Lecture 3

Deep Inelastic Scattering & Parton Distribution Functions

Ian Cloët

The University of Adelaide & Argonne National Laboratory

CSSM Summer School

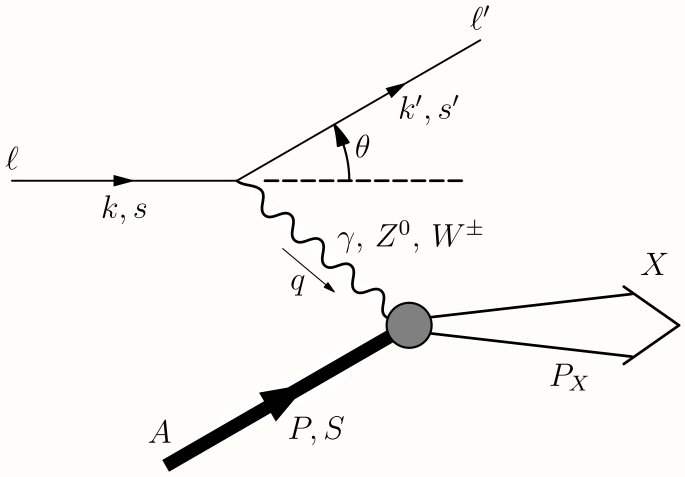
Non-perturbative Methods in Quantum Field Theory

$11^{th} - 15^{th}$ February 2013

Argonne NATIONAL LABORATORY

SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

Deep Inelastic Scattering



$$q^2 = (k' - k)^2 = -Q^2 \leq 0, \qquad s = (\ell + P)^2$$

$$x_A \equiv A\,\frac{Q^2}{2\,P\cdot q} = A\,\frac{Q^2}{2\,M_A\,\nu}, \quad 0 < x_A \leqslant A$$

$$y = \frac{Q^2}{x\,s}, \qquad W^2 = (P+q)^2 = Q^2\,\frac{1-x}{x}$$

- Unpolarized cross-section for DIS with single photon exchange is

$$\frac{d\sigma^\gamma}{dx_A\,dQ^2} = \frac{2\pi\,\alpha_e^2}{x_A\,Q^4}\left[\left(1+(1+y)^2\right)F_2^\gamma(x,Q^2) - y^2 F_L^\gamma(x,Q^2)\right]$$

  - ✦ $F_L^\gamma(x,Q^2) = F_2^\gamma(x,Q^2) - 2\,x\,F_1^\gamma(x,Q^2)$

- The longitudinally polarized cross-section is

$$\frac{d\,\Delta_L\sigma^\gamma(\lambda)}{dx_A\,dQ^2} = \frac{4\pi\,\alpha_e^2}{x_A\,Q^4}\left[-2\lambda\left(1-(1-y)^2\right)x\,g_1^\gamma(x,Q^2) + y^2 g_L^\gamma(x,Q^2)\right]$$

- Also structure functions for γZ, Z^0 & $W^\pm$ exchange

Bjorken Limit and Scaling

- The Bjorken limit is defined as:

$$Q^2,\ \nu \to \infty \ \mid \ x = \text{fixed}$$

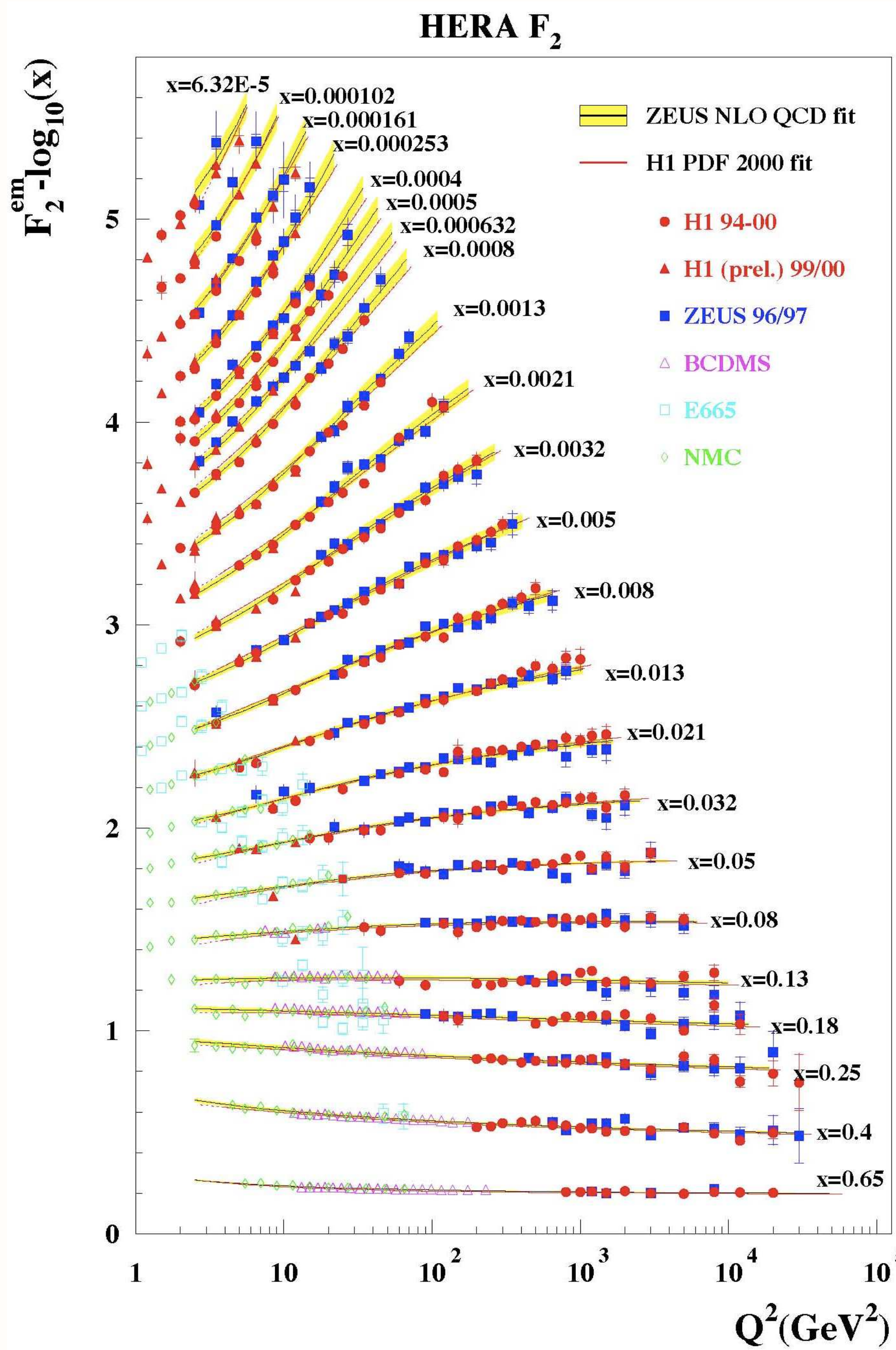
- In 1968 J. D. Bjorken argued that in this limit the photon interactions with the target constituents (partons) involves no dimensional scale, therefore

$$F_2^{\gamma}(x, Q^2) \to F_2^{\gamma}(x)$$
$$g_1^{\gamma}(x, Q^2) \to g_1^{\gamma}(x) \quad etc$$

  - ✦ Bjorken scaling

- Confirmation from SLAC in 1968 was the first evidence for pointlike constituents inside proton

- Scaling violation ⇔ perturbative QCD



Physical meaning of Bjorken x

- Choose a frame where $\vec{q}_\perp = 0$ then photon moment is

$$q = \left[\nu, 0, 0, -\sqrt{\nu^2 + Q^2}\right] \xrightarrow{\text{Bjorken limit}} q = \left[\nu, 0, 0, -\nu - x\, M_N\right]$$

- Lightcone coordinates: $q^\pm = \frac{1}{\sqrt{2}}\left(q^0 \pm q^3\right) \Rightarrow a \cdot b = a^+ b^- + a^- b^+ - \vec{a}_\perp \cdot \vec{b}_\perp$
- Therefore in Bjorken limit: $q^- \to \infty \quad q^+ \to -x\, M_N/\sqrt{2}$ and

$$x = \frac{Q^2}{2\, p \cdot q} = -\frac{q^+ q^-}{q^- p^+ + q^+ p^-} \to -\frac{q^+}{p^+}$$

- The lightcone dispersion relation reads: $k^- = \dfrac{m^2 + \vec{k}^{\,2}}{2\, k^+}$
- Can only be satisfied for $k'^- \; (= k^- + q^-)$ if $k'^+ = 0 \implies k^+ = -q^+$
- *Therefore x has physical meaning of the lightcone momentum fraction carried by the struck quark before it is hit by photon*

$$x = \frac{k^+}{p^+}$$

Parton Distribution Functions

- Factorization theorems in QCD prove that the structure functions can be expressed in terms of *universal* parton distribution functions (PDFs)
  - that is, the cross-sections can be factorized into process depend perturbative pieces, determined by pQCD (Wilson coefficients) and the innately non-perturbative *universal* PDFs
- For example at LO and leading twist the structure functions are given by

$$F_2^\gamma(x,Q^2) = \sum\nolimits_{q=u,d,s,\ldots} e_q^2 \left[x\, q(x,Q^2) + x\, \bar{q}(x,Q^2)\right]$$

$$g_1^\gamma(x,Q^2) = \frac{1}{2}\sum\nolimits_{q=u,d,s,\ldots} e_q^2 \left[\Delta q(x,Q^2) + \Delta\bar{q}(x,Q^2)\right]$$

- These PDFs have a probability interpretation:

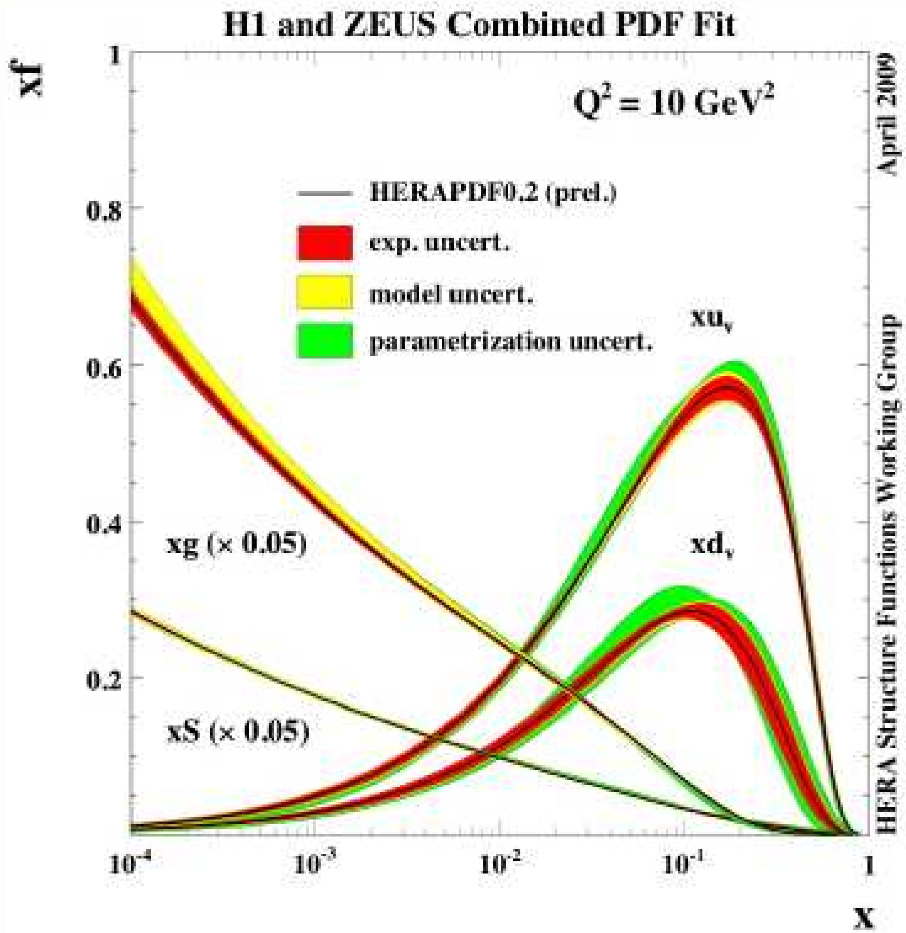
$q(x) = q_+(x) + q_-(x)$ [spin-independent PDF]

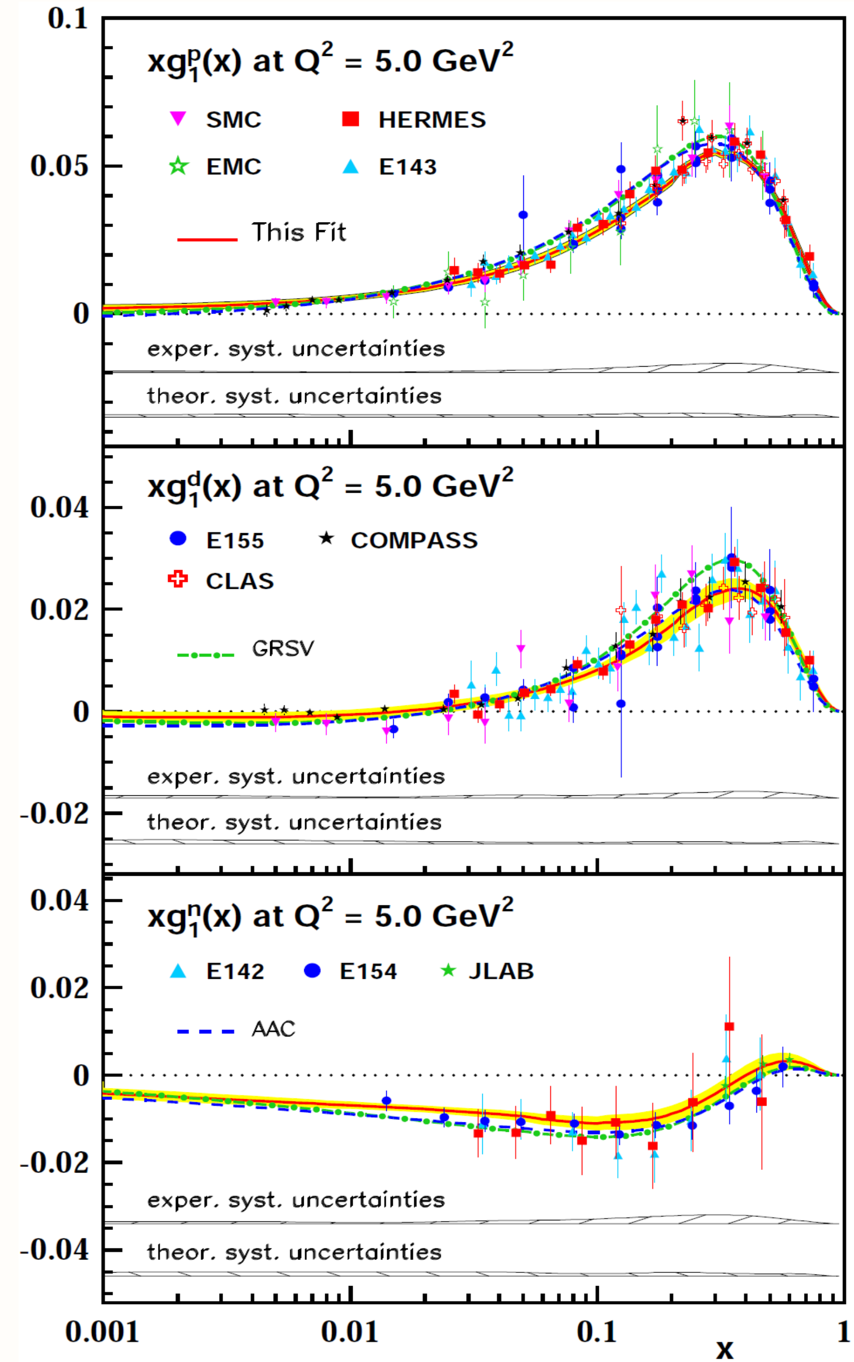
"probability to strike a quark of flavour q with lightcone momentum fraction x of the target momentum"

$\Delta q(x) = q_+(x) - q_-(x)$ [spin-dependent PDF]

"helicity weighted probability to strike a quark of flavour q with lightcone momentum fraction x of the target momentum"

Experimental Status: Nucleon PDFs

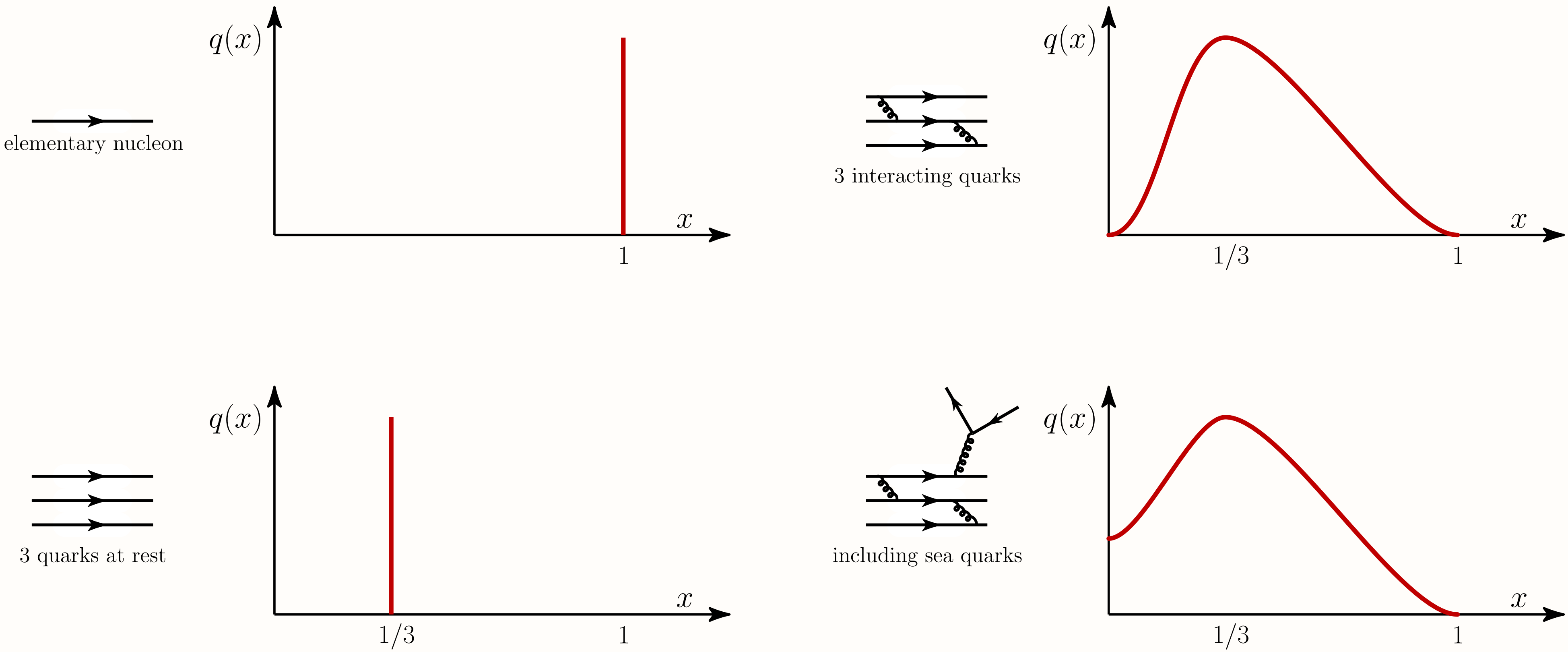




- The distance scales, ξ, probed in DIS are given by: $\xi \sim 1/(x\,M_N)$
  - $x = 0.5 \quad \Longrightarrow \quad \xi = 0.4$ fm
  - $x = 0.05 \quad \Longrightarrow \quad \xi = 4$ fm

Physical Interpretation

- PDFs tells us how *particle number*, *momentum* and *spin* are distributed



- Sum rules

$$\int dx\,[q(x)-\bar{q}(x)] = N_q; \qquad \int dx\, x\,[u(x)+d(x)+\ldots] = 1; \qquad \int dx\,\Delta q(x) = \Sigma_q$$

baryon number — momentum — spin sum

- Nucleon angular momentum: $J = \frac{1}{2} = \frac{1}{2}\Sigma + L_q + J_g$

Frame Dependence

- Quark distributions are Lorentz invariants (just like form factors)
  - ✦ therefore PDF measurements in different frames give the same result
- However the interpretation of the PDFs are frame dependence
  - ✦ consider $J = \frac{1}{2}\Sigma + L_q + J_g$ only J is a Lorentz invariant
  - ✦ therefore in what frame does $\int dx\, \Delta q(x) = \Sigma_q$ give the spin sum? Consider:

$$\langle P,S|\overline{\psi}_q(0)\,\gamma^\mu\gamma_5\,\psi_q(0)|P,S\rangle = 2\sigma\, S^\mu, \qquad p^\mu = \frac{\Lambda}{\sqrt{2}}(1,0,0,1), \qquad n^\mu = \frac{1}{\sqrt{2}\Lambda}(1,0,0,-1),$$

$$\int \frac{d\xi}{2\pi} e^{i\xi\, x}\langle P,S|\overline{\psi}_q(0)\,\gamma^\mu\gamma_5\,\psi_q(\xi\, n)|P,S\rangle = 2\Delta q(x)\,(S\cdot n)p^\mu + 2\, g_T(x)\, S^\mu_\perp + 2\, M_N^2\, \Delta q_3(x)\,(S\cdot n)\, n^\mu$$

- Integrate 2nd equation over x & take $+$ component of current

$$\langle P,S|\overline{\psi}_q(0)\,\gamma^+\gamma_5\,\psi_q(0)|P,S\rangle = 2\, p^+(S\cdot n)\int dx\, \Delta q(x) = 2\sigma\, S^+ = 2\sigma\,(S\cdot n)p^+$$

- Meaning of σ can be seem in the rest frame where $S^\mu = (0, \vec{r})$

$$\langle P,S|\overline{\psi}_q\,\gamma^i\gamma_5\,\psi_q|P,S\rangle = 2\sigma\, r_i \qquad \overline{\psi}_q\,\gamma^i\gamma_5\,\psi_q = \psi_q^\dagger\,\gamma^0\gamma^i\gamma_5\,\psi_q = \psi_q^\dagger\,\Sigma^i\,\psi_q = \psi_q^\dagger\,\text{diag}[\sigma^i, \sigma^i]\,\psi_q,$$

- Hence, σ is the expectation value of the spin operator in rest frame

Moments of PDFs

- Low moments of PDFs are related to conservation laws and observables
  - recall baryon and momentum sum rules; spin carried by quarks
- Most PDFs moments dependent on the resolving scale Q^2
- PDFs are usually obtained by fitting a chosen functional form to data
  - see MRST/MSTW, GRV/GJR, CTEQ, NNPDF (neural network), etc
- Typical values for proton PDF moments $(Q^2_{0\,\text{NLO}} = 0.5\,\text{GeV}^2)$

$$\langle x\,u\rangle = 0.404 \quad \langle x\,d\rangle = 0.194 \quad \langle x\,\bar{u}\rangle = 0.029 \quad \langle x\,\bar{d}\rangle = 0.039 \quad \langle x\,g\rangle = 0.334$$

  - gluon carry 33% of proton momentum [GJR, Eur. Phys. J. C53 (2008) 355]
- Typical polarized PDF moments $(Q^2_{0\,\text{NLO}} = 1\,\text{GeV}^2)$ [LSS2010]:

$$\langle \Delta u\rangle = 0.78 \quad \langle \Delta d\rangle = -0.38 \quad \langle \Delta \bar{u}\rangle = 0.043 \quad \langle \Delta \bar{d}\rangle = -0.069 \quad \langle \Delta g\rangle \simeq 0.30$$

- For spin sum have [LSS2010]: $\Sigma = 0.42 \pm 0.19 \quad Q^2 = 4\,\text{GeV}^2$
- Recall *"proton spin crisis"*: $\Sigma_{u+d} = 0.14 \pm 0.9 \pm 0.21$ [Ashman, *et al., PLB, 1987]*

Extracting Proton Spin Content

- Ellis–Jaffe sum rule $\left[\frac{1}{2} = \frac{1}{2}\Sigma + L_q + J_g\right]$

$$\int dx\, g_{1p}^{\gamma}(x, Q^2) = \frac{1}{36}\left[3\,\Delta q_3 + \Delta q_8\right] + \frac{1}{9}\,\Delta q_0,$$

$$\Sigma = \Delta q_0 = \Delta u^+ + \Delta d^+ + \Delta s^+ \qquad \text{[singlet]}$$

$$g_A = \Delta q_3 = \Delta u^+ - \Delta d^+ \qquad \text{[triplet]}$$

$$\Delta q_8 = \Delta u^+ + \Delta d^+ - 2\,\Delta s^+ \qquad \text{[octet]}$$

- To help extract Σ usual to use semi-leptonic hyperon decays and assume $SU(3)$ flavour symmetry to relate Δq_3 and Δq_8

$$\Delta q_3 = F + D \qquad \Delta q_8 = 3\,F - D$$

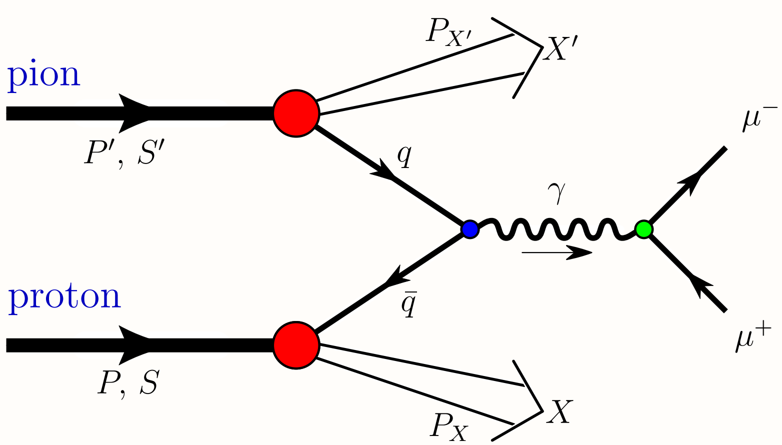
$$n\,p \to F + D, \qquad \Lambda\,p \to F + \tfrac{1}{3}\,D, \qquad \Sigma\,n \to F - D, \quad \text{etc}$$

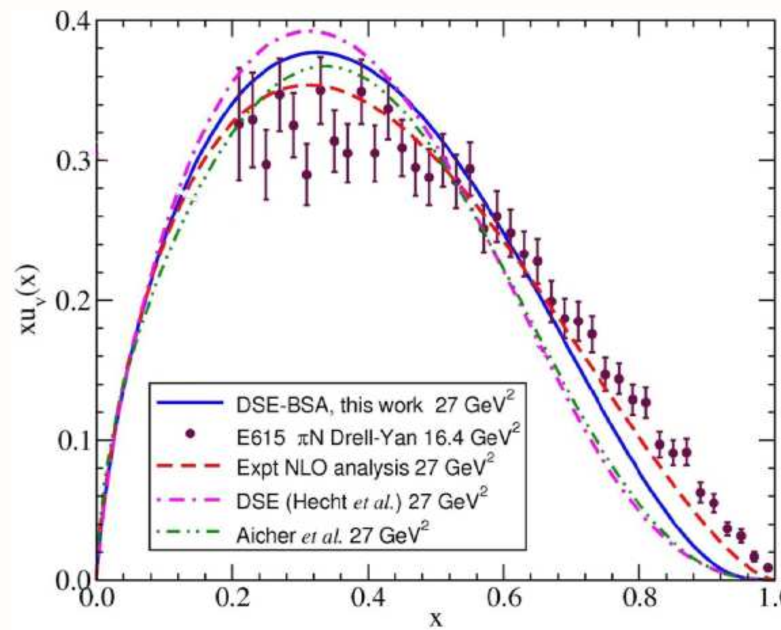
- Spin sum can also be determined via

$$\int dx\, g_{1p}^{\gamma Z}(x, Q^2) = \frac{1}{36}\left(1 - 4\,\sin^2\theta_W\right)\left[3\,\Delta q_3 + \Delta q_8\right] + \frac{2}{9}\left(1 - 2\,\sin^2\theta_W\right)\Delta q_0$$

The Pion PDF

- In QCD alone the pion is a stable particle, however in the real world it decays via the electroweak interaction with a mean lifetime of 2.6×10^{-8} s
- Therefore in nature there are no pion targets, however because of time dilation it is possible to create a beam of pions: e.g. $p +$ Be $\rightarrow \pi^- + X$
- Can measure pion PDFs via a process called pion-induced Drell-Yan: $\pi\, p \rightarrow \mu^+\, \mu^-\, X$





- There have been three experiments: CERN 1983 & 1985, Fermilab 1989

$$q_\pi(x) \overset{x\to 1}{\longrightarrow} (1-x)^{1+\varepsilon} \qquad \text{pQCD} \implies q_\pi(x) \sim (1-x)^{2+\gamma}$$

Theory Definition of Pion PDFs

- Pion is a spin zero particle $\Longrightarrow$ only has spin-independent PDFs: $q_\pi(x, Q^2)$
- The pion quark distribution function is defined by

$$q_\pi(x) = p^+ \int \frac{d\xi^-}{2\pi} e^{i\,x\,p^+\xi^-} \langle p, s|\overline{\psi}_q(0)\gamma^+\psi_q(\xi^-)|p, s\rangle_c,$$

- The *moments* of PDFs are defined by

$$\langle x^{n-1}\, q_\pi \rangle = \int_0^1 dx\; x^{n-1}\; q_\pi(x)$$

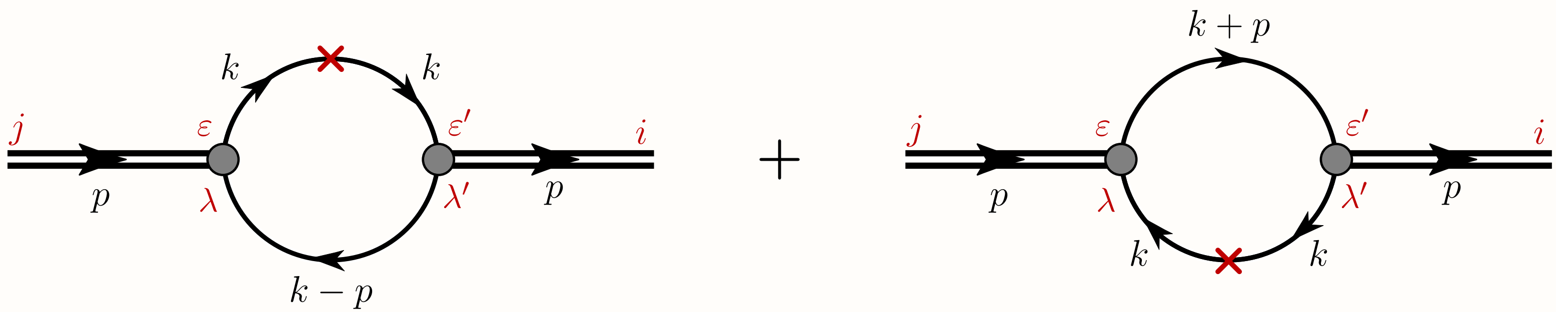
- The moments of these PDFs must satisfy the baryon number & momentum sum rules
- For example the $\pi^+ = u\bar{d}$ PDFs must satisfy

$$\langle u_\pi - \bar{u}_\pi \rangle = 1 \qquad \langle \bar{d}_\pi - d_\pi \rangle = 1 \qquad \langle x\, u_\pi + x\, \bar{d}_\pi + \ldots \rangle = 1$$

baryon number sum rules — momentum sum rule

✦ the baryon number sum rule is equivalent to charge conservation

Pion PDF in the NJL Model



- The pion quark distribution functions can be obtains from a Feynman diagram calculation
- The needed ingredients are
  - ✦ the pion Bethe-Salpeter amplitude: $\Gamma_\pi = \sqrt{g_\pi}\,\gamma_5\,\tau_i$
  - ✦ dressed quark propagator: $S(p)^{-1} = \not{p} - M + i\varepsilon$
- The operator insertion is given by

$$\gamma^+\,\delta\left(x - \frac{k^+}{p^+}\right)\frac{1}{2}\left(1 \pm \tau_3\right)$$

  - ✦ plus sign projects out u-quarks and minus d-quarks
  - ✦ recall x is the lightcone momentum fraction carried by struck quark

Pion PDF Results in NJL

- PDFs are scale – Q^2 – dependent, however within the NJL model there is no way to determine the model scale Q_0^2
- Standard method is to fit the proton valence u-quark distribution to empirical results, best fit determines Q_0^2
- The NJL model result for π^+ PDFs at $Q^2 = Q_0^2 = 0.16\,\text{GeV}^2$

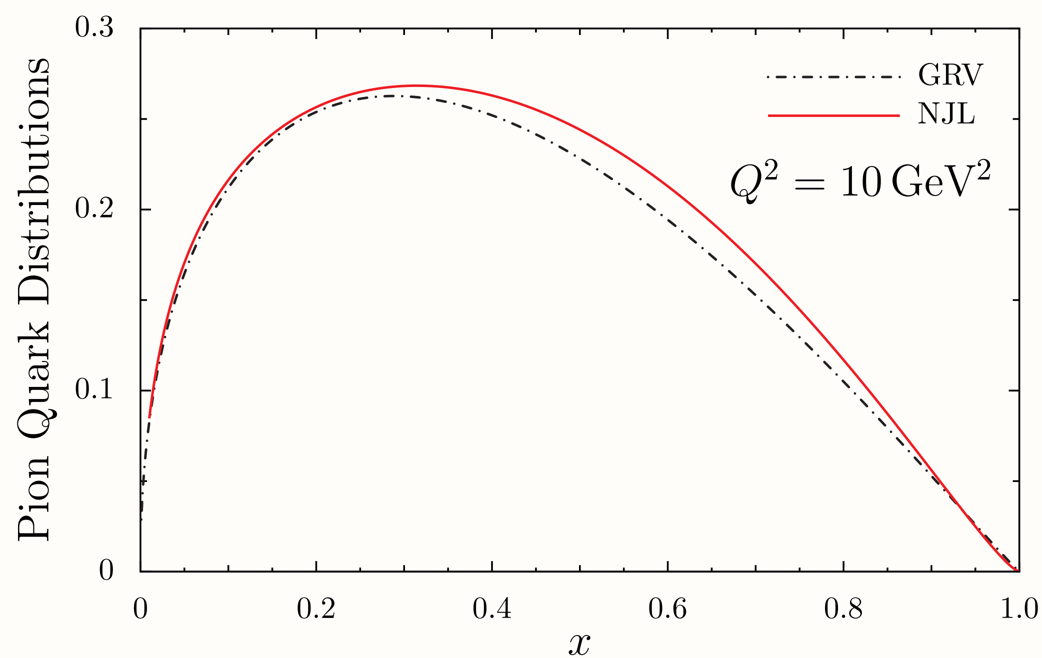
$$u_\pi(x) = \bar{d}_\pi(x) = \frac{3\,g_\pi}{4\pi^2} \int d\tau \left[\frac{1}{\tau} + x\,(1-x)\,m_\pi^2\right] e^{-\tau\left[x(x-1)m_\pi^2 + M^2\right]}.$$

- Agreement with data excellent
- At large x NJL finds

$$u_\pi(x) \overset{x\to 1}{\simeq} (1-x)^1$$

- Disagrees with pQCD result

$$u_\pi(x) \overset{x\to 1}{\simeq} (1-x)^{2+\gamma}$$



Pion PDF in DSEs

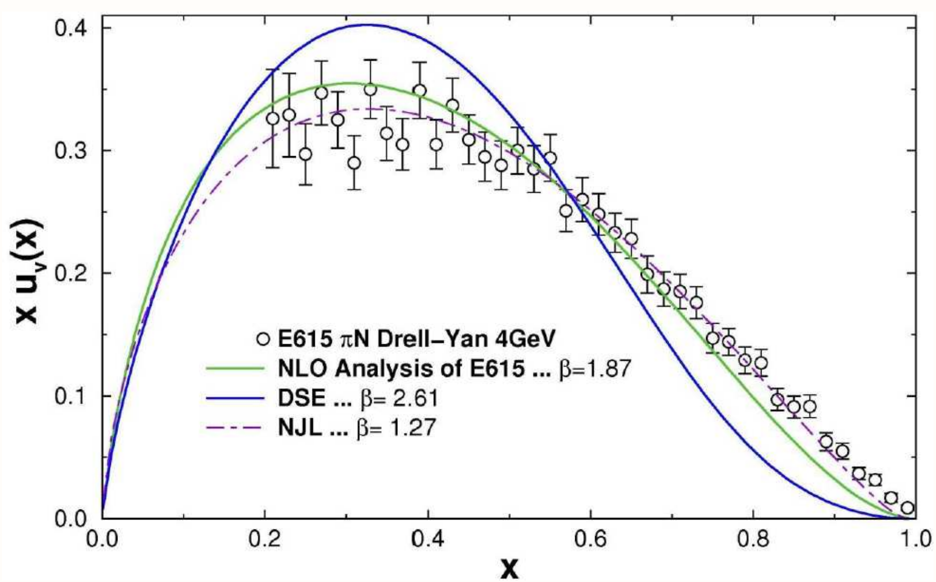
- DSE calculations – fully dressed quark propagator and BS vertex function

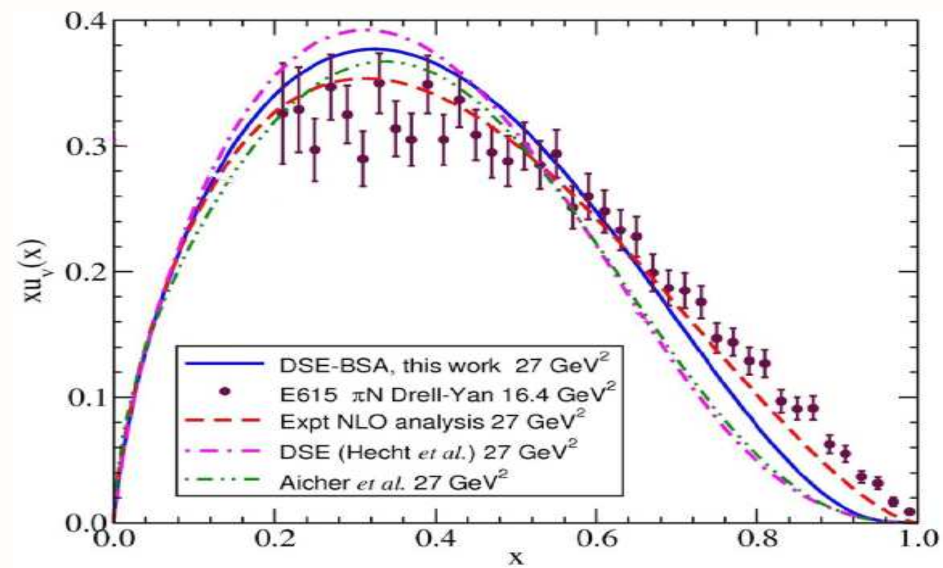
$$S(p)^{-1} = \not{p}\, A(p^2) + B(p^2)$$

$$\Gamma_\pi(p,k) = \gamma_5\Big[E_\pi(p,k) + \not{p}\, F_\pi(p,k) + \not{k}\, k\cdot p\, \mathcal{G}(p,k) + \sigma^{\mu\nu} k_\mu p_\nu\, \mathcal{H}(p,k)\Big]$$

- At large x DSE and pQCD results agree: $u_\pi(x) \overset{x\to1}{\simeq} (1-x)^{2+\gamma}$
  - ✦ this 2001 result seemed to disagree with experiment for a decade

- Recent reanalysis of data by *Aicher et al.* now finds excellent agreement with DSEs





QCD Evolution Equations

- The DGLAP evolution equations are one of the greatest successes of perturbative QCD
  - ✦ DGLAP $\Longleftrightarrow$ Dokshitzer (1977), Gribov-Lipatov (1972), Altarelli-Parisi (1977)
- These QCD evolution equations relate the PDFs at one scale, Q_0^2, to another scale, Q^2, provided $Q_0^2, Q^2 \gg \Lambda_{QCD}$.
- Evolution equation for minus type – $q^- \equiv q - \bar{q}$ – PDFs is

$$\frac{\partial}{\partial \ln Q^2} q^-(x, Q^2) = \alpha_s(Q^2)\, P(z) \otimes q^-(y, Q^2) \qquad \text{[non-singlet]}$$

  - ✦ $P(z)$: probability for quark to emit gluon leaving quark with momentum fraction z
  - ✦ note that the gluon PDF does not contribute to minus type PDF evolution
- Evolution equations for $q^+ \equiv q + \bar{q}$ and gluon, $g(x)$, PDFs are coupled

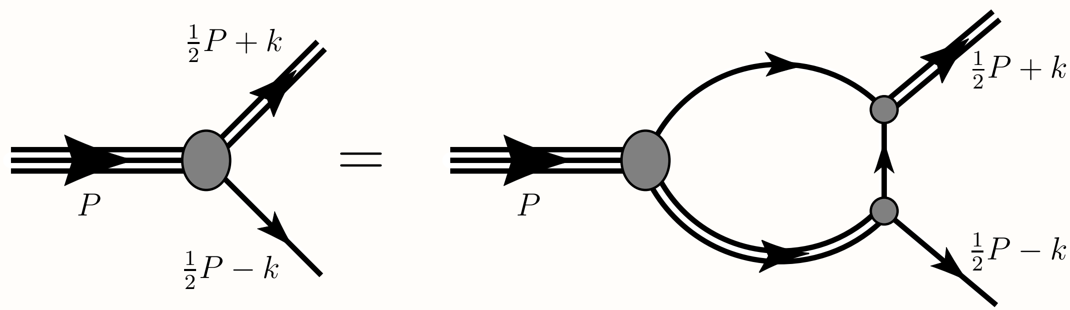
The physics behind these equations is that a valence quark can radiate gluons and a gluon can become a quark–antiquark pair, therefore momentum can be shifted between the valence quarks, sea quarks and gluons. The probability of this radiation is scale, Q^2, dependent.

Nucleon PDFs in the NJL model

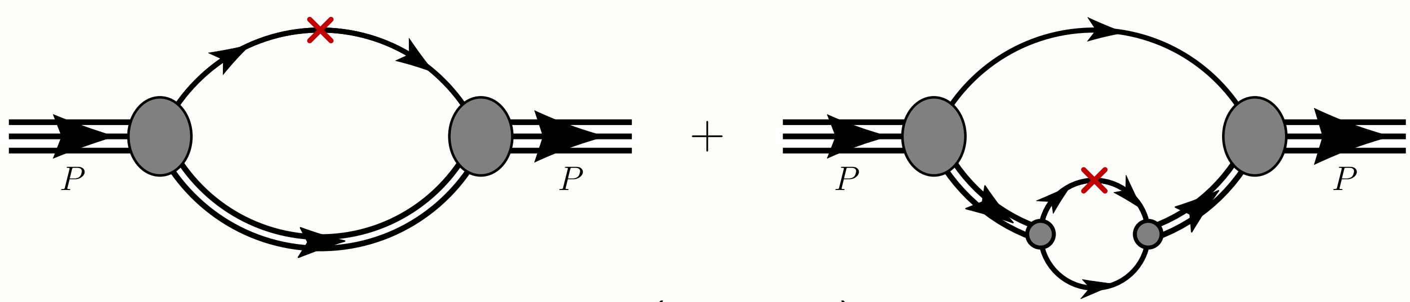
- Nucleon quark distributions are defined by

$$q(x) = p^+ \int \frac{d\xi^-}{2\pi} e^{i\,x\,p^+\xi^-} \langle p, s|\overline{\psi}_q(0)\gamma^+\psi_q(\xi^-)|p, s\rangle_c, \quad \Delta q(x) \quad = \langle \gamma^+\gamma_5 \rangle$$

- Nucleon bound state is obtained by solving the relativistic Faddeev equation in the quark-diquark approximation

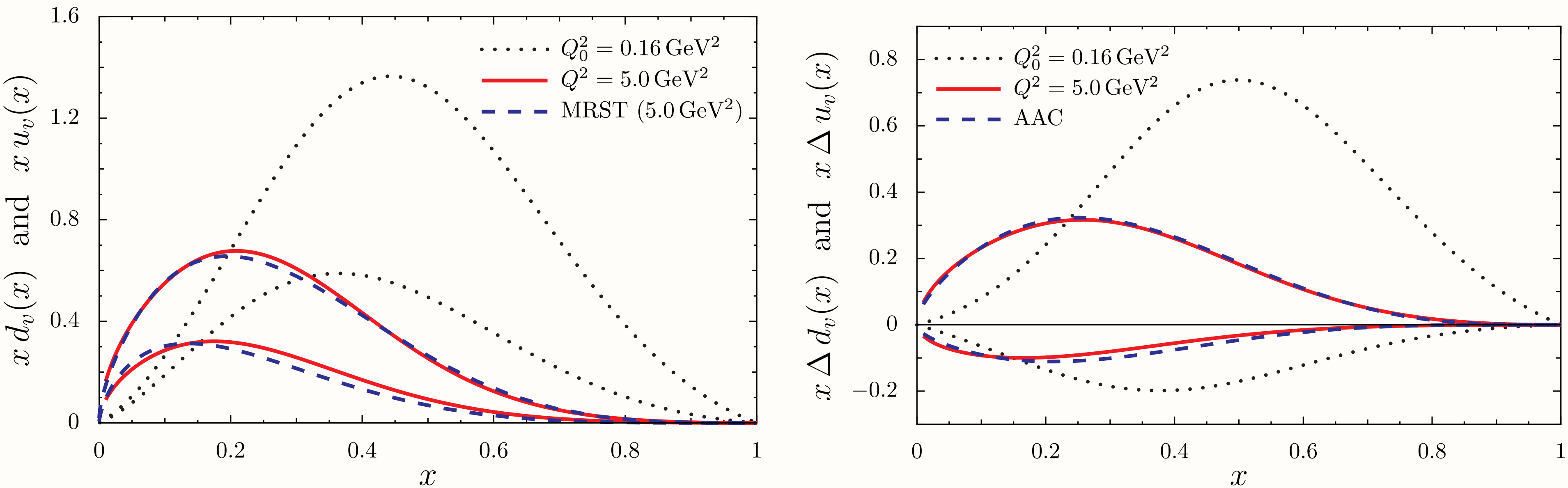


- PDFs are associated with the Feynman diagrams



✦ $[q(x), \Delta q(x), \Delta_T q(x)]$ → X $= \delta\left(x - \frac{k^+}{p^+}\right) [\gamma^+, \gamma^+\gamma_5, \gamma^+\gamma^1\gamma_5]$

Results: proton quark distributions



- Covariant, correct support, satisfies baryon and momentum sum rules

$$\int dx\,[q(x) - \bar{q}(x)] = N_q, \qquad \int dx\, x\,[u(x) + d(x) + \ldots] = 1$$

- Satisfies positivity constraints and Soffer bound

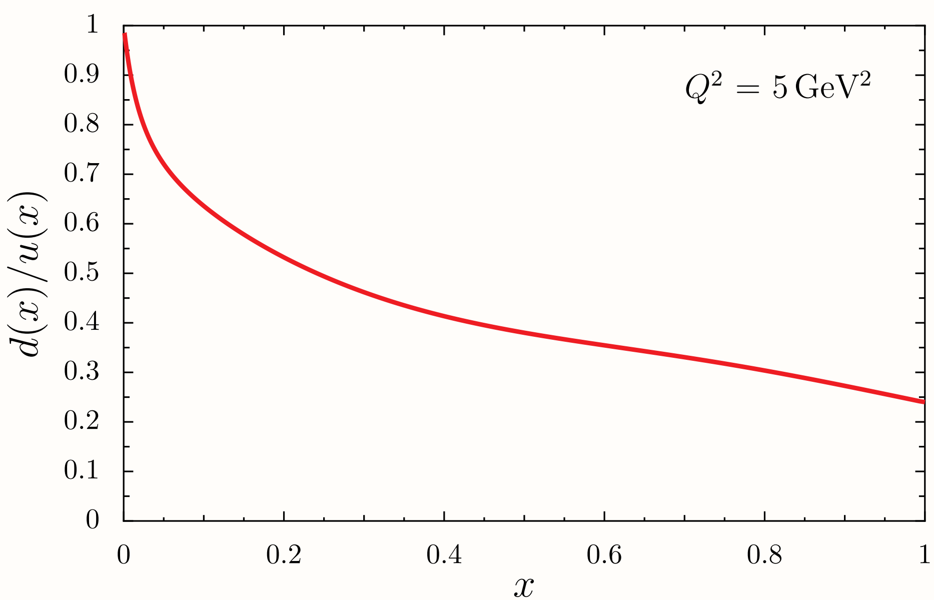
$$|\Delta q(x)|\,,\ |\Delta_T q(x)| \leqslant q(x), \qquad q(x) + \Delta q(x) \geqslant 2\,|\Delta_T q(x)|$$

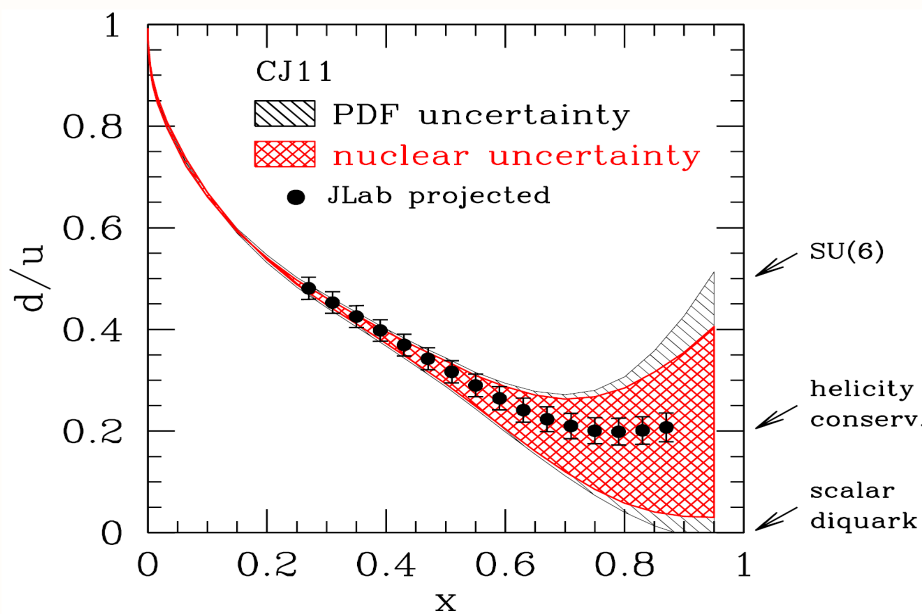
- Martin, Roberts, Stirling and Thorne, Phys. Lett. B **531**, 216 (2002).
- M. Hirai, S. Kumano and N. Saito, Phys. Rev. D **69**, 054021 (2004).

Proton d/u ratio

- The $d(x)/u(x)$ ratio as $x \to 1$ is of great interest because if does not change with QCD evolution and pQCD can make predictions

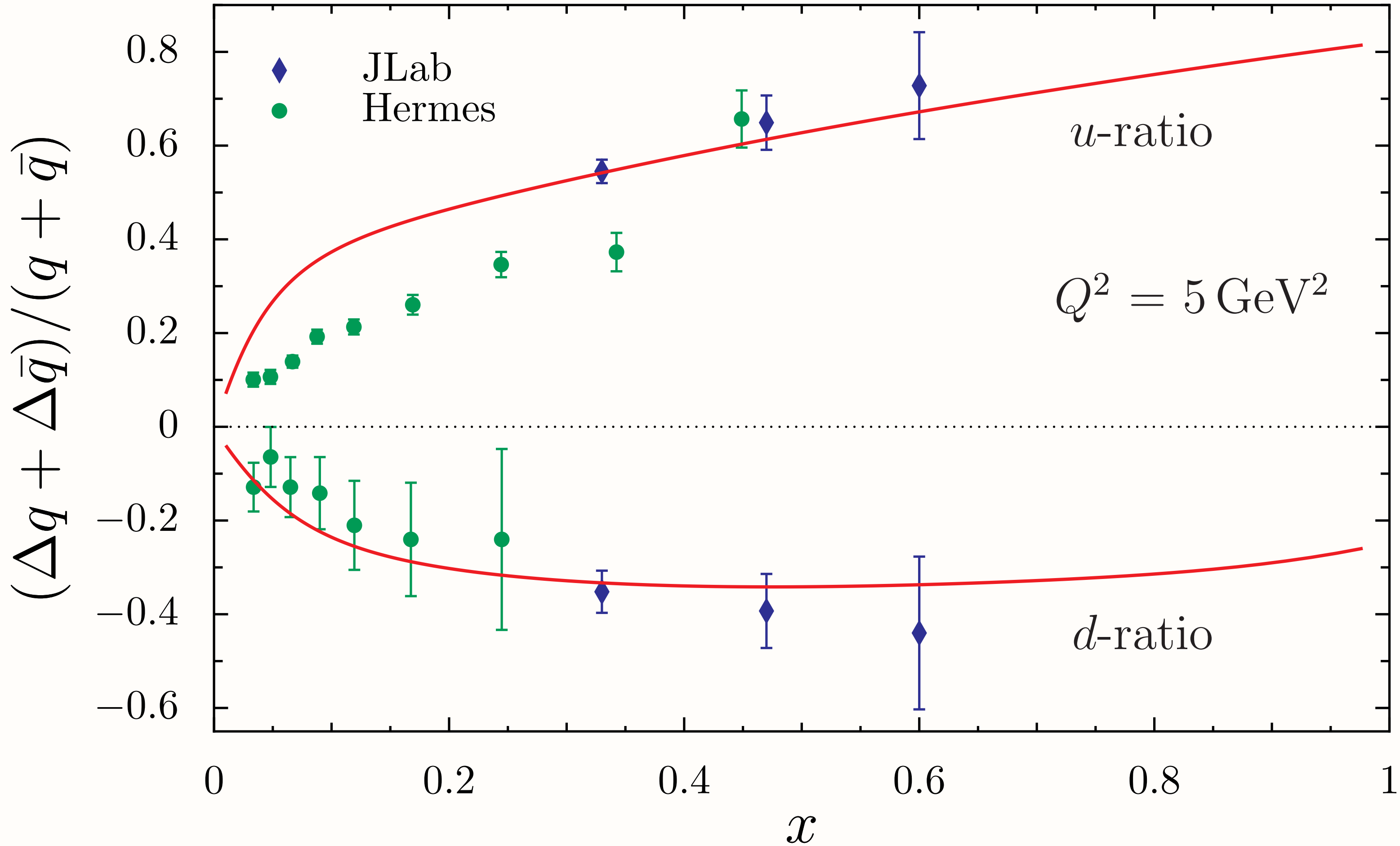
- There are three classes of predictions for this ratio
  - $SU(6)$ spin-flavour symmetry $\implies d(x)/u(x) \overset{x\to 1}{=} 1/2$
  - pQCD and helicity conservation $\implies d(x)/u(x) \overset{x\to 1}{=} 1/5$
  - scalar diquark dominance $\implies d(x)/u(x) \overset{x\to 1}{=} 0$





[Melnitchouk, et al.]

Perturbative QCD Predictions $x \to 1$



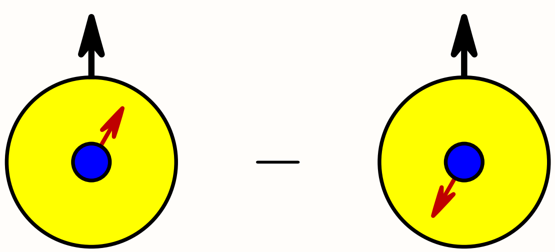
- The pQCD predictions for $\Delta q(x)/q(x)$ as $x \to 1$ are the most robust
- The pQCD prediction is: $\Delta q(x)/q(x) \stackrel{x\to 1}{=} 1$ for $q \in u,\, d$
- Realization would require a dramatic change in sign of $\Delta d(x)/d(x)$

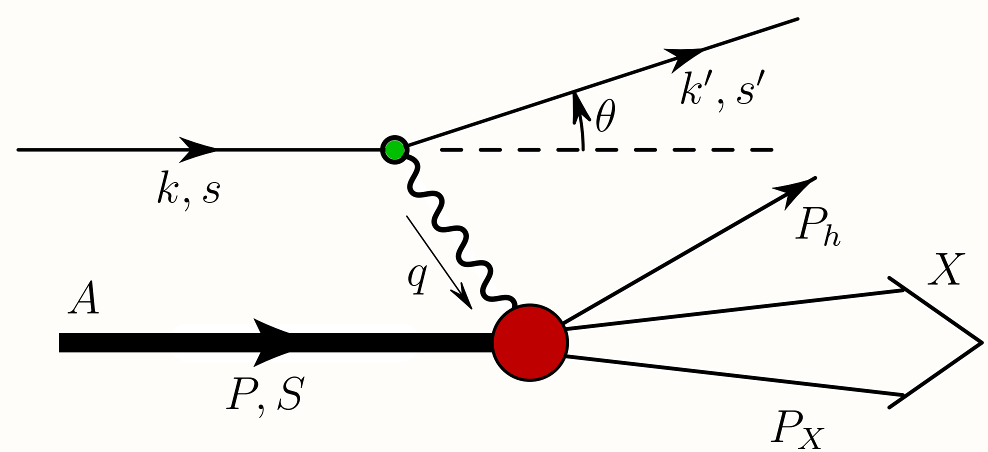
Transversity PDFs

- At leading twist there are three collinear PDFs
  - ✦ $q(x)$ – spin-independent
  - ✦ $\Delta q(x)$ – spin-dependent
  - ✦ $\Delta_T q(x)$ – transversity

- However transversity PDFs are chiral odd and therefore do not appear in deep inelastic scattering

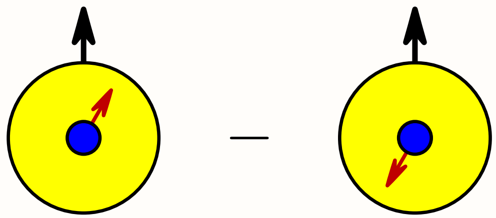
- Can be measured using semi-inclusive DIS on a transversely polarized target or certain Drell-Yan experiments

$\Delta_T q(x) =$ 



- Quarks in eigenstates of $\gamma^\perp \gamma_5$

Why is Transversity Interesting?

$\Delta_T q(x) =$ 

- Quarks in eigenstates of $\gamma^{\perp}\gamma_5$
- Tensor charge [c.f. Bjorken sum rule for g_A]

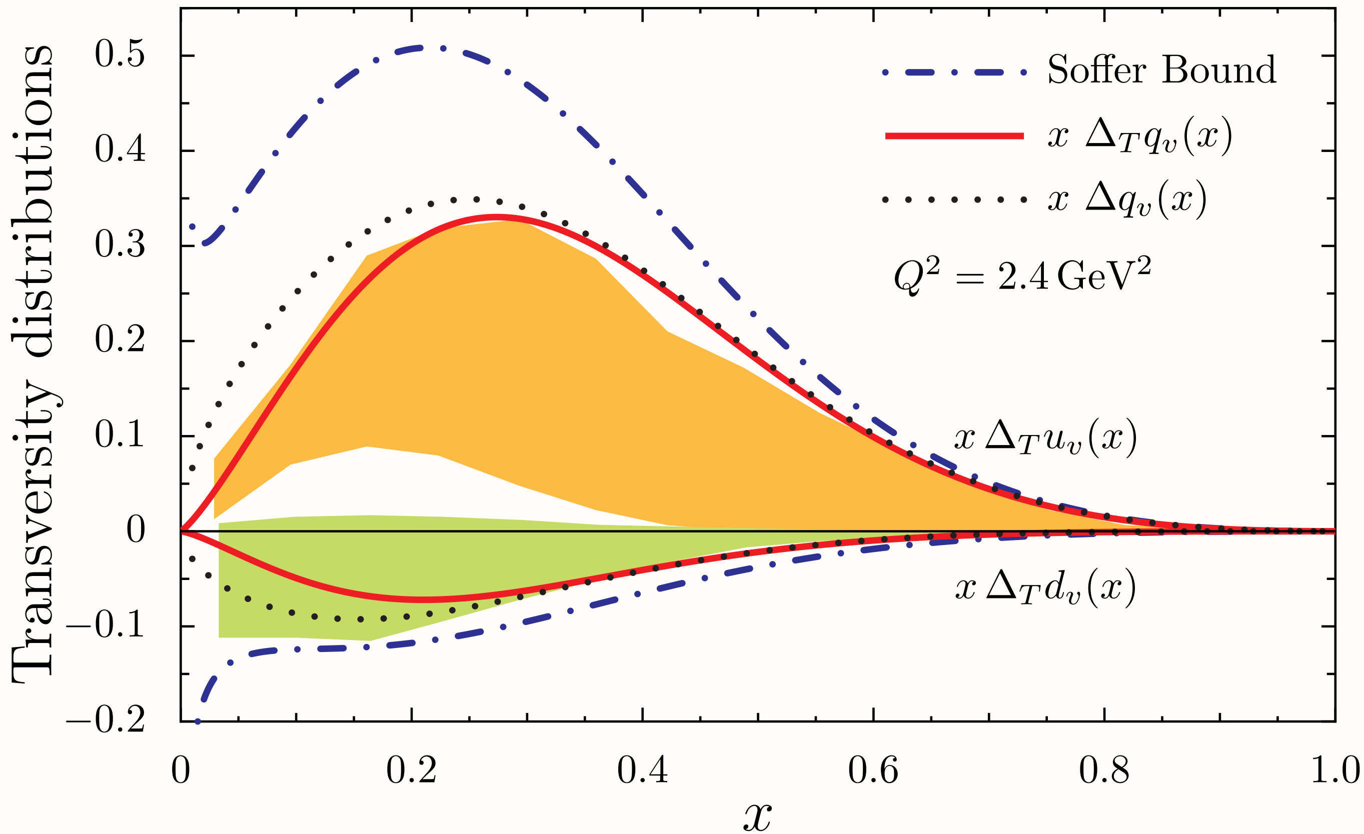
$$g_T = \int dx\,[\Delta_T u(x) - \Delta_T d(x)] \qquad g_A = \int dx\,[\Delta u(x) - \Delta d(x)]$$

- In non-relativistic limit: $\Delta_T q(x) = \Delta q(x)$
  - therefore $\Delta_T q(x)$ is a measure of relativistic effects
- Helicity conservation $\Longrightarrow$ no mixing between $\Delta_T q$ & $\Delta_T g$
- For $J \leqslant \frac{1}{2}$ we have $\Delta_T g(x) = 0$
- Therefore for the nucleon $\Delta_T q(x)$ is valence quark dominated
- *Important!!* A very common mistake – transverse spin sum:

$$\int dx\,\Delta_T q(x) = \langle \overline{\psi}_q \gamma^+ \gamma^1 \gamma_5 \psi_q \rangle \neq \langle \psi_q^\dagger \gamma^0 \gamma^1 \gamma_5 \psi_q \rangle = \Sigma_T^q$$

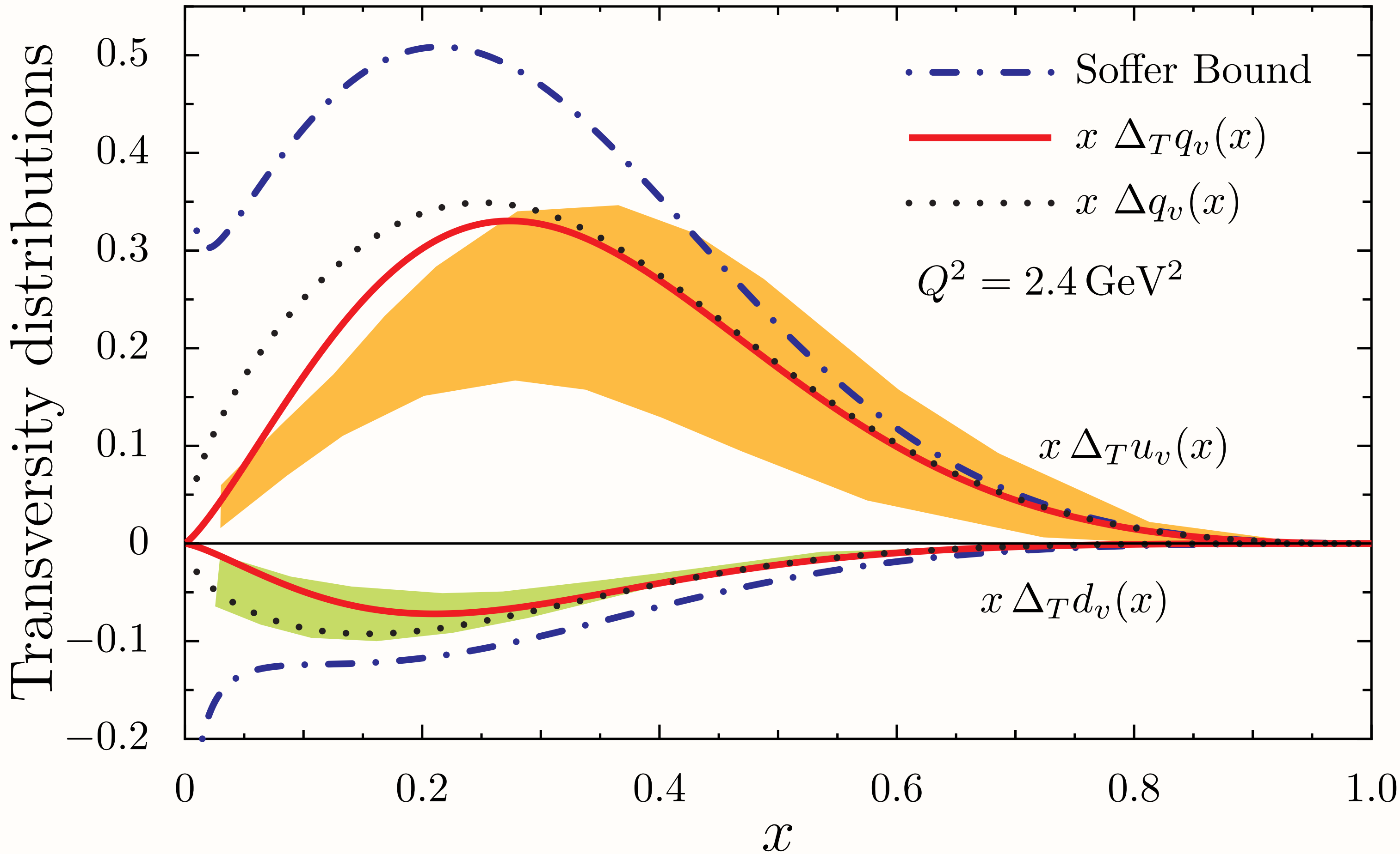
  - transversity moment $\neq$ spin quarks in transverse direction [c.f. $g_T(x)$]

$\Delta_T u_v(x)$ and $\Delta_T d_v(x)$ distributions



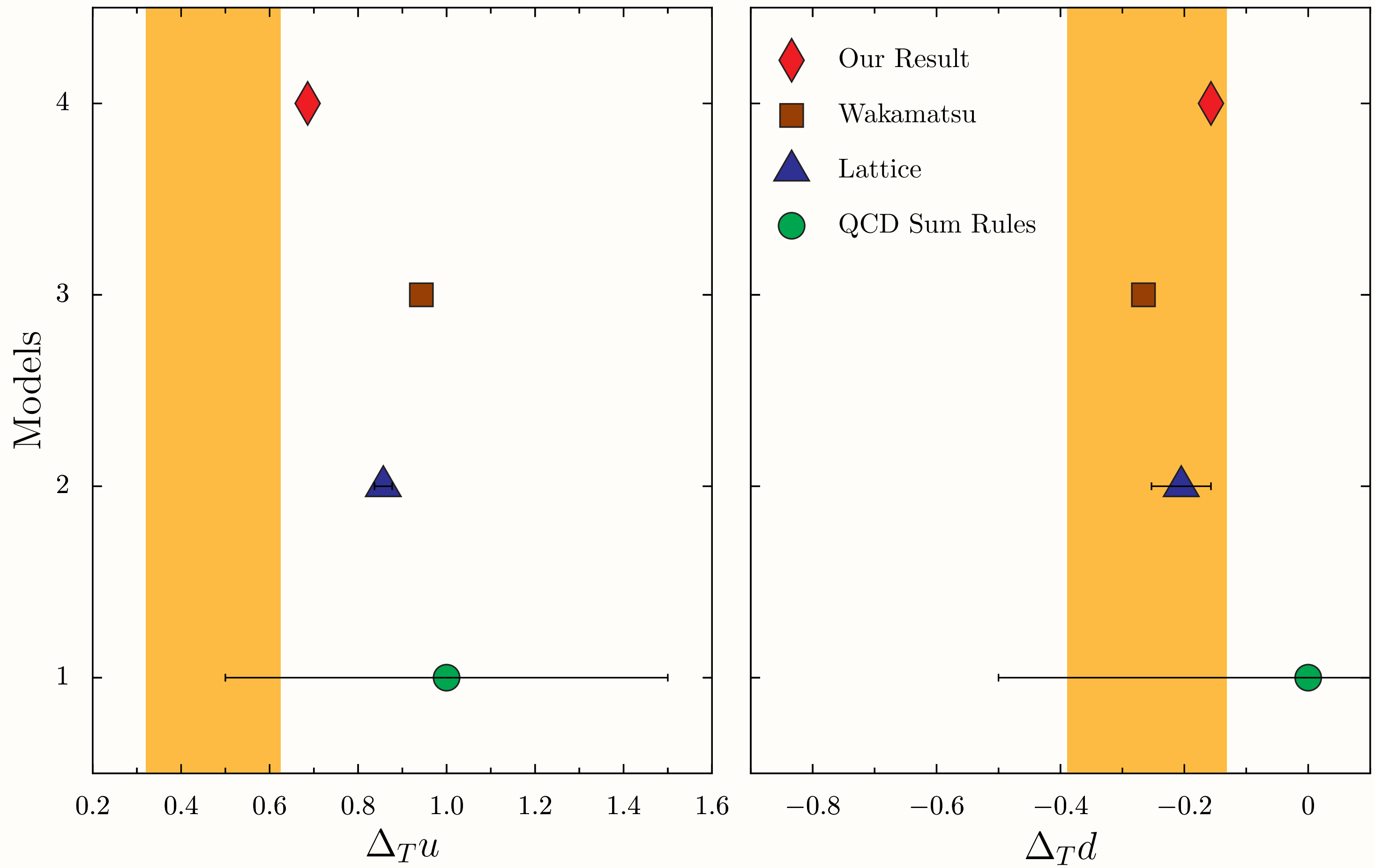
- Predict small relativistic corrections
- Empirical analysis *potentially* found large relativistic corrections
  - M. Anselmino *et. al.*,Phys. Rev. D **75**, 054032 (2007).
- Large effects difficult to support with quark mass ~ 0.4 GeV
  - maybe running quark mass is needed

Transversity: Reanalysis



- M. Anselmino *et al*, Nucl. Phys. Proc. Suppl. **191**, 98 (2009)
- Our results are now in better agreement updated distributions
- Concept of constituent quark models safe . . . for now

Transversity Moments

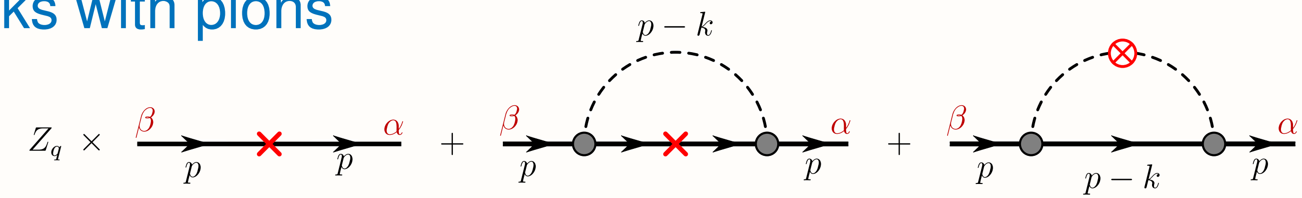


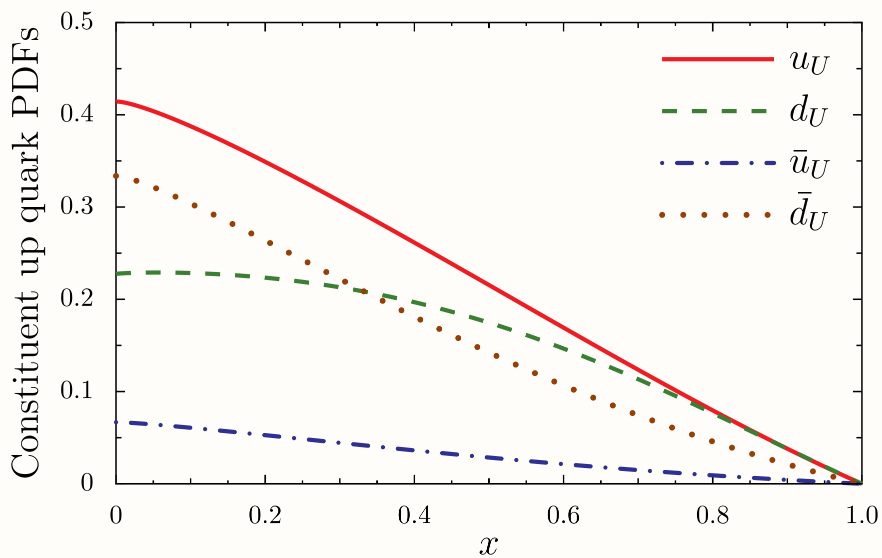
- M. Anselmino *et al*, Nucl. Phys. Proc. Suppl. **191**, 98 (2009)
- At model scale we find tensor charge

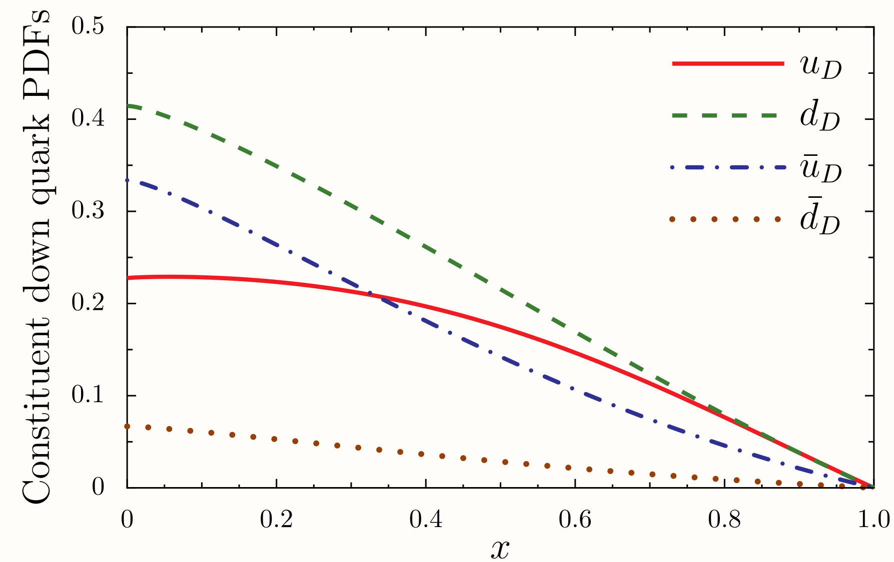
$g_T = 1.28$ compared with $g_A = 1.267$

Including Anti-quarks

- Dress quarks with pions







- Gottfried Sum Rule: NMC 1994: $S_G = 0.258 \pm 0.017\ [Q^2 = 4\,\text{GeV}^2]$

$$S_G = \int_0^1 \frac{dx}{x}\,[F_{2p}(x) - F_{2n}(x)] = \frac{1}{3} - \frac{2}{3}\int_0^1 dx\,\left[\bar{d}(x) - \bar{u}(x)\right]$$

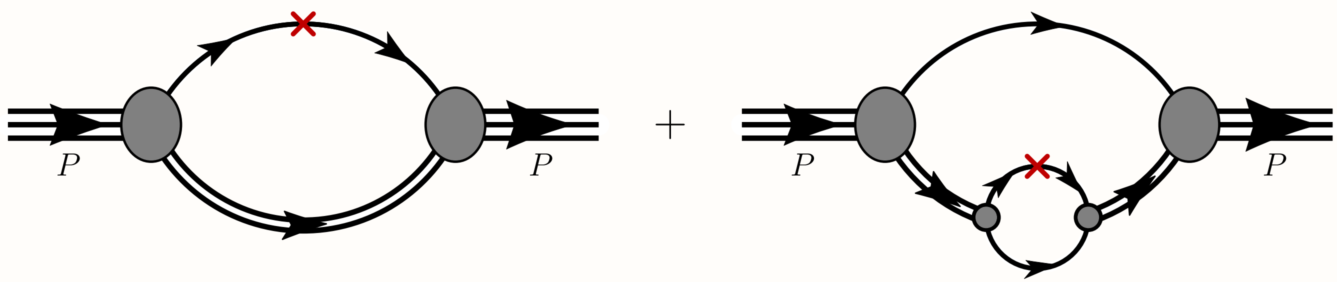
- We find: $S_G = \frac{1}{3} - \frac{4}{9}\,(1 - Z_q) = 0.252 \qquad [Z_q = 0.817]$

Spin Sum in NJL Model

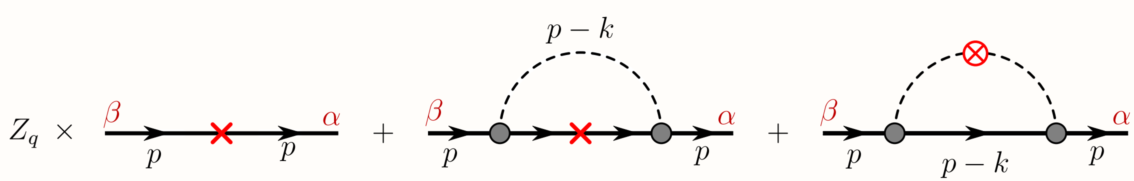
- Nucleon angular momentum must satisfy: $J = \frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$

$\Delta\Sigma = 0.33 \pm 0.03(stat.) \pm 0.05(syst.)$ [COMPASS & HERMES]

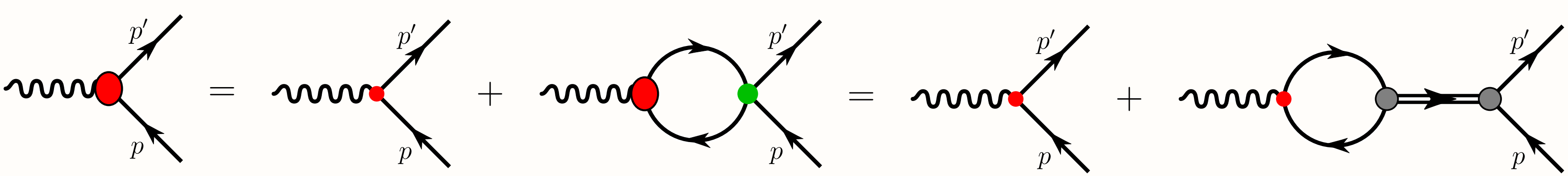
- Result from Faddeev calculation: $\Delta\Sigma = 0.66$



- Correction from pion cloud: $\Delta\Sigma = 0.79 \times 0.66 = 0.52$

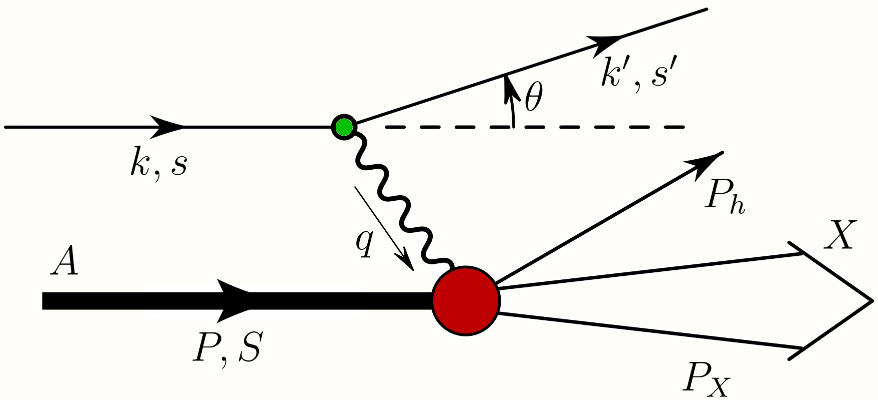


- Bare operator $\gamma^\mu\gamma_5$ gets renormalized: $\Delta\Sigma = 0.91 \times 0.52 = 0.47$



A 3D image of the nucleon – TMD PDFs

- Measured in semi-inclusive DIS



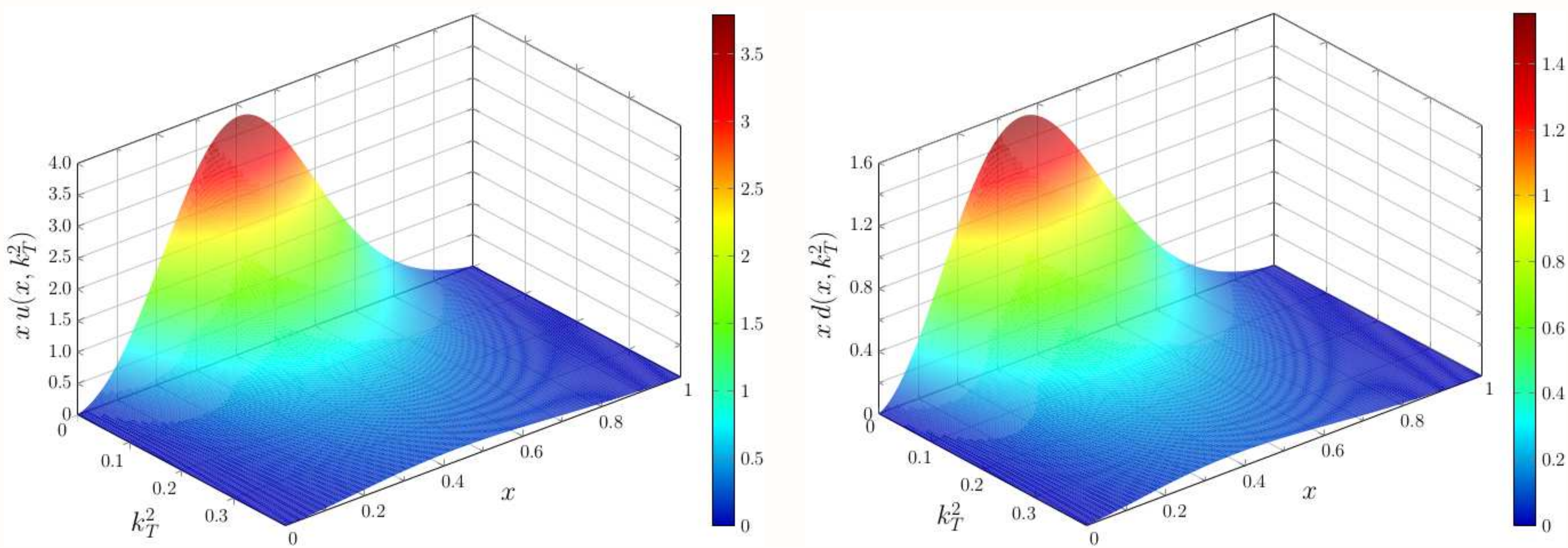
- Leading twist 6 T-even TMD PDFs

$$q(x,k_\perp^2),\quad \Delta q(x,k_\perp^2),\quad \Delta_T q(x,k_\perp^2)$$
$$g_{1T}^q(x,k_\perp^2),\quad h_{1L}^{\perp q}(x,k_\perp^2),\quad h_{1T}^{\perp q}(x,k_\perp^2)$$

- $$\langle p_T\rangle(x) \equiv \frac{\int d\vec{k}_\perp\, k_\perp\, q(x,k_\perp^2)}{\int d\vec{k}_\perp\, q(x,k_\perp^2)}$$

[H. Avakian, *et al.*, Phy. Rev. D81, 074035 (2010).]

- $\langle p_T\rangle^{Q^2=Q_0^2} = 0.36$ GeV c.f. $\langle p_T\rangle_{\text{Gauss}} = 0.56$ GeV [HERMES], 0.64 GeV [EMC]



[H. H. Matevosyan, ICC *et al.*, Phys. Rev. D **85**, 014021 (2012).]

NJL robust conclusions

- Diquark correlations in nucleon are very important
  - ✦ $d(x)$ is softer than $u(x)$ $\iff$ scalar diquark $(ud)_{0+}$
  - ✦ $d(x)/u(x) \overset{x\to 1}{\simeq} 0.2$ – sensitive to strength of axial-vector diquark

- Can *almost* reproduce measured spin sum: $\Delta\Sigma = 0.366^{+0.042}_{-0.062}$ [DSSV]
  - ✦ relativistic effects + pions + vertex renormalization $\Longrightarrow \Delta\Sigma = 0.47$
  - ✦ perturbative gluon dressing on quarks will reduce $\Delta\Sigma$ further

- Perturbative pions $\Rightarrow$ Gottfried Sum Rule: [$S_G = 0.258 \pm 0.017$ ($Q^2 = 4$ GeV^2) – NMC 1994]

$$S_G = \int_0^1 \frac{dx}{x}\left[F_{2p}(x) - F_{2n}(x)\right] = \frac{1}{3} - \frac{2}{3}\int_0^1 dx\left[\bar{d}(x) - \bar{u}(x)\right] \overset{NJL}{\to} \frac{1}{3} - \frac{4}{9}\left(1 - Z_q\right) = 0.252$$

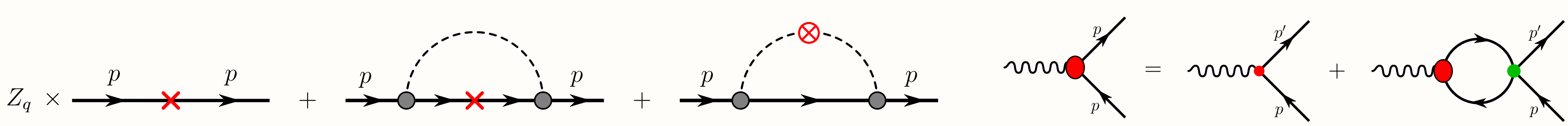


Table of Contents