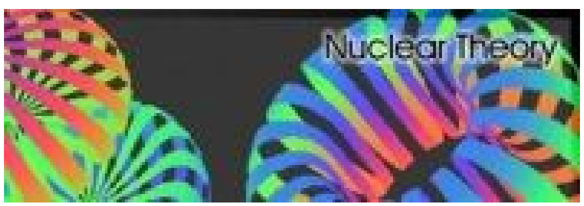
Argonne NATIONAL LABORATORY

Continuum strong QCD

Craig Roberts



Physics Division

Office of Nuclear Physics
Exploring Nuclear Matter - Quarks to Stars

U.S. DEPARTMENT OF ENERGY

Chiral QCD

- Action, in terms of local Lagrangian density:

$$S[G^a_\mu,\bar q,q]=\int d^4x\,\{\frac{1}{4}G^a_{\mu\nu}(x)G^a_{\mu\nu}(x)+\frac{1}{2\xi}\partial_\mu G^a_\mu(x)\,\partial_\nu G^a_\nu(x)+\bar q(x)[\gamma_\mu D_\mu+M]q(x)\}$$

  - Chromomagnetic Field Strength Tensor:
    $\partial_\mu G^a_\nu(x)-\partial_\nu G^a_\mu(x)+gf^{abc}G^b_\mu(x)G^c_\nu(x)$
  - Covariant Derivative: $D_\mu=\partial_\mu-ig\frac{\lambda^a}{2}G^a_\mu(x)$
  - Current-quark Mass matrix: $\begin{pmatrix} m_u & 0 & 0 & \ldots \\ 0 & m_d & 0 & \ldots \\ 0 & 0 & m_s & \ldots \\ \vdots & \vdots & \vdots & \end{pmatrix}$

$m_t = 40{,}000\, m_u$

Why?

- Understanding Hadron Physics means knowing all that this Action predicts.

➢ Current-quark masses
  - External paramaters in QCD
  - Generated by the Higgs boson, within the Standard Model
  - Raises more questions than it answers

Chiral Symmetry

- Interacting gauge theories, in which it makes sense to speak of massless fermions, have a nonperturbative chiral symmetry
- A related concept is *Helicity*, which is the projection of a particle's spin, J, onto it's direction of motion:

$$\lambda \propto J \bullet p$$

- For a massless particle, helicity is a Lorentz-invariant *spin-observable* $\lambda = \pm$; i.e., it's parallel or antiparallel to the direction of motion
  - Obvious:
    - massless particles travel at speed of light
    - hence no observer can overtake the particle and thereby view its momentum as having changed sign

Chiral Symmetry

- Chirality operator is γ_5
  - Chiral transformation: $\Psi(x) \to \exp(i\,\gamma_5\,\vartheta)\,\Psi(x)$
  - Chiral rotation through $\vartheta = \frac{1}{4}\pi$
    - Composite particles: $J^{P=+} \leftrightarrow J^{P=-}$
    - Equivalent to the operation of parity conjugation
- *Therefore, a prediction of chiral symmetry is the existence of degenerate parity partners in the theory's spectrum*

Chiral Symmetry

- Perturbative QCD: *u*- & *d*- quarks are very light

  $m_u/m_d \approx 0.5$ & $m_d \approx 4$ MeV

  (a generation of high-energy experiments)

  H. Leutwyler, *0911.1416 [hep-ph]*

- However, splitting between parity partners is greater-than 100-times this mass-scale; e.g.,

J^P	$\frac{1}{2}^+$ (p)	$\frac{1}{2}^-$
Mass	940 MeV	1535 MeV

Dynamical
Chiral Symmetry Breaking

- Something is happening in QCD
  - some inherent dynamical effect is dramatically changing the pattern by which the Lagrangian's chiral symmetry is expressed
- Qualitatively different from spontaneous symmetry breaking *aka* the Higgs mechanism
  - *Nothing is added to the QCD*
  - *Have only fermions & gauge-bosons that define the theory*

  Yet, the mass-operator generated by the theory produces a spectrum with no sign of chiral symmetry

Chiral symmetry

- Chiral symmetry is something that one studies in connection with fermion mass terms
- In order to understand it, therefore, the quantum field theory equation describing dynamical dressing of fermion mass is a sensible place to start

Fermion Self-Energy

Photon vacuum polarisation was introduced to re-express the DSE for the gauge boson propagator, Eq. (85). Analogue, one can define a fermion self-energy:

$$\Sigma^f(x,z) = i(e_0^f)^2 \int d^4u\, d^4w\; D^{\mu\nu}(x,z)\,\gamma_\mu\, S(x,u)\,\Gamma_\nu(u,w;z)\,, \tag{98}$$

so that Eq. (97) assumes the form

$$\int d^4z\,[(i\partial\!\!\!/_x - m_0^f)\delta^4(x-z) - \Sigma^f(x,z)]\,S(z,y) = \delta^4(x-y)\,. \tag{99}$$

Using property that Green functions are translationally invariant in the absence of external sources:

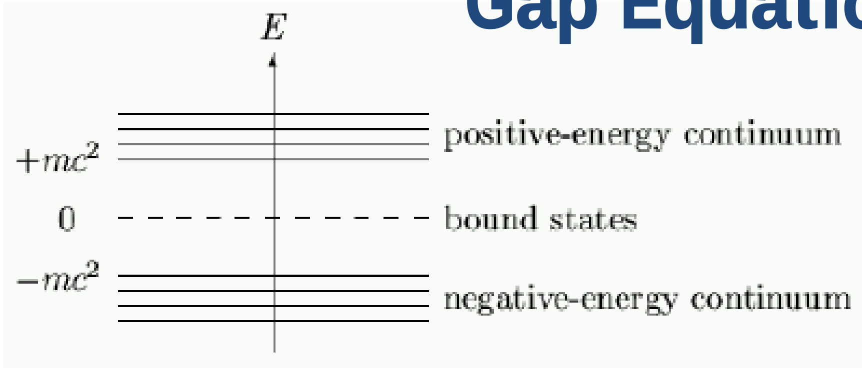
$$-i\Sigma^f(p) = (e_0^f)^2 \int \frac{d^4\ell}{(2\pi)^4}\,[iD^{\mu\nu}(p-\ell)]\,[i\gamma_\mu]\,[iS^f(\ell)]\,[i\Gamma_\nu^f(\ell,p)]\,. \tag{100}$$

Now follows from Eq. (99) that connected fermion 2-point function in momentum space is

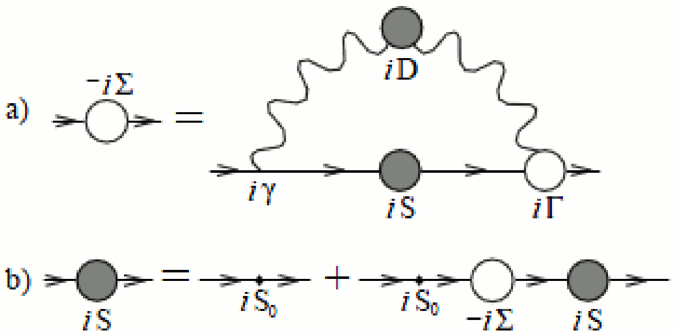
$$S^f(p) = \frac{1}{p\!\!\!/ - m_0^f - \Sigma^f(p) + i\eta^+}\,. \tag{101}$$

Gap Equation

Equation (100) is the **exact** *Gap Equation*.



Describes manner in which propagation characteristics of a fermion moving through ground state of QED (the QED vacuum) is altered by the repeated emission and reabsorption of virtual photons.



- Equation can also describe the real process of Bremsstrahlung. Furthermore, solution of analogous equation in QCD provides information about dynamical chiral symmetry breaking and also quark confinement.

Perturbative Calculation of Gap

Keystone of strong interaction physics is dynamical chiral symmetry breaking (DCSB). In order to understand DCSB one must first come to terms with explicit chiral symmetry breaking. Consider then the DSE for the quark self-energy in QCD:

$$-i\,\Sigma(p) = -g_0^2 \int \frac{d^4\ell}{(2\pi)^4}\, D^{\mu\nu}(p-\ell)\,\frac{i}{2}\lambda^a\gamma_\mu\, S(\ell)\, i\Gamma^a_\nu(\ell,p)\,, \tag{102}$$

where the flavour label is suppressed.

Form is precisely the same as that in QED, Eq. (100) but . . .

- colour (Gell-Mann) matrices: $\{\lambda^a; a=1,\ldots,8\}$ at the fermion-gauge-boson vertex
- $D^{\mu\nu}(\ell)$ is the connected gluon 2-point function
- $\Gamma^a_\nu(\ell,\ell')$ is the proper quark-gluon vertex

One-loop contribution to quark's self-energy obtained by evaluating r.h.s. of Eq. (102) using the free quark and gluon propagators, and the quark-gluon vertex:

$$\Gamma^{a\,(0)}_\nu(\ell,\ell') = \frac{1}{2}\lambda^a\gamma_\nu\,. \tag{103}$$

Explicit Leading-Order Computation

$$-i\,\Sigma^{(2)}(p) \quad = \quad -g_0^2 \int \frac{d^4k}{(2\pi)^4} \left[-g^{\mu\nu} + (1-\lambda_0)\frac{k^\mu k^\nu}{k^2+i\eta^+} \right] \frac{1}{k^2+i\eta^+} \times \frac{i}{2}\lambda^a\gamma_\mu \, \frac{1}{\not k + \not p - m_0 + i\eta^+} \, \frac{i}{2}\lambda^a\gamma_\mu \,. \qquad (104)$$

To proceed, first observe that Eq. (104) can be re-expressed as

$$-i\,\Sigma^{(2)}(p) = -g_0^2\, C_2(R) \int \frac{d^4k}{(2\pi)^4} \, \frac{1}{(k+p)^2 - m_0^2 + i\eta^+} \, \frac{1}{k^2+i\eta^+} \times \left\{ \gamma^\mu \,(\not k + \not p + m_0)\,\gamma_\mu - (1-\lambda_0)\,(\not k - \not p + m_0) - 2\,(1-\lambda_0)\,\frac{(k,p)\,\not k}{k^2+i\eta^+} \right\}, \qquad (105)$$

where we have used $\boxed{\frac{1}{2}\lambda^a \, \frac{1}{2}\lambda^a = C_2(R)\, \boldsymbol{I}_c \,;\; C_2(R) = \frac{N_c^2-1}{2N_c}}$, with N_c the number of colours ($N_c = 3$ in QCD), and $\boldsymbol{I}_c$ is the identity matrix in colour space.

Explicit Leading-Order Computation

Now note that $2\,(k,p) = [(k+p)^2 - m_0^2] - [k^2] - [p^2 - m_0^2]$ and hence

$$\begin{aligned}
-\,i\,\Sigma^{(2)}(p) \quad = \quad & -g_0^2\,C_2(R)\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{(k+p)^2-m_0^2+i\eta^+}\,\frac{1}{k^2+i\eta^+} \\
& \Big\{\gamma^\mu\,(\not k+\not p+m_0)\,\gamma_\mu+(1-\lambda_0)\,(\not p-m_0) \\
& +(1-\lambda_0)\,(p^2-m_0^2)\,\frac{\not k}{k^2+i\eta^+} \\
& -(1-\lambda_0)\,[(k+p)^2-m_0^2]\,\frac{\not k}{k^2+i\eta^+}\Big\}. \qquad (106)
\end{aligned}$$

Focus on the last term:

$$\begin{aligned}
& \int\frac{d^4k}{(2\pi)^4}\,\frac{1}{(k+p)^2-m_0^2+i\eta^+}\,\frac{1}{k^2+i\eta^+}\,[(k+p)^2-m_0^2]\,\frac{\not k}{k^2+i\eta^+} \\
= \quad & \int\frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2+i\eta^+}\,\frac{\not k}{k^2+i\eta^+}=0 \qquad (107)
\end{aligned}$$

because the integrand is odd under $k \to -k$, and so this term in Eq. (106) vanishes.

Explicit Leading-Order Computation

$$-i\,\Sigma^{(2)}(p) = -g_0^2\,C_2(R)\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{(k+p)^2-m_0^2+i\eta^+}\,\frac{1}{k^2+i\eta^+}$$
$$\left\{\gamma^\mu\,(\not k+\not p+m_0)\,\gamma_\mu+(1-\lambda_0)\,(\not p-m_0)+(1-\lambda_0)\,(p^2-m_0^2)\,\frac{\not k}{k^2+i\eta^+}\right\}.$$

Consider the second term:

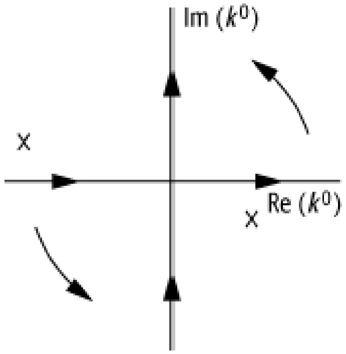
$$(1-\lambda_0)\,(\not p-m_0)\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{(k+p)^2-m_0^2+i\eta^+}\,\frac{1}{k^2+i\eta^+}\,.$$

In particular, focus on the behaviour of the integrand at large k^2:

$$\frac{1}{(k+p)^2-m_0^2+i\eta^+}\,\frac{1}{k^2+i\eta^+}\overset{k^2\to\pm\infty}{\sim}\frac{1}{(k^2-m_0^2+i\eta^+)\,(k^2+i\eta^+)}\,. \tag{108}$$

Wick Rotation

Integrand has poles in the second and fourth quadrant of the complex-k_0-plane but vanishes on any circle of radius $R \to \infty$ in this plane. That means one may rotate the contour anticlockwise to find



$$\begin{aligned}
&\int_0^\infty dk^0 \, \frac{1}{(k^2 - m_0^2 + i\eta^+)\,(k^2 + i\eta^+)} \\
&= \int_0^{i\infty} dk^0 \, \frac{1}{([k^0]^2 - \vec{k}^2 - m_0^2 + i\eta^+)([k^0]^2 - \vec{k}^2 + i\eta^+)} \\
&\stackrel{k^0 \to ik_4}{=} i\int_0^\infty dk_4 \, \frac{1}{(-k_4^2 - \vec{k}^2 - m_0^2)\,(-k_4^2 - \vec{k}^2)} \,. \qquad (109)
\end{aligned}$$

Performing a similar analysis of the $\int_{-\infty}^0$ part, one obtains the complete result:

$$\begin{aligned}
&\int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_0^2 + i\eta^+)\,(k^2 + i\eta^+)} \\
&= i\int \frac{d^3k}{(2\pi)^3} \int_{-\infty}^\infty \frac{dk_4}{2\pi} \frac{1}{(-\vec{k}^2 - k_4^2 - m_0^2)\,(-\vec{k}^2 - k_4^2)} \,. \qquad (110)
\end{aligned}$$

These two steps constitute what is called a *Wick rotation*.

Euclidean Integral

The integral on the r.h.s. is defined in a four-dimensional Euclidean space; i.e., $k^2 := k_1^2 + k_2^2 + k_3^2 + k_4^2 \geq 0$, with k^2 nonnegative.

A general vector in this space can be written in the form:

$$(k) = |k| \left(\cos\phi \ \sin\theta \ \sin\beta, \sin\phi \ \sin\theta \ \sin\beta, \cos\theta \ \sin\beta, \cos\beta\right); \qquad (111)$$

i.e., using hyperspherical coordinates, and clearly $k^2 = |k|^2$.

In this Euclidean space using these coordinates the four-vector measure factor is

$$\begin{aligned} &\int d_E^4 k \, f(k_1, \ldots, k_4) \\ &= \quad \frac{1}{2} \int_0^\infty dk^2 \, k^2 \int_0^\pi d\beta \, sin^2\beta \int_0^\pi d\theta \sin\theta \int_0^{2\pi} d\phi \, f(k, \beta, \theta, \phi) \, . \end{aligned} \qquad (112)$$

Euclidean Integral

Returning to Eq. (108) and making use of the material just introduced, the large k^2 behaviour of the integral can be determined via

$$\begin{aligned}
&\int \frac{d^4k}{(2\pi)^4} \frac{1}{(k+p)^2 - m_0^2 + i\eta^+} \frac{1}{k^2 + i\eta^+} \\
\approx \quad & \frac{i}{16\pi^2} \int_0^\infty dk^2 \frac{1}{(k^2 + m_0^2)} \\
= \quad & \frac{i}{16\pi^2} \lim_{\Lambda\to\infty} \int_0^{\Lambda^2} dx \, \frac{1}{x + m_0^2} \\
= \quad & \frac{i}{16\pi^2} \lim_{\Lambda\to\infty} \ln(1 + \Lambda^2/m_0^2) \to \infty \, ;
\end{aligned} \tag{113}$$

After all this work, the result is meaningless: the one-loop contribution to the quark's self-energy is divergent!

Regularisation and Renormalisation

- Such "ultraviolet" divergences, and others which are more complicated, arise whenever loops appear in perturbation theory. (The others include "infrared" divergences associated with the gluons' masslessness; e.g., consider what would happen in Eq. (113) with $m_0 \to 0$.)
- In a *renormalisable* quantum field theory there exists a well-defined set of rules that can be used to render perturbation theory sensible.
  - First, however, one must *regularise* the theory; i.e., introduce a cutoff, or use some other means, to make finite every integral that appears. Then each step in the calculation of an observable is rigorously sensible.
  - *Renormalisation* follows; i.e, the absorption of divergences, and the redefinition of couplings and masses, so that finally one arrives at S-matrix amplitudes that are finite and physically meaningful.
- The regularisation procedure must preserve the Ward-Takahashi identities (the Slavnov-Taylor identities in QCD) because they are crucial in proving that a theory can sensibly be renormalised.
- A theory is called renormalisable if, and only if, number of different types of divergent integral is finite. Then only finite number of masses & couplings need to be renormalised; i.e., *a priori* the theory has only a finite number of undetermined parameters that must be fixed through comparison with experiments.

Renormalised One-Loop Result

Don't have time to explain and illustrate the procedure. Interested?

Read . . . Pascual, P. and Tarrach, R. (1984), Lecture Notes in Physics, Vol. **194**, *QCD: Renormalization for the Practitioner* (Springer-Verlag, Berlin).

Answer, in Momentum Subtraction Scheme:

$$\Sigma_R^{(2)}(\not p) = \Sigma_{VR}^{(2)}(p^2)\,\not p + \Sigma_{SR}^{(2)}(p^2)\,1_D;$$

$$\Sigma_{VR}^{(2)}(p^2;\zeta^2) = \frac{\alpha(\zeta)}{\pi}\,\lambda(\zeta)\,\frac{1}{4}\,C_2(R)\left\{-m^2(\zeta)\left(\frac{1}{p^2}+\frac{1}{\zeta^2}\right)\right.$$
$$\left.+\left(1-\frac{m^4(\zeta)}{p^4}\right)\ln\left(1-\frac{p^2}{m(\zeta)^2}\right)-\left(1-\frac{m^4(\zeta)}{\zeta^4}\right)\ln\left(1+\frac{\zeta^2}{m^2(\zeta)}\right)\right\},$$

$$\Sigma_{SR}^{(2)}(p^2;\zeta^2) = m(\zeta)\,\frac{\alpha(\zeta)}{\pi}\,\frac{1}{4}\,C_2(R)\left\{-[3+\lambda(\zeta)]\right.$$
$$\left.\times\left[\left(1-\frac{m^2(\zeta)}{p^2}\right)\ln\left(1-\frac{p^2}{m^2(\zeta)}\right)-\left(1+\frac{m^2(\zeta)}{\zeta^2}\right)\ln\left(1+\frac{\zeta^2}{m^2(\zeta)}\right)\right]\right\},$$

where the renormalised quantities depend on the point at which the renormalisation has been conducted;

e.g., $\alpha(\zeta)$ is the running coupling, $m(\zeta)$ is the running quark mass.

Observations on perturbative quark self-energy

- QCD is Asymptotically Free. Hence, at some large spacelike $p^2 = \zeta^2$ the propagator is exactly the free propagator *except* that the bare mass is replaced by the renormalised mass.
- At one-loop order, the vector part of the dressed self energy is proportional to the running gauge parameter. In Landau gauge, that parameter is zero. Hence, the vector part of the renormalised dressed self energy vanishes at one-loop order in perturbation theory.
- The scalar part of the dressed self energy is proportional to the renormalised current-quark mass.
  - This is true at one-loop order, and at every order in perturbation theory.
  - Hence, if current-quark mass vanishes, then $\Sigma_R \equiv 0$ in perturbation theory. That means if one starts with a chirally symmetric theory, one ends up with a chirally symmetric theory.

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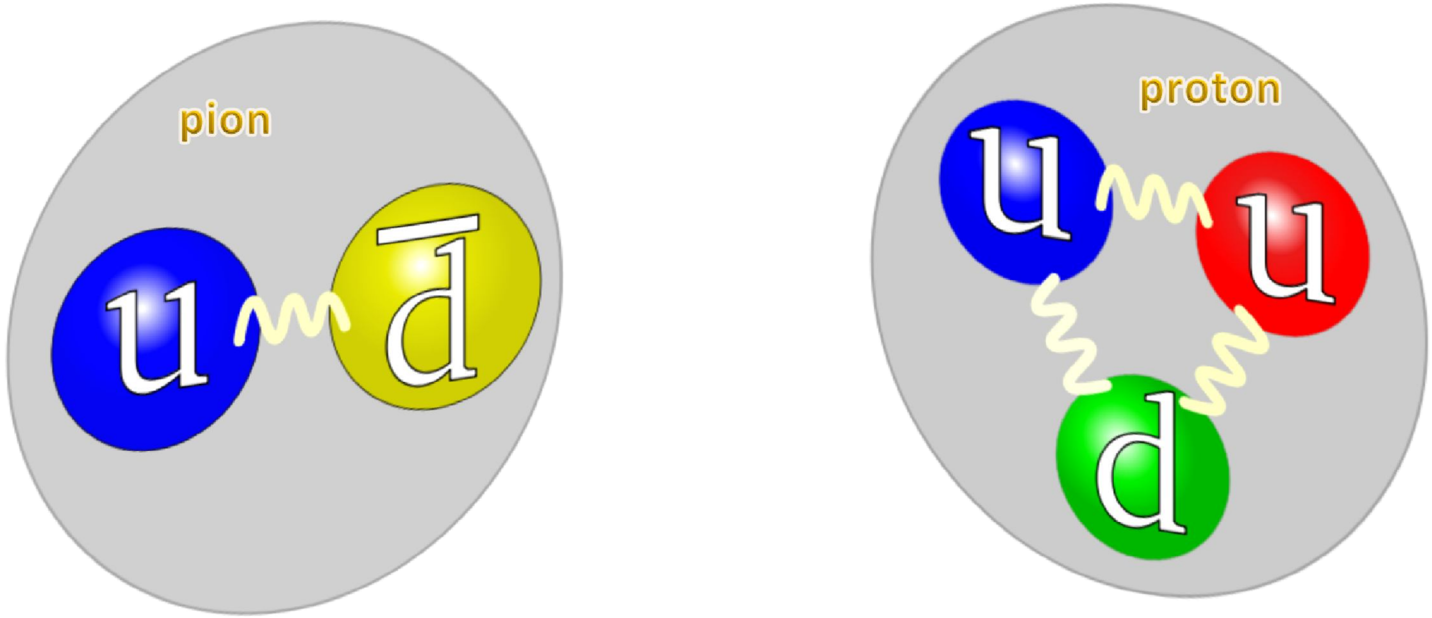
No Dynamical Chiral Symmetry Breaking in perturbation theory

Overarching Science Questions for the coming decade: 2013-2022

- Discover meaning of confinement;
- its relationship to DCSB;
- and the nature of the transition between the nonperturbative & perturbative domains of QCD

… coming lectures



The structure of matter

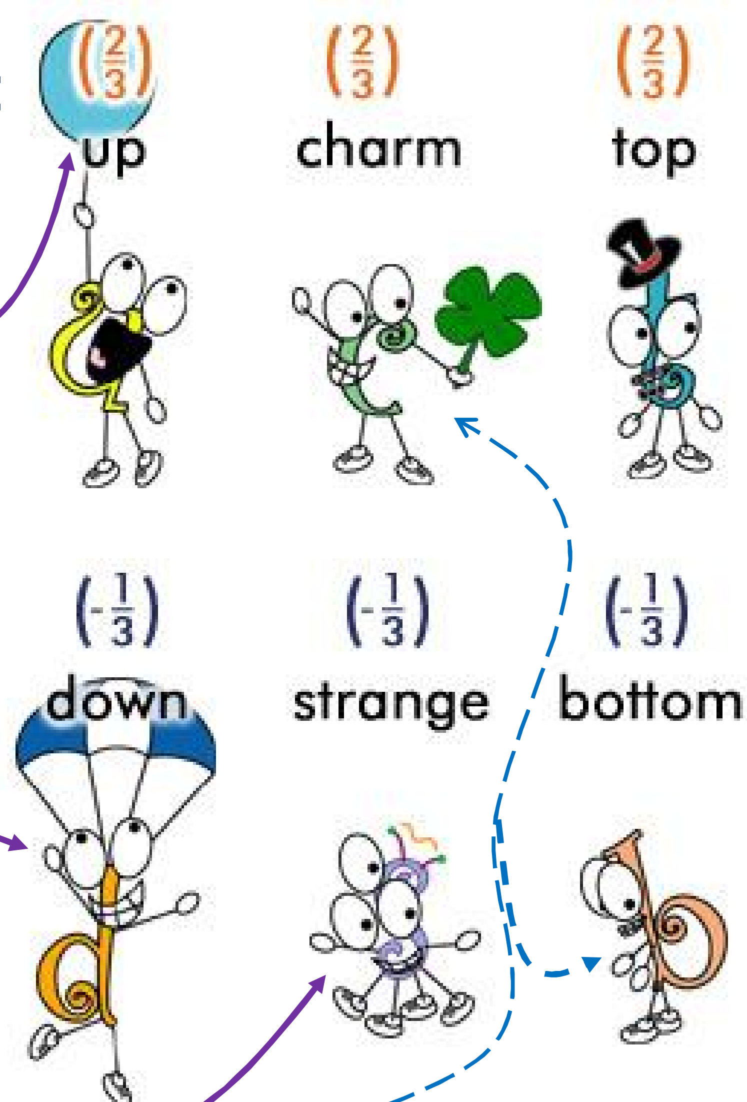
Hadron Theory

Quarks and Nuclear Physics

Standard Model of Particle Physics:
- ➢ Six quark flavours

Real World
- ➢ Normal matter – only two light-quark flavours are active
- ➢ Or, perhaps, three

For numerous good reasons, much research also focuses on accessible heavy-quarks
- ➢ Nevertheless, I will mainly focus on the light-quarks; i.e., *u* & *d*.



Problem: Nature chooses to build things, us included, from matter fields instead of gauge fields.

Quarks & QCD

- Quarks are the problem with QCD
- Pure-glue QCD is far simpler
  - Bosons are the only degrees of freedom
    - Bosons have a classical analogue – see Maxwell's formulation of electrodynamics
  - Generating functional can be formulated as a discrete probability measure that is amenable to direct numerical simulation using Monte-Carlo methods
    - No perniciously nonlocal fermion determinant
- Provides the Area Law & Linearly Rising Potential between static sources, so long identified with confinement

In perturbation theory, quarks don't seem to do much, just a little bit of very-normal charge screening.

K.G. Wilson, formulated lattice-QCD in 1974 paper: "Confinement of quarks".

Wilson Loop

Nobel Prize (1982): *"for his theory for critical phenomena in connection with phase transitions"*.

Contrast with Minkowksi metric:
infinitely many four-vectors satisfy
$p^2 = p^0p^0 - p^ip^i = 0$;
e.g., $p= \mu\,(1,0,0,1)$, μ any number

Formulating QCD Euclidean Metric

- In order to translate QCD into a computational problem, Wilson had to employ a *Euclidean Metric*

  $x^2 = 0$ possible if and only if $x=(0,0,0,0)$

  because Euclidean-QCD action defines a probability measure, for which many numerical simulation algorithms are available.
- However, working in Euclidean space is more than simply pragmatic:
  - Euclidean lattice field theory is currently a primary candidate for the rigorous definition of an interacting quantum field theory.
  - This relies on it being possible to define the generating functional via a proper limiting procedure.

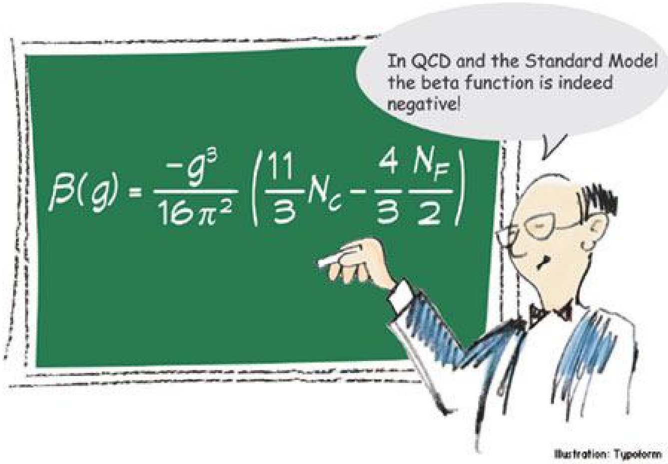
Formulating QCD Euclidean Metric

- The moments of the measure; i.e., "vacuum expectation values" of the fields, are the n-point Schwinger functions; and the quantum field theory is completely determined once all its Schwinger functions are known.
- The time-ordered Green functions of the associated Minkowski space theory can be obtained in a formally well-defined fashion from the Schwinger functions.

This is all *formally* true.

Formulating Quantum Field Theory Euclidean Metric

- Constructive Field Theory Perspectives:
  - Symanzik, K. (1963) in *Local Quantum Theory (Academic, New York) edited* by R. Jost.
  - Streater, R.F. and Wightman, A.S. (1980), *PCT, Spin and Statistics, and All That (Addison-Wesley, Reading, Mass, 3rd edition).*
  - Glimm, J. and Jaffee, A. (1981), *Quantum Physics. A Functional Point of View* (Springer-Verlag, New York).
  - Seiler, E. (1982), *Gauge Theories as a Problem of Constructive Quantum Theory and Statistical Mechanics (Springer-Verlag, New York).*
- For some theorists, interested in essentially nonperturbative QCD, this is always in the back of our minds

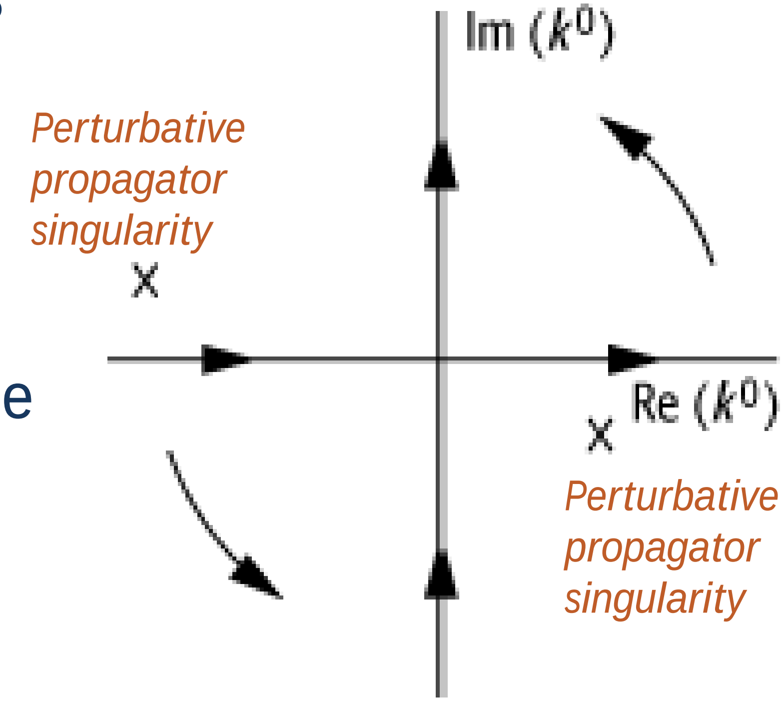


Formulating QCD Euclidean Metric

- However, there is another very important reason to work in Euclidean space; viz.,
  Owing to asymptotic freedom, all results of perturbation theory are strictly valid only at spacelike-momenta.
  - The set of spacelike momenta correspond to a Euclidean vector space
- The continuation to Minkowski space rests on many assumptions about Schwinger functions that are demonstrably valid only in perturbation theory.

Euclidean Metric & Wick Rotation

- It is assumed that a Wick rotation is valid; namely, that QCD dynamics don't nonperturbatively generate anything *unnatural*
- This is a brave assumption, which turns out to be *very, very false* in the case of coloured states.
- Hence, QCD *MUST* be defined in Euclidean space.
- The properties of the real-world are then determined only from a continuation of colour-singlet quantities.



Aside: QED is only defined perturbatively. It possesses an infrared stable fixed point; and masses and couplings are regularised and renormalised in the vicinity of $k^2=0$. *Wick rotation is always valid in this context.*

The Problem with QCD

- This is a RED FLAG in QCD because
  nothing elementary is a colour singlet
- Must somehow solve real-world problems
  - the spectrum and interactions of complex two- and three-body bound-states

  before returning to the real world
- This is going to require a little bit of imagination and a very good toolbox:

Dyson-Schwinger equations

Euclidean Metric Conventions

- To make clear our conventions: for 4-vectors a, b: $a \cdot b := a_\mu\, b_\nu\, \delta_{\mu\nu} := \sum_{i=1}^{4} a_i\, b_i$,

  Hence, a spacelike vector, Q_μ, has $Q^2 > 0$.
- Dirac matrices:
  - Hermitian and defined by the algebra $\{\gamma_\mu, \gamma_\nu\} = 2\,\delta_{\mu\nu}$;
  - we use $\gamma_5 := -\gamma_1\gamma_2\gamma_3\gamma_4$, so that $\mathrm{tr}\,[\gamma_5\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma] = -4\,\varepsilon_{\mu\nu\rho\sigma}$, $\varepsilon_{1234} = 1$.
  - The Dirac-like representation of these matrices is:

$$\vec{\gamma} = \begin{pmatrix} 0 & -i\vec{\tau} \\ i\vec{\tau} & 0 \end{pmatrix}, \quad \gamma_4 = \begin{pmatrix} \tau^0 & 0 \\ 0 & -\tau^0 \end{pmatrix}, \tag{2}$$

where the 2×2 Pauli matrices are:

$$\tau^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{3}$$

Euclidean Transcription Formulae

It is possible to derive every equation of Euclidean QCD by assuming certain analytic properties of the integrands. However, the derivations can be sidestepped using the following *transcription rules*:

Configuration Space

1. $\int^M d^4x^M \to -i\int^E d^4x^E$
2. $\not\partial \to i\gamma^E \cdot \partial^E$
3. $\not A \to -i\gamma^E \cdot A^E$
4. $A_\mu B^\mu \to -A^E \cdot B^E$
5. $x^\mu \partial_\mu \to x^E \cdot \partial^E$

Momentum Space

1. $\int^M d^4k^M \to i\int^E d^4k^E$
2. $\not k \to -i\gamma^E \cdot k^E$
3. $\not A \to -i\gamma^E \cdot A^E$
4. $k_\mu q^\mu \to -k^E \cdot q^E$
5. $k_\mu x^\mu \to -k^E \cdot x^E$

These rules are valid in perturbation theory; i.e., the correct Minkowski space integral for a given diagram will be obtained by applying these rules to the Euclidean integral: they take account of the change of variables and rotation of the contour. However, for diagrams that represent DSEs which involve dressed n-point functions, whose analytic structure is not known *a priori*, the Minkowski space equation obtained using this prescription will have the right appearance but it's solutions may bear no relation to the analytic continuation of the solution of the Euclidean equation. Any such differences will be nonperturbative in origin.

Never before seen by the human eye

Nature's strong messenger – Pion

- 1947 – Pion discovered by Cecil Frank Powell
- Studied tracks made by cosmic rays using photographic emulsion plates
- Despite the fact that Cavendish Lab said method is incapable of "*reliable and reproducible precision measurements*."
- Mass measured in scattering $\approx 250\text{-}350\, m_e$

Nuclear capture of pion

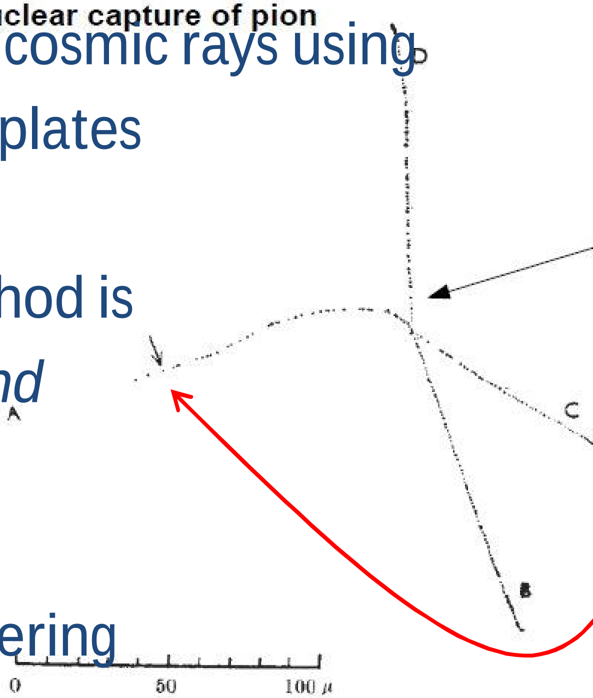
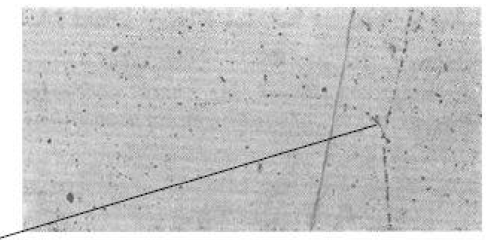


Fig. 1 *b*. Trace of complete star on screen of projection microscope, showing projection of the tracks in the plane of the emulsion. Track *A* cannot be traced with certainty beyond the arrow



g. 1 *a*. Photomicrograph of centre of star, showing track of eson producing disintegration. (Leitz 2 mm. oil-immersion objective. × 500)

- A is the new meson
- B,D,C are likely protons
- Track C goes into the page

Why A is a new meson:
electron: range too large
proton: scattering too large
muon: frequent nuclear interaction

11

Nature's strong messenger – Pion

- The beginning of Particle Physics
- Then came
  - Disentanglement of confusion between $_{(1937)}$ muon and pion – similar masses
  - Discovery of particles with "strangeness" (e.g., kaon$_{1947\text{-}1953}$)
- Subsequently, a complete spectrum of mesons and baryons with mass below ≈1 GeV
  - 28 states

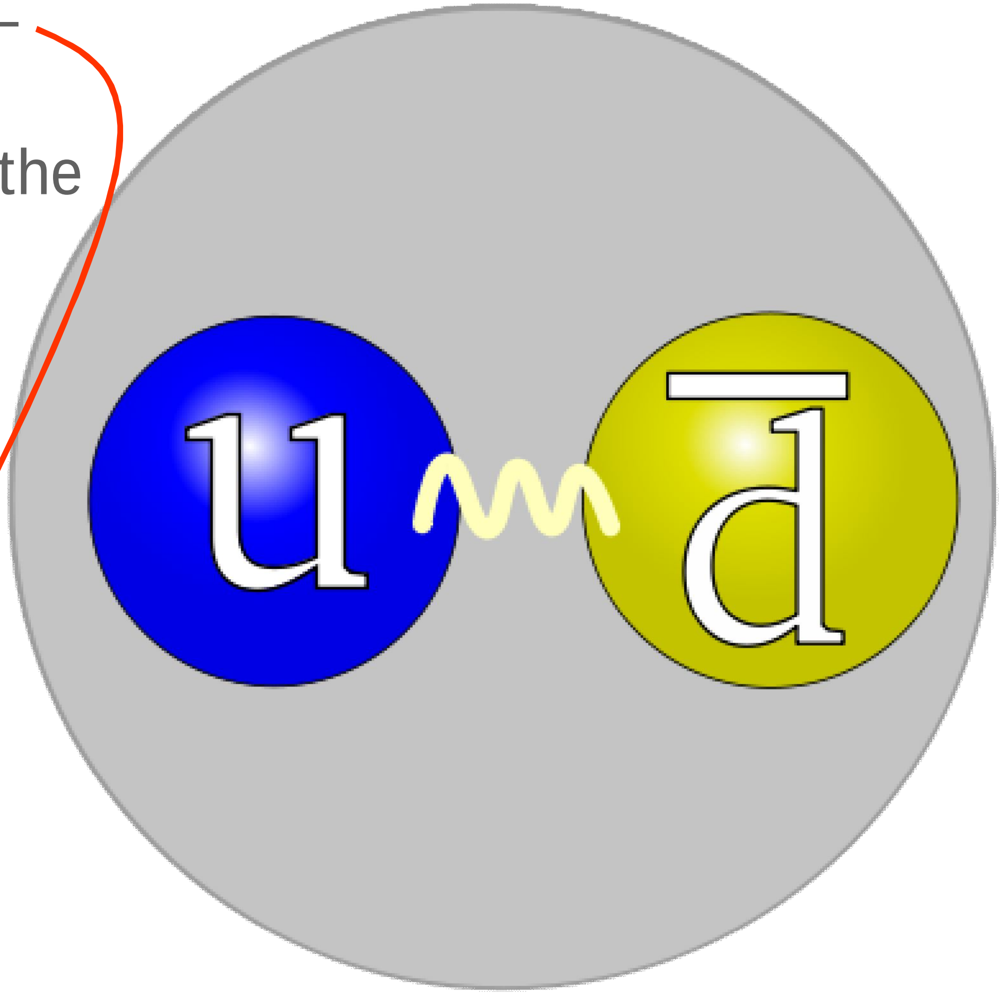
π	140 MeV
ρ	780 MeV
P	940 MeV

- Became clear that pion is "too light"

\- *hadrons* supposed to be heavy, yet ...

Simple picture - Pion

- Gell-Mann and Ne'eman:
  - Eightfold way$_{(1961)}$ – a picture based on group theory: *SU(3)*
  - Subsequently, quark model – where the *u*-, *d*-, *s*-quarks became the basis vectors in the fundamental representation of *SU(3)*
- Pion =
  Two *quantum-mechanical*
  constituent-quarks -
  particle+antiparticle -
  interacting via a *potential*



Some of the Light Mesons

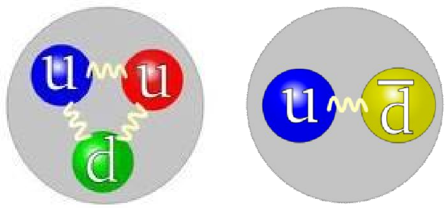
LIGHT UNFLAVORED MESONS ($S = C = B = 0$)

For $I=1$ (π, b, ρ, a): $u\bar{d}$, $(u\bar{u}-d\bar{d})/\sqrt{2}$, $d\bar{u}$;
for $I=0$ ($\eta, \eta', h, h', \omega, \varphi, f, f'$): $c_1(u\bar{u}+d\bar{d}) + c_2(s\bar{s})$

		$I^G(J^{PC})$				
$\pi^\pm$	140 MeV	$1^-(0^-)$	$\eta(1475)$	$0^+(0^{-+})$	▪ $f_2(1910)$	$0^+(2^{++})$
π^0		$1^-(0^{-+})$	$f_0(1500)$	$0^+(0^{++})$	$f_2(1950)$	$0^+(2^{++})$
η		$0^+(0^{-+})$	▪ $f_1(1510)$	$0^+(1^{++})$	▪ $\rho_3(1990)$	$1^+(3^{--})$
$f_0(600)$ or σ		$0^+(0^{++})$	$f_2'(1525)$	$0^+(2^{++})$	$f_2(2010)$	$0^+(2^{++})$
$\rho(770)$	780 MeV	$1^+(1^{--})$	▪ $f_2(1565)$	$0^+(2^{++})$	▪ $f_0(2020)$	$0^+(0^{++})$
$\omega(782)$		$0^-(1^{--})$	▪ $\rho(1570)$	$1^+(1^{--})$	$a_4(2040)$	$1^-(4^{++})$
$\eta'(958)$		$0^+(0^{-+})$	▪ $h_1(1595)$	$0^-(1^{+-})$	$f_4(2050)$	$0^+(4^{++})$
$f_0(980)$		$0^+(0^{++})$	$\pi_1(1600)$	$1^-(1^{-+})$	▪ $\pi_2(2100)$	$1^-(2^{-+})$
$a_0(980)$		$1^-(0^{++})$	▪ $a_1(1640)$	$1^-(1^{++})$	▪ $f_0(2100)$	$0^+(0^{++})$
$\varphi(1020)$		$0^-(1^{--})$	▪ $f_2(1640)$	$0^+(2^{++})$	▪ $f_2(2150)$	$0^+(2^{++})$
$h_1(1170)$		$0^-(1^{+-})$	$\eta_2(1645)$	$0^+(2^{-+})$	▪ $\rho(2150)$	$1^+(1^{--})$
$b_1(1235)$		$1^+(1^{+-})$	$\omega(1650)$	$0^-(1^{--})$	$\varphi(2170)$	$0^-(1^{--})$
$a_1(1260)$		$1^-(1^{++})$	$\omega_3(1670)$	$0^-(3^{--})$	▪ $f_0(2200)$	$0^+(0^{++})$
$f_2(1270)$		$0^+(2^{++})$	$\pi_2(1670)$	$1^-(2^{-+})$	▪ $f_J(2220)$	$0^+(2^{++}$ or $4^{++})$
$f_1(1285)$		$0^+(1^{++})$	$\varphi(1680)$	$0^-(1^{--})$	▪ $\eta(2225)$	$0^+(0^{-+})$
$\eta(1295)$		$0^+(0^{-+})$	$\rho_3(1690)$	$1^+(3^{--})$	▪ $\rho_3(2250)$	$1^+(3^{--})$
$\pi(1300)$		$1^-(0^{-+})$	$\rho(1700)$	$1^+(1^{--})$	$f_2(2300)$	$0^+(2^{++})$
$a_2(1320)$		$1^-(2^{++})$	▪ $a_2(1700)$	$1^-(2^{++})$	▪ $f_4(2300)$	$0^+(4^{++})$
$f_0(1370)$		$0^+(0^{++})$	$f_0(1710)$	$0^+(0^{++})$	▪ $f_0(2330)$	$0^+(0^{++})$
▪ $h_1(1380)$		$?^-(1^{+-})$	▪ $\eta(1760)$	$0^+(0^{-+})$	$f_2(2340)$	$0^+(2^{++})$
$\pi_1(1400)$		$1^-(1^{-+})$	$\pi(1800)$	$1^-(0^{-+})$	▪ $\rho_5(2350)$	$1^+(5^{--})$
$\eta(1405)$		$0^+(0^{-+})$	▪ $f_2(1810)$	$0^+(2^{++})$	▪ $a_6(2450)$	$1^-(6^{++})$
$f_1(1420)$		$0^+(1^{++})$	▪ $X(1835)$	$?^?(?^{-+})$	▪ $f_6(2510)$	$0^+(6^{++})$
$\omega(1420)$		$0^-(1^{--})$	$\varphi_3(1850)$	$0^-(3^{--})$		
▪ $f_2(1430)$		$0^+(2^{++})$	▪ $\eta_2(1870)$	$0^+(2^{-+})$		
$a_0(1450)$		$1^-(0^{++})$	$\pi_2(1880)$	$1^-(2^{-+})$		
$\rho(1450)$		$1^+(1^{--})$	▪ $\rho(1900)$	$1^+(1^{--})$		

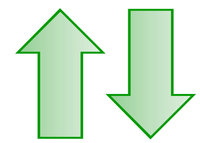
▪ — OMITTED FROM SUMMARY TABLE

Craig Roberts: Continuum strong QCD (II.60p)

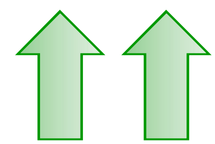


Modern Miracles in Hadron Physics

- proton = three constituent quarks
  - $M_{proton} \approx$ 1GeV
  - Therefore guess $M_{constituent-quark} \approx$ ⅓ × GeV ≈ 350MeV
- pion = constituent quark + constituent antiquark



  - Guess $M_{pion} \approx$ ⅔ × $M_{proton} \approx$ 700MeV
- WRONG . . . . M_{pion} = 140MeV
- Rho-meson



  - Also *constituent quark + constituent antiquark*

    – just pion with spin of one constituent flipped
  - $M_{rho} \approx 770MeV \approx 2 \times M_{constituent-quark}$

What is “wrong” with the pion?

Dichotomy of the pion

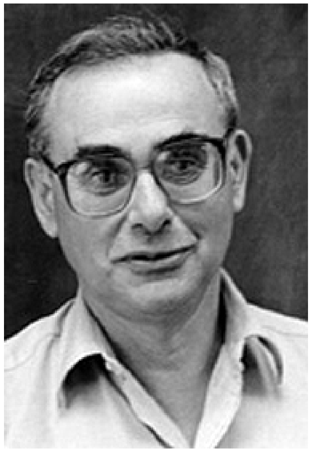
- How does one make an almost massless particle from two massive constituent-quarks?
- Naturally, one *could* always tune a potential in quantum mechanics so that the ground-state is massless

  – but some are still making this mistake

- However: $$m_\pi^2 \propto m$$

  current-algebra (1968)

- This is *impossible in quantum mechanics*, for which one always finds: $m_{bound-state} \propto m_{constituent}$



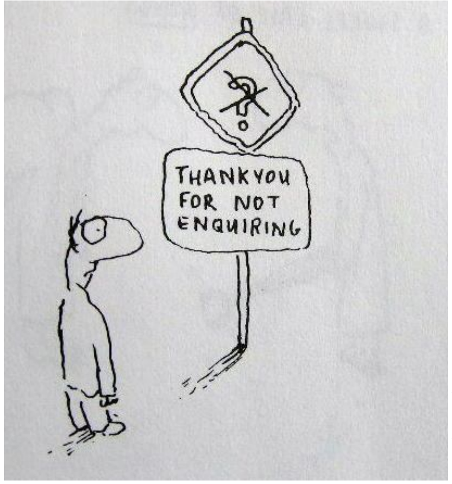
Dichotomy of the pion Goldstone mode and bound-state

- The *correct understanding* of pion observables; e.g. mass, decay constant and form factors, requires an approach to contain a
  - well-defined and valid chiral limit;
  - and an accurate realisation of dynamical chiral symmetry breaking.

HIGHLY NONTRIVIAL

Impossible in quantum mechanics

Only possible in asymptotically-free gauge theories



QCD's Challenges

Understand emergent phenomena

- *Quark and Gluon Confinement*
  No matter how hard one strikes the proton,
  one cannot liberate an individual quark or gluon
- *Dynamical Chiral Symmetry Breaking*
  Very unnatural pattern of bound state masses;
  e.g., Lagrangian (pQCD) quark mass is small but
  . . . no degeneracy between $J^P=J^+$ and $J^P=J^-$ *(parity partners)*
- *Neither of these phenomena is apparent in QCD's Lagrangian*
  Yet they are the dominant determining characteristics of real-world QCD.
- QCD
  – Complex behaviour arises from apparently simple rules.

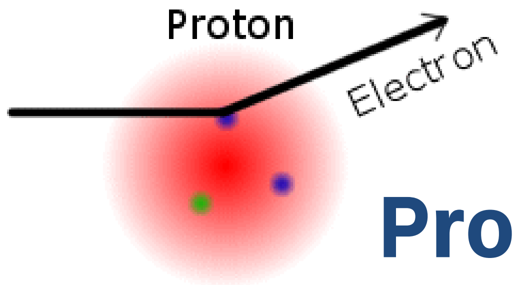
The study of nonperturbative QCD is the puriew of …

Hadron Physics

Nucleon … Two Key Hadrons Proton and Neutron

- Fermions – two static properties:
  proton electric charge = +1; and magnetic moment, μ_p
- Magnetic Moment discovered by Otto Stern and collaborators in 1933;
  Stern awarded Nobel Prize (1943): *"for his contribution to the development of the molecular ray method and his discovery of the magnetic moment of the proton"*.
- Dirac (1928) – pointlike fermion: $\mu_p = \frac{e\hbar}{2M}$
- Stern (1933) – $\mu_p = (1 + 1.79)\frac{e\hbar}{2M}$
- Big Hint that Proton is not a point particle
  - Proton has constituents
  - These are Quarks and Gluons
- Quark discovery via e-p-scattering at SLAC in 1968
  - the elementary quanta of QCD

Friedman, Kendall, Taylor, Nobel Prize (1990): "for their pioneering investigations concerning deep inelastic scattering of electrons on protons and bound neutrons, which have been of essential importance for the development of the quark model in particle physics"



Nucleon Structure
Probed in scattering experiments

- Electron is a good probe because it is structureless

  Electron's relativistic current is

$$j_\mu(P',P) = ie\,\bar{u}_e(P')\,\Lambda_\mu(Q,P)\,u_e(P)\,,\quad Q = P' - P$$
$$= ie\,\bar{u}_e(P')\,\gamma_\mu(-1)\,u_e(P)$$

Structureless fermion, or simply-structured fermion, $F_1=1$ & $F_2=0$, so that $G_E=G_M$ and hence distribution of charge and magnetisation within this fermion are identical

- Proton's electromagnetic current

$$J_\mu(P',P) = ie\,\bar{u}_p(P')\,\Lambda_\mu(Q,P)\,u_p(P)\,,$$
$$= ie\,\bar{u}_p(P')\left(\gamma_\mu F_1(Q^2) + \frac{1}{2M}\,\sigma_{\mu\nu}\,Q_\nu\,F_2(Q^2)\right)u_p(P)$$

F_1 = Dirac form factor

F_2 = Pauli form factor

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2)\,,\quad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

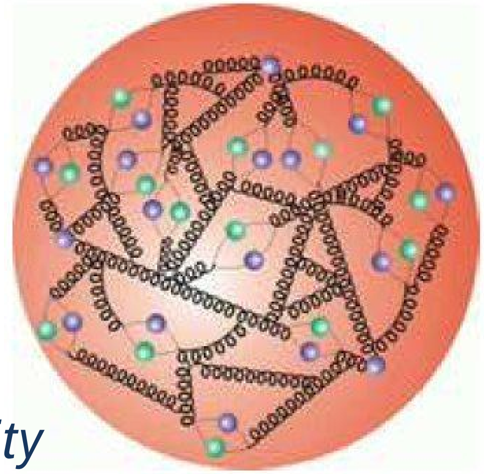
G_E = Sachs Electric form factor
If a nonrelativistic limit exists, this relates to the charge density

G_M = Sachs Magnetic form factor
If a nonrelativistic limit exists, this relates to the magnetisation density

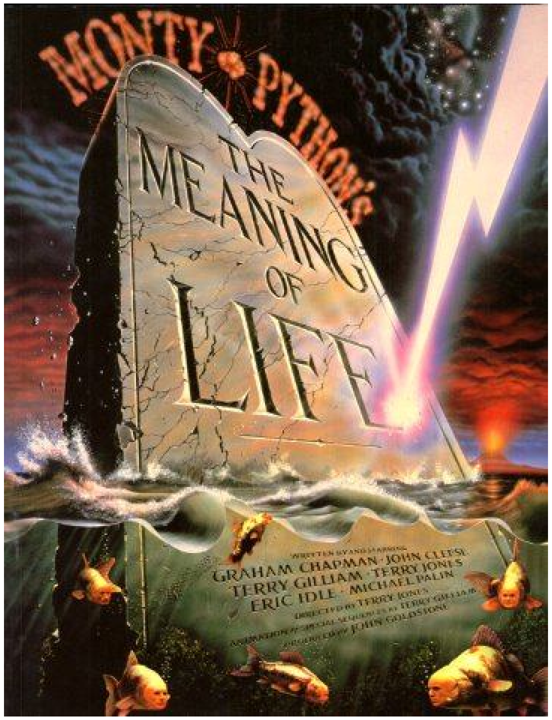
Nuclear Science Advisory Council Long Range Plan

- A central goal of nuclear physics is to understand the structure and properties of protons and neutrons, and ultimately atomic nuclei, in terms of the quarks and gluons of QCD
- So, what's the problem?

  They are legion …
  - *Confinement*
  - *Dynamical chiral symmetry breaking*
  - *A fundamental theory of unprecedented complexity*



- QCD defines the difference between nuclear and particle physicists:
  - Nuclear physicists try to solve this theory
  - Particle physicists run away to a place where tree-level computations are all that's necessary; *perturbation theory is the last refuge of a scoundrel*



Understanding NSAC's Long Range Plan

- *What are the quarks and gluons of QCD?*
- *Is there such a thing as a constituent quark, a constituent-gluon?*

  After all, these are the concepts for which Gell-Mann won the Nobel Prize.

- *Do they – can they – correspond to well-defined quasi-particle degrees-of-freedom?*
- *If not, with what should they be replaced?*

What is the meaning of the NSAC Challenge?

Recall the dichotomy of the pion

- How does one make an almost massless particle from two massive constituent-quarks?
- One can always tune a potential in quantum mechanics so that the ground-state is massless

  – *and some are still making this mistake*

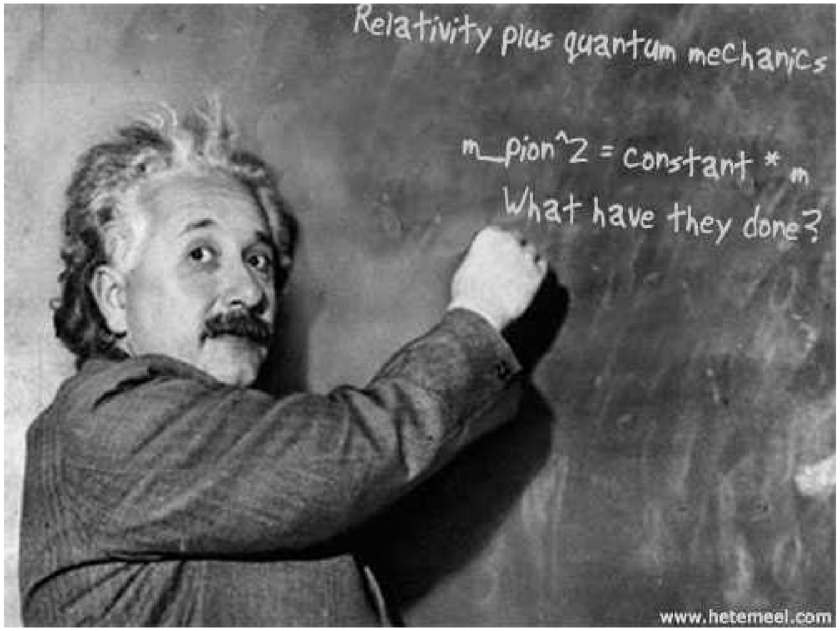
- However:

  current-algebra (1968)

$$m_\pi^2 \propto m$$

Models based on constituent-quarks cannot produce this outcome. They must be fine tuned in order to produce the empirical splitting between the π & ρ mesons

- This is *impossible in quantum mechanics*, for which one always finds: $m_{bound-state} \propto m_{constituent}$



What is the meaning of all this?

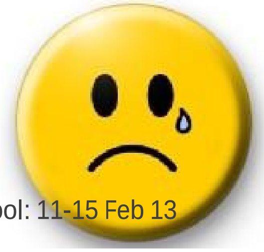
If $m_\pi = m_\rho$, then repulsive and attractive forces in the Nucleon-Nucleon potential have the SAME range and there is NO intermediate range attraction.

Under these circumstances:

- *Can* ^{12}C *be produced, can it be stable?*
- *Is the deuteron stable; can Big-Bang Nucleosynthesis occur?*

(Many more existential questions …)

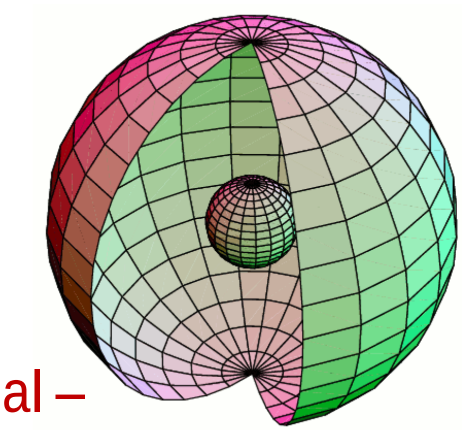
Probably not … but it wouldn't matter because we wouldn't be around to worry about it.



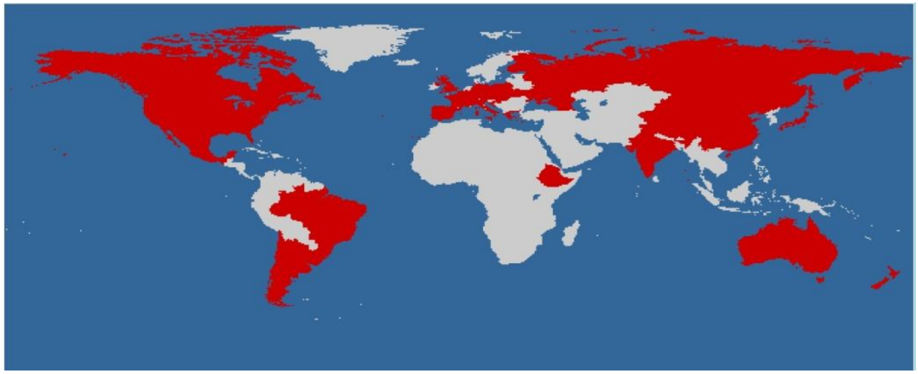
Why don't we just stop talking and solve the problem?

Just get on with it!

- But … QCD's emergent phenomena can't be studied using perturbation theory
  - *So what? Same is true of bound-state problems in quantum mechanics!*
- Differences:
  - Here relativistic effects are crucial – *virtual particles*
    Quintessence of Relativistic Quantum Field Theory
  - Interaction between quarks – the Interquark Potential –
    Unknown throughout > 98% of the pion's/proton's volume!



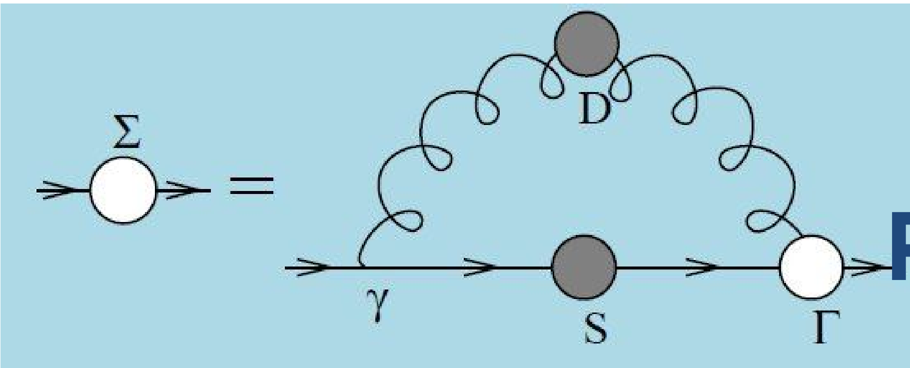
- Understanding requires *ab initio* nonperturbative solution of fully-fledged interacting relativistic quantum field theory, something which Mathematics and Theoretical Physics are a long way from achieving.



Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory . . . Materially Reduces Model-Dependence … Statement about long-range behaviour of quark-quark interaction
- NonPerturbative, Continuum approach to QCD
- Hadrons as Composites of Quarks and Gluons
- Qualitative and Quantitative Importance of:
  - Dynamical Chiral Symmetry Breaking
    - Generation of fermion mass from *nothing*
  - Quark & Gluon Confinement
    - Coloured objects not detected, *Not detectable?*

- Approach yields Schwinger functions; i.e., propagators and vertices
- Cross-Sections built from Schwinger Functions
- Hence, method connects observables with long-range behaviour of the running coupling
- Experiment ↔ Theory comparison leads to an understanding of long-range behaviour of strong running-coupling



Mass from Nothing?! Perturbation Theory

- QCD is asymptotically-free (2004 Nobel Prize)
  - Chiral-limit is well-defined;
    i.e., one can truly speak of a massless quark.
  - NB. This is nonperturbatively *impossible* in QED.
- Dressed-quark propagator: $$S(p) = \frac{Z(p^2)}{i\gamma\cdot p + M(p^2)}$$
- Weak coupling expansion of gap equation yields every diagram in perturbation theory
- In perturbation theory:
  If *m=0*, then *M(p²)=0*
  Start with no mass,
  Always have no mass.

$$M(p^2) = m\left(1 - \frac{\alpha}{\pi}\ln\left[\frac{p^2}{\Lambda^2}\right] + \ldots\right)$$

0

Dynamical Chiral Symmetry Breaking

Nambu—Jona-Lasinio Model

- Recall the gap equation

$$S^{-1}(p) = i\gamma\cdot p\,A(p^2) + B(p^2) = i\gamma\cdot p + m$$
$$+ \int^{\Lambda} \frac{d^4\ell}{(2\pi)^4}\, g^2\, D_{\mu\nu}(p-\ell)\,\gamma_\mu \frac{\lambda^a}{2} \frac{1}{i\gamma\cdot\ell A(\ell^2) + B(\ell^2)} \Gamma^a_\nu(\ell,p)$$

NJL: $\Gamma^a_\mu(k,p)_{\rm bare} = \gamma_\mu\, \dfrac{\lambda^a}{2}$;

$$g^2 D_{\mu\nu}(p-\ell) \to \delta_{\mu\nu}\, \frac{1}{m_G^2}\, \theta(\Lambda^2 - \ell^2)$$

- Model is not renormalisable

$\Rightarrow$ regularisation parameter (Λ) plays a dynamical role.

- NJL gap equation

$$i\gamma\cdot p\,A(p^2) + B(p^2)$$
$$= \quad i\gamma\cdot p + m + \frac{4}{3}\,\frac{1}{m_G^2} \int \frac{d^4\ell}{(2\pi)^4}\, \theta(\Lambda^2 - \ell^2)\,\gamma_\mu\, \frac{-i\gamma\cdot\ell A(\ell^2) + B(\ell^2)}{\ell^2 A^2(\ell^2) + B^2(\ell^2)}\,\gamma_\mu$$

Nambu—Jona-Lasinio Model

- Multiply the NJL gap equation by $(-i\gamma\cdot p)$; trace over Dirac indices:

$$p^2 A(p^2) = p^2 + \frac{8}{3}\frac{1}{m_G^2}\int \frac{d^4\ell}{(2\pi)^4}\,\theta(\Lambda^2-\ell^2)\,p\cdot\ell\,\frac{A(\ell^2)}{\ell^2 A^2(\ell^2)+B^2(\ell^2)}$$

  - Angular integral vanishes, therefore $A(p^2) = 1$.
  - This owes to the fact that the NJL model is defined by a four-fermion contact-interaction in configuration space, which entails a momentum-independent interaction in momentum space.

- Simply take Dirac trace of NJL gap equation:

$$B(p^2) = m + \frac{16}{3}\frac{1}{m_G^2}\int \frac{d^4\ell}{(2\pi)^4}\,\theta(\Lambda^2-\ell^2)\,\frac{B(\ell^2)}{\ell^2+B^2(\ell^2)}$$

  - Integrand is p^2-independent, therefore the only solution is $B(p^2)$ = constant = M.

- General form of the propagator for a fermion dressed by the NJL interaction: $S(p) = 1/[\, i\,\gamma\cdot p + M\,]$

Critical coupling for dynamical mass generation?

NJL model & a mass gap?

- Evaluate the integrals

$$M = m + M \frac{1}{3\pi^2} \frac{1}{m_G^2} \mathcal{C}(M^2, \Lambda^2)\,,$$
$$\mathcal{C}(M^2, \Lambda^2) = \Lambda^2 - M^2 \ln\left[1 + \Lambda^2/M^2\right]\,.$$

- Λ defines the model's mass-scale. Henceforth set $\Lambda = 1$, then all other dimensioned quantities are given in units of this scale, in which case the gap equation can be written

$$M = M \frac{1}{3\pi^2} \frac{1}{m_G^2} \mathcal{C}(M^2, 1)$$

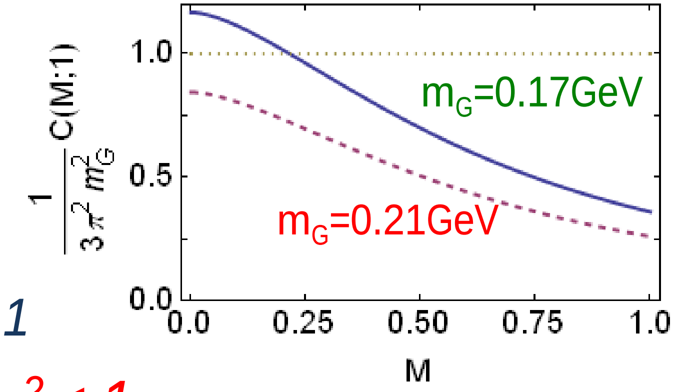
- Chiral limit, *m=0*
  - Solutions?
    - One is obvious; viz., *M=0*
      This is the *perturbative result*
      … start with no mass, end up with no mass

- Chiral limit, *m=0*
  - Suppose, on the other hand that $M \neq 0$, and thus may be cancelled
    - This nontrivial solution can exist if-and-only-if one may satisfy

$$3\pi^2\, m_G^2 = \mathcal{C}(M^2,1)$$

Critical coupling for dynamical mass generation!

NJL model & a mass gap?



- Can one satisfy $3\pi^2 m_G^2 = C(M^2,1)$?
  - $C(M^2, 1) = 1 - M^2 \ln [1 + 1/M^2]$
    - Monotonically decreasing function of M
    - Maximum value at $M = 0$; viz., $C(M^2=0, 1) = 1$
- Consequently, there is a solution *iff* $3\pi^2 m_G^2 < 1$
  - Typical scale for hadron physics: $\Lambda = 1$ GeV
    - There is a M≠0 solution iff $m_G^2 < (\Lambda/(3\pi^2)) = (0.2 \text{ GeV})^2$
- Interaction strength is proportional to $1/m_G^2$
  - Hence, if interaction is strong enough, then one can start with no mass but end up with a massive, perhaps very massive fermion

Dynamical Chiral Symmetry Breaking

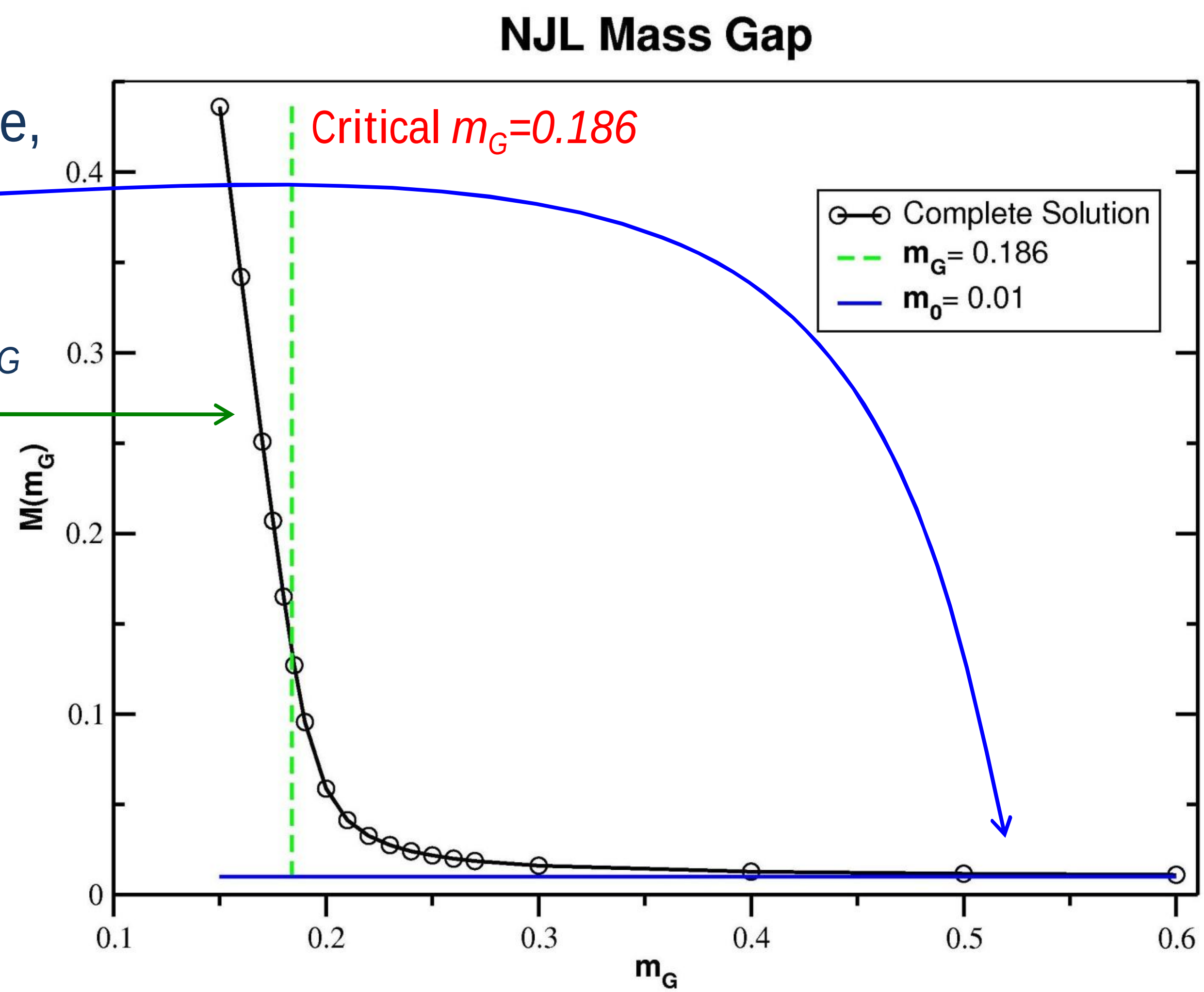
Solution of gap equation

$$M = m + M\,\frac{1}{3\pi^2}\,\frac{1}{m_G^2}\,\mathcal{C}(M^2,1)$$

NJL Model Dynamical Mass

- Weak coupling corresponds to m_G large, in which case $M \approx m$
- On the other hand, strong coupling; i.e., m_G small, $M >> m$

This is the defining characteristic of dynamical chiral symmetry breaking



NJL Model
and Confinement?

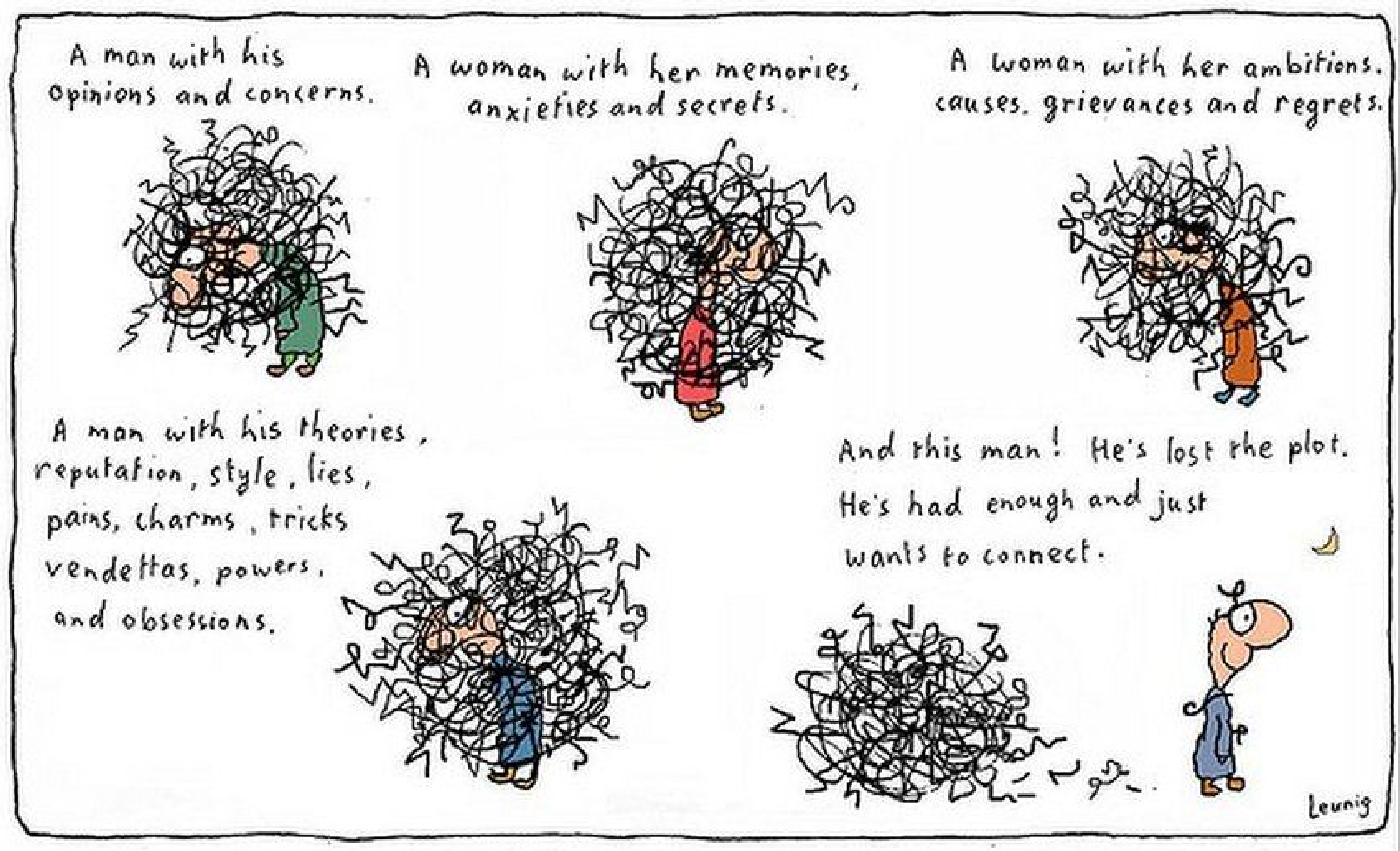
- **Confinement**: no free-particle-like quarks
- Fully-dressed NJL propagator

$$S(p)^{\mathrm{NJL}} = \frac{1}{i\gamma\cdot p[A(p^2)=1]+[B(p^2)=M]} = \frac{-i\gamma\cdot p+M}{p^2+M^2}$$

- This is merely a free-particle-like propagator with a shifted mass

  $p^2 + M^2 = 0 \rightarrow$ Minkowski-space mass = M
- Hence, whilst **NJL model** exhibits dynamical chiral symmetry breaking it **does not confine**.

NJL-fermion still propagates as a plane wave



Any Questions?