Hadron Structure using the Feynman-Hellmann Theorem

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QCDSF-UKQCD/CSSM Collaborations

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Overview

Want to demonstrate how a Feynman-Hellmann approach can be used to tackle two important problems in lattice QCD

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Disconnected contributions to matrix elements

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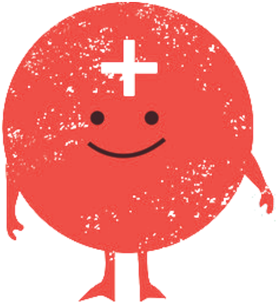
Disconnected contributions to matrix elements

High-momentum form factors

Proton Spin Structure

The proton is a spin-$\frac{1}{2}$ baryon



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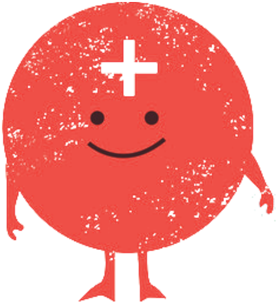
Where does this spin come from?



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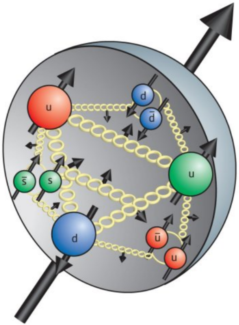

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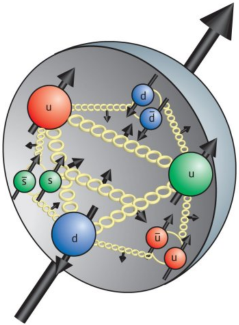

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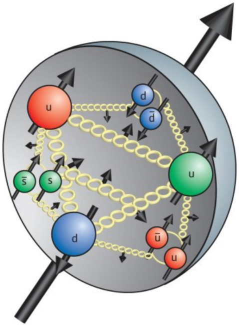

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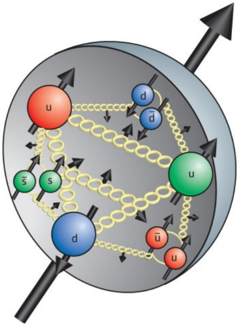

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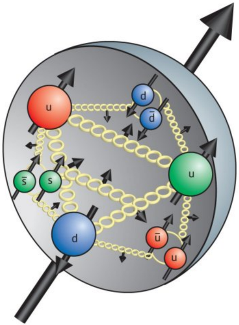

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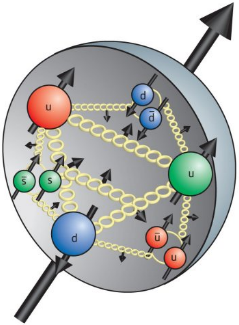

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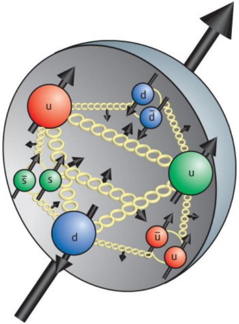

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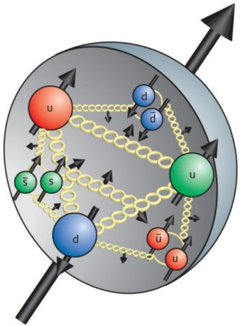

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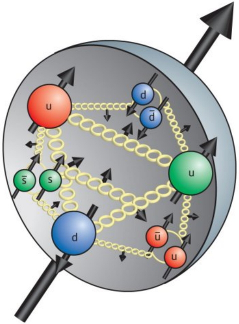

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Can we calculate these spin fractions using lattice QCD?

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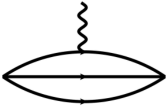
Running theme: Complete calculations in both cases require access to disconnected contributions

Disconnected Contributions

Two groups of contributions to matrix elements

Two groups of contributions to matrix elements

Connected



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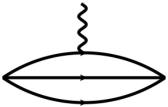

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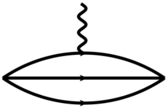

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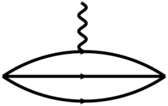

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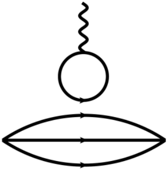

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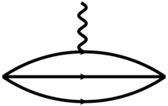

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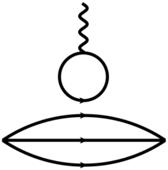

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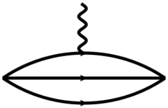

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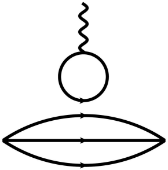

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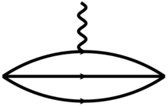

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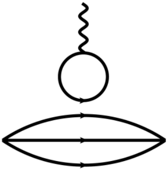

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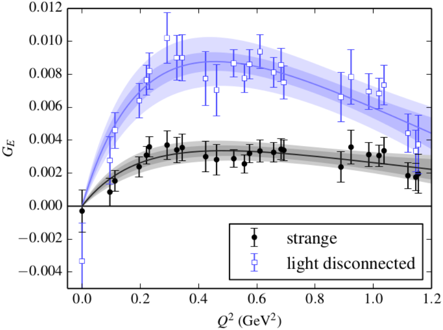
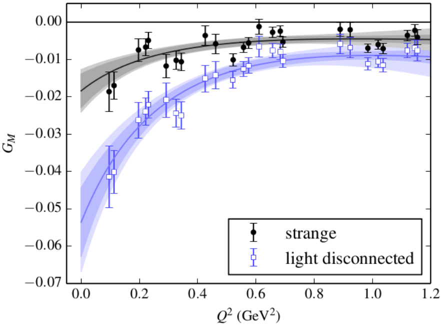
J. Green, S. Meinel et al.





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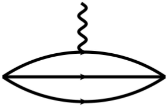

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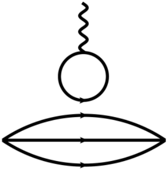

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Feynman-Hellmann method can access both contributions

Feynman-Hellmann Recipe

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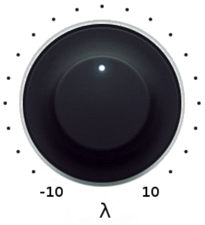

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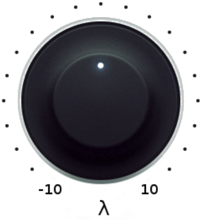

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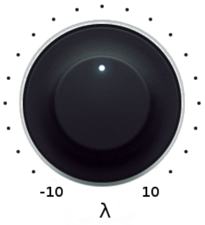

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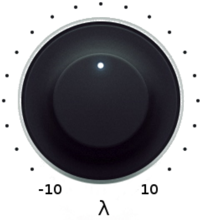

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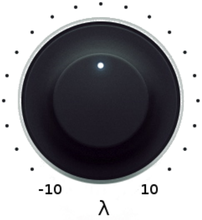

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Calculation of matrix element $\to$ hadron spectroscopy

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Modification to can be made during
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Feynman-Hellmann Recipe

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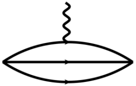

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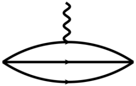

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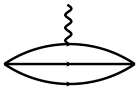

- Operator insertion encoded in quark propagator
- Easy to implement

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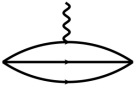
Access connected contributions



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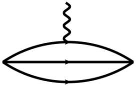

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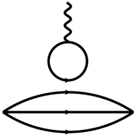

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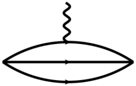

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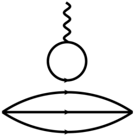

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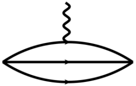

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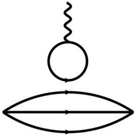

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Demonstrate technique by calculating
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Energy shifts are proportional to the quark axial charges

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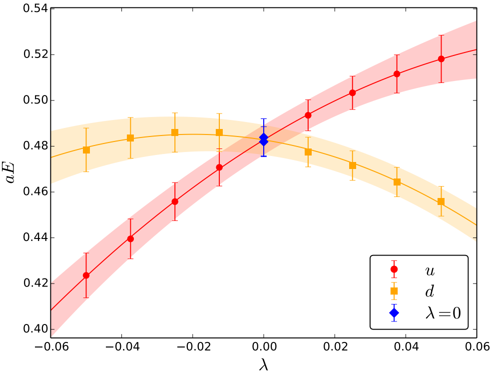
$m_\pi \approx 470$ MeV   350 configs $32^3 \times 64$



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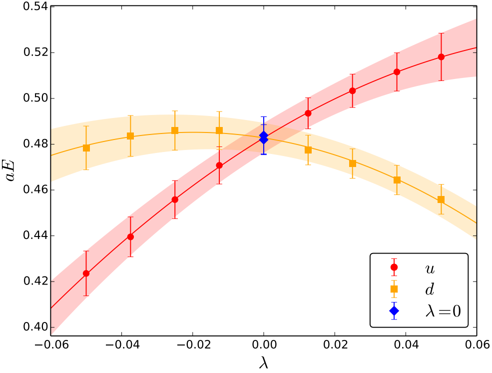

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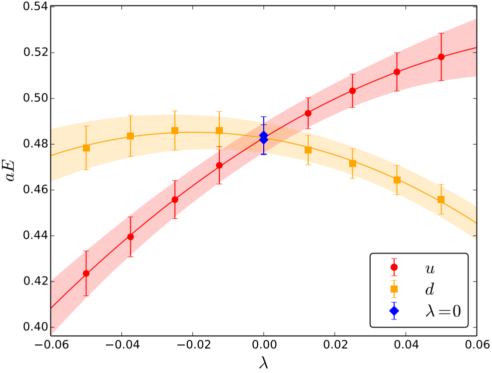

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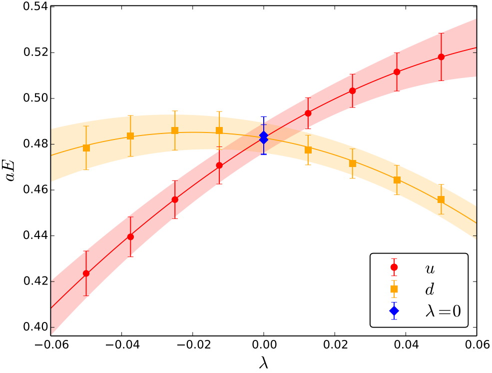

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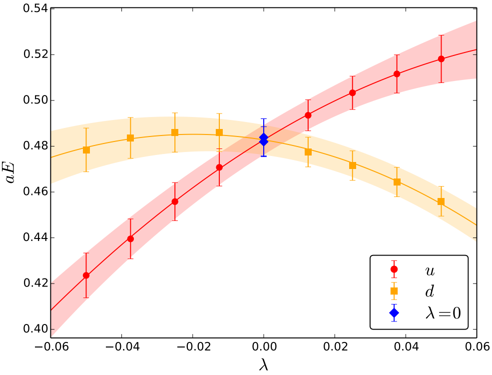

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Feynman-Hellmann approach in perfect agreement with existing methods

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Improve signal by allowing statistical fluctuations to cancel

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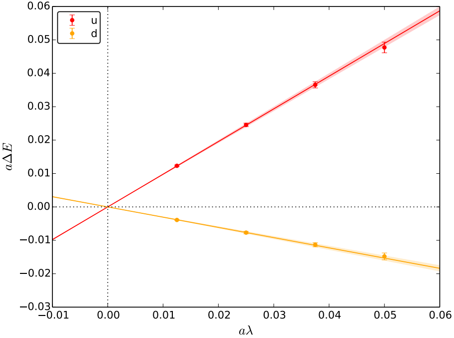

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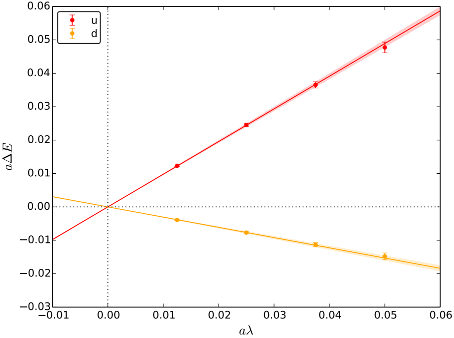

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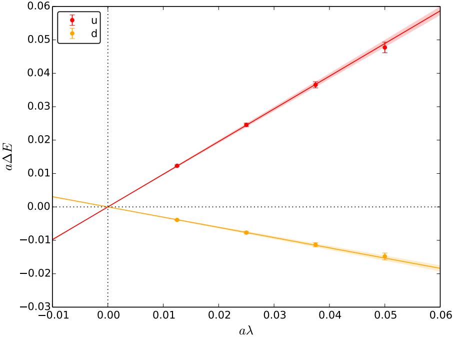

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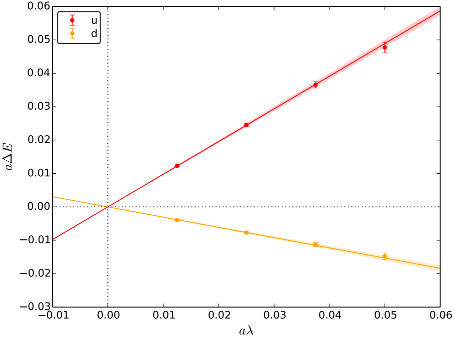

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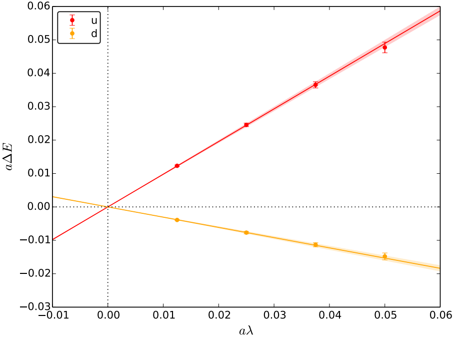

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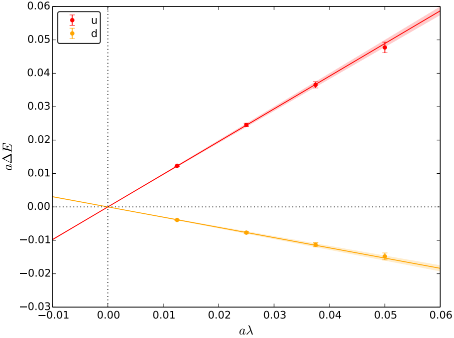

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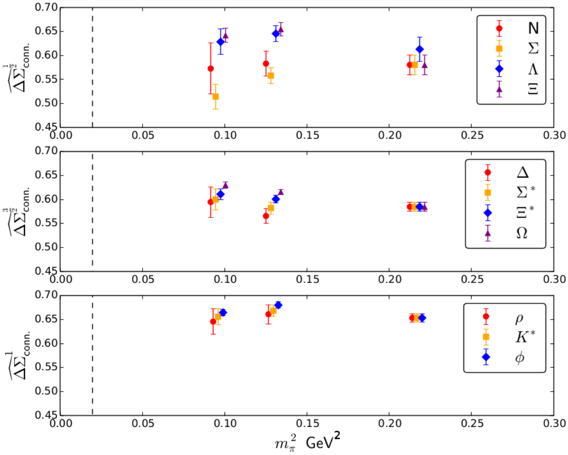

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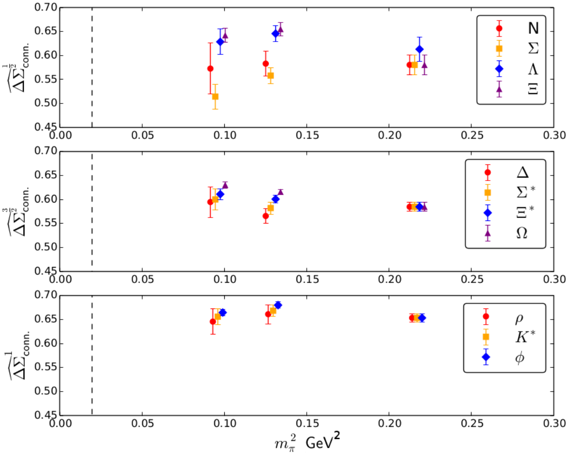

A. J. Chambers et al., Phys. Rev. D 90, 014510 (2014), 1405.3019

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We need a strategy to extract signal from imaginary part of correlation functions

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Energy shifts manifest as a phase in the correlation functions

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Demonstrate procedure by recalculating <u>connected contributions</u> to nucleon axial charges

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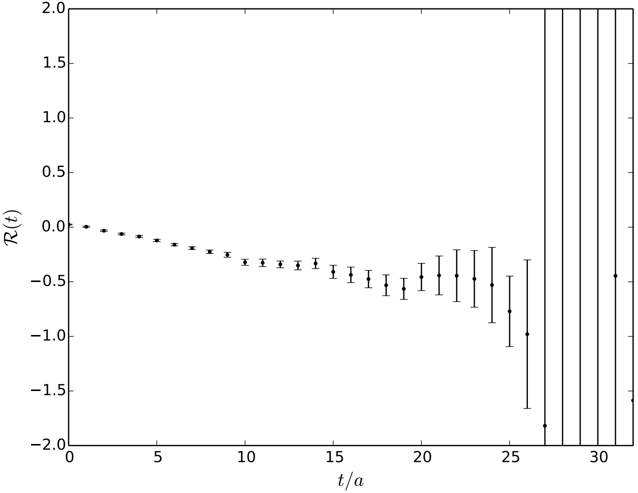

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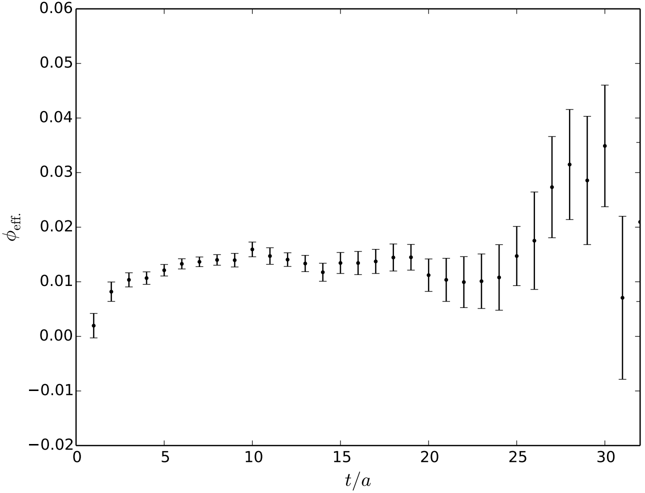

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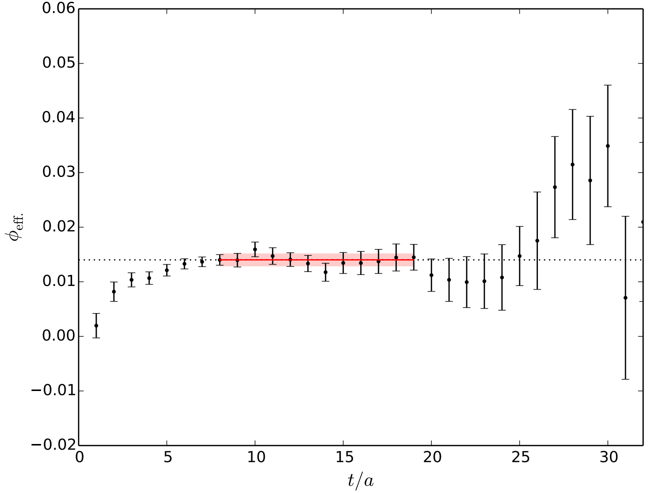

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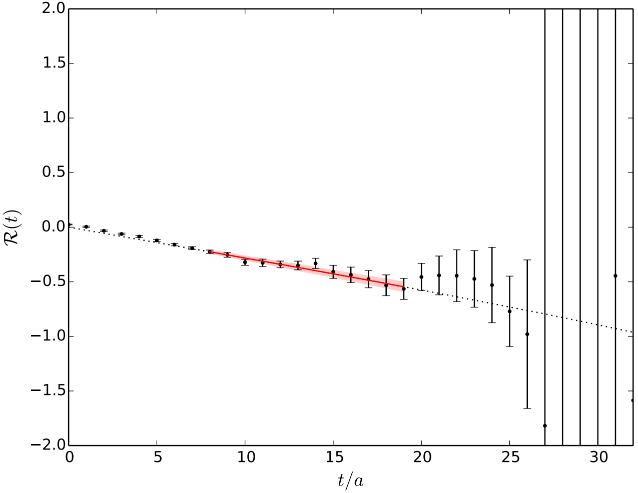

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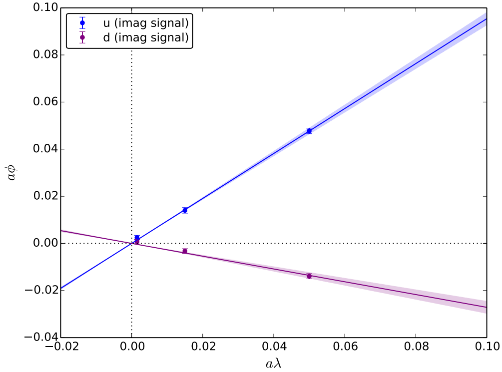

	Imag.
$\Delta u_{\text{conn.}}$	0.811(24)
$\Delta d_{\text{conn.}}$	-0.232(24)
$\Delta\Sigma_{\text{conn.}}$	0.579(34)
$g_{A\text{conn.}}$	1.043(34)

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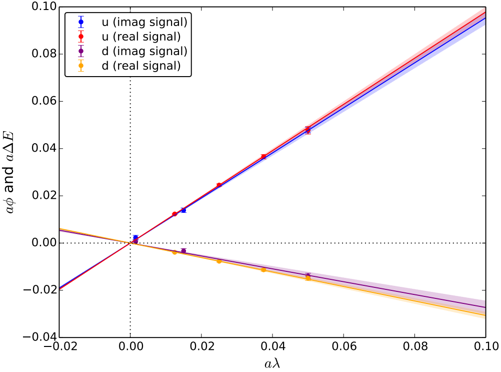

	Imag.	Real
$\Delta u_{\mathrm{conn.}}$	0.811(24)	0.832(18)
$\Delta d_{\mathrm{conn.}}$	-0.232(24)	-0.260(13)
$\Delta \Sigma_{\mathrm{conn.}}$	0.579(34)	0.572(22)
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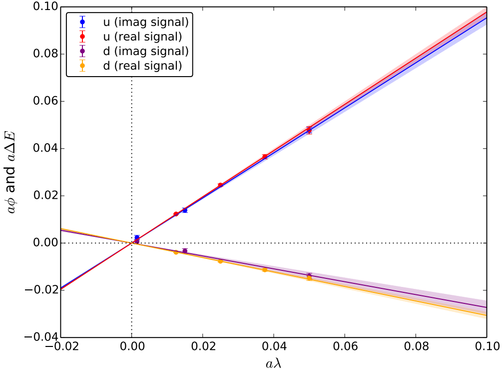

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Reliable extraction from imaginary signal

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Let's do the disconnected for real now

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Feynman-Hellmann relation gives phase shifts

$$E \to E(\lambda) + i\phi(\lambda) \qquad \left.\frac{\partial \phi}{\partial \lambda}\right|_{\lambda=0} = \Delta\Sigma_{\text{disconn.}}$$

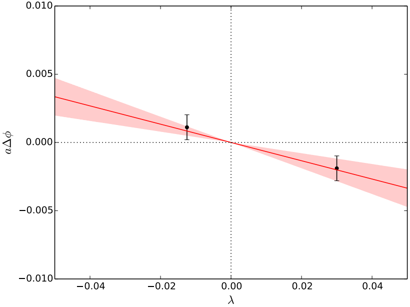
Shift in phase with respect to λ gives total disconnected quark spin contribution

Nucleon Axial Charges (Disconnected)

Flavour symmetric point

$m_\pi \approx 470$ MeV $\quad 32^3 \times 64$

500 configs each



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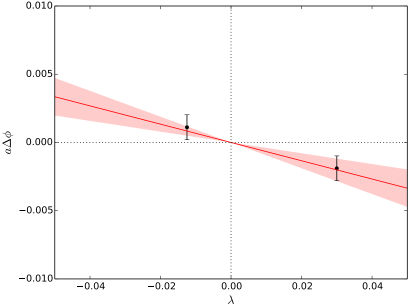

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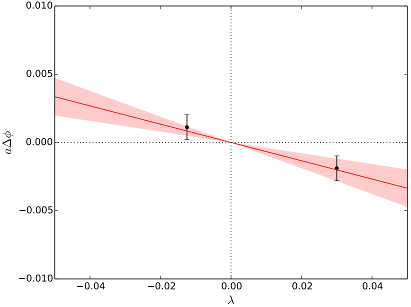

$$\Delta\Sigma_{\text{disconn.}} = -0.057(23)$$

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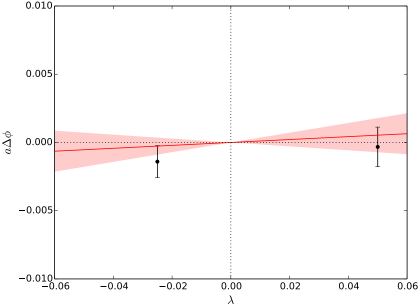
$m_\pi \approx 310$ MeV $\quad 32^3 \times 64$

750 configs each



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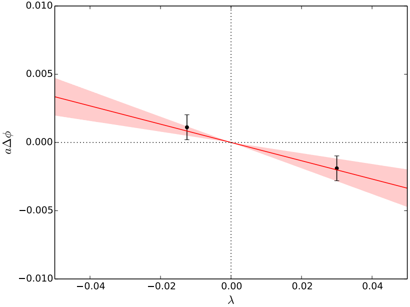

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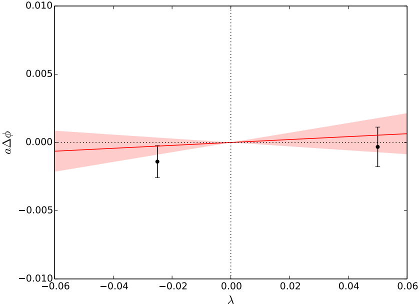

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Statistics too low, λ too small?

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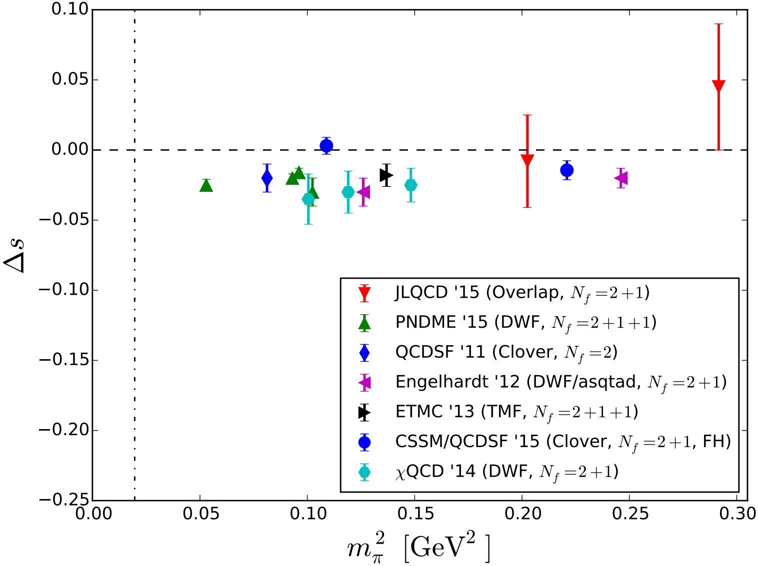

Overview

Want to demonstrate how a Feynman-Hellmann approach can be used to tackle two important problems in lattice QCD

Disconnected contributions to matrix elements

High-momentum form factors

Pion and Nucleon Form Factors

Hadrons are not point-like particles

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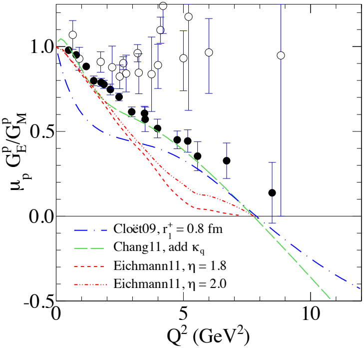

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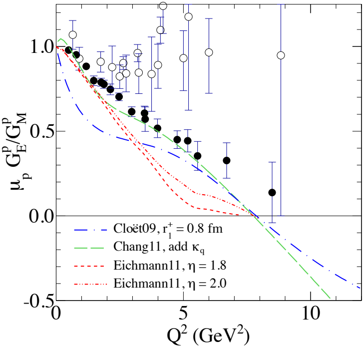

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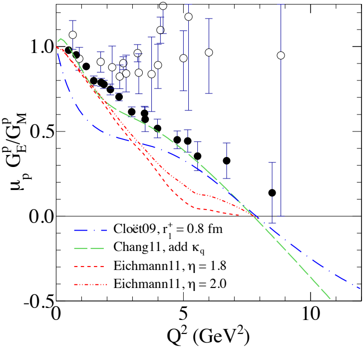

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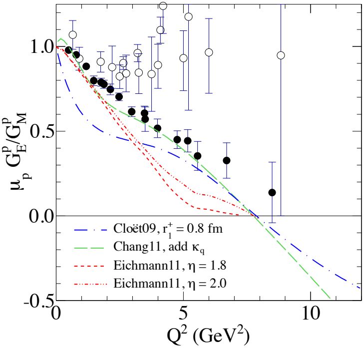

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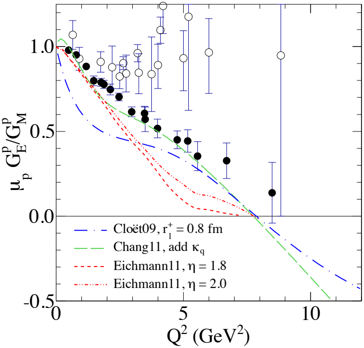

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Want to extend Feynman-Hellmann approach to non-forward matrix elements

Feynman-Hellmann Recipe (Non-Forward Case)

Want to calculate some non-forward matrix element

$$\langle\, H(\vec{p}\,')\,|\,\mathcal{O}(0)\,|\,H(\vec{p})\,\rangle$$

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$$\langle\, H(\vec{p}\,') \,|\, \mathcal{O}(0) \,|\, H(\vec{p}) \,\rangle$$

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$$\mathcal{L}(y) \to \mathcal{L}(y) + \lambda e^{i\vec{q}\cdot(\vec{y}-\vec{x})}\mathcal{O}(y)$$

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$$\vec{p} + \vec{q} = \vec{p}\,' \qquad\qquad E(\vec{p}) = E(\vec{p}\,')$$

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$$\vec{p} + \vec{q} = \vec{p}' \qquad\qquad E(\vec{p}) = E(\vec{p}')$$

3. **Calculate matrix element from energy shifts with respect to λ**

$$\left.\frac{\partial E}{\partial \lambda}\right|_{\lambda=0} = \frac{1}{2E}\langle H(\vec{p}') \,|\, \mathcal{O}(0) \,|\, H(\vec{p}) \rangle$$

Breit Frame Kinematics

Quick digression about Breit Frame kinematics. . .

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Restricted to Breit frame kinematics for Feynman-Hellmann method

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So we prefer to use kinematics corresponding to

$$\vec{p}\,' = -\vec{p}$$

Pion Form Factor

Demonstrate technique by calculating pion form factor

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Pion form factor defined by non-forward matrix element

$$\langle\, \vec{p}\,' \,|\, \bar{q}(0)\gamma_\mu q(0) \,|\, \vec{p}\,\rangle = (p_\mu + p'_\mu) F_\pi(Q^2)$$

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Make modification to the Lagrangian

$$\mathcal{L}(y) \to \mathcal{L}(y) + \lambda \, e^{i\vec{q}\cdot(\vec{y}-\vec{x})} \, \bar{q}(y) \, \gamma_\mu \, q(y)$$

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Choose this option

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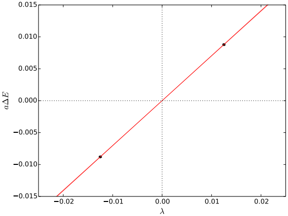
$Q^2 = 0.5\ \text{GeV}^2$



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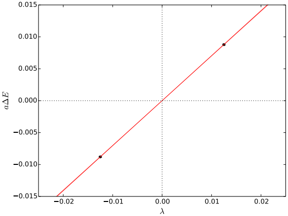
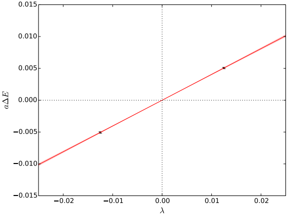


$Q^2 = 1.6\ \mathrm{GeV}^2$



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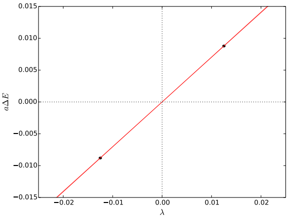

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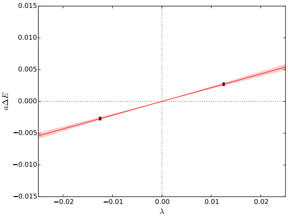

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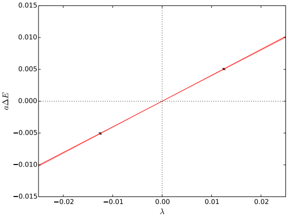

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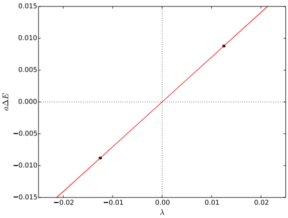

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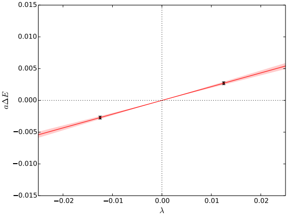

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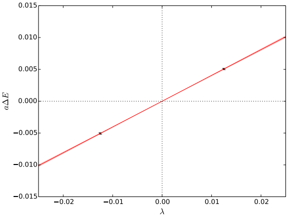
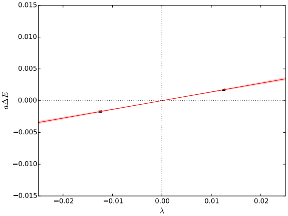


$Q^2 = 6.6$ GeV^2



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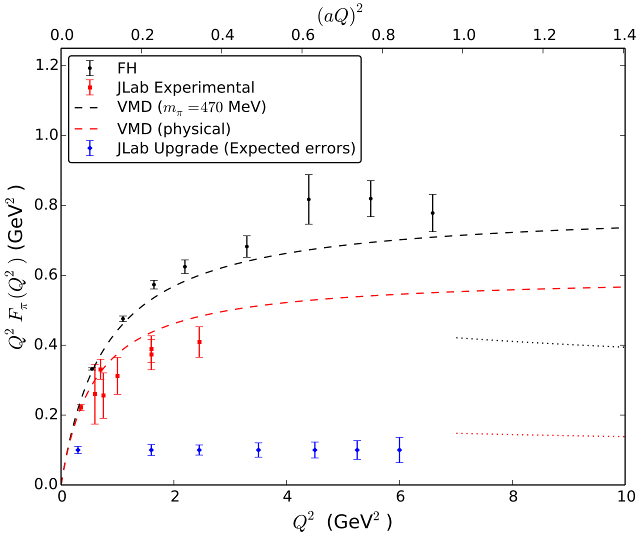

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Carry out similar analysis for nucleon form factors

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Dirac and Pauli form factors defined by

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Related to the Sachs electromagnetic form factors by

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m^2}F_2(Q^2)$$
$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

Carry out similar analysis for nucleon form factors

Dirac and Pauli form factors defined by

$$\langle\, \vec{p}'\ \vec{s}' \,|\, \bar{q}(0)\gamma_\mu q(0) \,|\, \vec{p}\ \vec{s}\,\rangle = \bar{u}(\vec{p}', \sigma') \left[\gamma_u F_1(Q^2) + \sigma_{\mu\nu}\frac{q_\nu}{2m} F_2(Q^2)\right] u(\vec{p}, \sigma)$$

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Make identical modification to the action as for the pion case

$$\mathcal{L}(y) \to \mathcal{L}(y) + \lambda\, e^{i\vec{q}\cdot\vec{y}}\, \bar{q}(y)\, \gamma_\mu\, q(y)$$

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Need to choose projection matrices for different currents

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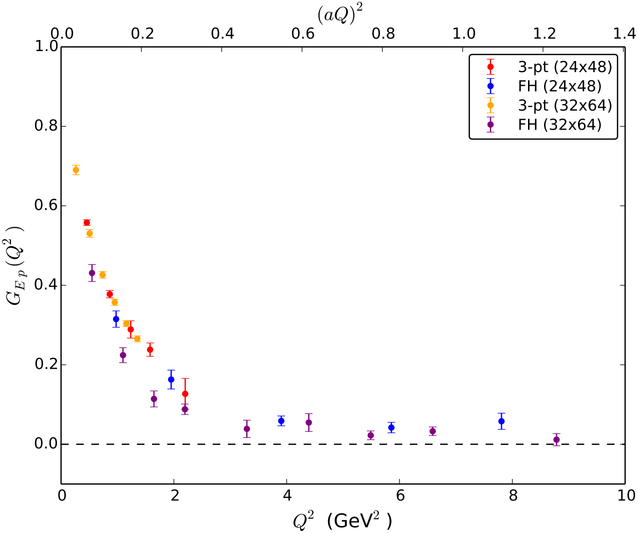
Then we get linear combinations of G_E and G_M

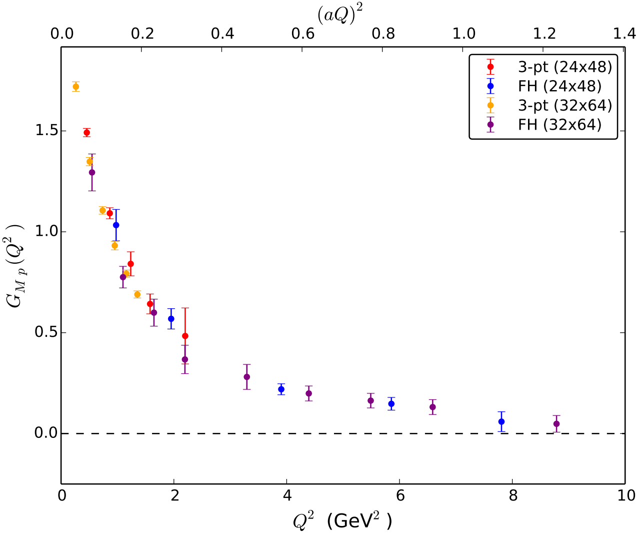
Nucleon Form Factors (Electric)

$m_\pi \approx 470$ MeV 500-1000 configs $32^3 \times 64$



Nucleon Form Factors (Magnetic)

$m_\pi \approx 470$ MeV 500-1000 configs $32^3 \times 64$



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