)\frac{\lambda^a}{2}\gamma_\nu \,.$$

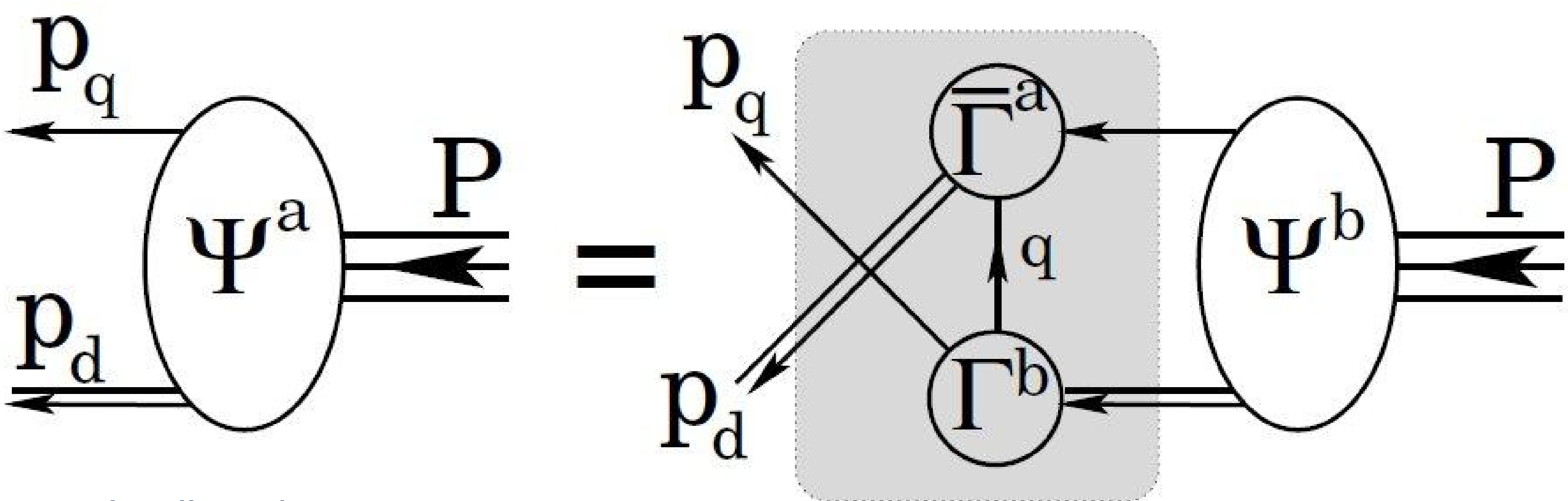
- In this symmetry-preserving truncation, colour-antitriplet quark-quark correlations (diquarks) are described by a very similar homogeneous Bethe-Salpeter equation

$$\Gamma_{qq}(k;P)C^\dagger = -\frac{1}{2}\int \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2}\gamma_\mu S(q+P)\Gamma_{qq}(q;P)C^\dagger S(q)\frac{\lambda^a}{2}\gamma_\nu$$

- Only difference is factor of ½
- Hence, an interaction that describes mesons also generates diquark correlations in the colour-antitriplet channel

Survey of nucleon electromagnetic form factors
I.C. Cloët et al, arXiv:0812.0416 [nucl-th],
Few Body Syst. **46** (2009) pp. 1-36

Faddeev Equation



scalar diquark component

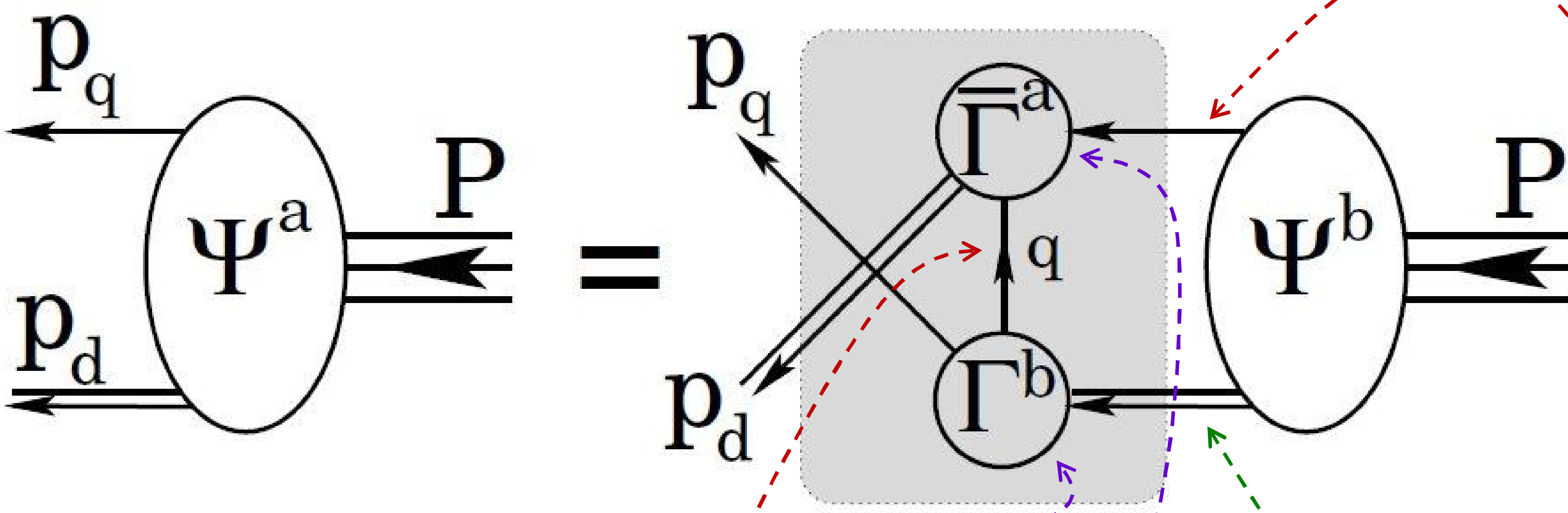
$$\begin{bmatrix} \mathcal{S}(k;P)\,u(P) \\ \mathcal{A}^i_\mu(k;P)\,u(P) \end{bmatrix} = -4\int\frac{d^4\ell}{(2\pi)^4}\,\mathcal{M}(k,\ell;P)\begin{bmatrix} \mathcal{S}(\ell;P)\,u(P) \\ \mathcal{A}^j_\nu(\ell;P)\,u(P) \end{bmatrix}$$

axial-vector diquark component

$$\mathcal{M}(k,\ell;P) = \begin{bmatrix} \mathcal{M}_{00} & (\mathcal{M}_{01})^j_\nu \\ (\mathcal{M}_{10})^i_\mu & (\mathcal{M}_{11})^{ij}_{\mu\nu} \end{bmatrix}$$

Survey of nucleon electromagnetic form factors
I.C. Cloët et al, [arXiv:0812.0416 [nucl-th]],
Few Body Syst. **46** (2009) pp. 1-36

Faddeev Equation



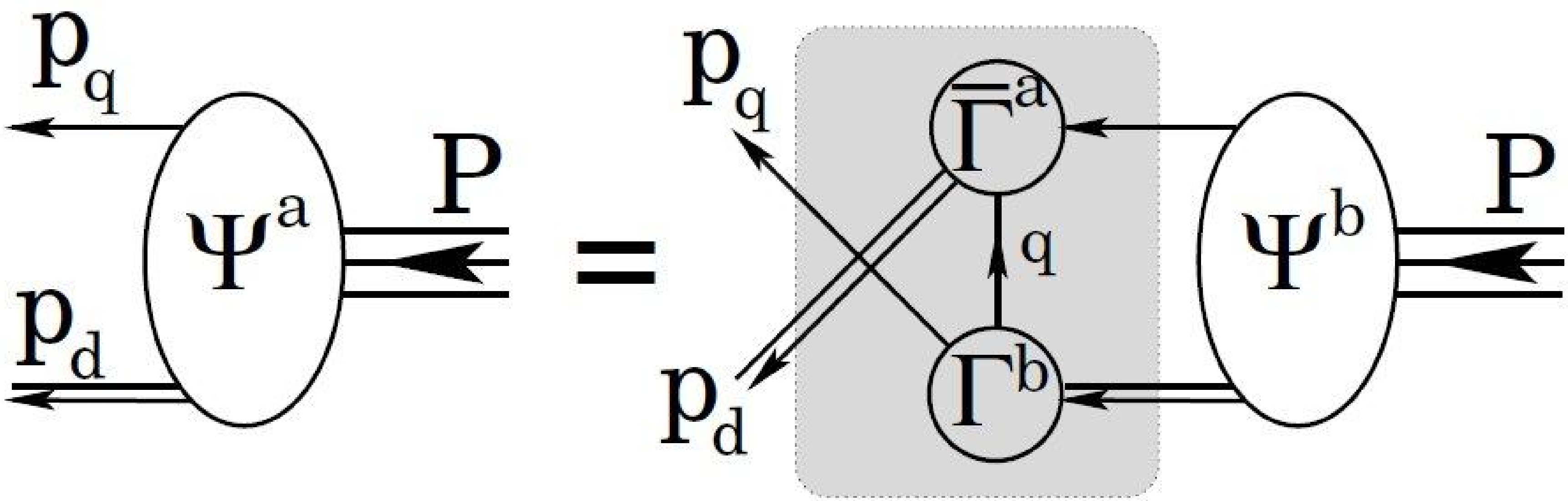
$$\mathcal{M}_{00} = \Gamma^{0^+}(k_q - \ell_{qq}/2; \ell_{qq})\, S^{\mathrm{T}}(\ell_{qq} - k_q) \times \bar{\Gamma}^{0^+}(\ell_q - k_{qq}/2; -k_{qq})\, S(\ell_q)\, \Delta^{0^+}(\ell_{qq})$$

$$\ell_q = \ell + P/3,\; k_q = k + P/3,$$

$$\ell_{qq} = -\ell + 2P/3,\; k_{qq} = -k + 2P/3$$

Survey of nucleon electromagnetic form factors
I.C. Cloët et al, arXiv:0812.0416 [nucl-th],
Few Body Syst. **46** (2009) pp. 1-36

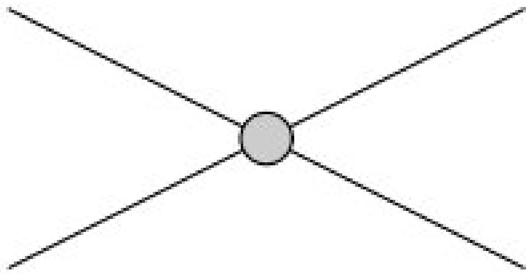
Faddeev Equation



- Every one of these entries has a simple matrix structure
- Similar form for the kernel entries that involve axial-vector diquark correlations
- Combining everything, one arrives at a linear homogeneous matrix equation for the amplitudes

  $S(k;P)u(P)$, $A(k;P)u(P)$

Voyage of Discovery



Contact-Interaction Kernel

- Vector-vector contact interaction

$$g^2 D_{\mu\nu}(p-q) = \delta_{\mu\nu}\frac{4\pi\,\alpha_{\rm IR}}{m_G^2}$$

m_G = 800MeV is a gluon mass-scale

– dynamically generated in QCD

- Gap equation: $M_f = m_f + M_f \frac{4\alpha_{\rm IR}}{3\pi m_G^2}\int_0^\infty ds\, s\,\frac{1}{s+M_f^2}$
- DCSB: $M \neq 0$ is possible so long as $\alpha_{\rm IR} > \alpha_{\rm IR}^{critical} = 0.4\pi$
- Observables require $\alpha_{\rm IR} = 0.93\pi$

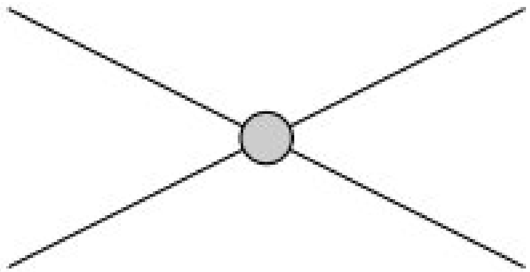
Contact Interaction

Symmetry-preserving treatment of vector-vector contact-interaction: series of papers establishes strengths & limitations.

- **arXiv:1212.2212 [nucl-th]**
  Features and flaws of a contact interaction treatment of the kaon
  Chen Chen, L. Chang, C. D. Roberts, S. M. Schmidt, Shaolong Wan and D. J. Wilson,
- **arXiv:1209.4352 [nucl-th]**, Phys. Rev. C *in press*
  Electric dipole moment of the rho-meson
  M. Pitschmann, C.-Y. Seng, M. J. Ramsey-Musolf, C. D. Roberts, S. M. Schmidt and D. J. Wilson
- **arXiv:1204.2553 [nucl-th],** Few Body Syst. (2012) DOI: 10.1007/s00601-012-0466-3
  Spectrum of Hadrons with Strangeness,
  Chen Chen, L. Chang, C.D. Roberts, Shaolong Wan and D.J. Wilson
- **arXiv:1112.2212 [nucl-th]**, Phys. Rev. C**85** (2012) 025205 [21 pages]
  Nucleon and Roper electromagnetic elastic and transition form factors,
  D. J. Wilson, I. C. Cloët, L. Chang and C. D. Roberts
- **arXiv:1102.4376 [nucl-th]**, Phys. Rev. C **83**, 065206 (2011) [12 pages] ,
  π- and ρ-mesons, and their diquark partners, from a contact interaction,
  H.L.L. Roberts, A. Bashir, L.X. Gutiérrez-Guerrero, C.D. Roberts and David J. Wilson
- **arXiv:1101.4244 [nucl-th]**, Few Body Syst. **51** (2011) pp. 1-25
  Masses of ground and excited-state hadrons
  H.L.L. Roberts, Lei Chang, Ian C. Cloët and Craig D. Roberts
- **arXiv:1009.0067 [nucl-th]**, Phys. Rev. C**82** (2010) 065202 [10 pages]
  Abelian anomaly and neutral pion production
  Hannes L.L. Roberts, C.D. Roberts, A. Bashir, L. X. Gutiérrez-Guerrero & P. C. Tandy
- **arXiv:1002.1968 [nucl-th]**, Phys. Rev. C **81** (2010) 065202 (5 pages)
  Pion form factor from a contact interaction, L. Xiomara Gutiérrez-Guerrero, A. Bashir, I. C. Cloët & C. D. Roberts

Contact Interaction

- Symmetry-preserving treatment of vector×vector contact interaction is useful tool for the study of phenomena characterised by probe momenta less-than the dressed-quark mass.
- Whilst this interaction produces form factors which are too hard, interpreted carefully, even they can be used to draw valuable insights; e.g., concerning relationships between different hadrons.
- Studies employing a symmetry-preserving regularisation of the contact interaction serve as a useful surrogate, exploring domains which analyses using interactions that more closely resemble those of QCD are as yet unable to enter.
- They're critical at present in attempts to use data as tool for charting nature of the quark-quark interaction at long-range; i.e., identifying signals of the running of couplings and masses in QCD.



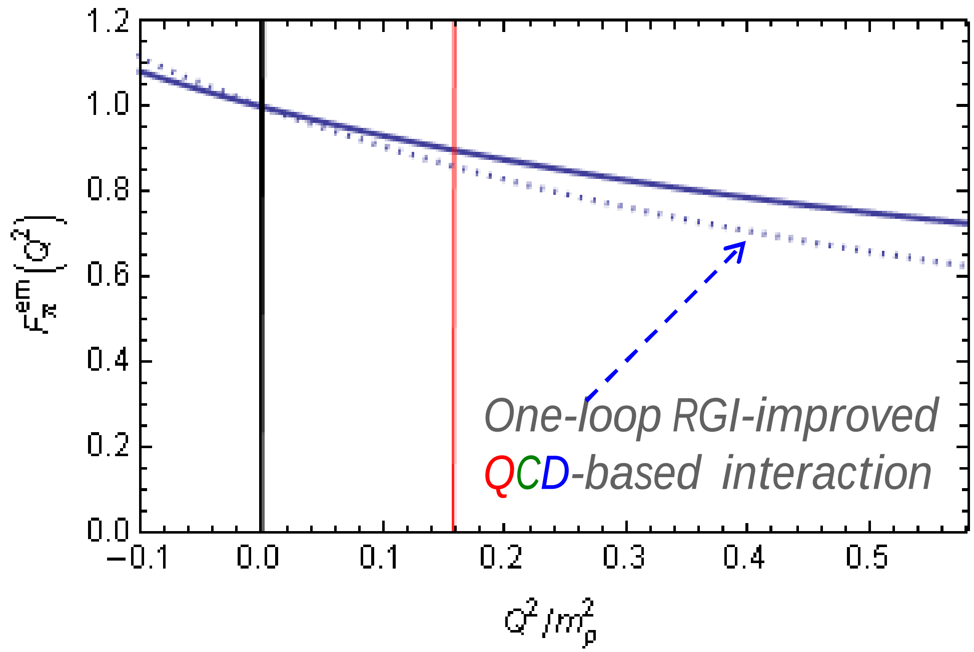
Interaction Kernel

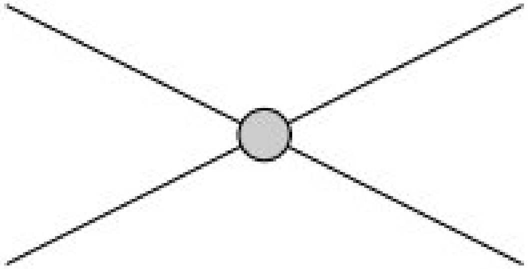
- Studies of π & ρ static properties and π form factor establish that contact-interaction results are not realistically distinguishable from those of renormalisation-group-improved one-gluon exchange for $Q^2<M^2$

	contact interaction	QCD 1-loop RGI gluon
M	0.37	0.34
κ_π	0.24	0.24
m_π	0.14	0.14
m_ρ	0.93	0.74
f_π	0.10	0.093
f_ρ	0.13	0.15

cf. expt.
rms rel.err.=13%

M^2/m_ρ^2





Interaction Kernel - Regularisation Scheme

- Contact interaction is not renormalisable
- Must therefore introduce regularisation scheme
  - Use confining proper-time definition

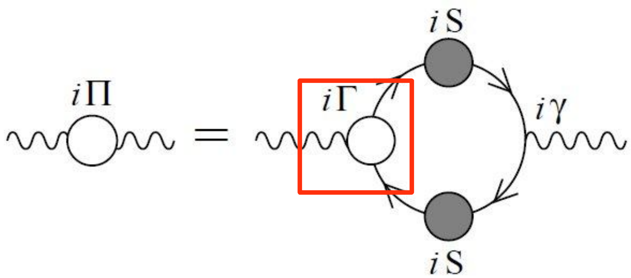
D. Ebert, T. Feldmann and H. Reinhardt, Phys. Lett. B 388 (1996) 154.

No pole in propagator – DSE realisation of confinement

$$\frac{1}{s+M^2} = \int_0^\infty d\tau\, \mathrm{e}^{-\tau(s+M^2)} \to \int_{\tau_{\rm uv}^2}^{\tau_{\rm ir}^2} d\tau\, \mathrm{e}^{-\tau(s+M^2)} = \frac{\mathrm{e}^{-(s+M^2)\tau_{\rm uv}^2} - e^{-(s+M^2)\tau_{\rm ir}^2}}{s+M^2}$$

- $\Lambda_{ir} = 0.24$GeV, $\tau_{ir} = 1/\Lambda_{ir} = 0.8$fm a confinement radius, which is not varied
- Two parameters: $m_G=0.13$GeV, $\Lambda_{uv}=0.91$GeV fitted to produce tabulated results

	contact interaction
M	0.37
κ_π	0.24
m_π	0.14
m_ρ	0.93
f_π	0.10
f_ρ	0.13



Regularisation & Symmetries

- In studies of the hadron spectrum it's critical that an approach satisfy the vector and axial-vector Ward-Green Takahashi identities.
  - Without this it is impossible to preserve the pattern of chiral symmetry breaking in QCD & hence a veracious understanding of hadron mass splittings is not achievable.
- Contact interaction should & can be regularised appropriately
- Example: dressed-quark-photon vertex
  - Contact interaction plus rainbow-ladder entails general form

$$\Gamma_\mu(k;Q) = \gamma_\mu^T P_T(Q^2) + \gamma_\nu^L P_L(Q^2)$$

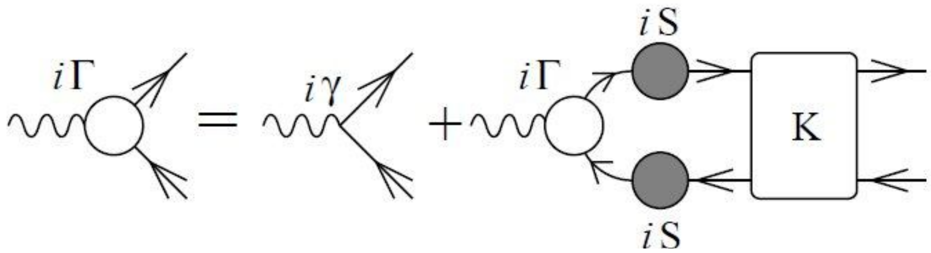
  - Vector Ward-Takahashi identity

$$Q_\mu i\Gamma_\mu(k;Q) = S^{-1}(k+Q/2) - S^{-1}(k-Q/2)$$

  - With symmetry-preserving regularisation of contact interaction, Ward Takahashi identity requires

$P_L(Q^2)=1$ & $P_T(Q^2=0)=1$

Interactions cannot generate an on-shell mass for the photon.



BSE – inhomogeneous vector vertex

$$\Gamma_\mu(Q) = \gamma_\mu - \frac{4}{3}\frac{1}{m_G^2}\int \frac{d^4q}{(2\pi)^4}\,\gamma_\alpha \chi_\mu(q_+,q)\gamma_\alpha$$

$$\Gamma_\mu(k;Q) = \gamma_\mu^T P_T(Q^2) + \gamma_\nu^L P_L(Q^2)$$

- Solution: $P_L(Q^2) = 1$
  Readily established using vector Ward-Green-Takahashi identity

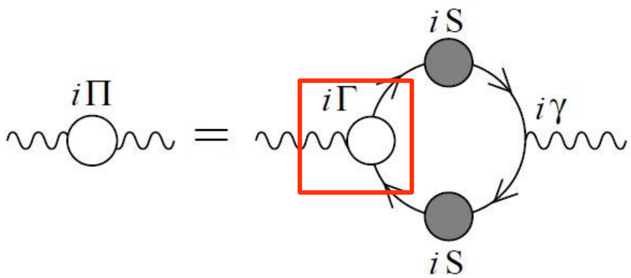
$$P_T(Q^2) = \frac{1}{1+K_\gamma(Q^2)}$$

$$K_\gamma(Q^2) = \frac{1}{3\pi^2 m_G^2} \times \int_0^1 d\alpha\, \alpha(1-\alpha) Q^2\, \overline{\mathcal{C}}_1^{iu}(\omega(M^2,\alpha,Q^2))$$

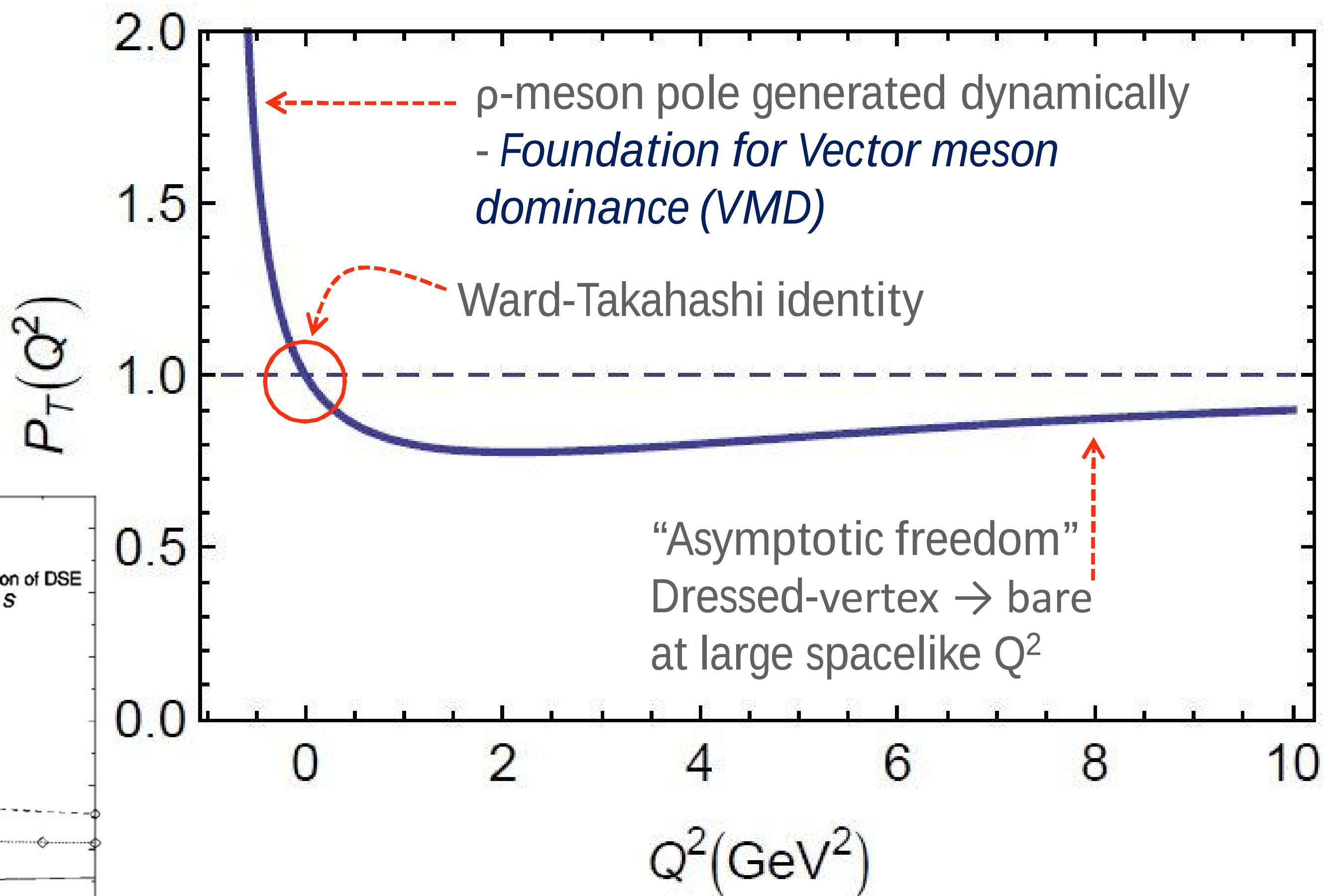
$\omega(x,\alpha,z) = x + \alpha(1-\alpha)z$

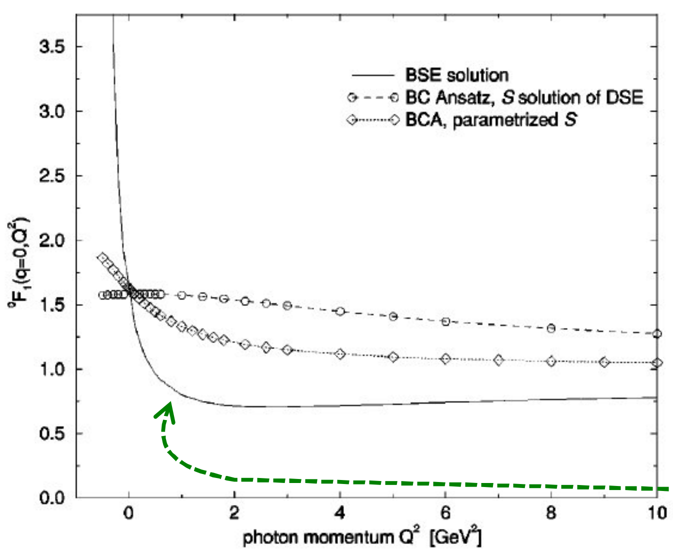
- Plainly: $P_T(Q^2=0) = 1$ because $K_\gamma(Q^2=0)=1$
  Again, power of vector Ward-Green-Takahashi identity revealed

Regularisation & Symmetries



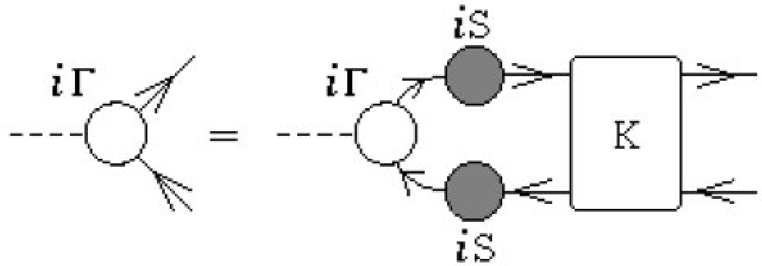
- Solved Bethe-Salpeter equation for dressed-quark photon vertex, regularised using symmetry-preserving scheme





Craig Roberts: Continuum strong QCD (V.83p)

Renormalisation-group-improved one-gluon exchange
Maris & Tandy prediction of $F_\pi(Q^2)$



Bethe-Salpeter Equations

- Ladder BSE for ρ-meson

$$1+K^{\rho}(-m_{1-}^2)=0\,,\;K^{\rho}(P^2)=\frac{1}{3\pi^2 m_G^2}\int_0^1 d\alpha\,\alpha(1-\alpha)P^2\,\overline{\mathcal{C}}_1^{iu}(\omega(M^2,\alpha,P^2))$$

$$\overline{C}_1^{iu}(\omega)=\Gamma(0,M^2r_{uv}^2)-\Gamma(0,M^2r_{ir}^2),C_1^{iu}(\omega)=\omega\overline{C}_1^{iu}(\omega)$$

$\omega(M^2,\alpha,P^2)= M^2 + \alpha(1- \alpha)P^2$

  - *Contact interaction, properly regularised,*
    provides a practical simplicity & physical transparency

- Ladder BSE for a_1-meson

$$1+K^{a_1}(-m_{1+}^2)=0\,,\;K^{a_1}(P^2)=-\frac{1}{3\pi^2 m_G^2}\int_0^1 d\alpha\,\mathcal{C}_1^{\rm iu}(\omega(M^2,\alpha,P^2))$$

- All BSEs are one- or –two dimensional eigenvalue problems, eigenvalue is P^2= - (mass-bound-state)^2

H.L.L. Roberts, L. Chang, I.C. Cloët and C.D. Roberts,
arXiv:1101.4244 [nucl-th], Few Body Syst. **51** (2011) pp. 1-25

Meson Spectrum -Ground-states

- Ground-state masses
  - Computed very often, always with same result

	m_π	m_ρ	m_σ	m_{a_1}
RL	0.14	0.93	0.74	1.08
experiment	0.14	0.78	0.4 – 1.2	1.24

But, we know how to fix that (Lecture IV)
viz., DCSB – beyond rainbow ladder
- increases scalar and axial-vector masses
- leaves π & ρ unchanged

- Namely, with rainbow-ladder truncation

$m_{a1} - m_\rho = 0.15$ GeV $\approx ⅓ \times 0.45_{\text{experiment}}$

	Experiment	Rainbow-ladder	One-loop corrected	Full vertex
a1	1230	759	885	1230
ρ	770	644	764	745
Mass splitting	455	115	121	485

H.L.L. Roberts, L. Chang, I.C. Cloët and C.D. Roberts,
arXiv:1101.4244 [nucl-th], Few Body Syst. **51** (2011) pp. 1-25

Meson Spectrum -Ground-states

- Ground-state masses
  - Correct for omission of DCSB-induced spin-orbit repulsion

	m_π	m_ρ	m_σ	m_{a_1}
RL	0.14	0.93	0.74	1.08
experiment	0.14	0.78	0.4 – 1.2	1.24

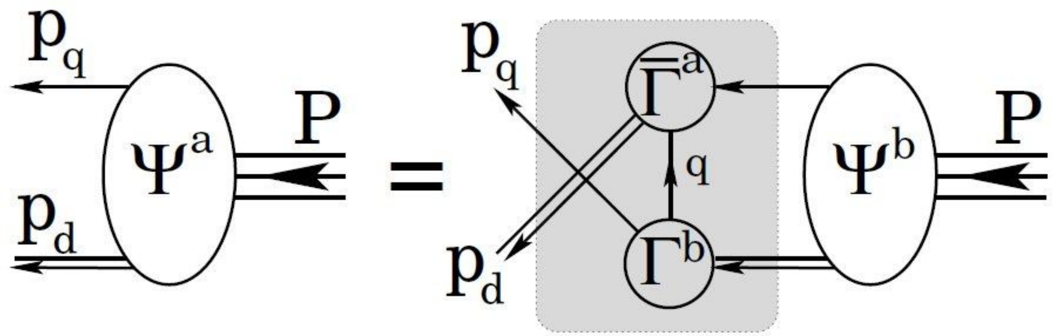
$m_\sigma{}^{qq} \approx 1.2$ GeV is location of quark core of σ-resonance:
- Pelaez & Rios (2006)
- Ruiz de Elvira, Pelaez, Pennington & Wilson (2010)

First novel post-diction

- Leave π- & ρ-meson BSEs unchanged but introduce repulsion parameter in scalar and axial-vector channels; viz.,

$$1 + K^{a_1}(-m_{1+}^2) = 0\,,\ K^{a_1}(P^2) = -\frac{\boxed{g_{SO}^2}}{3\pi^2 m_G^2}\int_0^1 d\alpha\, \mathcal{C}_1^{\rm iu}(\omega(M^2,\alpha,P^2))$$

- $g_{SO}=0.24$ fitted to produce $m_{a1} - m_\rho = 0.45_{\rm experiment}$



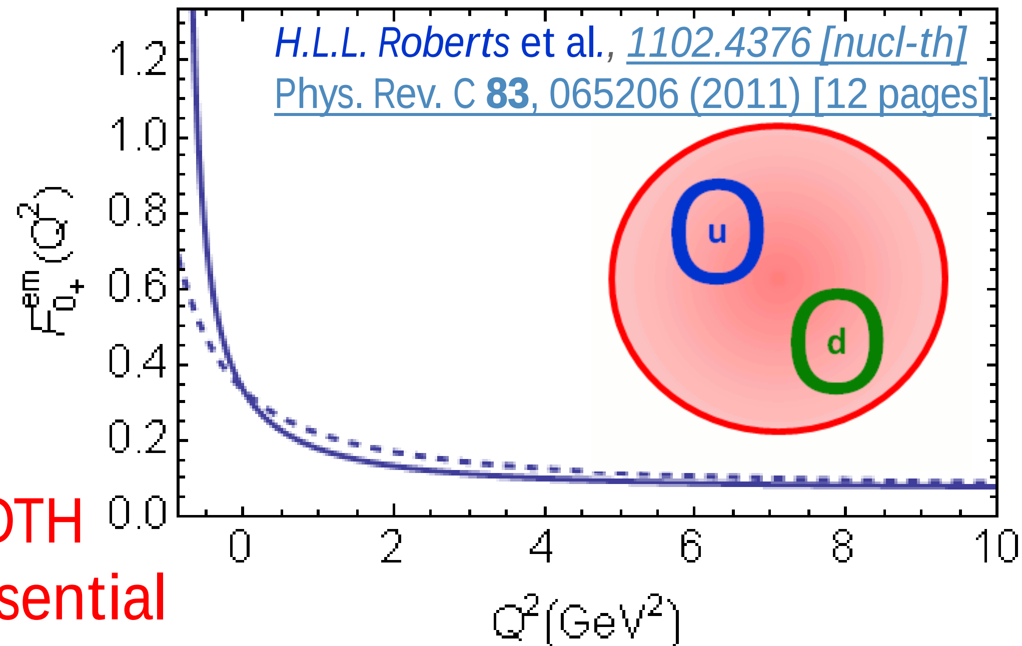
Diquarks in QCD

Masses of ground and excited-state hadrons
Hannes L.L. Roberts, Lei Chang, Ian C. Cloët and Craig D. Roberts, arXiv:1101.4244 [nucl-th]
Few Body Systems (2011) pp. 1-25

	$m_{qq_{0+}}$	$m_{qq_{1+}}$	$m_{qq_{0-}}$	$m_{qq_{1-}}$	$m_{qq^*_{0+}}$	$m_{qq^*_{1+}}$	$m_{qq^*_{0-}}$	$m_{qq^*_{1-}}$
RL	0.78	1.06	0.93	1.16	1.39 ± 0.06	1.32 ± 0.05	1.42 ± 0.05	1.33 ± 0.05
RL $* g^2_{SO}$	0.78	1.06	1.37	1.45	1.39 ± 0.06	1.32 ± 0.05	1.50 ± 0.03	1.52 ± 0.02

- "Spectrum" of *nonpointlike* quark-quark correlations
- Observed in
  - DSE studies in QCD
  - 0^+ & 1^+ in Lattice-QCD
- Scalar diquark form factor
  - $r_{0+} \approx r_\pi$
- Axial-vector diquarks
  - $r_{1+} \approx r_\rho$

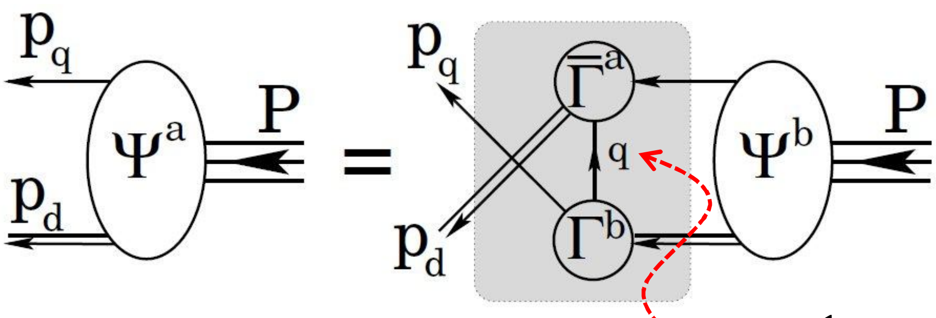
BOTH essential



- *Zero* relation with old notion of pointlike constituent-like diquarks

H.L.L. Roberts, L. Chang, I.C. Cloët and C.D. Roberts,
[arXiv:1101.4244 [nucl-th]](arXiv:1101.4244), Few Body Syst. **51** (2011) pp. 1-25

Spectrum of Baryons



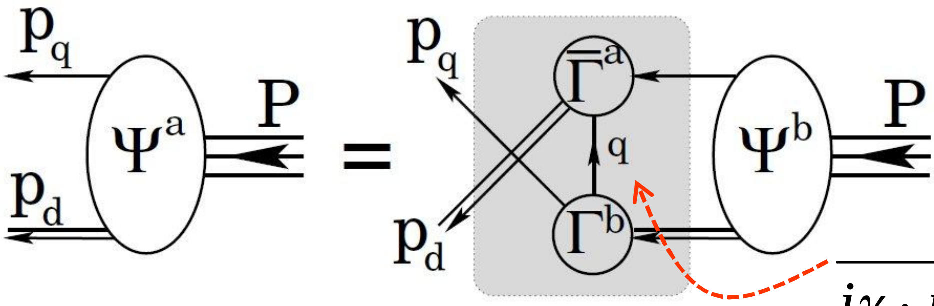
$$\frac{1}{i\gamma\cdot p + M} \to \frac{g_B^2}{M}$$

Variant of:
A. Buck, R. Alkofer & H. Reinhardt, Phys. Lett. ***B286*** *(1992) 29.*

- Static "approximation"
  - Implements analogue of contact interaction in Faddeev-equation
- In combination with contact-interaction diquark-correlations, generates Faddeev equation kernels which themselves are momentum-independent
- The merit of this truncation is the *dramatic simplifications* which it produces
- Used widely in hadron physics phenomenology; e.g., Bentz, Cloët, Thomas *et al*.

$$\overline{C}_1^{iu}(\omega) = \Gamma(0, M^2 r_{uv}^2) - \Gamma(0, M^2 r_{ir}^2), C_1^{iu}(\omega) = \omega \overline{C}_1^{iu}(\omega)$$

Spectrum of Baryons



$$\frac{1}{i\gamma \cdot p + M} \rightarrow \frac{g_B^2}{M}$$

With the right glasses; i.e., those polished by experience with the DSEs, one can look at this equation and see that increasing the current-quark mass will boost the mass of the bound-state

- Faddeev equation for Δ-resonance

$$1 = 8 \frac{g_\Delta^2}{M} \frac{E_{qq_1+}^2}{m_{qq_1+}^2} \int \frac{d^4\ell'}{(2\pi)^4} \int_0^1 d\alpha \, \frac{(m_{qq_{1+}}^2 + (1-\alpha)^2 m_\Delta^2)(\alpha m_\Delta + M)}{[\ell'^2 + \sigma_\Delta(\alpha, M, m_{qq_{1+}}, m_\Delta)]^2}$$

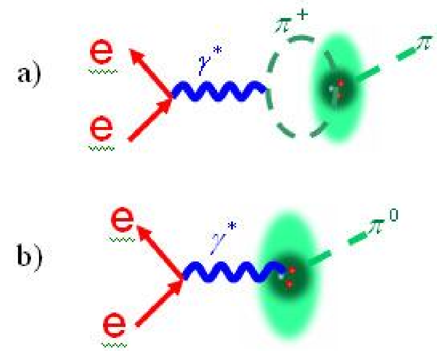
$$= \frac{g_\Delta^2}{M} \frac{E_{qq_1+}^2}{m_{qq_1+}^2} \frac{1}{2\pi^2} \int_0^1 d\alpha \, (m_{qq_{1+}}^2 + (1-\alpha)^2 m_\Delta^2)(\alpha m_\Delta + M) \overline{\mathcal{C}}_1^{\mathrm{iu}}(\sigma_\Delta(\alpha, M, m_{qq_{1+}}, m_\Delta))$$

- *One-dimensional eigenvalue problem, to which only the axial-vector diquark contributes*
- *Nucleon has scalar & axial-vector diquarks. It is a five-dimensional eigenvalue problem*

What's missing?

H.L.L. Roberts, L. Chang, I.C. Cloët and C.D. Roberts, arXiv:1101.4244 [nucl-th], Few Body Syst. **51** (2011) pp. 1-25

Pion cloud



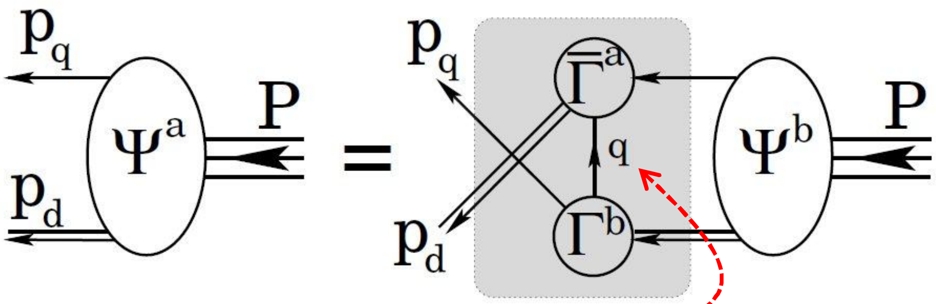
- Kernels constructed in the rainbow-ladder truncation do not contain any long-range interactions
  - These kernels are built only from dressed-quarks and -gluons
- But, QCD produces a very potent long-range interaction; namely that associated with the pion, without which no nuclei would be bound and we wouldn't be here
- The rainbow-ladder kernel produces what we describe as the hadron's dressed-quark core
- The contribution from pions is omitted, and can be added without "double counting"
- The pion contributions must be thoughtfully considered before any comparison can be made with the real world

H.L.L. Roberts, L. Chang, I.C. Cloët and C.D. Roberts,
arXiv:1101.4244 [nucl-th], Few Body Syst. **51** (2011) pp. 1-25
M.B. Hecht, M. Oettel, C.D. Roberts, S.M. Schmidt,
P.C.Tandy and A.W. Thomas, Phys.Rev. C65 (2002) 055204

Pion cloud

Pion cloud typically reduces a hadron's mass

- The body of results described hitherto suggest that whilst corrections to our truncated DSE kernels may have a material impact on m_N and m_Δ separately, the modification of each is approximately the same, so that the mass-difference, δm, is largely unaffected by such corrections.
- This is consistent with analyses that considers the effect of pion loops, which are explicitly excluded in the rainbow-ladder truncation: whilst the individual masses are reduced by roughly 300MeV, the mass difference, δm, increases by only 50MeV.
- With the educated use of appropriate formulae, one finds that pion-loops yields a shift of (-300MeV) in m_N and (-270MeV) in m_Δ, from which one may infer that the uncorrected Faddeev equations should produce $m_N = 1.24$GeV and $m_\Delta = 1.50$GeV



$$\frac{1}{i\gamma\cdot p + M} \to \frac{g_B^2}{M}$$

Pion cloud

All three spectrum parameters now fixed ($g_{SO}=0.24$)

- One can actually do better owing to the existence of the Excited Baryon Analysis Center (EBAC), which for five years has worked to understand and quantify the effects of a pseudoscalar meson cloud on baryons
- For the Δ-resonance, EBAC's calculations indicate that the dressed-quark core of this state should have

$$m_\Delta^{qqq} = 1.39\,\text{GeV}$$

- These observations indicate that the dressed-quark core Faddeev equations should yield

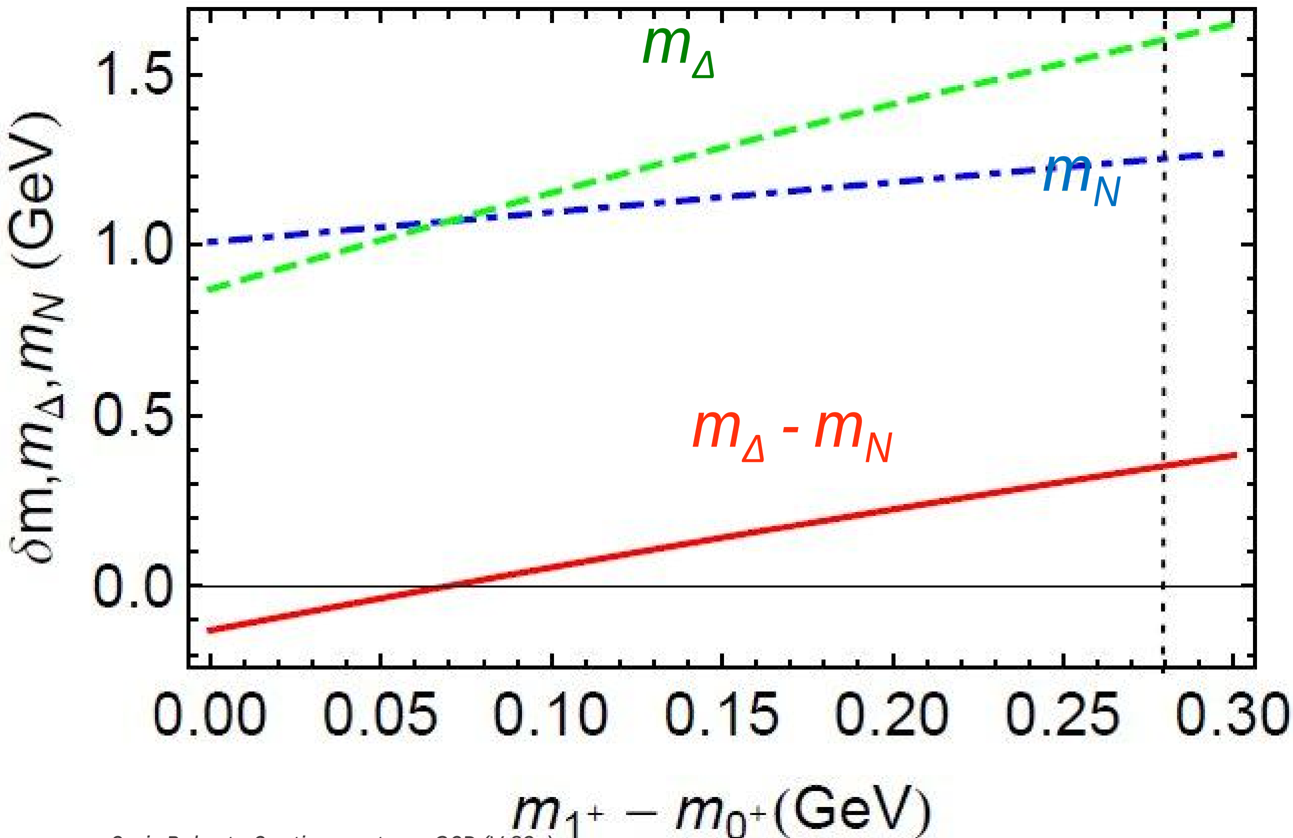
$$m_N = 1.14\,\text{GeV},\ m_\Delta = 1.39\,\text{GeV},\ \delta m = 0.25\,\text{GeV}$$

which requires $g_N = 1.18$, $g_\Delta = 1.56$

Masses of ground and excited-state hadrons
Hannes L.L. Roberts, Lei Chang, Ian C. Cloët
and Craig D. Roberts, arXiv:1101.4244 [nucl-th]
Few Body Systems (2011) pp. 1-25

Baryons & diquarks

- From apparently simple material, one arrives at a powerful elucidative tool, which provides numerous insights into baryon structure; e.g.,
  - *There is a causal connection between m_Δ - m_N & m_{1+}- m_{0+}*



Physical splitting grows rapidly with increasing diquark mass difference

Baryons & diquarks

- Provided numerous insights into baryon structure; e.g.,
  - $m_N \approx 3\,M$ & $m_\Delta \approx M + m_{1+}$

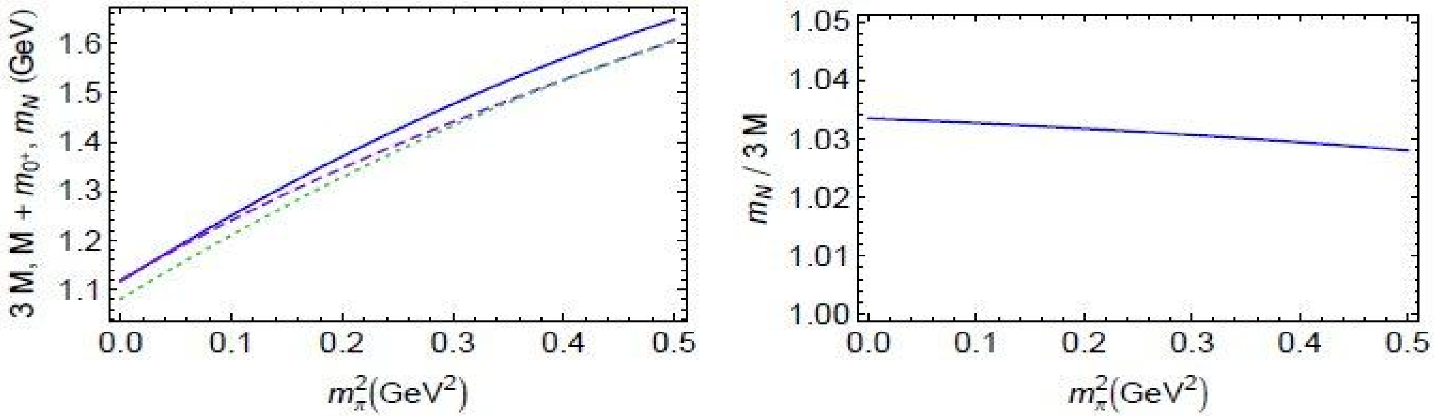


Fig. 3 *Left panel* – Evolution with current-quark mass of the: nucleon mass, m_N (*solid curve*); the sum $[M + m_{qq_{0^+}}]$ (*dashed curve*); and $3M$ (*dotted curve*). *Right panel* – Evolution with current-quark mass of the ratio $m_N/[3M]$, which varies by less-than 1% on the domain depicted.

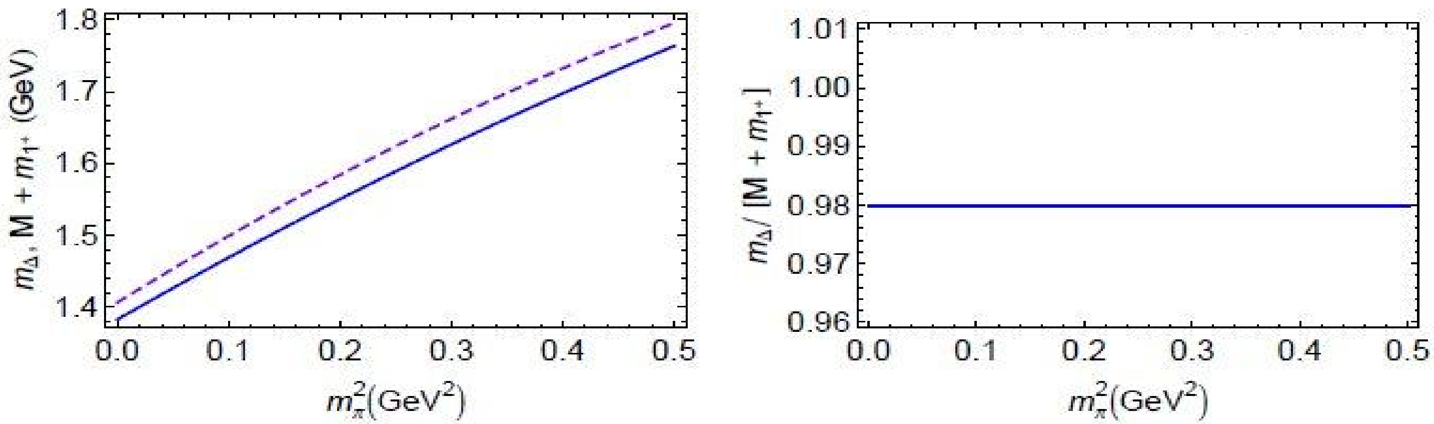
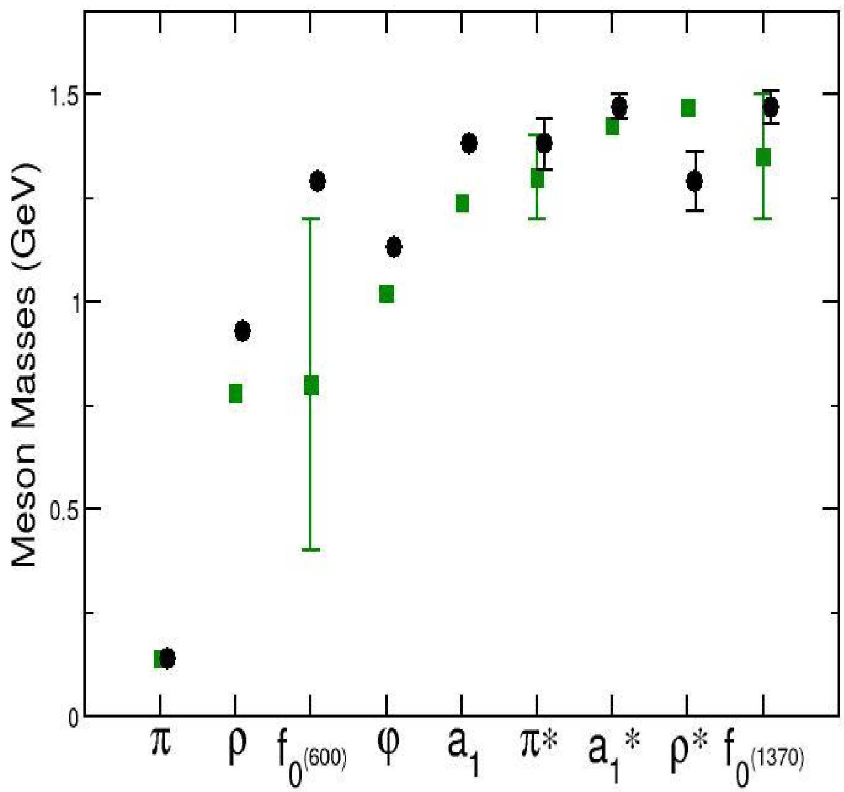


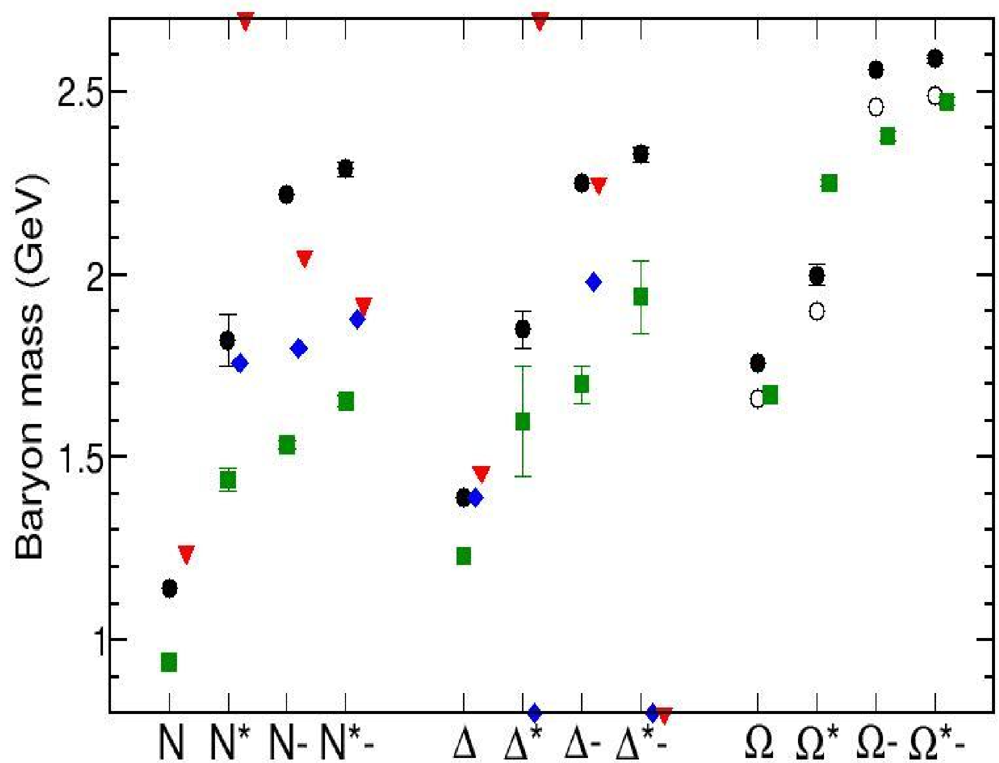
Fig. 4 *Left panel* – Evolution with current-quark mass of the: Δ mass, m_Δ (*solid curve*); and $[M + m_{qq_{1^+}}]$ (dashed curve). *Right panel* – Evolution with current-quark mass of the ratio $m_\Delta/[M + m_{qq_{1^+}}]$, which does not vary noticeably on the domain depicted.

Legend:
Particle Data Group
H.L.L. Roberts *et al*.
EBAC
Jülich

Hadron Spectrum

- o Symmetry-preserving unification of the computation of meson & baryon masses
- o rms-rel.err./deg-of-freedom = 13%
- o PDG values (almost) uniformly overestimated in both cases - room for the pseudoscalar meson cloud?!





Masses of ground and excited-state hadrons
H.L.L. Roberts *et al*., arXiv:1101.4244 [nucl-th]
Few Body Systems (2011) pp. 1-25

Baryon Spectrum

Table 4 *Row-1*: Dressed-quark-core masses for nucleon and Δ, their first radial excitations (denoted by "$*$"), and the parity-partners of these states, computed with $g_N = 1.18$, $g_\Delta = 1.56$, and the parameter values in Eq. (25) and Table 1. The errors on the masses of the radial excitations indicate the effect of shifting the location of the zero according to Eq. (30). *Row-2*: Bare-masses inferred from a coupled-channels analysis at the Excited Baryon Analysis Center (EBAC) [65]. EBAC's method does not provide a bare nucleon mass. *Row-3*: Bare masses inferred from the coupled-channels analysis described in Ref. [67], which describes the Roper resonance as dynamically-generated. In both these rows, "..." indicates states not found in the analysis. A visual comparison of these results is presented in Fig. 7.

	m_N	m_{N^*}	$m_{N\frac{1}{2}^-}$	$m_{N^*\frac{1}{2}^-}$	m_Δ	m_{Δ^*}	$m_{\Delta\frac{3}{2}^-}$	$m_{\Delta^*\frac{3}{2}^-}$
PDG label	N	$N(1440)\,P_{11}$	$N(1535)\,S_{11}$	$N(1650)\,S_{11}$	$\Delta(1232)\,P_{33}$	$\Delta(1600)\,P_{33}$	$\Delta(1700)\,D_{33}$	$\Delta(1940)\,D_{33}$
This work	1.14	1.82±0.07	2.22	2.29 ± 0.02	1.39	1.85 ± 0.05	2.25	2.33 ± 0.02
EBAC		1.76	1.80	1.88	1.39	...	1.98	...
Jülich	1.24	none	2.05	1.92	1.46	...	2.25	...

- In connection with EBAC's analysis, dressed-quark Faddeev-equation predictions for bare-masses agree within rms-relative-error of 14%.
  - *Notably, EBAC finds a dressed-quark-core for the Roper resonance, at a mass which agrees with Faddeev Eq. prediction.*

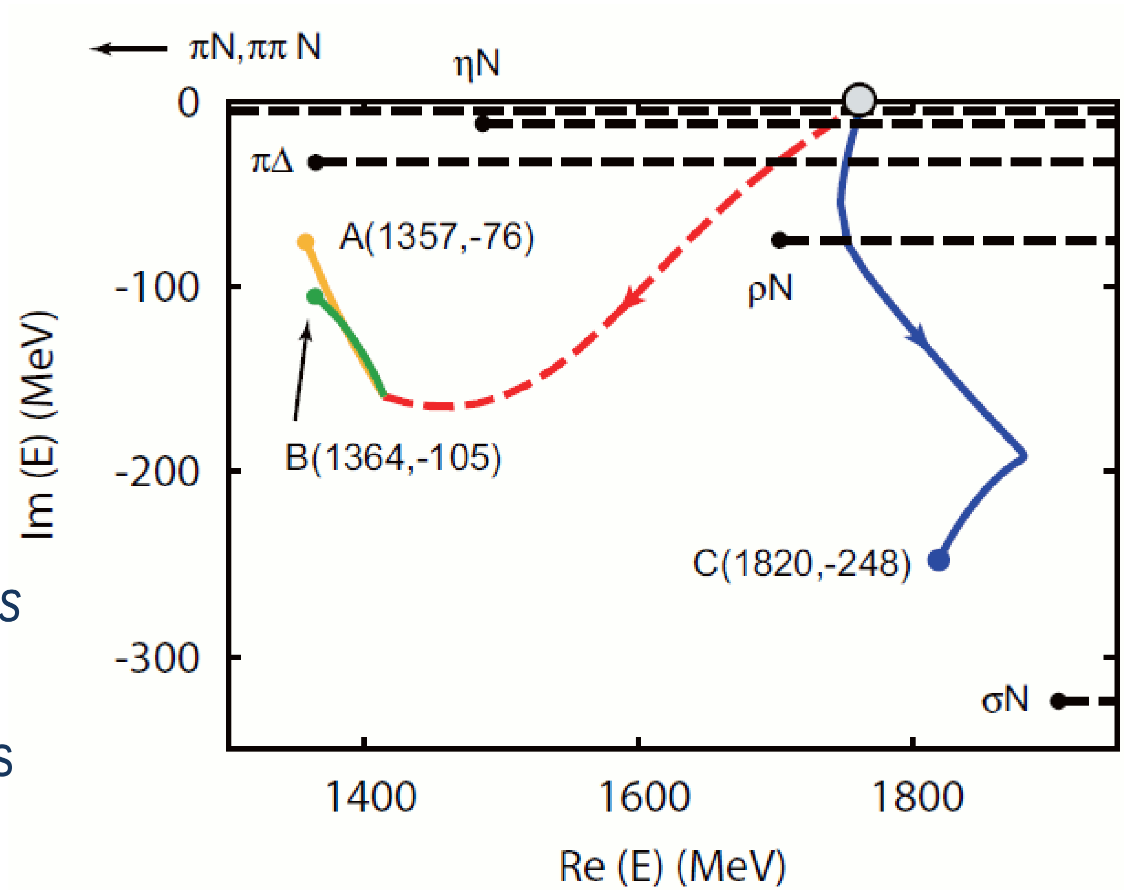
Roper Resonance

- Consider the $N(1440)P_{11}$, $J^P = (1/2)^+$ "Roper resonance," whose discovery was reported in 1964 – part of Roper's PhD thesis
- In important respects the Roper appears to be a copy of the proton. However, its (Breit-Wigner) mass is 50% greater.
- Features of the Roper have long presented a problem within the context of constituent-quark models formulated in terms of color-spin potentials, which typically produce a mass of $2\ M_N$ and the following level ordering:
  - ground state, $J^P = (1/2)^+$ with radial quantum number $n = 0$ and angular momentum $l = 0$;
  - first excited state, $J^P = (1/2)^-$ with $(n, l) = (0, 1)$;
  - second excited state, $J^P = (1/2)^+$, with $(n, l) = (1, 0)$; *etc.*
- The difficulty is that the lightest $l = 1$ *baryon appears to be the* $N(1535)S_{11}$, which is heavier than the Roper!

N. *Suzuki* et al., [Phys.Rev.Lett. 104 (2010) 042302]

EBAC & the Roper resonance

- EBAC examined the dynamical origins of the two poles associated with the Roper resonance are examined.
- Both of them, together with the next higher resonance in the P_{11} partial wave were found to have the same originating bare state
- Coupling to the meson-baryon continuum induces multiple observed *resonances* from the same bare state.
- All PDG identified resonances consist of a core state and meson-baryon components.



EBAC
& the Roper resonance

- *Nuclear Physics: Exploring the Heart of Matter* Decadal Report, issued 2012, by the National Academy of Sciences
  - In a recent breakthrough, theorists at the Excited Baryon Analysis Center (EBAC) at Jefferson Lab

    – led by T.-S. H. Lee, Argonne –

    demonstrated that the Roper resonance is the proton's first radial excitation, with its lower-than-expected mass coming from a quark core shielded by a dense cloud of pions and other mesons.
  - This breakthrough was enabled by both new analysis tools and new high quality data from the CLAS-Collaboration.

Recapitulation

- One method by which to validate QCD is computation of its hadron spectrum and subsequent comparison with modern experiment. Indeed, this is an integral part of the international effort in nuclear physics.
- For example, the N^* programme and the search for hybrid and exotic mesons together address the questions:
  - which hadron states and resonances are produced by QCD?
  - how are they constituted?
- This intense effort in hadron spectroscopy is a motivation to extend the research just described and treat ground- and excited-state hadrons with *s*-quark content. (New experiments planned in Japan)
- Key elements in a successful spectrum computation are:
  - symmetries and the pattern by which they are broken;
  - the mass-scale associated with confinement and DCSB;
  - and full knowledge of the physical content of bound-state kernels.

All this is provided by the DSE approach.

arXiv:1204.2553 [nucl-th], *Spectrum of hadrons with strangeness,* Chen Chen, L. Chang, C.D. Roberts, Shaolong Wan and D.J. Wilson, Few Body Syst. (2012) DOI: 10.1007/s00601-012-0466-3

Spectrum of Hadrons with Strangeness

- Solve gap equation for *u* & *s*-quarks

Table 1 Computed dressed-quark properties, required as input for the Bethe-Salpeter and Faddeev equations, and computed values for in-hadron condensates [52; 53; 54]. All results obtained with $\alpha_{\mathrm{IR}} = 0.93\pi$ and (in GeV) $\Lambda_{\mathrm{ir}} = 0.24$, $\Lambda_{\mathrm{uv}} = 0.905$. N.B. These parameters take the values determined in the spectrum calculation of Ref. [6]; and we assume isospin symmetry throughout. (All dimensioned quantities are listed in GeV.)

m_u	m_s	m_s/m_u	M_0	M_u	M_s	M_s/M_u	$\kappa_0^{1/3}$	$\kappa_\pi^{1/3}$	$\kappa_K^{1/3}$
0.007	0.17	24.3	0.36	0.37	0.53	1.43	0.241	0.243	0.246

- Input ratio $m_s/m_u = 24$ is consistent with modern estimates
- Output ratio $M_s/M_u = 1.43$ shows dramatic impact of DCSB, even on the *s*-quark
- κ = in-hadron condensate rises slowly with mass of hadron

[arXiv:1204.2553 [nucl-th]](arXiv:1204.2553), *Spectrum of hadrons with strangeness,* Chen Chen, L. Chang, C.D. Roberts, Shaolong Wan and D.J. Wilson

Spectrum of Mesons with Strangeness

- Solve Bethe-Salpeter equations for **mesons** and diquarks

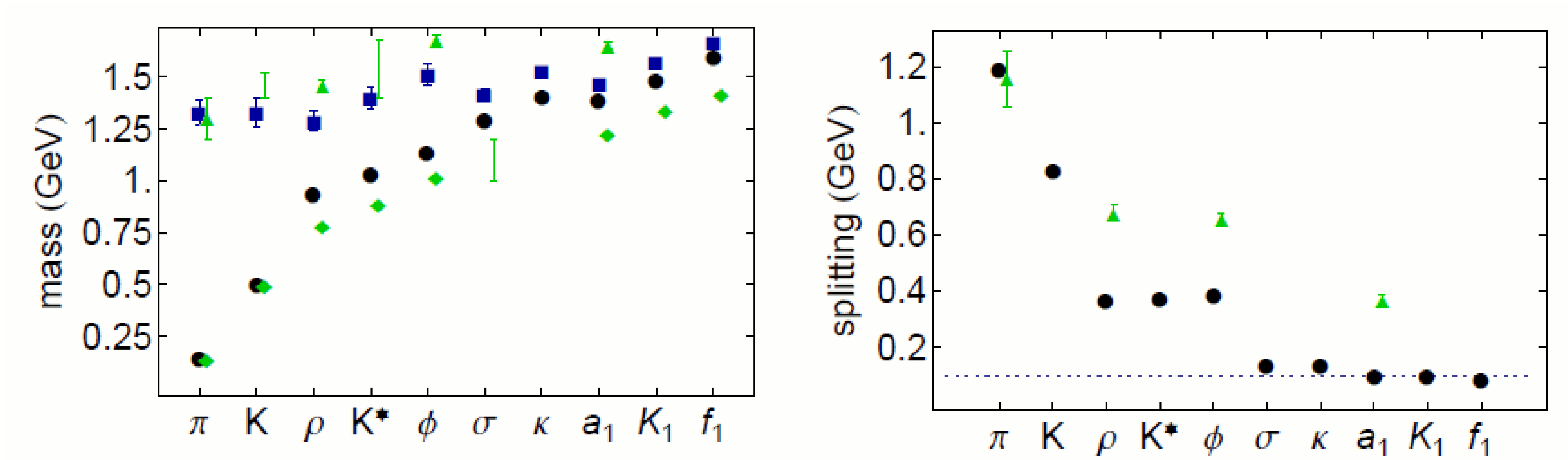
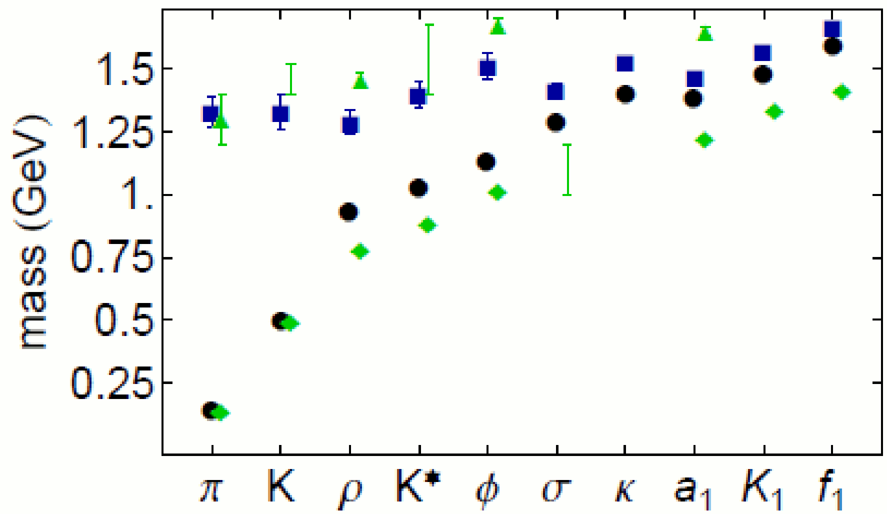


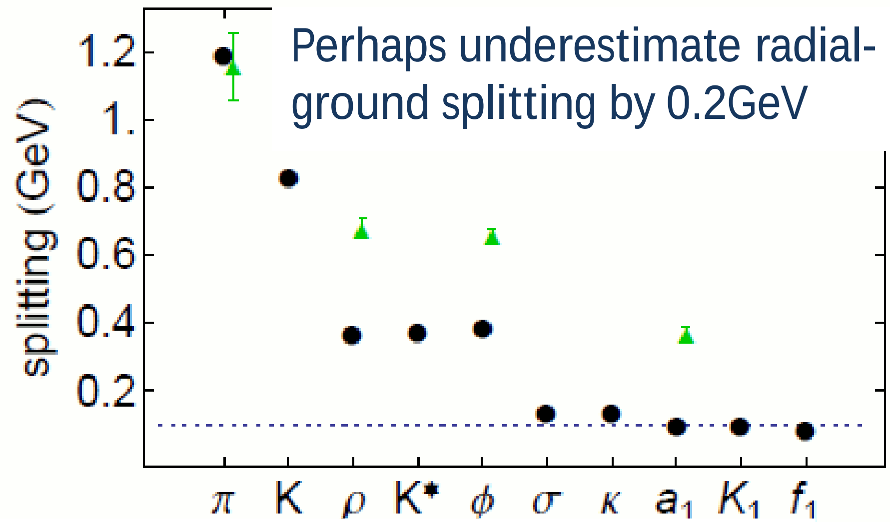
Fig. 2 Left panel: Pictorial representation of Table 2. *Circles* – computed ground-state masses; *squares* – computed masses of radial excitations; *diamonds* – empirical ground-state masses in Row 2; and *triangles* – empirical radial excitation masses in Row 4. Right panel: *Circles* – computed splitting between the first radial excitation and ground state in each channel; and *triangles* – empirical splittings, where they are known. The *dashed line* marks a splitting of 0.1 GeV.

arXiv:1204.2553 [nucl-th], *Spectrum of hadrons with strangeness,* Chen Chen, L. Chang, C.D. Roberts, Shaolong Wan and D.J. Wilson

Spectrum of Mesons with Strangeness

- Solve Bethe-Salpeter equations for **mesons** and diquarks





Perhaps underestimate radial-ground splitting by 0.2GeV

Fig. 2 comput empiric excitat dashed

res — les — adial The

- ✓ Computed values for ground-states are greater than the empirical masses, where they are known.
- ✓ Typical of DCSB-corrected kernels that omit resonant contributions; i.e., do not contain effects that may phenomenologically be associated with a meson cloud.

[arXiv:1204.2553 [nucl-th]](), *Spectrum of hadrons with strangeness,* Chen Chen, L. Chang, C.D. Roberts, Shaolong Wan and D.J. Wilson

Spectrum of Diquarks with Strangeness

- Solve Bethe-Salpeter equations for mesons and **diquarks**

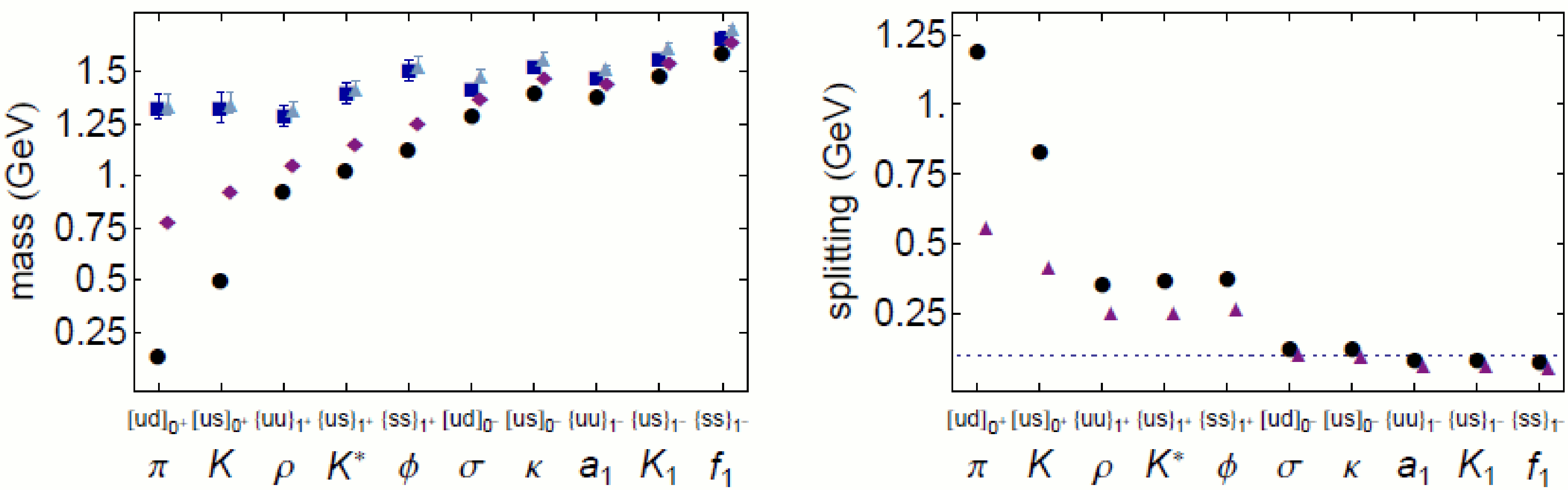
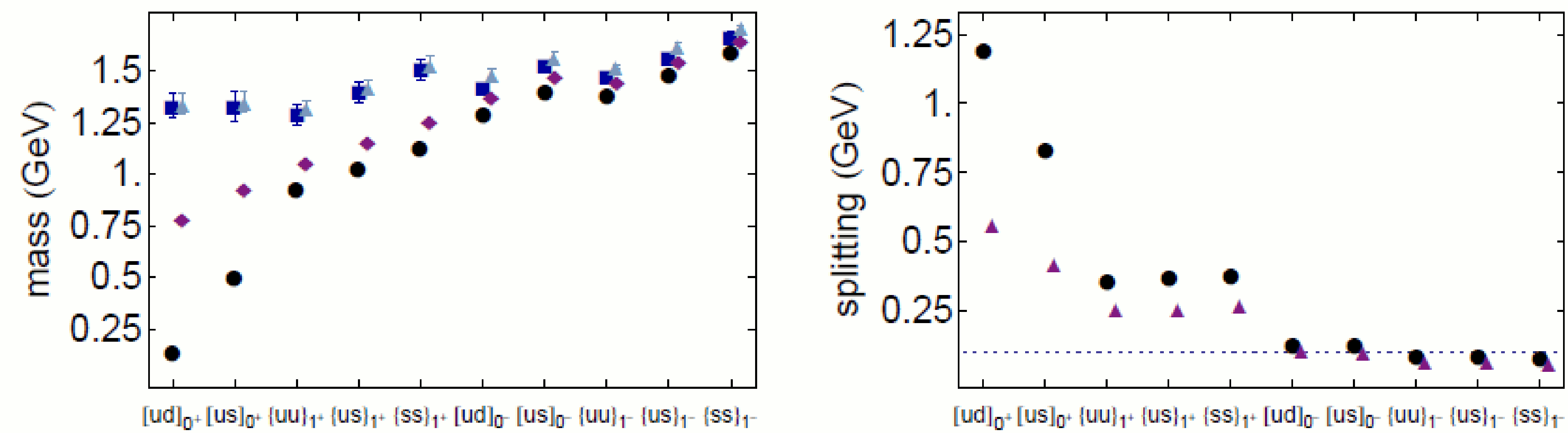


Fig. 3 Left panel: Pictorial representation of Table 4. *Diamonds* – ground-state diquark masses in Row 1; *circles* – ground-state meson masses in Row 2; *triangles* – masses of diquark first radial excitations in Row 3; and *squares* – masses of meson radial excitations in Row 4. Right panel: *Diamonds* – for diquarks, computed splittings between first radial excitation and ground state; and *circles* – for mesons, computed splitting between the first radial excitation and ground state in each channel. The *dashed line* marks a splitting of 0.1 GeV.

arXiv:1204.2553 [nucl-th], *Spectrum of hadrons with strangeness*, Chen Chen, L. Chang, C.D. Roberts, Shaolong Wan and D.J. Wilson

Spectrum of Diquarks with Strangeness

➢ Solve Bethe-Salpeter equations for mesons and **diquarks**



✓ Level ordering of diquark correlations is same as that for mesons.

✓ In all diquark channels, except scalar, mass of diquark's partner meson is a fair guide to the diquark's mass:

- o Meson mass bounds the diquark's mass from below;
- o Splitting always less than 0.13GeV and decreases with increasing meson mass

✓ Scalar channel "special" owing to DCSB

arXiv:1204.2553 [nucl-th], *Spectrum of hadrons with strangeness,* Chen Chen, L. Chang, C.D. Roberts, Shaolong Wan and D.J. Wilson

Bethe-Salpeter amplitudes

- Bethe-Salpeter amplitudes are couplings in Faddeev Equation

Table 3 The structure of meson Bethe-Salpeter amplitudes is described in Sect. 2.2.1 and App. B. Here we list the canonically normalised amplitude associated with each of the BSE eigenstates in Table 2. Only pseudoscalar mesons involve two independent amplitudes when a vector×vector contact interaction is treated systematically in rainbow-ladder truncation.

		m_π	m_K	m_ρ	m_{K^*}	m_ϕ	m_σ	m_κ	m_{a_1}	m_{K_1}	m_{f_1}
n=0	$E_{q\bar q}$	3.60	3.86	1.53	1.62	1.74	0.47	0.47	0.31	0.31	0.31
	$F_{q\bar q}$	0.48	0.60								
n=1	$E_{q\bar q}$	0.83	0.76	0.72	0.70	0.66	0.34	0.35	0.28	0.28	0.28
	$F_{q\bar q}$	0.05	1.18								

- Magnitudes for diquarks follow precisely the meson pattern

Table 5 The structure of diquark Bethe-Salpeter amplitudes is described in Sect. 2.2.2 and App. B. Here we list all canonically normalised amplitudes that are relevant to the baryons we consider. Only scalar diquarks involve two independent amplitudes.

	$[u,d]_{0^+}$	$[s,u]_{0^+}$	$\{u,u\}_{1^+}$	$\{s,u\}_{1^+}$	$\{s,s\}_{1^+}$	$[u,d]_{0^-}$	$[s,u]_{0^-}$	$\{u,u\}_{1^-}$	$\{s,u\}_{1^-}$	$\{s,s\}_{1^-}$
E_{qq}	2.74	2.91	1.30	1.36	1.42	0.40	0.39	0.27	0.27	0.26
F_{qq}	0.31	0.40								

Owing to DCSB, FE couplings in ½⁻ channels are 25-times weaker than in ½⁺ !

arXiv:1204.2553 [nucl-th], *Spectrum of hadrons with strangeness,* Chen Chen, L. Chang, C.D. Roberts, Shaolong Wan and D.J. Wilson

Spectrum of Baryons with Strangeness

- Solved all Faddeev equations, obtained masses and eigenvectors of the octet and decuplet baryons.

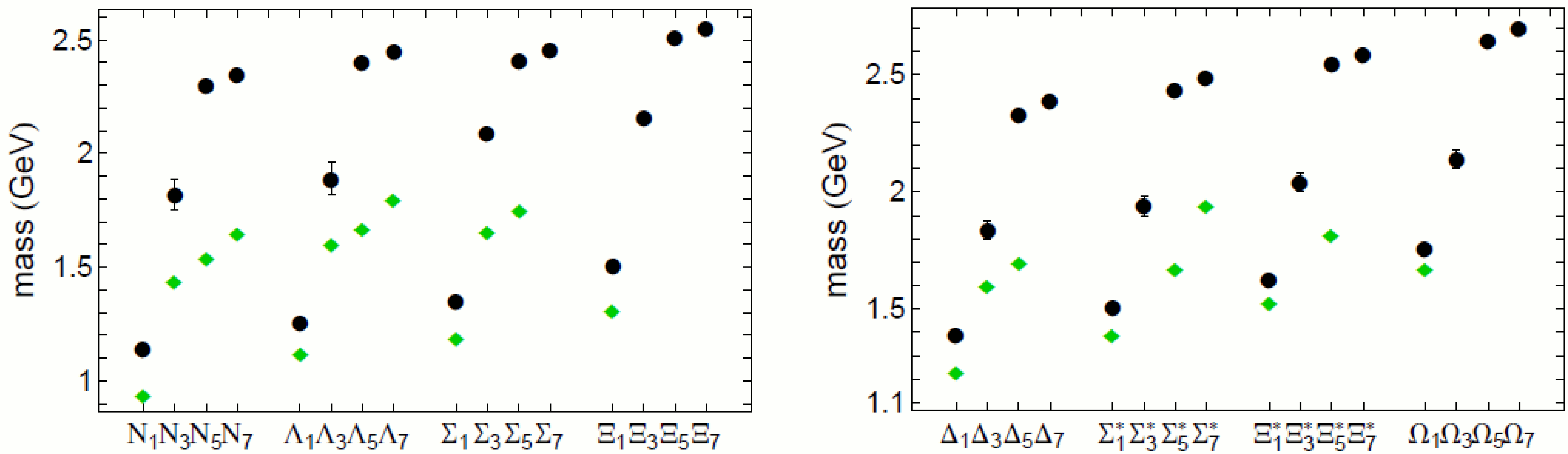


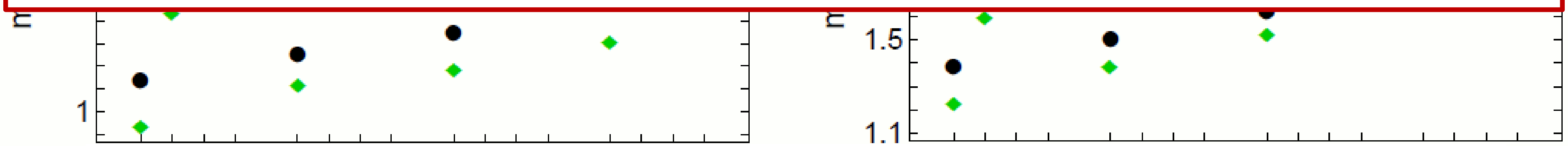
Fig. 4 Left panel: Pictorial representation of octet masses in Table 6. *Circles* – computed masses; and *diamonds* – empirical masses. On the horizontal axis we list a particle name with a subscript that indicates its row in the table; e.g., N_1 means nucleon column, row 1. In this way the labels step through ground-state, radial excitation, parity partner, parity partner's radial excitation. Right panel: Analogous plot for the decuplet masses in Table 6.

arXiv:1204.2553 [nucl-th], *Spectrum of hadrons with strangeness,* Chen Chen, L. Chang, C.D. Roberts, Shaolong Wan and D.J. Wilson

Spectrum of Baryons with Strangeness

- Solved all Faddeev equations, obtained masses and eigenvectors of the octet and decuplet baryons.

	$N_{940}P_{11}$	$N_{1440}P_{11}$	$N_{1535}S_{11}$	$N_{1650}S_{11}$	$\Delta_{1232}P_{33}$	$\Delta_{1700}D_{33}$
Table 6 (DSE)	1.14	$1.82_{0.07}$	2.30	$2.35_{0.01}$	1.39	2.33
M_B^0 Jülich])	1.24		2.05	1.92	1.46	2.25
M_B^0 EBAC])		1.76	1.80	1.88	1.39	1.98



- As with mesons, computed baryon masses lie uniformly above the empirical values.
- Success because our results are those for the baryons' dressed-quark cores, whereas empirical values include effects associated with meson-cloud, which typically produce sizable reductions.

arXiv:1204.2553 [nucl-th], *Spectrum of hadrons with strangeness,* Chen Chen, L. Chang, C.D. Roberts, Shaolong Wan and D.J. Wilson

Structure of Baryons with Strangeness

- Baryon structure is flavour-blind

Table 7 Contact interaction Faddeev amplitudes for each of the octet baryons and their low-lying excitations. The superscript in the expression s^i or a^i is a diquark enumeration label associated with Eq. (31), except for $[2,3]$ and $[6,8]$, which are the $I=0$ combinations in Eq. (49).

Diquark content

		s^1	s^2	$s^{[2,3]}$	a_1^4	a_2^4	a_1^5	a_2^5	a_1^6	a_2^6	$a_1^{[6,8]}$	$a_2^{[6,8]}$	a_1^9	a_2^9	$P_{J=0}$
$(P=+,n=0)$	N	0.88			-0.38	0.27	-0.06	0.04							78%
	Λ	0.67		-0.27							-0.45	-0.09			79%
	Σ		0.85		-0.45	0.26			0.12	0.02					72%
	Ξ		0.91		0.14	0.08							0.39	0.00	82%
$(P=+,n=1)$	N	-0.02			0.52	-0.37	-0.63	0.44							0%
	Λ	0.03		0.06							-0.78	0.63			0%
	Σ		0.00		-0.04	0.02			0.83	-0.55					0%
	Ξ		0.00		0.01	-1.00							-0.02	0.06	0%
$(P=-,n=0)$	N	0.71			-0.41	0.29	0.41	-0.29							50%
	Λ	0.64		0.44							-0.47	0.42			61%
	Σ		0.61		-0.47	0.23			0.55	-0.21					38%
	Ξ		0.76		-0.34	0.35							0.33	-0.28	58%
$(P=-,n=1)$	N	0.66			-0.41	0.29	0.45	-0.32							44%
	Λ	0.60		0.43							-0.48	0.47			55%
	Σ		0.57		-0.47	0.23			0.58	-0.24					33%
	Ξ		0.73		-0.34	0.37							0.33	-0.31	54%

arXiv:1204.2553 [nucl-th], *Spectrum of hadrons with strangeness,* Chen, Chang, Roberts, Wan and Wilson & *Nucleon and Roper em elastic and transition form factors,* D. J. Wilson, I. C. Cloët, L. Chang and C. D. Roberts, arXiv:1112.2212 [nucl-th], Phys. Rev. C**85** (2012) 025205 [21 pages]

Structure of Baryons with Strangeness

- Baryon structure is flavour-blind

Table 7 Contact interaction Faddeev amplitudes for each of the octet baryons and their low-lying excitations. The superscript in the expression s^i or a^i is a diquark enumeration label associated with Eq. (31), except for [2, 3] and [6, 8], which are the $I = 0$ combinations in Eq. (49).

Diquark content

		s^1	s^2	$s^{[2,3]}$	a_1^4	a_2^4	a_1^5	a_2^5	a_1^6	a_2^6	$a_1^{[6,8]}$	$a_2^{[6,8]}$	a_1^9	a_2^9	$P_{J=0}$
$(P=+,n=0)$	N														78%
80%	Λ														79%
	Σ														72%
	Ξ														82%
$(P=+,n=1)$	N														0%
0%	Λ														0%
	Σ														0%
	Ξ														0%
$(P=-,n=0)$	N														50%
50%	Λ														61%
	Σ														38%
	Ξ														58%
$(P=-,n=1)$	N														44%
50%	Λ														55%
	Σ														33%
	Ξ														54%

- $J_{qq}=0$ content of $J=½$ baryons is almost independent of their flavour structure
- *Radial excitation of ground-state octet possess zero scalar diquark content!*
- *This is a consequence of DCSB*
  - *Ground-state $(1/2)^+$ possess unnaturally large scalar diquark content*
  - *Orthogonality forces radial excitations to possess (almost) none at all!*

arXiv:1204.2553 [nucl-th], *Spectrum of hadrons with strangeness,* Chen Chen, L. Chang, C.D. Roberts, Shaolong Wan and D.J. Wilson

Spectrum of Hadrons with Strangeness

- Solved all Faddeev equations, obtained masses and eigenvectors of the octet and decuplet baryons.

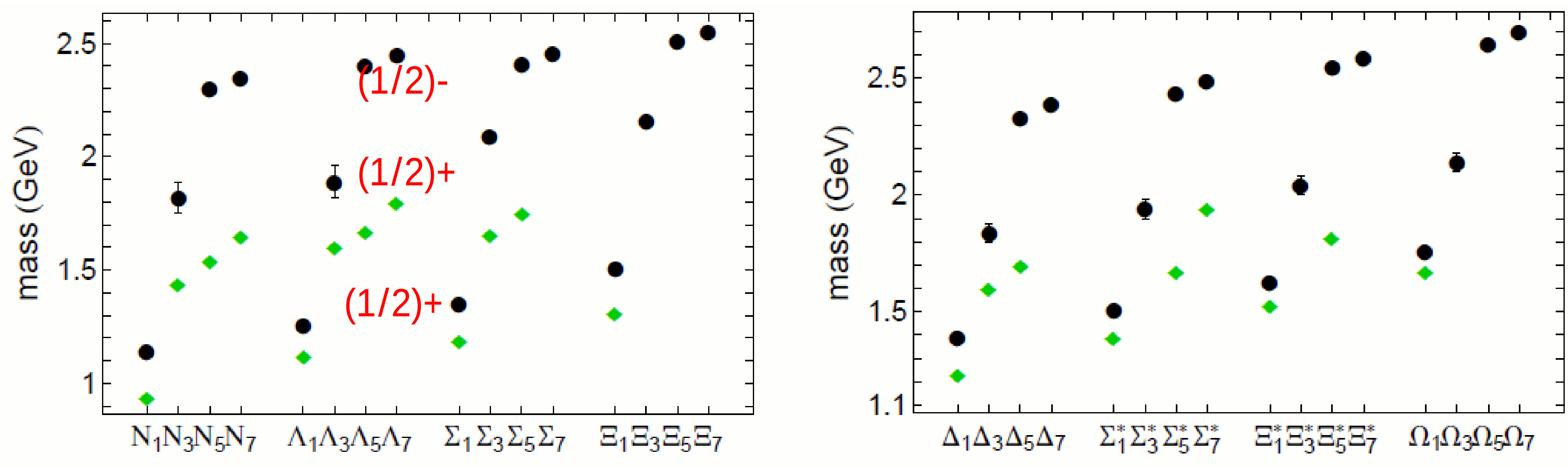


Fig. 4 Left panel: Pictorial representation of octet masses in Table 6. *Circles* – computed masses; and *diamonds* – empirical masses. On the horizontal axis we list a particle name with a subscript that indicates its row in the table; e.g., N_1 means nucleon column, row 1. In this way the labels step through ground-state, radial excitation, parity partner, parity partner's radial excitation. Right panel: Analogous plot for the decuplet masses in Table 6.

[arXiv:1204.2553 [nucl-th]](arXiv:1204.2553), *Spectrum of hadrons with strangeness,* Chen Chen, L. Chang, C.D. Roberts, Shaolong Wan and D.J. Wilson

Spectrum of Hadrons with Strangeness

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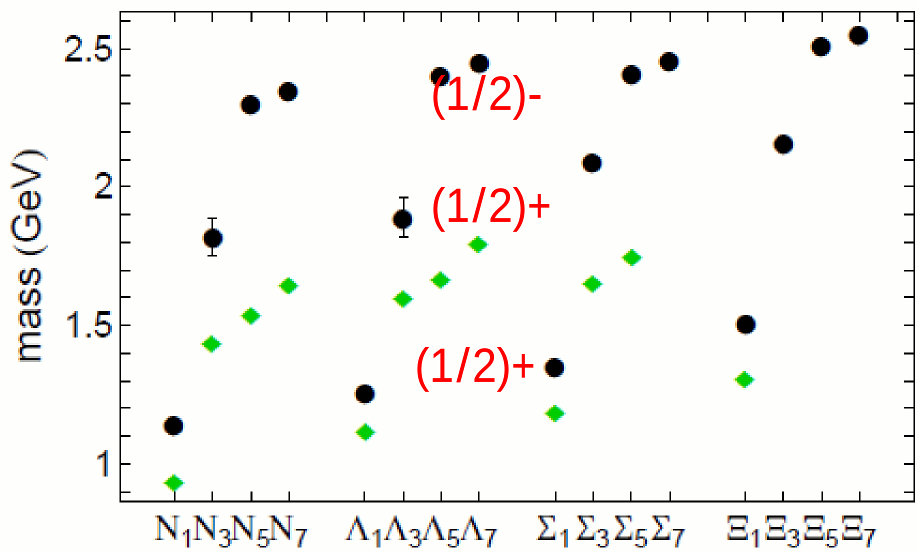
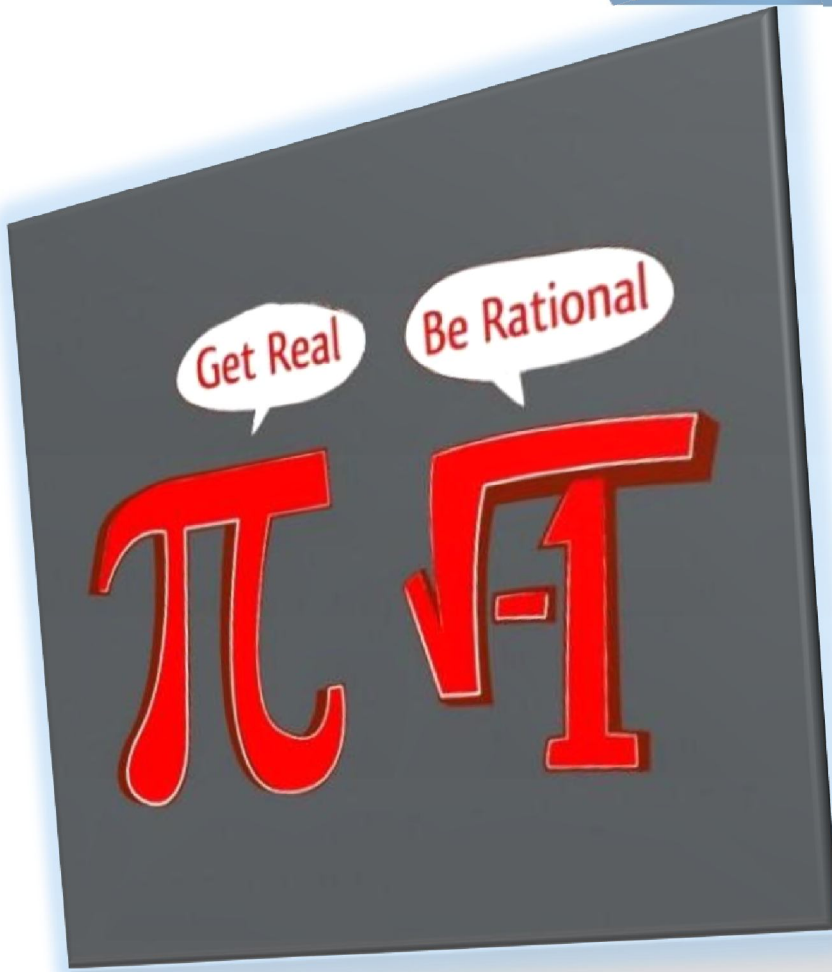


Fig. 4 Left panel: Pictorial representation of octet *diamonds* – empirical masses. On the horizontal axis its row in the table; e.g., N_1 means nucleon column, r radial excitation, parity partner, parity partner's radial masses in Table 6.

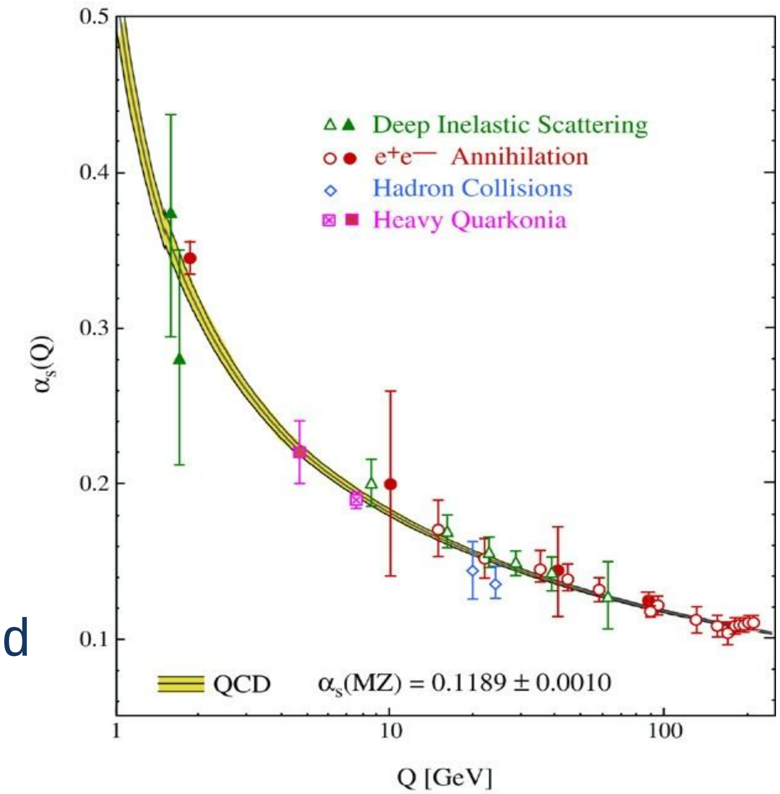
- This level ordering has long been a problem in CQMs with linear or HO confinement potentials
- *Correct ordering owes to DCSB*
  - *Positive parity diquarks have Faddeev equation couplings 25-times greater than negative parity diquarks*
- Explains why approaches within which DCSB cannot be realised (CQMs) or simulations whose parameters suppress DCSB will both have difficulty reproducing experimental ordering



Getting real

Charting the Interaction

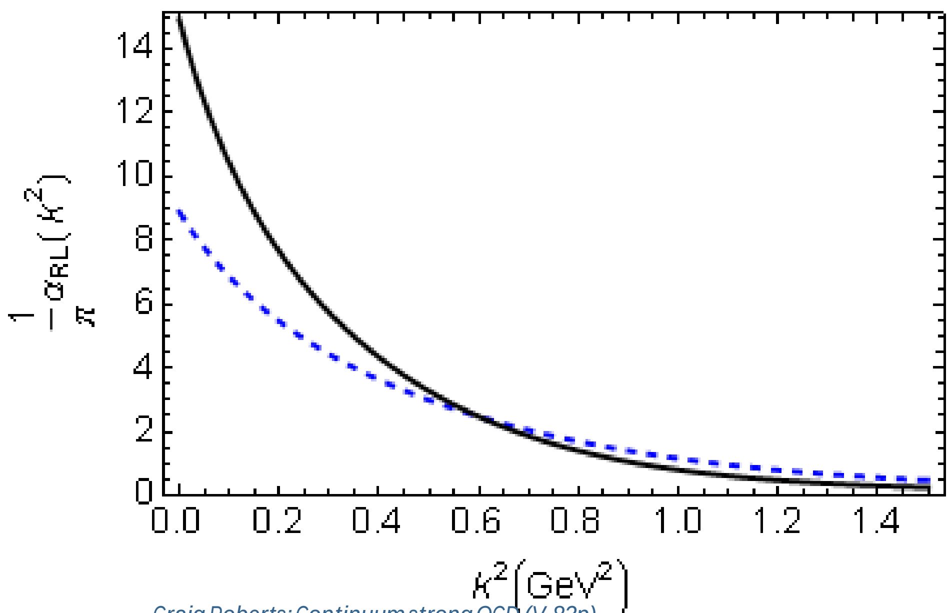
- Interaction in QCD is not momentum-independent
  - Behaviour for Q^2>2GeV^2 is well know; namely, renormalisation-group-improved one-gluon exchange
  - Computable in perturbation theory
- Known = there is a “freezing” of the interaction below a scale of roughly 0.4GeV, which is why momentum-independent interaction works
- Unknown
  - Infrared behavior of the interaction, which is responsible for
    - Confinement
    - DCSB
  - How is the transition to pQCD made and is it possible to define a transition boundary?

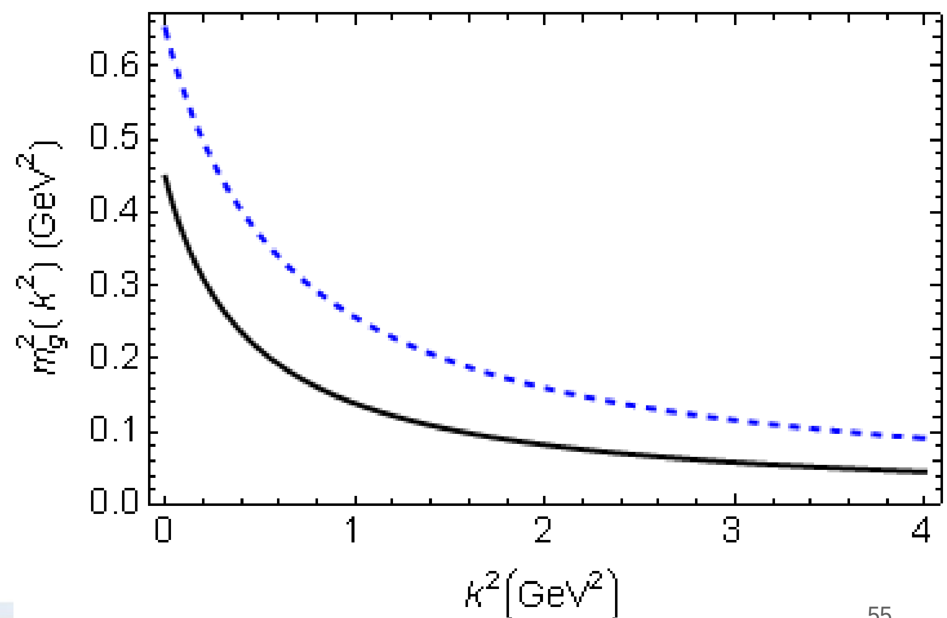


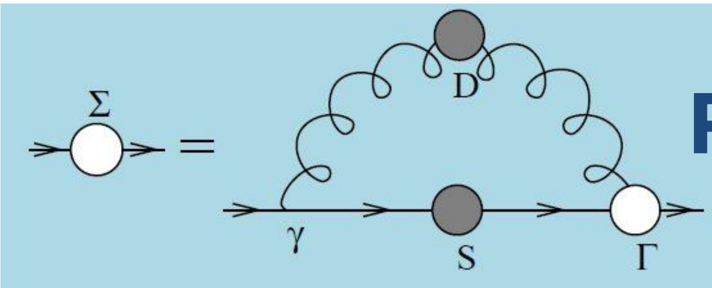
Qin et al., *Phys. Rev. C 84 042202(Rapid Comm.) (2011)*
Rainbow-ladder truncation

DSE Studies
- Phenomenology of gluon

- Wide-ranging study of π & ρ properties
- Effective coupling
  - Agrees with pQCD in ultraviolet
  - Saturates in infrared
    - α(0)/π = 8-15
    - $\alpha(m_G^2)/\pi$ = 2-4
- Running gluon mass
  - Gluon is massless in ultraviolet in agreement with pQCD
  - Massive in infrared
    - $m_G(0)$ = 0.67-0.81 GeV
    - $m_G(m_G^2)$ = 0.53-0.64 GeV



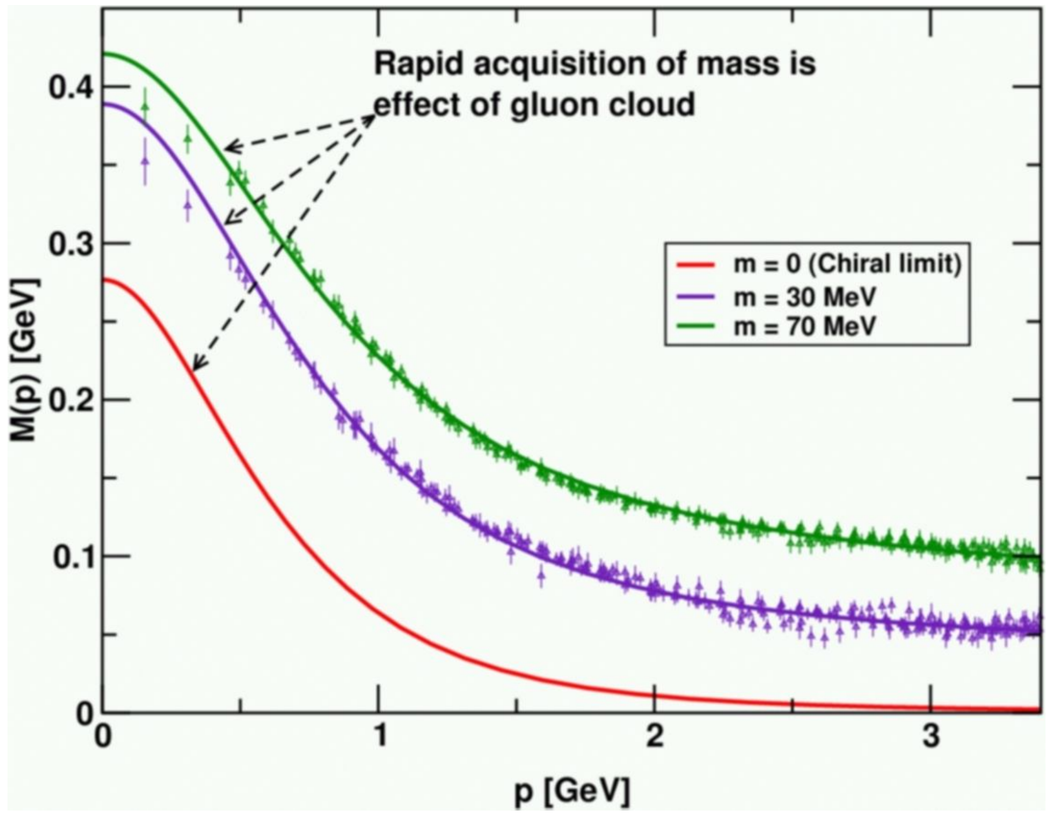




Frontiers of Nuclear Science: Theoretical Advances

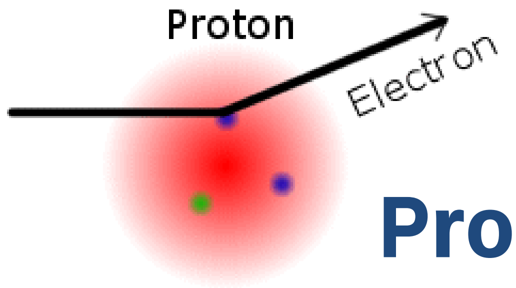
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies (m =0, red curve) acquires a large constituent mass at low energies.



C.D. Roberts, Prog. Part. Nucl. Phys. 61 (2008) 50

M. Bhagwat & P.C. Tandy, AIP Conf.Proc. 842 (2006) 225-227



Nucleon Structure Probed in scattering experiments

- Electron is a good probe because it is structureless

Structureless fermion, or simply structured fermion, $F_1=1$ & $F_2=0$, so that $G_E=G_M$ and hence distribution of charge and magnetisation within this fermion are identical

- Proton's electromagnetic current

$$J_\mu(P',P) = ie\,\bar{u}_p(P')\,\Lambda_\mu(Q,P)\,u_p(P)\,,$$
$$= \;\; ie\,\bar{u}_p(P')\left(\gamma_\mu F_1(Q^2) + \frac{1}{2M}\,\sigma_{\mu\nu}\,Q_\nu\,F_2(Q^2)\right)u_p(P)$$

F_1 = Dirac form factor

F_2 = Pauli form factor

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2)\,,\;\; G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

G_E = Sachs Electric form factor
If a nonrelativistic limit exists, this relates to the charge density

G_M = Sachs Magntic form factor
If a nonrelativistic limit exists, this relates to the magnetisation density

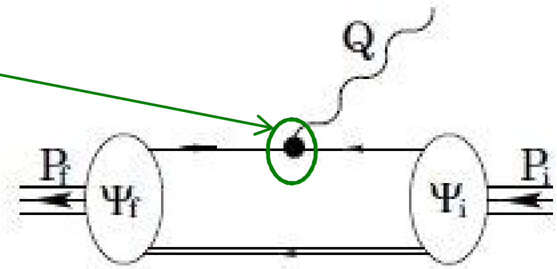
Nucleon form factors

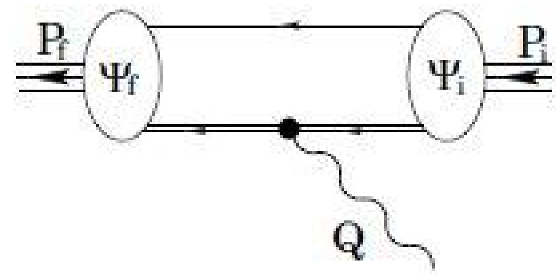
- For the nucleon & Δ-resonance, studies of the Faddeev equation exist that are based on the 1-loop renormalisation-group-improved interaction that was used efficaciously in the study of mesons
  - *Toward unifying the description of meson and baryon properties*
    G. Eichmann, I.C. Cloët, R. Alkofer, A. Krassnigg and C.D. Roberts
    arXiv:0810.1222 [nucl-th], Phys. Rev. C **79** (2009) 012202(R) (5 pages)
  - *Survey of nucleon electromagnetic form factors*
    I.C. Cloët, G. Eichmann, B. El-Bennich, T. Klähn and C.D. Roberts
    arXiv:0812.0416 [nucl-th], Few Body Syst. **46** (2009) pp. 1-36
  - *Nucleon electromagnetic form factors from the Faddeev equation*
    G. Eichmann, arXiv:1104.4505 [hep-ph]
- These studies retain the scalar and axial-vector diquark correlations, which we know to be necessary and sufficient for a reliable description
- In order to compute form factors, one needs a photon-nucleon current

Photon-nucleon current

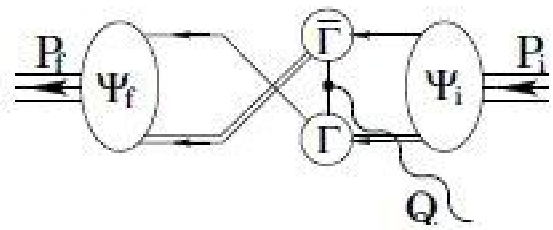
L. Chang, Y. –X. Liu and C.D. Roberts
[arXiv:1009.3458 [nucl-th]]
Phys. Rev. Lett. ***106*** *(2011) 072001*

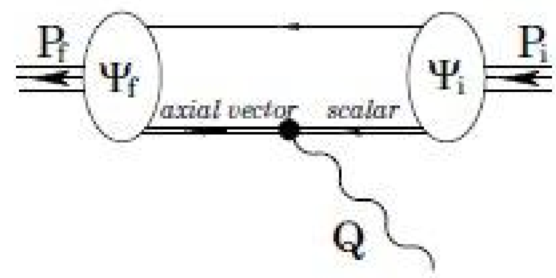
Vertex must contain the dressed-quark anomalous magnetic moment: Lecture IV



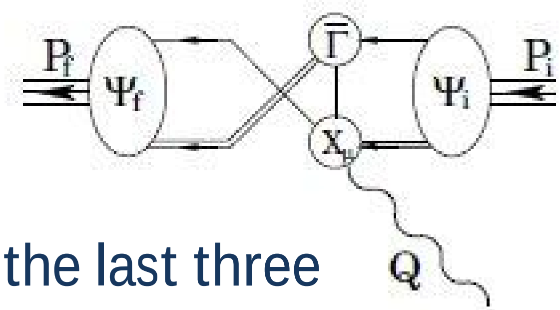


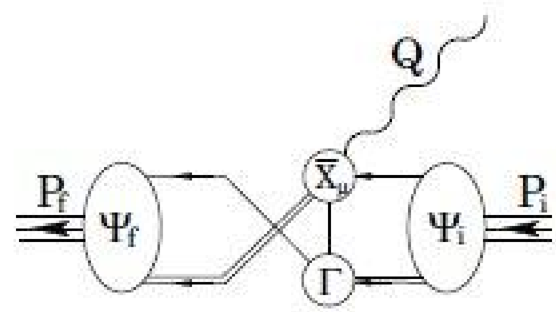
- Composite nucleon must interact with photon via nontrivial current constrained by Ward-Green-Takahashi identities





- DSE → BSE → Faddeev equation plus current → nucleon form factors



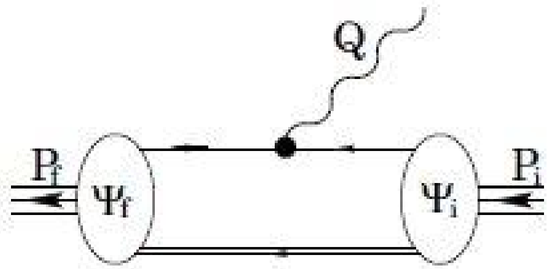


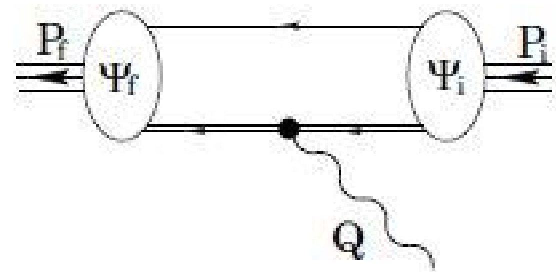
- In a realistic calculation, the last three diagrams represent 8-dimensional integrals, which can be evaluated using Monte-Carlo techniques

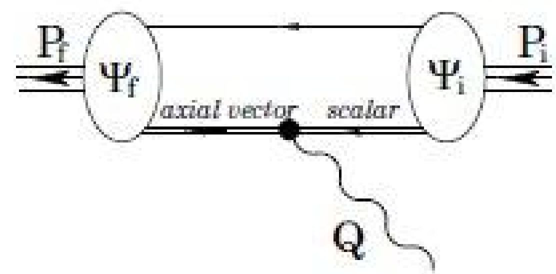
Oettel, Pichowsky, Smekal
Eur.Phys.J. A8 (2000) 251-281

Photon-nucleon current

- Owing to momentum-independence of the diquark Bethe-Salpeter and Faddeev amplitudes using the contact interaction in “static approximation”, the nucleon photon current simplifies
- Comparison between results from contact-interaction and realistic interaction can reveal a great deal







Just three terms survive

- arXiv:1112.2212 [nucl-th], Phys. Rev. C**85** (2012) 025205 [21 pages] , *Nucleon and Roper electromagnetic elastic and transition form factors*, D. J. Wilson, I. C. Cloët, L. Chang and C. D. Roberts

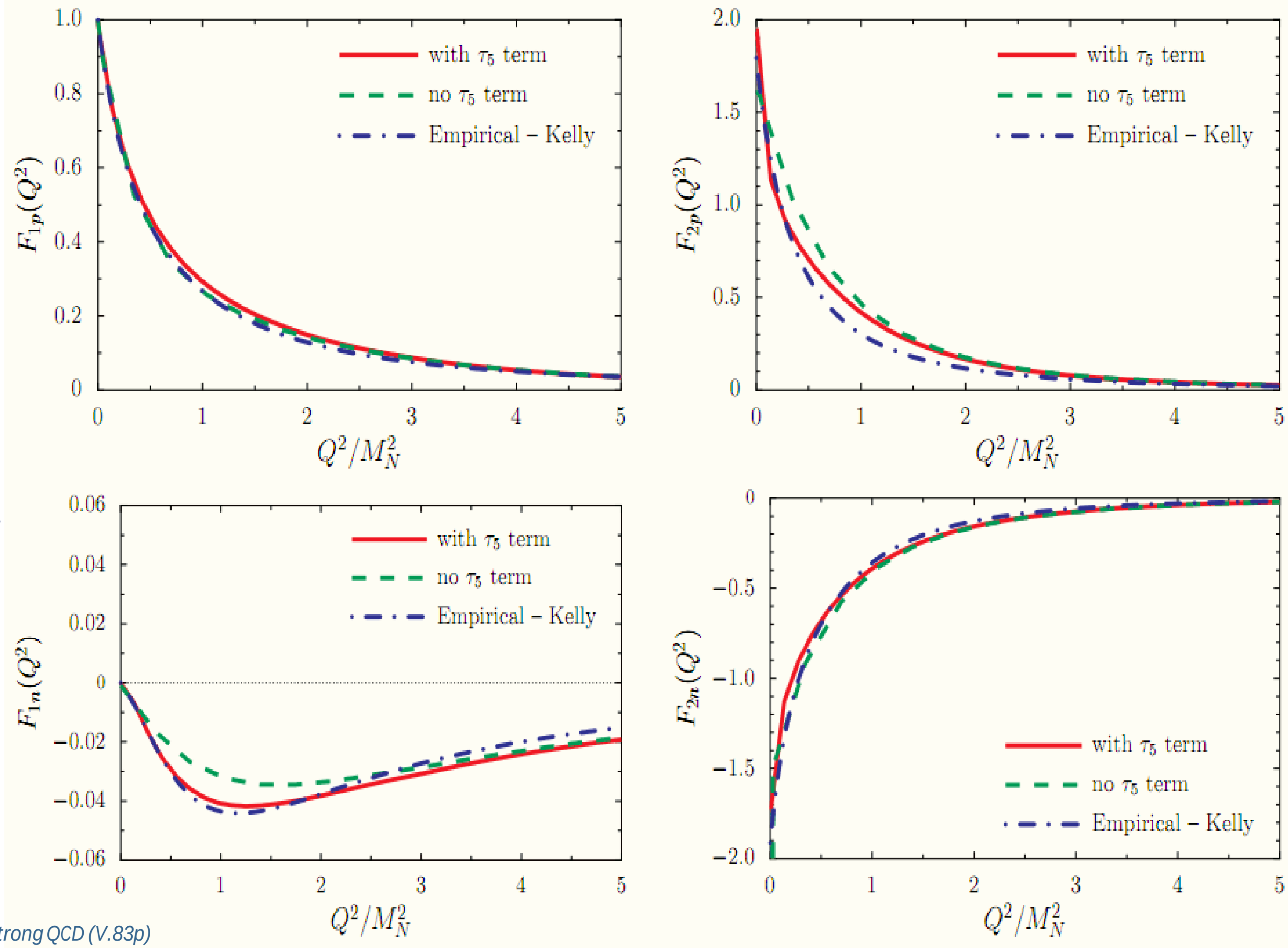
Survey of nucleon electromagnetic form factors
I.C. Cloët et al, arXiv:0812.0416 [nucl-th],
Few Body Syst. **46** (2009) pp. 1-36

Nucleon Form Factors

Unification of meson and nucleon form factors.

Very good description.

Quark's momentum-dependent anomalous magnetic moment has observable impact & materially improves agreement in all cases.



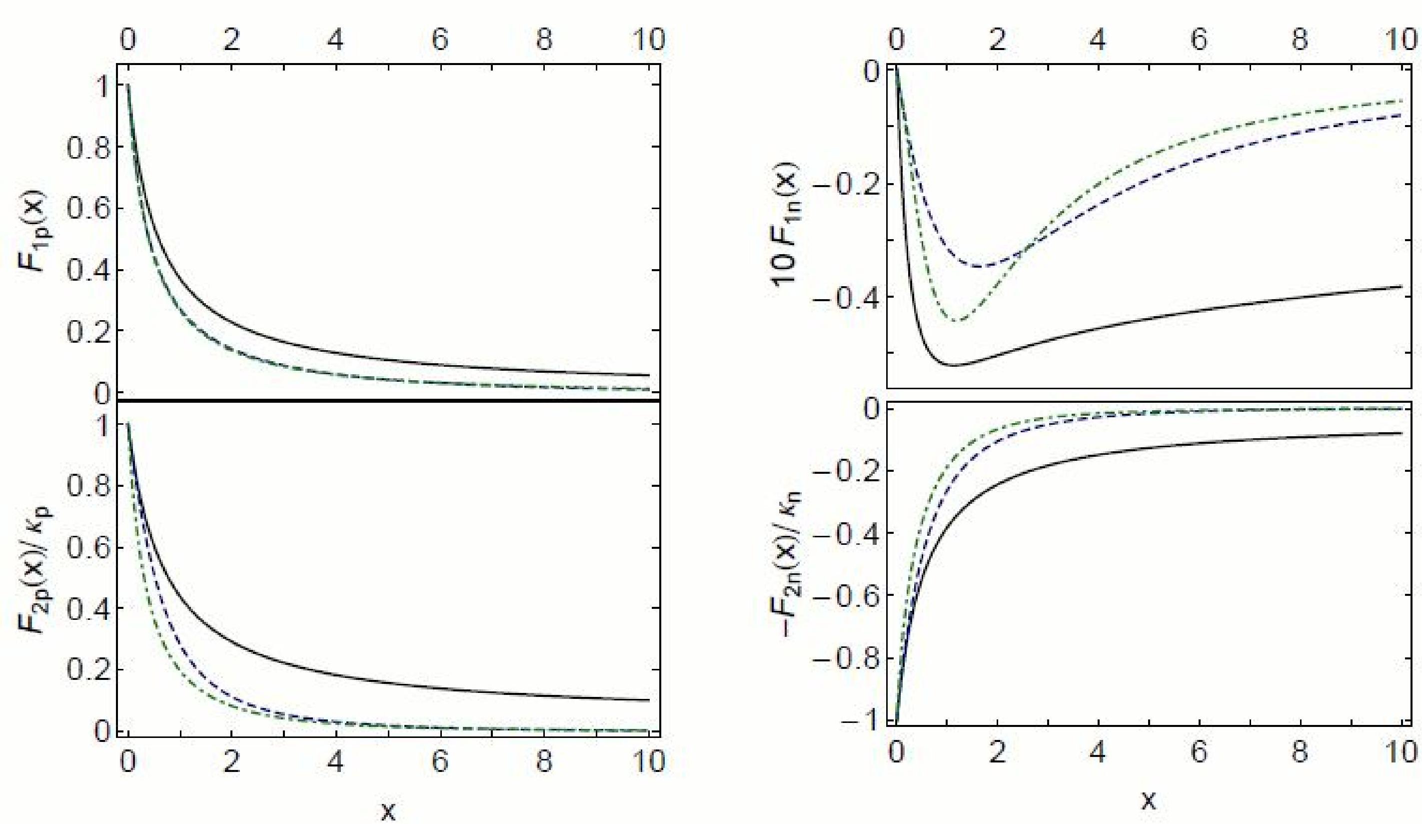
Nucleon and Roper electromagnetic elastic and transition form factors, D. J. Wilson, I. C. Cloët, L. Chang and C. D. Roberts, arXiv:1112.2212 [nucl-th], Phys. Rev. C**85** (2012) 025205 [21 pages]

Nucleon Form Factors

Black solid curve = contact interaction

Blue dashed curve = momentum-dependent interaction

Green dot-dashed curve = parametrisation of experimental data



Momentum independent Faddeev amplitudes, paired with momentum-independent dressed-quark mass and diquark Bethe-Salpeter amplitudes, produce harder form factors, which are readily distinguished from experiment

Nucleon and Roper electromagnetic elastic and transition form factors, D. J. Wilson, I. C. Cloët, L. Chang and C. D. Roberts, [arXiv:1112.2212 [nucl-th]], [Phys. Rev. C**85** (2012) 025205 [21 pages]]

Nucleon Form Factors

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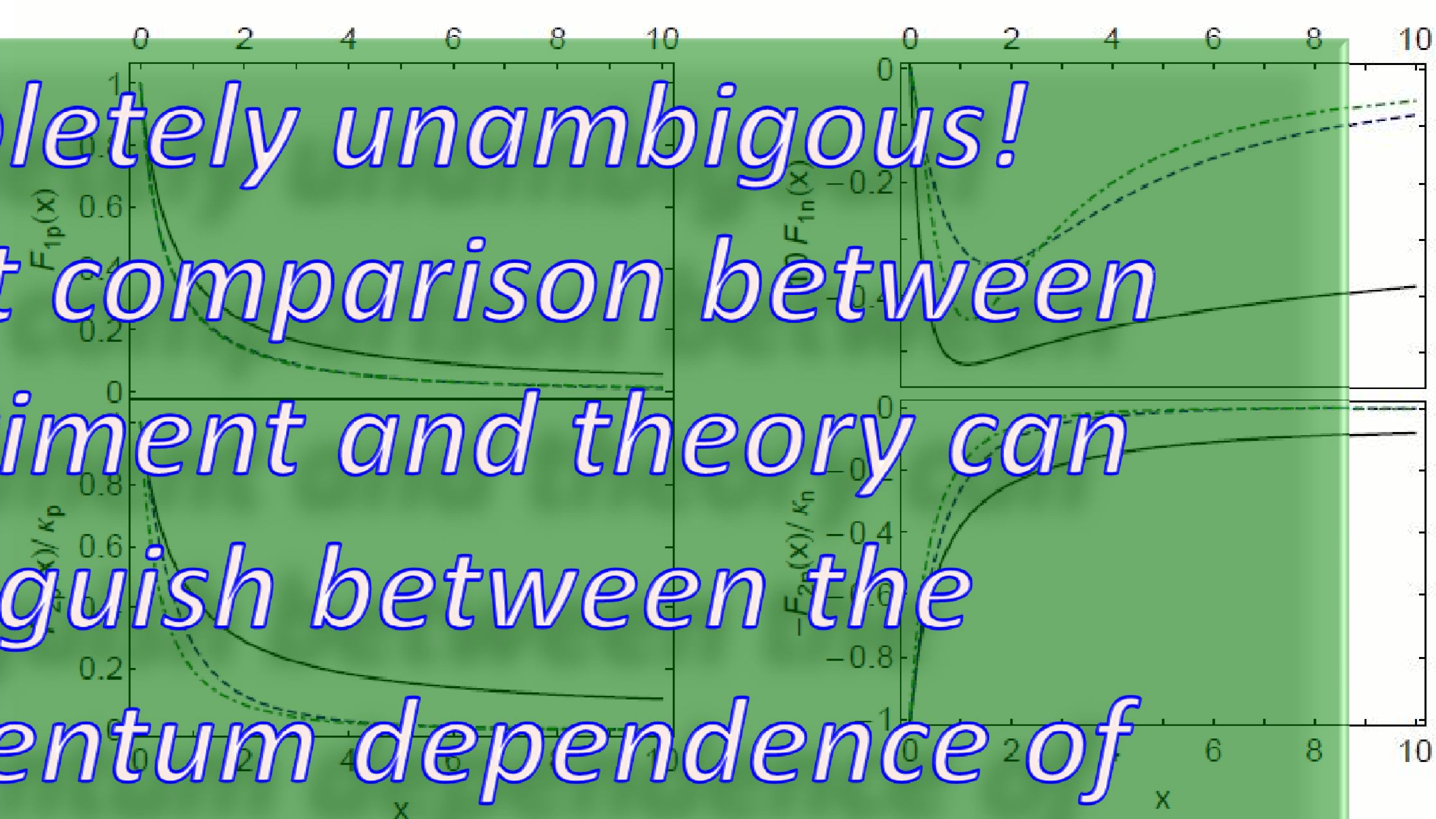
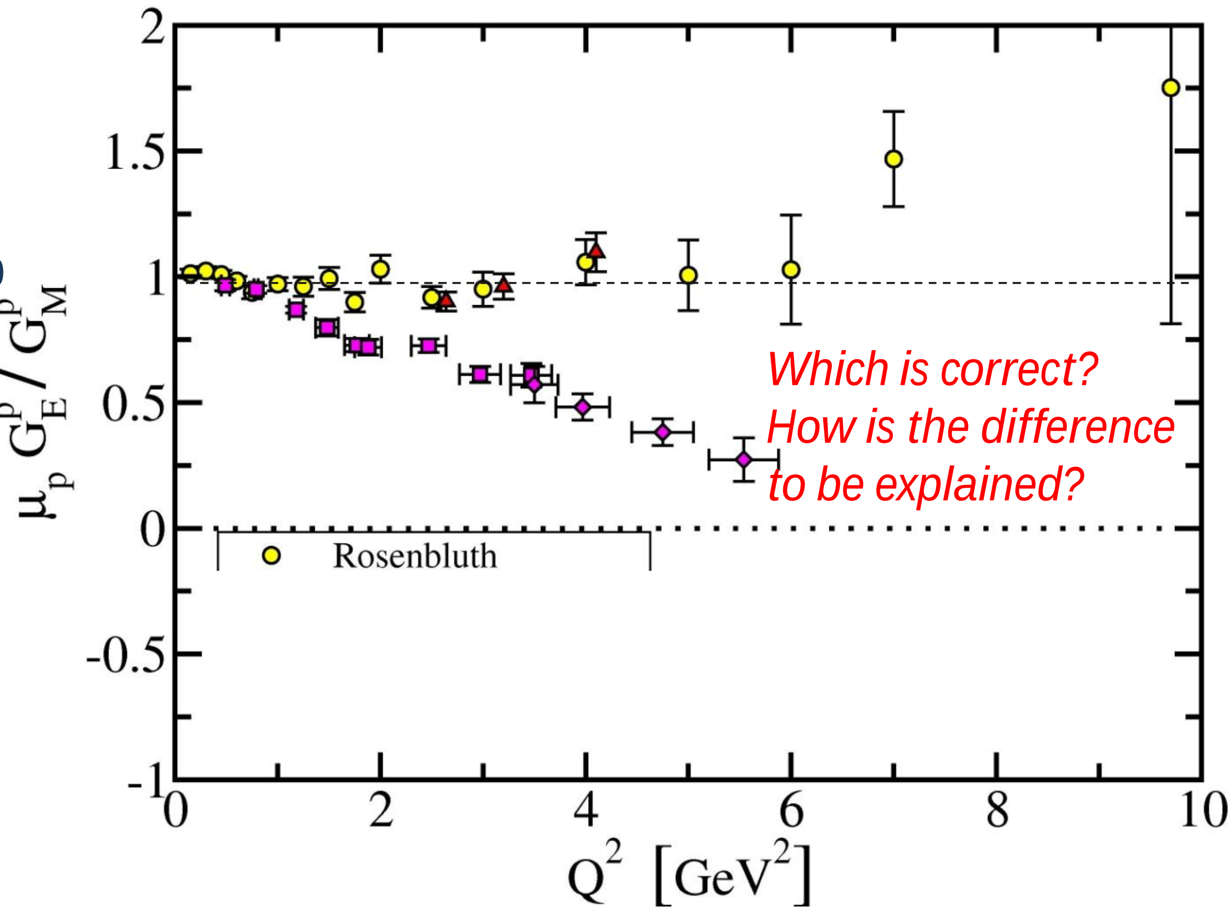


Completely unambigous! Direct comparison between experiment and theory can distinguish between the momentum dependence of strong-interaction theory

Momentum independent Faddeev amplitudes, paired with momentum-independent dressed-quark and diquark Bethe-Salpeter amplitudes, produce harder form factors, which are readily distinguished from experiment

$$\frac{\mu_p G_E^p(Q^2)}{G_M^p(Q^2)}$$

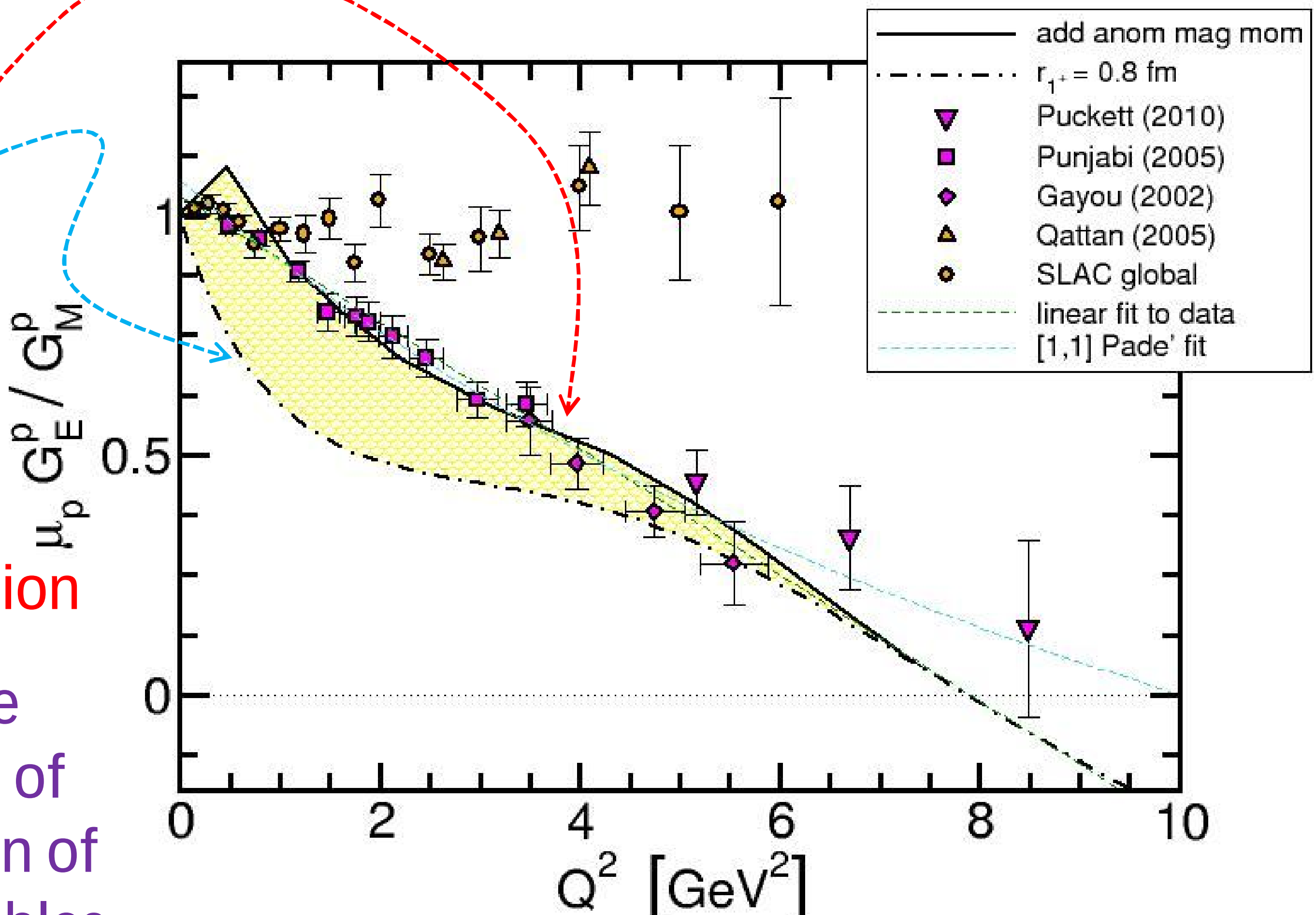
- Data before 1999
  - Looks like the structure of the proton is simple
- The properties of JLab (high luminosity) enabled a new technique to be employed.
- First data released in 1999 and paint a VERY DIFFERENT PICTURE



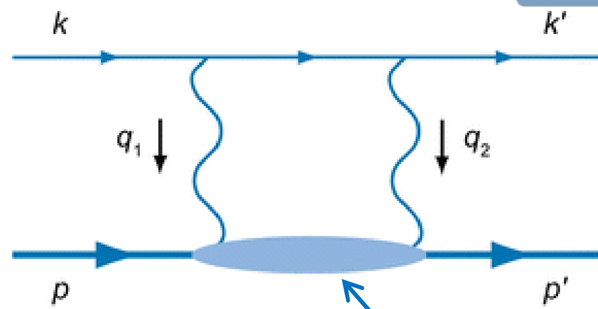
I.C. Cloët, C.D. Roberts, *et al.*
arXiv:0812.0416 [nucl-th]
I.C. Cloët, C.D. Roberts, *et al.*
In progress

$$\frac{\mu_p G_E^p(Q^2)}{G_M^p(Q^2)}$$

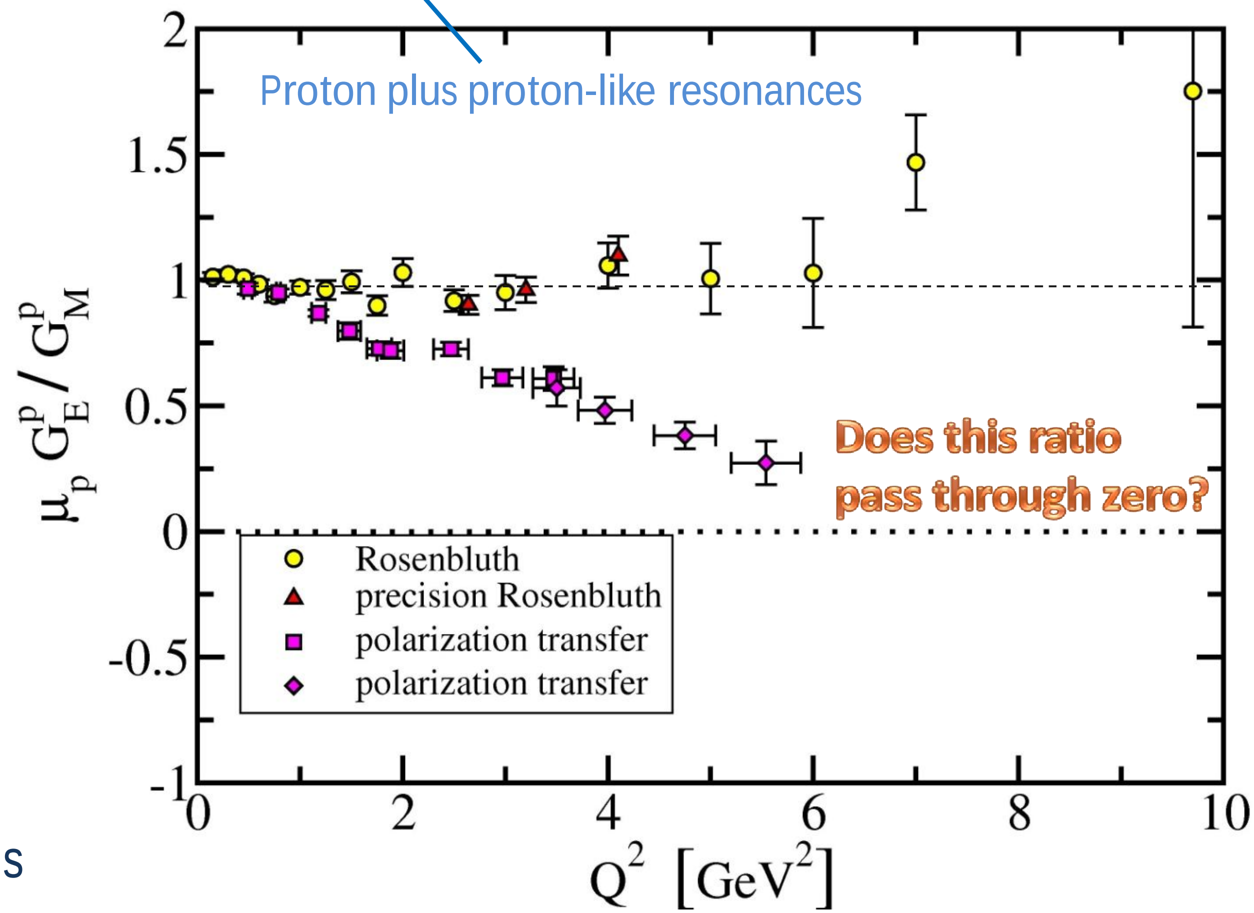
- ➢DSE result Dec 08
- ➢DSE result
  – including the anomalous magnetic moment distribution
- ➢Highlights again the critical importance of DCSB in explanation of real-world observables.



- DSE studies indicate that the *proton has a very rich internal structure*
- The JLab data, obtained using the polarisaton transfer method, are an accurate indication of the behaviour of this ratio
- The pre-1999 data (Rosenbluth) receive large corrections from so-called 2-photon exchange contributions



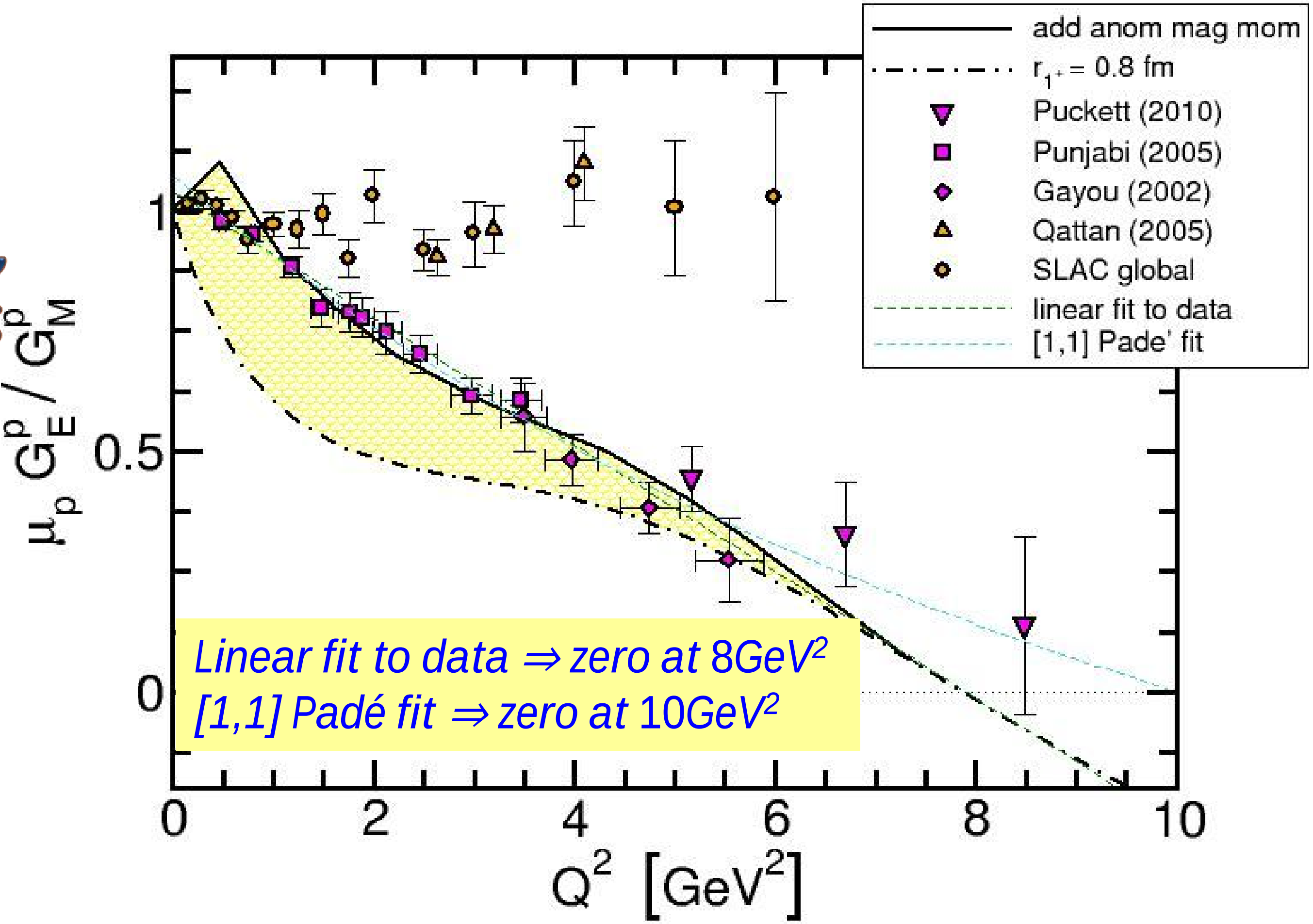
$$\frac{\mu_p G_E^p(Q^2)}{G_M^p(Q^2)}$$



I.C. Cloët, C.D. Roberts, *et al.*
arXiv:0812.0416 [nucl-th]
I.C. Cloët, C.D. Roberts, *et al.*
In progress

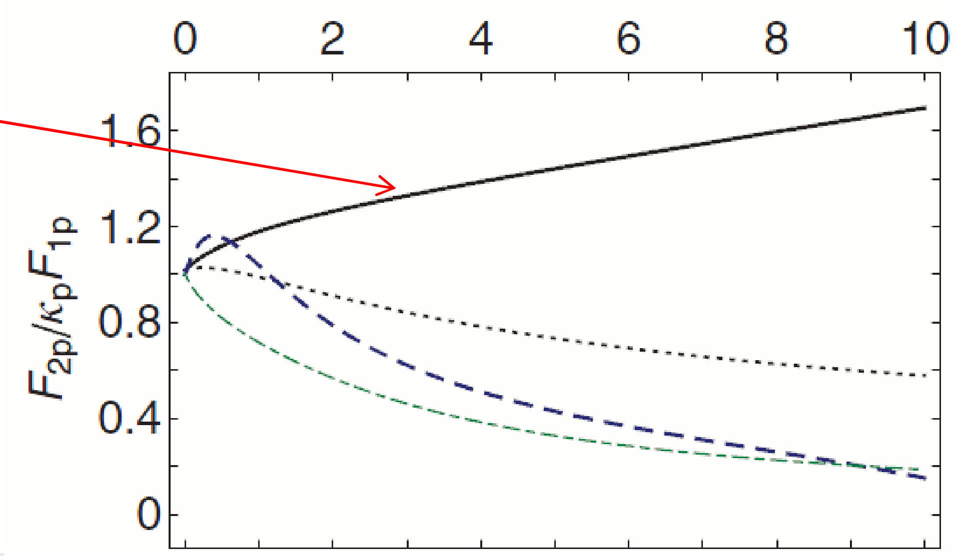
$$\frac{\mu_p G_E^p(Q^2)}{G_M^p(Q^2)}$$

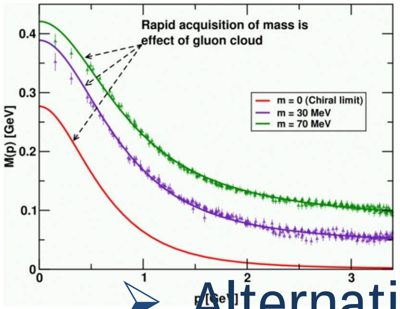
- **Does this ratio pass through zero?**
- **DSE studies say YES, with a zero crossing at *8*GeV², as a consequence of strong correlations within the nucleon**
- **Experiments at the upgraded JLab facility will provide the answer**
- **In the meantime, the DSE studies will be refined**



Origin of the zero & its location

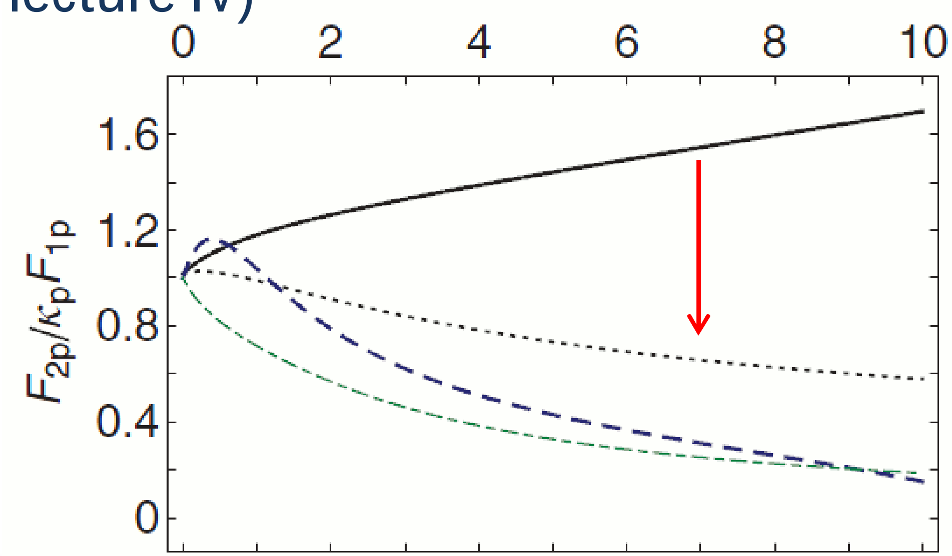
- The Pauli form factor is a gauge of the distribution of magnetization within the proton. Ultimately, this magnetisation is carried by the dressed quarks and influenced by correlations amongst them, which are expressed in the Faddeev wave function.
- If the dressed quarks are described by a momentum-independent mass function, *M*=constant, then they behave as Dirac particles with constant Dirac values for their magnetic moments and produce a hard Pauli form factor





Origin of the zero & its location

- Alternatively, suppose that the dressed quarks possess a momentum-dependent mass function, $M=M(p^2)$, which is large at infrared momenta but vanishes as their momentum increases.
- At small momenta they will then behave as constituent-like particles with a large magnetic moment, but their mass and magnetic moment will drop toward zero as the probe momentum grows. (Remember: Massless fermions do not possess a measurable magnetic moment – lecture IV)
- Such dressed quarks produce a proton Pauli form factor that is large for $Q^2 \sim 0$ but drops rapidly on the domain of transition between nonperturbative and perturbative QCD, to give a very small result at large Q^2.



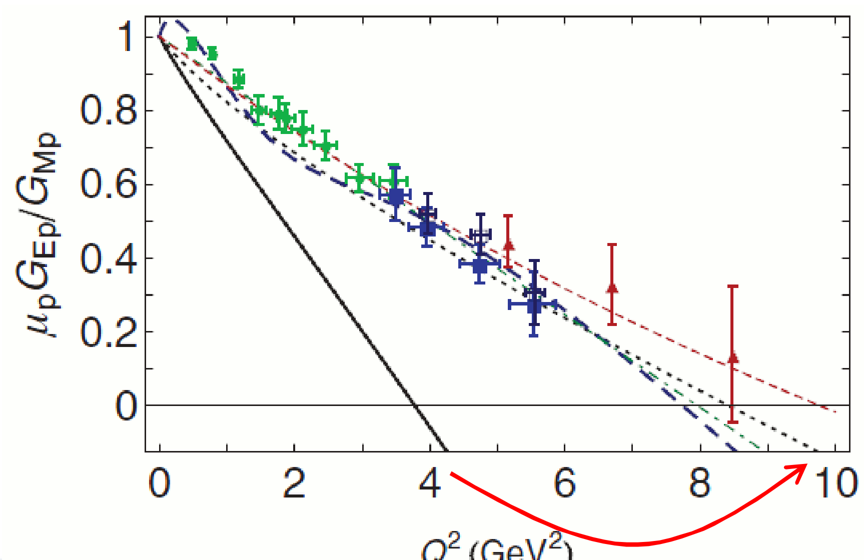
Origin of the zero & its location

- The precise form of the Q^2 dependence will depend on the evolving nature of the angular momentum correlations between the dressed quarks.
- From this perspective, existence, and location if so, of the zero in

  $$\mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2)$$

  are a fairly direct measure of the location and width of the transition region between the nonperturbative and perturbative domains of QCD as expressed in the momentum dependence of the dressed-quark mass function.
- Hard, M=constant

  → Soft, $M=M(p^2)$



I.C. Cloët & C.D. Roberts
In progress

Origin of the zero & its location

- One can anticipate that a mass function which rapidly becomes partonic—namely, is very soft—will not produce a zero
- We've seen that a constant mass function produces a zero at a small value of Q^2
- And also seen and know that a mass function which resembles that obtained in the best available DSE studies and via lattice-QCD simulations produces a zero at a location that is consistent with extant data.
- There is opportunity here for very constructive feedback between future experiments and theory.

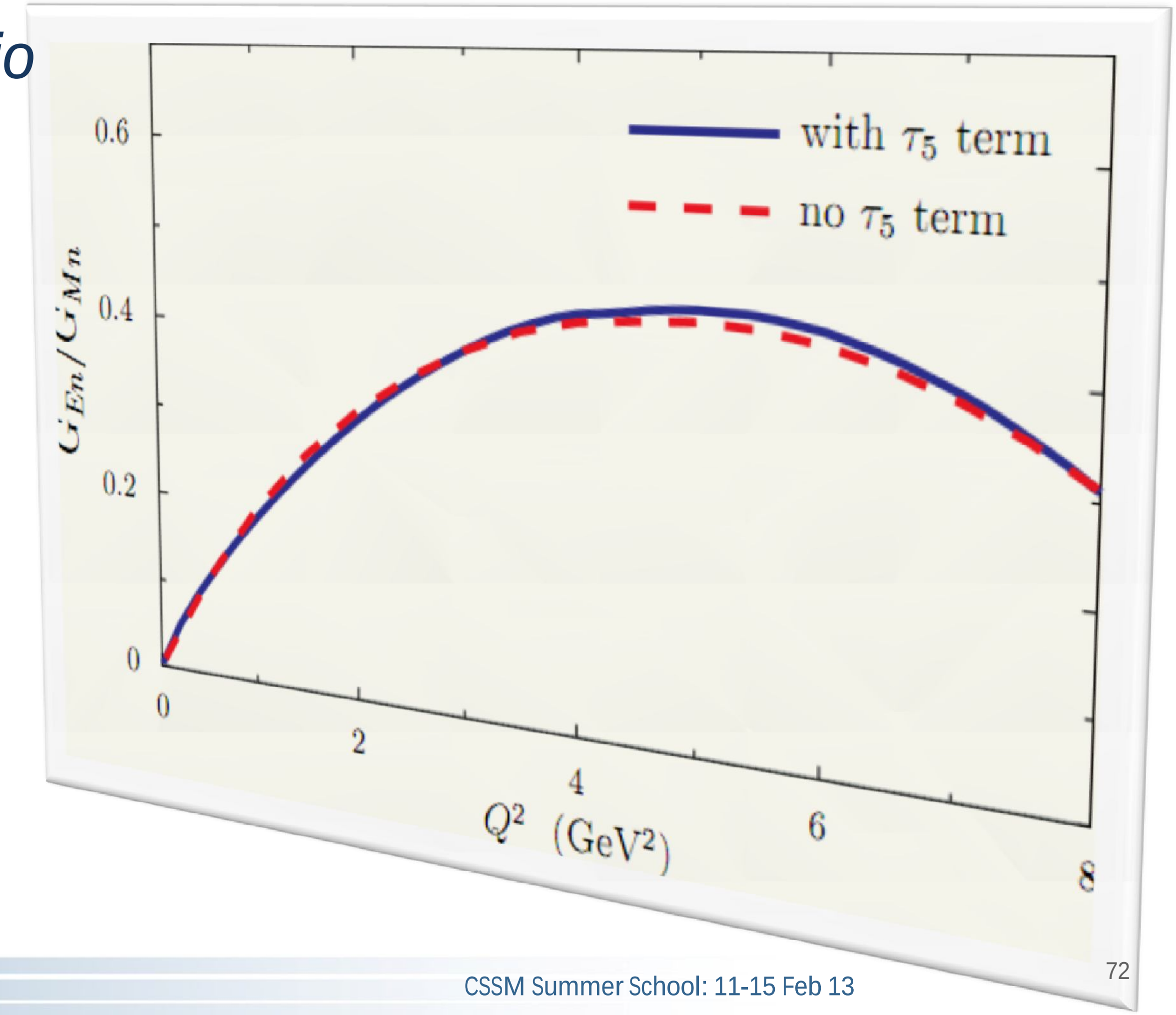
I.C. Cloët & C.D. Roberts
In progress

$$\frac{\mu_n G_E^n(Q^2)}{G_M^n(Q^2)}$$

What about the same ratio for the neutron?

Quark anomalous magnetic moment has big impact on proton ratio

But little impact on the neutron ratio ... because effect is focused near $Q^2=0$, at which G_{En} vanishes

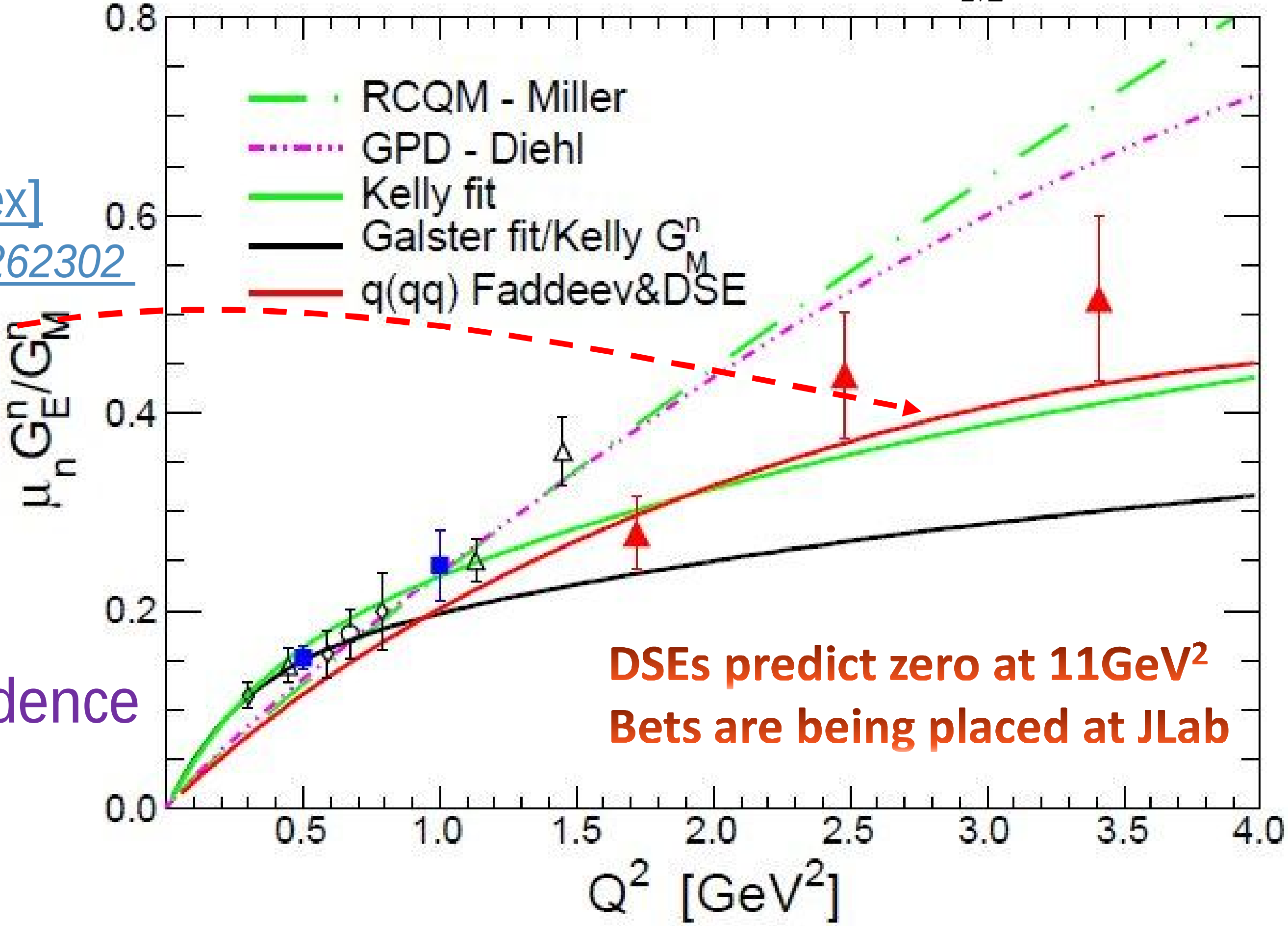


I.C. Cloët, C.D. Roberts, *et al.*
arXiv:0812.0416 [nucl-th]

$$\frac{\mu_n G_E^n(Q^2)}{G_M^n(Q^2)}$$

- New JLab data: S. Riordan *et al.*, ▲ arXiv:1008.1738 [nucl-ex] *Phys.Rev.Lett. 105 (2010) 262302*

- DSE-prediction
- This evolution is very sensitive to momentum-dependence of dressed-quark propagator

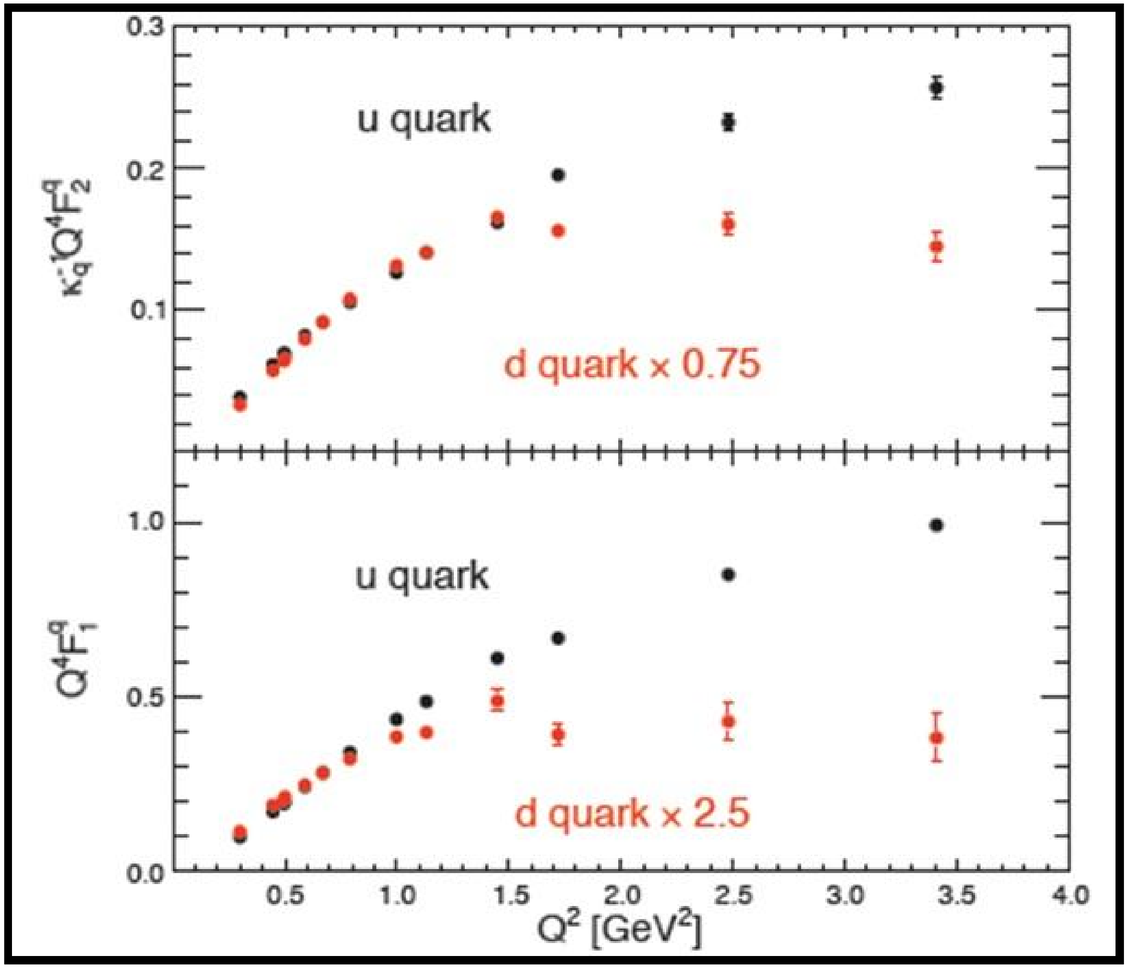


Flavor separation of proton form factors

$Q^4F_2^q/\kappa$

Cates, de Jager, Riordan, Wojtsekhowski, PRL 106 (2011) 252003

$Q^4\,F_1^q$



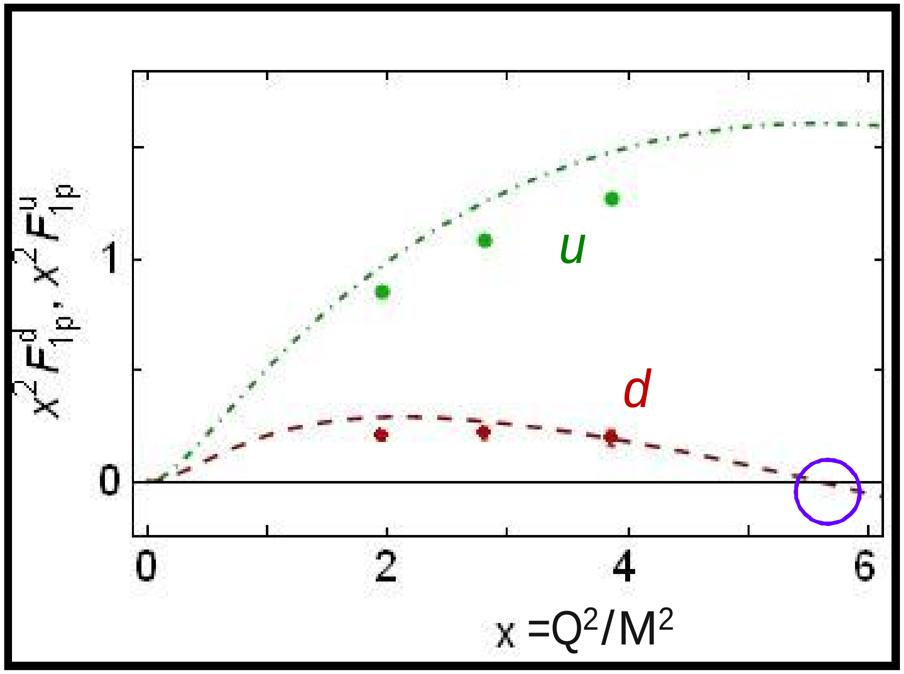
- Very different behavior for *u* & *d* quarks

  Means apparent scaling in proton F2/F1 is *purely accidental*

Cloët, Eichmann, El-Bennich, Klähn, Roberts, Few Body Syst. 46 (2009) pp.1-36

Wilson, Cloët, Chang, Roberts, PRC 85 (2012) 045205

Diquark correlations!



- Poincaré covariant Faddeev equation
  - Predicts scalar and axial-vector diquarks
- Proton's singly-represented *d*-quark more likely to be struck in association with 1^+ diquark than with 0^+
  - form factor contributions involving 1^+ diquark are softer
- Doubly-represented *u*-quark is predominantly linked with harder 0^+ diquark contributions
- Interference produces zero in Dirac form factor of *d*-quark in proton
  - Location of the zero depends on the relative probability of finding 1^+ & 0^+ diquarks in proton
  - Correlated, e.g., with valence *d/u* ratio at x=1

Nucleon Structure Functions

- Moments method will work here, too.
- Work about to begin: *Cloët, Roberts & Tandy*
  - Based on predictions for nucleon elastic form factors, which, e.g., predicted large-Q^2 behavior of $G_E^n(Q^2)$:

    Survey of nucleon electromagnetic form factors
    I.C. Cloët, G. Eichmann, B. El-Bennich, T. Klähn and C.D. Roberts
    arXiv:0812.0416 [nucl-th], Few Body Syst. **46** (2009) pp. 1-36
- Meantime, capitalise on connection between $x=1$ and $Q^2=0$ ...

I.C. Cloët, C.D. Roberts, *et al*.
arXiv:0812.0416 [nucl-th], Few Body Syst. 46 (2009) 1-36
D. J. Wilson, I. C. Cloët, L. Chang and C. D. Roberts
arXiv:1112.2212 [nucl-th], Phys. Rev. C**85** (2012) 025205 [21 pages]

Neutron structure function at high-*x*

- Valence-quark distributions at *x=1*
  - Fixed point under DGLAP evolution
  - Strong discriminator between models

$$\left.\frac{d_v(x)}{u_v(x)}\right|_{x\to 1}, \quad \text{where} \quad \frac{d_v(x)}{u_v(x)} = \frac{4\frac{F_2^n(x)}{F_2^p(x)} - 1}{4 - \frac{F_2^n(x)}{F_2^p(x)}}$$

- Algebraic formula

$$\left.\frac{d_v(x)}{u_v(x)}\right|_{x\to 1} = \frac{P_1^{p,d}}{P_1^{p,u}} = \frac{\frac{2}{3}P_1^{p,a} + \frac{1}{3}P_1^{p,m}}{P_1^{p,s} + \frac{1}{3}P_1^{p,a} + \frac{2}{3}P_1^{p,m}}$$

  - $P_1^{p,s}$ = contribution to the proton's charge arising from diagrams with a scalar diquark component in both the initial and final state
  - $P_1^{p,a}$ = kindred axial-vector diquark contribution
  - $P_1^{p,m}$ = contribution to the proton's charge arising from diagrams with a different diquark component in the initial and final state.

I.C. Cloët, C.D. Roberts, *et al.*
arXiv:0812.0416 [nucl-th], Few Body Syst. 46 (2009) 1-36
D. J. Wilson, I. C. Cloët, L. Chang and C. D. Roberts
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Contact interaction

	$P_1^{p,s}$	$P_1^{p,a}$	$P_1^{p,m}$	$\frac{d_v}{u_v}$	$\frac{F_2^n}{F_2^p}$
$M = \text{constant}$	0.78	0.22	0	0.18	0.41
$M(p^2)$	0.60	0.25	0.15	0.28	0.49

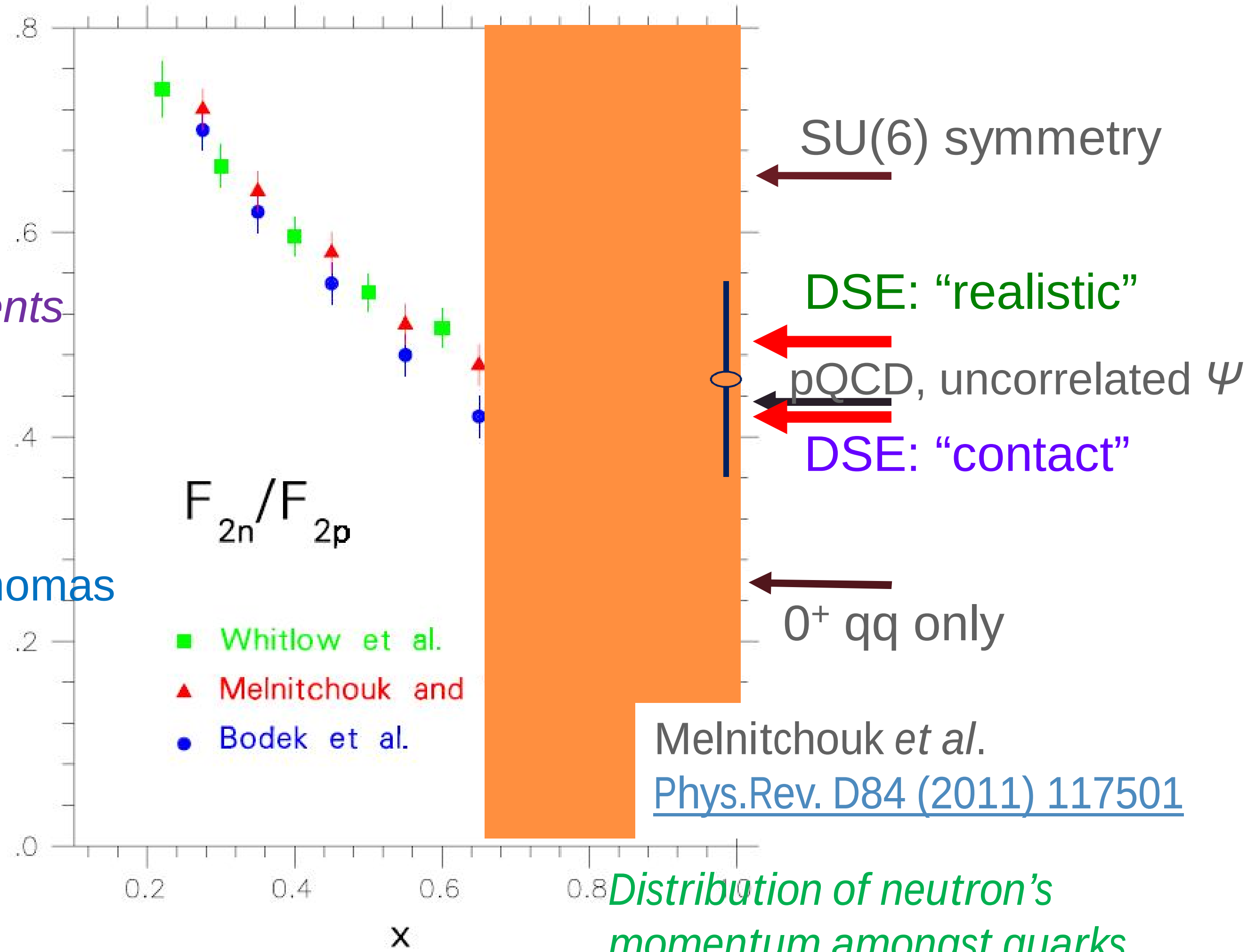
“Realistic” interaction

I.C. Cloët, C.D. Roberts, *et al*.
arXiv:0812.0416 [nucl-th]
D. J. Wilson, I. C. Cloët, L. Chang and C. D. Roberts
arXiv:1112.2212 [nucl-th], Phys. Rev. C**85** (2012) 025205 [21 pages]

Neutron Structure Function at high x

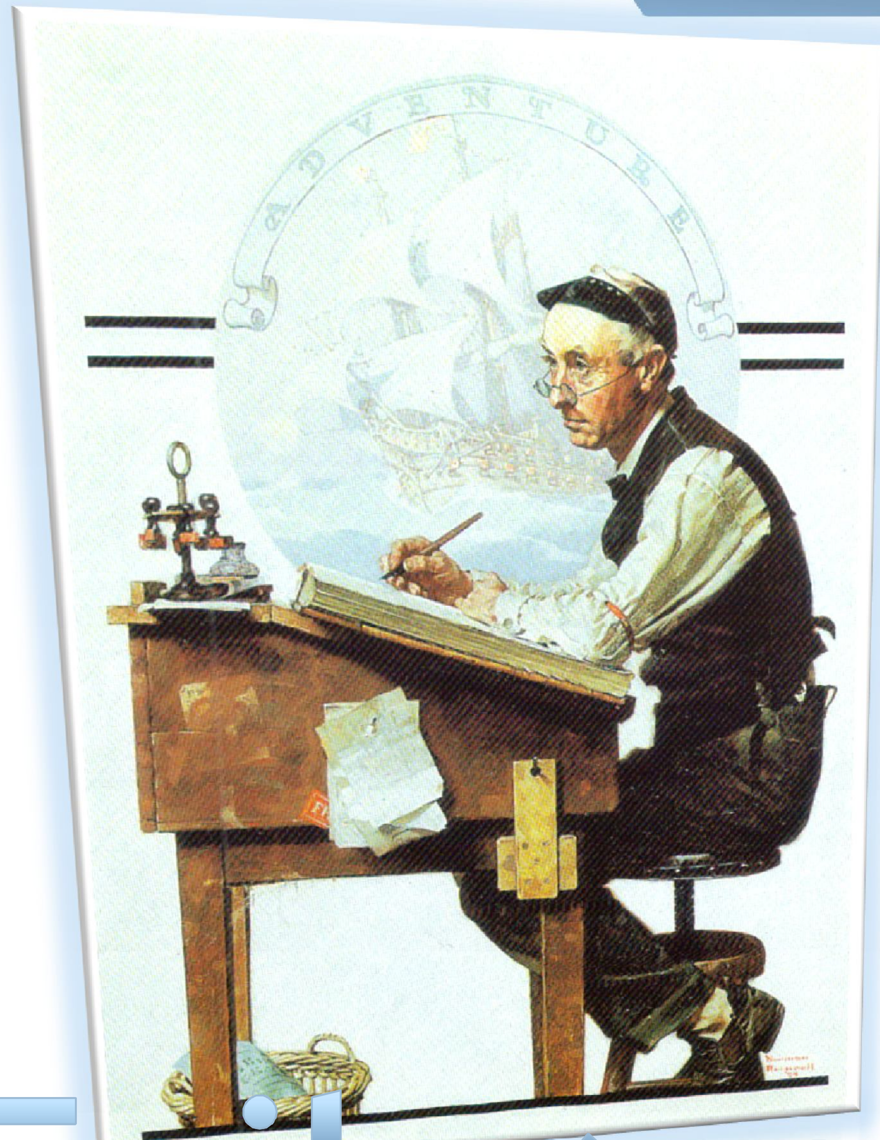
Deep inelastic scattering – the Nobel-prize winning quark-discovery experiments

Reviews:
- S. Brodsky *et al.* NP B441 (1995)
- W. Melnitchouk & A.W.Thomas PL B377 (1996) 11
- N. Isgur, PRD 59 (1999)
- R.J. Holt & C.D. Roberts RMP (2010)



Melnitchouk *et al*.
Phys.Rev. D84 (2011) 117501

Distribution of neutron's momentum amongst quarks on the valence-quark domain



Epilogue

DSEs: A practical, predictive, unifying tool for fundamental physics

- o Exact results proved in QCD, amongst them:
  - ✓ Quarks are not Dirac particles and gluons are nonperturbatively massive
  - ✓ Dynamical chiral symmetry breaking is a fact.
    It's responsible for 98% of the mass of visible matter in the Universe
  - ✓ Goldstone's theorem is fundamentally an expression of equivalence between the one-body problem and the two-body problem in the pseudoscalar channel
  - ✓ Confinement is a dynamical phenomenon
    It cannot in principle be expressed via a potential
  - ✓ The list goes on …

McLerran & Pisarski
arXiv:0706.2191 [hep-ph]

- ❑ DSEs are a single framework, with IR model-input turned to advantage, *"almost unique in providing an unambiguous path from a defined interaction → Confinement & DCSB → Masses → radii → form factors → distribution functions → etc."*

It's been a journey

This is not the end