Lecture 2

Relativistic Faddeev equation & Electromagnetic Form Factors

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Non-perturbative Methods in Quantum Field Theory

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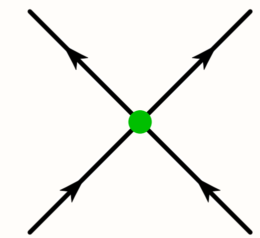
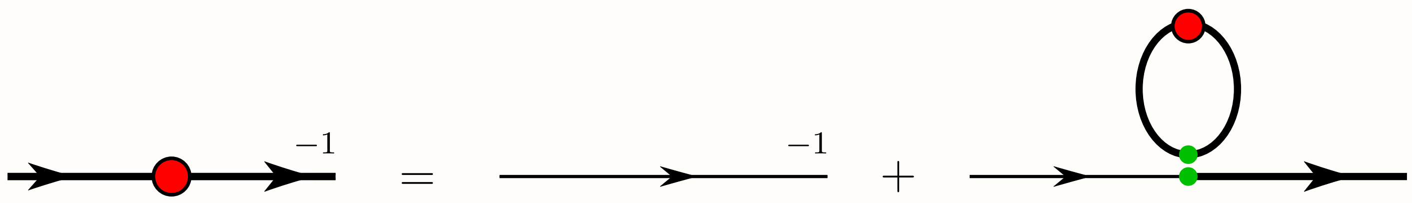
Recap

- NJL Lagrangian has the form

$$\mathcal{L}_{NJL} = \mathcal{L}_0 + \mathcal{L}_I = \overline{\psi}\left(i\,\partial\!\!\!/ - m\right)\psi + \sum_\alpha G_\alpha \left(\overline{\psi}\,\Gamma_\alpha\,\psi\right)^2$$

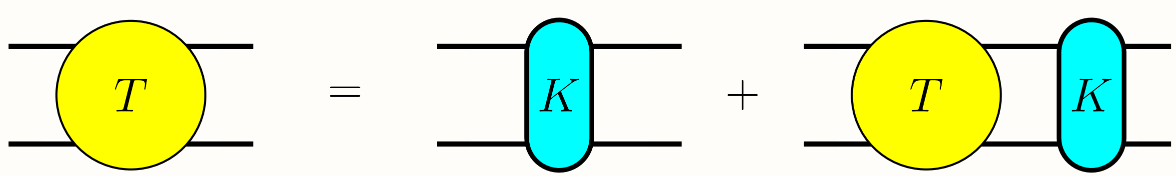


- Solution to the gap equation gives quark propagator



$$S_0(k) = [k\!\!\!/ - m + i\varepsilon]^{-1} \xrightarrow{DCSB} S(k) = [k\!\!\!/ - M + i\varepsilon]^{-1}$$

- Meson masses are obtained as poles in the two-body T-matrix



  - Bethe-Salpeter Equation
  - for the pion we obtain: $\mathcal{T}(q)^i_{\alpha\beta,\gamma\delta} = (\gamma_5\tau_i)_{\alpha\beta}\, \frac{-2i\,G_\pi}{1+2\,G_\pi\,\Pi_\pi(q^2)}\, (\gamma_5\tau_i)_{\gamma\delta}$

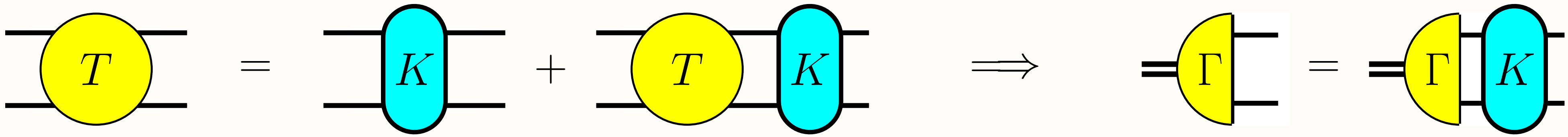
Chiral Partners

- If chiral symmetry was *NOT* dynamically broken in nature expect mass degenerate chiral partners, e.g., $m_\sigma \simeq m_\pi \quad \& \quad m_{a_1} \simeq m_\rho$
- The ρ and a_1 are the lowest lying vector ($J^P = 1^-$) and axial-vector ($J^P = 1^+$) $\bar{q}q$ bound states: $m_\rho^{\text{exp't}} \simeq 770\,\text{MeV}$ & $m_{a_1}^{\text{exp't}} \simeq 1260\,\text{MeV}$
- Solving the NJL BSE gives the following pole conditions:

$$1 + 2\,G_\rho\,\Pi_\rho(q^2 = m_\rho^2) = 0 \qquad \& \qquad 1 + 2\,G_\rho\,\Pi_{a_1}(q^2 = m_{a_1}^2) = 0$$

  - where $\Pi_{a_1}(q^2) = M^2\,I(q^2) + \Pi_\rho(q^2)$
- If $m = 0$ and there is *NO* DCSB ($M = 0$) would have: $m_\rho = m_{a_1}$
- In nature and NJL, DCSB splits chiral partner masses
  - NJL gives: $m_\rho \equiv 770\,\text{MeV} \quad \& \quad m_{a_1} \simeq 1098\,\text{MeV}$
  - good agreement with the Weinberg sum rule result: $m_{a_1} \simeq \sqrt{2}\,m_\rho$
- NJL BSE pole conditions for π and σ $\implies$ $m_\sigma^2 \simeq m_\pi^2 + 4\,M^2$

Homogeneous Bethe-Salpeter vertex functions



- Near a bound state pole of mass m a two-body T-matrix behaves as

$$\mathcal{T}(p,k) \to \frac{i\,\Gamma(p,k)\,\bar{\Gamma}(p,k)}{p^2-m^2} \qquad \text{where} \qquad p=p_1+p_2, \quad k=p_1-p_2$$

- $\Gamma(p,k)$ *is the homogeneous Bethe-Salpeter vertex & describes relative motion of the quark and anti-quark while they form the bound state*

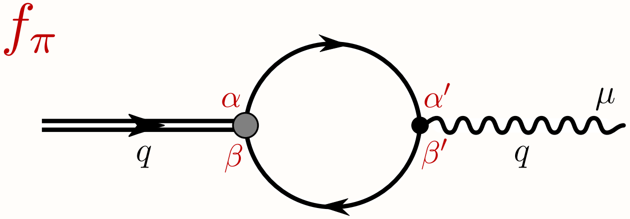
- Expanding the pion T-matrix about the pole gives

$$\mathcal{T} = \gamma_5\tau_i\,\frac{-2i\,G_\pi}{1+2\,G_\pi\,\Pi_\pi(q^2)}\,\gamma_5\tau_i \to \frac{i\,g_{\pi qq}}{q^2-m_\pi^2}\,(\gamma_5\tau_i)\,(\gamma_5\tau_i) \quad\Longrightarrow\quad \Gamma_\pi = \sqrt{g_{\pi qq}}\,\gamma_5\tau_i$$

  - ✦ $g_{\pi qq}$ is effective pion-quark coupling constant

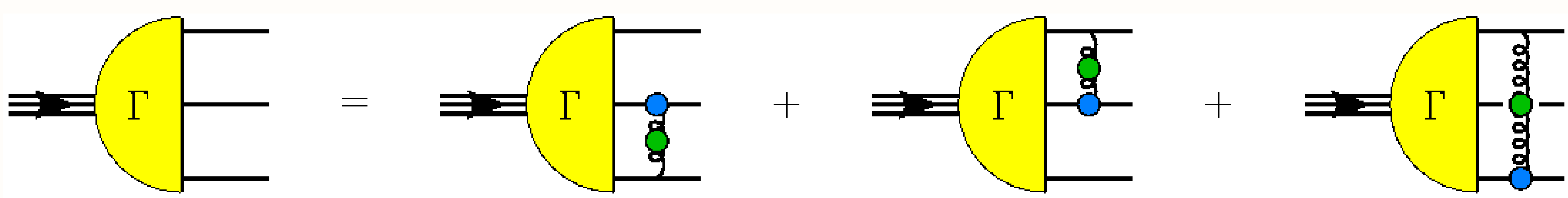
- Bethe-Salpeter vertex needed for calculations e.g. f_π

$$i\,f_\pi\,q^\mu\,\delta_{ij} = \int\frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\left[\tfrac{1}{2}\,\gamma^\mu\gamma_5\tau_j\,S(k)\,\Gamma_\pi^i\,S(k-q)\right]$$

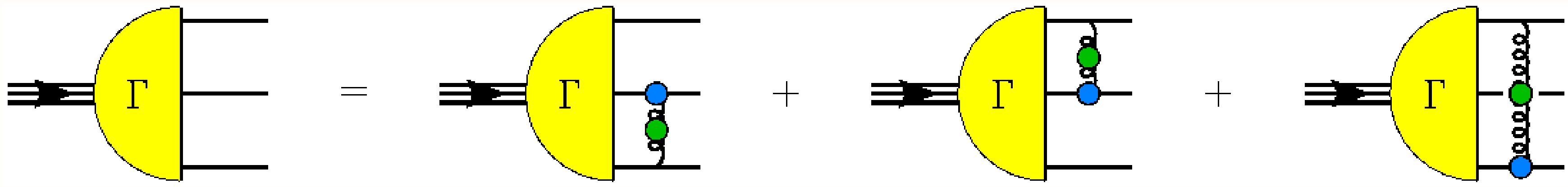


Baryons in the QFT

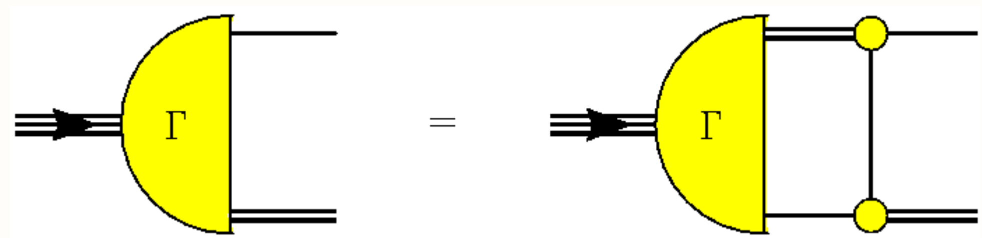
- Baryons are 3-quark bound states – with the proton (uud) and neutron (udd) being the most important examples
- In quantum field theory physical baryons appear as poles in six-point Green functions
  - recall that two-body bound states appear as poles in four-point Green functions, where solutions are obtained by solving Bethe-Salpeter equation
- The analogue of the Bethe-Salpeter equation for 3-quark bound states is called the Faddeev equation
- Faddeev kernel usually only contains two-body interactions
  - this is an approximation which is yet to be explored and could have important consequences for QCD
- Diagrammatically the homogeneous Faddeev equation is given by



Baryons in the QFT (2)



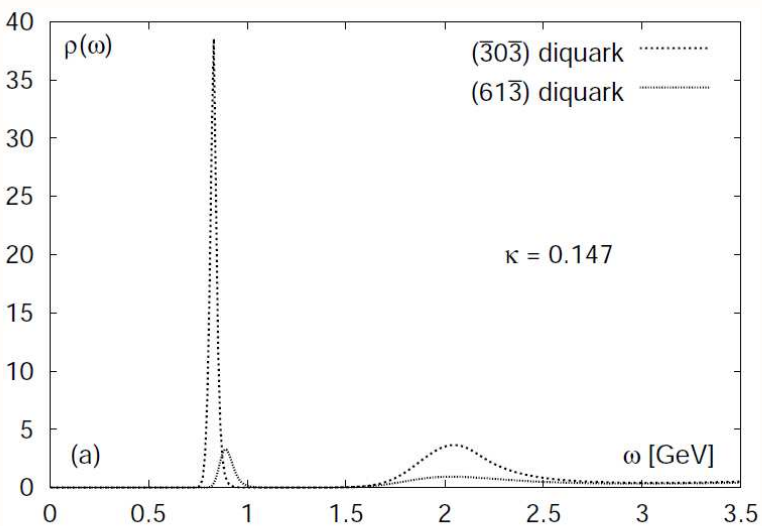
- Problem is rendered tractable by making the quark-diquark approximation



- This is a linear matrix equation, whose solution gives the "baryon wavefunction" – strictly the Poincaré covariant Faddeev amplitude
- Include scalar ($J^P = 0^+, T = 0$) and axial-vector ($J^P = 1^+, T = 1$) diquarks
  - ✦ in the non-relativistic limit parity dictates that pseudoscalar and vector diquarks must be in a $\ell = 1$ state and are therefore suppressed in the nucleon
  - ✦ for the negative parity $N^*(1535)$ the opposite is true
- The nucleon wavefunction contains S, P and D wave correlations
- Equation has discrete solutions at $p^2 = m_i^2$ – nucleon, Roper, etc

What is a Diquark

- A diquark is a correlated (interacting) quark-quark state
- Diquark interactions occur in colour $\bar{3}$ or colour 6 channels – only the colour $\bar{3}$ can exist inside a colour singlet nucleon
- Diquarks are analogous to mesons – colour singlet $\bar{q}q$ bound states
- Because diquarks are coloured they should not appear as physical states in QCD $\Longleftrightarrow$ confinement
- However in the NJL model and also the rainbow ladder approximation to QCDs DSE, diquarks do appear as poles in the qq scattering (t) matrix
- Lattice QCD also sees evidence for diquarks



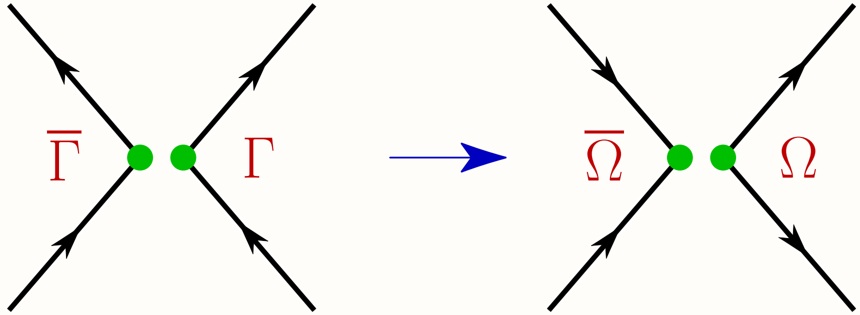
- I. Wetzorke, F. Karsch, hep-lat/0008008
- $(\bar{3}0\bar{3})$ implies scalar diquark: (flavour-$\bar{3}$, spin-0, colour-$\bar{3}$)
- $(60\bar{3})$ implies axial-vector diquark: (flavour-6, spin-0, colour-$\bar{3}$)

Diquarks in the NJL model

- To describe diquarks in the NJL model one usually rewrites the $\bar{q}q$ interaction Lagrangian into a qq interaction Lagrangian

$$\left(\bar{\psi}\,\Gamma\,\psi\right)^2 \to \left(\bar{\psi}\,\Omega\,\bar{\psi}^T\right)\left(\psi^T\,\bar{\Omega}\,\psi\right)$$

  - ✦ Ω has quantum numbers if interaction channel



- NJL qq Lagrangian in the scalar and axial-vector diquark channels reads

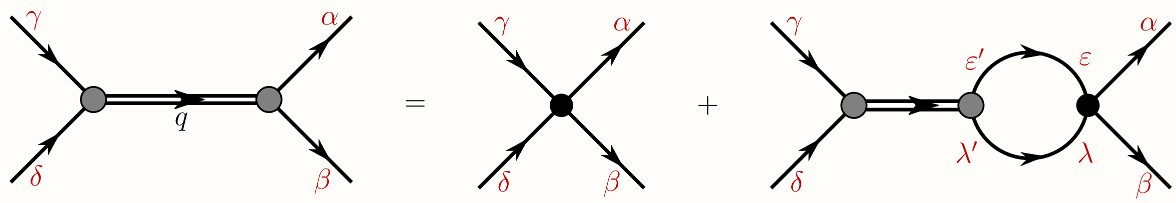
$$\begin{aligned}\mathcal{L}_I = G_s &\left[\overline{\psi}\,\gamma_5\,C\,\tau_2\,\beta^A\,\overline{\psi}^T\right]\left[\psi^T\,C^{-1}\,\gamma_5\,\tau_2\,\beta^{A'}\,\psi\right]\\ &+ G_a\left[\overline{\psi}\,\gamma_\mu\,C\,\tau_i\tau_2\,\beta^A\,\overline{\psi}^T\right]\left[\psi^T\,C^{-1}\,\gamma^\mu\,\tau_2\tau_j\,\beta^{A'}\,\psi\right] + \ldots.\end{aligned}$$

  - ✦ the first term is the scalar diquark channel ($J^P = 0^+, T = 0$)
  - ✦ τ_2 couples isospin of two quarks to $T = 0$, $C\gamma_5$ couples spin to $J = 0$, $\beta^A = \sqrt{\frac{3}{2}}\,\lambda^A \quad (A = 2, 5, 7)$ couples quarks to colour $\bar{3}$
  - ✦ the second the axial-vector diquark channel ($J^P = 1^+, T = 1$)

NJL diquark T-matrices

- Bethe-Salpeter equation for qq scattering matrix reads

$$\mathcal{T}(q)_{\alpha\beta,\gamma\delta} = K_{\alpha\beta,\gamma\delta} + \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\, K_{\alpha\beta,\varepsilon\lambda}\, S(k)_{\varepsilon\varepsilon'}\, S(q-k)_{\lambda\lambda'}\, \mathcal{T}(q)_{\varepsilon'\lambda',\gamma\delta},$$



  - ✦ note symmetry factor of $\frac{1}{2}$ (c.f. $\bar{q}q$ BSE)

- The Feynman rules for the interaction kernels are

$$\mathcal{K}_s = 4i\,G_s\left(\gamma_5\, C\,\tau_2\,\beta^A\right)_{\alpha\beta}\left(C^{-1}\,\gamma_5\,\tau_2\,\beta^A\right)_{\gamma\delta} \qquad \mathcal{K}_a = 4i\,G_a\left(\gamma_\mu\, C\,\tau_i\tau_2\,\beta^A\right)_{\alpha\beta}\left(C^{-1}\,\gamma^\mu\,\tau_2\tau_i\,\beta^A\right)_{\gamma\delta}$$

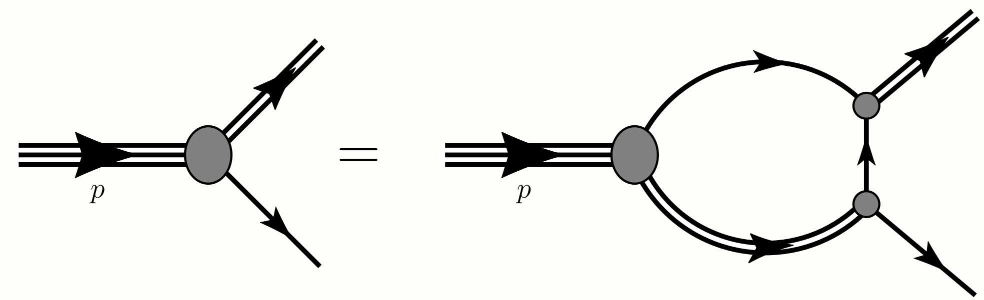
- The solution to the BSE is of the form: $\mathcal{T}(q)_{\alpha\beta,\gamma\delta} = \tau(q^2)\,\Omega_{\alpha\beta}\bar{\Omega}_{\gamma\delta}$

$$\tau_s(q^2) = \frac{4i\,G_s}{1+2\,G_s\,\Pi_s(q^2)} \qquad \tau_a^{\mu\nu}(q) = \frac{4\,i\,G_a}{1+2\,G_a\,\Pi_a(q^2)}\left[g^{\mu\nu} + 2\,G_a\,\Pi_a(q^2)\,\frac{q^\mu q^\nu}{q^2}\right]$$

  - ✦ these reduced t-matrices are the diquark propagators

NJL Faddeev Equation

- To describe nucleon Faddeev equation kernel must be projected onto colour singlet, spin one-half, isospin one-half & positive parity



- Make the "static approximation" to quark exchange kernel: $S(p) \to -\frac{1}{M}$
- Homogeneous Faddeev amplitude with static approximation does not depend of relative momentum between the quark and diquark
- The Faddeev equation and vertex have the form

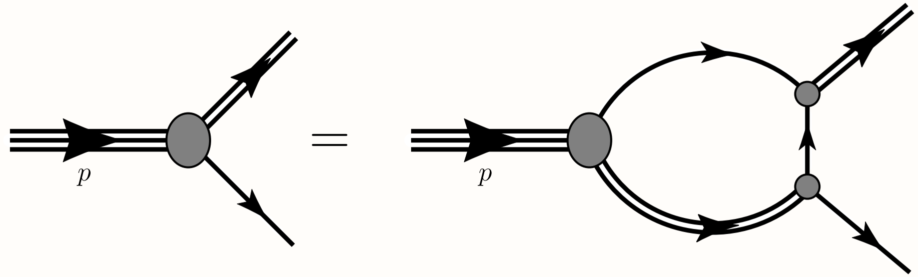
$$\Gamma_N(p,s) = K(p)\,\Gamma_N(p,s)$$

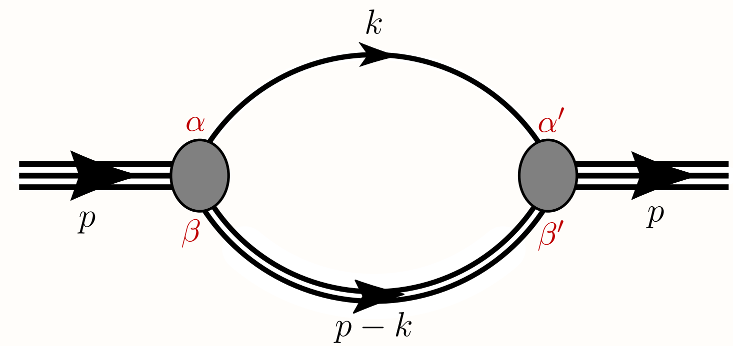
$$\Gamma_N(p,s) = \sqrt{-Z_N \frac{M_N}{p_0}}\left[\overset{\alpha_1}{\alpha_2 \frac{p^\mu}{M_N}\gamma_5} + \alpha_3\,\gamma^\mu\gamma_5\right] u_N(p,s)$$

  - ✦ $K(p)$ is the Faddeev kernel

- Faddeev equation describes the continual recombination of the three quarks into quark-diquark configurations

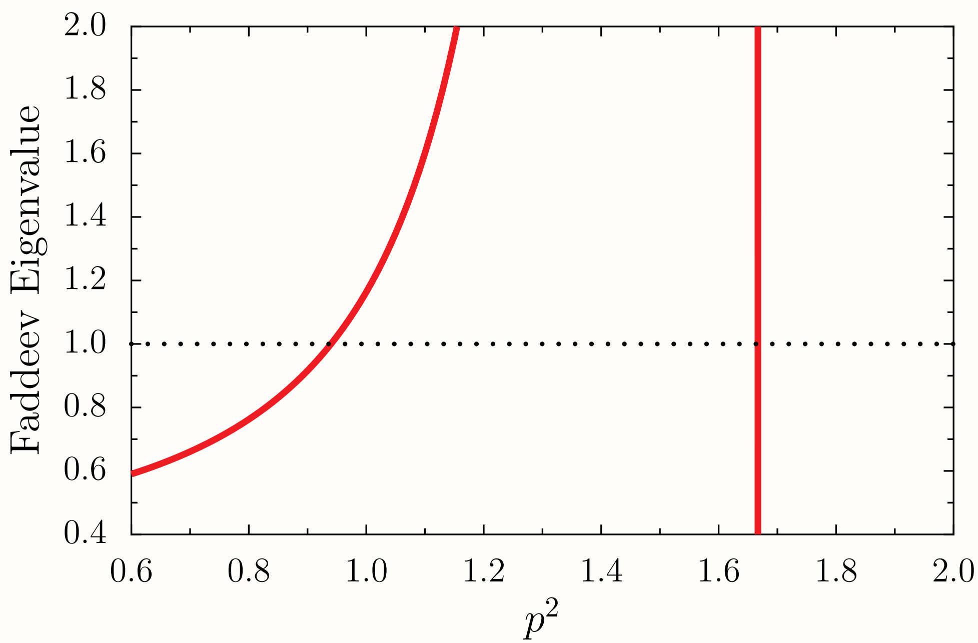
NJL Faddeev Equation (2)





- The kernel of this NJL Faddeev eq – $\Gamma_N(p,s) = K(p)\,\Gamma_N(p,s)$ – is

$$\begin{bmatrix} \Gamma_s \\ \Gamma_a^\mu \end{bmatrix} = \frac{3}{M} \begin{bmatrix} \Pi_{Ns} & \sqrt{3}\gamma_\alpha\gamma_5\,\Pi_{Na}^{\alpha\beta} \\ \sqrt{3}\gamma_5\gamma^\mu\,\Pi_{Ns} & -\gamma_\alpha\gamma^\mu\,\Pi_{Na}^{\alpha\beta} \end{bmatrix} \begin{bmatrix} \Gamma_s \\ \Gamma_{a,\beta} \end{bmatrix}$$



- First solution is the nucleon $M_N = 940$ MeV
- Second solution is 1st excited state of the nucleon $\Longleftrightarrow$ Roper $M_{\text{Roper}} = 1670$ MeV

Nucleon Static Properties

- If the proton was a point particle its electromagnetic properties would be characterized by two observables

$$\text{charge: } e_p = +1 \quad \& \quad \text{magnetic moment } (\mu_p)$$

- In 1933 Otto Stern measured the proton magnetic moment and found that it differed from one $\Longleftrightarrow$ anomalous magnetic moment

$$\text{Dirac: } \mu_p = \frac{e_p\,\hbar}{2\,M_P} \qquad\qquad \text{Stern: } \mu_p = (1 + 1.79)\,\frac{e_p\,\hbar}{2\,M_P}$$

  - ✦ this was strong evidence that the proton was not a point particle
  - ✦ later of course quarks were discovered at SLAC in 1968 via deep inelastic experiments

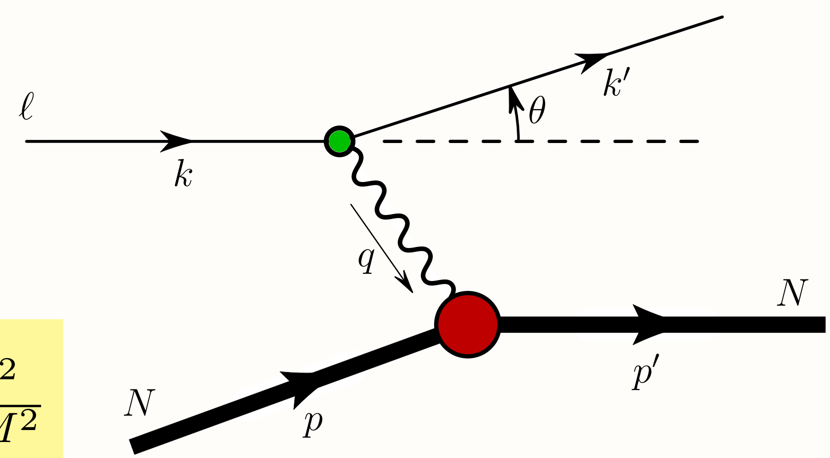
- In 1943 Otto Stern would receive the Nobel Prize in part for this discovery

Nucleon electromagnetic form factors

- The electromagnetic structure of the nucleon is best determined by electron elastic scattering
- The electron makes a good probe because its interaction with the electromagnetic current is very well understood
  - ✦ the electron anomalous magnetic moment is known experimentally to 1 part in a trillion $a = 0.00115965218085(76)$
  - ✦ theory agrees almost perfectly with experiment
- The interaction of the electromagnetic with the nucleon is characterized by two form factors

$$\langle J^\mu \rangle = u(p')\left[\gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2\,M} F_2(Q^2)\right] u(p)$$

Dirac *Pauli*

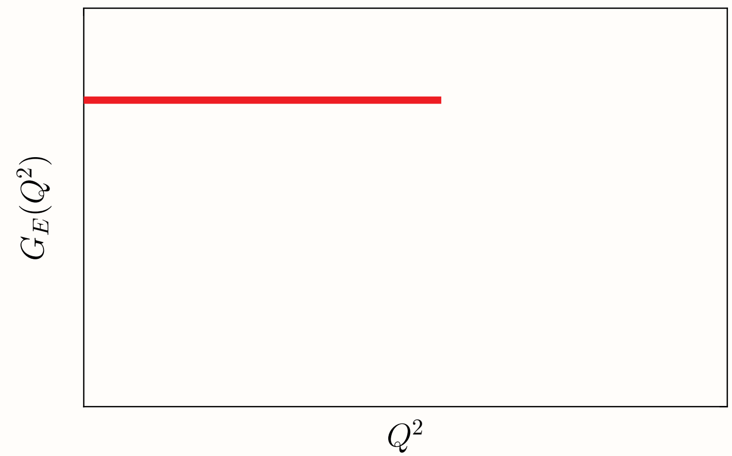


$$\frac{d\sigma}{d\Omega} = \frac{\sigma_{\text{Mott}}}{1+\tau}\left[G_E^2(Q^2) + \frac{\tau}{\varepsilon} G_M^2(Q^2)\right]; \quad \tau = \frac{Q^2}{4\,M^2}$$

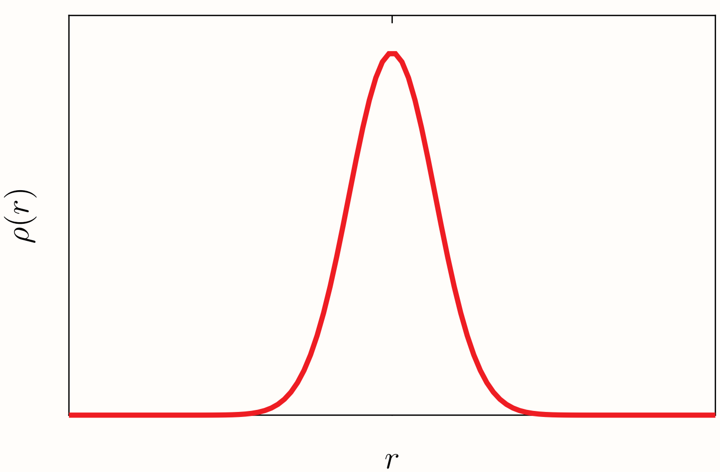
- Sachs form factors: $G_E = F_1 - \frac{Q^2}{4\,M^2} F_2, \quad G_M = F_1 + F_2$

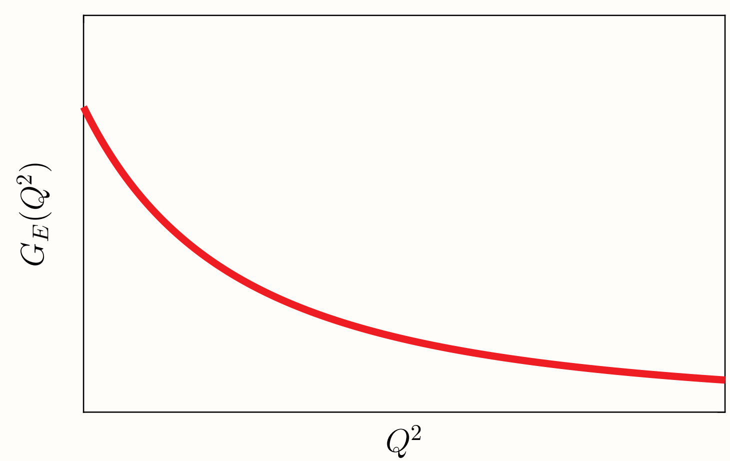
Physical Interpretation of Form Factors

- $G_E(0) = F_1(0) =$ charge, $\quad G_M(0) = F_1(0) + F_2(0) =$ magnetic moment
- Textbooks teach that in the Breit frame – $\vec{p}\,' = -\vec{p}$ – Sachs form factors can be interpreted as 3-d Fourier transforms of the charge and magnetization densities



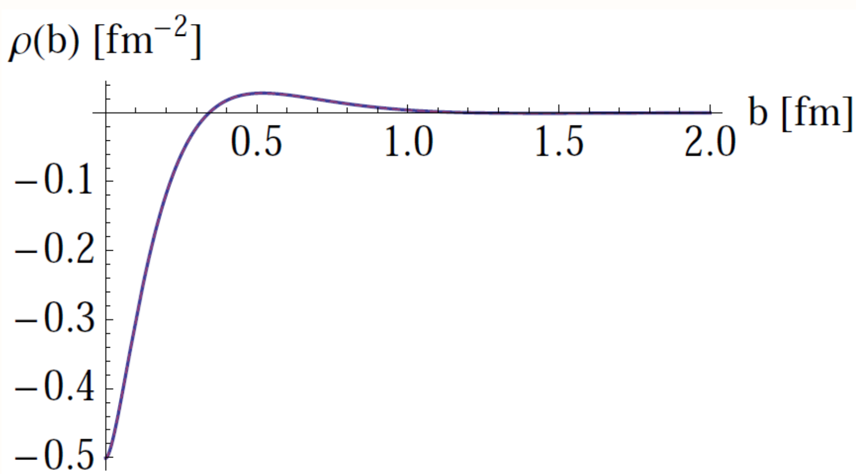
- Deviation from a constant provides information on target structure



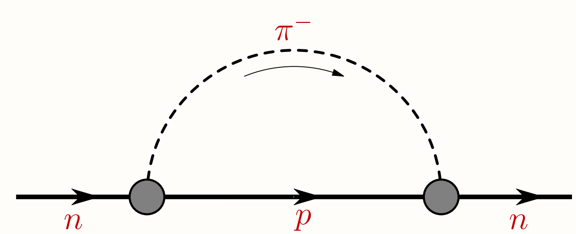


Physical Interpretation of Form Factors (2)

- There maybe problems with the interpretation of the Sachs form factors as 3-d Fourier transforms of charge and magnetization densities
  - non-relativistically Sachs form factors are FTs of rest frame densities; initial and final states are essentially the same ($M \to \infty$).
  - in relativistic QFT initial and final states are different – as $p' \neq p$ – therefore a density cannot be defined; states are not easily related by Lorentz boosts
  - also infinite number of Breit frames, one for each Q^2
- New interpretation: form factors provide information on the IMF transverse densities – transverse structure invariant under z-direction boosts
  - transverse charge densities are given by 2-d Fourier transforms of the Dirac and Pauli form factors

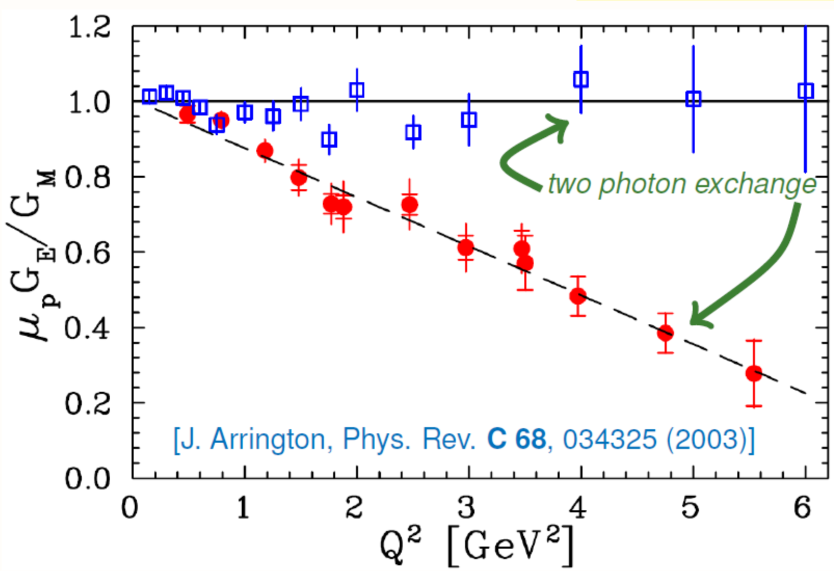


- Neutron negative central charge density contradicts pion cloud picture



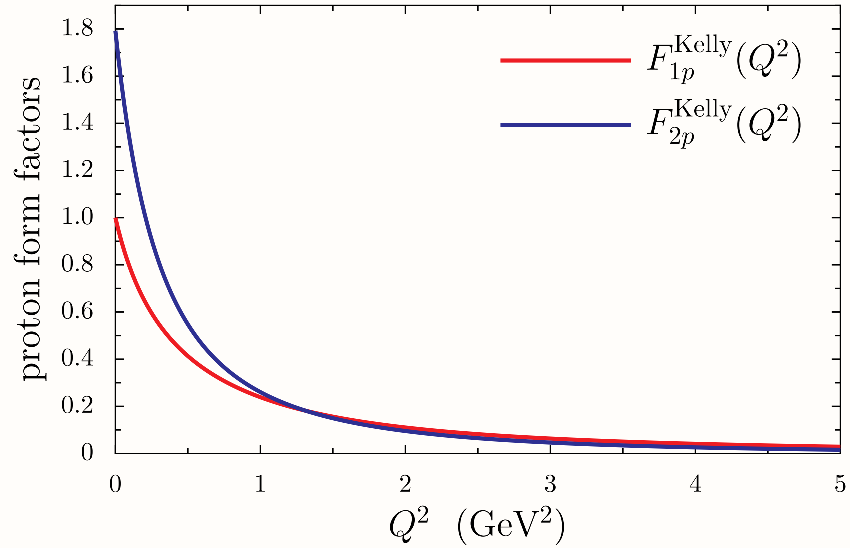
Experimental Status

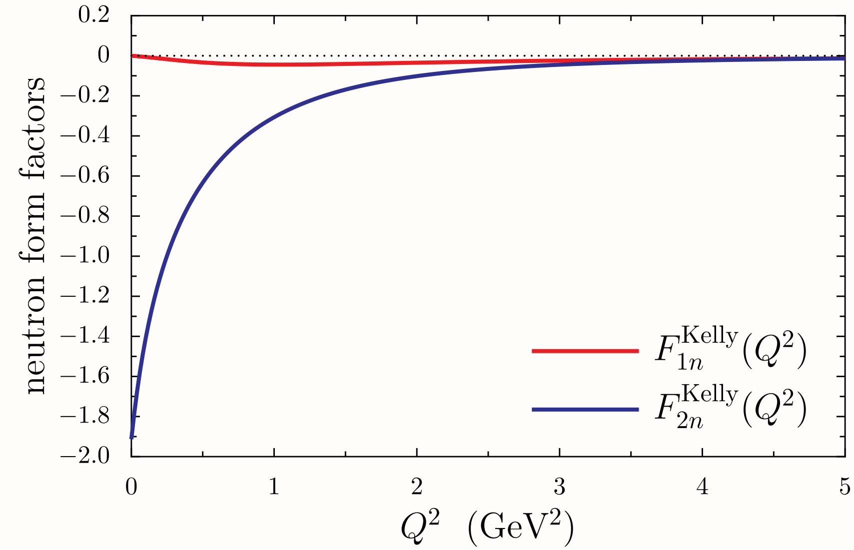
- Proton form factors were first measured by Hofstadter *et al.* in 1953
  - deviation from constant gives information on nucleon structure e.g. radii
- Many new things are still being learnt about nucleon EM structure
- A recent atomic experiment discovered the "Proton Radius puzzle"
  - $r_{Ep} = 0.84184 \pm 0.00067$ fm   muonic hydrogen [Pohl *et al.*]
  - $r_{Ep} = 0.8768 \pm 0.0069$ fm ep elastic scattering & hydrogen [PDG]
  - radius is defined by: $\langle r_E^2 \rangle = -6 \frac{\partial}{\partial Q^2} G_E(Q^2)\Big|_{Q^2=0}$

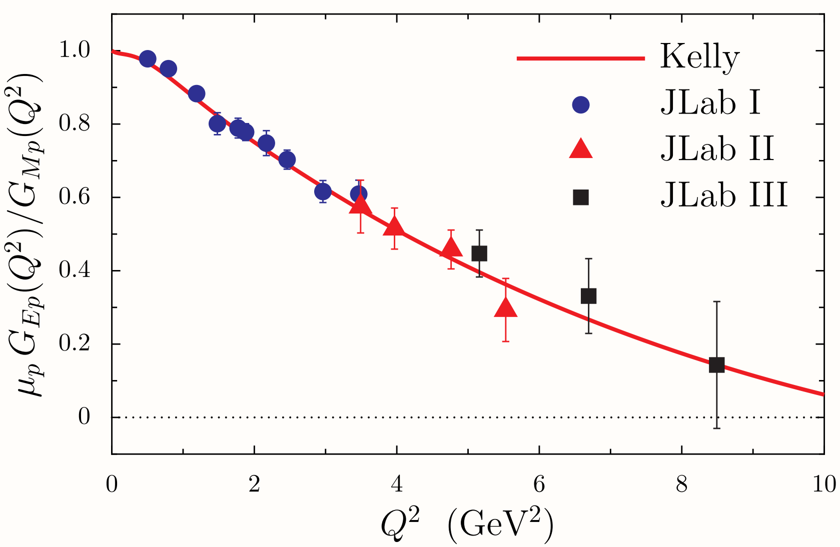


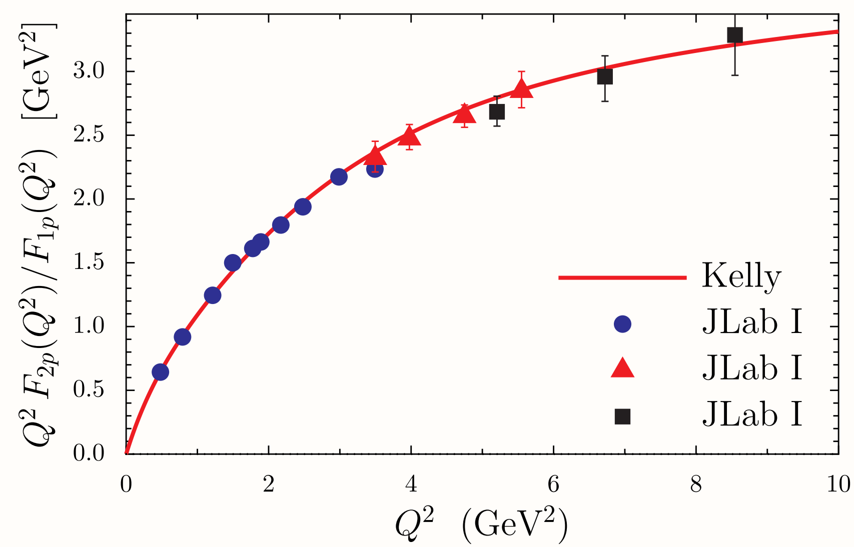
- Until the late 90s Rosenbulth experiments found that the G_{Ep}/G_{Mp} ratio was flat
- However JLab polarization transfer experiments which are directly sensitive to this ratio, found a slope toward zero

Experimental Status (2)





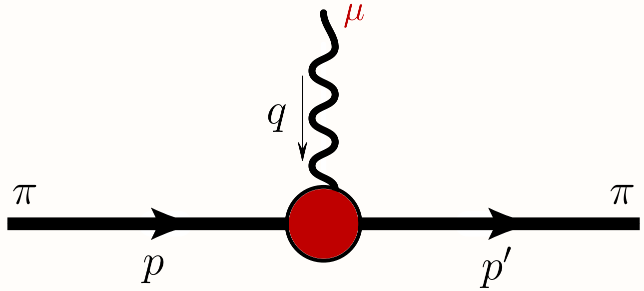




- pQCD $F_1 \sim 1/Q^4 \qquad F_2 \sim 1/Q^6 \qquad \Longrightarrow \qquad Q^2\, F_2/F_1 \sim$ constant
  - ✦ this behaviour is not seen in the data yet: $Q\, F_2/F_1 \sim$ constant

Deriving a general form for a Photon-Hadron Vertex

- Deriving the most general form of a photon-hadron vertex:
  - write down the most general structure consistent with Lorentz covariance
  - multiply each Lorentz structure by a scalar function
  - used symmetries to derive constrains on these functions
  - most importantly use CPT invariance and Ward-Takahashi identities



- For example consider the pion:

$$\langle \pi \,| J^{\mu}_{\text{em}} |\, \pi \rangle = (p' + p)^{\mu} \; F_1\left(p'^2, p^2, q^2\right) + (p' - p)^{\mu} \; F_2\left(p'^2, p^2, q^2\right)$$

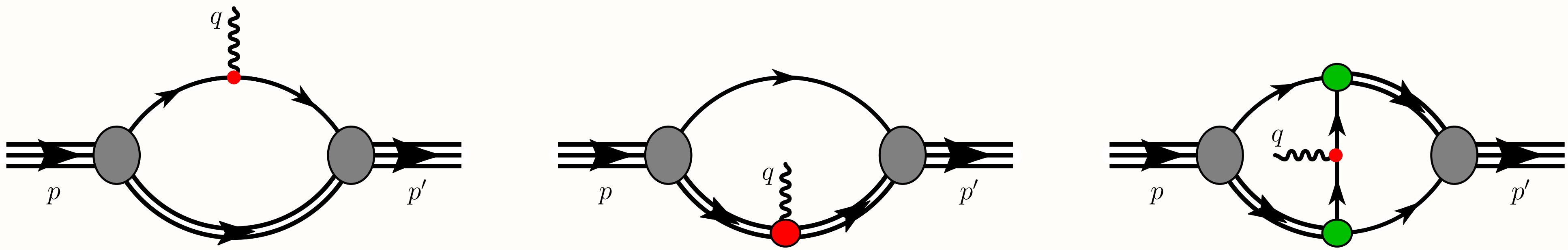
- WT identity states: $q_{\mu}\, \Gamma^{\mu}_{\gamma\pi\pi}(p', p) = \hat{Q}_{\pi}\left[\tau_{\pi}(p') - \tau_{\pi}(p)\right] \xrightarrow{\text{on-shell}} 0$

- Now $q \cdot (p' + p) = 0$ & $q \cdot (p' - p) = q^2$ implies $F_2 = 0$, therefore

$$\langle \pi \,| J^{\mu}_{\text{em}} |\, \pi \rangle = (p' + p)^{\mu} \; F_{\pi}\left(Q^2\right) \qquad Q^2 = -q^2, \;\; p'^2 = p^2 = m_{\pi}^2$$

Nucleon Form Factors in the NJL model

- The Feynman diagrams that give the nucleon form factors in our NJL are



- Ingredients are:
  - nucleon Faddeev amplitude $\Longleftrightarrow$ Faddeev equation
  - diquark propagators $\Longleftrightarrow$ Bethe-Salpeter equation
  - diquark BS vertex $\Longleftrightarrow$ homogeneous Bethe-Salpeter equation
  - quark propagator $\Longleftrightarrow$ gap equation
  - quark photon vertex $\Longleftrightarrow$ inhomogeneous Bethe-Salpeter equation
- A separate calculation gives diquark form factors
- We also make the "static approximation" to the quark exchange kernel:

$$S(p) = \left[\not{p} - M + i\varepsilon\right]^{-1} \longrightarrow M^{-1}$$

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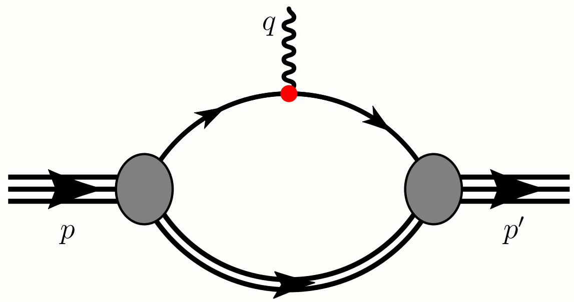

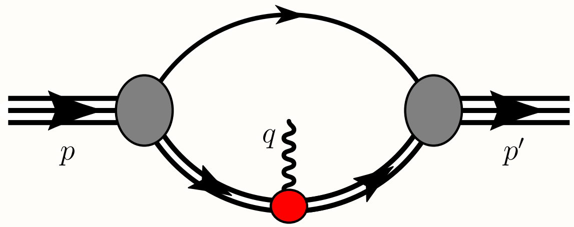

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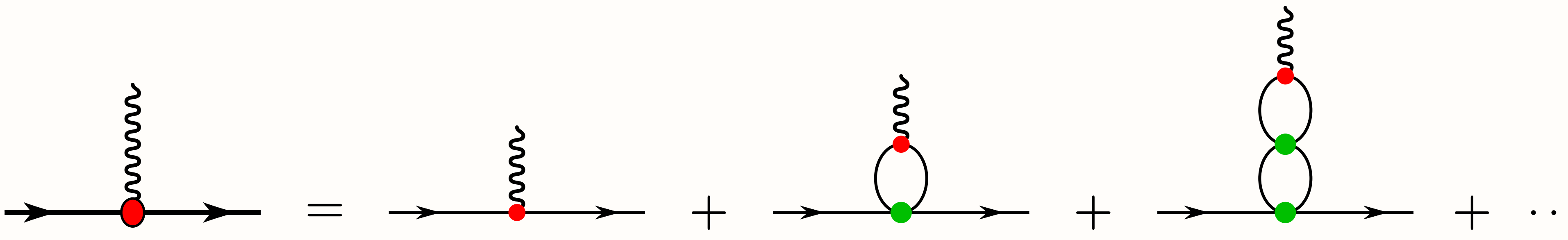
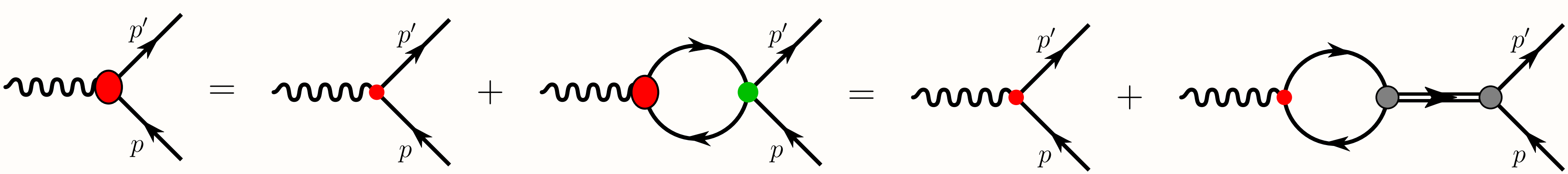
From Current to Constituent Quarks

- Recall that the NJL gap equation takes the current quarks and dresses them non-perturbatively so that they become constituent quarks
- Constituent quarks are extended non-trivial quasi-particles
- Consider an arbitrary current interacting with a contituent quark



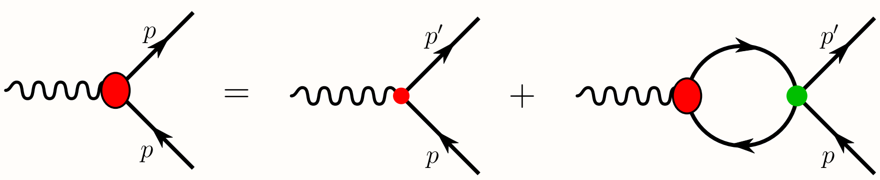
- This series can be summed by an integral equation

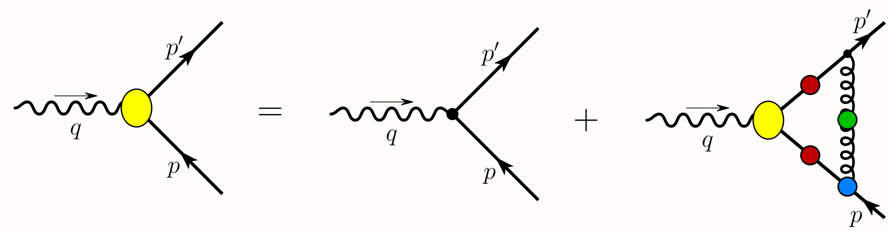


✦ *this is the inhomogeneous Bethe-Salpeter equation (BSE)*

Constituent Quark EM Form Factors

- Quark-photon vertex is given by the *inhomogeneous Bethe-Salpeter equation* – driving term is an external vector current: $\gamma^\mu\left(\frac{1}{6}+\frac{\tau_3}{2}\right)$





- Lorentz covariance implies that the quark–photon vertex has the structure

$$\Gamma^\mu_{\gamma qq}(p',p) = \sum_{i=1}^{12} \lambda_i^\mu\ f_i(p'^2,p^2,q^2) = \Gamma^\mu_L(p',p) + \Gamma^\mu_T(p',p)$$

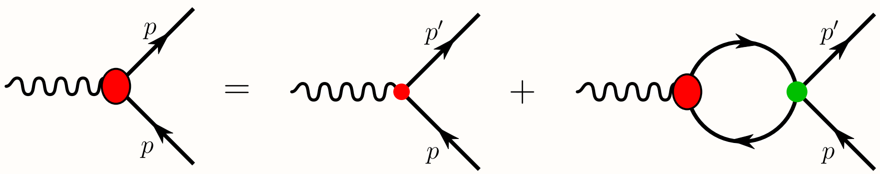
- In QCD the properties of the quark–photon vertex are governed by the quark propagator and the quark–gluon vertex

- A Ward-Takahashi identity constrains Γ^μ_L piece of quark–photon vertex

$$q_\mu\,\Gamma^\mu_{\gamma qq} = q_\mu\,\Gamma^\mu_L = \hat{Q}\,\left[S^{-1}(p') - S^{-1}(p)\right], \qquad q_\mu\,\Gamma^\mu_T = 0$$

  - ✦ these identities are a consequence of local $U(1)_V$ gauge invariance

NJL Constituent Quark Form Factors



$$K_{\alpha\beta,\gamma\delta} = -\,2i\,G_\omega\,(\gamma_\mu)_{\alpha\beta}\,(\gamma^\mu)_{\gamma\delta} \\ -\,2i\,G_\rho\,(\gamma_\mu\boldsymbol{\tau})_{\alpha\beta}\,(\gamma^\mu\boldsymbol{\tau})_{\gamma\delta}$$

- In general the quark–photon vertex has form

$$\Gamma^\mu_{\gamma qq}(p',p) = \frac{1}{6}\,\Lambda^\mu_\omega(p',p) + \frac{\tau_3}{2}\,\Lambda^\mu_\rho(p',p).$$

- Recall Ward–Takahashi identity $\quad [S^{-1}(p) = \not{p} - M + i\varepsilon]$

$$q_\mu\,\Gamma^\mu_{\gamma qq}(p',p) = \left(\frac{1}{6} + \frac{\tau_3}{2}\right)\left[S^{-1}(p') - S^{-1}(p)\right] \xrightarrow{NJL} \left(\frac{1}{6} + \frac{\tau_3}{2}\right)\not{q}$$

- NJL the vertex must be of form $\quad \Lambda^\mu_{\omega,\rho} = \gamma^\mu +$ transverse terms
- Solving the NJL inhomogeneous BSE for the quark–photon vertex gives

$$\Lambda^\mu_\omega(p',p) = \gamma^\mu + \left(\gamma^\mu - \frac{q^\mu\not{q}}{q^2}\right)\hat{F}_{1\omega}(q^2), \qquad \Lambda^\mu_\rho(p',p) = \gamma^\mu + \left(\gamma^\mu - \frac{q^\mu\not{q}}{q^2}\right)\hat{F}_{1\rho}(q^2)$$

NJL Results

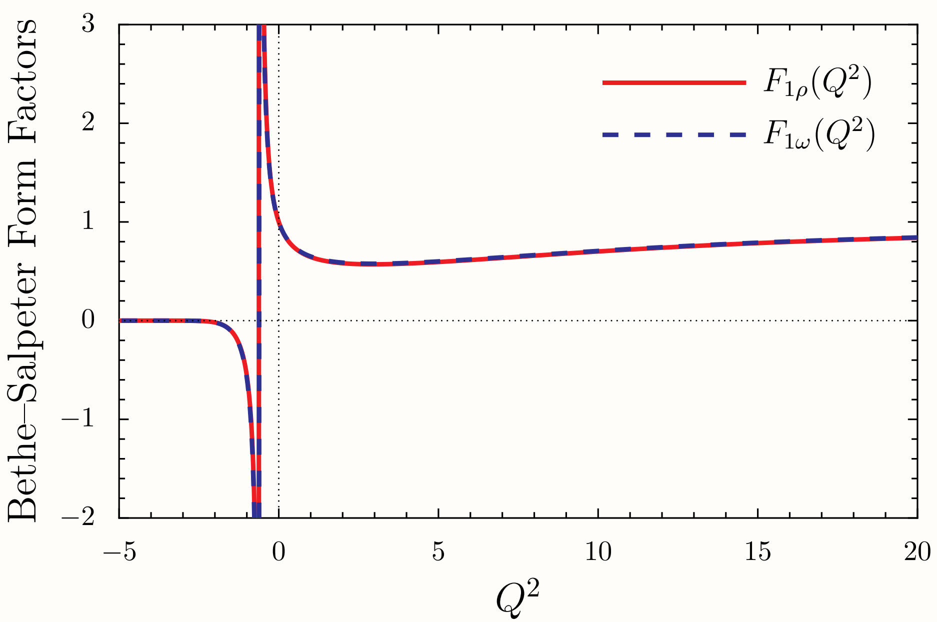
- Current conservation implies $q^\mu \not{q}$ cannot contribute; vertex becomes

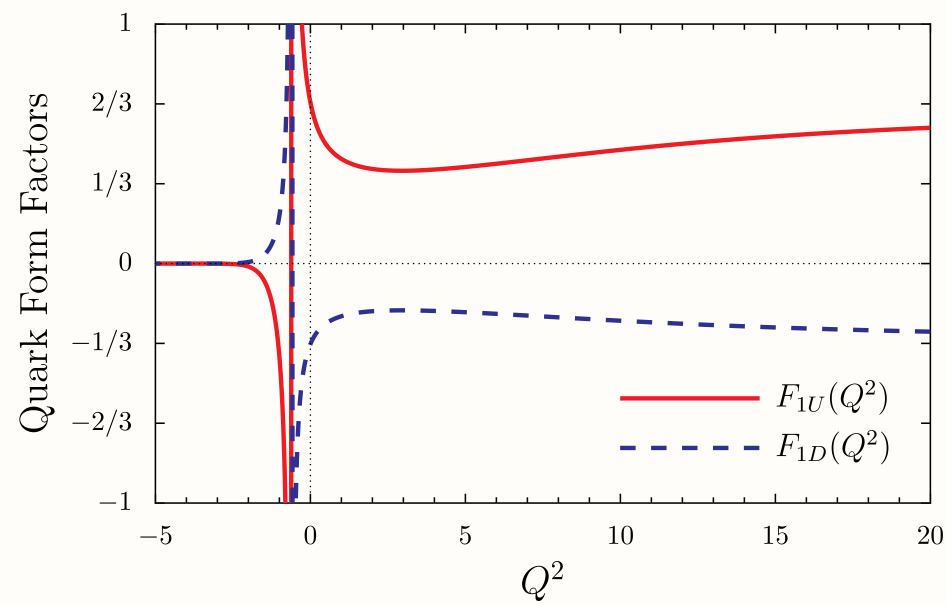
$$\langle J^\mu \rangle = \gamma^\mu \left[\tfrac{1}{6}\, F_{1\omega}(Q^2) + \tfrac{\tau_3}{2}\, F_{1\rho}(Q^2)\right]$$

- The up and down constituent quark form factors are given by $[Q^2 = -q^2]$

$$F_{1U}(Q^2) = \tfrac{1}{6}\, F_{1\omega}(Q^2) + \tfrac{1}{2}\, F_{1\rho}(Q^2) \quad \& \quad F_{1D}(Q^2) = \tfrac{1}{6}\, F_{1\omega}(Q^2) - \tfrac{1}{2}\, F_{1\rho}(Q^2)$$

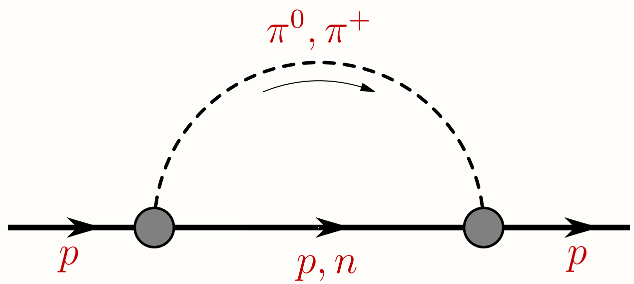
- Timelike poles at: $F_{1\omega}(Q^2 = -m_\omega^2)$ & $F_{1\rho}(Q^2 = -m_\rho^2)$

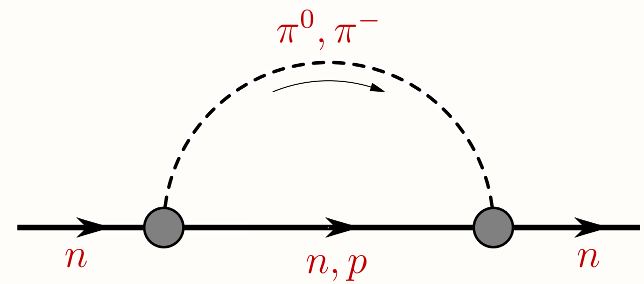




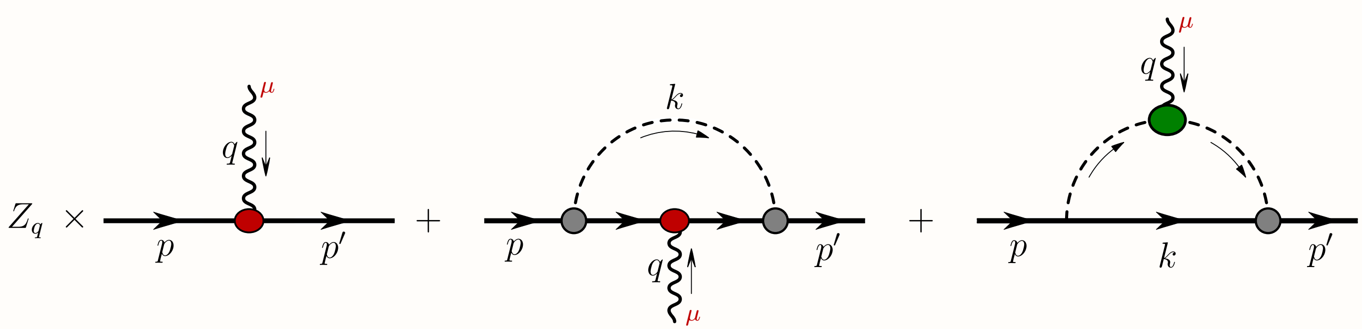
The role of Pions

- Pions are the lightest hadrons, therefore, because of quantum fluctuations we expect them to play an important role in many observables

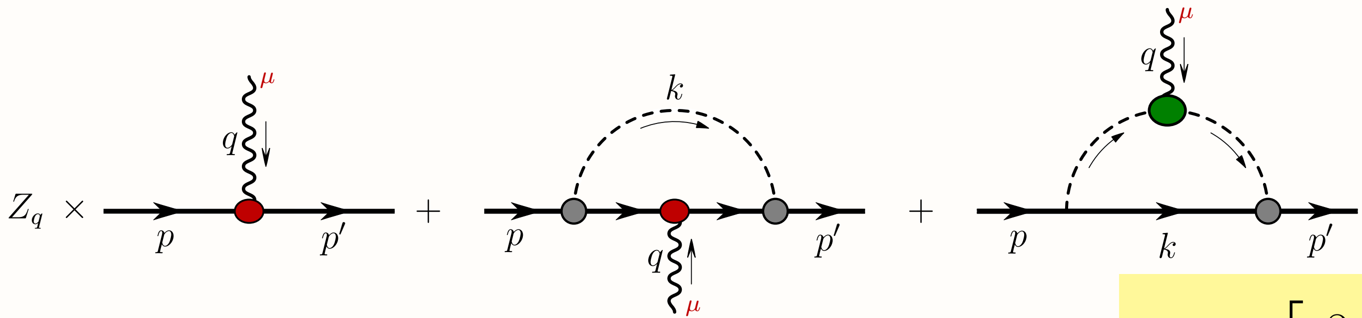




- Because the pion is light it is long range
  - expect proton and neutron charge and magnetic radii to be increased
  - the nucleon magnetic moments are also sensitive to pion cloud effects
- To include pions in NJL we dress the constituent quarks with a pion cloud



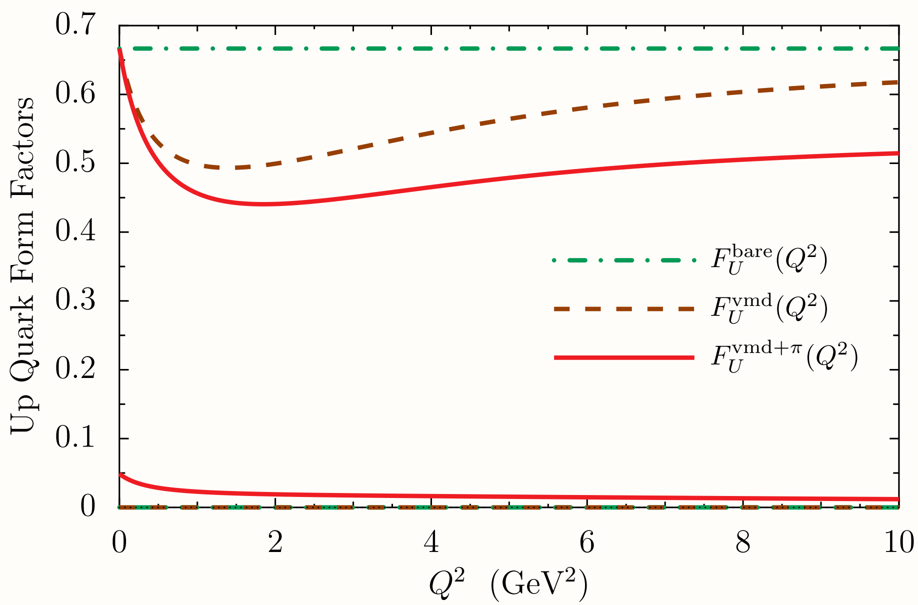
Quark Form Factors with Pion Cloud

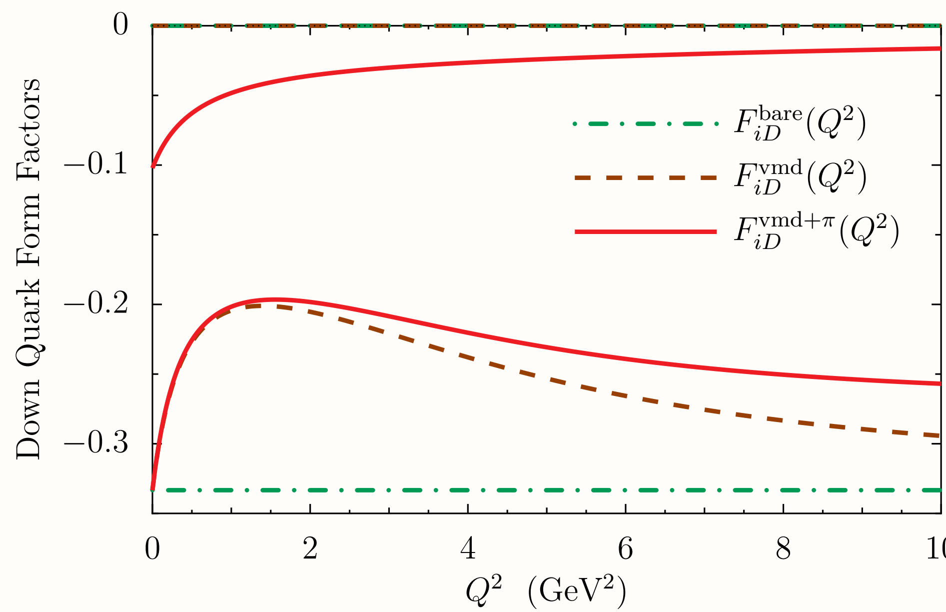


- Z_q is the probability to find a bare constituent quark: $Z_q = \left[\frac{\partial}{\partial \not{p}} S(p)\right]^{-1}_{\not{p}=M}$
- Pion cloud induces an anomalous magnetic moment for the quarks

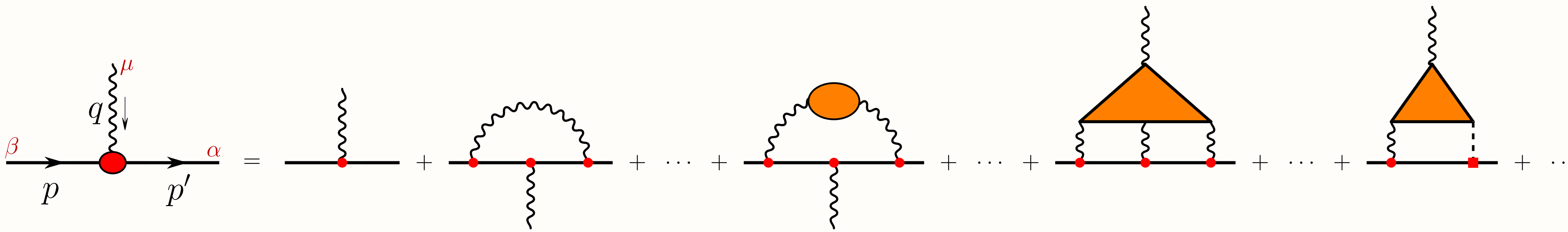
$$F_{1q}(Q^2) = Z_q \left[\frac{1}{6} F_\omega(Q^2) + \frac{1}{2} \tau_3 F_\rho(Q^2)\right] + \left[F_\omega(Q^2) - \tau_3 F_\rho(Q^2)\right] F_{1q}^{(q)}(Q^2) + \tau_3 F_\rho \; F_{1q}^{(\pi)}(Q^2)$$

$$F_{2q}(Q^2) = \left[F_\omega(Q^2) - \tau_3 F_\rho(Q^2)\right] F_{2q}^{(q)}(Q^2) + \tau_3 F_\rho \; F_{2q}^{(\pi)}(Q^2)$$

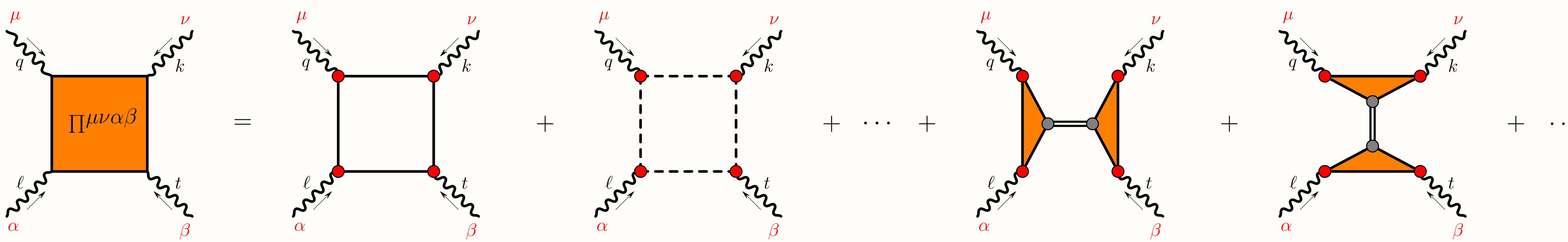




An Aside – Muon Anomalous Magnetic Moment

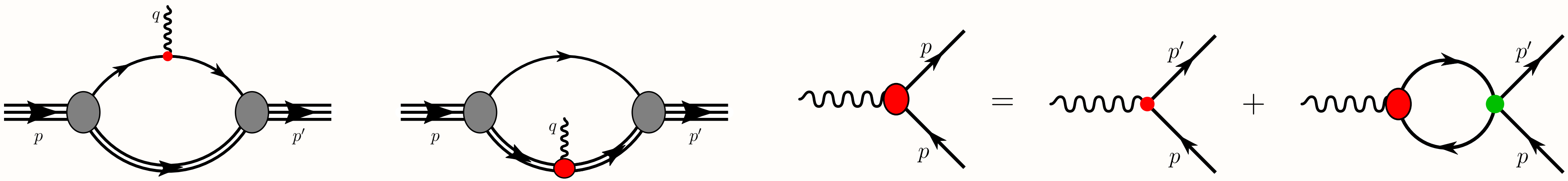


- $a_\mu^{\text{exp}} = 11659208.0 \pm 6.3 \times 10^{-10}$; $a_\mu^{\text{theory}} = 11659179.0 \pm 6.5 \times 10^{-10}$
- largest theory error come from HLBL scattering contribution

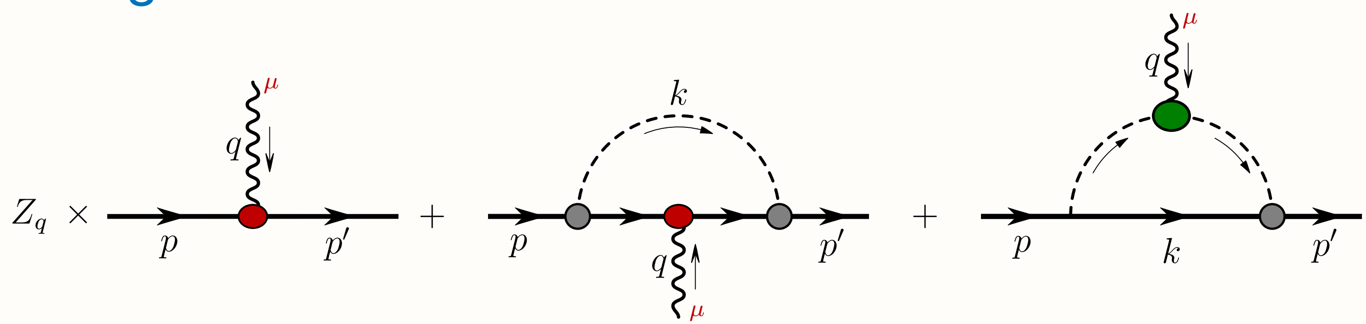


- Box diagram contribution is least know
  - only γ^μ coupling and VMD has been considered so far
  - we argue that the anomalous magnetic moment term cannot be ignored
- At least error on $a_\mu^{\text{HLBL}} = 8.3 \pm 3.2 \times 10^{-10}$ should be much larger
- Fred Jegerlehner, Andreas Nyffeler, Physics Reports 477 (2009) 1–110

Nucleon Electromagnetic Form Factors



- Now have all ingredients needed to determine NJL nucleon form factors



- The nucleon electromagnetic current is given by

$$\langle J^\mu \rangle = u(p') \left[\gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2\,M} F_2(Q^2) \right] u(p)$$

- Include both scalar and axial-vector diquarks

$$\tau_s(q) = \frac{-4i\,G_s}{1 + 2\,G_s\,\Pi_s(q^2)},$$

$$\tau_a^{\mu\nu}(q) = \frac{-4i\,G_a}{1 + 2\,G_a\,\Pi_a(q^2)} \left[g^{\mu\nu} + 2\,G_a\,\Pi_a(q^2)\,\frac{q^\mu q^\nu}{q^2} \right],$$

Proton Form Factor Results

- For the proton magnetic moment $(\mu = 1 + \kappa)$ find

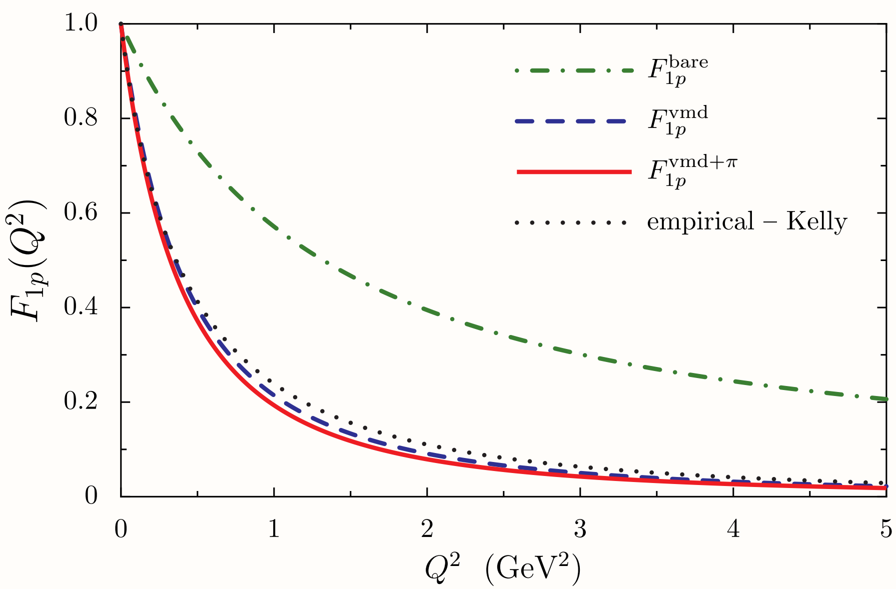
$$\mu_p^{\text{bare}} = 2.37\,\mu_N, \qquad \mu_p^{\text{vmd}+\pi} = 2.78\,\mu_N, \qquad \mu_p^{\text{experiment}} = 2.79\,\mu_N$$

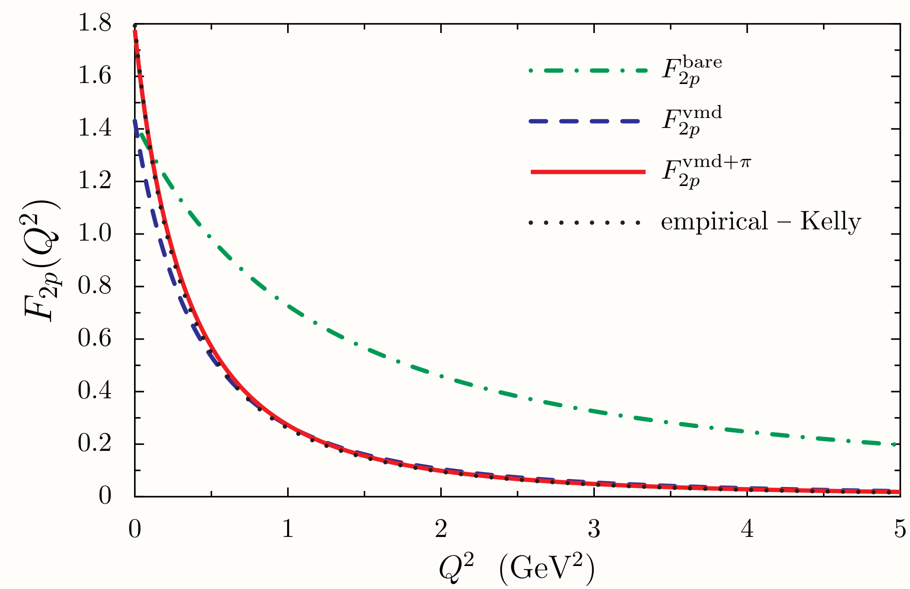
  - ✦ pion increases anomalous magnetic moment by $\sim 30\%$
  - ✦ results use a consistent treatment for meson and diquark T-matrices

- For the proton charge and magnetic radii find

$$\langle r_E\rangle_p^{\text{vmd}+\pi} = 0.86\,\text{fm} \qquad \langle r_E\rangle_p^{\text{experiment}} = 0.85\,\text{fm}$$

$$\langle r_M\rangle_p^{\text{vmd}+\pi} = 0.83\,\text{fm} \qquad \langle r_M\rangle_p^{\text{experiment}} = 0.84\,\text{fm}$$





Neutron Form Factor Results

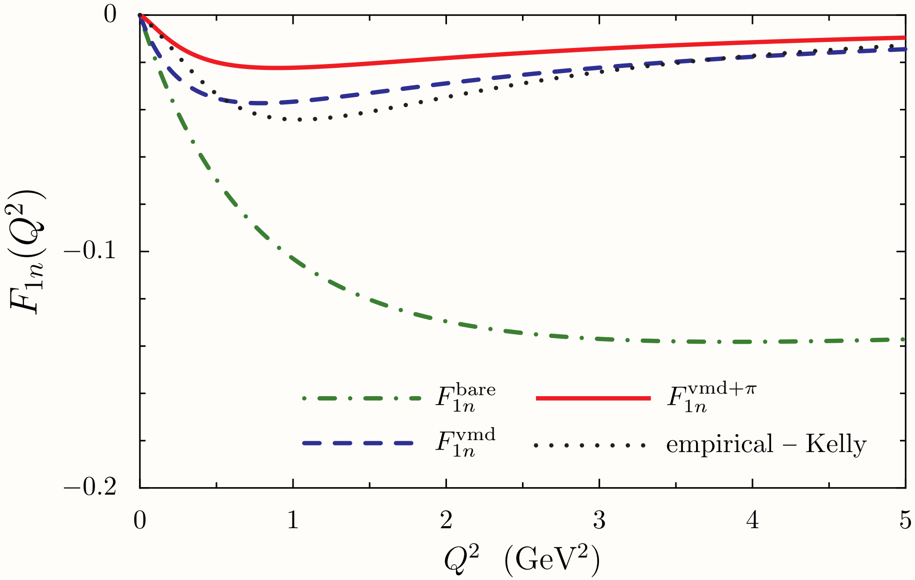
- For the neutron magnetic moment $(\mu = \kappa)$ find

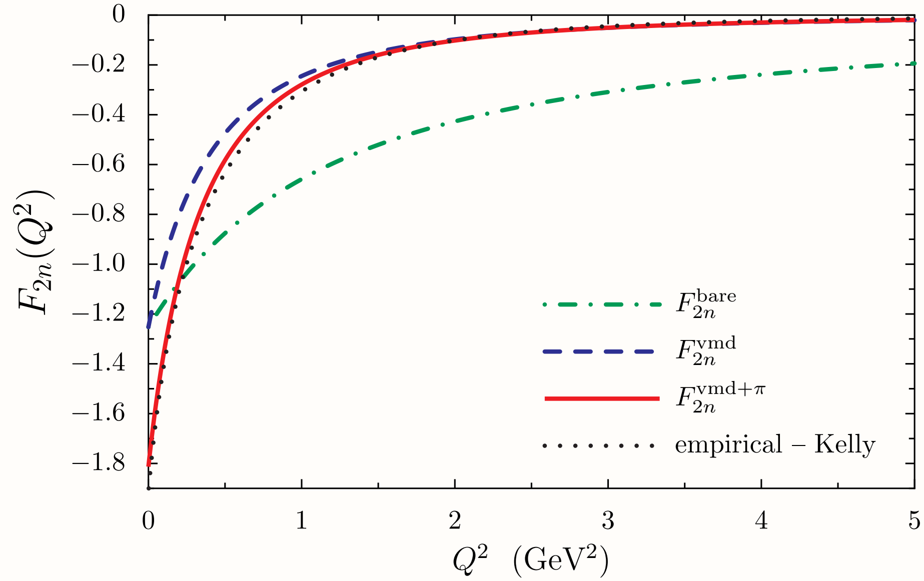
$$\mu_n^{\text{bare}} = 1.25\,\mu_N, \qquad \mu_n^{\text{vmd}+\pi} = 1.81\,\mu_N, \qquad \mu_n^{\text{experiment}} = 1.91\,\mu_N$$

  - ✦ pion increases anomalous magnetic moment by $\sim 45\%$
  - ✦ results use a consistent treatment for meson and diquark T-matrices

- For the neutron charge and magnetic radii find

$$\langle r_E\rangle_n^{\text{vmd}+\pi} = -0.34\,\text{fm} \qquad \langle r_E\rangle_n^{\text{experiment}} = -0.35\,\text{fm}$$
$$\langle r_M\rangle_n^{\text{vmd}+\pi} = 0.86\,\text{fm} \qquad \langle r_M\rangle_n^{\text{experiment}} = 0.89\,\text{fm}$$





Model Parameters

- Free Parameters:
  - ✦ $\Lambda_{IR}, \ \Lambda_{UV}, \ M_0, \ G_\pi, \ G_s, \ G_a, \ G_\omega, \ G_\rho$
- Constraints:
  - ✦ $f_\pi = 93$ MeV, $m_\pi = 140$ MeV, $m_\rho = 770$ MeV & $m_\omega = 782$ MeV
  - ✦ $M_N = 940$ MeV & $M_\Delta = 1232$ MeV
  - ✦ $\Lambda_{IR} = 240$ MeV, $M_0 = 400$ MeV
- Obtain:
  - ✦ $\Lambda_{UV} = 645$ MeV
  - ✦ $M_s = 768$ MeV, $M_a = 928$ MeV, $\ldots$
- Can now study a large array of observables:
  - ✦ e.g. meson and baryon quark distributions, form factors, GPDs, TMDs, properties at finite temperature and density; neutron stars, etc

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