

Determining Sigma – Lambda mass mixing

R. Horsley, J. Najjar, Y. Nakamura, H. Perlt, D. Pleiter,
P. E. L. Rakow, G. Schierholz, A. Schiller, H. Stüben and
J. M. Zanotti

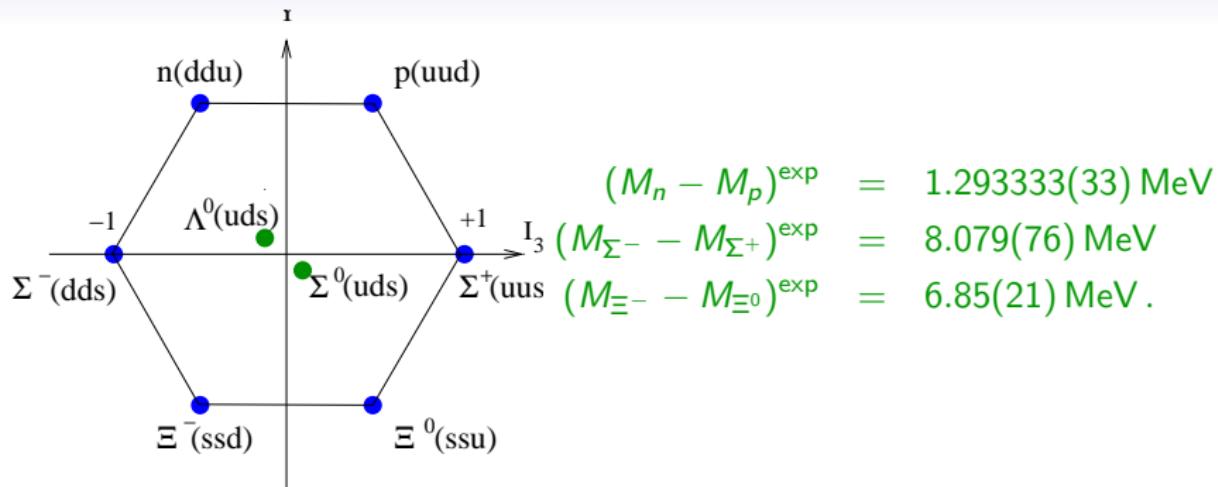
– QCDSF-UKQCD Collaboration –

Edinburgh – Regensburg – RIKEN (Kobe) – Leipzig – FZ (Jülich) – Liverpool – DESY – Hamburg – Adelaide

[LHP 2015, Cairns, Australia]



Introduction

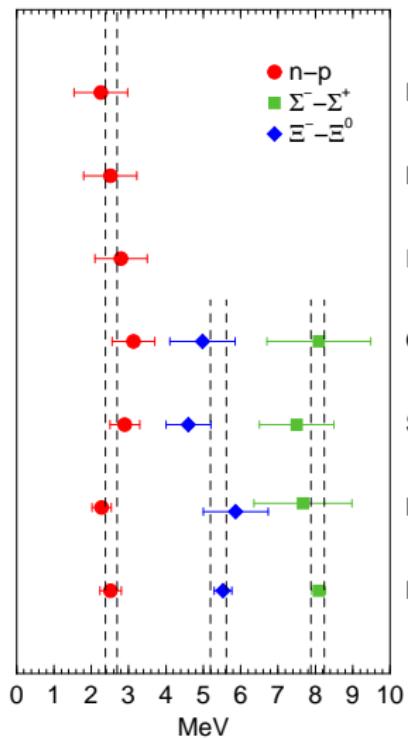


- Baryon octet ‘Outer’ ring
 - Small mass differences [expt precision way beyond what achieve here]
 - Isospin breaking effects:
 - QED component [not considered here]
 - $m_d - m_u$ component — pure QCD
- Baryon octet ‘centre’
 - $\Lambda^0(uds)$, $\Sigma^0(uds)$ have the same quark content (quantum numbers) but different wavefunctions

Previous status

Baryon octet ‘outer’ ring

QCDSF-UKQCD 1206.3156



NPLQCD [arXiv:hep-lat/0605014]

Blum et al [arXiv:1006.1311]

RM123 [arXiv:1110.6294]

QCDSF-UKQCD [arXiv:1206.3156]

Shanahan et al [arXiv:1209.1892]

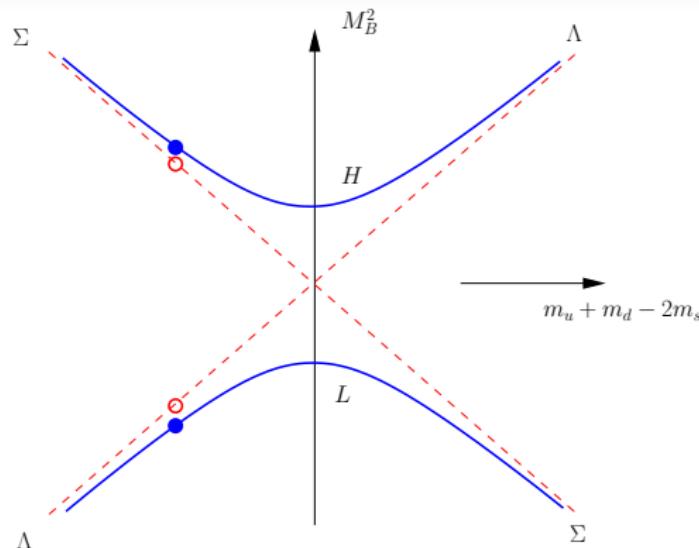
BMW [arXiv:1306.2287]

BMW [arXiv:1406.4088]

‘pure’ QCD part; weighted average

To complete the job: now consider octet ‘centre’

Baryon octet 'centre'



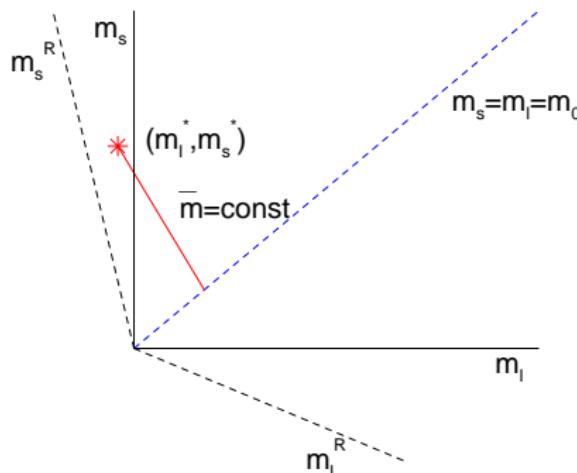
- $\Lambda^0(uds)$, $\Sigma^0(uds)$ have the same quark content (quantum numbers) but different wavefunctions
- Sketch shows mass difference lines for constant $m_u - m_d$:
 - $m_u = m_d \equiv m_l$
 - $m_u \neq m_d$
 - mixing
 - avoided level crossing (Heavy H , Light L particles)

QCDSF strategy:

[arXiv:1102.5300]

$2 + 1$ simulations: many paths to approach the physical point

$[m_u = m_d \equiv m_l \text{ case}]$



QCDSF: extrapolate from a point on the $SU(3)_F$ flavour symmetry line to the physical point

$$(m_0, m_0) \longrightarrow (m_l^*, m_s^*)$$

Choice here: keep the singlet quark mass \bar{m} constant

$$\bar{m} = m_0 = \frac{1}{3} (2m_l + m_s)$$

QCDSF strategy

[arXiv:1102.5300]

- develop $SU(3)_F$ flavour symmetry breaking expansion for hadron masses
- expansion in: $SU(3)$ flavour symmetric point $\delta m_q = 0$

$$\delta m_q = m_q - \bar{m}, \quad \bar{m} = \frac{1}{3}(m_u + m_d + m_s) = m_0$$

- expansion coefficients are functions of \bar{m}
- trivial constraint

$$\delta m_u + \delta m_d + \delta m_s = 0$$

- path called ‘unitary line’ as expand in both sea and valence quarks

Main observations:

- Provided \bar{m} kept constant, then expansion coefficients $A(\bar{m}), \dots$ remain unaltered whether
 - $1 + 1 + 1$
 - $2 + 1$
- Opens possibility of determining quantities that depend on $1 + 1 + 1$ from just $2 + 1$ simulations
- Furthermore can generalise to different valence quark masses, μ_q to sea quark masses m_q without increasing number of expansion coefficients

$$\delta\mu_q = \mu_q - \bar{m}$$

Mass matrix

Quark mass matrix

$$\mathcal{M} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

Baryon mass matrix

$$M^2(\mathcal{M}) = \begin{pmatrix} M_n^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & M_p^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & M_{\Sigma^-}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & M_{\Sigma\Sigma}^2 & M_{\Sigma\Lambda}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & M_{\Lambda\Sigma}^2 & M_{\Lambda\Lambda}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & M_{\Sigma^+}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & M_{\Xi^-}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_{\Xi^0}^2 \end{pmatrix}$$

Demand that under all $SU(3)$ transformations

$$\mathcal{M} \rightarrow \mathcal{M}' = U \mathcal{M} U^\dagger \quad \leftrightarrow \quad M^2(\mathcal{M}') = U M^2(\mathcal{M}) U^\dagger$$

Physically no change

[to eigenvalues]

eg $m_d \leftrightarrow m_s$ equivalent to relabelling $M_n \leftrightarrow M_{\Xi^0}, \dots$

N matrices

$$M^2 = \sum_{i=1}^{10} K_i(m_q, \mu_q) N_i$$

N_i classified under S_3 , $SU(3)$ symmetry; $K(m_q, \mu_q)$ coefficients

									S_3	$SU(3)$
1	1	1	1	1	1	1	1	A_1	1	
-1	-1	0	0	0	0	1	1	E^+	8_a	
-1	1	-2	0	0	2	-1	1	E^-	8_a	
1	1	-2	-2	2	-2	1	1	E^+	8_b	
-1	1	0	mix		0	1	-1	E^-	8_b	
1	1	1	-3	-3	1	1	1	A_1	27	
1	1	-2	3	-3	-2	1	1	E^+	27	
-1	1	0	mix		0	1	-1	E^-	27	
1	-1	-1	0	0	1	1	-1	A_2	$10, \overline{10}$	
0	0	0	mix		0	0	0	A_2	$10, \overline{10}$	

- N_i mostly diagonal, eg $N_1 = \text{diag}(1, 1, 1, 1, 1, 1, 1, 1, 1)$, except N_5 , N_8 , N_{10}
- S_3 symmetry group (equilateral triangle C_{3v}); 3 irreducible representations:
 - two singlets A_1 , A_2
 - one doublet E , elements E^\pm

Demand that under all $SU(3)$ transformations

$$\mathcal{M} \rightarrow U\mathcal{M}U^\dagger \quad \leftrightarrow \quad M^2(\mathcal{M}') = UM^2(\mathcal{M})U^\dagger$$

giving

[for B not at octet centre ie $\neq \Sigma, \Lambda$]

$$M_B^2 = P_{A_1} + P_{E^+}$$

and

$$\begin{pmatrix} M_{\Sigma\Sigma}^2 & M_{\Sigma\Lambda}^2 \\ M_{\Lambda\Sigma}^2 & M_{\Lambda\Lambda}^2 \end{pmatrix} = P_{A_1} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + P_{E^+} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + P_{E^-} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + P_{A_2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

P_G are functions of the quark masses with the symmetry G under the S_3 permutation group

$$\begin{aligned}
 P_{A_1} = & M_0^2 + 3A_1\delta\bar{\mu} \\
 & + \frac{1}{6}B_0(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) + B_1(\delta\mu_u^2 + \delta\mu_d^2 + \delta\mu_s^2) \\
 & + \frac{1}{4}(B_3 + B_4) \left[(\delta\mu_s - \delta\mu_u)^2 + (\delta\mu_s - \delta\mu_d)^2 + (\delta\mu_u - \delta\mu_d)^2 \right] + O(3)
 \end{aligned}$$

$$\begin{aligned}
 P_{E^+} = & \frac{3}{2}A_2(\delta\mu_s - \delta\bar{\mu}) \\
 & + \frac{1}{2}B_2(2\delta\mu_s^2 - \delta\mu_u^2 - \delta\mu_d^2) \\
 & + \frac{1}{4}(B_3 - B_4) \left[(\delta\mu_s - \delta\mu_u)^2 + (\delta\mu_s - \delta\mu_d)^2 - 2(\delta\mu_u - \delta\mu_d)^2 \right] + O(3)
 \end{aligned}$$

$$\begin{aligned}
 P_{E^-} = & \frac{\sqrt{3}}{2}A_2(\delta\mu_d - \delta\mu_u) \\
 & + \frac{\sqrt{3}}{2}B_2(\delta\mu_d^2 - \delta\mu_u^2) + \frac{\sqrt{3}}{4}(B_3 - B_4) \left[(\delta\mu_s - \delta\mu_d)^2 - (\delta\mu_s - \delta\mu_u)^2 \right] + O(3)
 \end{aligned}$$

$$P_{A_2} = 0 + O(3)$$

[$O(3)$ terms have also been determined]

Diagonalisation

$$\begin{aligned} M_H^2 &= P_{A_1} + \sqrt{P_{E^+}^2 + P_{E^-}^2 + P_{A_2}^2} \\ M_L^2 &= P_{A_1} - \sqrt{P_{E^+}^2 + P_{E^-}^2 + P_{A_2}^2} \end{aligned}$$

write eigenvectors as

$$e_H = \begin{pmatrix} \cos \theta \\ e^{-i\phi} \sin \theta \end{pmatrix}, \quad e_L = \begin{pmatrix} -e^{i\phi} \sin \theta \\ \cos \theta \end{pmatrix}$$

giving for the mixing angle, θ , and phase, ϕ

$$\tan 2\theta = \frac{\sqrt{P_{E^-}^2 + P_{A_2}^2}}{P_{E^+}}, \quad \tan \phi = \frac{P_{A_2}}{P_{E^-}}$$

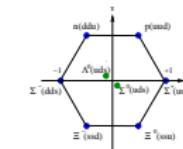
$$M_{\Sigma^0} = M_H^*, \quad M_{\Lambda^0} = M_L^*, \quad \theta^*$$

Dimensionless quantities

- octet baryons (centre of mass):

$$\begin{aligned} X_N^2 &= \frac{1}{6}(M_p^2 + M_n^2 + M_{\Sigma^+}^2 + M_{\Sigma^-}^2 + M_{\Xi^0}^2 + M_{\Xi^-}^2) = (1.160 \text{ GeV})^2 \\ &= M_0^2 + \frac{1}{6}(B_0 + B_1 + B_3)(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) = M_0^2 + O(\delta m_q^2) \end{aligned}$$

- all singlet quantities



stable under strong ints.

$$X_S^2 = \# + \#(\delta m_q^2)$$

(almost) constant

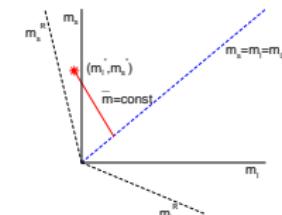
[\implies scale determination]

- form dimensionless ratios (within a multiplet):

$$\tilde{M}^2 \equiv \frac{M^2}{X_S^2}, \quad S = \pi, N, \dots, \quad \tilde{A}_i \equiv \frac{A_i}{M_0^2}, \dots \quad \text{in expansions}$$

Lattice

- $O(a)$ NP improved clover action
 - tree level Symanzik glue
 - mildly stout smeared $2+1$ clover fermion
 - $\beta = 5.50, 32^3 \times 64$
- κ_0 is start point on $SU(3)_F$ symmetric line
 κ_{0c} is chiral limit along $SU(3)_F$ symmetric line



$$m_q = \frac{1}{2} \left(\frac{1}{\kappa_q} - \frac{1}{\kappa_{0c}} \right)$$

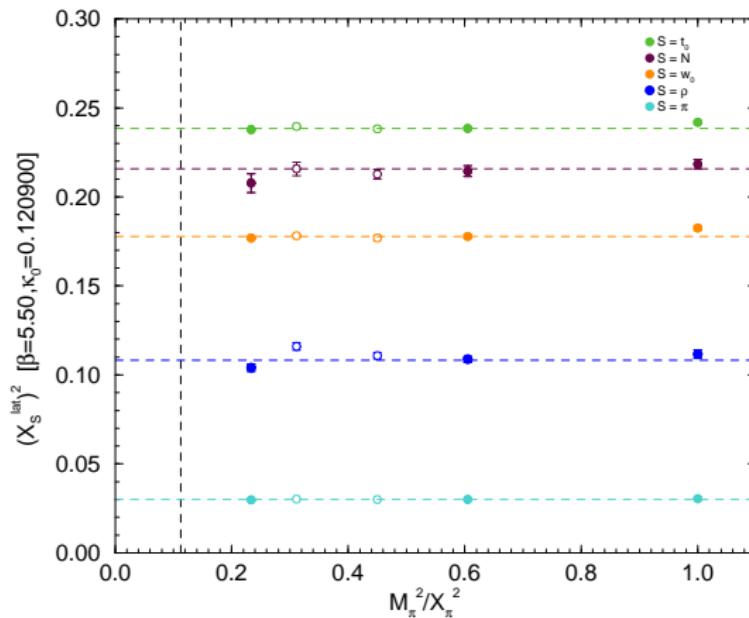
giving

$$m_0 = \frac{1}{2} \left(\frac{1}{\kappa_0} - \frac{1}{\kappa_{0c}} \right) = \bar{m} = \frac{1}{3}(2m_l + m_s) = \frac{1}{2} \left(\frac{2}{\kappa_l} + \frac{1}{\kappa_s} - \frac{1}{\kappa_{0c}} \right)$$

So $1/\kappa_{0c}$ cancels: given κ_0 and κ_l gives κ_s

$$\delta m_q = m_q - m_0 = \frac{1}{2} \left(\frac{1}{\kappa_q} - \frac{1}{\kappa_0} \right)$$

χ_s^2 determination:



- $\beta = 5.50, \kappa_0 = 0.120900$
- $X_{t_0}^2, X_{w_0}^2, X_\pi^2, X_\rho^2, X_N^2 \approx X_\Lambda^2$ along the $\bar{m} = \text{const.}$ line

[in plot $M_\pi \sim 460 \text{ MeV} - 225 \text{ MeV}$]

Method

1. Use PQ data to determine expansion coefficients:

$$\tilde{A} \quad \tilde{B} \quad \tilde{C}$$

2. Determine physical quark masses:

$$\delta m_u^*, \quad \delta m_d^*, \quad \delta m_s^*$$

First consider 1

Correlation functions

[wf $\Sigma(abc)$, $\Lambda(abc)$]

- General: $M_H(abc)$, $M_L(abc)$

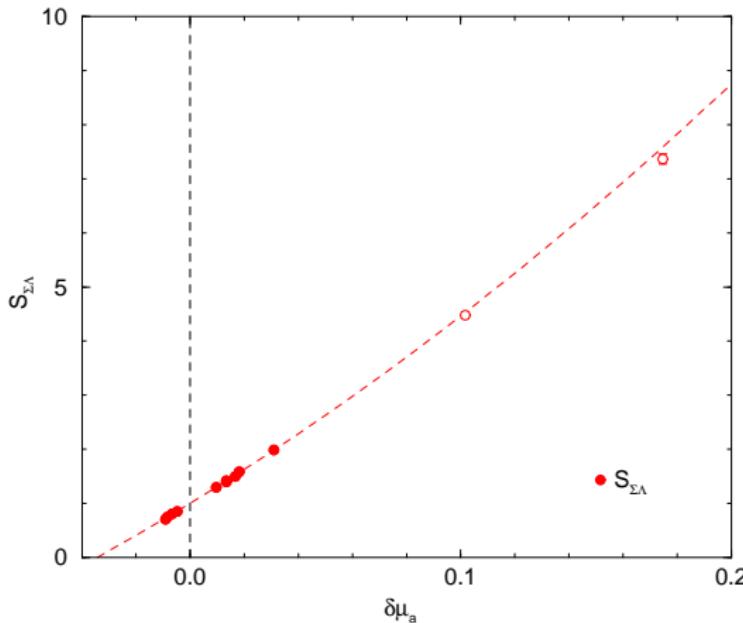
$$\begin{aligned} C_{ij}(t) &= \frac{1}{V_s} \text{Tr}_D \Gamma_{\text{unpol}} \left\langle \sum_{\vec{y}} \mathcal{B}_i(\vec{y}, t) \sum_{\vec{x}} \bar{\mathcal{B}}_j(\vec{x}, 0) \right\rangle \\ &\propto A_i A_j e^{-M_L t} + B_i B_j e^{-M_H t} \quad i, j = H, L \end{aligned}$$

diagonalise, C_{ij} yielding M_H and M_L

- Outer ring: $M_H(aab)$, $M_L(aa'b)$
already diagonal – much simpler

Check I – mass degenerate case

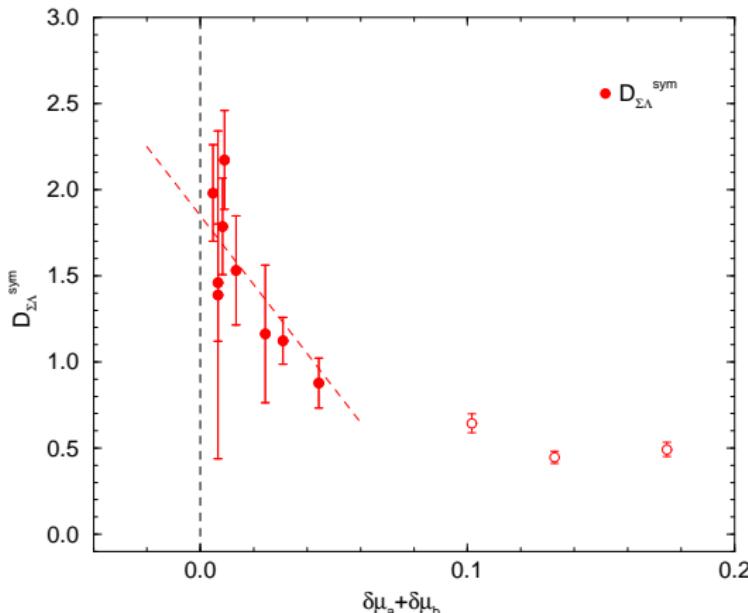
$$S_{\Sigma\Lambda} \equiv \tilde{M}_{\Sigma}^2(aaa'') = 1 + 3\tilde{A}_1\delta\mu_a + 3\tilde{B}_1\delta\mu_a^2$$



- $\beta = 5.50, \kappa_0 = 0.120900$

Check II – ‘symmetric’ difference

$$\begin{aligned} D_{\Sigma\Lambda}^{\text{sym}} &\equiv \frac{\tilde{M}_{\Sigma}^2(aab) - \tilde{M}_{\Lambda}^2(aa'b) - \tilde{M}_{\Sigma}^2(bba) + \tilde{M}_{\Lambda}^2(bb'a)}{4(\delta\mu_b - \delta\mu_a)} \\ &= \tilde{A}_2 + \tilde{B}_2(\delta\mu_a + \delta\mu_b) + \dots \end{aligned}$$



Method

1. Use PQ data to determine expansion coefficients:

$$\tilde{A} \quad \tilde{B} \quad \tilde{C}$$

2. Determine physical quark masses:

$$\delta m_u^*, \quad \delta m_d^*, \quad \delta m_s^*$$

Now secondly consider 2

Same procedure as before: pseudoscalar mesons

- $SU(3)$ flavour breaking expansion

$$\begin{aligned} M^2(a\bar{b}) &= M_{0\pi}^2 + \alpha(\delta\mu_a + \delta\mu_b) \\ &\quad + \beta_0 \frac{1}{6}(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) + \beta_1(\delta\mu_a^2 + \delta\mu_b^2) + \beta_2(\delta\mu_a - \delta\mu_b)^2 \\ &\quad + \gamma_0 \delta m_u \delta m_d \delta m_s + \gamma_1(\delta\mu_a + \delta\mu_b)(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) \\ &\quad + \gamma_2(\delta\mu_a + \delta\mu_b)^3 + \gamma_3(\delta\mu_a + \delta\mu_b)(\delta\mu_a - \delta\mu_b)^2 \end{aligned}$$

- $X_\pi^2 = \frac{1}{6}(M_{K^+}^2 + M_{K^0}^2 + M_{\pi^+}^2 + M_{\pi^-}^2 + M_{\bar{K}^0}^2 + M_{K^-}^2)$
form ratios: $\alpha \rightarrow \tilde{\alpha}, \dots;$; determine coefficients
- $M^{\text{exp}} = M^{\text{QCD}} + M^{\text{QED}}$

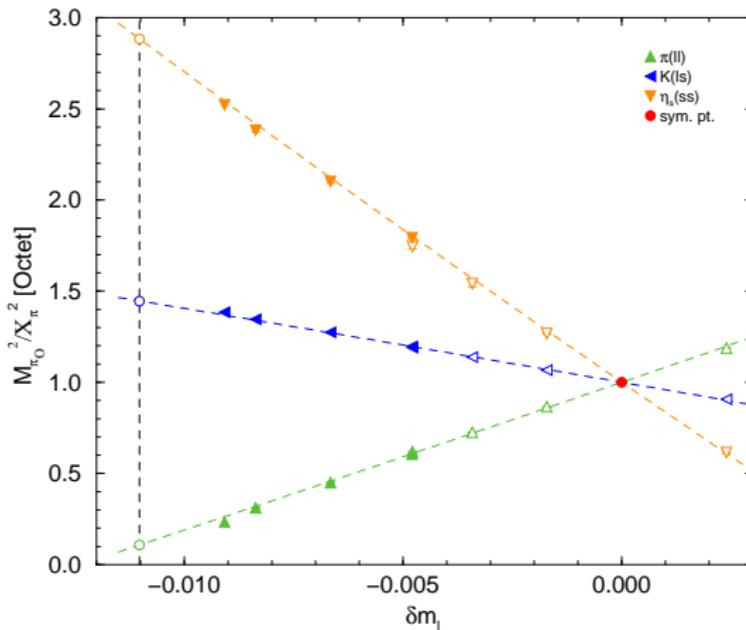
Dashen's theorem:

EM effects for charged mesons K^+ , π^+ same; for neutral mesons π^0 , K^0 vanish

$$\begin{aligned} M_{\pi^+}^{\text{exp}\,2} &= M_{\pi^+}^{*\,2} + \mu_\gamma, & M_{\pi^0}^{\text{exp}\,2} &= M_{\pi^0}^{*\,2} \approx M_{\pi^+}^{*\,2}, \\ M_{K^+}^{\text{exp}\,2} &= M_{K^+}^{*\,2} + \mu_\gamma, & M_{K^0}^{\text{exp}\,2} &= M_{K^0}^{*\,2} \end{aligned}$$

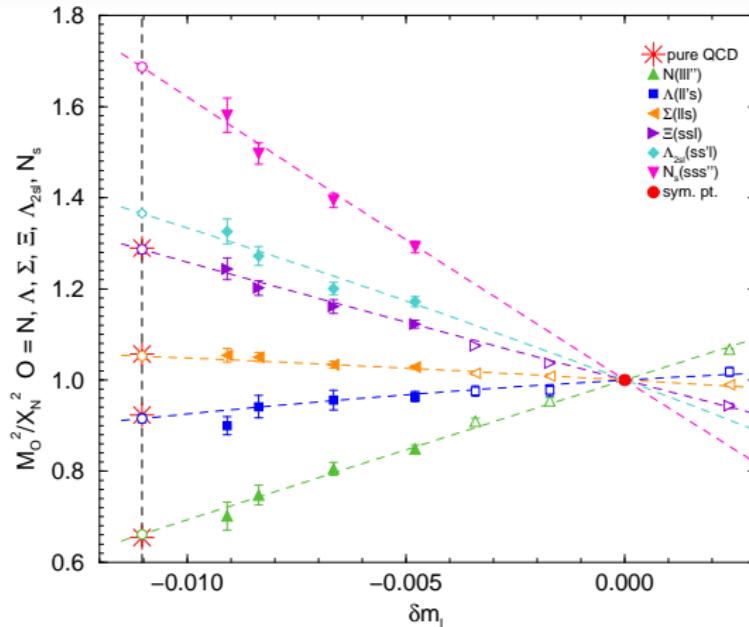
- determines δm_u^* , δm_d^* , δm_s^*

'Fan' plot for the 2 + 1: $\tilde{M}_{\pi_0}(a\bar{b})^2$



- $\pi(\text{II}')$, $K(\text{ls})$, $\eta(\text{ss})$
- $\delta m_l^* = (\delta m_u^* + \delta m_d^*)/2$,

'Fan' plot for the 2 + 1: $\tilde{M}_\Sigma(aab)^2$, $\tilde{M}_\Lambda(aa'b)^2$



- $N(III'')[= \Lambda_{3I}(I'I'')]$, $\Sigma(IIs)$, $\Xi(ssl)$, $N_s(sss'')[= \Lambda_{3s}(ss's'')$, $\Lambda(I'I's)$, $\Lambda_{1/2s}(ss'I)$
- As diagonal: $\tilde{M}_N^2 = P_{A_1} + P_{E^+}$, $\tilde{M}_\Lambda^2 = P_{A_1} - P_{E^+}$
- $\delta m_l^* = (\delta m_u^* + \delta m_d^*)/2$, $M_N^{*2}(III'') = (M_N^{\text{exp}}{}^2(ddu) + M_P^{\text{exp}}{}^2(uud))/2$, $M_\Lambda^{*2}(I'I's) = M_{\Lambda 0}^{\text{exp}}{}^2(uds)$,
 $M_\Sigma^{*2}(IIs) = (M_\Sigma^{\text{exp}}{}^2(dds) + M_{\Sigma^+}^{\text{exp}}{}^2(uus))/2$, $M_\Xi^{*2}(ssl) = (M_{\Xi^-}^{\text{exp}}{}^2(ssd) + M_{\Xi 0}^{\text{exp}}{}^2(ssu))/2$

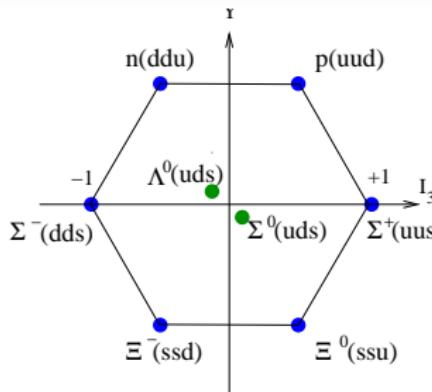
$\Sigma^0 - \Lambda^0$ mixing at NLO – unitary limit

$$\tilde{M}_{\Sigma^0} - \tilde{M}_{\Lambda^0} = \sqrt{\frac{3}{2}} \tilde{A}_2 \sqrt{\delta m_u^2 + \delta m_d^2 + \delta m_s^2} \times \\ \left[1 + \frac{3}{2} \left(\frac{2\tilde{B}_2 + 3\tilde{B}_3 - 3\tilde{B}_4}{\tilde{A}_2} \right) \frac{\delta m_u \delta m_d \delta m_s}{\delta m_u^2 + \delta m_d^2 + \delta m_s^2} \right]$$

$$\tan 2\theta = \frac{(\delta m_d - \delta m_u)}{\sqrt{3}\delta m_s} \times \\ \left[1 - \frac{1}{3} \left(\frac{2\tilde{B}_2 + 3\tilde{B}_3 - 3\tilde{B}_4}{\tilde{A}_2} \right) \frac{(\delta m_s - \delta m_u)(\delta m_s - \delta m_d)}{\delta m_s} \right]$$

- mixing only when $\delta m_d \neq \delta m_u$, so mixing contribution to $\tilde{M}_{\Sigma^0} - \tilde{M}_{\Lambda^0}$ very small
- small electromagnetic effects

cf Baryon 'Outer' ring



$$\tilde{M}_n - \tilde{M}_p = (\delta m_d - \delta m_u) \left[\tilde{A}'_1 - 2\tilde{A}'_2 + (\tilde{B}'_1 - 2\tilde{B}'_2)(\delta m_d + \delta m_u) \right]$$

$$\tilde{M}_{\Sigma^-} - \tilde{M}_{\Sigma^+} = (\delta m_d - \delta m_u) \left[2\tilde{A}'_1 - \tilde{A}'_2 + (2\tilde{B}'_1 - \tilde{B}'_2 + 3\tilde{B}'_3)(\delta m_d + \delta m_u) \right]$$

$$\tilde{M}_{\Xi^-} - \tilde{M}_{\Xi^0} = (\delta m_d - \delta m_u) \left[\tilde{A}'_1 + \tilde{A}'_2 + (\tilde{B}'_1 + \tilde{B}'_2 + 3\tilde{B}'_3)(\delta m_d + \delta m_u) \right]$$

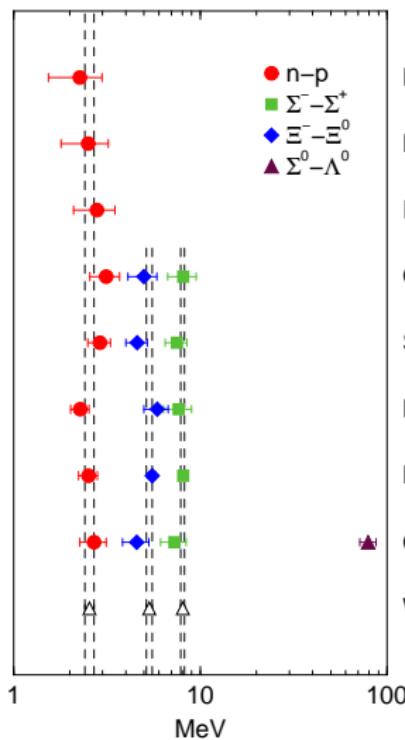
- A', B' simply related to A, B
- structure very different – LO also depends on \tilde{A}_1

[numerically $\tilde{A}_1 \gg \tilde{A}_2$]

Present status: including additional results

Baryon octet 'outer' ring

QCDSF-UKQCD 1411.7665



NPLQCD [arXiv:hep-lat/0605014]

Blum et al [arXiv:1006.1311]

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BMW [arXiv:1306.2287]

BMW [arXiv:1406.4088]

QCDSF-UKQCD [arXiv:1411.7665]

Weighted average

'pure' QCD part; weighted average

$$M_{\Sigma^0} - M_{\Lambda^0}$$

- Mixing angle

[as anticipated very small $\theta \underset{\sim}{<} 1^\circ$]

$$\tan 2\theta = 0.0123(45)(25)$$

- Mass difference

[mixing contribution to mass difference $\underset{\sim}{<} 1 \text{ MeV}$]

$$M_{\Sigma^0} - M_{\Lambda^0} = 79.44(7.37)(3.37) \text{ MeV}$$

$$[(M_{\Sigma^0} - M_{\Lambda^0})^{\text{exp}} = 76.959(23) \text{ MeV}]$$

Comments on matrix elements

- LO mass difference between isospin breaking and isospin symmetric cases

$$\left(\tilde{M}_{\Sigma^0} - \tilde{M}_{\Lambda^0} \right) - \left(\tilde{M}_\Sigma - \tilde{M}_\Lambda \right) \Big|_{\delta m_I} = \frac{1}{8} \tilde{A}_2 \frac{(\delta m_d - \delta m_u)^2}{|\delta m_I|} \propto \tan^2 \theta_{\Sigma\Lambda}$$

- second order
- Semi-leptonic decays $\Sigma^- \rightarrow \Lambda^0 e \bar{\mu}$, $\Sigma^+ \rightarrow \Lambda^0 e^+ \bar{\mu}$ also mix [Karl]

$$\Sigma^+ \rightarrow \Lambda^0 \quad A = \left\{ \sqrt{2} \gamma_\mu \sin \theta_{\Sigma\Lambda} + \left(\sqrt{\frac{2}{3}} D \cos \theta_{\Sigma\Lambda} + \sqrt{2} F \sin \theta_{\Sigma\Lambda} \right) \gamma_\mu \gamma_5 \right\} V_{ud}$$

$$\Sigma^- \rightarrow \Lambda^0 \quad A = \left\{ -\sqrt{2} \gamma_\mu \sin \theta_{\Sigma\Lambda} + \left(\sqrt{\frac{2}{3}} D \cos \theta_{\Sigma\Lambda} - \sqrt{2} F \sin \theta_{\Sigma\Lambda} \right) \gamma_\mu \gamma_5 \right\} V_{ud}$$

- first order
- vector pieces have developed
- potentially more significant

Conclusions

- Programme: Developed a $SU(3)$ flavour symmetry breaking expansion keeping the average quark mass \bar{m} constant advantages:
 - can use $2+1$ simulations, ie $m_u = m_d = m_l$
 - can use cheap PQ results
- Generalised previous method to now also include mixing

$$\Sigma^0 - \Lambda^0$$

mass splitting as well as

$$n - p, \quad \Sigma^- - \Sigma^+, \quad \Xi^- - \Xi^0$$

mass splittings due to difference in $u - d$ quark masses
(‘pure’ QCD isospin effects)

Now can include $B(abc)$ as well as $B(aa'b)$ PQ quarks

- Encouraging first results
- Repeat for $\eta - \eta'$ mixing (but then disconnected pieces to compute)