CSSM Summer School
Non-perturbative Methods in Quantum Field Theory
February 15, 2013.

A BRIEF INTRODUCTION TO FRAGMENTATION FUNCTIONS

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Literature

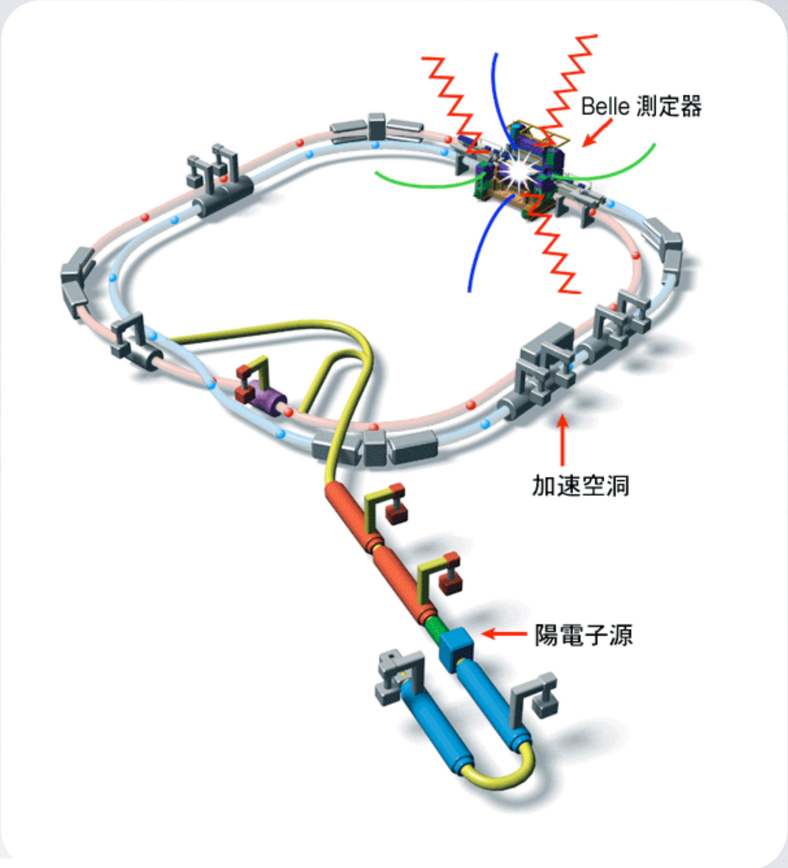
- **Books**
  - Collins, "*Foundations of Perturbative QCD*", 2011.
  - Halzen and Martin, "*Quarks and Leptons*", 1984.
- **Reviews**
  - Collins et. al., "*Factorization of Hard Processes in QCD*", arXiv:hep-ph/0409313
  - Barone et. al., "*Transverse-Spin and Transverse-Momentum Effects in High-Energy Processes*", arXiv:1011.0909
  - Sterman, "*Partons, factorization and resummation, TASI 95*", arXiv:hep-ph/9606312

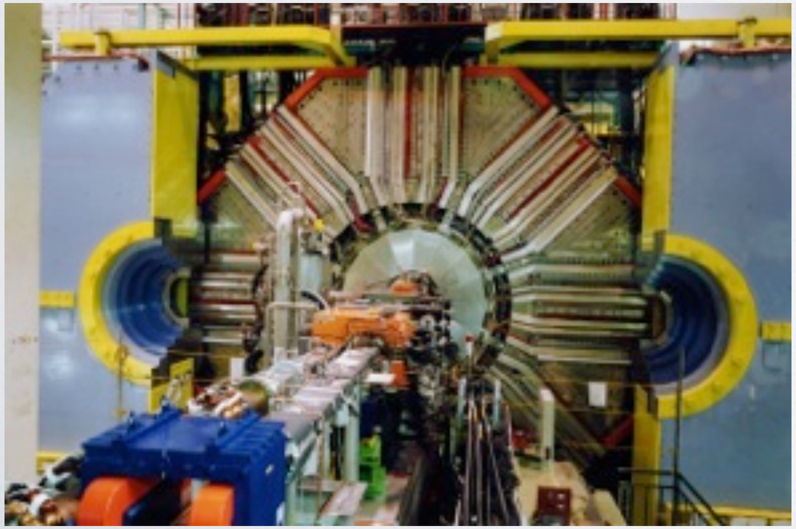
PROBING THE STRUCTURE OF MATTER: DEEP INELASTIC SCATTERING

❖*Deep inelastic scattering (DIS) probes partonic structure of hadrons.*

THE EXPERIMENTAL TOOLBOX

- e^+e^- colliders produce quark - antiquark pairs: a QCD lab.



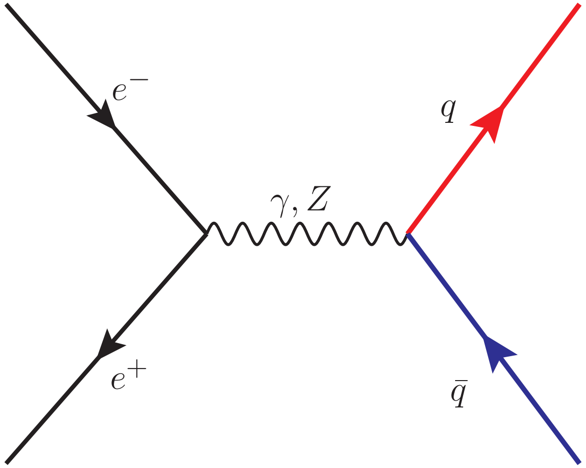


e^+e^- ANNIHILATION

- Let's consider creation of quark-antiquark pairs.

$$\boxed{\sigma(e^-e^+ \to q\bar{q}) = 3e_q^2\sigma(e^-e^+ \to \mu^-\mu^+)}$$

N_c



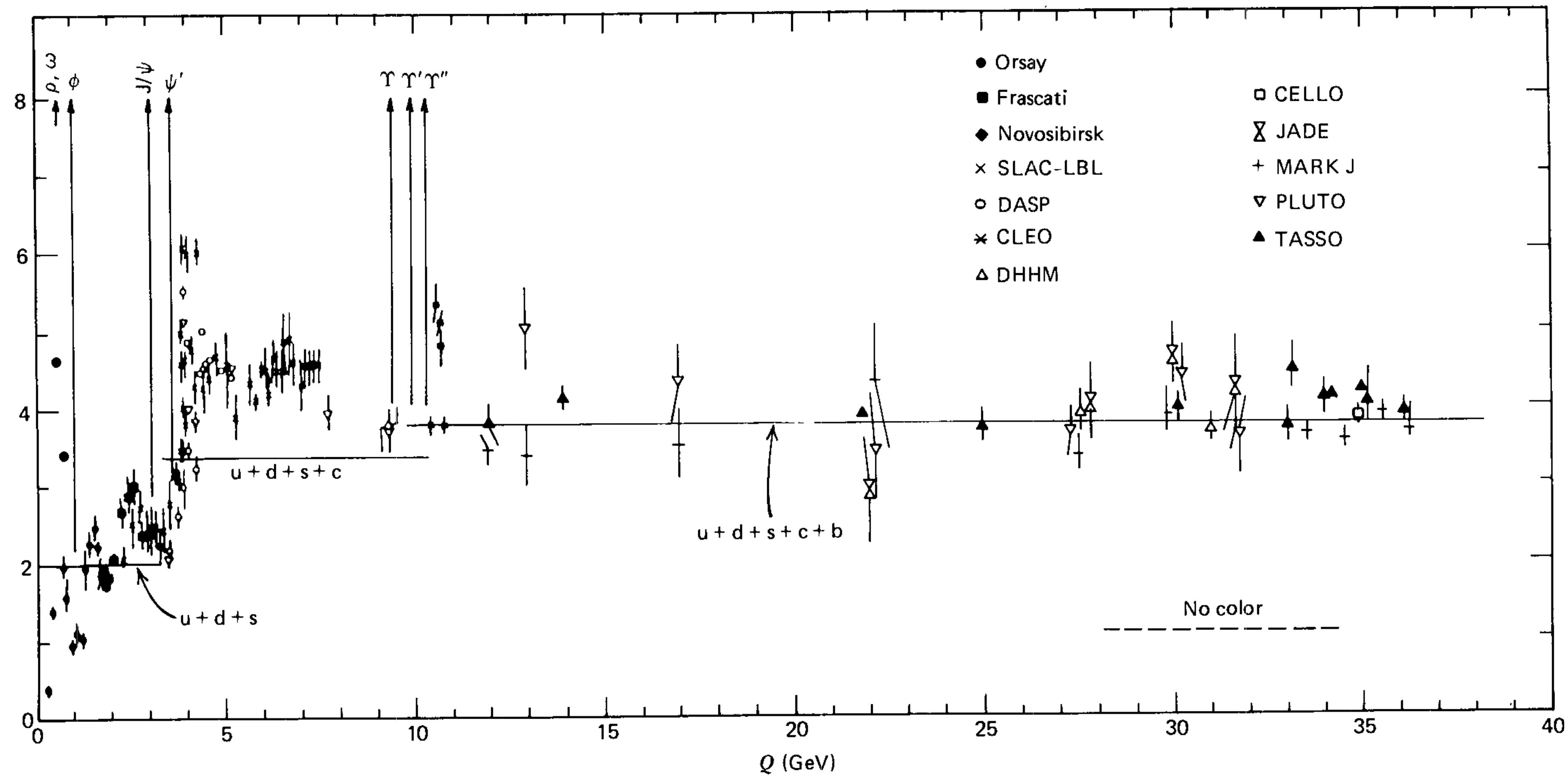
- The conjecture of Confinement:
NO free quarks or gluons have been directly observed: only HADRONS.

$$\boxed{\sigma(e^-e^+ \to hadrons) = \sum_q \sigma(e^-e^+ \to q\bar{q})}$$

$$\boxed{R \equiv \frac{\sigma(e^-e^+ \to hadrons)}{\sigma(e^-e^+ \to \mu\bar{\mu})} = 3\sum_q e_q^2}$$

TESTING SIMPLISTIC QCD PREDICTIONS

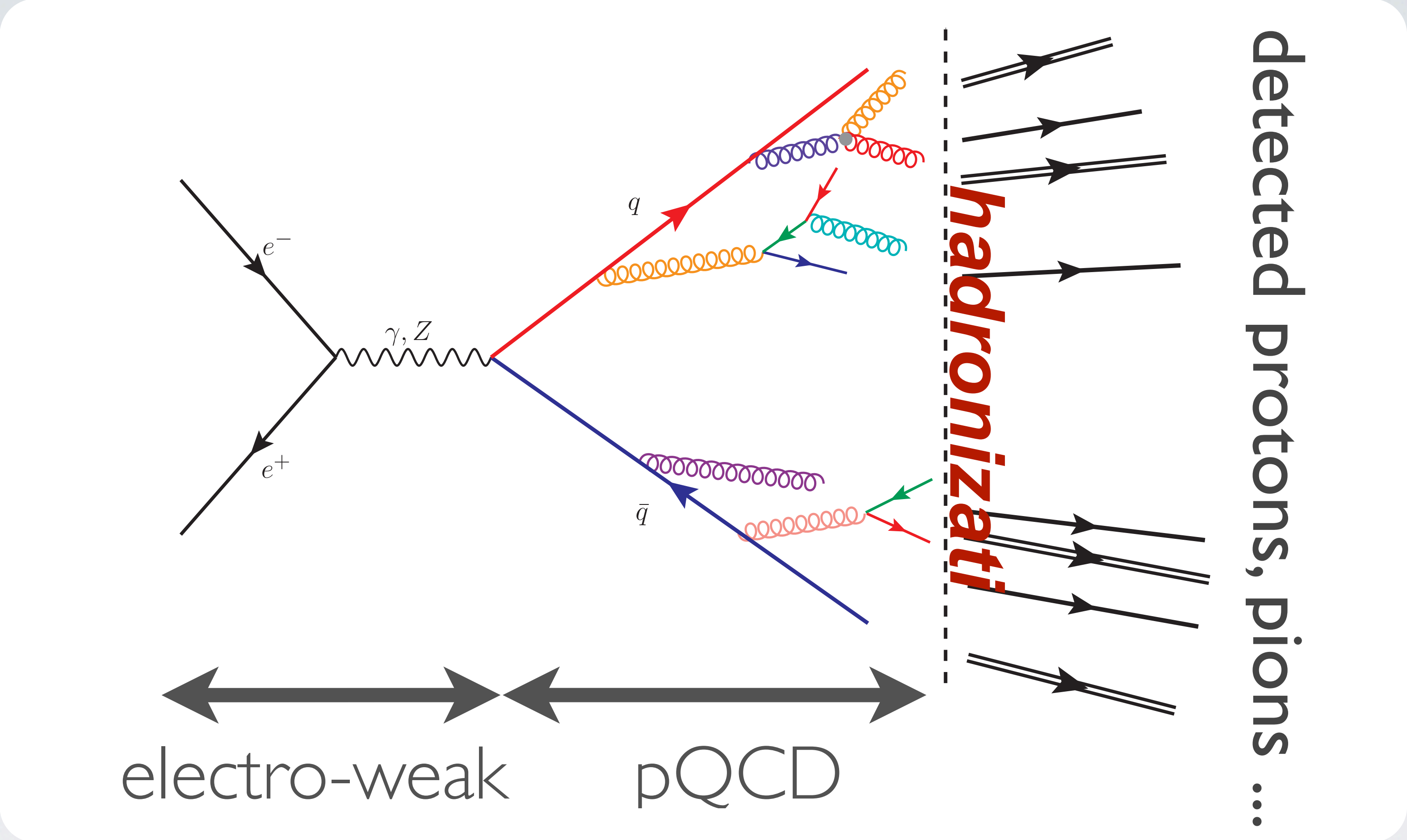
$$R = \begin{cases} 3[(2/3)^2 + (-1/3)^2 + (-1/3)^2] = 2 & : \text{u, d, s} \\ 2 + 3\,(-1/3)^2 = 10/3 & : \text{u, d, s, c} \\ 10/3 + 3(-1/3)^2 = 11/3 & : \text{u, d, s, c, b} \end{cases}$$



$$m_c \approx 1.2 \text{ GeV}, \;\; m_b \approx 4.1 \text{ GeV}$$

HADRONIZATION: $e^-e^+ \to hX$

- **Factorization:** *pQCD "hard" partonic scattering seaprated from "soft", universal fragmentation functions* at renormalization scale.



$$\frac{d}{dz}\sigma(e^-e^+ \to hX) = \sum_q \sigma(e^-e^+ \to q\bar{q}) \left[D_q^h(z_h) + D_{\bar{q}}^h(z_h)\right], \; z_h = \frac{2E_h}{Q}$$

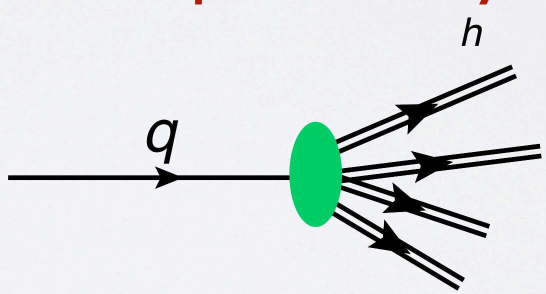
FRAGMENTATION FUNCTIONS

- The cross-sections of DIS processes can be factorized into "hard scattering" parts calculable in pQCD and "soft", non-perturbative universal functions encoding parton distribution in hadrons (PDFs) and parton hadronization: Fragmentation Functions (FF).

$$\boxed{\frac{1}{\sigma}\frac{d}{dz}\sigma(e^-e^+\to hX) = \sum_i \mathcal{C}_i(z,Q^2)\otimes D_i^h(z,Q^2)}$$

- Unpolarized, Integrated FF is the probability density for quark q to produce hadron h:

$$\boxed{D_q^h(z,Q^2)}$$

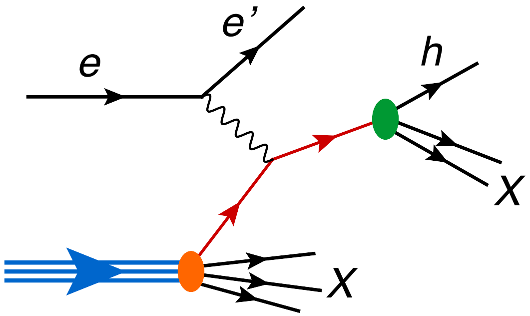


- where z is the light-cone momentum fraction of the parton carried by the hadron and Q is the scale of factorization.

$$\boxed{z=\frac{p^-}{k^-}\approx z_h=\frac{2E_h}{Q}}$$

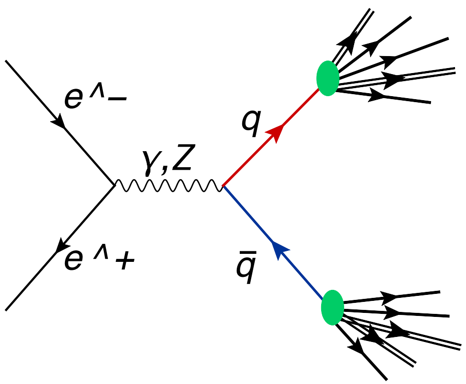
$$\boxed{a^\pm=\frac{1}{\sqrt{2}}(a^0\pm a^3)}$$

COLLINEAR **FACTORIZATION** AND **UNIVERSALITY**



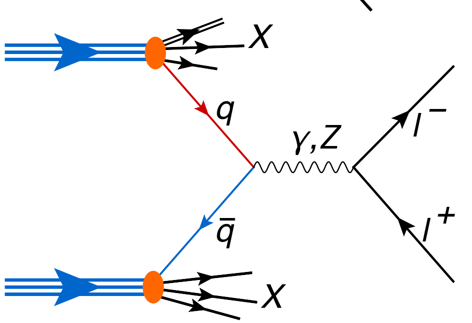
- SEMI INCLUSIVE DIS (SIDIS)

$$\boxed{\sigma^{eP\to ehX} = \sum_q f_q^P \otimes \sigma^{eq\to eq} \otimes D_q^h}$$



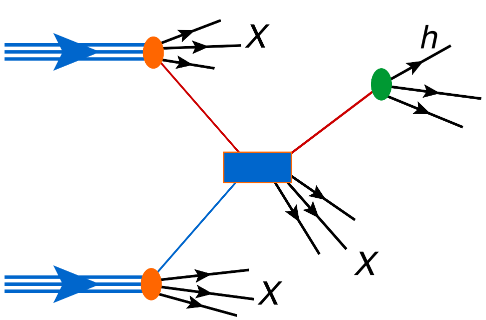
- e^+e^-

$$\boxed{\sigma^{e^+e^-\to hX} = \sum_q \sigma^{e^+e^-\to q\bar{q}} \otimes (D_q^h + D_{\bar{q}}^h)}$$



- DRELL-YAN (DY)

$$\boxed{\sigma^{PP\to l^+l^-X} = \sum_{q,q'} f_q^P \otimes f_{\bar{q}}^P \otimes \sigma^{q\bar{q}\to l^+l^-}}$$



- Hadron Production

$$\boxed{\sigma^{PP\to hX} = \sum_{q,q'} f_q^P \otimes f_{q'}^P \otimes \sigma^{qq'\to qq'} \otimes D_q^h}$$

QCD EVOLUTION

- PDFs and FFs depend on the factorization scale: $Q^2 \gg \Lambda_{QCD}$
- This dependence can be described by EVOLUTION *Dokshitzer-Gribov-Lipatov-Altarelli-Parisi* (DGLAP) equations.

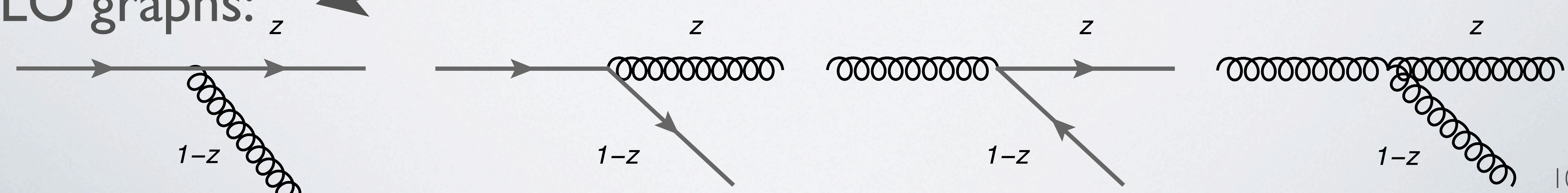
$$\boxed{\frac{\partial}{\partial \ \log Q^2} D_i^h(z,Q^2) = \sum_j [P_{ij}\left(Q^2\right) \otimes D_j^h\left(Q^2\right)](z)}$$

$$\boxed{[f \otimes g](z) \equiv \int_z^1 \frac{dy}{y} f(y)\, g\left(\frac{z}{y}\right) = \int_z^1 \frac{dy}{y} f\left(\frac{z}{y}\right)\ g(y)}$$

- Splitting functions from pQCD

$$\boxed{P_{ij}\left(z,Q^2\right) = \frac{\alpha_s(Q^2)}{2\pi} P_{ij}^{(0)}(z) + \left(\frac{\alpha_s(Q^2)}{2\pi}\right)^2 P_{ij}^{(1)}(z) + \ldots}$$

- LO graphs:



PROPERTIES

Sum Rules:

✦ Average Multiplicity:

$$\boxed{\int_{z_0}^{1} dz\ D_q^h(z,Q^2) = \langle N_h \rangle}$$

✦ Momentum Conservation:

$$\boxed{\sum_h \int_0^1 dz\ z\ D_q^h(z,Q^2) = 1}$$

All produced hadrons

Symmetries:

✦ Charge Conjugation: $q \Leftrightarrow \bar{q}$

$$\boxed{\begin{array}{l} D_u^{\pi^+} = D_{\bar{u}}^{\pi^-} \\ D_s^{K^-} = D_{\bar{s}}^{K^+} \end{array}}$$

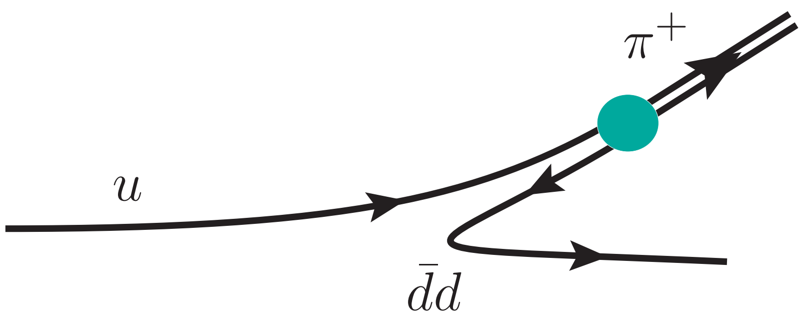
✦ Isospin: $u \Leftrightarrow d$

$$\boxed{\begin{array}{l} D_u^{\pi^+} = D_d^{\pi^-} \\ D_u^{K^+} = D_d^{K^0} \end{array}}$$

FAVORED AND UNFAVORED FFS

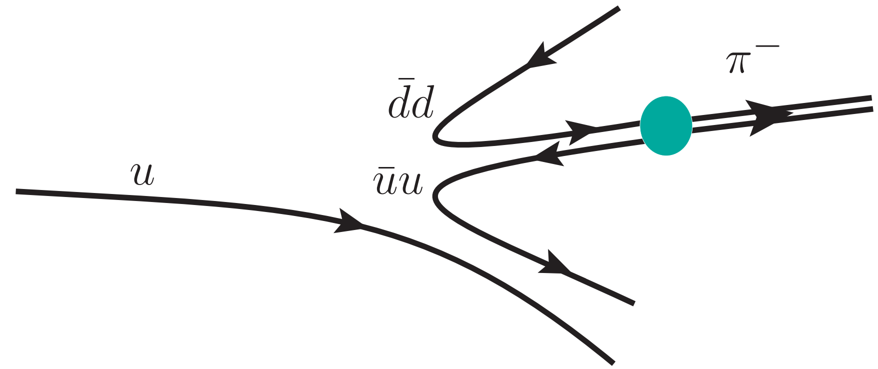
Using a naive quark model picture:

- Favored: the produced hadron has a valence quark of the same flavor.



$\pi^+(u\bar{d})$ $K^+(u\bar{s})$

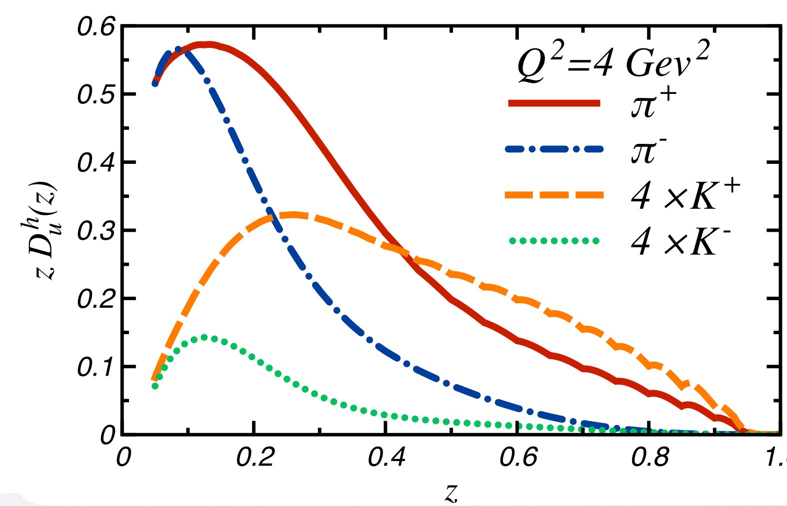
- Unfavored (disfavored): NO valence quark of the same flavor.



$\pi^-(\bar{u}d)$ $K^-(\bar{u}s)$

DSS Parametrization

- Favored FFs are dominant in large z.
- In small z region both favored and unfavored FFs are comparable
- Light quark to Kaon FFs are suppressed compared to pions.



EMPIRICAL PARAMETRIZATIONS OF DATA

- Experimentally measured cross-sections are convolution of PDFs and/or FFs: *need to separate flavor dependence, etc*.

- Use UNIVERSALITY: perform a combined fit.

- Measurements are at different Q^2: DGLAP evolution.

EMPIRICAL PARAMETRIZATIONS OF DATA

Choose a functional form at some initial scale

$$D_q^h(z, Q_0^2) = N_i z^{\alpha_i}(1-z)^{\beta_i}$$

Perform QCD evolution to the scale of the data and Calculate the Chi-square.

Is the Fit Good Enough?

No

Adjust the parameters

Yes

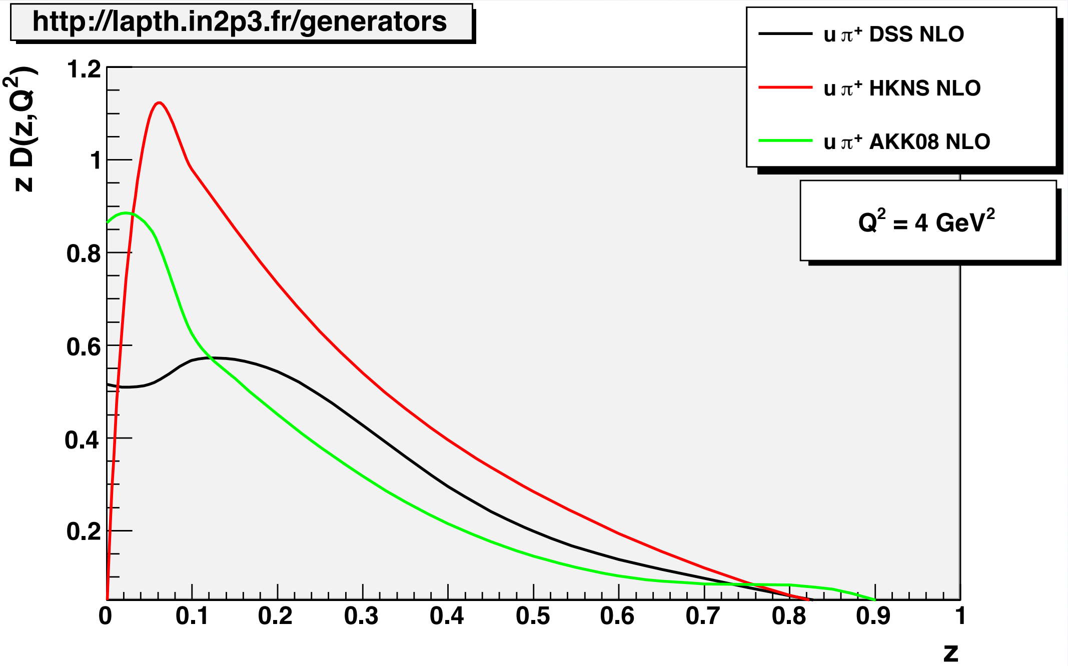
New Parametrization is ready!

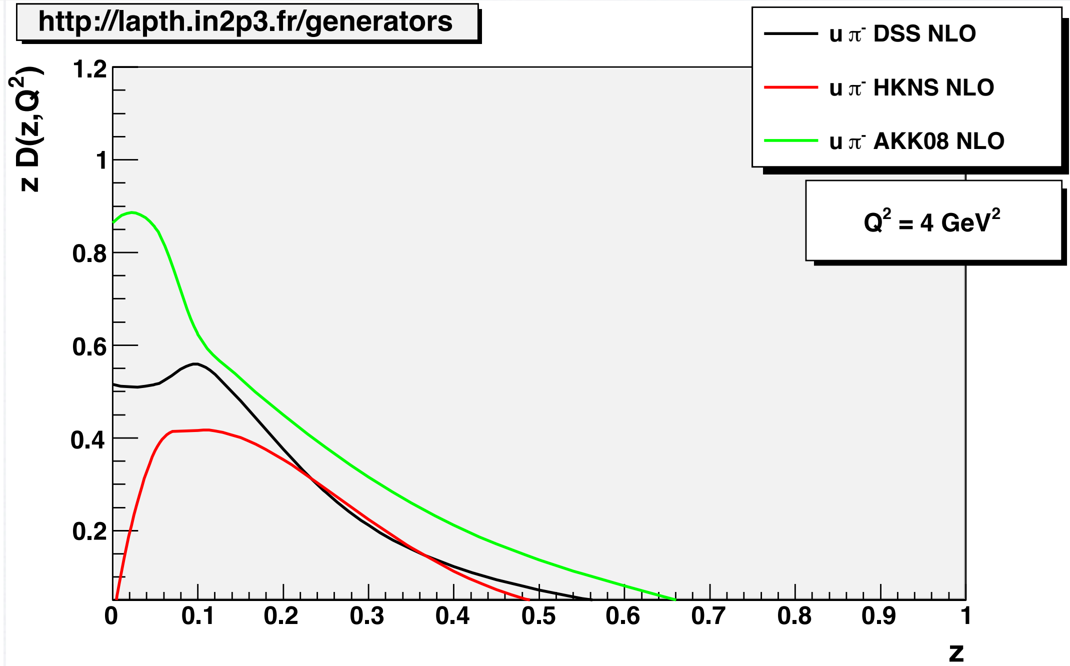
EMPIRICAL PARAMETRIZATIONS OF DATA

New Parametrization is ready!

Any Problems? YES!!!

- Many fragmentation channels, Huge number of parameters: *need to make approximations.*
- Large experiments uncertainties.
- Uncertainties from PDFs.



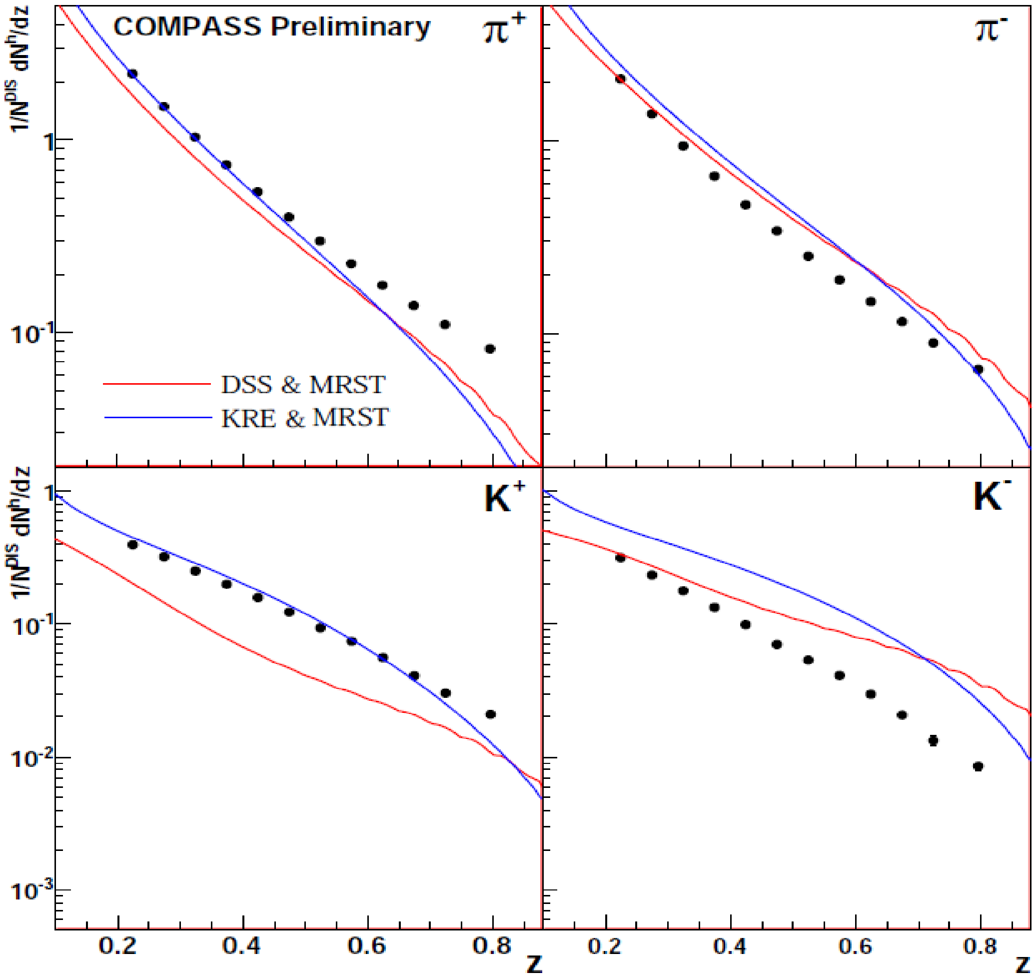


Unfavored FFs NOT well known!

- Recent SIDIS results from COMPASS collaboration.

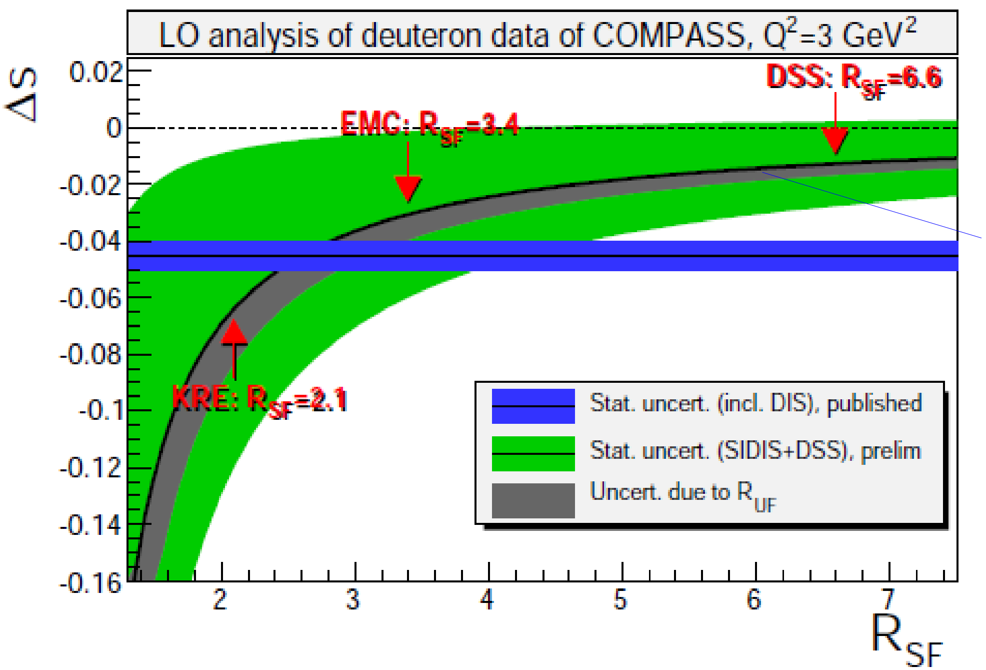
Hadron Multiplicities

$$\frac{dM^h(x,z,Q^2)}{dx\ dz} = \frac{\sum_q f_1^q(x,Q^2) D_q^h(z,Q^2)}{\sum_q f_1^q(x,Q^2)}$$



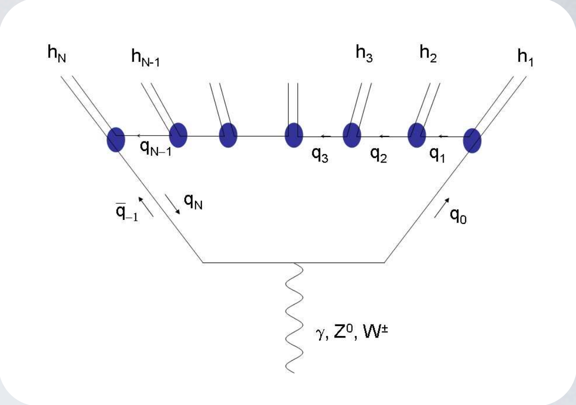
Impact on Extraction of Δs

$$R_{UF} = \frac{\int_{0.2}^{0.85} D_d^{K^+}(z)\,dz}{\int_{0.2}^{0.85} D_u^{K^+}(z)\,dz}, \quad R_{SF} = \frac{\int_{0.2}^{0.85} D_{\bar{s}}^{K^+}(z)\,dz}{\int_{0.2}^{0.85} D_u^{K^+}(z)\,dz}$$

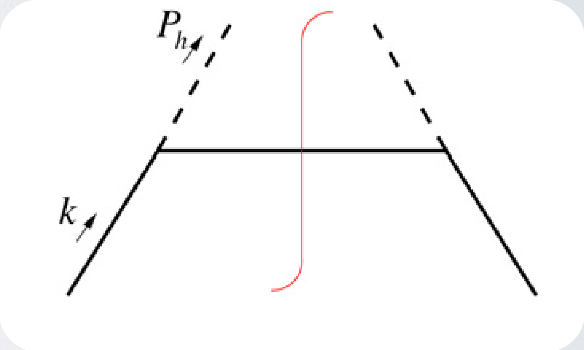


MODELS FOR FRAGMENTATION

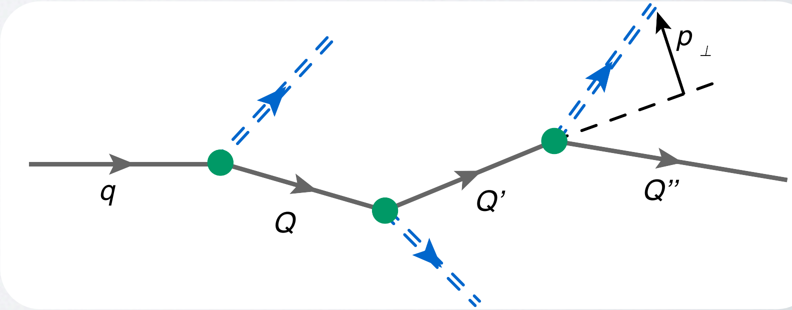
- Lund *String* Model
  - *Very Successful* implementation in ***JETSET, PYTHIA***.
  - Highly Tunable - Limited Predictive Power.
  - No Spin Effects - Formal developments by X. Artru et al but no quantitative results!



- *Spectator Model*
  - *Quark model calculations with empirical form factors.*
  - *No unfavored fragmentations.*
  - *Need to tune parameters for small z dependence.*



- *NJL-jet Model*
  - ***Multi-hadron*** *emission framework with effective quark model input.*
  - ***Monte-Carlo framework*** *allows flexibility in including the transverse momentum, spin effects, etc.*

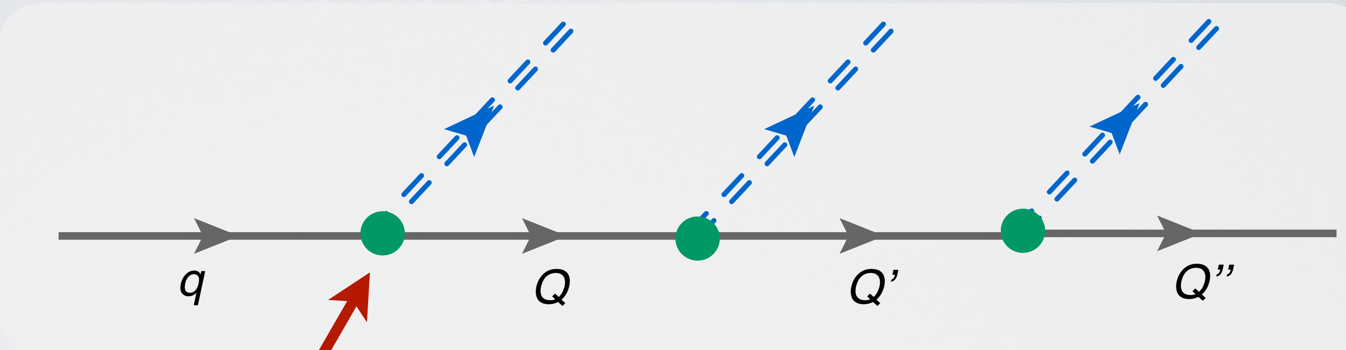


THE QUARK JET MODEL

Field, Feynman, Nucl.Phys.B136:1,1978.

Assumptions:

- Number Density interpretation
- No re-absorption
- ∞ hadron emissions



The probability of finding hadron h with mom. fraction z in a jet of quark q

$$\hat{d}_q^Q(z) = \hat{d}_q^h(1-z)|_{h=q\bar{Q}}$$

$$D_q^h(z)dz = \hat{d}_q^h(z)dz + \sum_Q \int_z^1 \hat{d}_q^Q(y)dy \cdot D_Q^h\left(\frac{z}{y}\right)\frac{dz}{y}$$

Probability of emitting the hadron at link 1

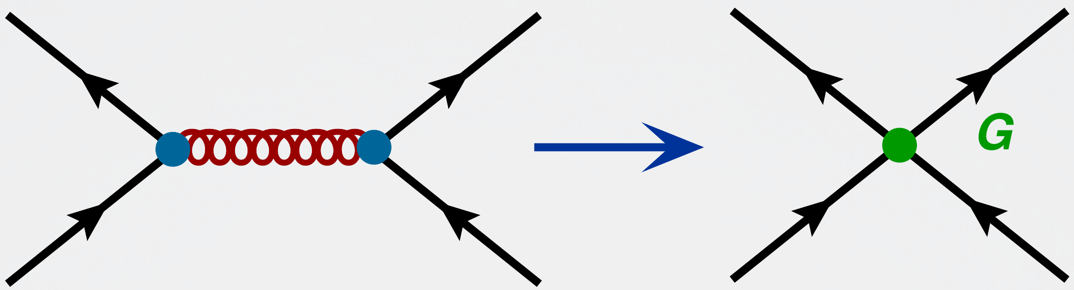
Probability of Momentum fraction y is transferred to jet at step 1

The probability scales with mom. fraction

NAMBU--JONA-LASINIO MODEL

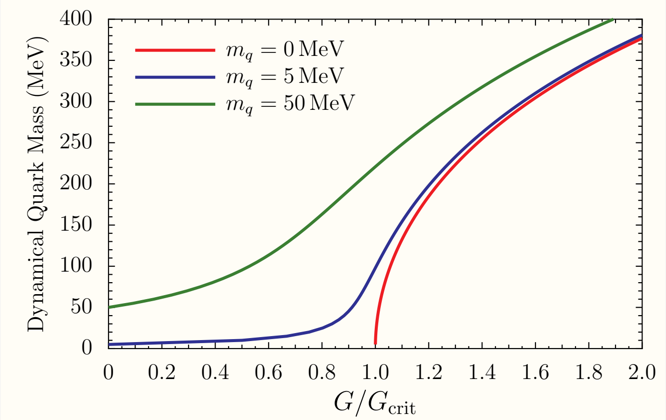
Effective Quark model of QCD

- Effective Quark Lagrangian $\mathcal{L}_{NJL} = \overline{\psi}_q(i\partial\!\!\!/ - m_q)\psi_q + G(\overline{\psi}_q\Gamma\psi_q)^2$



- Low energy chiral effective theory of QCD.
- Covariant, has the same flavor symmetries as QCD.

- Dynamically Generated Quark Mass from GAP Eqn.

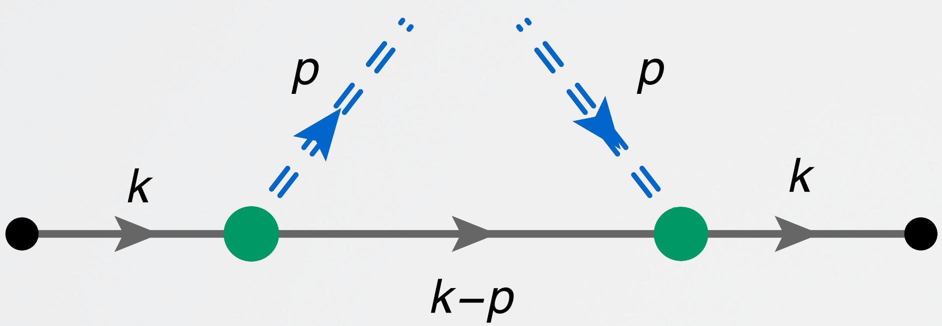


NJL-JET: ELEMENTARY SPLITTINGS

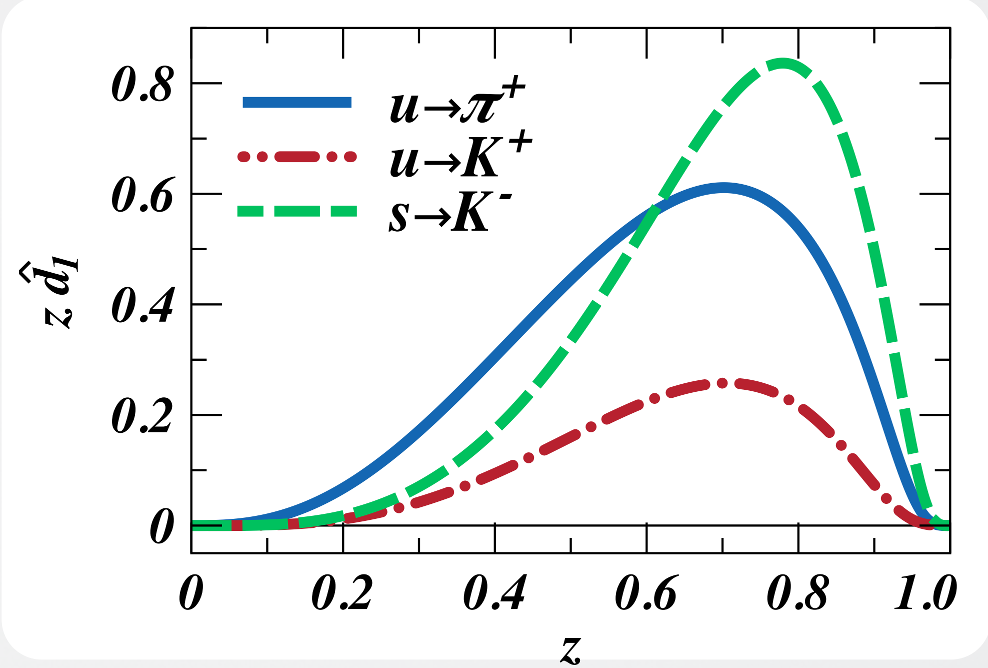
H.M., Thomas, Bentz, PRD. 83:074003, 2011

$$D_q^h(z) = \frac{1}{6} dp_- \int d^2p_\perp \sum_\alpha \frac{\left\langle k(\alpha)|a_h^\dagger(p)a_h(p)|k(\alpha)\right\rangle}{\langle k(\alpha)|k(\alpha)\rangle}$$

- One-quark truncation of the wavefunction: $d_q^h(z) : q \to Qh$



$$d_1^{h/q}(z, p_\perp^2) = \frac{1}{2}\mathrm{Tr}\left[\Delta_0(z, p_\perp^2)\gamma^+\right]$$

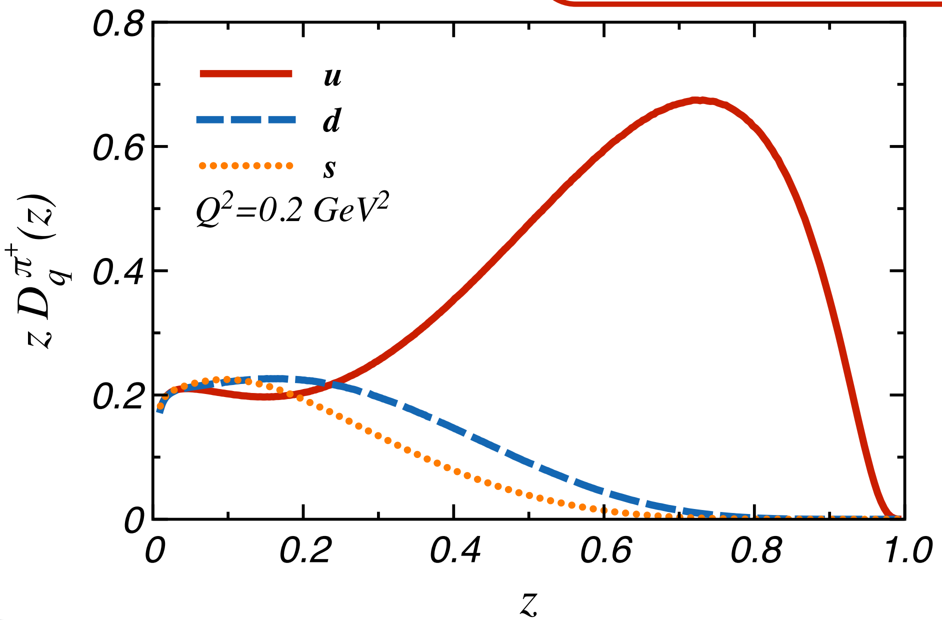


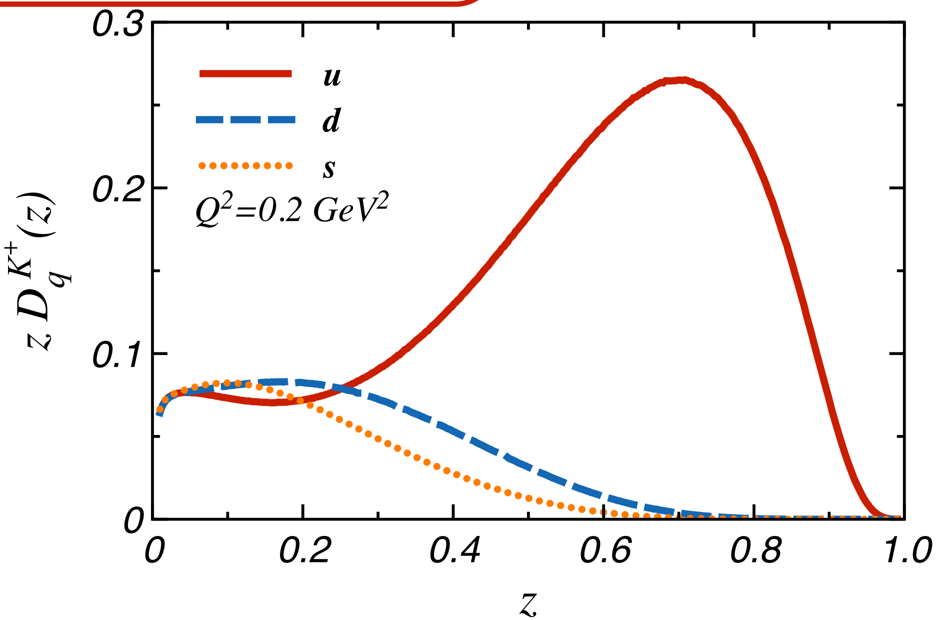
SOLUTIONS OF THE INTEGRAL EQUATIONS

$$\boxed{D_q^h(z) = \hat{d}_q^h(z) + \sum_Q \int_z^1 \hat{d}_q^Q\left(\frac{z}{y}\right)\frac{dy}{y}\cdot D_Q^h(y)}$$

- Discretize z in [0,1]: $z \to \{z_1 = 0, z_2, \ldots, z_N = 1\}$
- Approximate the integral over y as a sum: $\int_{z_i}^1 f(y)dy \approx \sum_{j=i+1}^{N} f(z_j)\Delta z_j$
- System of $N_q \times N$ linear eqns.: $D_q^h(i) = \hat{d}_q^h(i) + M_q^Q(i,j)D_Q^h(j)$

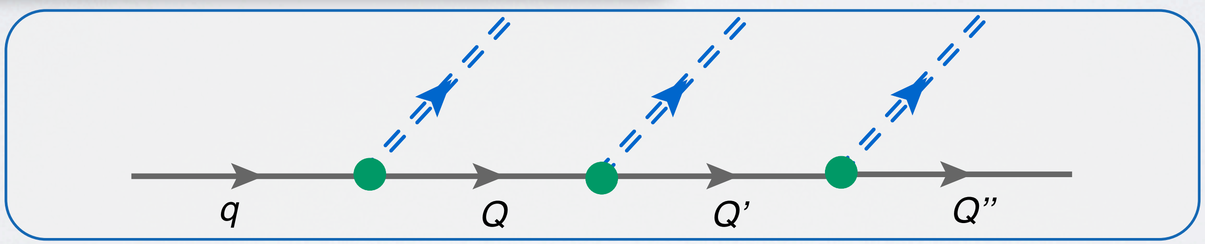
$$\boxed{D_q^h(i) = \hat{d}_q^h(j)[\mathbf{1} - M_q^Q(i,j)]^{-1}}$$





MONTE-CARLO (MC) APPROACH

H.M., Thomas, Bentz, PRD.83:114010, 2011

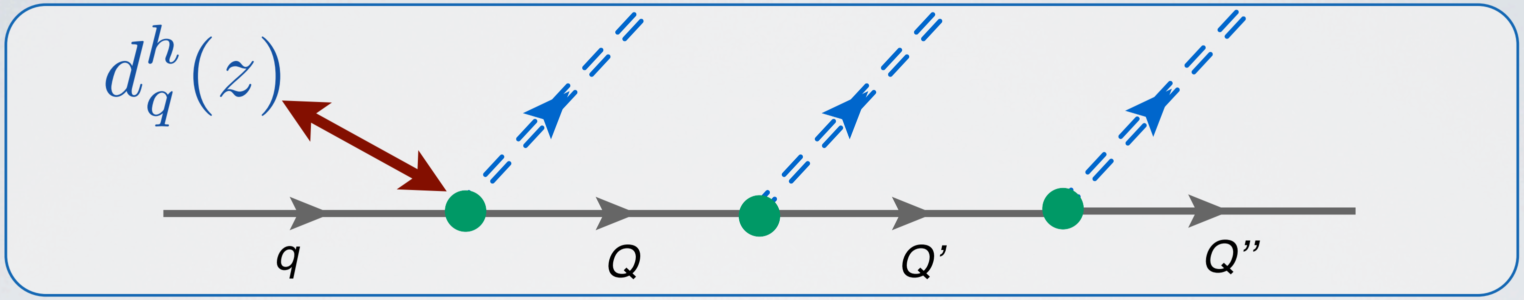


- Using the ***probabilistic*** interpretation of fragmentation funcs. to include the effect of ***multiple hadron emissions***.

$$D_q^h(z)\Delta z = \langle N_q^h(z, z+\Delta z)\rangle \equiv \frac{\sum_{N_{Sims}} N_q^h(z, z+\Delta z)}{N_{Sims}}$$

INTEGRATED FRAGMENTATIONS FROM MC

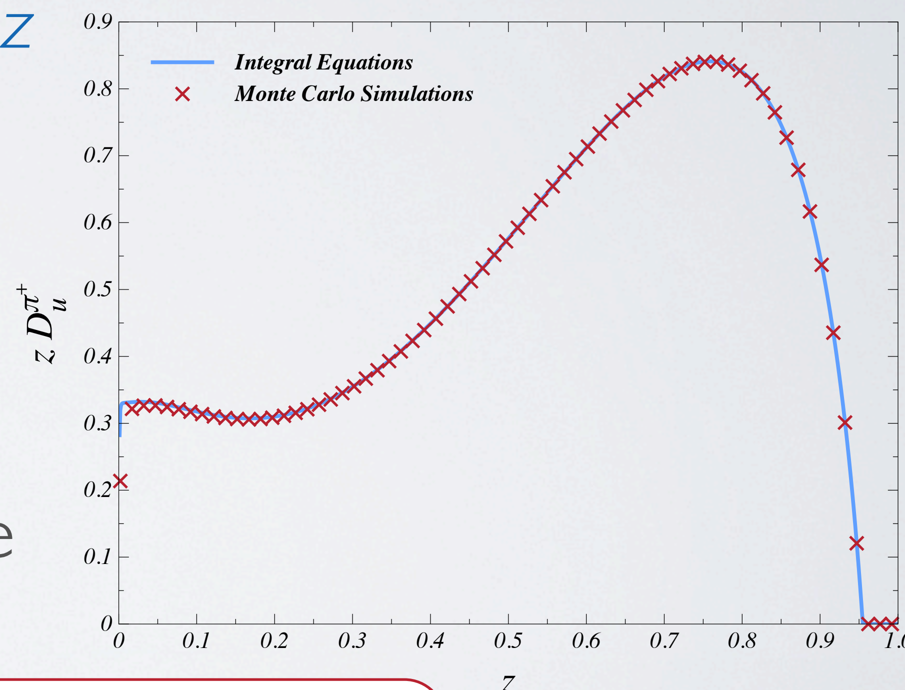
- Input: One hadron emission probability



- Sample the emitted hadron type and z according to input splitting.

- ***CONSERVE***: Momentum and Quark Flavor in each step.

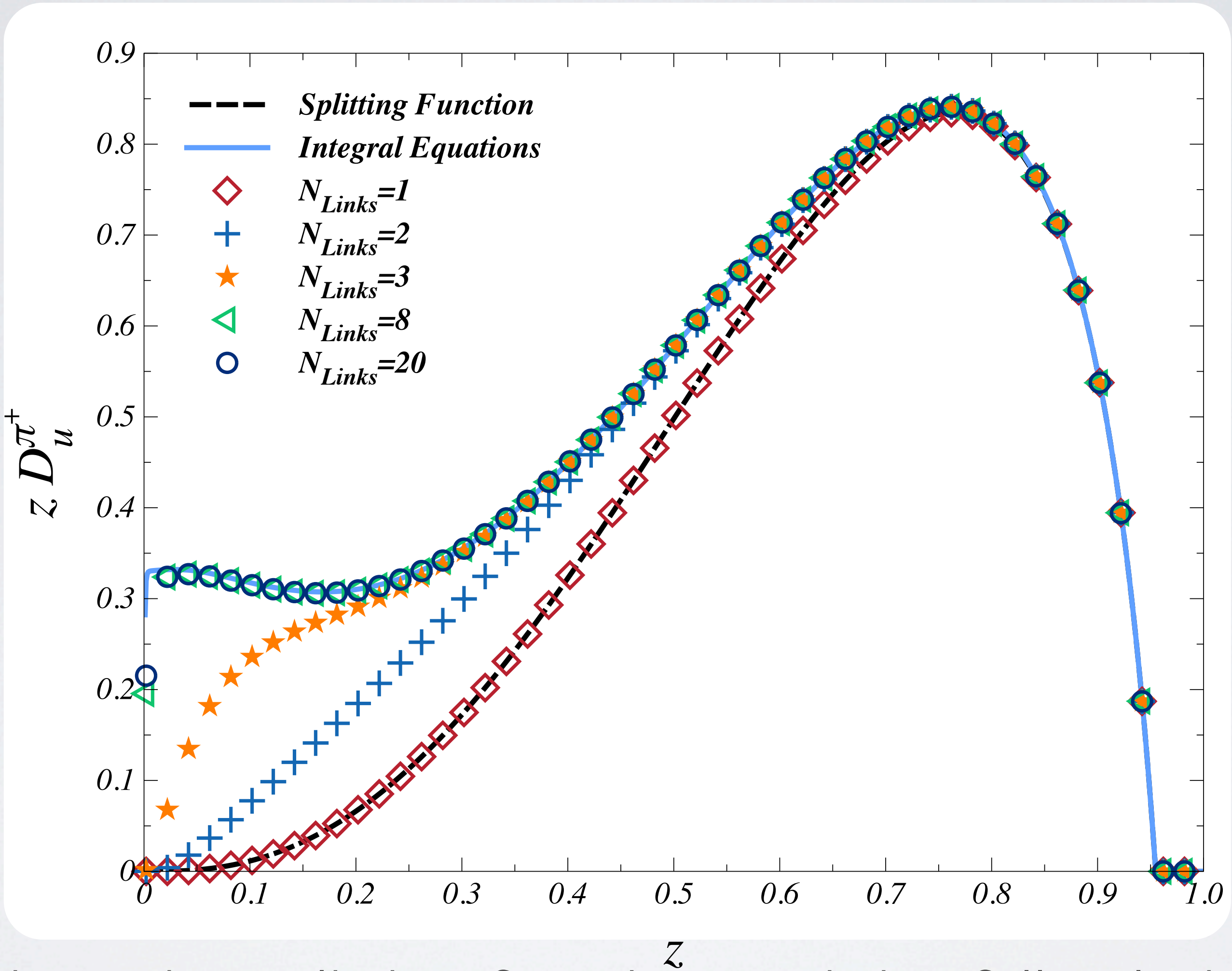
- Repeat for decay chains with the same initial quark.



$$D_q^h(z)\Delta z = \langle N_q^h(z, z+\Delta z)\rangle \equiv \frac{\sum_{N_{Sims}} N_q^h(z, z+\Delta z)}{N_{Sims}}$$

DEPENDENCE ON CHAIN CUTOFF

- Restrict the number of emitted hadrons, N_{Links} in MC.



- We reproduce the splitting function and the full solution perfectly.

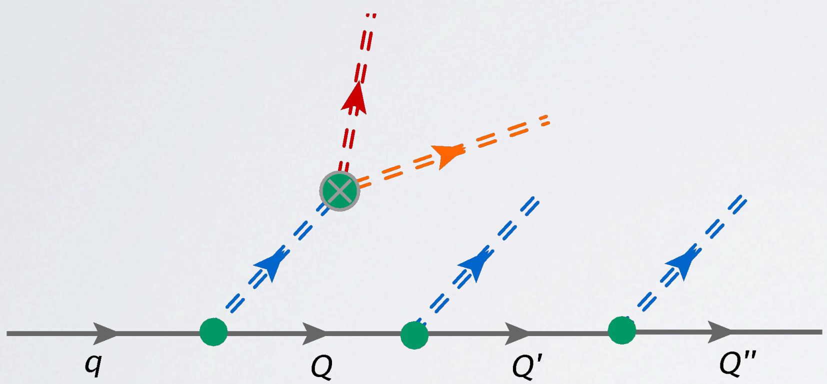
- The low z region is saturated with just a few emissions.

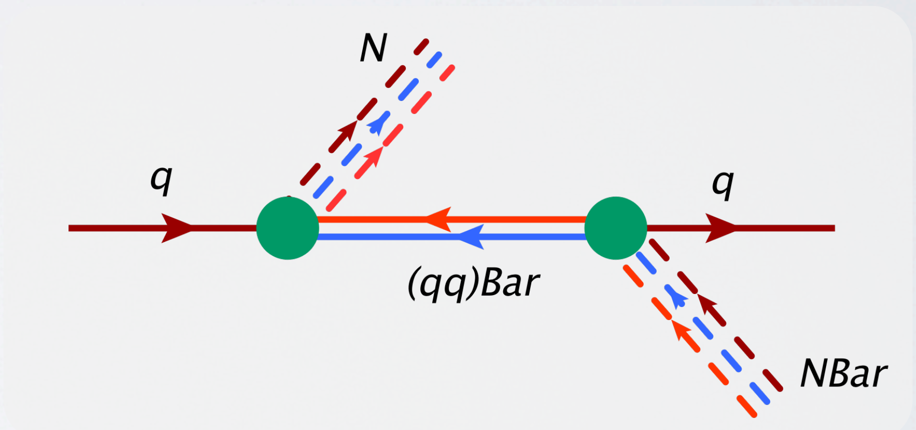
MORE CHANNELS

- Calculate quark splittings to vector mesons, Nucleon Anti-Nucleon: $d_q^h(z)$

$$\boxed{h = \rho^0, \rho^\pm, K^{*0}, \overline{K}^{*0}, K^{*\pm}, \phi, N, \bar{N}}$$

- Add the decay of the resonances:

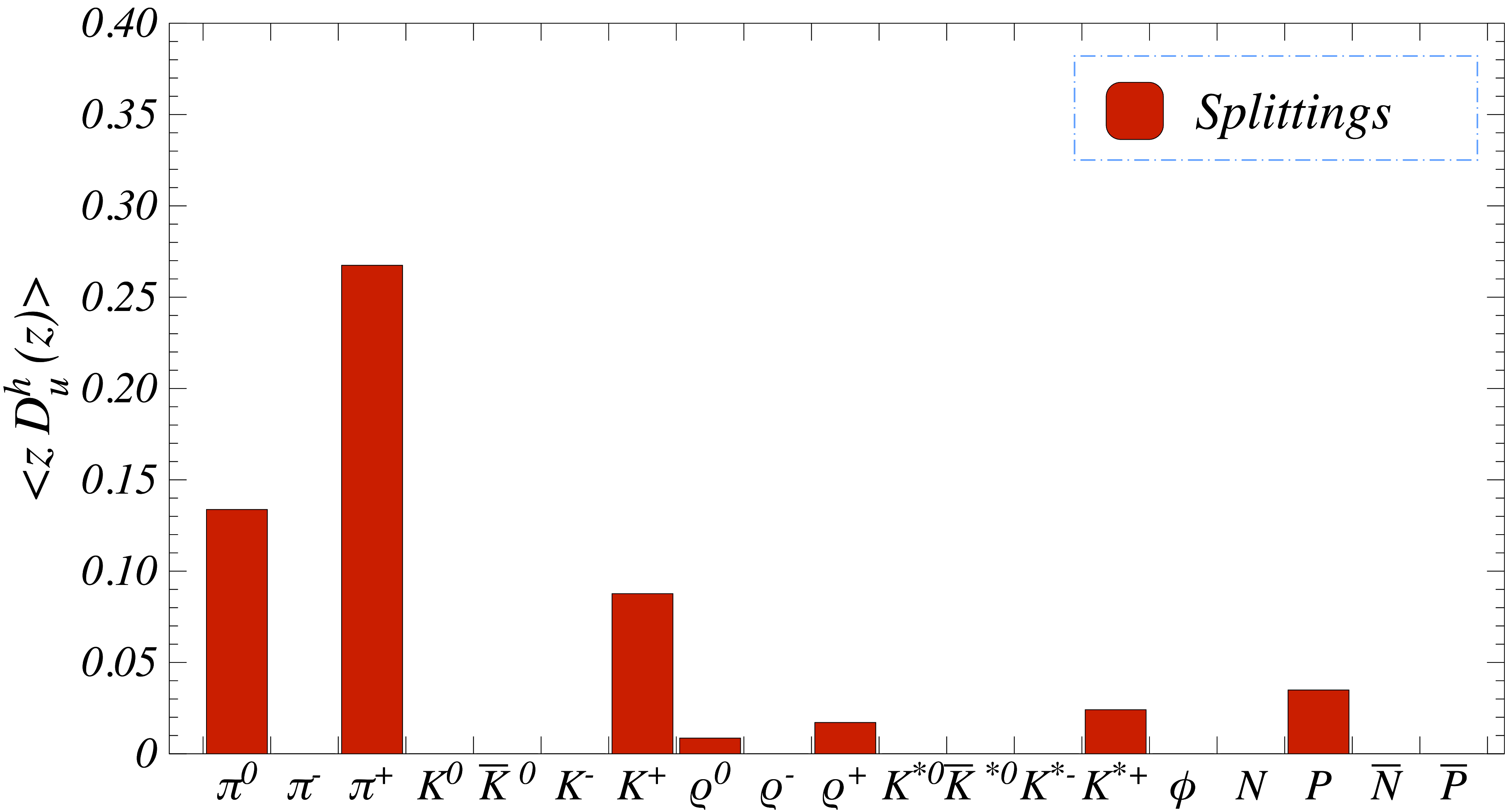




- Decay cross-section in light-front variables:

$$dP^{h\to h_1,h_2}(z_1) = \begin{cases} \frac{C_h^{h_1 h_2}}{8\pi}\, dz_1 & \text{if } z_1 z_2\, m_h^2 - z_2 m_{h1}^2 - z_1 m_{h2}^2 \geq 0;\ z_1 + z_2 = 1, \\ 0 & \text{otherwise.} \end{cases}$$

Results: Momentum Fractions



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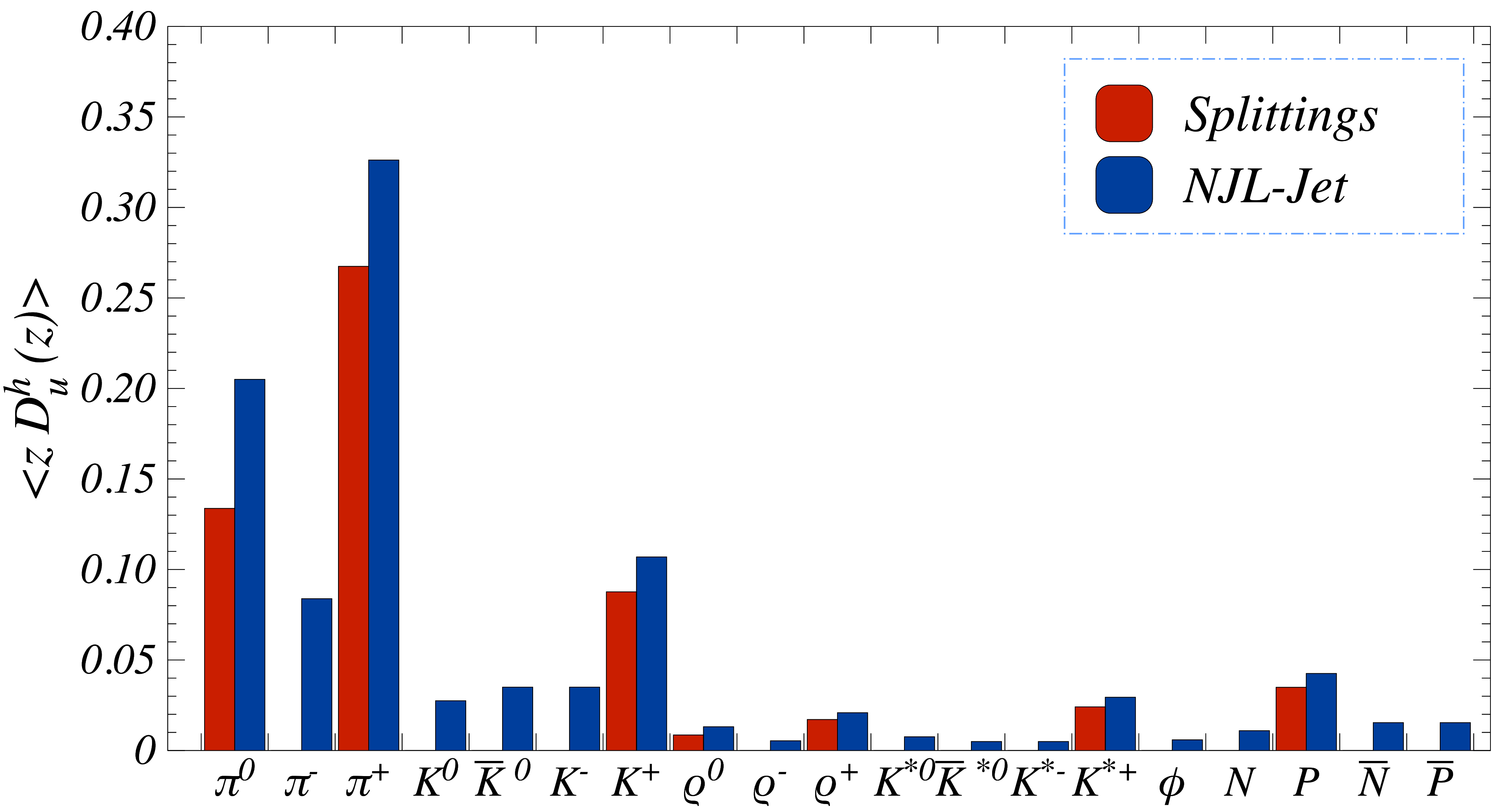
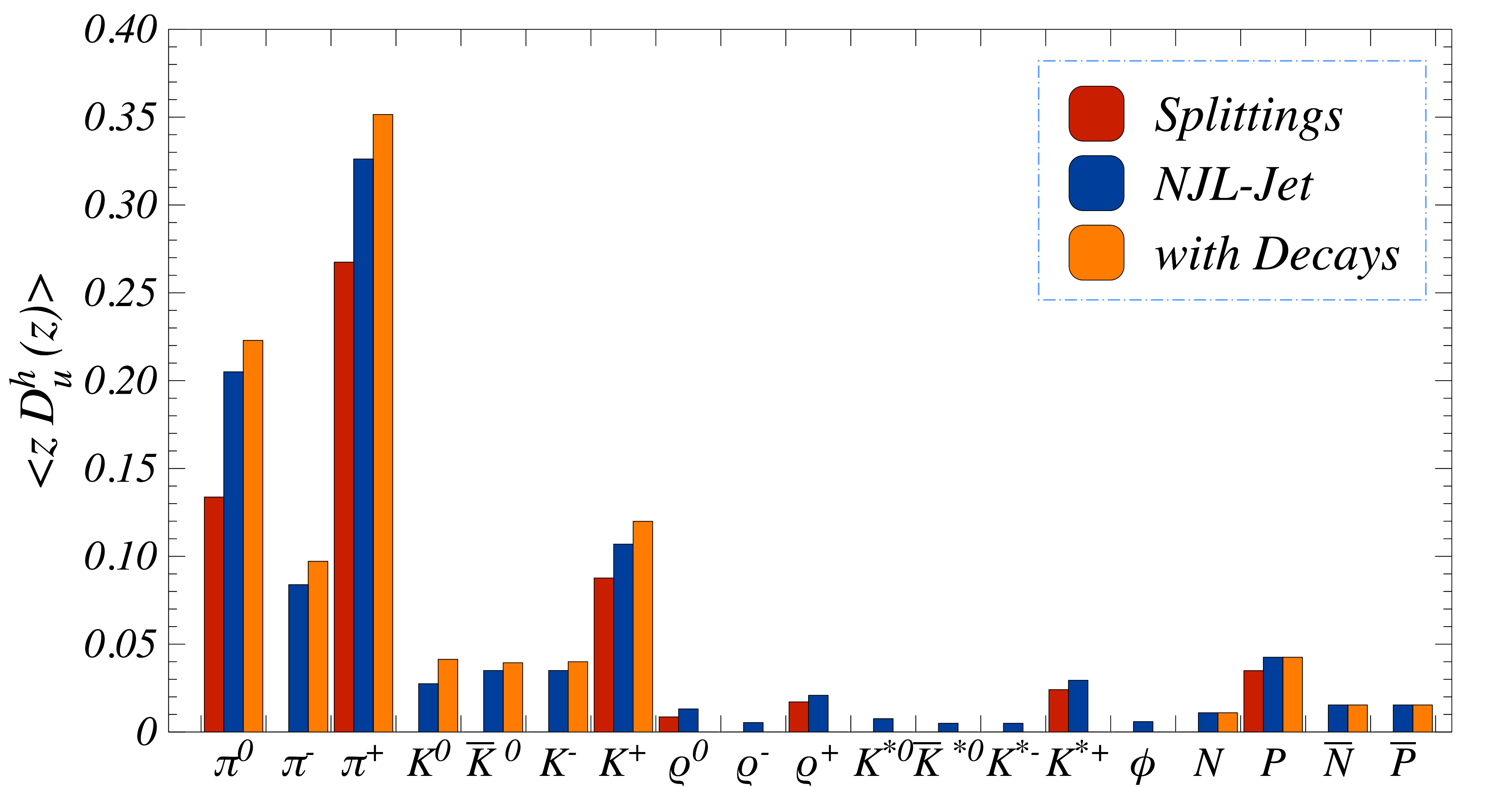
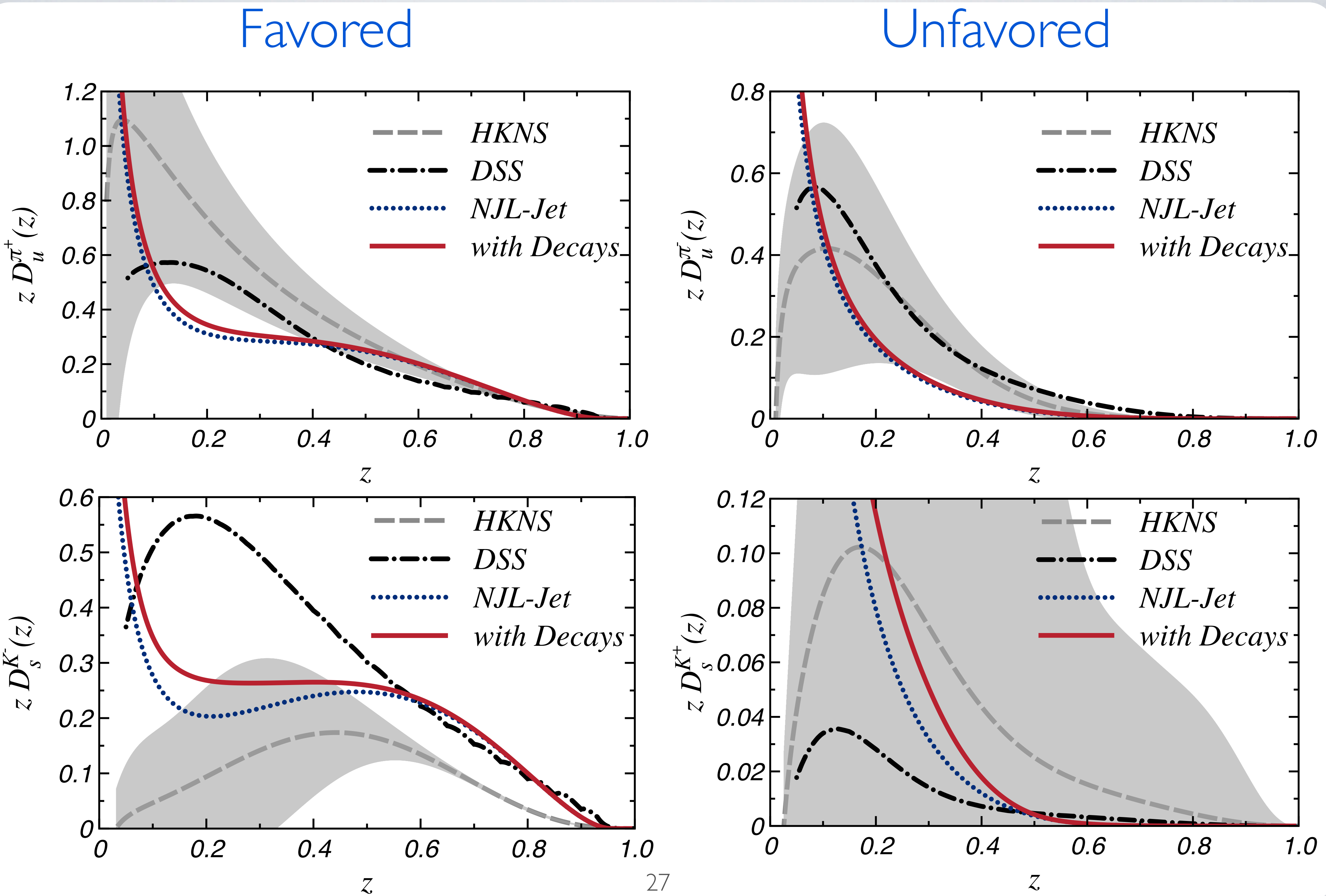


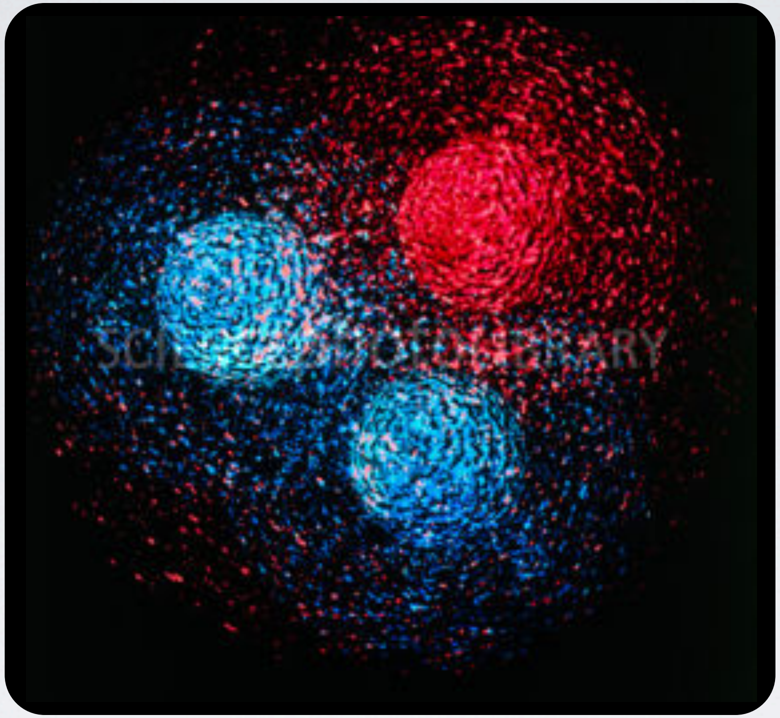
Results: Momentum Fractions



The Momentum (and Isospin) sum rules satisfied within numerical precision (less than 0.1 %)!

Results with vector mesons, N-Nbar: $Q^2 = 4\ \mathrm{GeV}^2$





3 DIMENSIONAL PICTURE OF NUCLEON FROM SIDIS:

TRANSVERSE MOMENTUM

EXPLORING HADRON STRUCTURE

A. Kotzinian, Nucl. Phys. B441, 234 (1995).

- Semi-inclusive deep inelastic scattering (SIDIS): $e\,N \rightarrow e\,h\,X$

- Cross-section factorizes: $P_T^2 \ll Q^2$

$$\frac{d\sigma^{lN\to l'hX}}{dx\,dQ^2\,dz\,d^2P_T} = \sum_q f_1^q(x, k_T^2, Q^2) \otimes d\sigma^{lq\to lq} \otimes D_q^h(z, p_\perp^2, Q^2)$$

Distribution

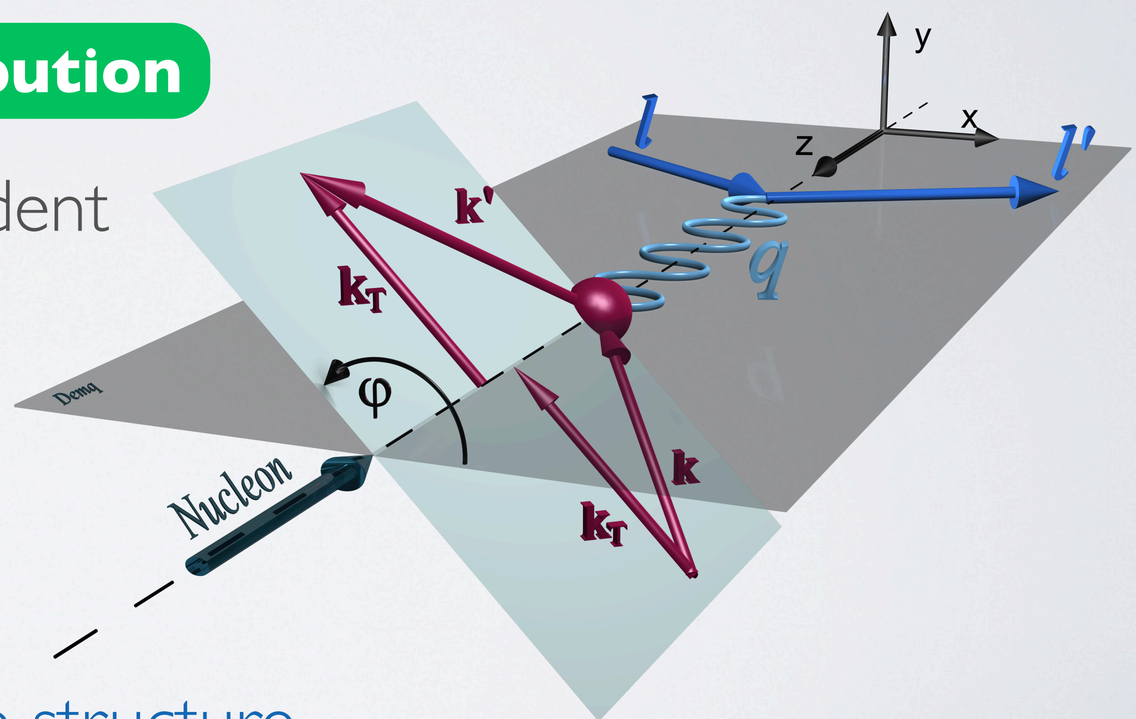
Fragmentation

$$\boxed{\mathbf{P_T} = \mathbf{P}_\perp + z\mathbf{k_T}}$$

- Transverse Momentum Dependent (TMD) PDFs and FFs:

$$\int d^2\mathbf{k}_\perp f(x, k_\perp^2) = f(x)$$

$$\int d^2\mathbf{P}_\perp D(z, P_\perp^2) = D(z)$$

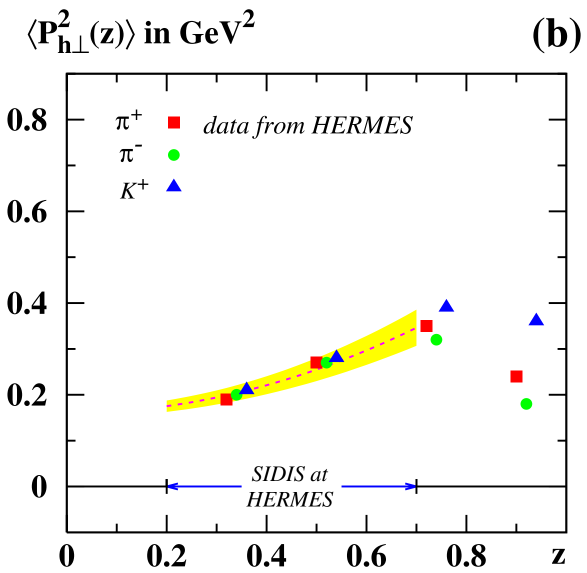


  - Access to nucleon's transverse structure.

  - NJL provides microscopic description of TMD PDFs and FFs!

AVERAGE TRANSVERSE MOMENTA

$$\langle k_T^2\rangle \equiv \frac{\int d^2\mathbf{k_T}\; k_T^2 f(x,k_T^2)}{\int d^2\mathbf{k_T}\; f(x,k_T^2)} \qquad \langle P_\perp^2\rangle \equiv \frac{\int d^2\mathbf{P}_\perp\; P_\perp^2 D(z,P_\perp^2)}{\int d^2\mathbf{P}_\perp\; D(z,P_\perp^2)}$$



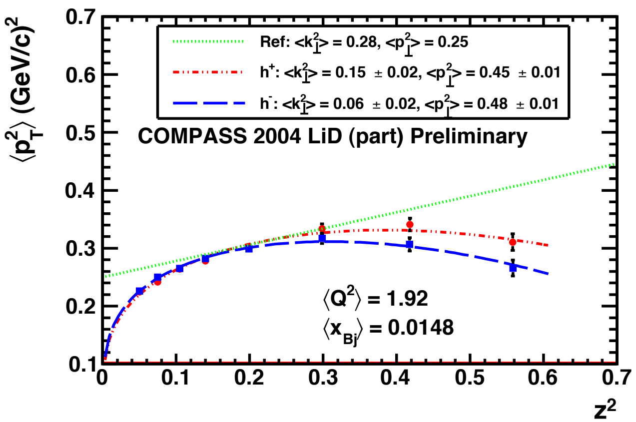
P. Schweitzer et al., Phys.Rev. D81, 094019 (2010).

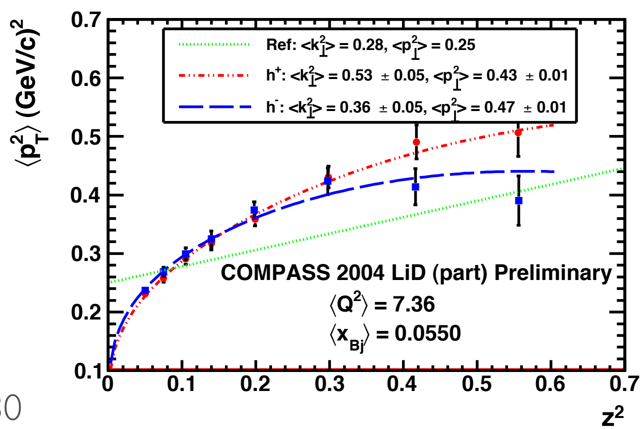
Using Gaussian Ansatz and:

$$D(z,P_\perp^2) = D(z)\frac{e^{-P_\perp^2/\langle P_\perp^2\rangle}}{\pi\langle P_\perp^2\rangle}$$

$$\boxed{\langle P_T^2\rangle(z) = \langle P_\perp^2\rangle + z^2\langle k_T^2\rangle}$$

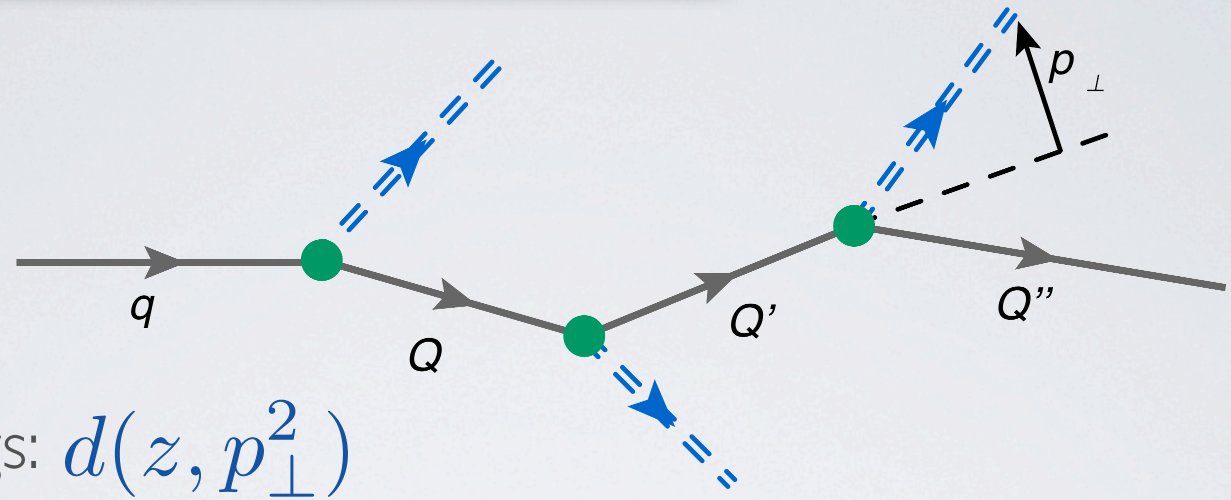
Non-trivial z dependence from COMPASS: **Rajotte arXiv:1008.5125**





INCLUDING THE TRANSVERSE MOMENTUM

H.M.,Bentz, Cloet, Thomas, PRD.85:014021, 2012

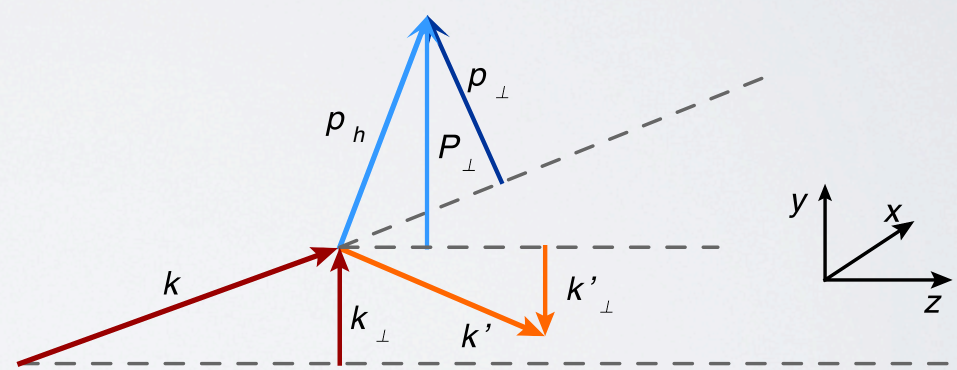


- TMD splittings: $d(z, p_\perp^2)$

- Conserve transverse momenta at each link.

$$\boxed{\mathbf{P}_\perp = \mathbf{p}_\perp + z\mathbf{k}_\perp}$$

$$\boxed{\mathbf{k}_\perp = \mathbf{P}_\perp + \mathbf{k}'_\perp}$$



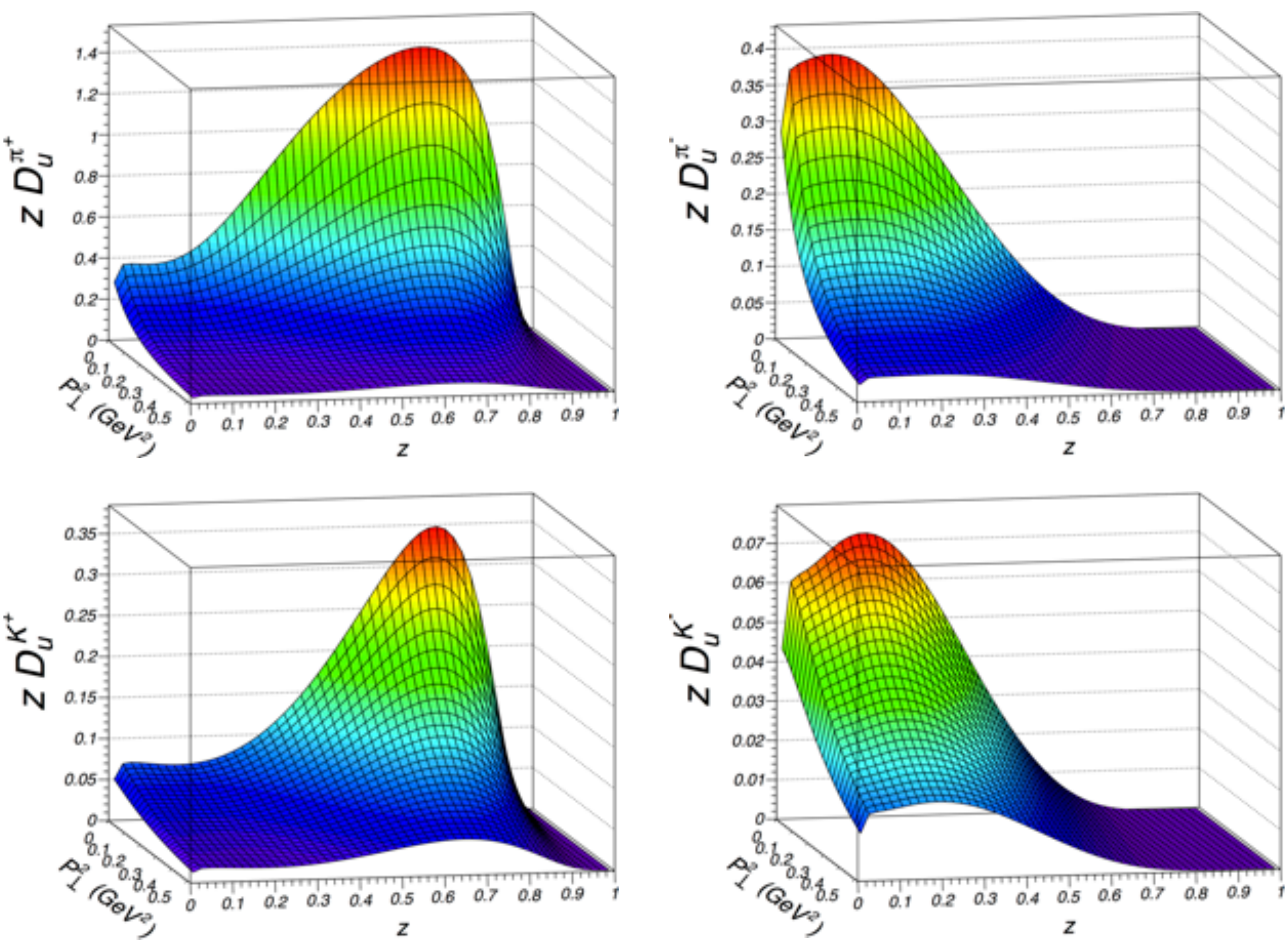
- Calculate the Number Density

$$D_q^h(z, P_\perp^2)\Delta z\ \pi\Delta P_\perp^2 = \frac{\sum_{N_{Sims}} N_q^h(z, z+\Delta z, P_\perp^2, P_\perp^2 + \Delta P_\perp^2)}{N_{Sims}}.$$

TMD FRAGMENTATION FUNCTIONS

FAVORED

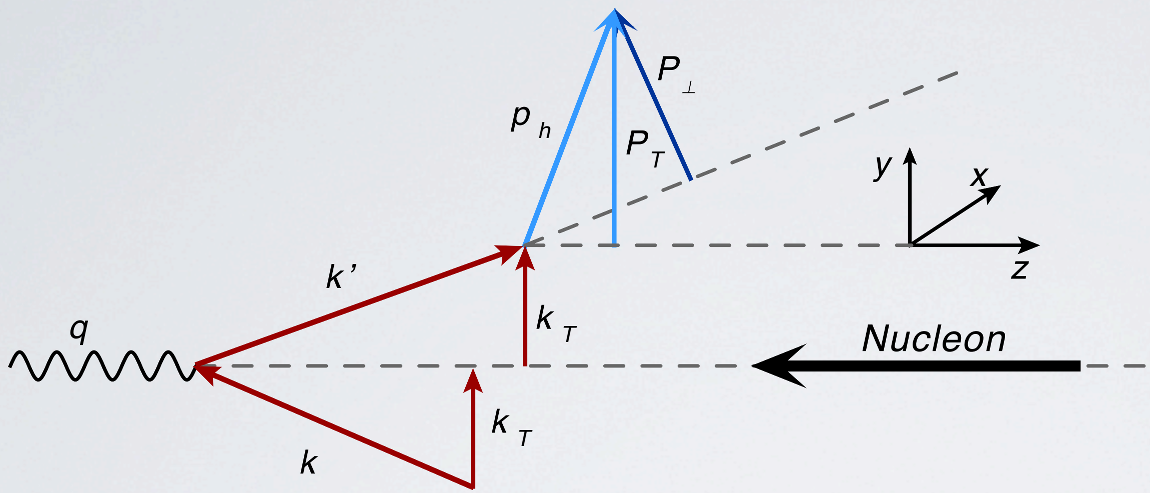
- UNFAVORED



π

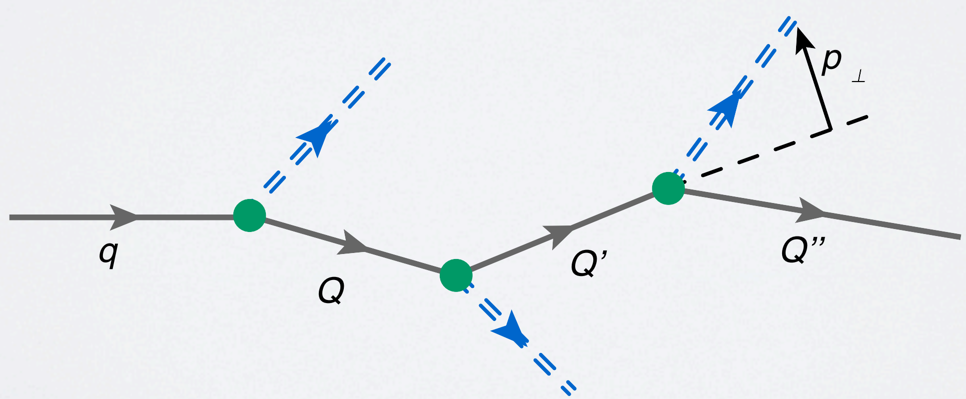
K

THE TRANSVERSE MOMENTA OF HADRONS IN SIDIS



$$\mathbf{P_T} = \mathbf{P}_\perp + z\mathbf{k_T}$$

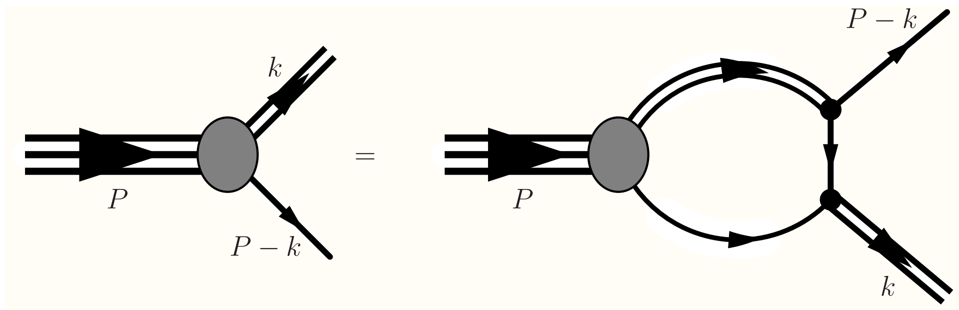
- Use TMD quark distribution functions from the NJL model .
- Use Quark-jet hadronization model and NJL splittings.



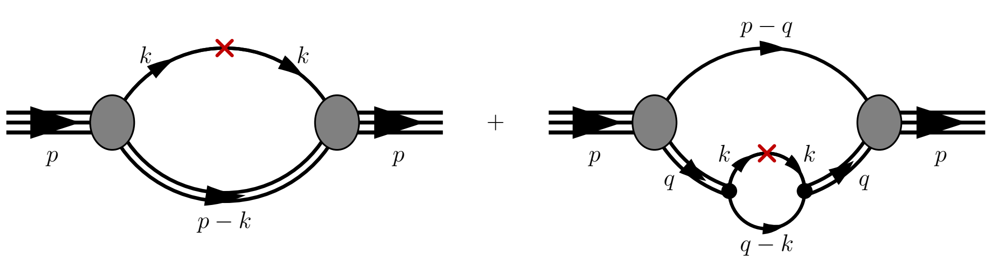
- Evaluate the cross-section using MC simulation.

NJL: NUCLEON PDFS

- Quark-diquark description of Nucleon using relativistic Faddeev approach



- PDFs from Feynman diagrams

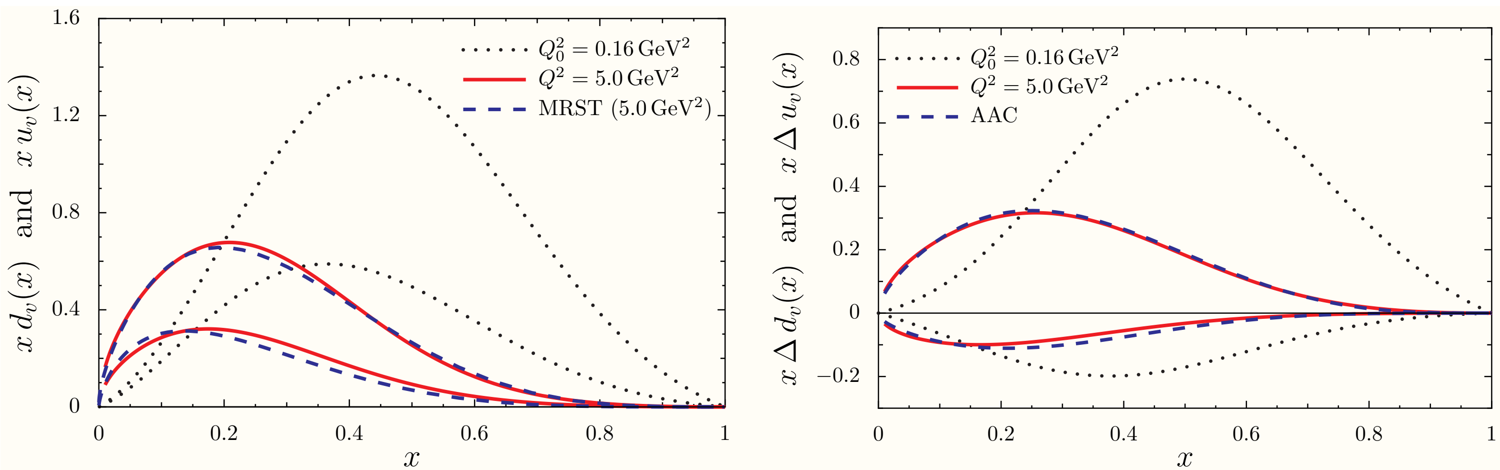


$$\mathcal{Q}(x,\boldsymbol{k_T}) = p^+ \int \frac{d\xi^- d\boldsymbol{\xi}_T}{(2\pi)^3}\, e^{ix\,p^+\,\xi^-}\, e^{-i\,\boldsymbol{k_T}\cdot\boldsymbol{\xi_T}} \left\langle N,S \left|\bar{\psi}_q(0)\,\gamma^+\,\mathcal{W}(\xi)\,\psi_q(\xi^-,\xi_T)\right| N,S\right\rangle\Big|_{\xi^+=0}$$

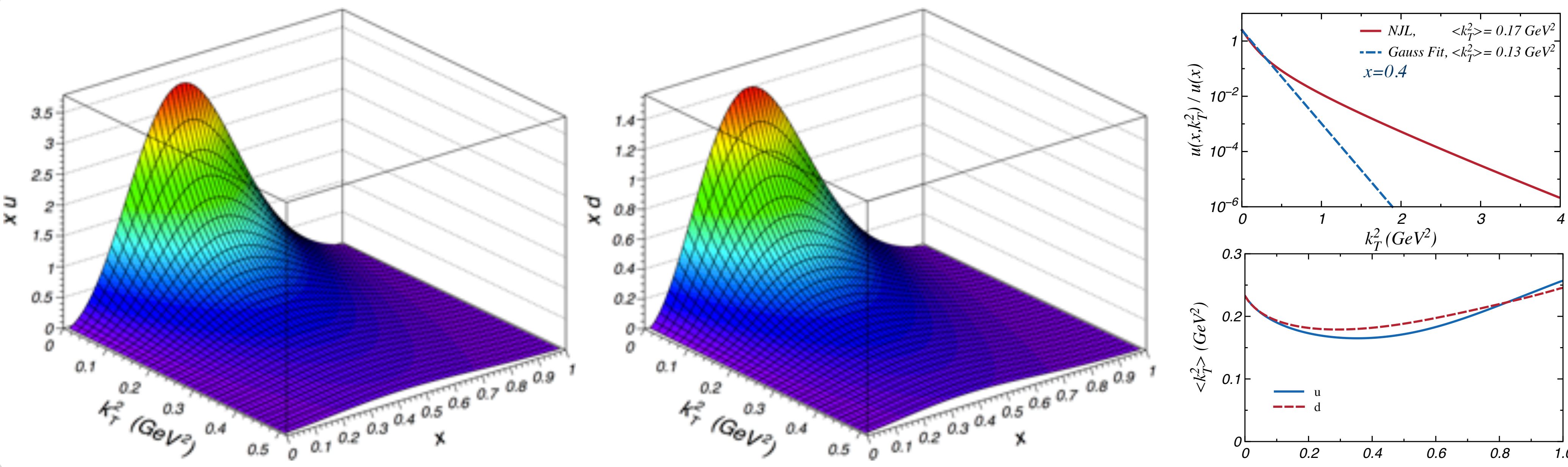
$$\mathcal{Q}(x,\boldsymbol{k_T}) = q(x,k_T^2) - \frac{\varepsilon^{-+ij}\,k_T^i\,S_T^j}{M}\,q_{1T}^{\perp}(x,k_T^2)$$

NJL: NUCLEON PDFS - RESULTS

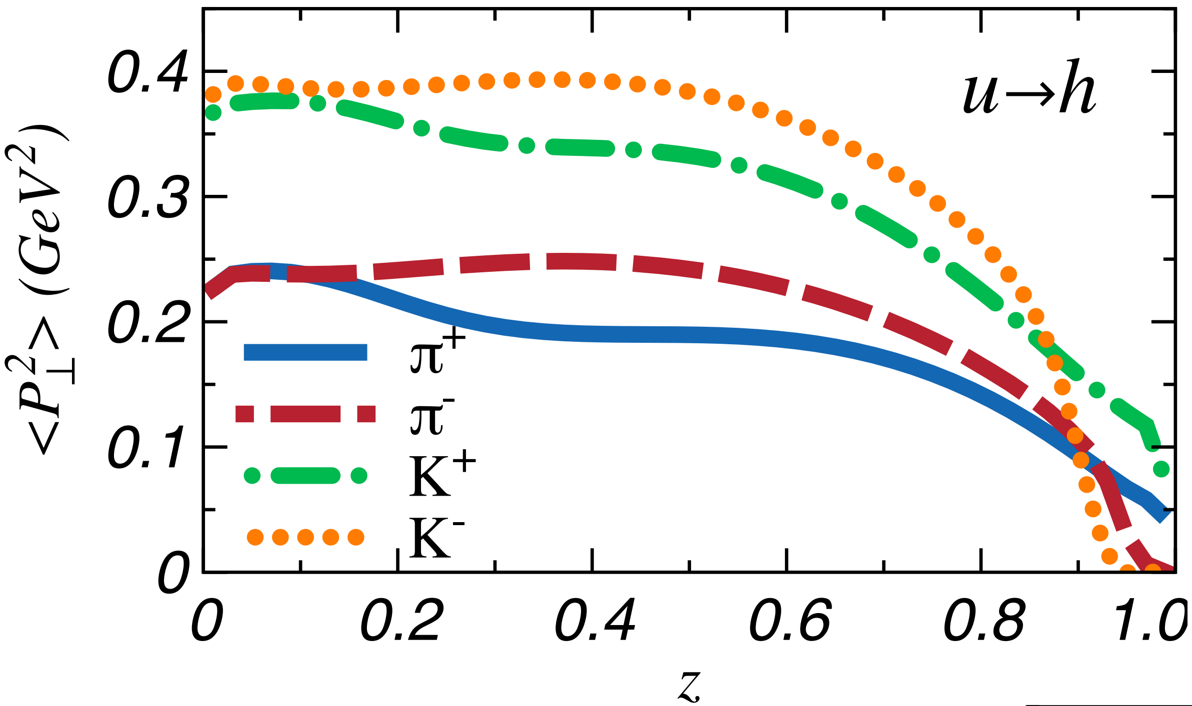
- Integrated PDFs



- TMD PDFs

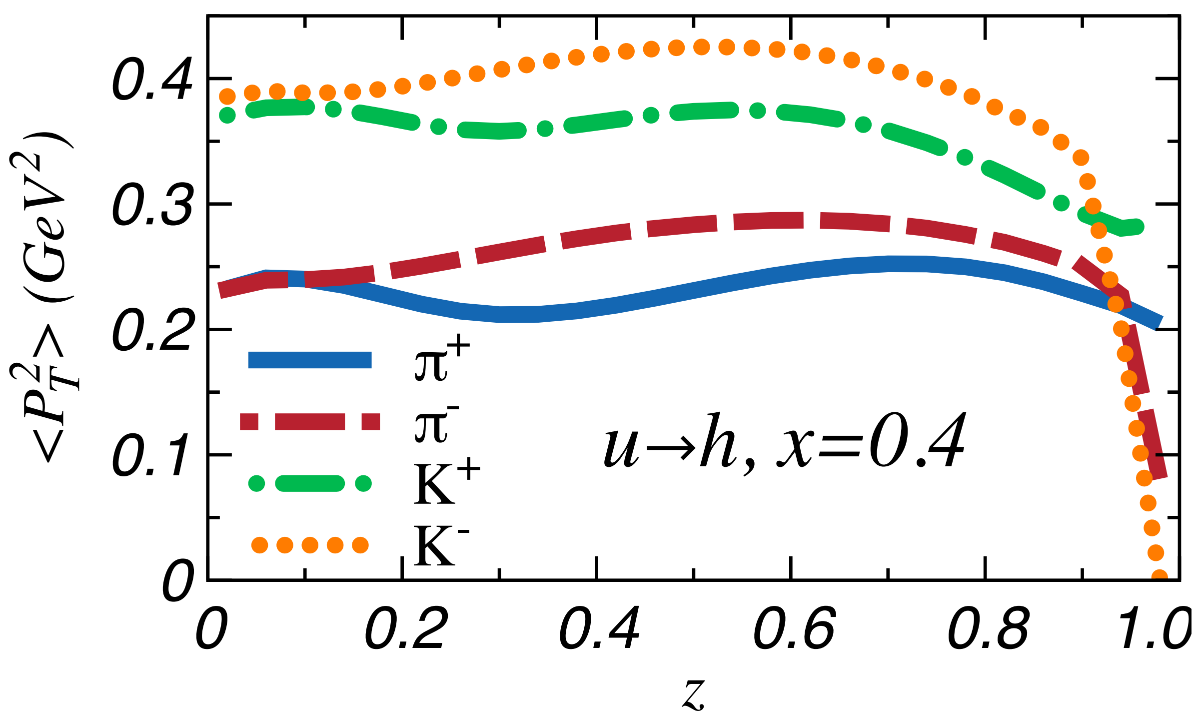


AVERAGE TRANSVERSE MOMENTA VS Z



FRAGMENTATION

SIDIS



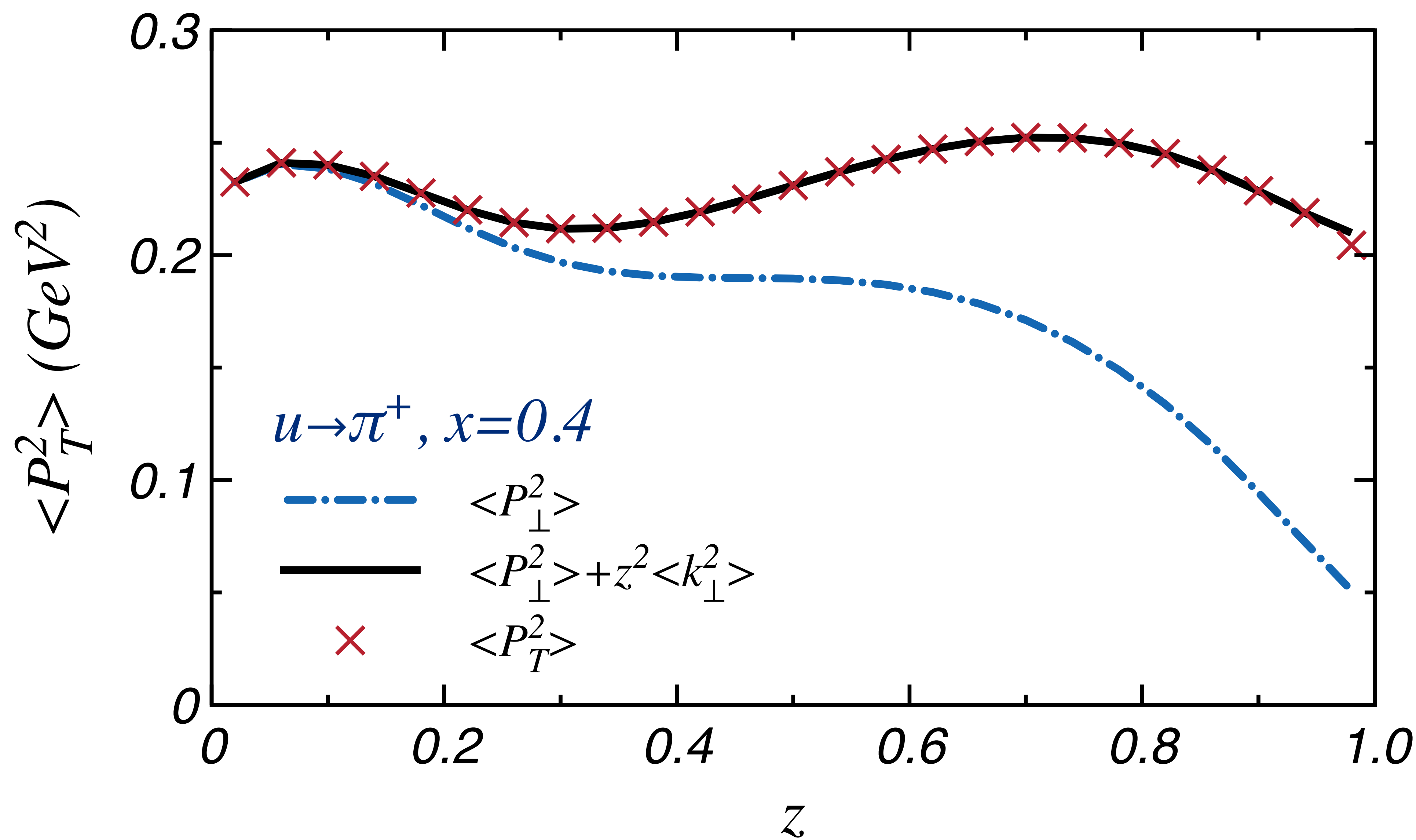
CROSS-CHECK OF MC FRAMEWORK

Input:

$$\mathbf{P_T} = \mathbf{P}_\perp + z\mathbf{k_T}$$

Output:

$$\langle P_T^2 \rangle(x,z) = \langle P_\perp^2 \rangle(z) + z^2 \langle k_T^2 \rangle(x)$$

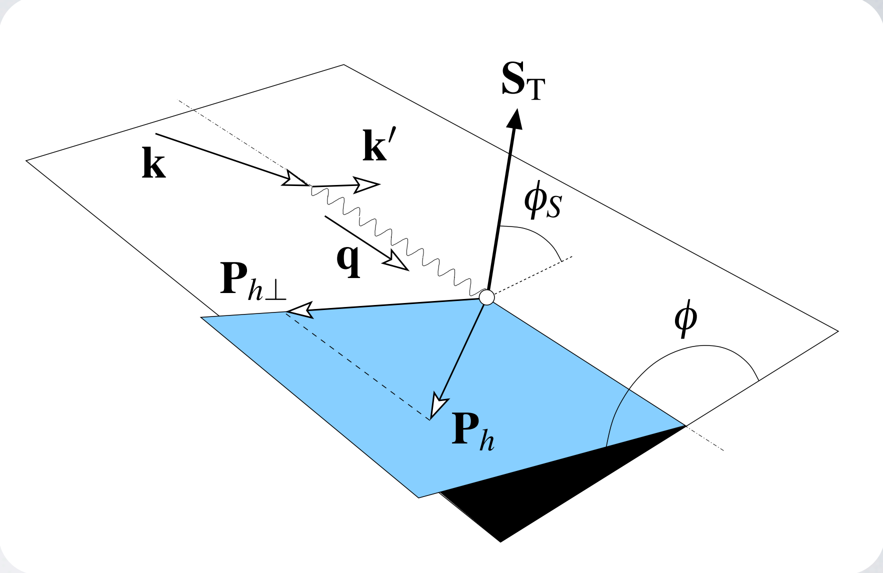


COLLINS EFFECT

SIDIS POLARIZED CROSS-SECTION

A. Bacchetta, JHEP08, 023 (2008).

- **For polarized SIDIS cross-section there are *18 terms* in leading twist expansion:**



$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_S\,d\phi_h\,dP_{h\perp}^2} \sim F_{UU,T} + \varepsilon F_{UU,L} + \ldots$$

$$+ |\boldsymbol{S}_\perp| \left[\sin(\phi_h - \phi_S) \left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\, F_{UT,L}^{\sin(\phi_h-\phi_S)} \right) + \varepsilon\, \sin(\phi_h + \phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)} + .. \right]$$

Sivers Effect

Collins Effect

- Extract the specific harmonics:

$$F_{UU} \sim \mathcal{C}[f_1\ D_1] \qquad F_{UT}^{\sin(\phi_h+\phi_S)} \sim \mathcal{C}[h_1\ H_1^\perp]$$

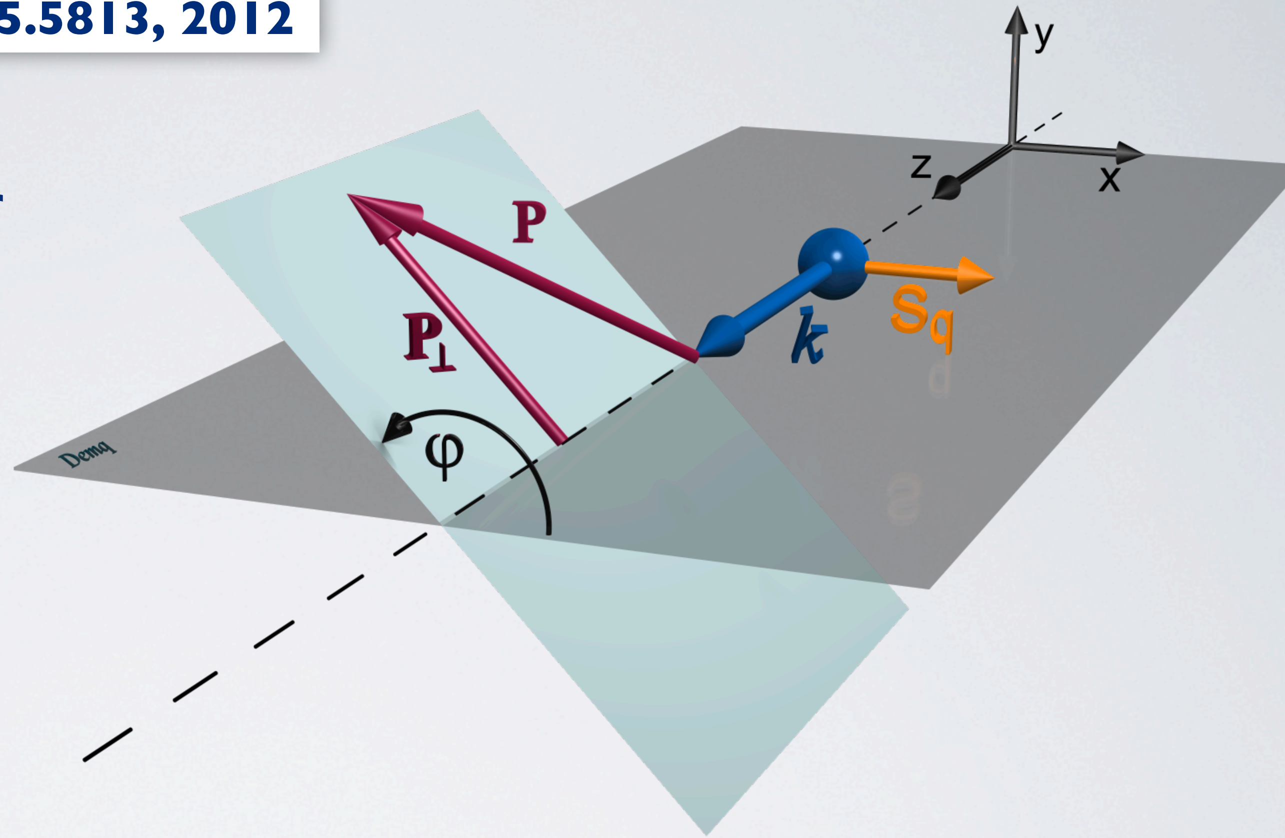
- NEED Collins Function to access the Transversity from SIDIS!

COLLINS FRAGMENTATION FUNCTION

H.M., Thomas, Bentz, arXiv:1205.5813, 2012

- **Collins Effect:**

  Azimuthal Modulation of Transversely Polarized Quark' Fragmentation Function.



Unpolarized

$$D_{h/q^\uparrow}(z, P_\perp^2, \varphi) = D_1^{h/q}(z, P_\perp^2) - H_1^{\perp h/q}(z, P_\perp^2)\frac{P_\perp S_q}{z m_h}\sin(\varphi)$$

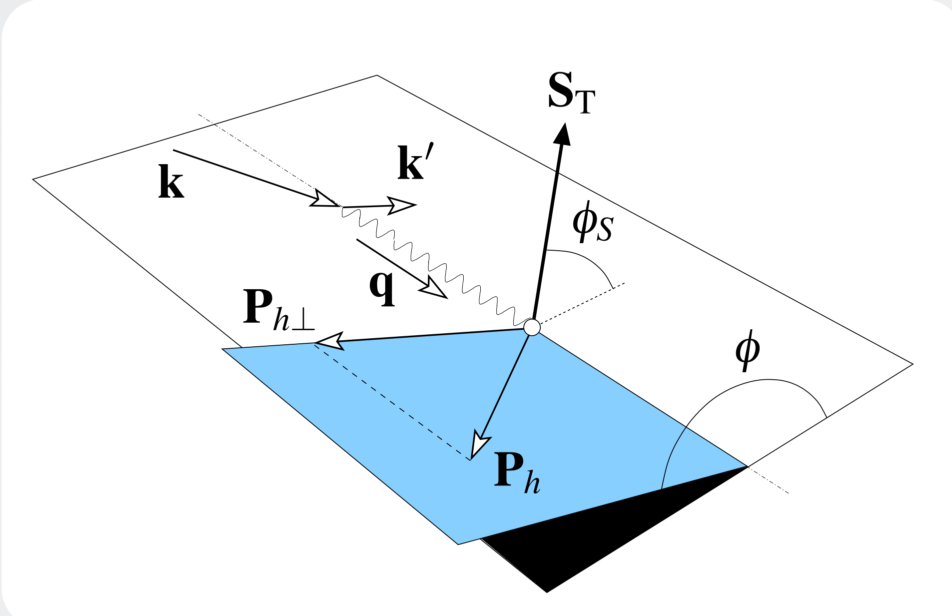
Collins

- **Chiral-ODD: Needs to be coupled with another chiral-odd quantity to be observed.**

EXPERIMENTAL MEASUREMENTS IN HERMES $l\ \vec{p} \to l'\ h\ X$

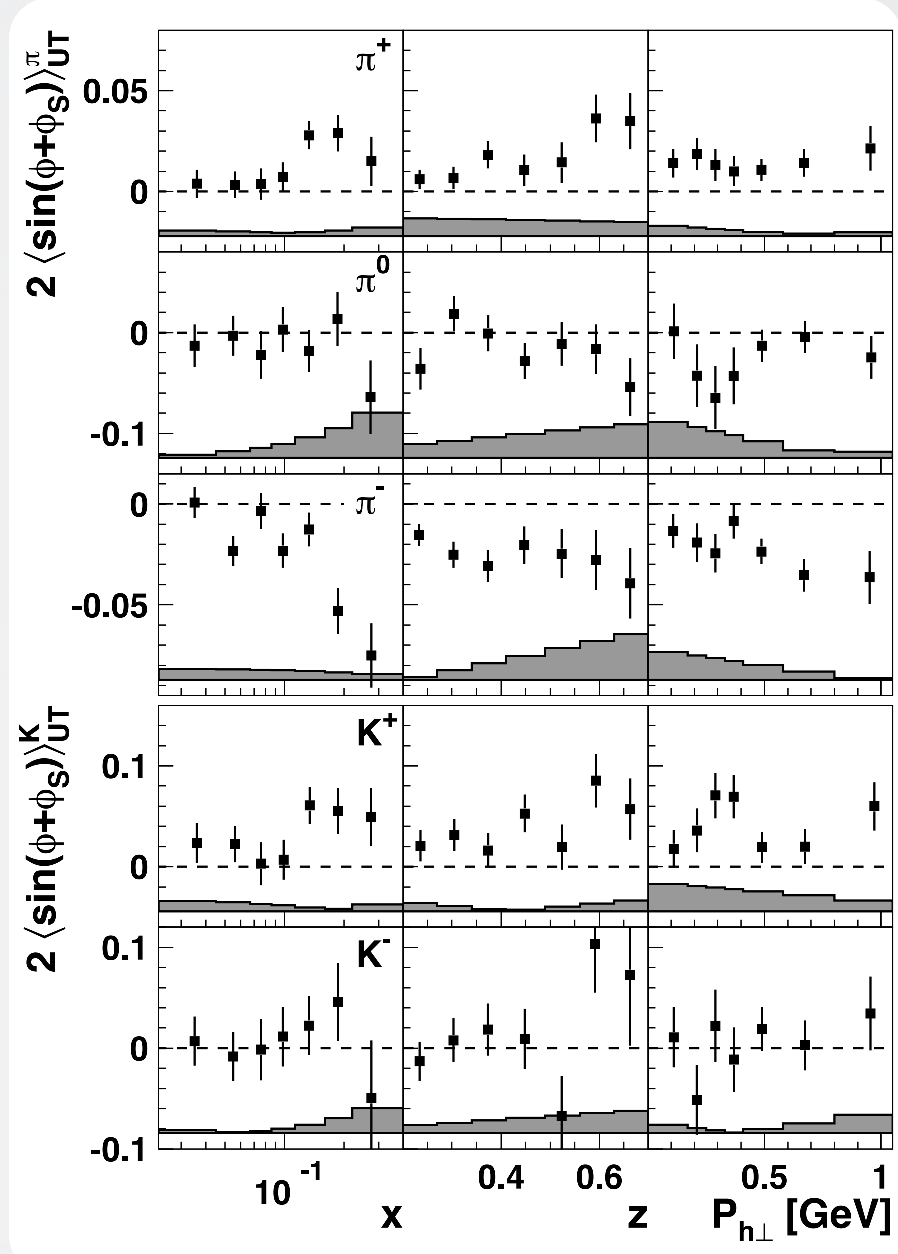
Airapetian et al, Phys.Lett. B693 (2010) 11-16.

- SIDIS with transversely polarized target:



$$\langle \sin(\phi + \phi_S) \rangle_{UT}^{h} \sim \frac{\mathcal{C}[h_1^q\ H_{1q}^{\perp h/q}]}{\mathcal{C}[f_1^q\ D_1^{h/q}]}$$

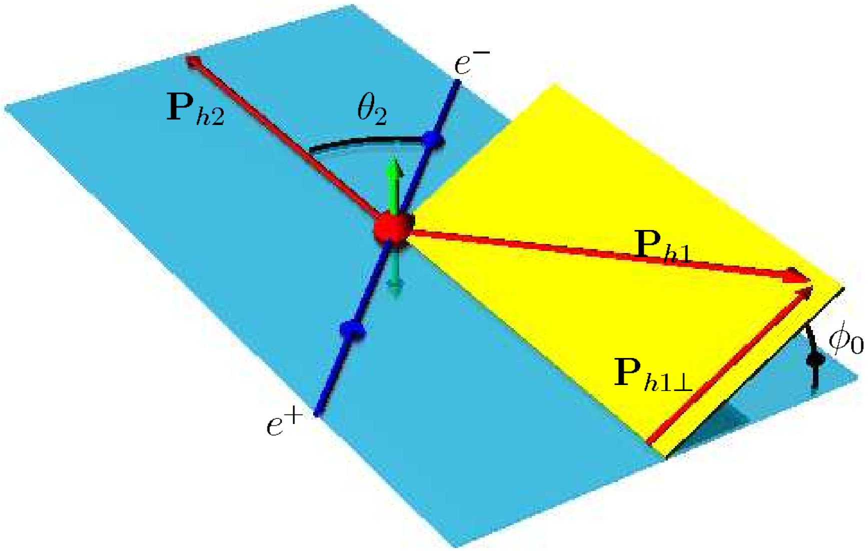
- Opposite sign for the charged pions.
- Large positive signal for K^+.
- Consistent with 0 for π^0 and K^-.

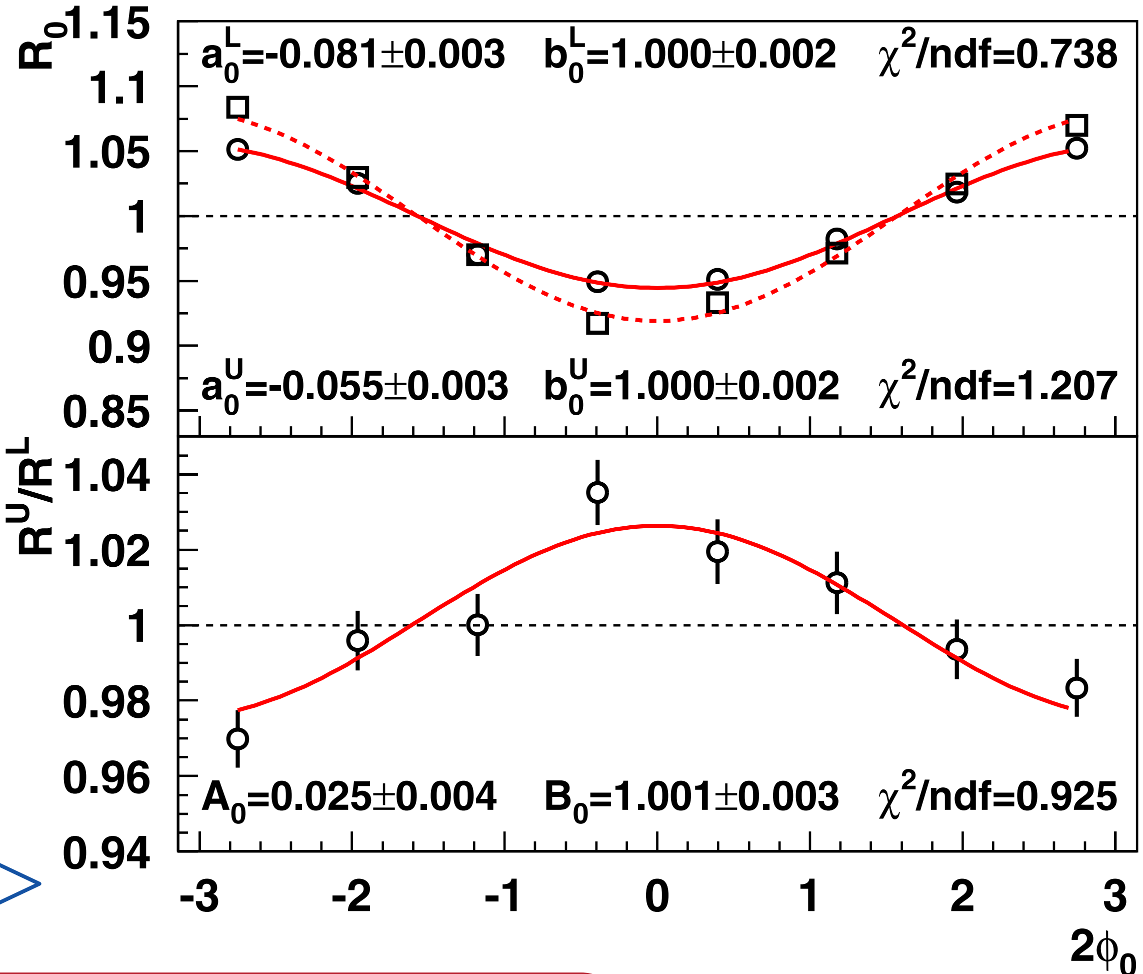


DIRECT EXPERIMENTAL CONFIRMATION

BELLE, R. Seidl et al.,Phys.Rev.Lett. 96, 232002 (2006).

$$e^+e^- \to h_1 h_2 X$$





$$R_0 = N_0(2\phi_0)/< N_0 >$$

$$\boxed{R_0 = a_0 \cos(2\phi_0) + b_0}$$

$$a_0(\theta_2, z_1, z_2) = \frac{\sin^2\theta}{1+\cos^2\theta}\frac{f(H^{\perp}_{1,q}(z_1)H^{\perp}_{1,\bar{q}}(z_2))}{D^q_1(z_1)D^{\bar{q}}_1(z_2)}$$

GLOBAL FITS TO EXPERIMENTAL DATA

Anselmino et al., Nuclear Physics B (Proc. Suppl.) 191 (2009) 98–107.

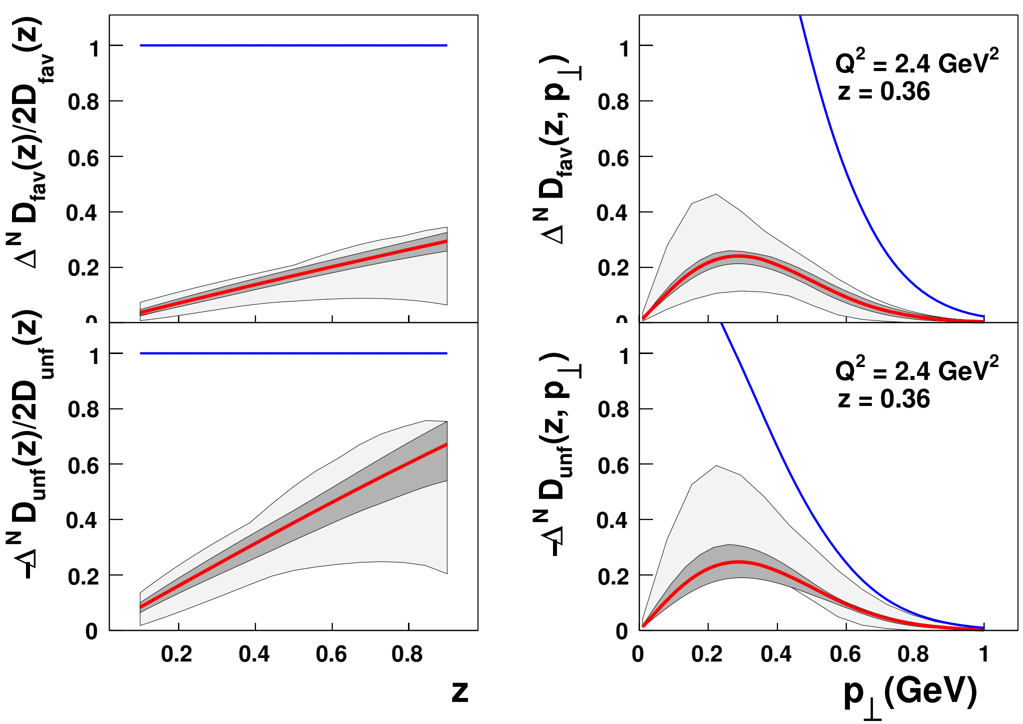
Consider e^+e^- and SIDIS

BELLE, R. Seidl et al., Phys. Rev. D78 (2008) 032011.
HERMES, M. Diefenthaler, Proc. of DIS2007 (2007).
COMPASS, M. Alekseev et al., arXiv:0802.2160.

$$\Delta^N D_{h/q^\uparrow}(z, p_\perp) = \frac{2p_\perp}{zm_h} H_1^{\perp h/q}(z, p_\perp) \qquad \Delta^N D_{h/q^\uparrow}(z) = \int d^2\boldsymbol{p}_\perp \Delta^N D_{h/q^\uparrow}(z, p_\perp) = 4H_1^{\perp(1/2)}(z)$$

$$\Delta^N D_{h/q^\uparrow}(z)/2D_1(z) = 2H_1^{\perp(1/2)}(z)/D_1(z)$$

Parametrizations and the fits.



Using Gaussian Ansatz:

$$D_1^{h/q}(z, p_\perp) \sim \frac{e^{-p_\perp^2/<p_\perp^2>}}{\pi <p_\perp^2>}$$

$$\Delta^N D_{h/q^\uparrow}(z, p_\perp) \sim \frac{p_\perp}{M} e^{-p_\perp^2/M^2} \frac{e^{-p_\perp^2/<p_\perp^2>}}{\pi <p_\perp^2>}$$

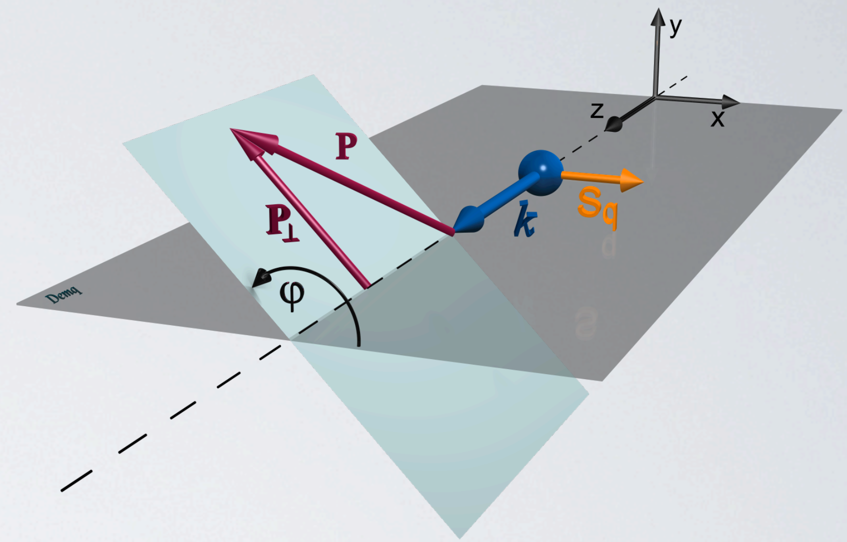
$$D_{\pi^+/u,\bar{d}} = D_{\pi^-/d,\bar{u}} = D_{fav}$$

$$D_{\pi^+/d,\bar{u}} = D_{\pi^-/u,\bar{d}} = D_{\pi^\pm/s,\bar{s}} = D_{unf}$$

COLLINS FRAGMENTATION FUNCTION

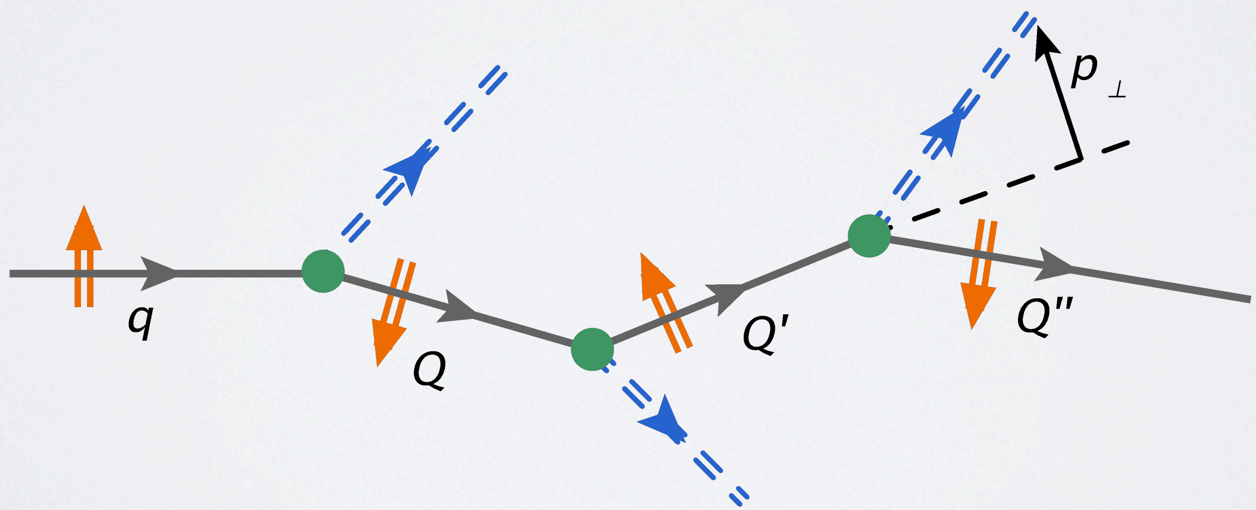
- **Collins Effect:**

  Azimuthal Modulation of the Fragmentation Function of a Transversely Polarized Quark.



$$D_{h/q^\uparrow}(z, P_\perp^2, \varphi) = D_1^{h/q}(z, P_\perp^2) - H_1^{\perp h/q}(z, P_\perp^2)\frac{P_\perp S_q}{z m_h}\sin(\varphi)$$

- **Extend the NJL-jet Model to Include the Quark's Spins.**



- **Model Calculated Elementary Collins Function as Input**

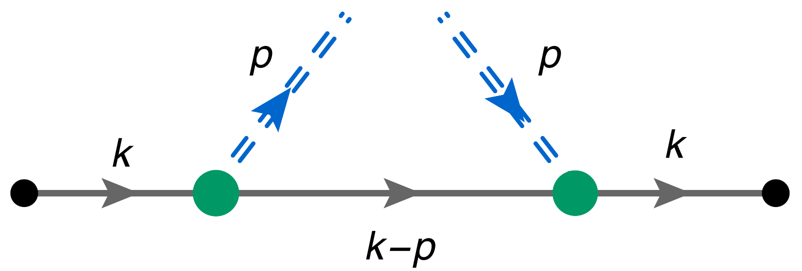
ELEMENTARY POLARIZED SPLITTINGS

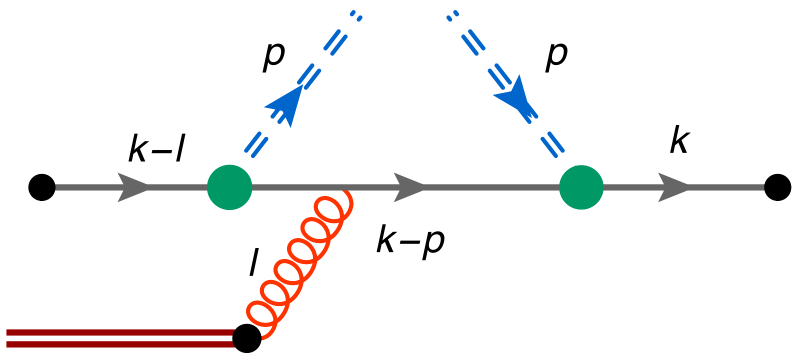
- One-quark truncation of the wavefunction:

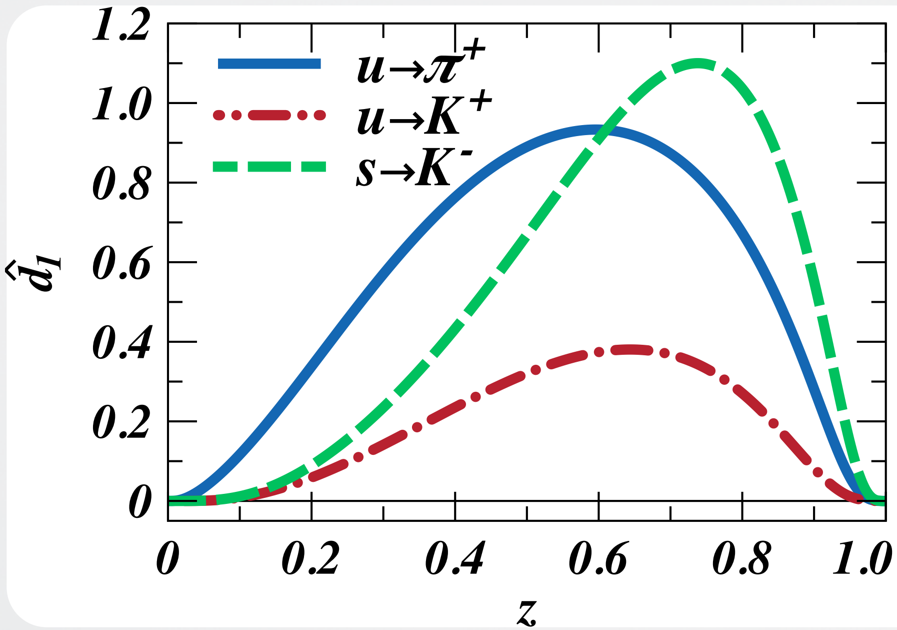
Bacchetta et. al., Phys. Lett. B659, 234 (2008).
Gamberg et. al.,Phys.Rev. D68, 051501 (2003).

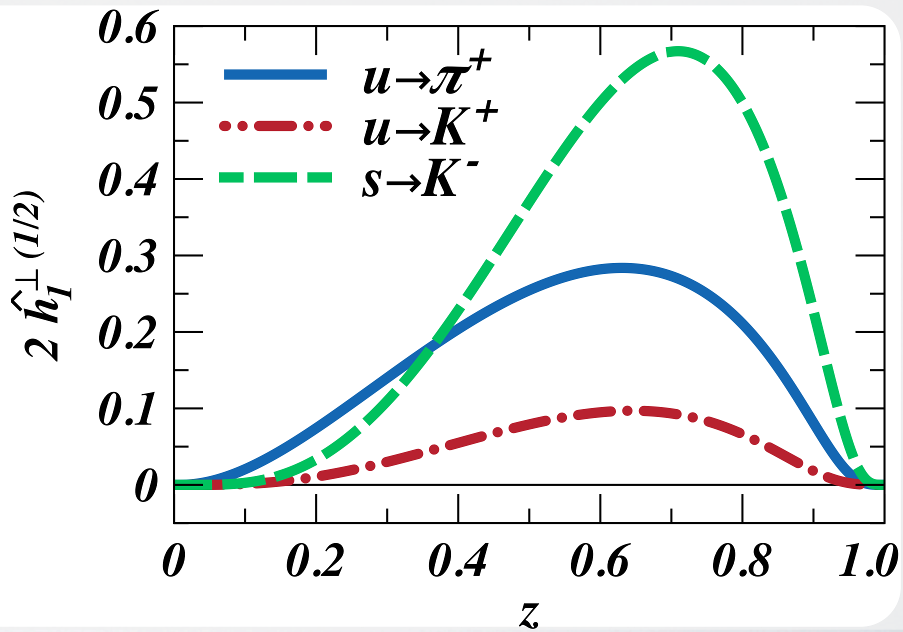
$$d_1^{h/q}(z,p_\perp^2) = \frac{1}{2}\mathrm{Tr}\left[\Delta_0(z,p_\perp^2)\gamma^+\right]$$

$$\frac{\epsilon_T^{ij} p_{\perp j}}{z m_h}\hat{H}_1^\perp(z,p_\perp^2) = \mathrm{Tr}[\Delta_0(z,\boldsymbol{p}_\perp^2)\imath\sigma^{i-}\gamma_5]$$

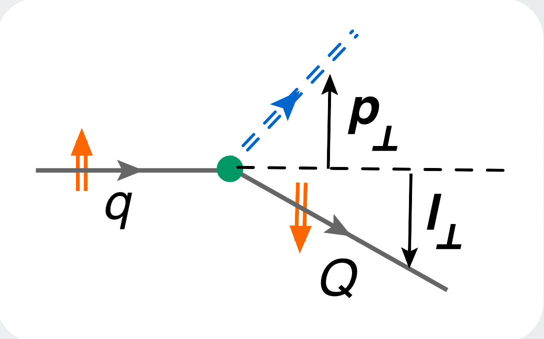








QUARK SPIN FLIP PROBABILITY



- Consider Elementary Splitting.
- *Approximation: only tree-level amplitude!*

Y.V. Kovchegov and M. D. Sievert (2012), 1201.5890.

- Use Lepage-Brodsky Spinors in helicity base to construct the transversely polarized quark spinors:

$$U_\chi \equiv \frac{1}{\sqrt{2}}\left[U_{(+z)} + \chi\, U_{(-z)}\right] \qquad \bar{U}_\chi(k,m)U_{\chi'}(k,m) = \delta_{\chi,\chi'}2m$$

$$(\not{k} - m)\ U_\chi = 0 \qquad W_1\ U_\chi = \chi\frac{m}{2}U_\chi$$

- Where Pauli-Lubanski vector as Lorentz-covariant spin operator:

$$W_\mu \equiv -\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}S^{\nu\rho}k^\sigma \qquad S^{\nu\rho} \equiv \frac{\imath}{4}\left[\gamma^\nu, \gamma^\rho\right]$$

- The corresponding matrix elements between *in* and *out* states: $\Psi_{out} = a_1\ U_1(l, M_2) + a_{-1}\ U_{-1}(l, M_2)$

$$\left|\bar{U}_{\chi'}(l, M_2)\gamma^5 U_\chi(k, M_1)\right|^2 = \delta_{\chi,\chi'}\frac{l_x^2}{1-z} + \delta_{\chi,-\chi'}\frac{l_y^2 + (M_2 - (1-z)M_1)^2}{1-z}$$

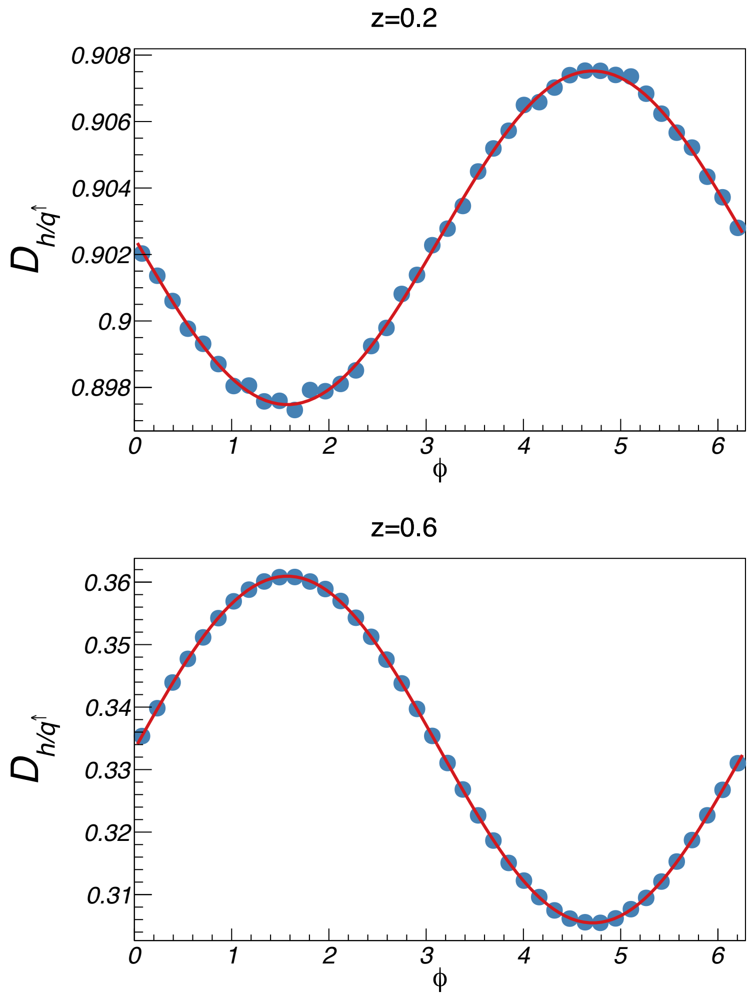
- ***Spin non-flip and flip probabilities are proportional to:***

$$|a_1|^2 \sim l_x^2,\ \ |a_{-1}|^2 \sim l_y^2 + (M_2 - (1-z)M_1)^2$$

INTEGRATED POLARIZED FRAGMENTATIONS

- First: Integrate Polarized Fragmentations over $P_\perp^2$

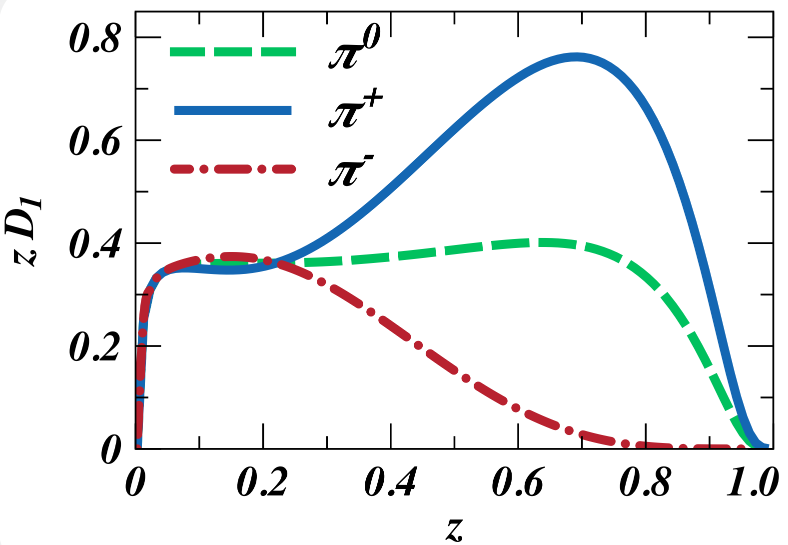
$$D_{h/q^\uparrow}(z,\varphi) \equiv \int_0^\infty dP_\perp^2 \; D_{h/q^\uparrow}(z,P_\perp^2,\varphi) = \frac{1}{2\pi}\left[D_1^{h/q}(z) \; - 2H_{1(h/q)}^{\perp(1/2)}(z) S_q \sin(\varphi)\right]$$



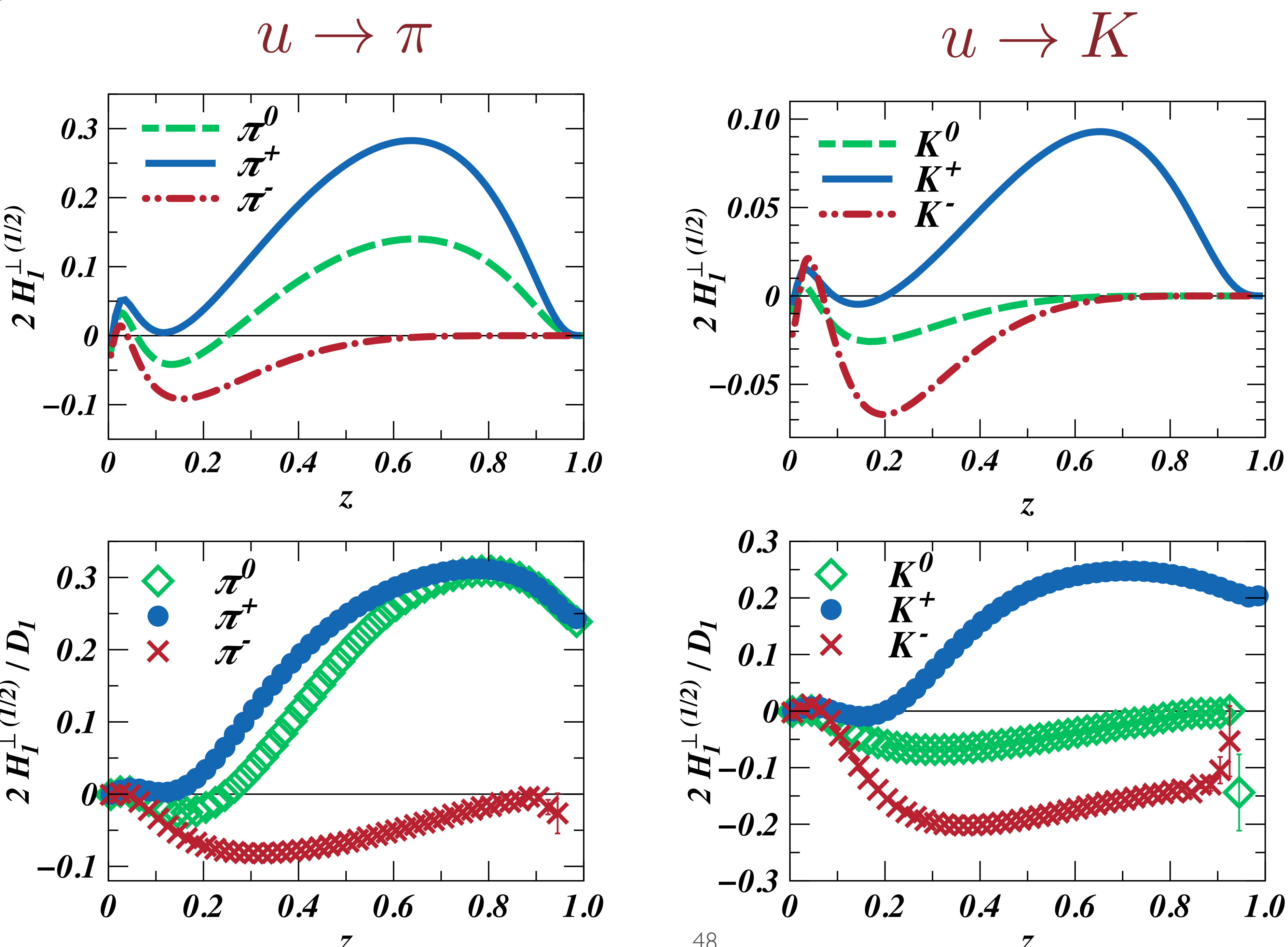
$$D_1^{h/q}(z) \equiv \pi \int_0^\infty dP_\perp^2 \; D_1^{h/q}(z,P_\perp^2)$$

$$H_{1(h/q)}^{\perp(1/2)}(z) \equiv \pi \int_0^\infty dP_\perp^2 \frac{P_\perp}{2zm_h} H_1^{\perp h/q}(z,P_\perp^2)$$

- Fit with form: $c_0 + c_1 \sin(\varphi)$



1/2 MOMENT OF COLLINS FUNC.



- Collins
- Ratio

FINAL REMARKS

- DIS is the tool of the modern subatomic physics.
- The *non-perturbative* QCD information is encoded in *UNIVERSAL* parton distribution and fragmentation functions.
- Fragmentation functions are essential to probe the structure of hadrons experimentally.
- **FFs are much worse determined than PDFs!**
- Models are needed to guide the extraction from experimental data.
- NJL-jet is an exciting new framework for modeling DIS processes using effective quark model description of both FFs and PDFs.