Properties of Thermal Matter: Conductivity, Parity Restoration and the Charmonium Potential

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FASTSUM Collaboration

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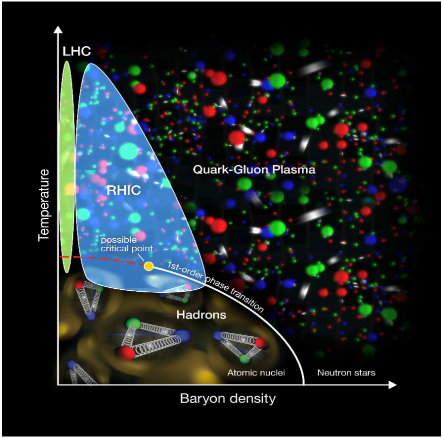
[10] Trinity College Dublin
[11] University of Mainz

Setting the scene

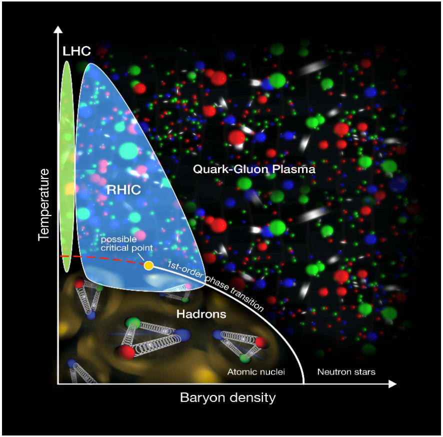


quarks & gluons
pressure
viscosity
plasma physics

hadrons
masses
mx els
atomic physics

[http://www.bnl.gov/rhic/news]

Setting the scene



quarks & gluons
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Correlation Functions ↔ Spectral Functions

Particle Data Book

$\sim 1,500$ pages

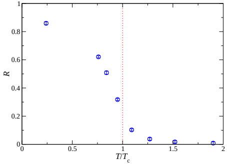
zero pages on Quark-Gluon Plasma...

Overview

- Parity Restoration in the Baryon Sector

  SYMMETRIES

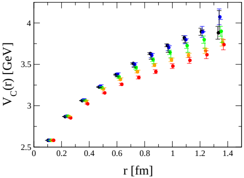
  arXiv/1505.06616



- Charmonium Potential

  INTERACTIONS

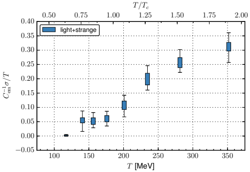
  arXiv/1502.03603



- Conductivity, Susceptibility and Diffusion Coefficient

  PHENOMENOLOGY

  arXiv/1412.6411

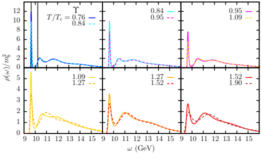


Other Work

- Bottomonium and Charmonium Spectral Functions

  MELTING

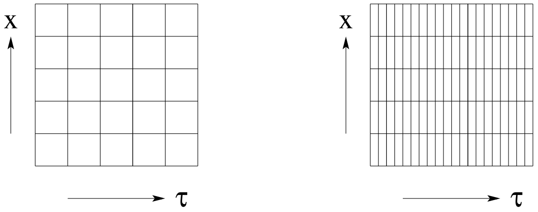
  arXiv:1402.6210



FASTSUM set up

- anisotropic lattices $a_\tau < a_s$
  - allowing better resolution, particularly at finite temperatures

since $$T = \frac{1}{N_\tau a_\tau}$$



- "2nd" generation lattice ensembles
  - moving towards continuum, infinite volume, realistic light quark masses

Physics/lattice parameters

2nd Generation

2+1 flavours

larger volume: $(3\text{fm})^3 - (4\text{fm})^3$
finer lattices: $a_s = 0.123$ fm
quark mass: $M_\pi / M_\rho \sim 0.45$
temporal cut-off: $a_\tau \sim 5.6$ GeV

N_s	N_τ	T(MeV)	T/T_c
24, 32	16	352	1.90
24	20	281	1.52
24, 32	24	235	1.27
24, 32	28	201	1.09
24, 32	32	176	0.95
24	36	156	0.84
24	40	141	0.76
32	48	117	0.63
16	128	44	0.24

Gauge Action:

Symanzik-improved, tree-level tadpole

Fermion Action:

clover, stout-links, tree-level tadpole

(Hadron Spectrum Collaboration)

Parity Restoration in the Baryon Spectrum

arXiv/1505.06616

Baryons at Finite Temperature

- little work on Baryons @ $T \neq 0$
  - DeTar and Kogut (1987) screening masses
  - QCD-TARO (2005) $\mu \neq 0$
  - Datta et al (2013) quenched

We use a standard baryon operator:

$$O_N(\mathbf{x},\tau) = \epsilon_{abc} u_a(\mathbf{x},\tau) \left[u_b^T(\mathbf{x},\tau) \mathcal{C}\gamma_5 d_c(\mathbf{x},\tau) \right]$$

and parity project it:

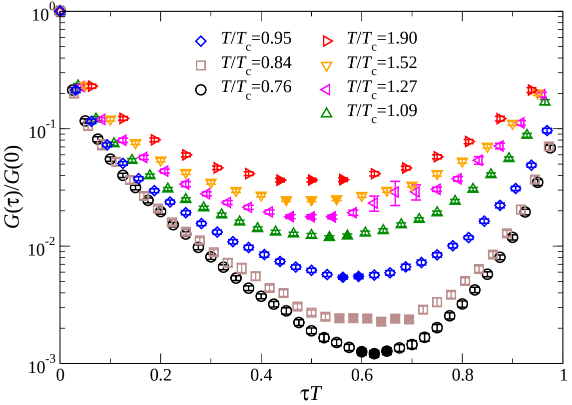
$$O_{N_\pm}(\mathbf{x},\tau) = P_\pm O_{N_\pm}(\mathbf{x},\tau)$$

Forward (+ve) and backward (−ve) parity states in correlator:

$$\begin{aligned} G(\tau) &= \int d^3x \, \langle O_{N_+}(\mathbf{x},\tau) \overline{O}_{N_+}(\mathbf{0},0) \rangle \\ &= \int_0^\infty \frac{d\omega}{2\pi} \left[\frac{e^{-\omega\tau}}{1+e^{-\omega/T}} \, \rho_+(\omega) - \frac{e^{-\omega(1/T-\tau)}}{1+e^{-\omega/T}} \, \rho_-(\omega) \right] \end{aligned}$$

Baryon Correlators

(Using Gaussian smeared baryon operators)



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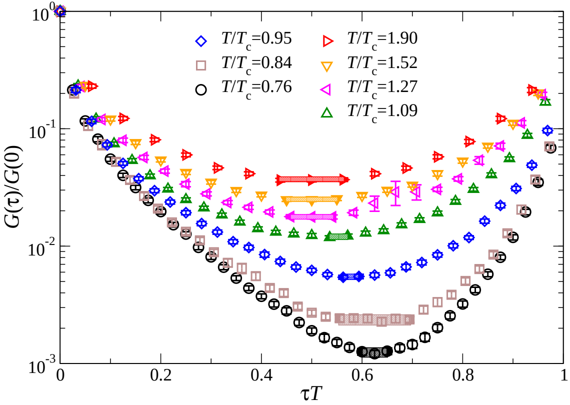
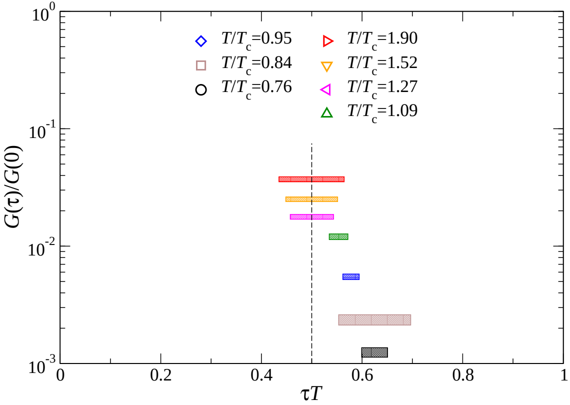


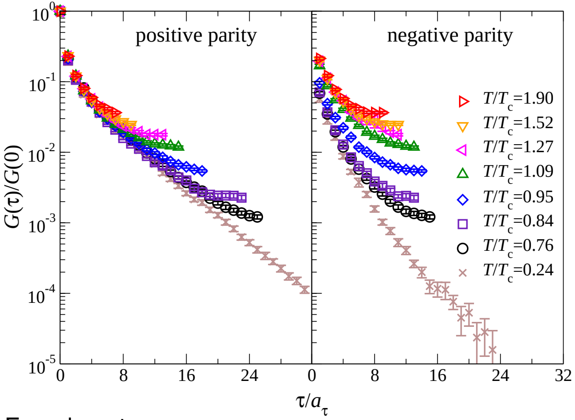
Baryon Correlators

(Using Gaussian smeared baryon operators)

$\longrightarrow$ parity doubling for $T \gtrsim T_C$ observed at correlator level



Correlators - Parity Comparison



Experiment:

$+$ve parity: $M_N = 939$ MeV $-$ve parity: $M_{N*} = 1535$ MeV

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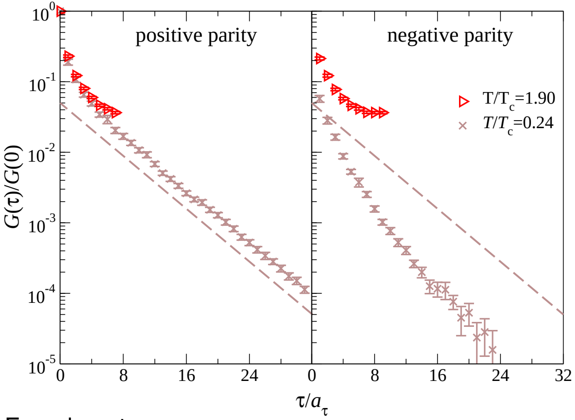


Experiment:

+ve parity: $M_N = 939$ MeV −ve parity: $M_{N*} = 1535$ MeV

Naive Exponential Fits

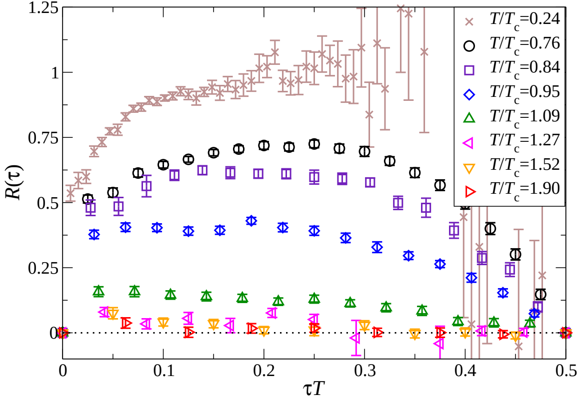
T/T_c	m_+ [GeV]	m_- [GeV]	$m_+ - m_-$ [MeV]	
0.24	1.20(3)	1.9(3)	$\sim$700	*cf* expt: $\sim$600
0.76	1.18(9)	1.6(2)		
0.84	1.08(9)	1.6(1)		
0.95	1.12(14)	1.3(2)		

Parity Comparison

Define $R(t) = \dfrac{G(\tau) - G(N_\tau - \tau)}{G(\tau) + G(N_\tau - \tau)}$ Datta et al, arXiv:1212.2927

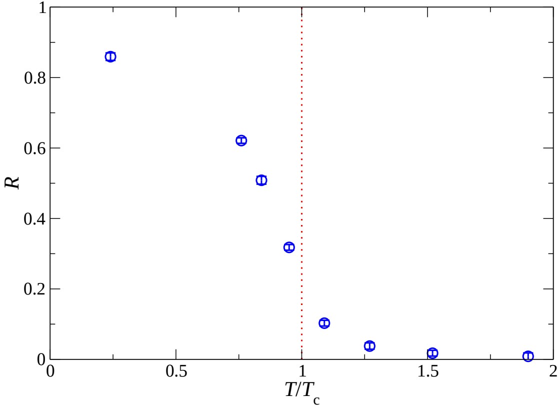
Note: $R(1/2T) \equiv 0$

with: $R(\tau) \equiv 0$ for parity symmetry



Parity Restoration

Define $\quad R = \dfrac{\sum_{\tau=1}^{N_\tau/2-1} R(\tau)/\sigma^2(\tau)}{\sum_{\tau=1}^{N_\tau/2-1} 1/\sigma^2(\tau)}$



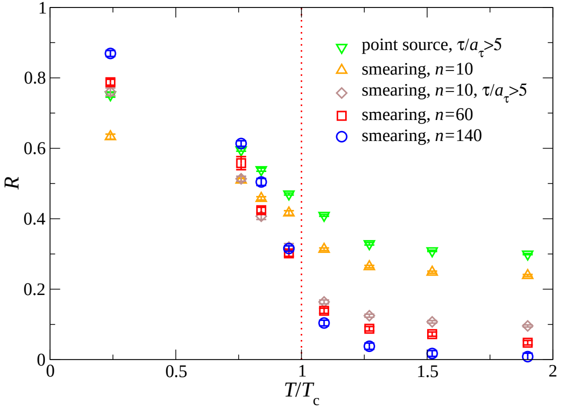
Effects of Smearing

Systematics checks of smearing:

- vary n
- vary τ-range

Implies parity doubling is:

- ground state feature (*recall* Wilson term breaks chiral symmetry)
- not an artefact of smearing



Maximum Entropy Method (MEM)

Cont: $G(\tau) = \int K(\tau,\omega)\rho(\omega)d\omega$ Lat: $G(\tau_i) = \sum_j K(\tau_i,\omega_j)\,\rho(\omega_j)$

Input data: $\tau_i,\ i = \{1,\ldots,\mathcal{O}(10)\}$ Output data : $\omega_j,\ j = \{1,\ldots,\mathcal{O}(10^3)\}$

$\longrightarrow$ ill-posed

Bayes Th'm: $P[\rho|DH] = \frac{P[D|\rho H]P[\rho|H]}{P[D|H]} \propto \exp(-\chi^2 + \alpha S)$

$H =$ prior knowledge $D =$ data

Shannon-Jaynes entropy: $S = \int_0^\infty \frac{d\omega}{2\pi}\left[\rho(\omega) - m(\omega) - \rho(\omega)\ln\frac{\rho(\omega)}{m(\omega)}\right]$

Competition between minimising χ^2 and maximising S

Asakawa, Hatsuda, Nakahara, Prog.Part.Nucl.Phys. 46 (2001) 459

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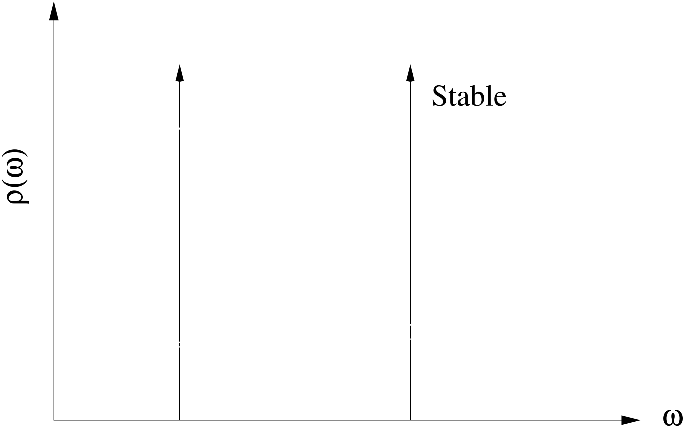
Example Spectral Functions

$$G_2(t) \sim \int \rho(\omega)\ e^{-\omega t}\ d\omega$$



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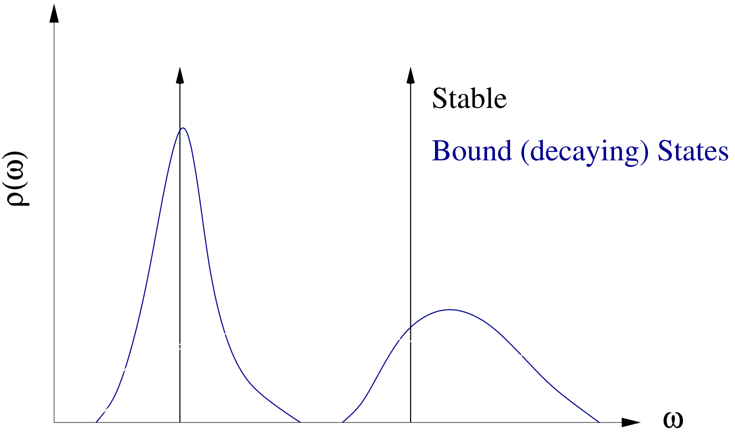

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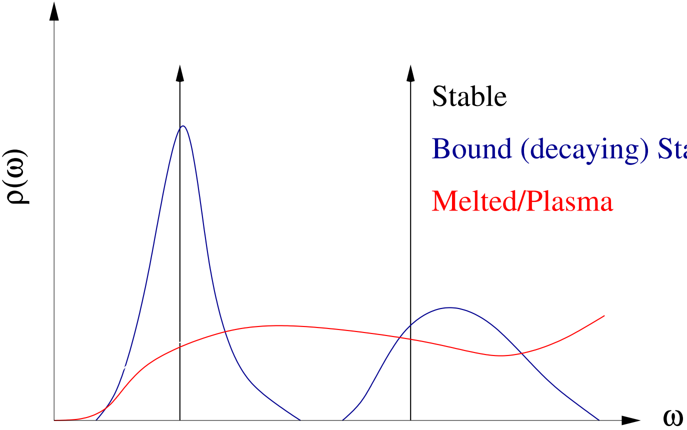


MEM for finite T baryons

Recall:
$$\begin{aligned} G(\tau) &= \int d^3x\, \langle O_{N_+}(\mathbf{x},\tau)\overline{O}_{N_+}(\mathbf{0},0)\rangle \\ &= \int_0^\infty \frac{d\omega}{2\pi}\left[\frac{e^{-\omega\tau}}{1+e^{-\omega/T}}\,\rho_+(\omega) - \frac{e^{-\omega(1/T-\tau)}}{1+e^{-\omega/T}}\,\rho_-(\omega)\right] \end{aligned}$$

So can define:
$$\begin{aligned} K(\tau,\omega) &= \frac{e^{-\omega\tau}}{1+e^{-\omega/T}} && \omega > 0 \\ &= \frac{e^{+\omega(1/T-\tau)}}{1+e^{+\omega/T}} && \omega < 0 \end{aligned}$$

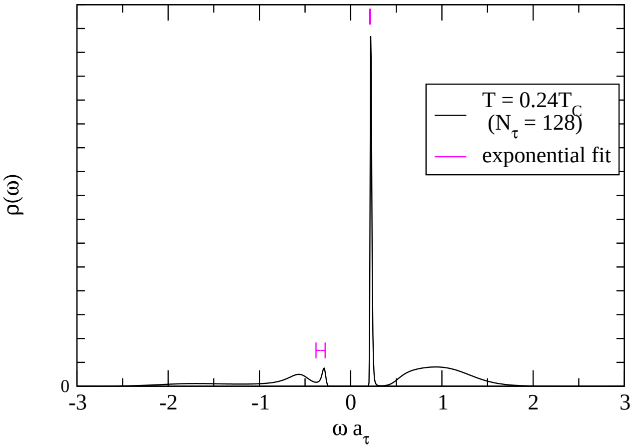
and use MEM with
$$G(\tau) \equiv \int_{-\infty}^{+\infty} K(\tau,\omega)\rho(\omega)d\omega$$

giving:
$$\begin{aligned} \rho_+(\omega) &\equiv \rho(\omega) && \omega > 0 \\ \rho_-(-\omega) &\equiv -\rho(\omega) && \omega < 0 \end{aligned}$$

(Need to assume $\rho(\omega)$ is positive definite for MEM to work)

Baryonic Spectral Functions

Preliminary



Baryonic Spectral Functions

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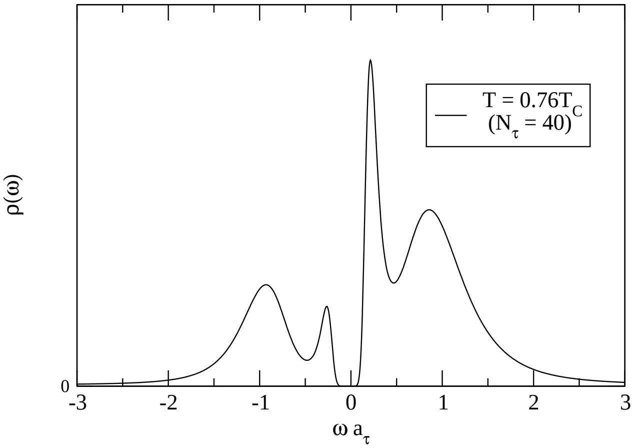

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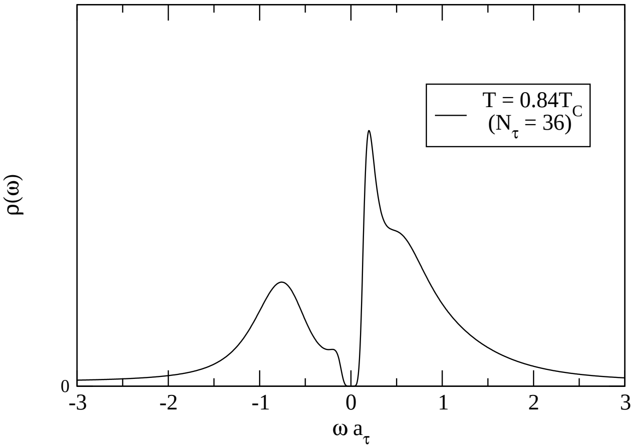

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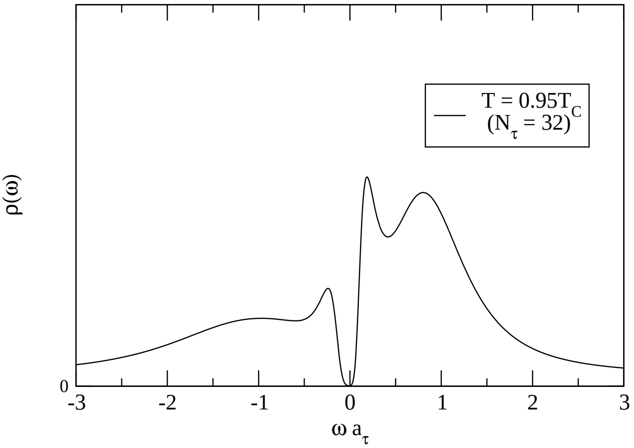

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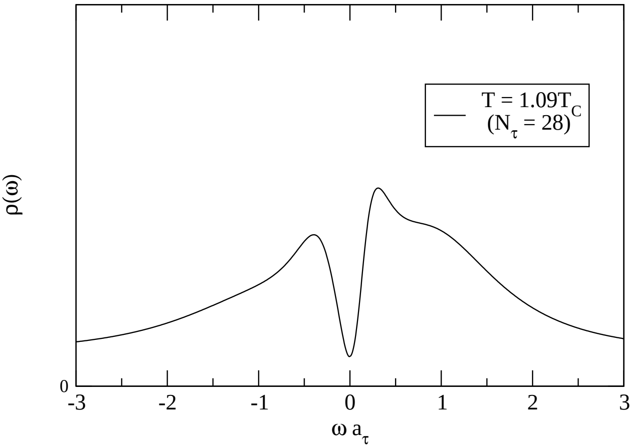

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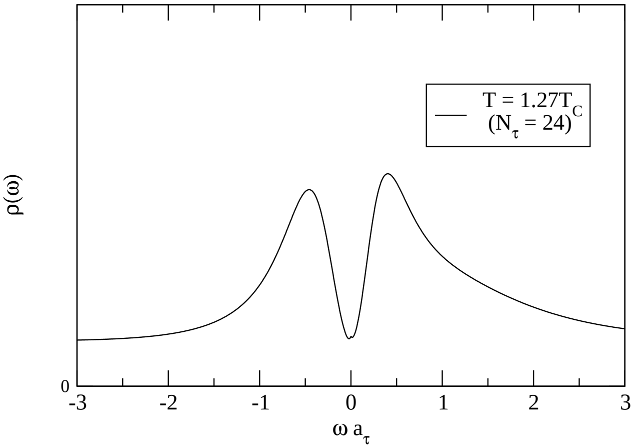

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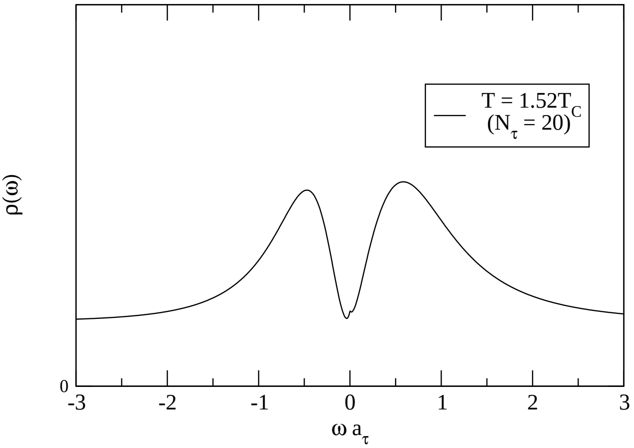

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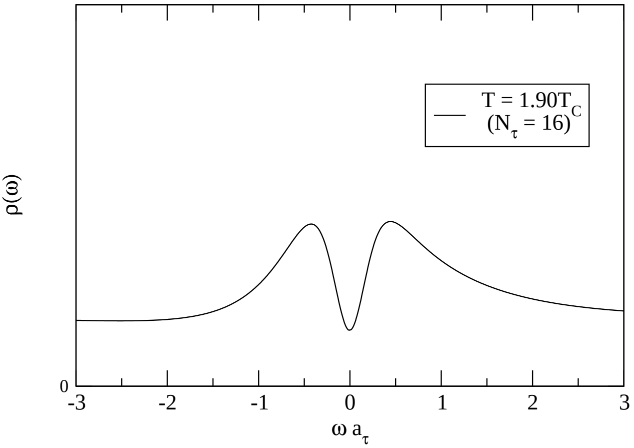

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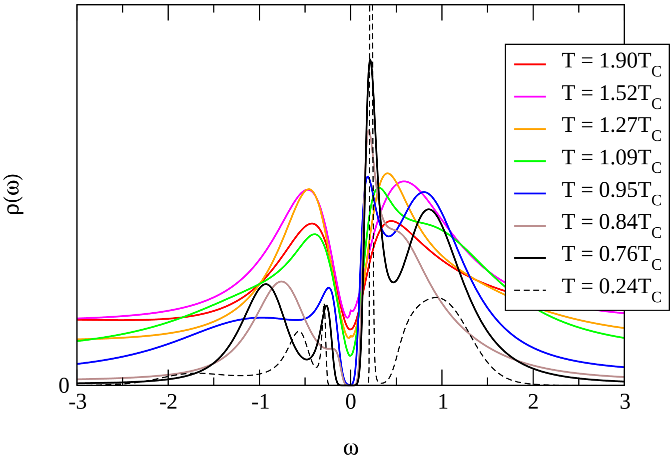
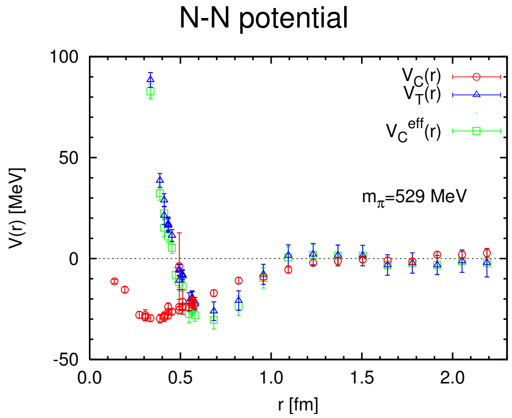


Charmonium Potential charmonium potential

arXiv/1502.03603

Lattice goes Nuclear



HAL QCD Collaboration, Aoki, Doi, Hatsuda, Ikeda, Inoue, Ishii, Murano, Nemura, Sasaki

Iida, Ikeda PoS LATTICE2011(2011)195

Schrödinger Equation Approach

HAL QCD Collaboration, S. Aoki et al. [arXiv:1206.5088]

Schrödinger equation used to “reverse engineer” the potential, $V(r)$, given the Nambu- Bethe-Salpeter wavefunction, $\psi(r)$:

$$\left(\frac{p^2}{2M} + V(r)\right) \psi(r) = E \;\; \psi(r)$$

$\psi(r)$ is determined from correlators of *non-local* operators,

$$\begin{aligned} J(x; \vec{r}) &= q(x)\, \Gamma\; U(x, x+\vec{r})\overline{q}(x+\vec{r}) \\ C(\vec{r}, t) &= \sum_{\vec{x}} \langle\, J(0; \vec{r} = \vec{0})\; J(x; \vec{r})\,\rangle \\ &\longrightarrow \;\; \psi(r)\; e^{-Mt} \quad \text{where} \quad \langle\, 0 \,|\, J(x; \vec{r}) \,|\, gnd \,\rangle \approx \psi(r) \end{aligned}$$

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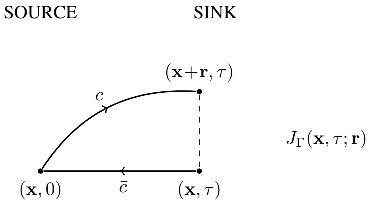
HAL QCD Time Dependent Method



$$J_\Gamma(x;\mathbf{r}) = \bar{q}(x)\,\Gamma\,U(x,x+\mathbf{r})\,q(x+\mathbf{r})$$

Local Extended Correlation Functions

$$C_\Gamma(\mathbf{r},\tau) = \sum_{\mathbf{x}} \langle J_\Gamma(\mathbf{x},\tau;\mathbf{r})\; J_\Gamma^\dagger(0;\mathbf{0})\rangle$$

$$C_\Gamma(\mathbf{r},\tau) = \sum_j \frac{\psi_j^*(\mathbf{0})\psi_j(\mathbf{r})}{2E_j}\left(e^{-E_j\tau} + e^{-E_j(N_\tau-\tau)}\right) \approx \sum_j \Psi_j(\mathbf{r})e^{-E_j\tau}$$

ignoring backward mover

Schrödinger Eqn $\quad E_j\Psi_j(r) = \left(-\frac{\nabla_r^2}{2\mu} + V_\Gamma(r)\right)\Psi_j(r)$

$$\frac{\partial C_\Gamma(\mathbf{r},\tau)}{\partial\tau} = -\sum_j E_j\Psi_j(\mathbf{r})e^{-E_j\tau} = \sum_j \left(\frac{\nabla_r^2}{2\mu} - V_\Gamma(r)\right)\Psi_j(r)e^{-E_j\tau}$$

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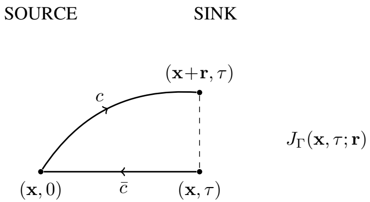

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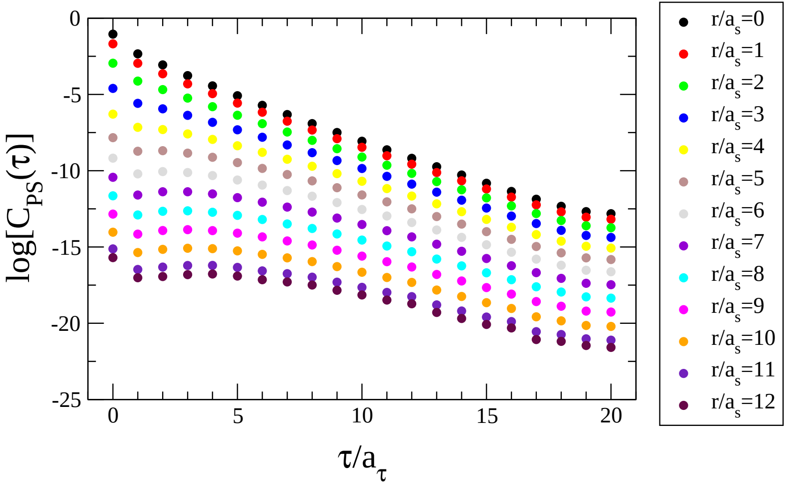
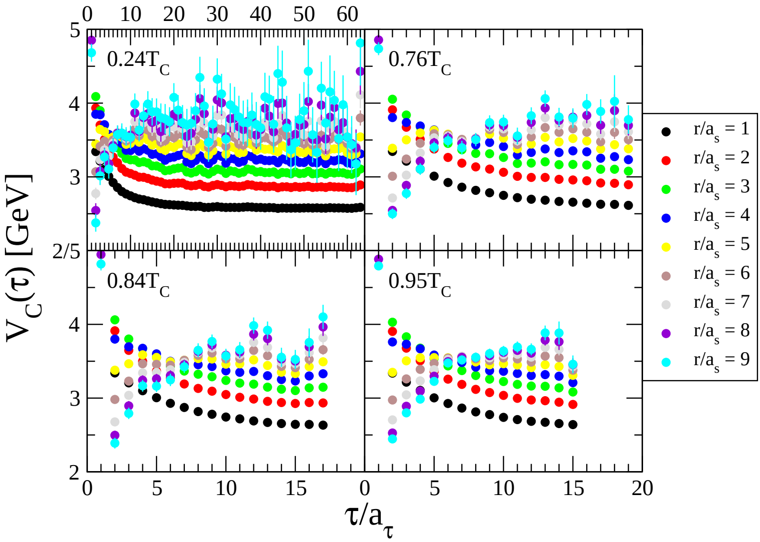
$$\boxed{\frac{\partial C_\Gamma(\mathbf{r},\tau)}{\partial\tau} = \left(\frac{\nabla_r^2}{2\mu} - V_\Gamma(r)\right)C_\Gamma(\mathbf{r},\tau)}$$

Correlation Functions



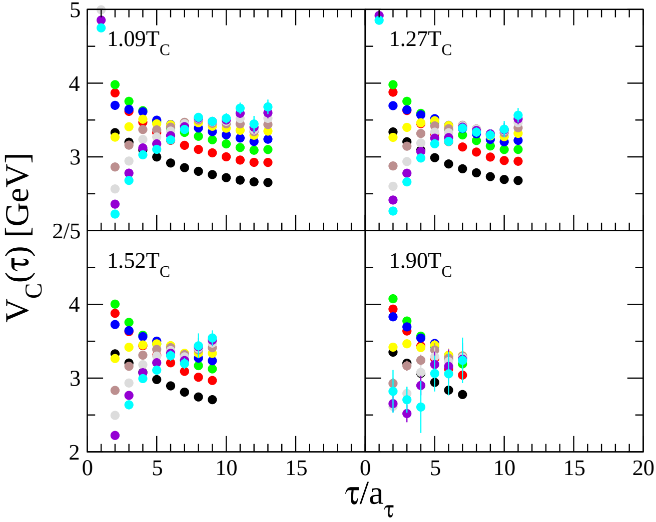
PS channel $0.76T_c$ ($N_\tau = 40$)

Central Potentials - cold



$$V_\Gamma(\mathbf{r}) = V_C(\mathbf{r}) + V_S(\mathbf{r})\, s_1 \cdot s_2 \quad \longrightarrow \quad V_C(\mathbf{r}) = \frac{1}{4} V_{PS} + \frac{3}{4} V_V \qquad V_S(\mathbf{r}) = V_V - V_{PS}$$

Central Potentials - hot



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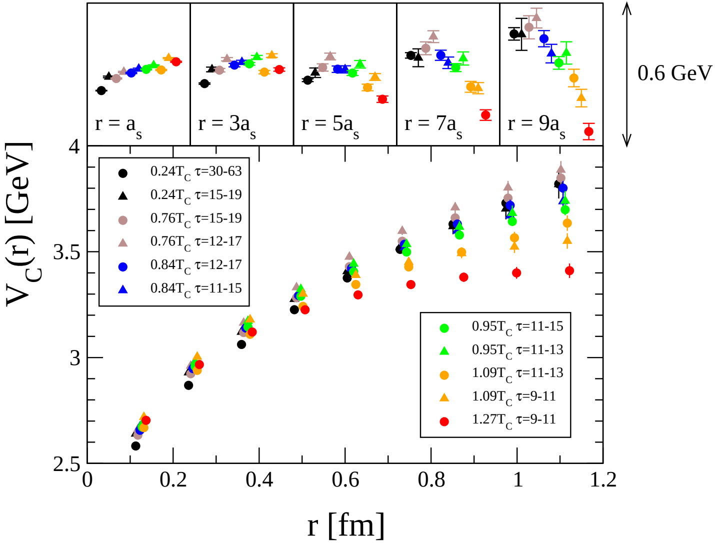
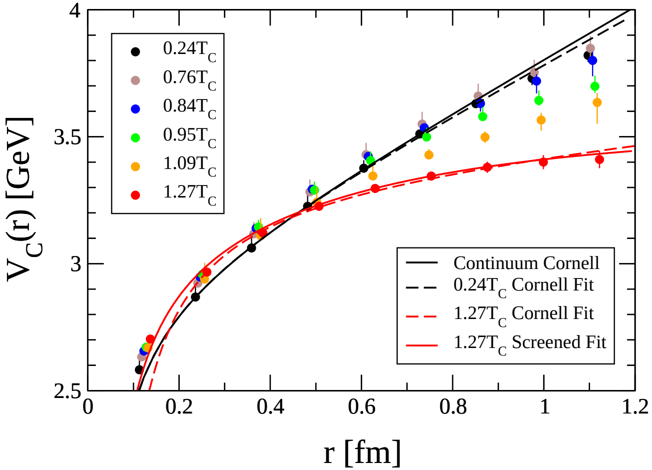
Fitting Ranges

T/T_C	N_τ	Best Range	Lower Range
0.24	128	30 – 63	15 – 19
0.76	40	15 – 19	12 – 17
0.84	36	12 – 17	11 – 15
0.95	32	11 – 15	11 – 13
1.09	28	11 – 13	9 – 11
1.27	24	9 – 11	N/A

Central Potential Results



Cornell Potential Comparison

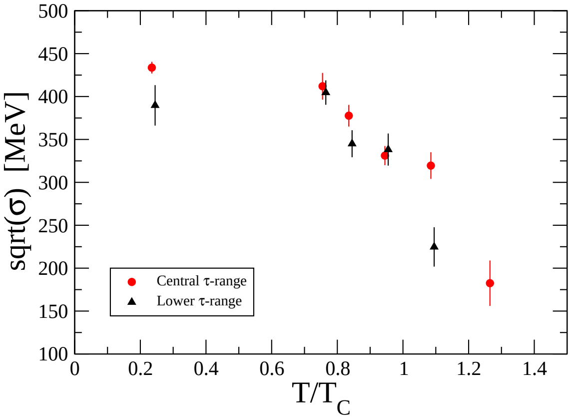


$$V(r) = -\frac{\alpha_c}{r} + \sigma r + C,$$

Karsch, hep-ph/0512217, "Continuum Cornell": $\alpha = \pi/12$, $\sqrt{\sigma} = 445$ GeV

String Tension

$$V(r) = -\frac{\alpha_c}{r} + \sigma r + C,$$



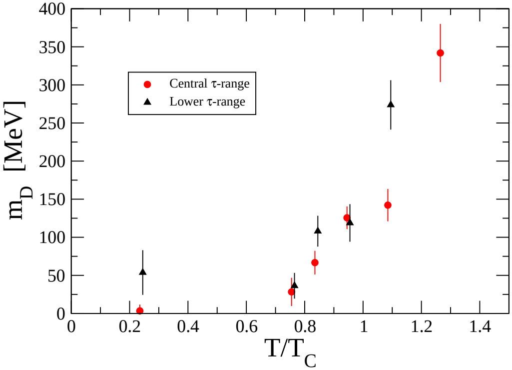
Debye Screening

Karsch, Mehr, Satz, Z.Phys. C37 (1988) 617

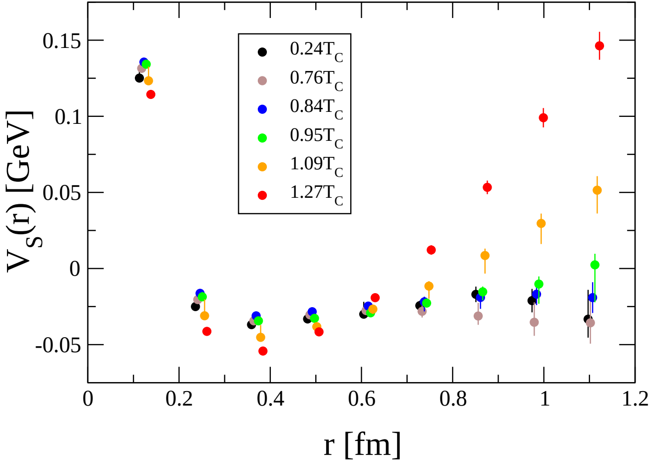
$$V(r,T) = -\frac{\alpha_s}{r} e^{-m_D(T) r} + \frac{\sigma}{m_D(T)} \left(1 - e^{-m_D(T) r}\right) + C$$

$m_D(T) =$ the Debye screening mass.

$\sigma = 434$ MeV (i.e. fixed to “zero” temperature value)

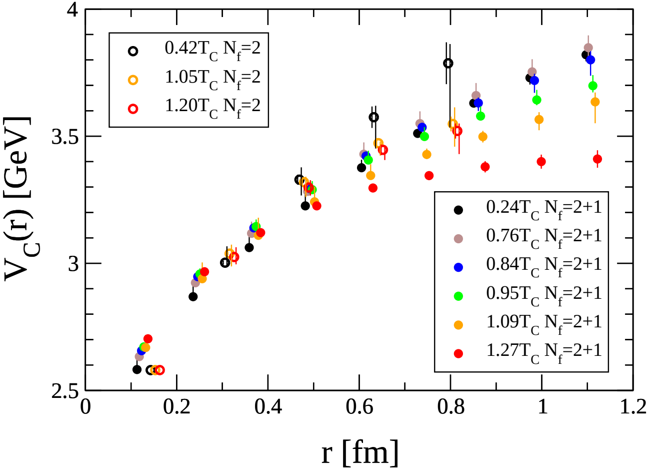


Spin-Dependent Potentials



$$V_\Gamma(\mathbf{r}) = V_\mathrm{C}(\mathbf{r}) + V_\mathrm{S}(\mathbf{r})\, s_1 \cdot s_2 \quad \longrightarrow \quad V_\mathrm{C}(\mathbf{r}) = \frac{1}{4} V_\mathrm{PS} + \frac{3}{4} V_\mathrm{V} \qquad V_\mathrm{S}(\mathbf{r}) = V_\mathrm{V} - V_\mathrm{PS}$$

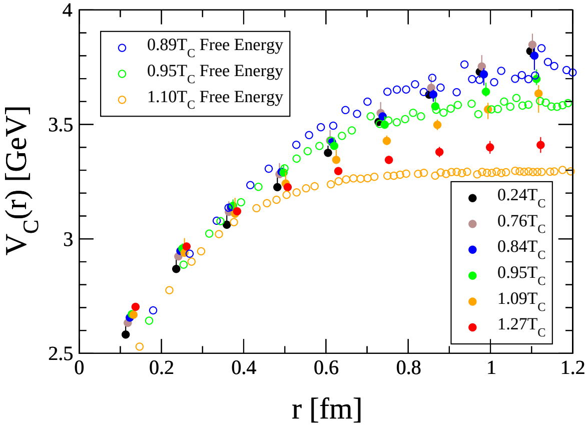
Comparison with 1st generation



Comparison with Static Quark Potential

$$\frac{F_1(r,T)}{T} = -\log\left[Tr\left(L_{\mathrm{ren}}(0)L_{\mathrm{ren}}(r)\right)\right] \qquad L_{\mathrm{ren}} = \text{ renormalised Polyakov loop}$$

Kaczmarek, arXiv:0710.0498



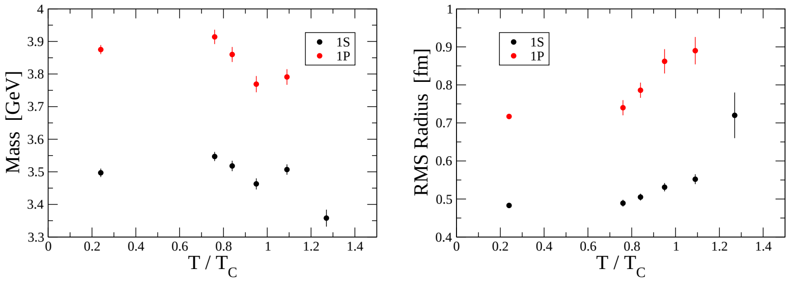
Charmonia Properties from the Potential: Radii

Using the parameterised screened potential from Lattice

$$V(r,T) = -\frac{\alpha_s}{r} e^{-m_D(T)r} + \frac{\sigma}{m_D(T)}\left(1 - e^{-m_D(T)r}\right) + C$$

and solve Schrödinger Equation

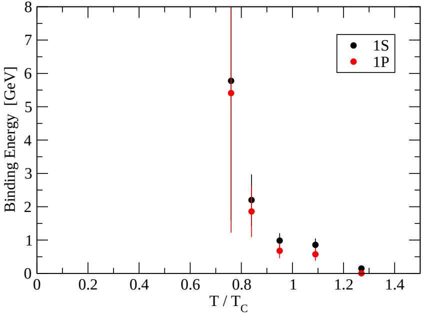
Preliminary



Charmonia Properties from Potential: Binding Energy

$$\text{Binding Energy} = M(T) - V(r \to \infty, T) = M(T) - \frac{m_D}{\sigma} - C$$

Preliminary

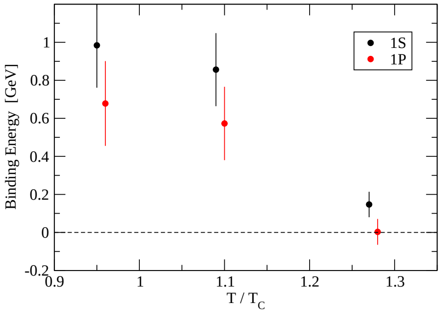


- 1P melts $\lesssim 1.2T_C$
- 1S remains bound up to at least $1.2T_C$

Charmonia Properties from Potential: Binding Energy

$$\text{Binding Energy} = M(T) - V(r \to \infty, T) = M(T) - \frac{m_D}{\sigma} - C$$

Preliminary



- 1P melts $\lesssim 1.2T_C$
- 1S remains bound up to at least $1.2T_C$

Conductivity & Light Quark Diffusivity

arXiv:1412.6411, arXiv:1307.6763

Electrical conductivity on the lattice

EM current: $j_\mu^{\text{em}} = \dfrac{2e}{3}\, j_\mu^{\text{u}} - \dfrac{e}{3}\, j_\mu^{\text{d}} - \dfrac{e}{3}\, j_\mu^{\text{s}},$

EM Correlator: $G_{\mu\nu}^{\text{em}}(\tau) = \displaystyle\int d^3x\, \langle j_\mu^{\text{em}}(\tau, \mathbf{x}) j_\nu^{\text{em}}(0, \mathbf{0})^\dagger \rangle$

Spectral decomposition:

$$G_{\mu\nu}^{\text{em}}(\tau) = \int_0^\infty \frac{d\omega}{2\pi}\, K(\tau,\omega)\, \rho_{\mu\nu}^{\text{em}}(\omega) \quad \text{with} \quad K(\tau,\omega) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh[\omega/2T]}$$

Conductivity: $\dfrac{\sigma}{T} = \dfrac{1}{6T} \lim\limits_{\omega\to 0} \dfrac{\rho^{\text{em}}(\omega)}{\omega}$

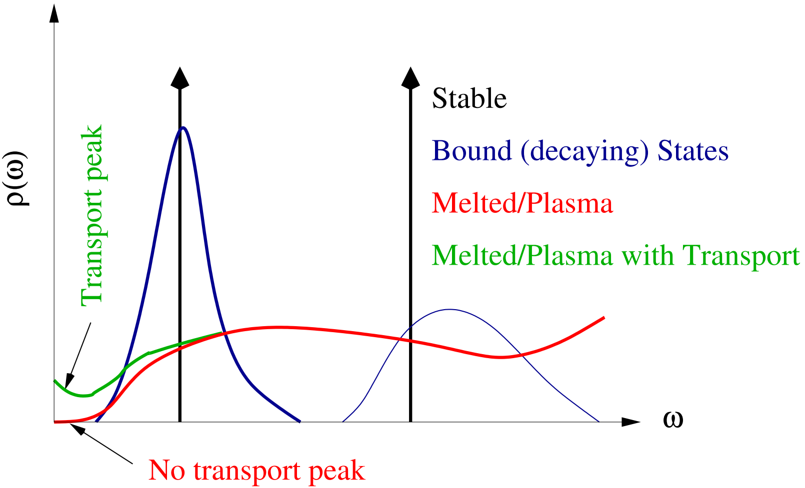
Relationship to Diffusivity: $D\chi_Q = \sigma$

Conserved (lattice) vector current used for j_μ^{em}

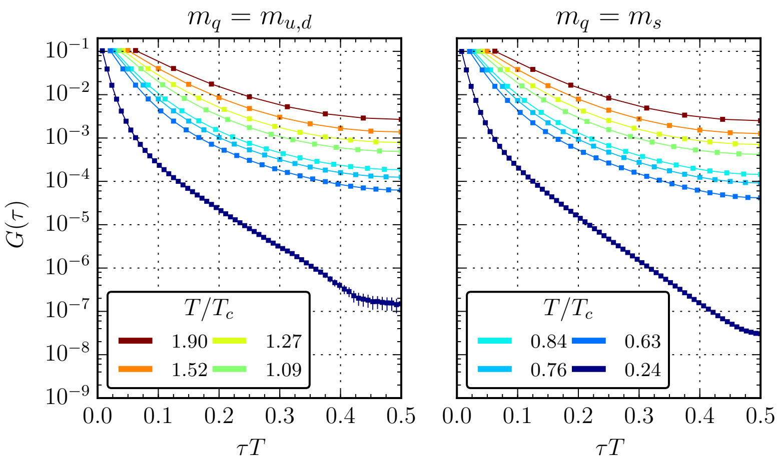
$$V_\mu^{\text{C}}(x) = \left[\bar\psi(x+\hat\mu)(1+\gamma_\mu)\, U_\mu^\dagger(x)\, \psi(x) - \bar\psi(x)(1-\gamma_\mu)\, U_\mu(x)\, \psi(x+\hat\mu)\right]$$

Example Spectral Functions

$$G_2(t) \sim \int \rho(\omega)\, e^{-\omega t}\, d\omega$$

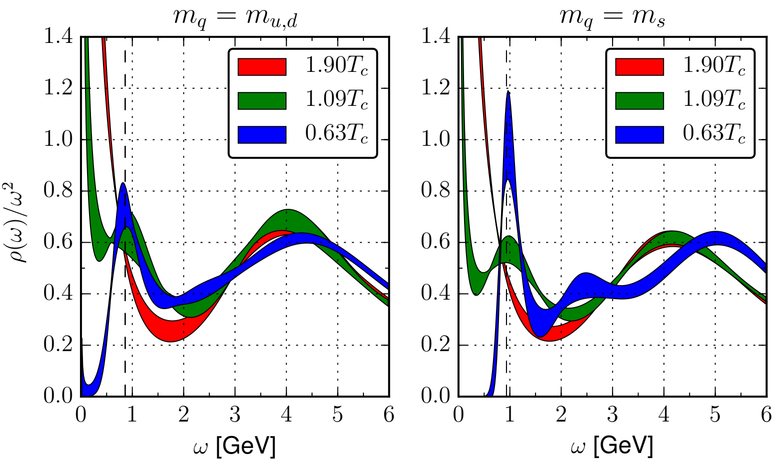


Conserved Vector Correlators

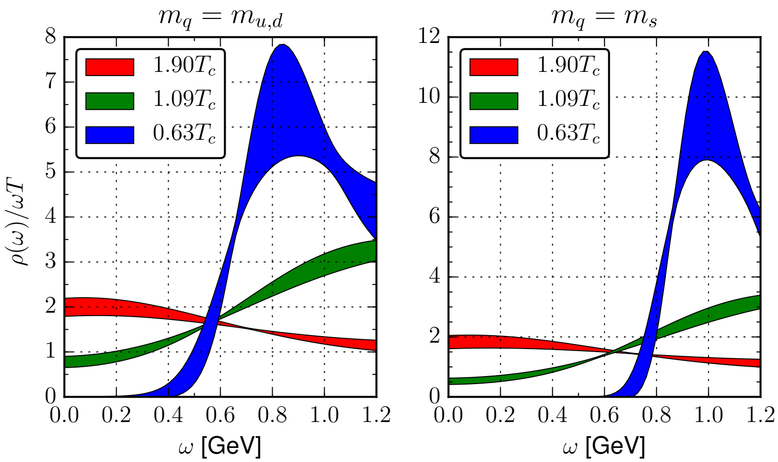


Vector Spectral Function

Using default model: $m(\omega) = m_0(b + \omega)\omega$

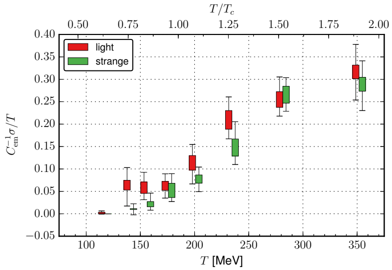


Vector Spectral Functions



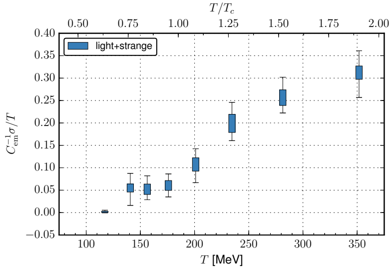
Recall $\sigma \sim \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}$

Conductivity Result



Useful to factor out charge:

$C_{\mathrm{em}} = e^2 \sum_f q_f^2$



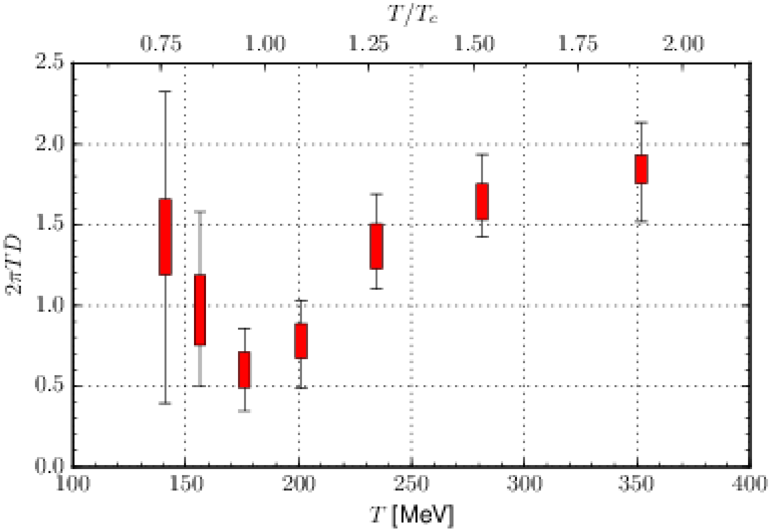
Rectangles = default model systematic (i.e. b). Recall :

$m(\omega) = m_0(b + \omega)\omega$

Whiskers = statistical error

Diffusion Coefficient

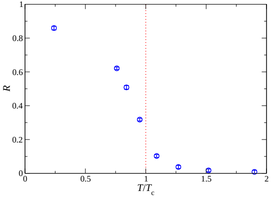
Relationship to Diffusivity: $D = \sigma/\chi_Q$

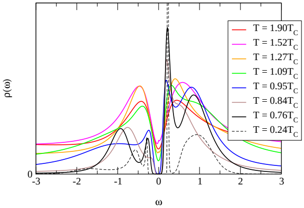


Summary 1

Baryonic Parity Restoration

- Signicant thermal effects in $-$ve parity nucleon
- No observed thermal modification of $+$ve parity mass below T_C
- Degeneracy in ground state of baryonic parity partners above T_c
- Finite temperature baryonic spectral functions determined

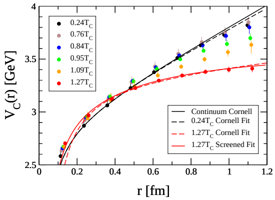




Summary 2

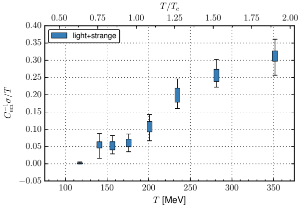
Charmonium Potential

- Relativistic quarks rather than static quarks
- Finite temperature rather than $T = 0$
- Clear temperature dependent effect
- Matches Debye-screened formula with $m_D \approx 0$ for $T < T_C$



Conductivity & Light Quark Diffusivity

- 2+1 flavour conductivity calculated as function of temperature
- Finite temperature diffusion coefficient determined



Physics/lattice parameters

2nd Generation

2+1 flavours

larger volume: (3fm)^3 – (4fm)^3
finer lattices: $a_s = 0.123$ fm
quark mass: $M_\pi/M_\rho \sim 0.45$
temporal cut-off: $a_\tau \sim 5.6$ GeV

N_s	N_τ	T(MeV)	T/T_c
24, 32	16	352	1.90
24	20	281	1.52
24, 32	24	235	1.27
24, 32	28	201	1.09
24, 32	32	176	0.95
24	36	156	0.84
24	40	141	0.76
32	48	117	0.63
16	128	44	0.24

3rd Generation

2+1 flavours

larger volume: (3fm)^3 – (4fm)^3
finer lattices: $a_s = 0.123$ fm
quark mass: $M_\pi/M_\rho \sim 0.45$
temporal cut-off: $a_\tau \sim 11.2$ GeV

N_s	N_τ	T(MeV)	T/T_c
24, 32	32	352	1.90
24	40	281	1.52
24, 32	48	235	1.27
24, 32	56	201	1.09
24, 32	64	176	0.95
24	72	156	0.84
24	80	141	0.76
32	96	117	0.63
16	256	44	0.24

Particle Data Book

$\sim 1,500$ pages

zero pages on Quark-Gluon Plasma...

SLIDES TO HELP ME ANSWER DUMB QUESTIONS

SLIDES TO HELP ME ANSWER TRICKY QUESTIONS

Physics/lattice parameters

1st Generation

2 flavours

smaller volume: $(2\text{fm})^3$
coarser lattices: $a_s = 0.167$ fm
quark mass: $M_\pi/M_\rho \sim 0.55$
temporal cut-off: $a_\tau \sim 7.4$ GeV

N_s	N_τ	T(MeV)	T/T_c
12	16	460	2.09
12	18	409	1.86
12	20	368	1.68
12	24	306	1.40
12	28	263	1.20
12	32	230	1.05
12	80	90	0.42

2nd Generation

2+1 flavours

larger volume: $(3\text{fm})^3 - (4\text{fm})^3$
finer lattices: $a_s = 0.123$ fm
quark mass: $M_\pi/M_\rho \sim 0.45$
temporal cut-off: $a_\tau \sim 5.6$ GeV

N_s	N_τ	T(MeV)	T/T_c
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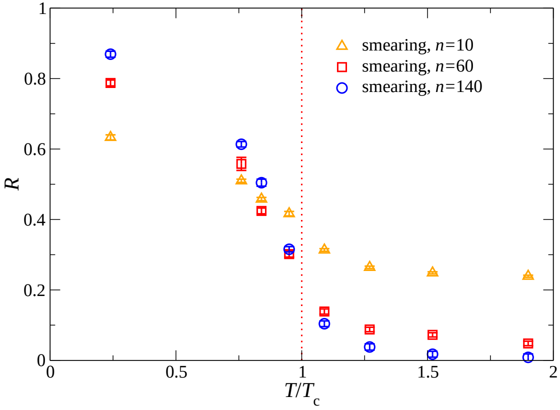
Effects of Smearing

Above results used Gaussian smearing with sources/sinks, η smeared with:

$\eta' = C\,(1 + \kappa H)^n\,\eta$ using $\kappa = 8.7$ and $n = 140$ Capitani et al [arXiv:1205.0180]

Systematics checks of smearing

- vary n
- vary τ-range



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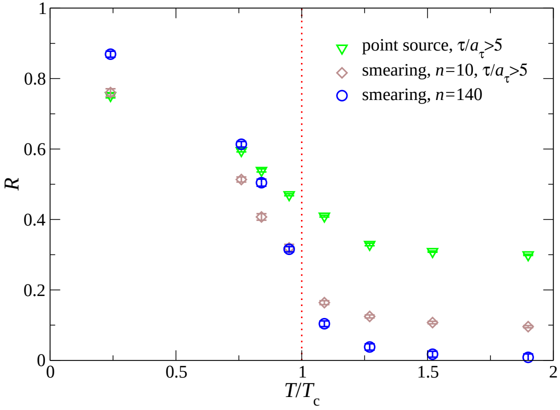
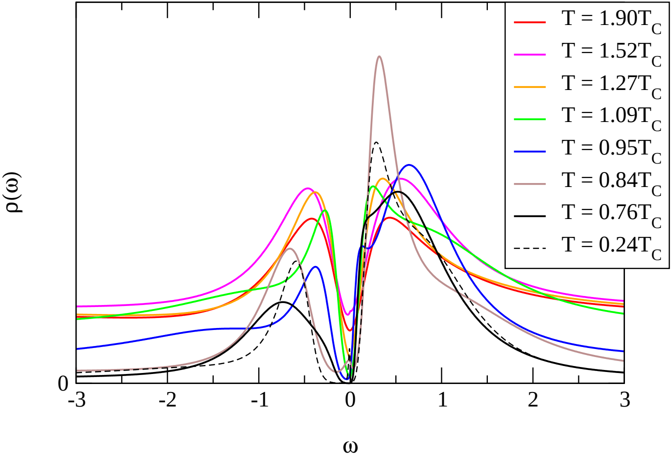


Baryonic Spectral Functions - Systematic Checks

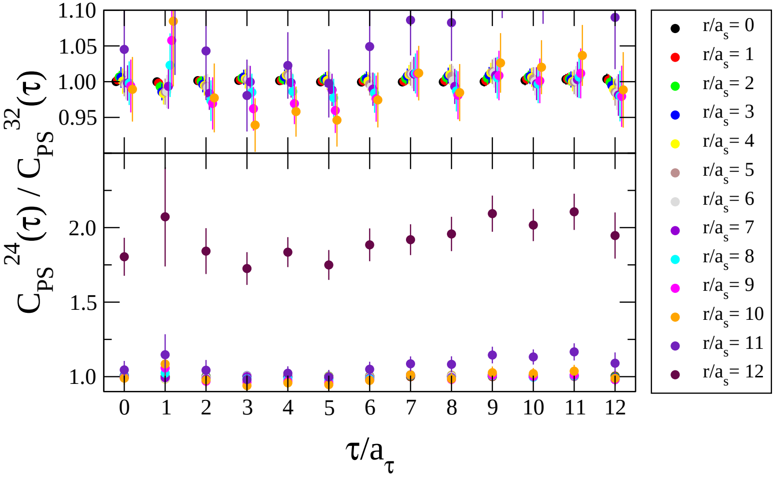
Checking systematics by using MEM on fixed τ windows:

$$\tau = 1, 2, \ldots 7, N_\tau - 7, N_\tau - 6, \ldots, N_\tau - 1$$

Preliminary

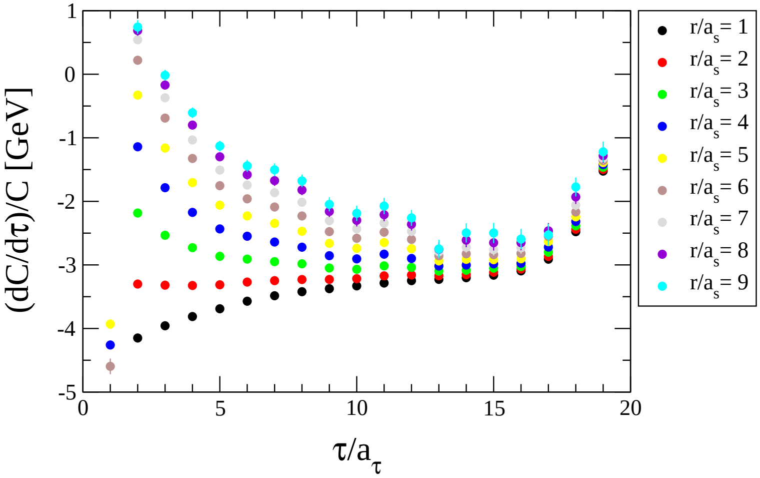


Volume Effects



$N_s = 24$ cf $N_s = 32$ for $1.27\,T_c$ $(N_\tau = 24)$

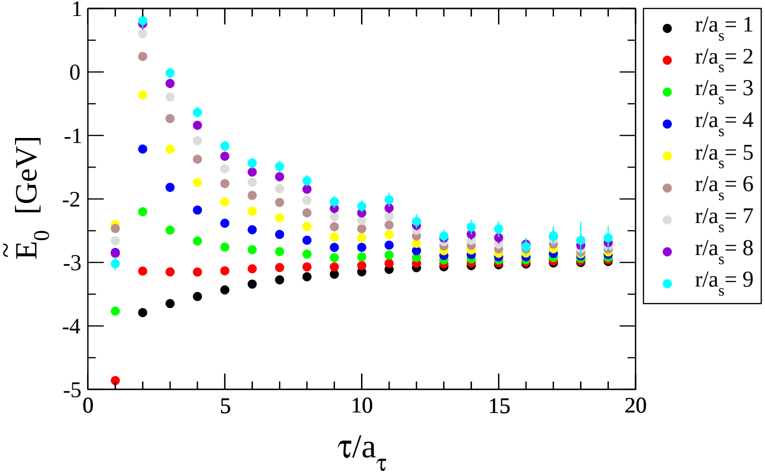
Naive Temporal Term in Potential



PS $0.76T_C$ using the naive form

$$\frac{\partial}{\partial \tau} f(\tau) \longrightarrow \left[\frac{f(\tau + a_\tau) - f(\tau - a_\tau)}{2a_\tau} \right]$$

Improved Temporal Term in Potential



PS 0.76T_C using the improved form Durr 1203.2560

$$\tilde{E}_0(\tau) = \frac{1}{2}\log\left(\frac{C_\Gamma(\tau-1)+\sqrt{C_\Gamma(\tau-1)^2 - C_\Gamma(N_\tau/2)^2}}{C_\Gamma(\tau+1)+\sqrt{C_\Gamma(\tau+1)^2 - C_\Gamma(N_\tau/2)^2}}\right)$$

Renormalising the Polyakov Loop

Polyakov Loop, L, related to free energy, F, via:

$$L(T) = e^{-F(T)/T}$$

But F defined up to addivitive constant $\Delta F = f(\beta, \kappa)$.
Imposing renormalisation condition:

$$L_R(T_R) \equiv \text{some number}$$

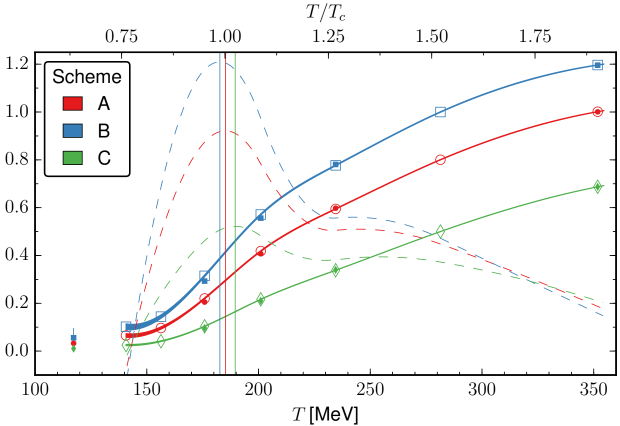
gives us

$$L_R(T) = e^{-F_R(T)/T} = e^{-(F_0(T)+\Delta F)/T} = L_0(T)e^{-\Delta F/T} = L_0(T){Z_L}^{N_\tau}$$

and Z_L defined from renormalisation condition.

Wuppertal-Budapest, PLB713(2012)342 [1204.4089]

T_C from Polyakov Loop



Scheme A: $L_R(Nt = 16) = 1.0$
Scheme B: $L_R(Nt = 20) = 1.0$
Scheme C: $L_R(Nt = 20) = 0.5$

Cubic spline, solid = 32^3, open = 24^3

$\longrightarrow$ $N_\tau^{\text{crit}} = 30.4(7)$ or $T_c = 171(4)$ MeV

Susceptibilities' Definitions

$$n_i = \frac{T}{V}\frac{\partial \ln Z}{\partial \mu_i} \qquad \chi_{ij} = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial \mu_i \partial \mu_j}$$

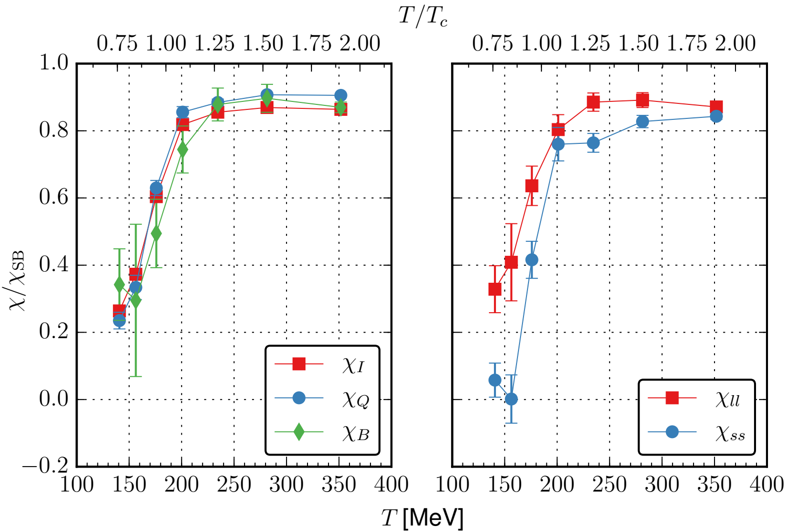
$$Q = \frac{T}{V}\frac{\partial \ln Z}{\partial \mu_Q} = \sum_{i=1}^{3} q_i n_i \qquad \chi_Q = \frac{\partial Q}{\partial \mu_Q} = \sum_{i=1}^{3} (q_i)^2 \chi_{ii} + \sum_{i \neq j}^{3} q_i q_j \chi_{ij}$$

$$B = \frac{T}{V}\frac{\partial \ln Z}{\partial \mu_B} = \sum_{i=1}^{3} n_i \qquad \chi_B = \frac{\partial B}{\partial \mu_B} = \sum_{i=1}^{3} \chi_{ii} + \sum_{i \neq j}^{3} \chi_{ij}$$

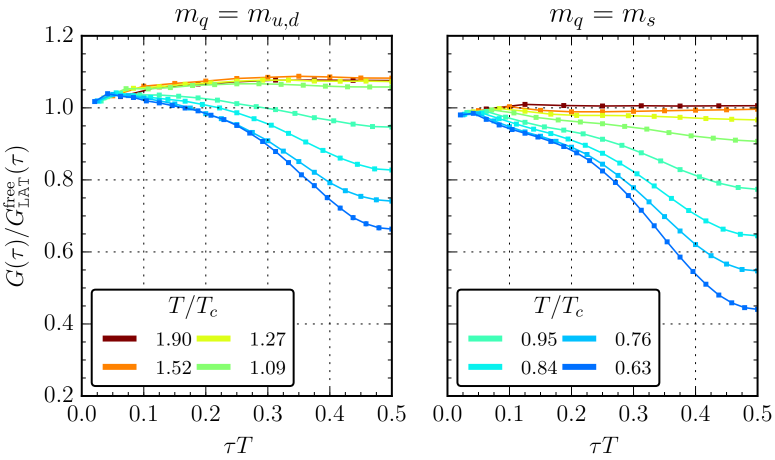
$$\mu_I = \mu_d - \mu_u \qquad \chi_I = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial \mu_I^2}$$

Susceptibilities

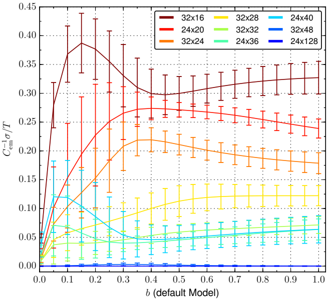
χ_{SB} is Stefan-Boltzman (free) result



Conserved Vector Correlators vs Free

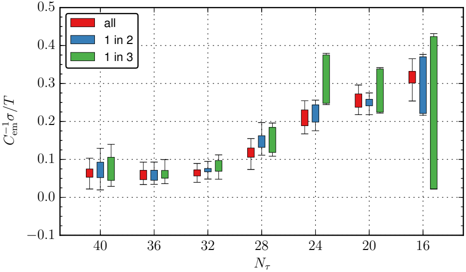


MEM Systematics I



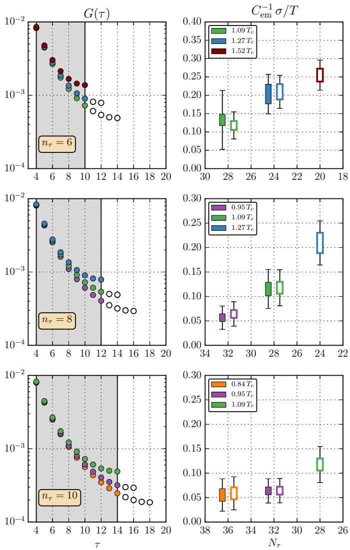
Variation with default model parameter b

Recall $m(\omega) = m_0(b + \omega)\omega$



Anisotropy check including:
all or 1 in 2 or 1 in 3
of the τ datapoints

MEM Systematics II



Stability tests:
discarding the last time slices:

Are we seeing a number-of-datapoints (N_τ) systematic or a true thermal effect?

MEM systematics

- default model
- time range
- energy discretisation: $\omega = \{\omega_{\text{min}}, \omega_{\text{min}} + \Delta\omega \ \ldots \omega_{\text{max}}\}$
- number of configs
- numerical precision

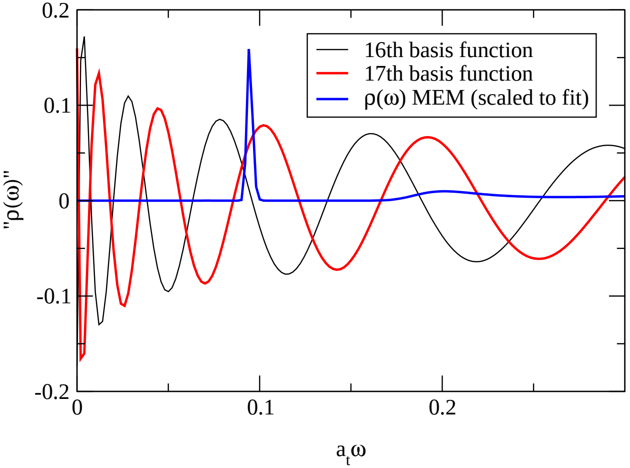
(All true also for BR)

Recall $\mathcal{I}(\rho) \leq N_t$ for MEM

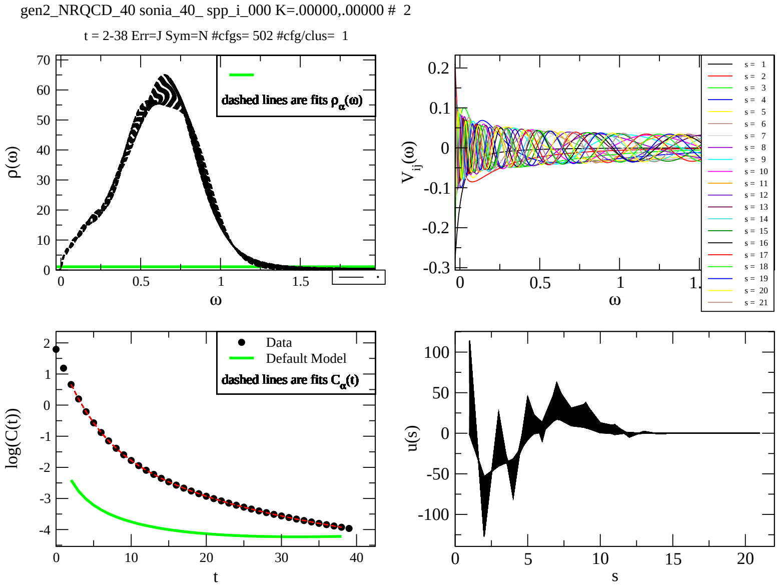
Can vary this in free case by varying N_t

Feature Resolution

MEM can reproduce features smaller than the characteristic size of its basis functions:



MEM: more than you ever wanted to know



The Task

Given data D

Find fit F by maximising $P(F|D)$

Bayes Theorem

Need to maximise $P(F|D)$

Bayes Theorem:

$$P(F|D)P(D) = P(D|F)P(F)$$

i.e. $$P(F|D) = \frac{P(D|F)P(F)}{P(D)}$$

But $P(D|F) \sim e^{-\chi^2}$ $\longrightarrow$ minimising χ^2 $\neq$ maximising $P(F|D)$

$\longrightarrow$ *Maximum Likelihood Method* wrong??

No! Since for simple $F(t) = Ze^{-Mt}$, $P(F) = P(Z, M) \sim$ const

Bayes Theorem

Need to maximise $P(F|D)$

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Priors

Actually $P(F = \text{elephant}) \equiv 0$

$\longrightarrow$ "priors" which encode any additional information

(a.k.a. predisposition, prejudices, impartialities, biases, prediliction, subjectivity, . . .)

E.g. in L.G.T. $P(M < 0) \equiv 0$

Maximum Likelihood Method applies this prior implicitly

Can encode prior information with "entropy"$= S$ (dis-information)

Define $\mathcal{I}(F) =$ "Information content" of F

"Bland" F has $\mathcal{I}(F) \sim 0$ and $S \gg 0$

"Spiky" F has $\mathcal{I}(F) \gg 0$ and $S \equiv 0$

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Entropy

	No Data	Data
No Prior	$\mathcal{I}(F) \equiv 0$	F from min χ^2
Prior	$F \equiv$ prior	F from max $P(F\|D)$

$$P(F) = e^{-S}$$