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Continuum strong QCD

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Office of Nuclear Physics
Exploring Nuclear Matter - Quarks to Stars

U.S. DEPARTMENT OF ENERGY

Munczek-Nemirovsky Model

- Munczek, H.J. and Nemirovsky, A.M. (1983),
  "The Ground State q-q.bar Mass Spectrum In QCD,"
  Phys. Rev. ***D 28, 181.***

Antithesis of NJL model; viz.,
Delta-function in momentum space
NOT in configuration space.

In this case, G sets the mass scale

- $$\Gamma^a_\mu(k,p)_{\rm bare} = \gamma_\mu \, \frac{\lambda^a}{2};$$

$$g^2 D_{\mu\nu}(k) \to (2\pi)^4 \, G \, \delta^4(k) \left[\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right]$$

- MN Gap equation

$$i\gamma\cdot p \, A(p^2) + B(p^2) = i\gamma\cdot p + m + G \, \gamma_\mu \, \frac{-i\gamma\cdot p \, A(p^2) + B(p^2)}{p^2 A^2(p^2) + B^2(p^2)} \, \gamma_\mu$$

MN Model's Gap Equation

- The gap equation yields the following pair of coupled, algebraic equations (set $G = 1$ GeV^2)

$$A(p^2) = 1 + 2\,\frac{A(p^2)}{p^2A^2(p^2) + B^2(p^2)}$$

$$\boxed{B(p^2) = 4\,\frac{B(p^2)}{p^2A^2(p^2) + B^2(p^2)}\,,}$$

- Consider the chiral limit form of the equation for $B(p^2)$
  - Obviously, one has the trivial solution $B(p^2) = 0$
  - However, is there another?

MN model and DCSB

- The existence of a $B(p^2) \neq 0$ solution; i.e., a solution that dynamically breaks chiral symmetry, requires (in units of G)

  $$p^2 A^2(p^2) + B^2(p^2) = 4$$

- Substituting this result into the equation for $A(p^2)$ one finds

  $$A(p^2) - 1 = \tfrac{1}{2} A(p^2) \rightarrow A(p^2) = 2,$$

  which in turn entails

  $$B(p^2) = 2\,(1 - p^2)^{1/2}$$

- Physical requirement: quark self-energy is real on the domain of spacelike momenta → complete chiral limit solution

$$A(p^2) = \begin{cases} 2\,; & p^2 \leq 1 \\ \frac{1}{2}\left(1 + \sqrt{1 + 8/p^2}\right)\,; & p^2 > 1 \end{cases}$$

$$B(p^2) = \begin{cases} \sqrt{1 - p^2}\,; & p^2 \leq 1 \\ 0\,; & p^2 > 1\,. \end{cases}$$

NB. Self energies are momentum-dependent because the interaction is momentum-dependent. Should expect the same in QCD.

MN Model and Confinement?

- Solution we've found is continuous and defined for all p^2, even $p^2 < 0$; namely, timelike momenta
- Examine the propagator's denominator

  $$p^2 A^2(p^2) + B^2(p^2) = 4$$

  This is greater-than zero for all p^2 …
  - There are no zeros
  - So, the *propagator has no pole*
- This is nothing like a free-particle propagator. It can be interpreted as describing a **confined degree-of-freedom**
- Note that, in addition there is no critical coupling:

  The nontrivial solution exists so long as $\boldsymbol{G > 0}$.
- Conjecture: ***All confining theories exhibit DCSB***
  - NJL model demonstrates that converse is not true.

Massive solution in MN Model

- In the chirally asymmetric case the gap equation yields

$$
\begin{aligned}
A(p^2) &= \frac{2\,B(p^2)}{m+B(p^2)}\,, \\
B(p^2) &= m + \frac{4\,[m+B(p^2)]^2}{B(p^2)([m+B(p^2)]^2+4p^2)}\,.
\end{aligned}
$$

- Second line is a quartic equation for *$B(p^2)$*.

  Can be solved algebraically with four solutions, available in a closed form.

- Only one solution has the correct $p^2 \to \infty$ limit; viz.,

  $B(p^2) \to m$.

  This is the *unique physical* solution.

- NB. The equations and their solutions always have a smooth $m \to 0$ limit, a result owing to the persistence of the DCSB solution.

Munczek-Nemirovsky Dynamical Mass

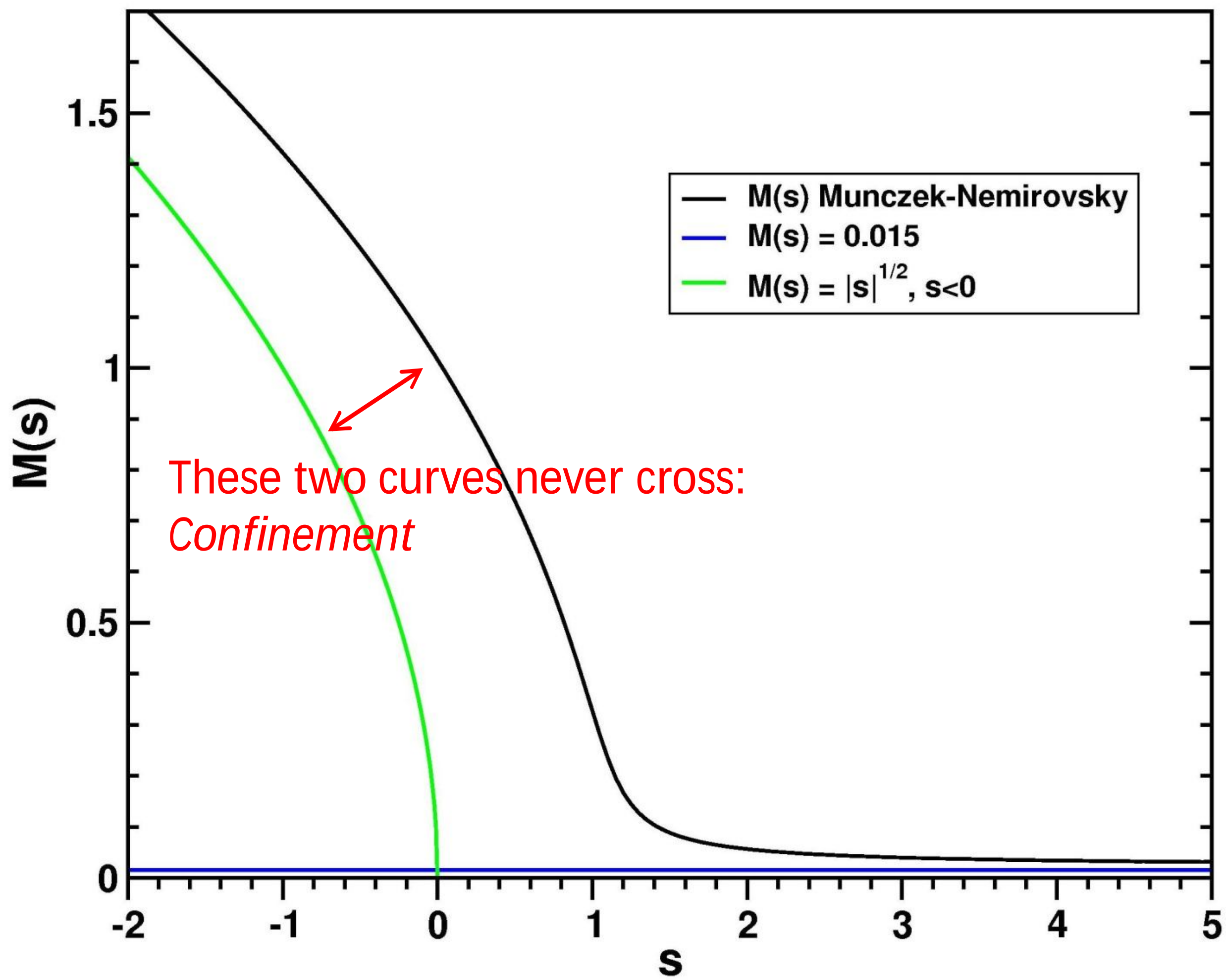
- Large-s: $M(s) \sim m$
- Small-s: $M(s) \gg m$

  This is the essential characteristic of DCSB
- We will see that p^2-dependent mass-functions are a quintessential feature of QCD.
- No solution of

  $$s + M(s)^2 = 0$$

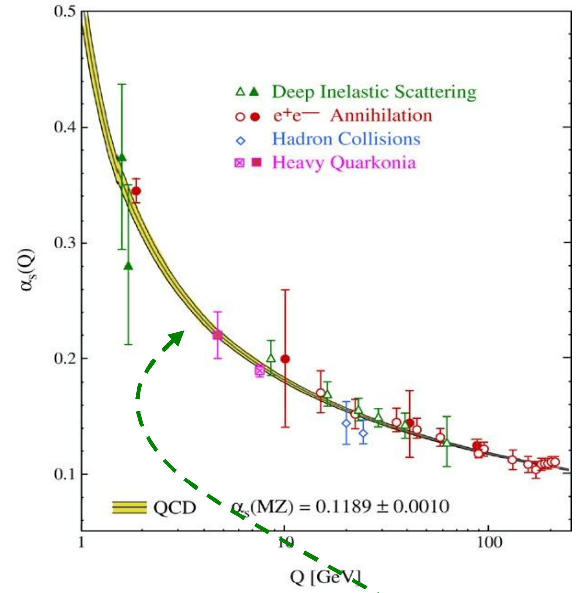
  $\rightarrow$ *No plane-wave propagation*

  Confinement?!

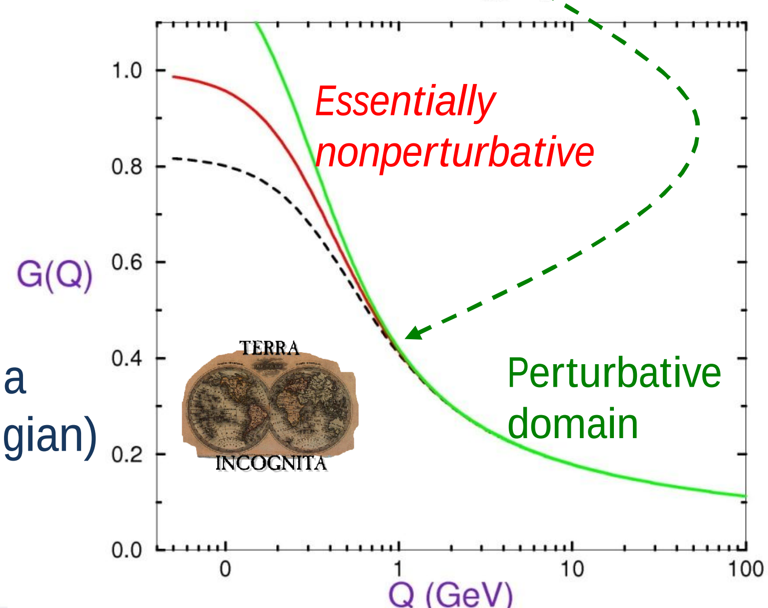


What happens in the real world?

- Strong-interaction: QCD
  - Asymptotically free
    - Perturbation theory is valid and accurate tool at large-Q^2 & hence chiral limit is defined
  - Essentially nonperturbative for $Q^2 < 2\ \mathrm{GeV}^2$
    - *Nature's only example of truly nonperturbative, fundamental theory*
    - *A-priori, no idea as to what such a theory can produce*



- Possibilities?
  - G(0) < 1: M(s) ≡ 0 is only solution for m = 0.
  - G(0) ≥ 1: M(s) ≠ 0 is possible and energetically favoured: DCSB.
  - M(0) ≠ 0 is a new, dynamically generated mass-scale. If it's large enough, can explain how a theory that is apparently massless (in the Lagrangian) possesses the spectrum of a massive theory.




IF YOU SEE ANYTHING MYSTERIOUS OR UNUSUAL. JUST ENJOY IT WHILE YOU CAN
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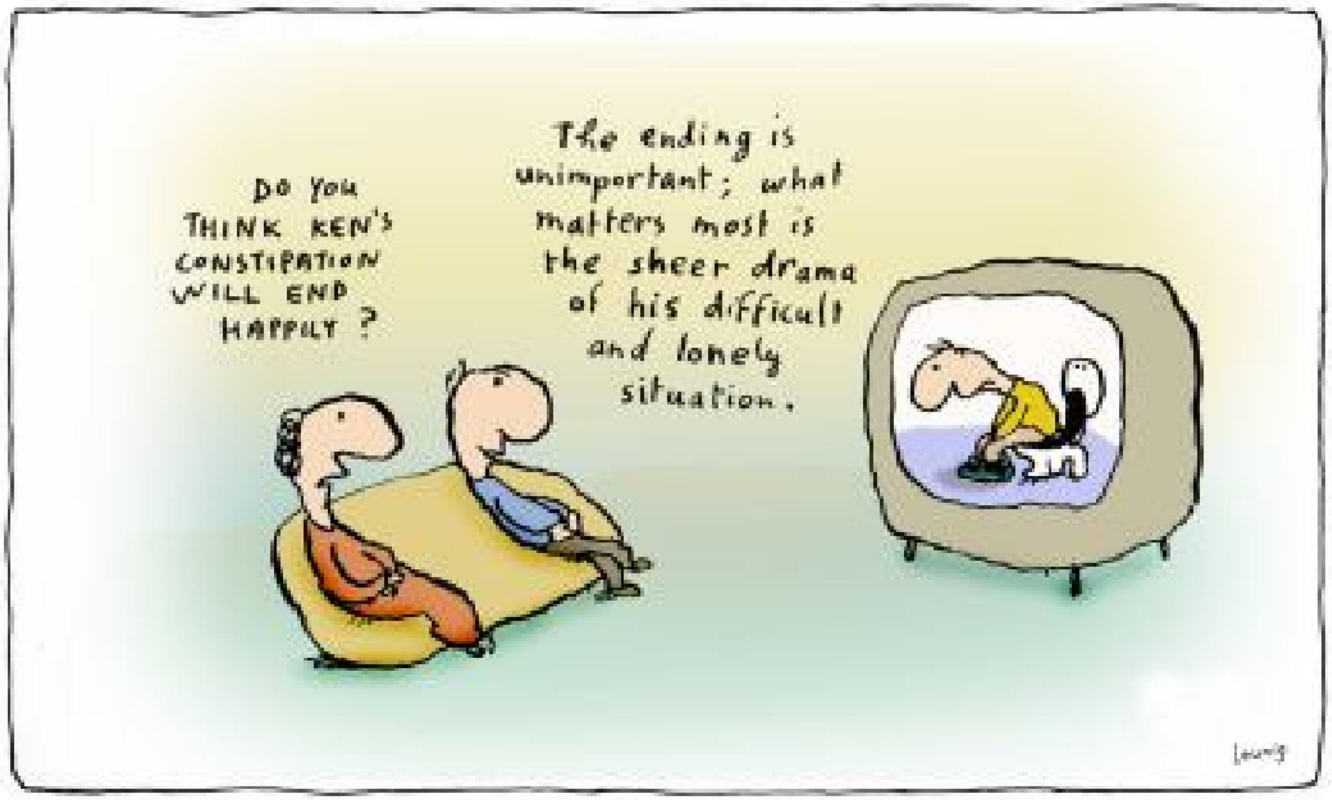
Big Picture

Overview

- Confinement and Dynamical Chiral Symmetry Breaking are Key Emergent Phenomena in QCD
- Understanding requires Nonperturbative Solution of Fully-Fledged Relativistic Quantum Field Theory
  - Mathematics and Physics still far from being able to accomplish that
- Confinement and DCSB are expressed in QCD's propagators and vertices
  - Nonperturbative modifications should have observable consequences
- Dyson-Schwinger Equations are a useful analytical and numerical tool for nonperturbative study of relativistic quantum field theory
- Simple models (NJL) can exhibit DCSB
  - DCSB $\not\Rightarrow$ Confinement
- Simple models (MN) can exhibit Confinement
  - Confinement $\Rightarrow$ DCSB

What's the story in QCD?



Confinement

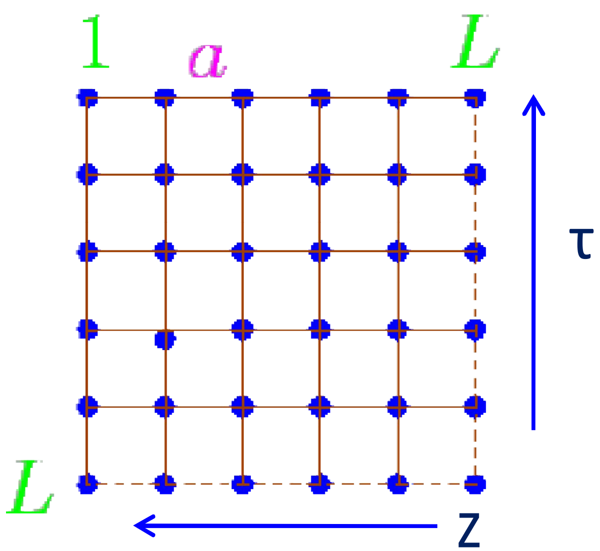
Confinement of quarks*

Kenneth G. Wilson
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850
(Received 12 June 1974)

A mechanism for total confinement of quarks, similar to that of Schwinger, is defined which requires the existence of Abelian or non-Abelian gauge fields. It is shown how to quantize a gauge field theory on a discrete lattice in Euclidean space-time, preserving exact gauge invariance and treating the gauge fields as angular variables (which makes a gauge-fixing term unnecessary). The lattice gauge theory has a computable strong-coupling limit; in this limit the binding mechanism applies and there are no free quarks. There is unfortunately no Lorentz (or Euclidean) invariance in the strong-coupling limit. The strong-coupling expansion involves sums over all quark paths and sums over all surfaces (on the lattice) joining quark paths. This structure is reminiscent of relativistic string models of hadrons.

Wilson Loop & the Area Law

$$\boxed{W_C := \mathrm{Tr}\left(\mathcal{P}\exp i\oint_C A_\mu dx^\mu\right)}$$



periodic lattice

- *C* is a closed curve in space,
  P is the path order operator
- Now, place static (infinitely heavy) fermionic sources of colour charge at positions

  $z_0=0$ & $z=\frac{1}{2}L$
- Then, evaluate $\langle W_C(z,\tau)\rangle$ as a functional integral over gauge-field configurations
- In the strong-coupling limit, the result can be obtained algebraically; viz.,

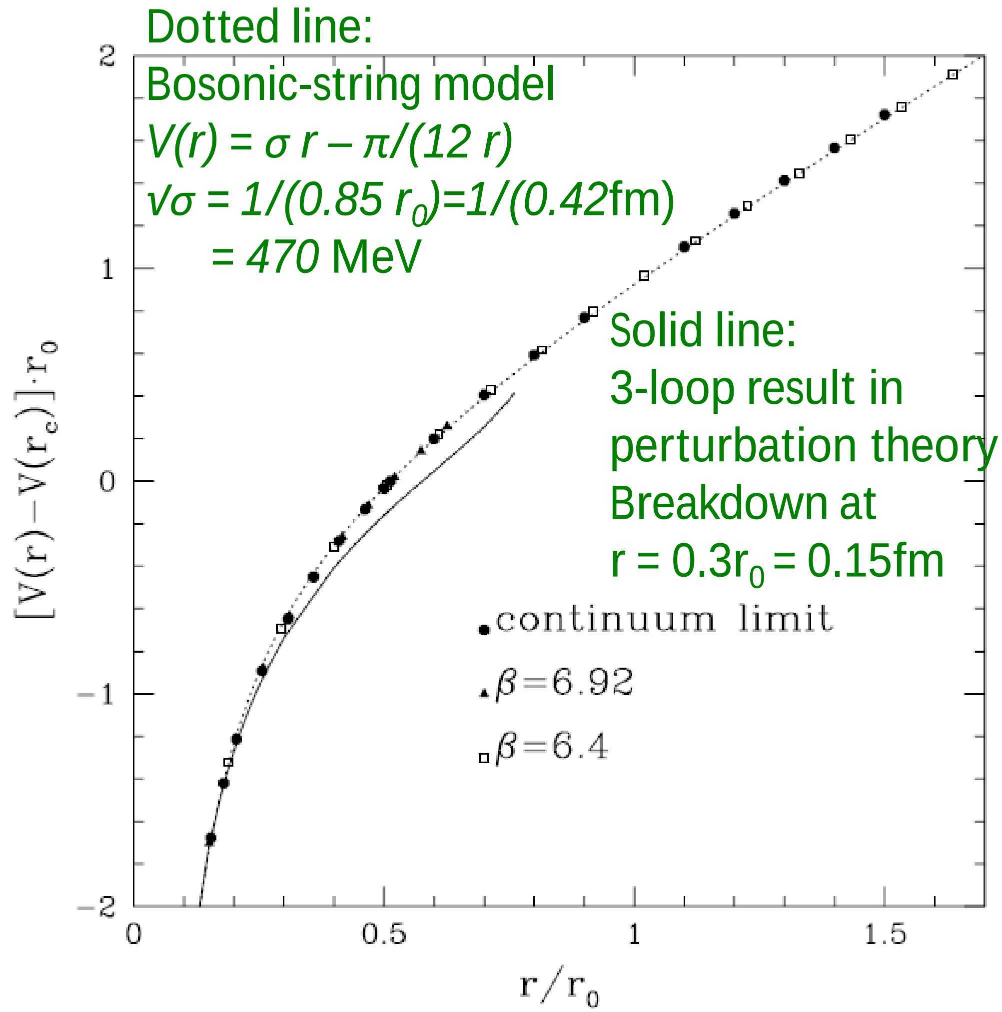
  $\langle W_C(z,\tau)\rangle = \exp(-V(z)\,\tau)$

  where $V(z)$ is the potential between the static sources, which behaves as $V(z) = \sigma z$

Linear potential
σ = String tension

Wilson Loop & Area Law

- Typical result from a numerical simulation of pure-glue QCD (hep-lat/0108008)
- r_0 is the Sommer-parameter, which relates to the force between static quarks at intermediate distances.
- The requirement
  $$r_0^2 F(r_0) = 1.65$$
  provides a connection between pure-glue QCD and potential models for mesons, and produces
  $$r_0 \approx 0.5 \text{ fm}$$

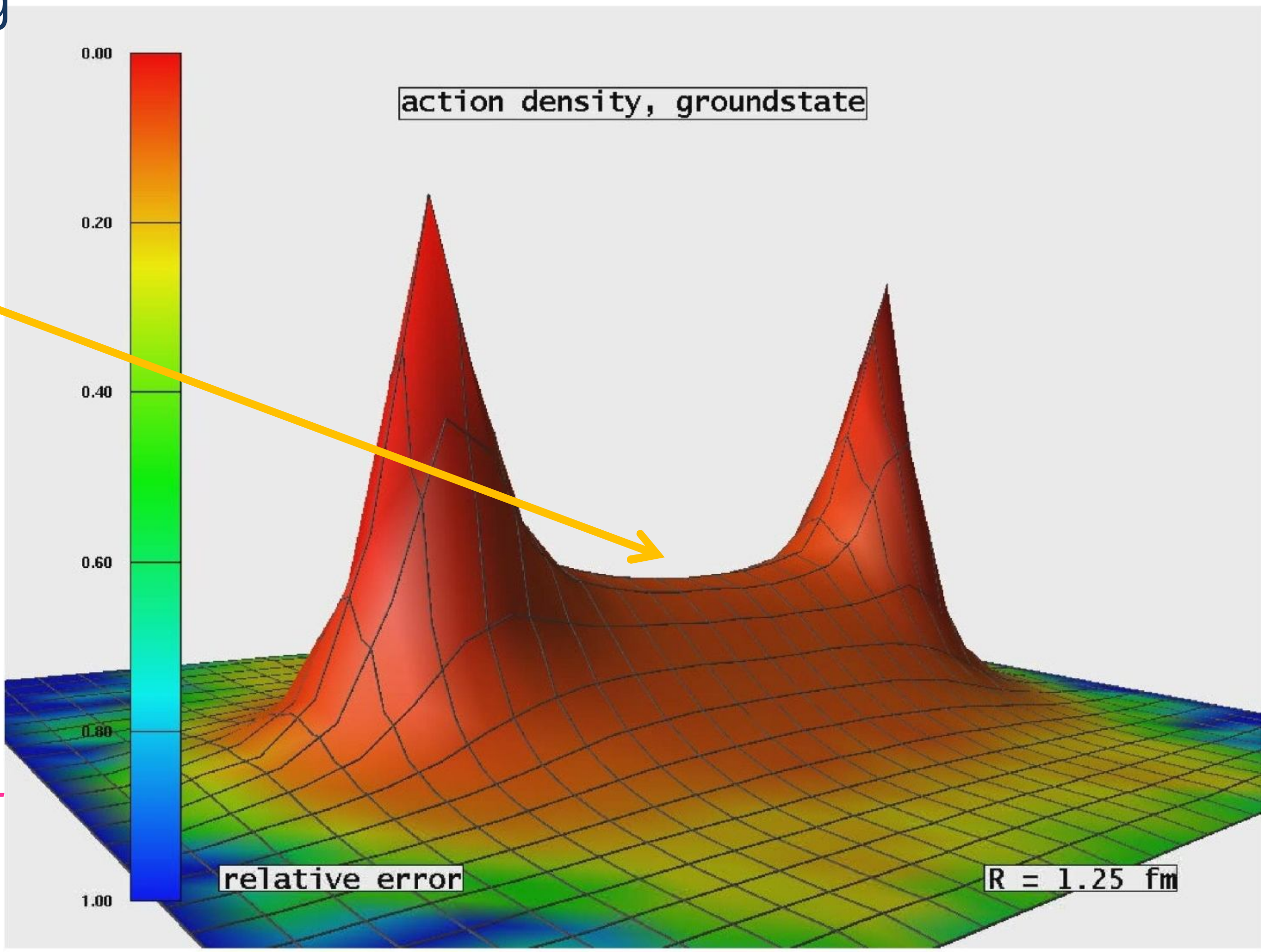


Flux Tube Models of Hadron Structure

- Illustration in terms of Action – density, which is analogous to plotting the force:

  $F(r) = \sigma - (\pi/12)(1/r^2)$

- It is pretty hard to overlook the flux tube between the static source and sink
- *Phenomenologists embedded in quantum mechanics and string theorists have been nourished by this result for many, many years.*



Light quarks & Confinement

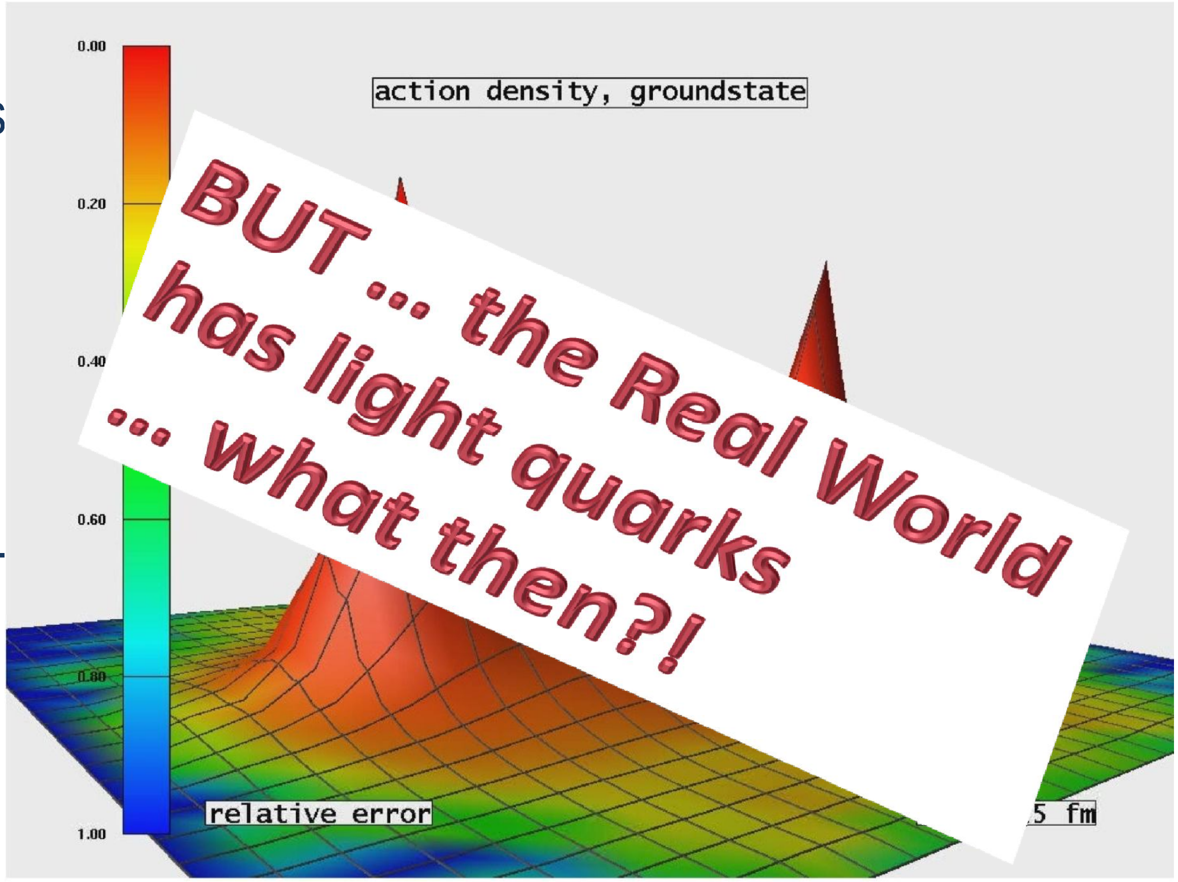
- Folklore

  "The color field lines between a quark and an anti-quark form flux tubes.

  A unit area placed midway between the quarks and perpendicular to the line connecting them intercepts a constant number of field lines, independent of the distance between the quarks.

  This leads to a constant force between the quarks – and a large force at that, equal to about 16 metric tons."

  Hall-D Conceptual-DR(5)



Light quarks & Confinement

- Problem:

  16 tonnes of force

  makes a lot of pions.

Light quarks & Confinement

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  16 tonnes of force
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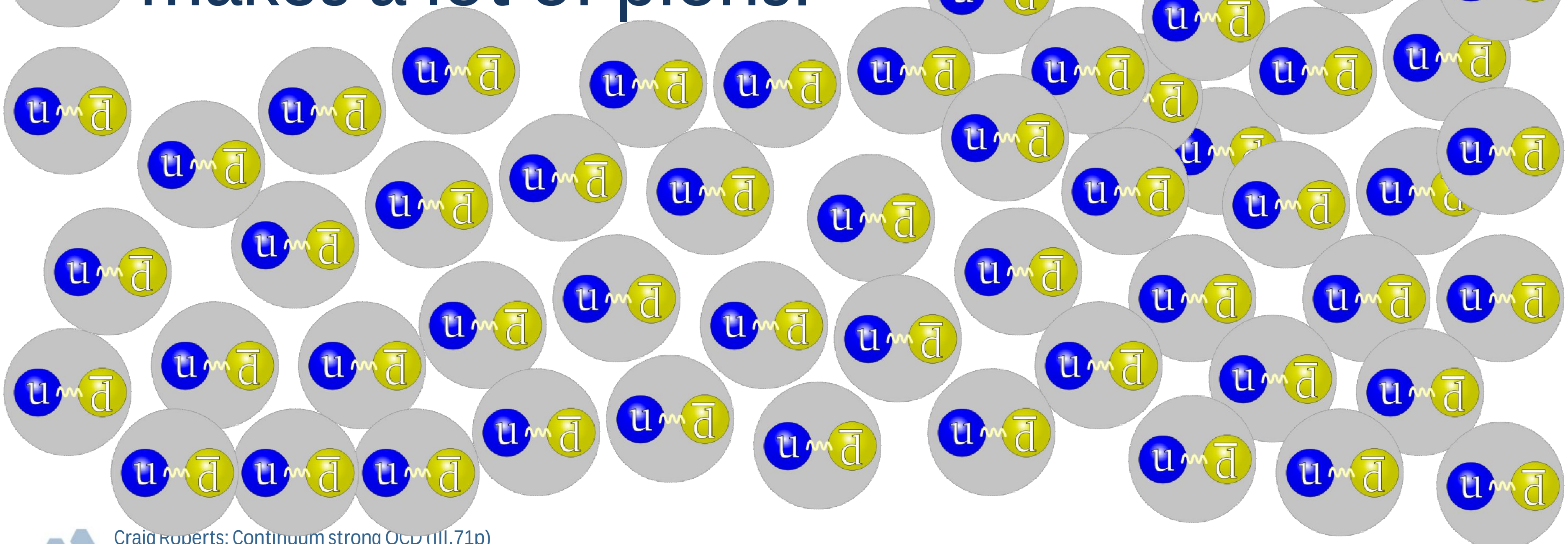
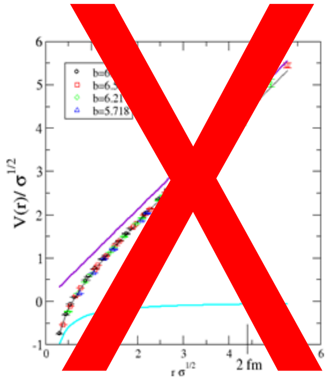
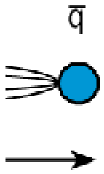






Confinement

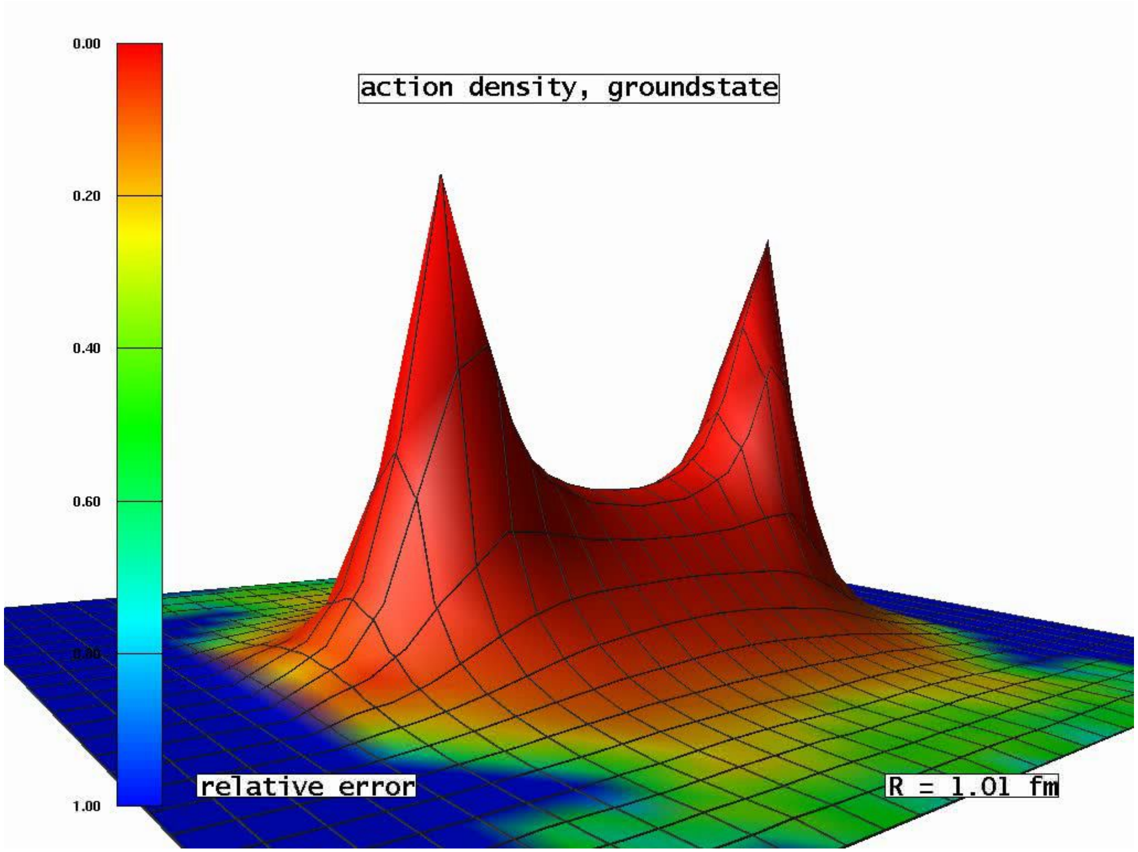
- Quark and Gluon Confinement
  - No matter how hard one strikes the proton, or any other hadron, one cannot liberate an individual quark or gluon
- Empirical fact. However
  - There is no agreed, theoretical definition of light-quark confinement
  - Static-quark confinement is irrelevant to real-world QCD
    - *There are no long-lived, very-massive quarks*
- Confinement entails *quark-hadron duality*; i.e., that all observable consequences of QCD can, in principle, be computed using an hadronic basis.

G. *Bali* et al., [PoS LAT2005 (2006) 308]

Confinement

"*Note that the time is not a linear function of the distance but dilated within the string breaking region. On a linear time scale string breaking takes place rather rapidly. [...] light pair creation seems to occur non-localized and instantaneously.*"

- Infinitely heavy-quarks *plus* 2 flavours with mass = m_s
  - Lattice spacing = 0.083fm
  - String collapses within one lattice time-step $R = 1.24 \ldots 1.32$ fm
  - Energy stored in string at collapse $E_c^{sb} = 2\, m_s$
  - (*mpg made via linear interpolation*)
- *No flux tube between light-quarks*



B_s anti-B_s

1993: *"for elucidating the quantum structure of electroweak interactions in physics"*

Regge Trajectories?

- Martinus Veltmann, “Facts and Mysteries in Elementary Particle Physics” (World Scientific, Singapore, 2003):

  In time the Regge trajectories thus became the cradle of string theory. Nowadays the Regge trajectories have largely disappeared, not in the least because these higher spin bound states are hard to find experimentally. At the peak of the Regge fashion (around 1970) theoretical physics produced many papers containing families of Regge trajectories, with the various (hypothetically straight) lines based on one or two points only!

Properties of Regge trajectories

Alfred Tang* and John W. Norbury†

Physics Department, University of Wisconsin–Milwaukee, P. O. Box 413, Milwaukee, Wisconsin 53201

(Received 30 November 1999; published 8 June 2000)

Early Chew-Frautschi plots show that meson and baryon Regge trajectories are approximately linear and non-intersecting. In this paper, we reconstruct all Regge trajectories from the most recent data. Our plots show that meson trajectories are non-linear and intersecting. We also show that all current meson Regge trajectories models are ruled out by data.

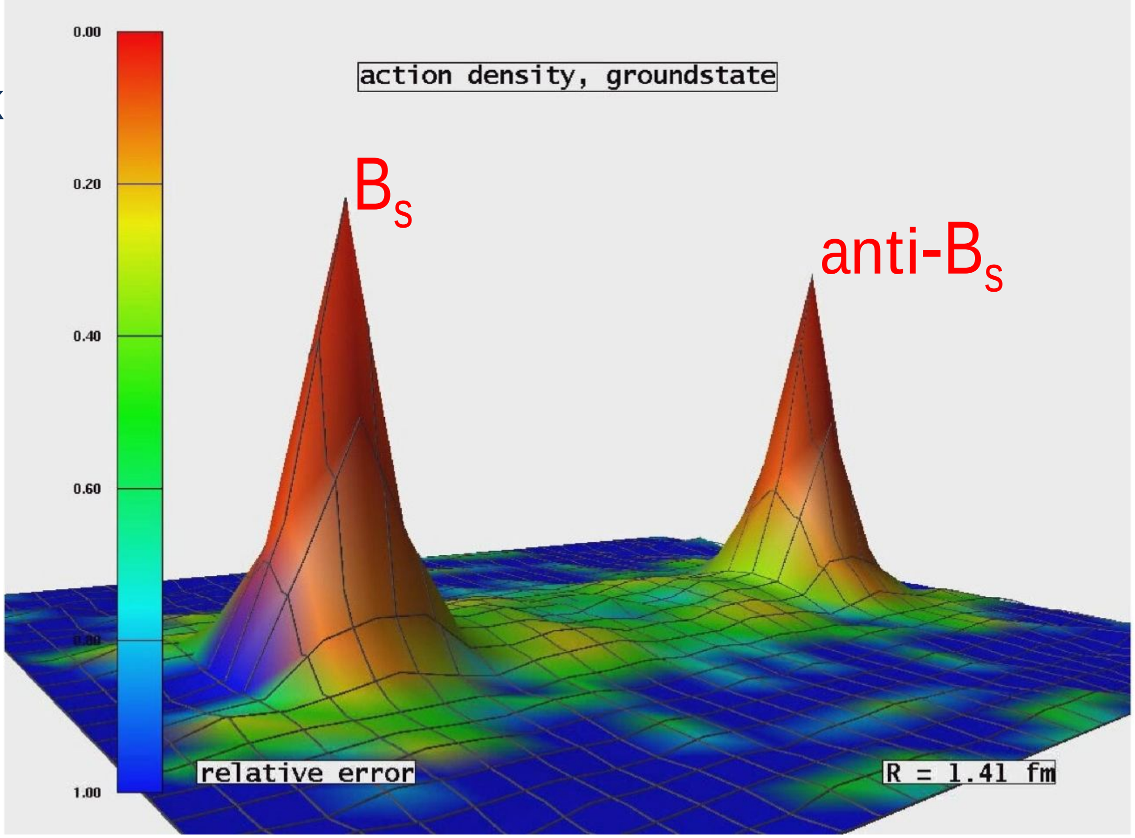
PACS number(s): 11.55.Jy, 12.40.Nn, 14.20.−c, 14.40.−n

Phys.Rev. D **62** (2000) 016006 [9 pages]

Confinement

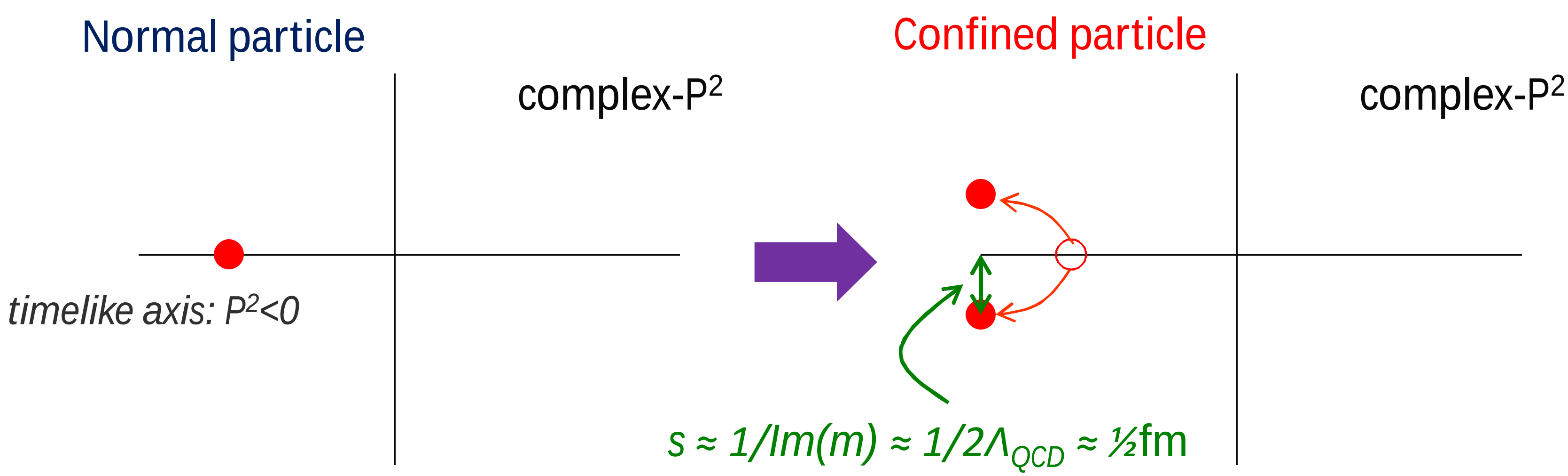
- Static-quark confinement is irrelevant to real-world QCD
  - *There are no long-lived, very-massive quarks*

- Indeed, potential models are irrelevant to light-quark physics, something which should have been plain from the start: *copious production of light particle-antiparticle pairs ensures that a potential model description is meaningless for light-quarks in QCD*



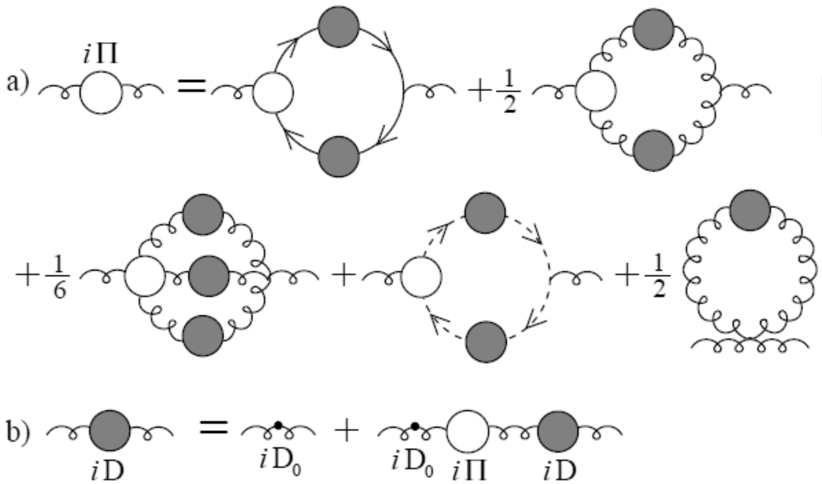
Confinement

- QFT Paradigm:
  - Confinement is expressed through a *dramatic* change in the analytic structure of propagators for coloured states
  - It can almost be read from a plot of the dressed-propagator for a coloured state



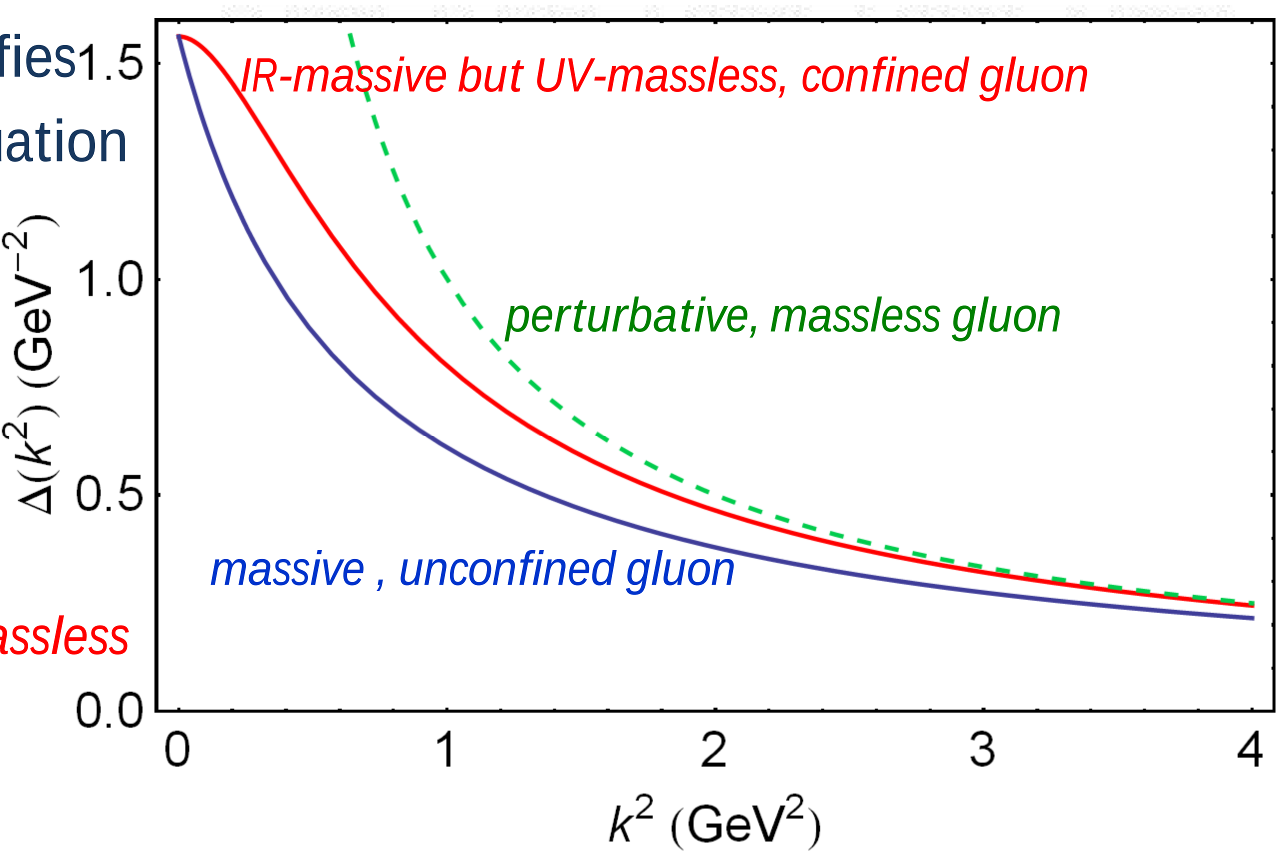
- *Real-axis mass-pole splits, moving into pair(s) of complex conjugate singularities*
- *State described by rapidly damped wave & hence state cannot exist in observable spectrum*

Dressed-gluon propagator



A.C. Aguilar et al., [Phys.Rev. D80 (2009) 085018]

- Gluon propagator satisfies a Dyson-Schwinger Equation
- Plausible possibilities for the solution
- DSE and lattice-QCD agree on the result
  - *Confined gluon*
  - *IR-massive but UV-massless*
  - *$m_G \approx$ 2-4 Λ_{QCD}*



Charting the interaction between light-quarks

This is a well-posed problem whose solution is an elemental goal of modern hadron physics. The answer provides QCD's running coupling.

- Confinement can be related to the analytic properties of QCD's Schwinger functions.
- Question of light-quark confinement can be translated into the challenge of charting the infrared behavior of QCD's universal β-function
  - This function may depend on the scheme chosen to renormalise the quantum field theory but it is unique within a given scheme.
  - Of course, the behaviour of the β-function on the perturbative domain is well known.

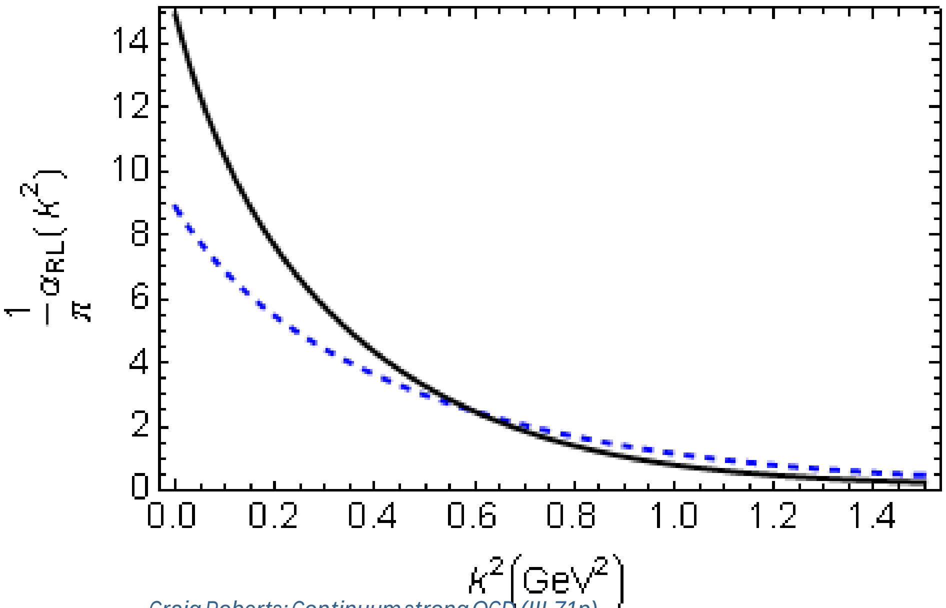
Charting the interaction between light-quarks

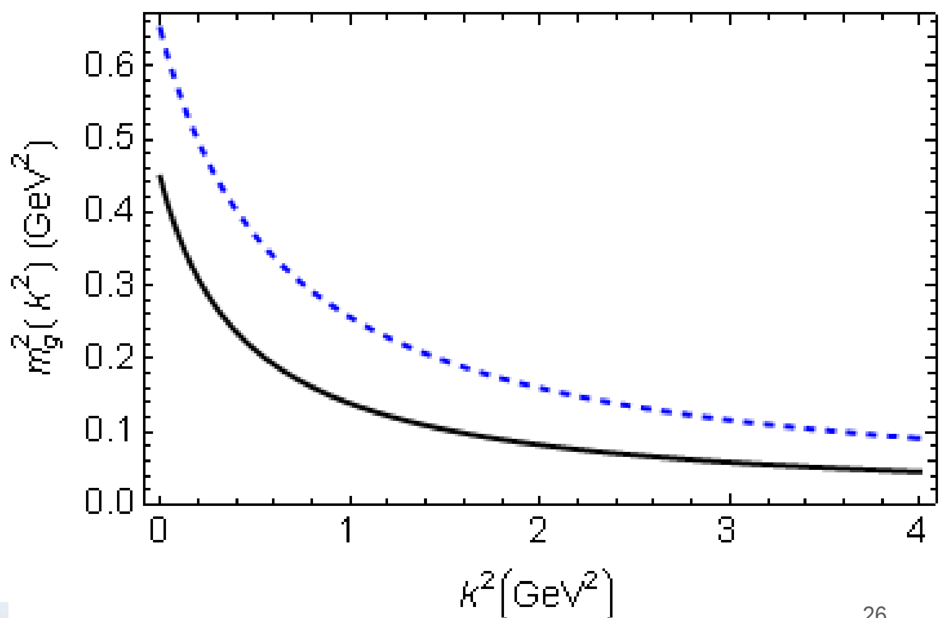
- Through QCD's Dyson-Schwinger equations (DSEs) the pointwise behaviour of the β-function determines the pattern of chiral symmetry breaking.
- DSEs connect β-function to experimental observables. Hence, comparison between computations and observations of
  - Hadron mass spectrum
  - Elastic and transition form factors
  - Parton distribution functions

  can be used to chart β-function's long-range behaviour.
- Extant studies show that the properties of hadron excited states are a great deal more sensitive to the long-range behaviour of the β-function than those of the ground states.

DSE Studies
- Phenomenology of gluon

- Wide-ranging study of π & ρ properties
- Effective coupling
  - Agrees with pQCD in ultraviolet
  - Saturates in infrared
    - α(0)/π = 8-15
    - $\alpha(m_G^2)/\pi$ = 2-4
- Running gluon mass
  - Gluon is massless in ultraviolet in agreement with pQCD
  - Massive in infrared
    - $m_G(0)$ = 0.67-0.81 GeV
    - $m_G(m_G^2)$ = 0.53-0.64 GeV

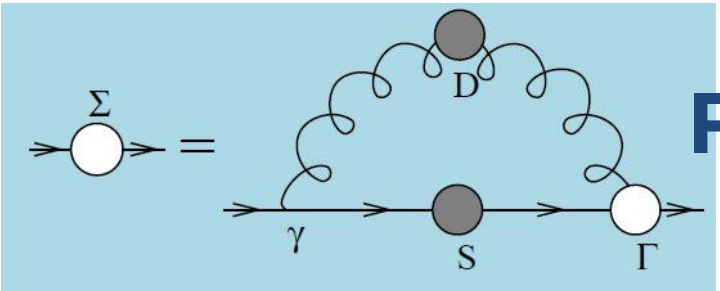




Dynamical Chiral Symmetry Breaking
Mass Gap

Dynamical Chiral Symmetry Breaking

- Whilst confinement is contentious …
- *DCSB is a fact in QCD*
  - ***Dynamical***, not spontaneous
    - Add nothing to *QCD* , *no Higgs field, nothing,* effect achieved purely through the dynamics of gluons and quarks.
  - It is the most important mass generating mechanism for visible matter in the Universe.
    - Responsible for approximately 98% of the proton's mass.
    - Higgs mechanism is (*almost*) irrelevant to light-quarks.

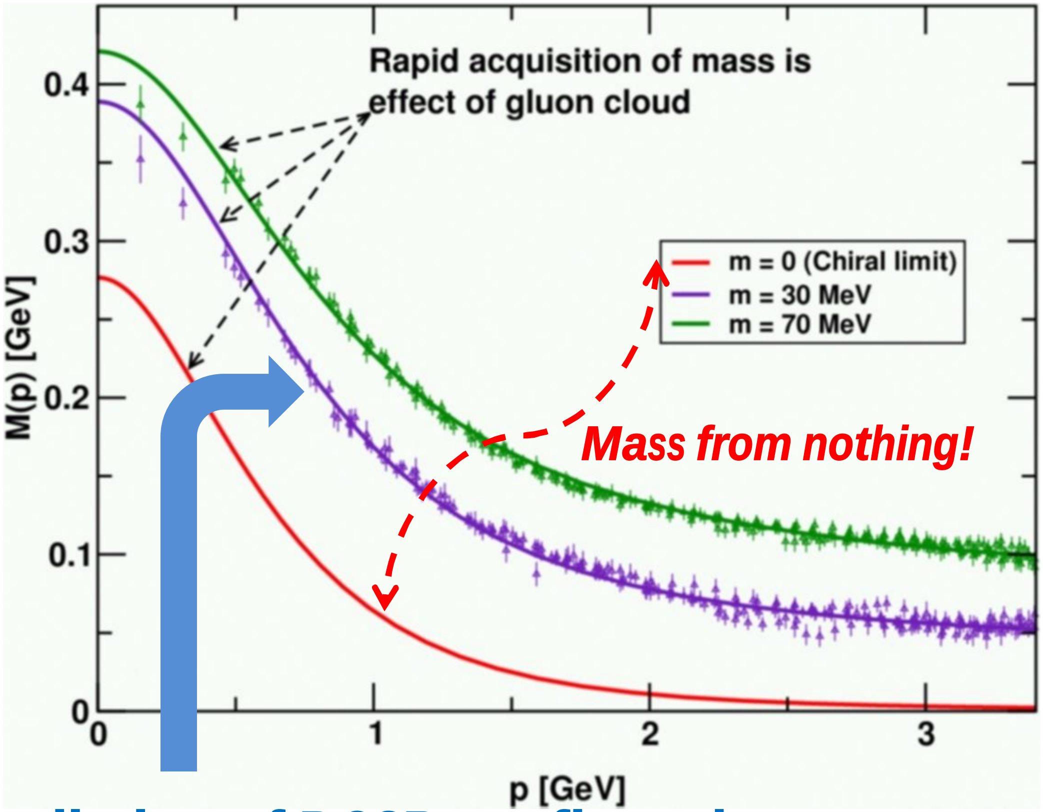


Frontiers of Nuclear Science: Theoretical Advances

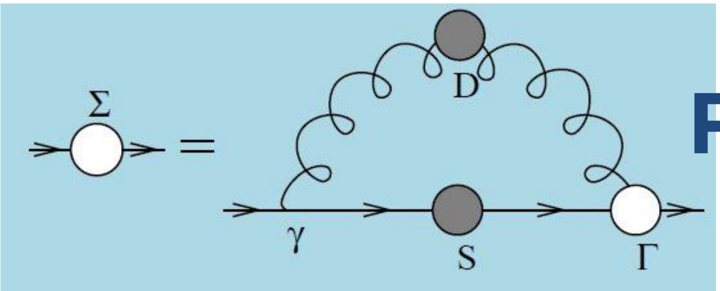
C.D. Roberts, [Prog. Part. Nucl. Phys. 61 (2008) 50]
M. Bhagwat & P.C. Tandy, [AIP Conf.Proc. 842 (2006) 225-227]

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies (m =0, red curve) acquires a large constituent mass at low energies.



DSE prediction of DCSB confirmed



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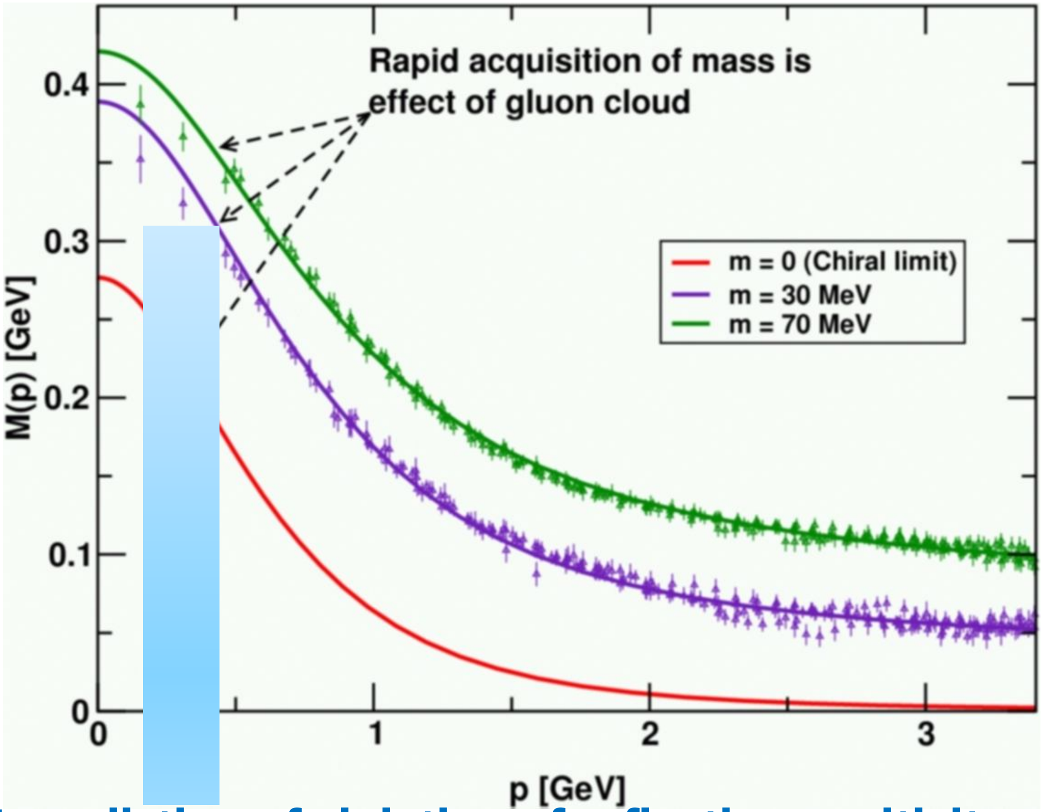
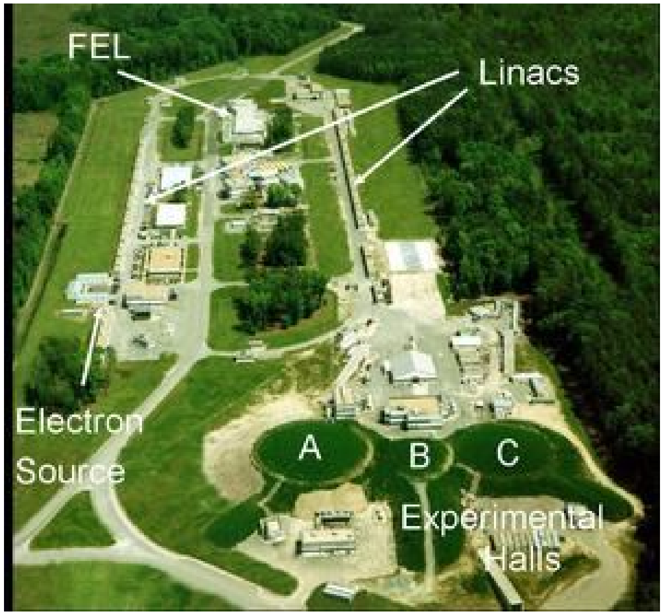


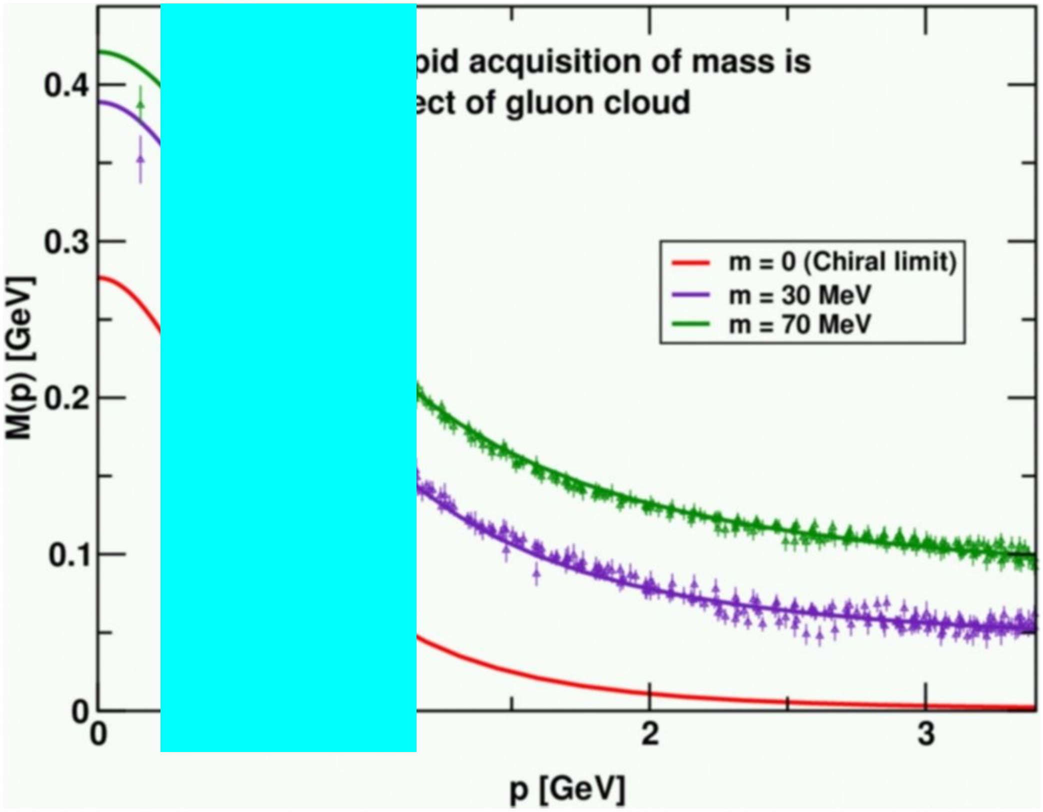
Hint of lattice-QCD support for DSE prediction of violation of reflection positivity

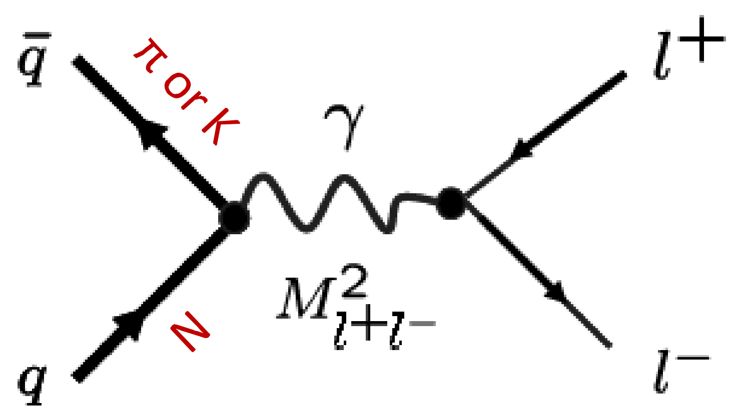


12GeV
The Future of JLab

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

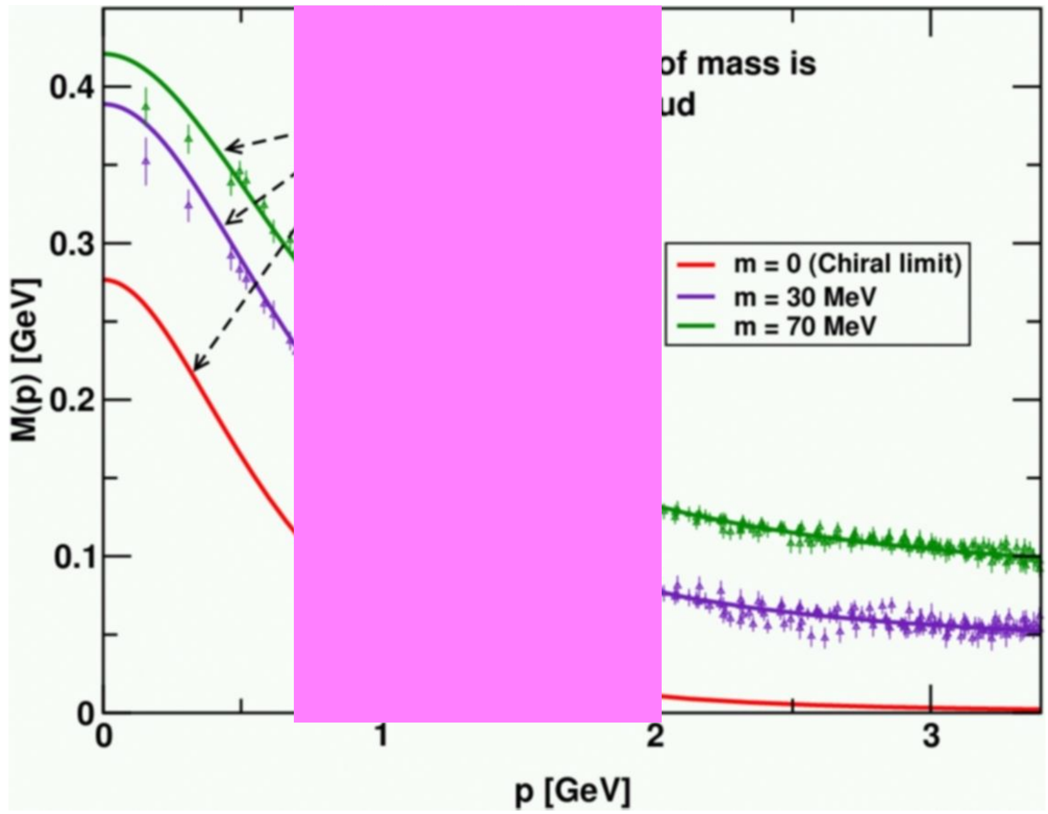
- **Jlab 12GeV: This region scanned by 2<Q^2<9 GeV^2 elastic & transition form factors.**



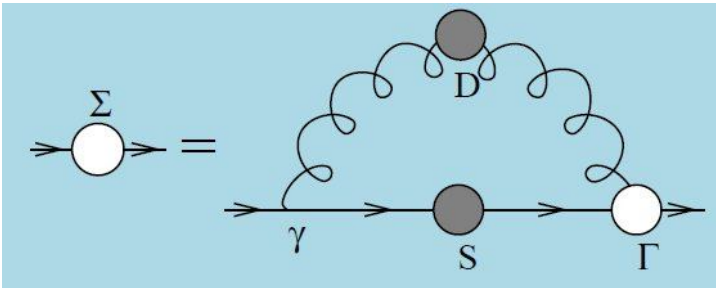


The Future of Drell-Yan

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

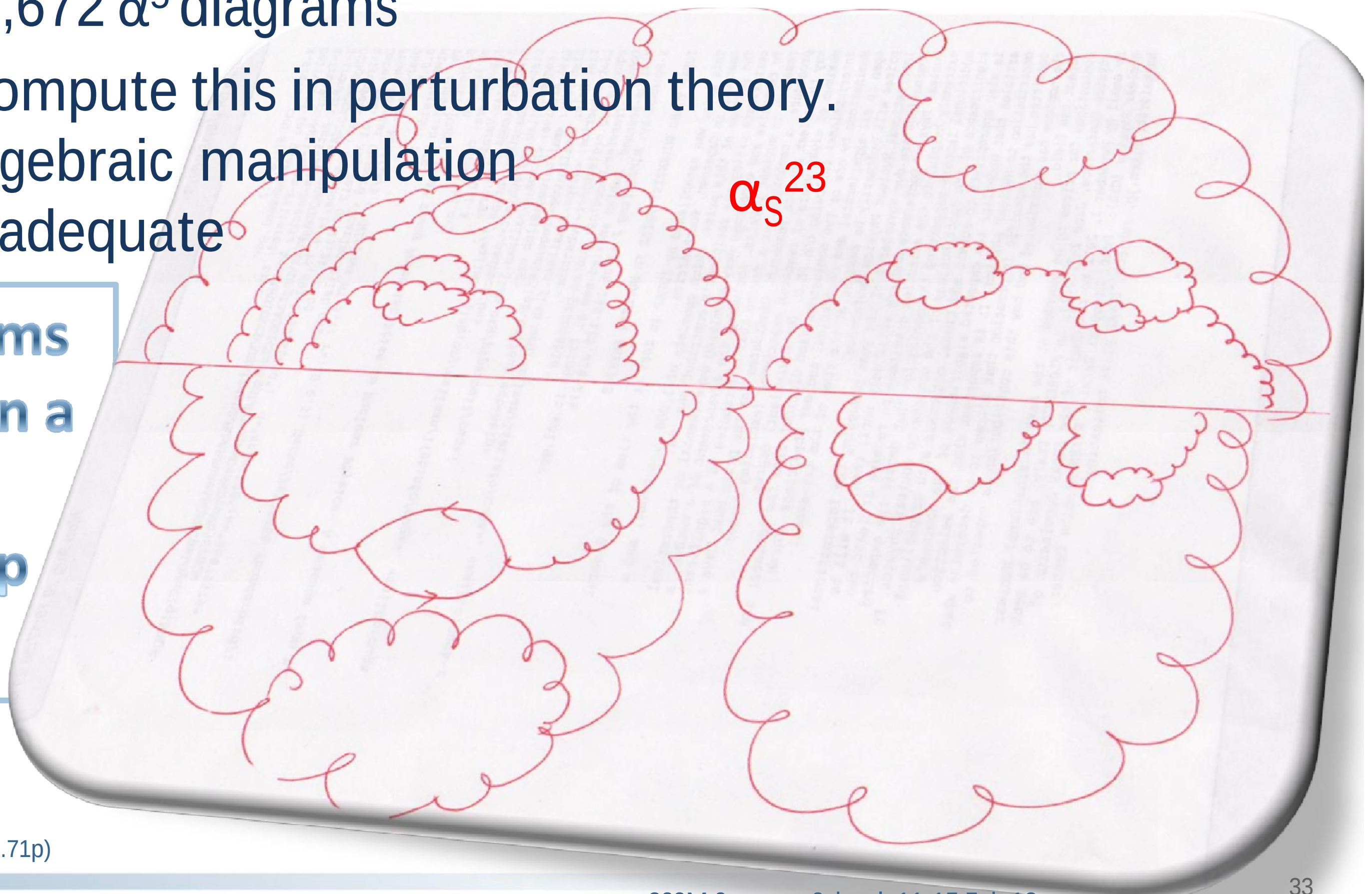


- Valence-quark PDFs and PDAs probe this critical and complementary region

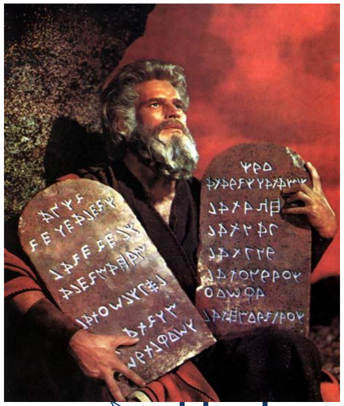


Where does the mass come from?

- Deceptively simply picture
  - Corresponds to the sum of a countable infinity of diagrams.
    NB. QED has 12,672 α^5 diagrams
  - Impossible to compute this in perturbation theory.
    The standard algebraic manipulation tools are just inadequate

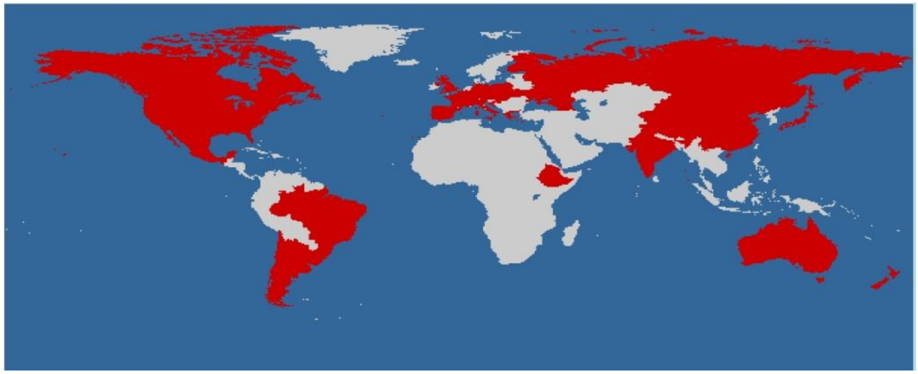


Just one of the terms that are summed in a solution of the rainbow-ladder gap equation



Universal Truths

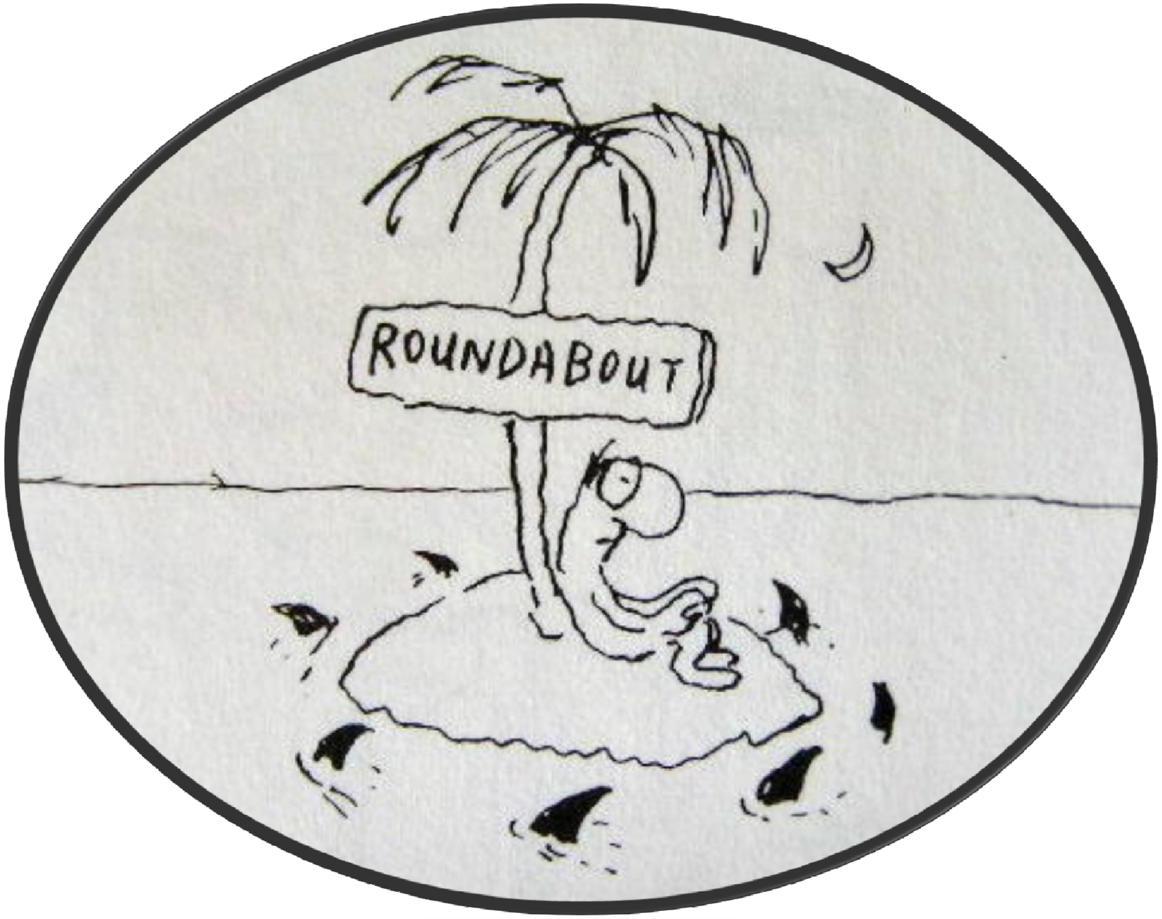
- Hadron spectrum, and elastic and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron's characterising properties amongst its QCD constituents.
- Dynamical Chiral Symmetry Breaking (DCSB) is most important mass generating mechanism for visible matter in the Universe.
  Higgs mechanism is (*almost*) irrelevant to light-quarks.
- Running of quark mass entails that calculations at even modest Q^2 require a Poincaré-covariant approach.
  Covariance + $M(p^2)$ require existence of quark orbital angular momentum in hadron's rest-frame wave function.
- Confinement is expressed through a violent change of the propagators for coloured particles & can almost be read from a plot of a states' dressed-propagator.
  It is intimately connected with DCSB.



Dyson-Schwinger Equations

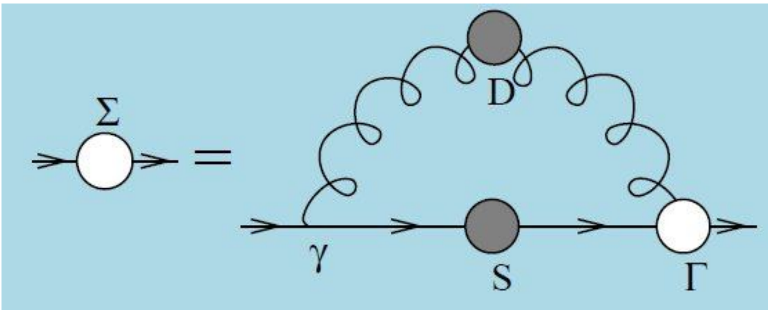
- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory . . . Materially Reduces Model-Dependence … Statement about long-range behaviour of quark-quark interaction
- NonPerturbative, Continuum approach to QCD
- Hadrons as Composites of Quarks and Gluons
- Qualitative and Quantitative Importance of:
  - Dynamical Chiral Symmetry Breaking
    - Generation of fermion mass from *nothing*
  - Quark & Gluon Confinement
    - Coloured objects not detected, *Not detectable?*

- Approach yields Schwinger functions; i.e., propagators and vertices
- Cross-Sections built from Schwinger Functions
- Hence, method connects observables with long-range behaviour of the running coupling
- Experiment ↔ Theory comparison leads to an understanding of long-range behaviour of strong running-coupling



Persistent Challenge

Truncation



Persistent challenge in application of DSEs

$$S_f(p)^{-1} = Z_2\,(i\gamma\cdot p + m_f^{\rm bm}) + \Sigma_f(p)\,,$$

$$\Sigma_f(p) = Z_1 \int_q^{\Lambda} g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S_f(q)\frac{\lambda^a}{2}\Gamma^f_\nu(q,p)$$

- Infinitely many coupled equations:
  Kernel of the equation for the quark self-energy involves:
  - ***$D_{\mu\nu}(k)$ – dressed-gluon propagator***
  - ***$\Gamma_\nu(q,p)$ – dressed-quark-gluon vertex***

  each of which satisfies its own DSE, etc…
- Coupling between equations *necessitates* a truncation
  - Weak coupling expansion
    ⇒ produces every diagram in perturbation theory
  - Otherwise ***useless***
    for the nonperturbative problems in which we're interested

Invaluable check on practical truncation schemes

Persistent challenge - truncation scheme

Relationship must be preserved by any truncation
Highly nontrivial constraint
FAILURE has an extremely high cost
– loss of any connection with QCD

- Symmetries associated with conservation of vector and axial-vector currents are critical in arriving at a veracious understanding of hadron structure and interactions
- Example: axial-vector W… …i identity
  – Statement of chi… …hich it's broken in quantum field …

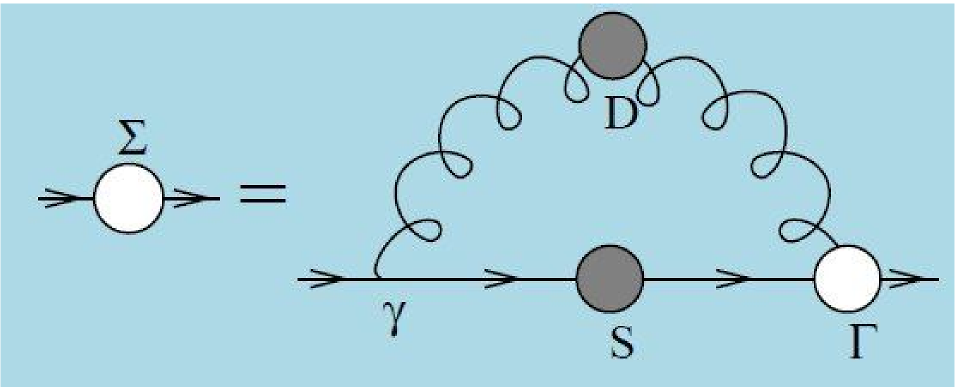
$P_\mu \Gamma^l_{5\mu}(k;P$ … $S^{-1}(k_-)$

… $_5(k;P)\, M_\zeta$

Axial-Vector vertex
Satisfies an inhomogeneous
Bethe-Salpeter equation

Quark
propagator
satisfies a
gap equation

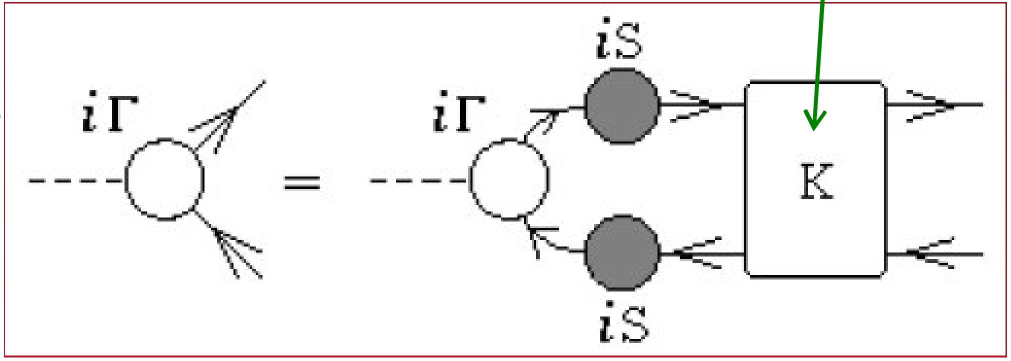
Kernels of these equations are completely different
But they must be intimately related

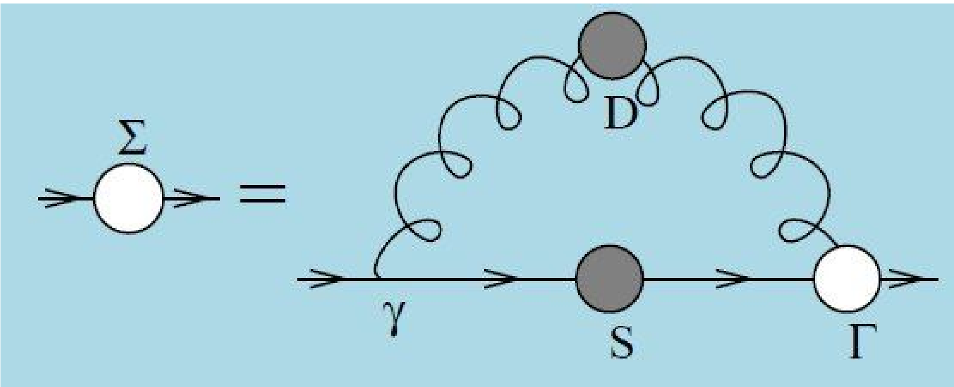


Persistent challenge - truncation scheme

- These observations show that symmetries relate the kernel of the gap equation – nominally a one-body problem, with that of the Bethe-Salpeter equation – considered to be a two-body problem
- Until 1995/1996 people had no idea what to do
- Equations were truncated, sometimes with good phenomenological results, sometimes with poor results
- Neither good nor bad could be explained

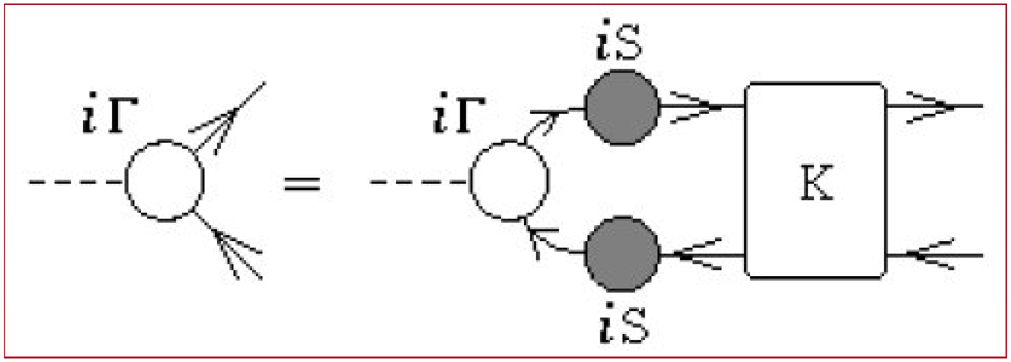
quark-antiquark scattering kernel





Persistent challenge - truncation scheme

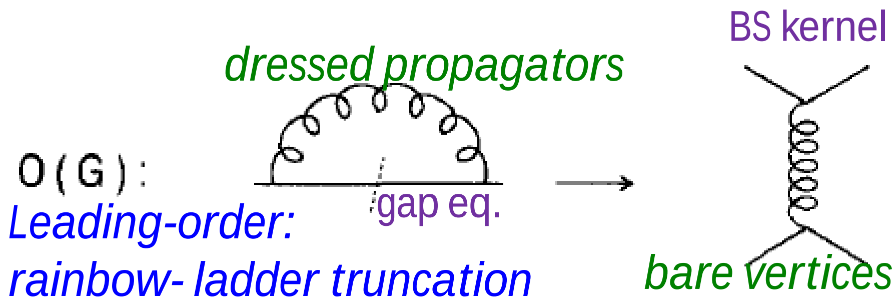
- Happily, that changed, and there is now at least one systematic, ***nonperturbative*** and symmetry preserving truncation scheme
  - H.J. Munczek, Phys. Rev. D **52** (1995) 4736, *Dynamical chiral symmetry breaking, Goldstone's theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations*
  - A. Bender, C.D. Roberts and L. von Smekal, Phys.Lett. B **380** (1996) 7, *Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation*



Modified skeleton expansion in which the propagators are fully-dressed but the vertices are constructed term-by-term

Cutting **scheme**

- The procedure generates a Bethe-Salpeter kernel from the kernel of any gap equation whose diagrammatic content is known
  - That this is possible and achievable systematically is necessary and sufficient to prove some exact results in QCD



- The procedure also enables the formulation of practical phenomenological models that can be used to illustrate the exact results and provide predictions for experiment with readily quantifiable errors.

Now able to explain the dichotomy of the pion

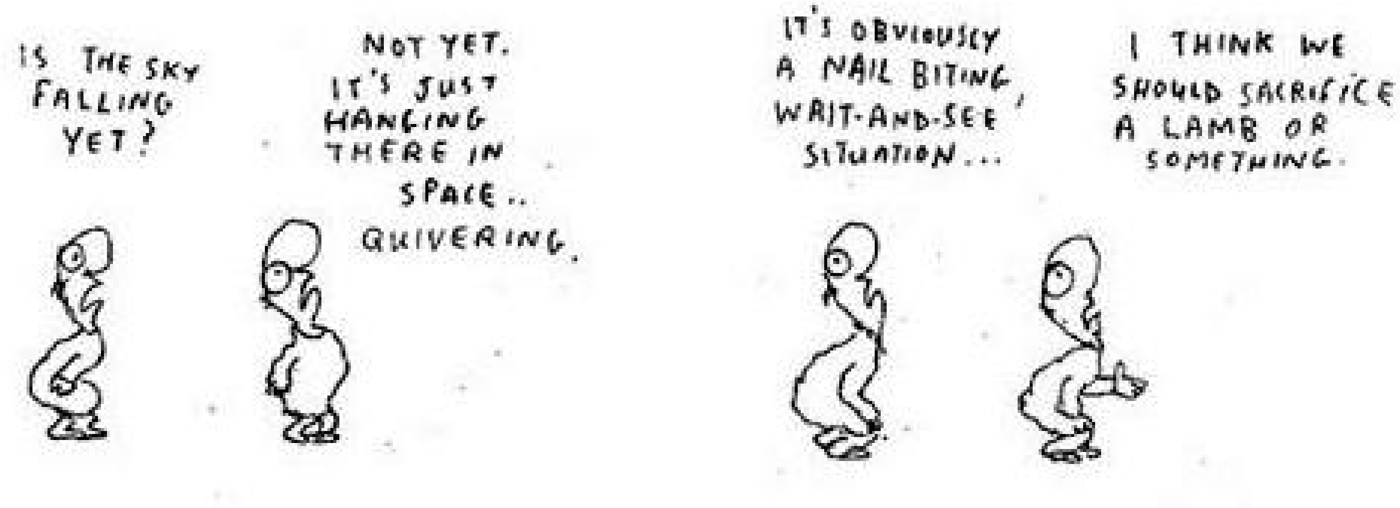
- How does one make an almost massless particle from two massive constituent-quarks?
- Naturally, one *could* always tune a potential in quantum mechanics so that the ground-state is massless

  – but some are still making this mistake

- However: $$m_\pi^2 \propto m$$

  current-algebra (1968)

- This is *impossible in quantum mechanics*, for which one always finds: $m_{bound-state} \propto m_{constituent}$



Some Exact Results

Maris, Roberts and Tandy
*nucl-th/9707003, Phys.Lett. B**420** (1998) 267-273*

Pion's Goldberger -Treiman relation

- Pion's Bethe-Salpeter amplitude

Solution of the Bethe-Salpeter equation

Pseudovector components necessarily nonzero. Cannot be ignored!

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\Big]$$

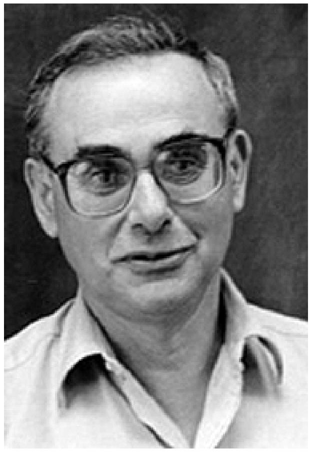
- Dressed-quark propagator $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$

- Axial-vector Ward-Takahashi identity entails

$$f_\pi E_\pi(k;P=0) = B(p^2)$$

Exact in Chiral QCD

***Miracle*: two body problem solved, almost completely, once solution of one body problem is known**



Dichotomy of the pion
Goldstone mode and bound-state

$$f_\pi \, E_\pi(p^2) = B(p^2)$$

- Goldstone's theorem
  has a pointwise expression in QCD;

  Namely, in the chiral limit the wave-function for the two-body bound-state Goldstone mode is intimately connected with, and almost completely specified by, the fully-dressed one-body propagator of its characteristic constituent

  - The one-body momentum is equated with the relative momentum of the two-body system

Maris, Roberts and Tandy
*nucl-th/9707003, Phys.Lett. B**420** (1998) 267-273*

Dichotomy of the pion
Mass Formula for 0⁻ Mesons

$$f_{H_5}\, m_{H_5}^2 = \rho_{H_5}^{\zeta}\, \mathcal{M}_{H_5}^{\zeta}$$

- Mass-squared of the pseudscalar hadron
- Sum of the current-quark masses of the constituents;
  e.g., pion = $m_u^{\varsigma} + m_d^{\varsigma}$, where "ς" is the renormalisation point

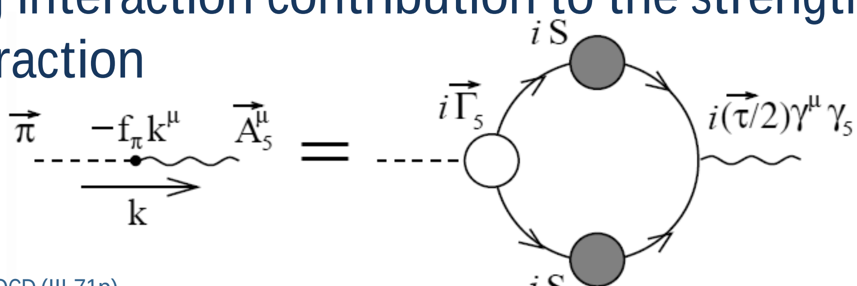
Maris, Roberts and Tandy
*nucl-th/9707003, Phys.Lett. B**420** (1998) 267-273*

Dichotomy of the pion Mass Formula for 0⁻ Mesons

$$f_{H_5}\, m_{H_5}^2 = \rho_{H_5}^{\zeta} \mathcal{M}_{H_5}^{\zeta}$$

$$f_{H_5} P_\mu = Z_2 \mathrm{tr} \int \frac{d^4q}{(2\pi)^4} \frac{1}{2} (T^{H_5})^{\mathrm{t}} \gamma_5 \gamma_\mu \boxed{S(q+\frac{1}{2}P)\Gamma_{H_5}(q;P)S(q-\frac{1}{2}P)}$$

- Pseudovector projection of the Bethe-Salpeter wave function onto the origin in configuration space
  - Namely, the pseudoscalar meson’s leptonic decay constant, which is the strong interaction contribution to the strength of the meson’s weak interaction



Maris, Roberts and Tandy
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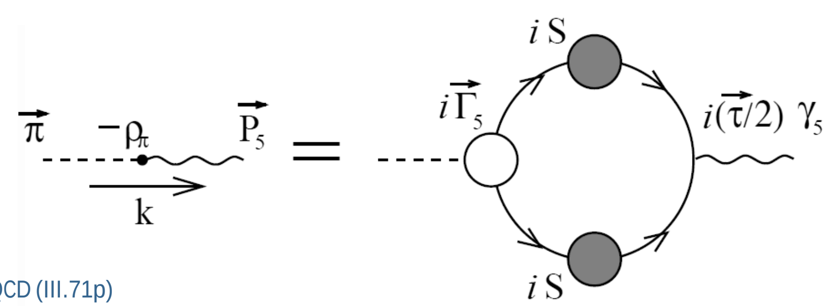
Dichotomy of the pion
Mass Formula for 0⁻ Mesons

$$f_{H_5}\, m_{H_5}^2 = \rho_{H_5}^{\zeta} \mathcal{M}_{H_5}^{\zeta}$$

$$i\rho_{H_5} = Z_4 \mathrm{tr} \int \frac{d^4q}{(2\pi)^4}\, \frac{1}{2}(T^{H_5})^{\mathrm{t}} \gamma_5 \boxed{S(q+\frac{1}{2}P)\Gamma_{H_5}(q;P)S(q-\frac{1}{2}P)}$$

- Pseudoscalar projection of the Bethe-Salpeter wave function onto the origin in configuration space
  - Namely, a pseudoscalar analogue of the meson's leptonic decay constant



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Dichotomy of the pion
Mass Formula for 0⁻ Mesons

$$f_{H_5}\, m^2_{H_5} = \rho^{\zeta}_{H_5} \mathcal{M}^{\zeta}_{H_5}$$

- Consider the case of light quarks; namely, $m_q \approx 0$
  - If chiral symmetry is dynamically broken, then
    - $f_{H5} \to f_{H5}^0 \neq 0$
    - $\rho_{H5} \to -\langle \bar{q} q\rangle / f_{H5}^0 \neq 0$

    both of which are independent of m_q

The so-called "vacuum quark condensate." More later about this.

- Hence, one arrives at the corollary

Gell-Mann, Oakes, Renner relation 1968

$$\boxed{m^2_{H_5} = 2 m_q \frac{-\langle \bar{q} q\rangle}{f^0_{H_5}}}$$

$$m_\pi^2 \propto m$$

Maris, Roberts and Tandy
*nucl-th/9707003, Phys.Lett. B**420** (1998) 267-273*

Dichotomy of the pion
Mass Formula for 0⁻ Mesons

$$f_{H_5}\, m^2_{H_5} = \rho^\zeta_{H_5} \mathcal{M}^\zeta_{H_5}$$

- Consider a different case; namely, one quark mass fixed and the other becoming very large, so that $m_q / m_Q << 1$
- Then
  - $f_{H5} \propto 1/\sqrt{m_{H5}}$
  - $\rho_{H5} \propto \sqrt{m_{H5}}$

  and one arrives at

$$m_{H5} \propto m_Q$$

Provides
QCD proof of
potential model result

Ivanov, Kalinovsky, Roberts
Phys. Rev. D **60**, 034018 (1999) [17 pages]

Radial excitations & Hybrids & Exotics
⇒ wave-functions with support at long-range
⇒ sensitive to confinement interaction
Understanding confinement "remains one of The greatest intellectual challenges in physics"

Radial excitations of Pseudoscalar meson

- Hadron spectrum contains 3 pseudoscalars [$I^G(J^P)L = 1^-(0-)S$] masses below 2GeV: π(140); π(1300); and π(1800)

the pion

- Constituent-Quark Model suggests that these states are the 1st three members of an n^1S_0 trajectory; i.e., ground state plus radial excitations
- But π(1800) is narrow ($\Gamma = 207 \pm 13$); i.e., surprisingly long-lived & decay pattern conflicts with usual quark-model expectations.
  - $S_{Q\text{-}barQ} = 1 \oplus L_{Glue} = 1 \Rightarrow J = 0$
    & $L_{Glue} = 1 \Rightarrow {}^3S_1 \oplus {}^3S_1$ (Q-bar Q) decays are suppressed
  - Perhaps therefore it's a hybrid?

exotic mesons: quantum numbers not possible for quantum mechanical quark-antiquark systems
hybrid mesons: normal quantum numbers but non-quark-model decay pattern
BOTH suspected of having "constituent gluon" content

Höll, Krassnigg and Roberts
*Phys.Rev. C**70** (2004) 042203(R)*

Radial excitations of Pseudoscalar meson

$$f_{H_5}\, m_{H_5}^2 = \rho_{H_5}^{\zeta} \mathcal{M}_{H_5}^{\zeta}$$

Flip side: if no DCSB, then all pseudoscalar mesons decouple from the weak interaction!

- Valid for ALL Pseudoscalar mesons
  - When chiral symmetry is dynamically broken, then
    - ρ_{H5} is finite and nonzero in the chiral limit, $M_{H5} \to 0$
  - A "radial" excitation of the π-meson, is not the ground state, so

    $m^2_{\pi \text{ excited state}} \neq 0 > m^2_{\pi \text{ ground state}} = 0$ (in chiral limit, $M_{H5} \to 0$)
- Putting this things together, it follows that

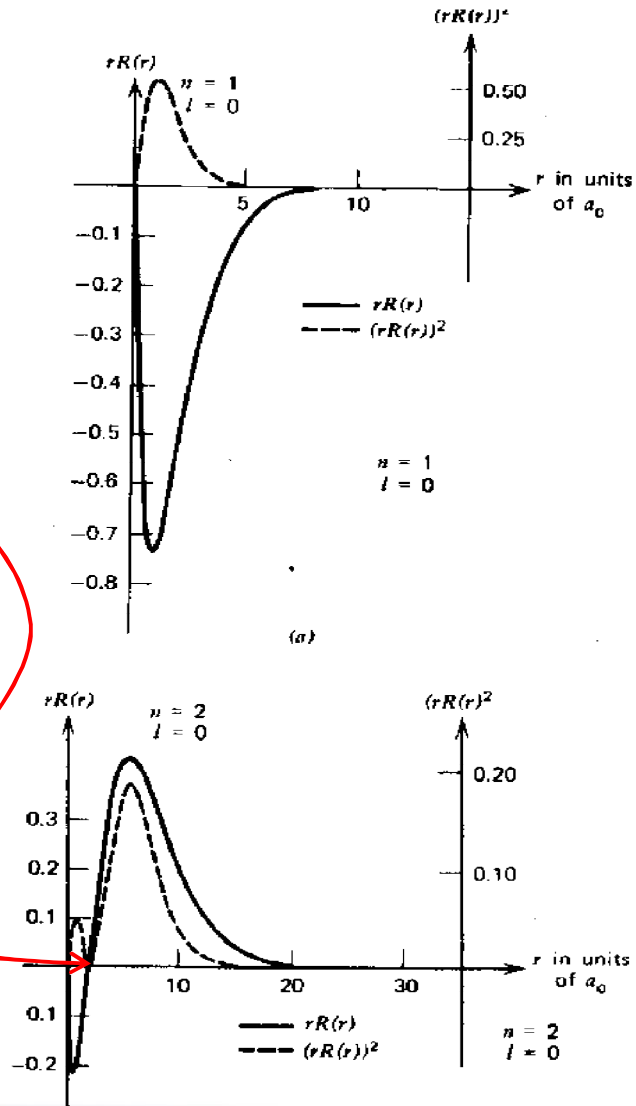
$$f_{H5} = 0$$

for ALL pseudoscalar mesons, except π(140), in the chiral limit

*Dynamical Chiral Symmetry Breaking – Goldstone's Theorem – impacts upon **every** pseudoscalar meson*

Radial excitations of Pseudoscalar meson

- This is fascinating because in quantum mechanics, decay constants of a radial excitation are suppressed by factor of roughly ⅓
  - Radial wave functions possess a zero
  - Hence, integral of "r $R_{n=2}(r)^2$" is quantitatively reduced compared to that of "r $R_{n=1}(r)^2$"



- **HOWEVER, ONLY A SYMMETRY CAN ENSURE THAT SOMETHING VANISHES COMPLETELY**

McNeile and Michael
*Phys.Lett. B**642** (2006) 244-247*

Lattice-QCD & radial excitations of pseudoscalar mesons

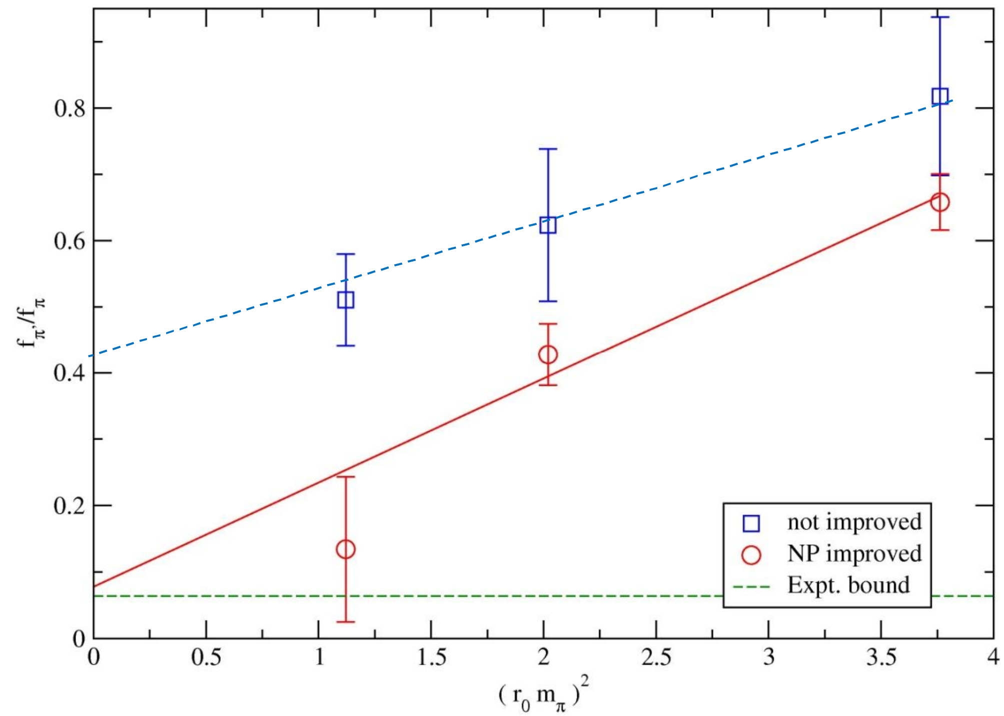
"The suppression of $f_{\pi 1}$ is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited state mesons."

- *When we first heard about [this result] our first reaction was a combination of "that is remarkable" and "unbelievable".*
- CLEO: $\tau \rightarrow \pi(1300) + \nu_\tau$

  $\Rightarrow f_{\pi 1} < 8.4\text{MeV}$

  Diehl & Hiller
  hep-ph/0105194
- Lattice-QCD check:
  $16^3 \times 32$-lattice, a ~ 0.1 fm,
  two-flavour, unquenched

  $\Rightarrow f_{\pi 1}/f_\pi = 0.078\ (93)$
- Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)



Bhagwat, Chang, Liu, Roberts, Tandy
*Phys.Rev. C**76** (2007) 045203*

Charge-neutral pseudoscalar mesons

non-Abelian Anomaly and η-η' mixing

- Neutral mesons containing s-bar & s are special, in particular

  η & η'

- Problem:

  η' is a pseudoscalar meson but it's much more massive than the other eight pseudoscalars constituted from light-quarks.

  $m_\eta = 548$ MeV
  $m_{\eta'} = 958$ MeV

  Splitting is 75% of η mass!

- Origin:

  While the classical action associated with QCD is invariant under $U_A(N_f)$ (non-Abelian axial transformations generated by $\lambda^0\gamma_5$), the quantum field theory is not!

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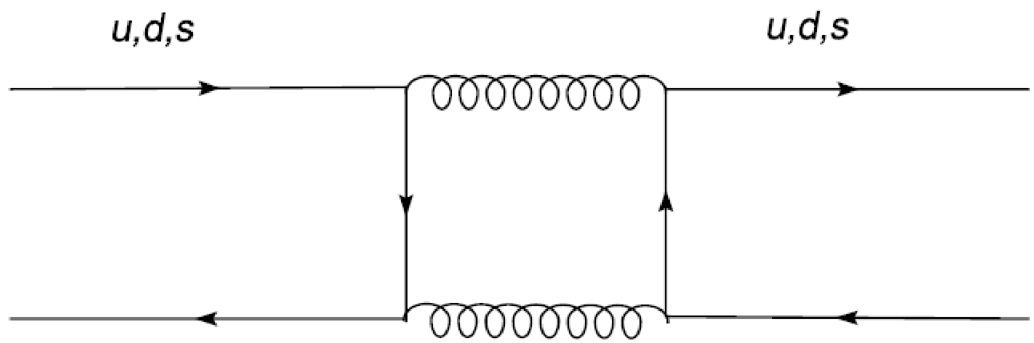
Charge-neutral pseudoscalar mesons

non-Abelian Anomaly and η-η' mixing

- Neutral mesons containing s-bar & s are special, in particular

  η & η'

- Flavour mixing takes place in singlet channel: $\lambda^0 \Leftrightarrow \lambda^8$



- Textbooks notwithstanding, this is a perturbative diagram, which has *absolutely nothing to do with the essence of the $\eta - \eta'$* problem

Bhagwat, Chang, Liu, Roberts, Tandy
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Charge-neutral pseudoscalar mesons

non-Abelian Anomaly and *η*-*η*′ mixing

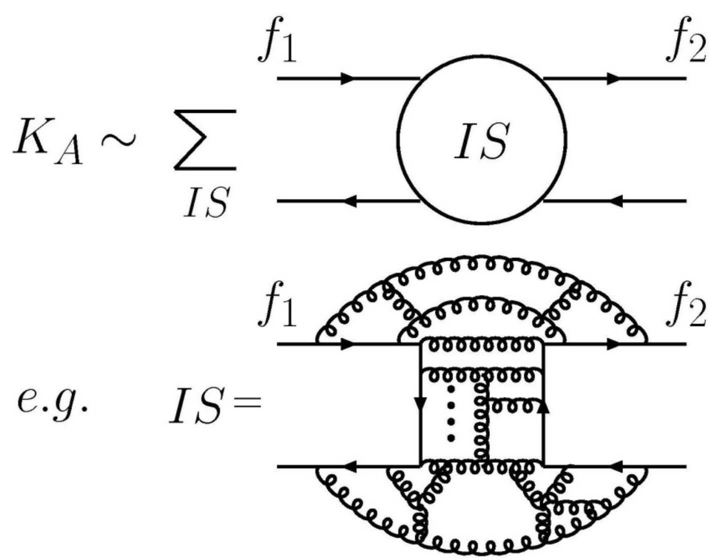
- Neutral mesons containing s-bar & s are special, in particular

  η & *η*′

- Driver is the non-Abelian anomaly
- Contribution to the Bethe-Salpeter kernel associated with the non-Abelian anomaly.

  All terms have the "hairpin" structure

- *No finite sum of such intermediate states is sufficient to veraciously represent the anomaly.*



Bhagwat, Chang, Liu, Roberts, Tandy
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Charge-neutral pseudoscalar mesons

- Axial-Vector Ward-Green-Takahashi identity

$$\begin{aligned} P_\mu \Gamma^a_{5\mu}(k;P) &= \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) \\ &\quad -2i\mathcal{M}^{ab}\Gamma^b_5(k;P) \end{aligned}$$

- $\{\mathcal{F}^a \,|\, a = 0, \ldots, N_f^2 - 1\}$ are the generators of $U(N_f)$
- $\mathcal{S} =$ diag$[S_u, S_d, S_s, S_c, S_b, \ldots]$
- $\mathcal{M}^{ab} = \mathrm{tr}_F\left[\{\mathcal{F}^a, \mathcal{M}\}\mathcal{F}^b\right]$,
  $\mathcal{M} = \mathrm{diag}[m_u, m_d, m_s, m_c, m_b, \ldots] =$ matrix of current-quark bare masses

- *Expresses the non-Abelian axial anomaly*

Bhagwat, Chang, Liu, Roberts, Tandy
Phys.Rev. C**76** (2007) 045203

Charge-neutral pseudoscalar mesons

- *Anomalous* Axial-Vector Ward-Green-Takahashi identity

$$\begin{aligned} P_\mu \Gamma^a_{5\mu}(k;P) &= \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) \\ &\quad -2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P) \end{aligned}$$

$$\mathcal{A}^a(k;P) = \mathcal{S}^{-1}(k_+)\,\delta^{a0}\,\mathcal{A}_U(k;P)\mathcal{S}^{-1}(k_-)$$

$$\mathcal{A}_U(k;P) = \int d^4x d^4y\, e^{i(k_+\cdot x - k_-\cdot y)} N_f \langle \mathcal{F}^0 q(x)\,\mathcal{Q}(0)\,\bar{q}(y)\rangle$$

Important that only A^0 is nonzero

$$\mathcal{Q}(x) = i\frac{\alpha_s}{4\pi}\mathrm{tr}_C\left[\epsilon_{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}(x)\right] = \partial_\mu K_\mu(x)$$

Anomaly expressed via a mixed vertex

NB. While Q(x) is gauge invariant, the associated Chern-Simons current, K_μ, is not ⇒ in QCD *no physical* boson can couple to K_μ and hence *no physical states can* contribute to resolution of $U_A(1)$ problem.

Bhagwat, Chang, Liu, Roberts, Tandy
*Phys.Rev. C**76** (2007) 045203*

Charge-neutral pseudoscalar mesons

- Only $A^0 \neq 0$ is interesting … otherwise there is no difference between η & η', and all pseudoscalar mesons are Goldstone mode bound states.
- General structure of the anomaly term:

$$\mathcal{A}^0(k;P) = \mathcal{F}^0\gamma_5\left[i\mathcal{E}_{\mathcal{A}}(k;P) + \gamma\cdot P\mathcal{F}_{\mathcal{A}}(k;P) + \gamma\cdot k k\cdot P\mathcal{G}_{\mathcal{A}}(k;P) + \sigma_{\mu\nu}k_\mu P_\nu \mathcal{H}_{\mathcal{A}}(k;P)\right]$$

- Hence, one can derive generalised Goldberger-Treiman relations

$$2f^0_{\eta'}E_{BS}(k;0) = 2B_0(k^2) - \mathcal{E}_{\mathcal{A}}(k;0),$$

Follows that $E_A(k;0)=2\,B_0(k^2)$ is necessary and sufficient condition for the absence of a massless η' bound state in the chiral limit, since this ensures $E_{BS} \equiv 0$.

A_0 and B_0 characterise gap equation's chiral-limit solution

Bhagwat, Chang, Liu, Roberts, Tandy
*Phys.Rev. C**76** (2007) 045203*

Charge-neutral pseudoscalar mesons

- $E_A(k; 0) = 2\, B_0(k^2)$

  We're discussing the chiral limit
  - $B_0(k^2) \neq 0$ if, and only if, chiral symmetry is dynamically broken.
  - Hence, absence of massless η' bound-state is only assured through existence of an intimate connection between DCSB and an expectation value of the topological charge density
- Further highlighted . . . proved

$$\langle \bar{q}q \rangle_\zeta^0 = -\lim_{\Lambda\to\infty} Z_4(\zeta^2, \Lambda^2) \mathrm{tr}_{\mathrm{CD}} \int_q^\Lambda S^0(q, \zeta)$$

$$= \frac{N_f}{2} \int d^4x \langle \bar{q}(x) i \gamma_5 q(x) \mathcal{Q}(0) \rangle^0 ,$$

So-called quark condensate linked inextricably with a mixed vacuum polarisation, which measures the topological structure within hadrons

Bhagwat, Chang, Liu, Roberts, Tandy
*Phys.Rev. C**76** (2007) 045203*

Charge-neutral pseudoscalar mesons

- AVWTI ⇒ QCD mass formulae for all pseudoscalar mesons, including those which are charge-neutral

$$m_{\pi_i}^2 f_{\pi_i}^a = 2\,\mathcal{M}^{ab}\rho_{\pi_i}^b + \delta^{a0} n_{\pi_i}$$

- Consider the limit of a *U(N_f)*-symmetric mass matrix, then this formula yields:

$$\begin{aligned} m_\eta^2 f_\eta &= 2m(\zeta)\rho_\eta^0(\zeta) \\ m_{\eta'}^2 f_{\eta'}^0 &= 2m(\zeta)\rho_{\eta'}^0(\zeta) + n_{\eta'} \end{aligned}$$

$$n_{\eta'} = \sqrt{\tfrac{N_f}{2}}\,\nu_{\eta'}\,,\ \nu_{\eta'} = \langle 0|\mathcal{Q}|\eta'\rangle$$

- Plainly, the $\eta - \eta'$ mass splitting is nonzero in the chiral limit so long as $\nu_{\eta'} \neq 0$ ... viz., so long as the topological content of the η' is nonzero!
- We know that, for large N_c,
  - $f_{\eta'} \propto N_c^{1/2} \propto \rho_{\eta'}^0$
  - $\nu_{\eta'} \propto 1/N_c^{1/2}$

Consequently, the $\eta - \eta'$ mass splitting vanishes in the large-N_c limit!

Bhagwat, Chang, Liu, Roberts, Tandy
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Charge-neutral pseudoscalar mesons

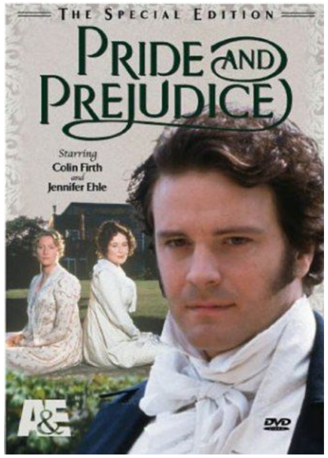
- AVWGTI ⇒ QCD mass formulae for neutral pseudoscalar mesons
- In "Bhagwat *et al*.," implications of mass formulae were illustrated using an elementary dynamical model, which includes a one-parameter *Ansatz* for that part of the Bethe-Salpeter kernel related to the non-Abelian anomaly
  - Employed in an analysis of pseudoscalar- and vector-meson bound-states
- Despite its simplicity, the model is elucidative and phenomenologically efficacious; e.g., it predicts
  - η–η' mixing angles of $\sim -15^\circ$ (Expt.: $-13.3^\circ \pm 1.0^\circ$)

$$|\eta\rangle \sim 0.55(\bar{u}u + \bar{d}d) - 0.63\bar{s}s,$$
$$|\eta'\rangle \sim 0.45(\bar{u}u + \bar{d}d) + 0.78\bar{s}s.$$

  - π^0–η angles of $\sim 1.2^\circ$ (Expt. from reaction $p\, d \to {}^3He\, \pi^0$: $0.6^\circ \pm 0.3^\circ$)

Dynamical Chiral Symmetry Breaking

Vacuum Condensates?



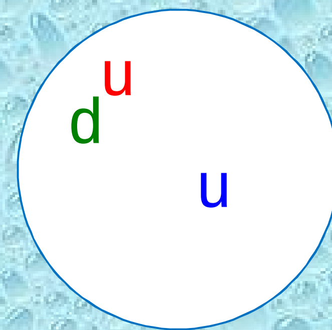
Universal *Conventions*

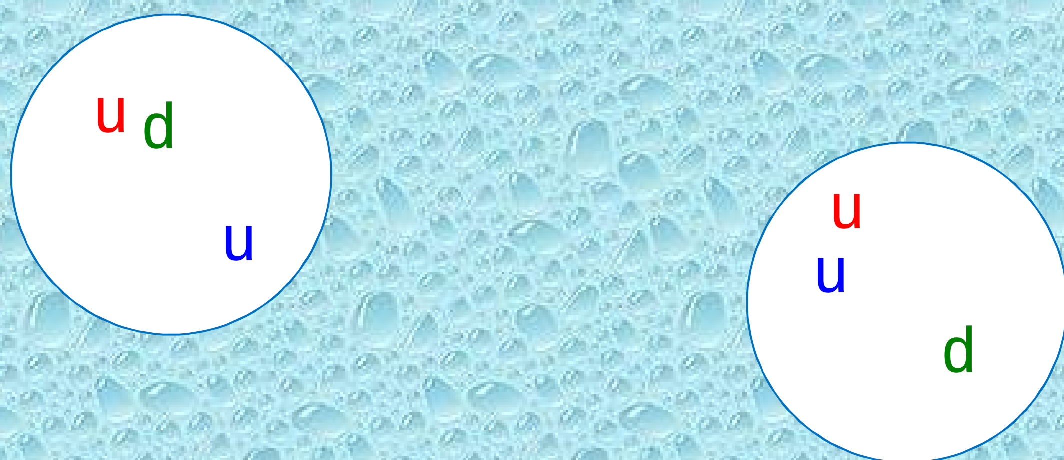
- Wikipedia: (http://en.wikipedia.org/wiki/QCD_vacuum)

  "The QCD vacuum is the vacuum state of quantum chromodynamics (QCD). It is an example of a non-perturbative vacuum state, characterized by many non-vanishing condensates such as the gluon condensate or the quark condensate. These condensates characterize the normal phase or the confined phase of quark matter."

"Orthodox Vacuum"

- Vacuum = "frothing sea"
- Hadrons = bubbles in that "sea", containing nothing but quarks & gluons interacting perturbatively, unless they're near the bubble's boundary, whereat they feel they're trapped!





Background

- Worth noting that nonzero vacuum expectation values of local operators in QCD—the so-called vacuum condensates—are phenomenological parameters, which were introduced at a time of limited computational resources in order to assist with the theoretical estimation of essentially nonperturbative strong-interaction matrix elements.
- A universality of these condensates was assumed, namely, that the properties of all hadrons could be expanded in terms of the same condensates. While this helps to retard proliferation, there are nevertheless infinitely many of them.
- As qualities associated with an unmeasurable state (the vacuum), such condensates do not admit direct measurement. Practitioners have attempted to assign values to them via an internally consistent treatment of many separate empirical observables.
- However, only one, the so-called quark condensate, is attributed a value with any confidence.

PHYSICAL REVIEW C **85**, 065202 (2012)

Confinement contains condensates

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(Received 27 February 2012; published 21 June 2012)

Dynamical chiral symmetry breaking and its connection to the generation of hadron masses has historically been viewed as a vacuum phenomenon. We argue that confinement makes such a position untenable. If quark-hadron duality is a reality in QCD, then condensates, those quantities that have commonly been viewed as constant empirical mass scales that fill all space-time, are instead wholly contained within hadrons; i.e., they are a property of hadrons themselves and expressed, e.g., in their Bethe-Salpeter or light-front wave functions. We explain that this paradigm is consistent with empirical evidence and incidentally expose misconceptions in a recent Comment.

DOI: 10.1103/PhysRevC.85.065202 PACS number(s): 12.38.Aw, 11.30.Rd, 11.15.Tk, 24.85.+p

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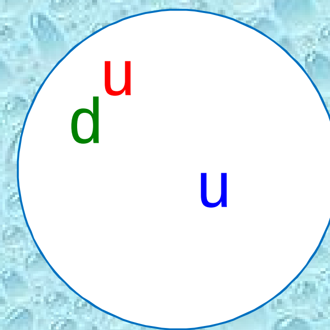
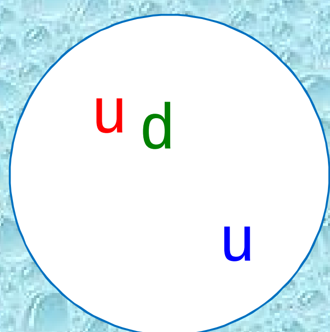
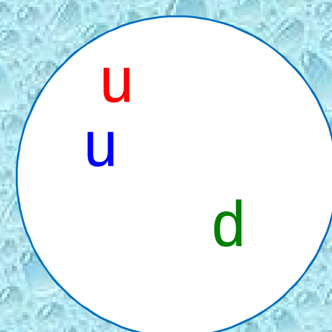
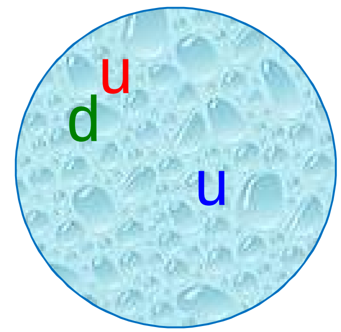


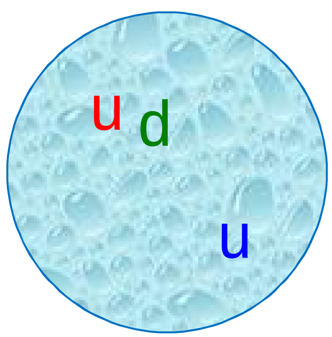


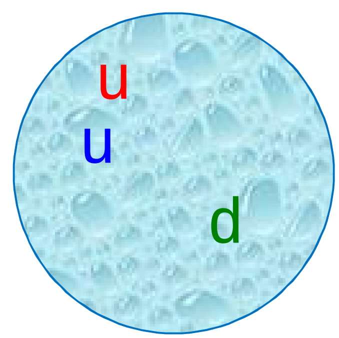


New Paradigm

- Vacuum = hadronic fluctuations
  but no condensates
- Hadrons = complex, interacting systems within which perturbative behaviour is restricted to just 2% of the interior







Any Questions?